Spacetime geometry of spin, polarization, and wavefunction collapse

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The Hitchhiker's Advanced Guide
to Quantum Collapse Models
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Aims (achievable or not):

(i) Incorporate quantum nonlocality into general relativity.

(ii) Resolve the measurement problem using spacetime geometry.

With these aims, we propose that *the preparation and measurement of a quantum system are simultaneous events.*

We investigate this proposal by modifying classical general relativity so that there are no distinct points in the worldlines of dust particles.

This new geometry recently arose in the study of nonnoetherian coordinate rings in algebraic geometry.

Definition

Let $(\tilde{M}, g)$ be an orientable Lorentzian manifold. Consider a set of dust particles on $\tilde{M}$ with worldlines $\beta_i \subset \tilde{M}$. We call the set

$$M := (\tilde{M} \setminus (\cup_i \beta_i)) \cup (\cup_j \{\beta_j\}),$$

where each β_j is a single point of M, an *internal spacetime*, or simply *spacetime*. We call $\tilde{M}$ the *external spacetime* of M, and the dust particles *pointons*.

Let β be the worldline of a pointon. Then

- *we cannot define a tangent vector along β in M, since β is a single point of M.*
- *β is a continuum of distinct 0-dimensional points in $\tilde{M}$, and a single '1-dimensional point' in M.*
- *Time does not advance along β.*

Therefore, to construct an 'internal metric' h_{ab} at a point $p \in \tilde{M}$ from the external metric g_{ab}, it must project out each vector v tangent to a (geodesic) pointon worldline $\beta \subset \tilde{M}$ at p.

Recall the orthogonal projection of a timelike unit vector v:

$$[v]^a{}_b = g^a{}_b - v^a v_b = \delta^a{}_b - v^a v_b.$$

We will describe the projection $[v]_{ab}$ in the case v is null shortly...

Definition
Fix $p \in \tilde{M}$.
We call the metric $g_{ab} : \tilde{M}_p \otimes \tilde{M}_p \to \mathbb{R}$ an *external metric* at p.
Let $v_1, \ldots, v_n \in \tilde{M}_p$ be the tangent vectors to the pointon worldlines $\beta_1, \ldots, \beta_n$ at p. We define the corresponding *internal metric* to be the degenerate symmetric rank-2 tensor given by the composition of projections

$$h = h_p = h^a{}_b := [v_1]^a{}_c [v_2]^c{}_d \cdots [v_n]^e{}_b : \tilde{M}_p^* \otimes \tilde{M}_p \to \mathbb{R}.$$

The *(internal) tangent space* at p is the image of h at p,

$$M_p := \operatorname{im} h = \{v^a \in \tilde{M}_p \mid h^a{}_b v^b = v^a\} \subseteq \tilde{M}_p.$$

Review: orientation and Hodge duals

• An orientation of a vector space V is given by fixing an ordered basis $\mathcal{B}$ of V, and declaring any ordered basis to be positive (resp. negative) if it can be obtained from $\mathcal{B}$ by a base change with a positive (resp. negative) determinant.

• Let $e_0, \ldots, e_m$ be an orthonormal basis for V. The exterior algebra $\bigwedge V^* := \oplus_{n=0}^{m} \bigwedge^n V^*$ has basis consisting of 1 and the set of volume forms

$$\mathrm{vol}(\mathrm{span}\{e_{j_1}, \ldots, e_{j_n}\}) := e^{j_1} \wedge \cdots \wedge e^{j_n} \in \bigwedge^n V^*$$

with $1 \leq j_1 < \cdots < j_n \leq m$. In particular,

$$\mathrm{vol}(V) = e^0 \wedge \cdots \wedge e^m \in \bigwedge^m V^*.$$

The Hodge dual $\star\psi$ of an element $\psi \in \bigwedge V^*$ is defined on the basis elements $\psi = e^{j_1} \wedge \cdots \wedge e^{j_n}$ by

$$\psi \wedge \star\psi = \det[\langle e_{j_i}, e_{j_k}\rangle]_{i,k}\, \mathrm{vol}(V),$$

and extended $\mathbb{R}$-multilinearly to $\bigwedge V^*$. For example, $\star 1 = \mathrm{vol}(V)$.

Hodge duals capture orthogonal subspaces: if $V = \mathbb{R}^3$, then

$$\star e_3 = e_1 \wedge e_2 \qquad \text{or} \qquad \star e_3 = -e_1 \wedge e_2 = e_2 \wedge e_1,$$

with the sign depending on a choice of orientation of V.

Hodge duals are pseudo-forms, that is, they depend on a choice of orientation, since they are defined using the volume form $\mathrm{vol}(V)$*.*

Internal 4-velocities

Consider a pointon with timelike worldline $\beta \subset \tilde{M}$ and 4-velocity v on $\tilde{M}$. Since $h(v) := h^a{}_b v^b = 0$, we want to replace v with a new geometric object $\check{v}$ that is intrinsic to spacetime M and independent of $\tilde{M}$.

We may replace v with the Hodge dual $\star\mathrm{vol}(\ker h)$ of the volume form of the kernel $\ker h \subset \tilde{M}_{\beta(t)}$ since each 1-form in $\star\mathrm{vol}(\ker h)$ lies in the internal tangent space $M_{\beta(t)} = \operatorname{im} h$. However, a Hodge dual is a *pseudo*-form, and thus depends on a choice of orientation of the kernel $\ker h$.

To eliminate this dependency, we allow an orientation of $\ker h$ to be freely chosen, independent of any (non-physical) choice of orientation of $\tilde{M}_{\beta(t)}$.

Definition

The *internal 4-velocity* of a pointon with worldline $\beta \subset \tilde{M}$ and 4-velocity v is the pseudo-form

$$\check{v}_{a\cdots b} := o_{\ker h} \star\mathrm{vol}(\ker h) \in \bigwedge^{\dim M_{\beta(t)}} M^*_{\beta(t)},$$

where $o_{\ker h} \in \{\pm 1\}$ is a free parameter independent of any orientation of $\tilde{M}_{\beta(t)}$. Note that the rank of $\check{v}$ changes along $\beta \subset \tilde{M}$ whenever the dimension of the tangent space $M_{\beta(t)}$ changes.

Let $\beta \subset \tilde{M}$ be a timelike pointon worldline with 4-velocity v, and let $e_0, \ldots, e_3$ be a vierbein along β for which $e_0 = v$. Fix $p = \beta(t) \in \tilde{M}$.

- If $\dim M_p = 3$, then the internal 4-velocity at p is

$$\breve{v}^{abc} = o_0\, e_1 \wedge e_2 \wedge e_3,$$

where $o_0 \in \{\pm 1\}$ is a free choice of time orientation (in the rest frame of the pointon), independent of any orientation of $\tilde{M}_p$.

We identify o_0 with the electric charge of the pointon.

Although this is similar to the Stückelberg-Feynman interpretation of antimatter in QFT, time does not flow along β: time does not flow backwards along β just as it does not flow forwards, since β is a single point of spacetime M.

- If $\dim M_p = 1$ and $\ker h$ is spanned by e_0, e_1, e_2, then

$$\breve{v}^{a} = o_0 o_{12}\, e_3,$$

where $o_{12} \in \{\pm 1\}$ is a free choice of orientation of the plane spanned by e_1 and e_2. We call $\breve{v}^a$ the *spin vector* of the pointon at p, and identify o_{12} as spin, up $\uparrow$ or down $\downarrow$, in the e_3 direction.

We then parallel transport $s := \breve{v}^a$ along β until it is projected under h onto a subsequent 1-dim'l tangent space $M_{\beta(t')}$.

Consider a pointon with spin vector $s = \breve{v}^a$ and worldline $\beta \subset \tilde{M}$ such that the dim of the tangent space at $\beta(0)$ is a local minimum along β.

Some notation: If $|h(s)| \neq 0$, denote by $\hat{h}(s) := h(s)/|h(s)|$ the normalization of $h(s)$, and denote by $M_{p \to q}$ the parallel transport of M_p to q along β.

$$
\begin{tikzcd}
\tilde{M}_{\beta(-\epsilon)} & \tilde{M}_{\beta(0)} \arrow[ddr, "h"] & \tilde{M}_{\beta(0)} & \tilde{M}_{\beta(0)} \arrow[ddl, "h"'] & \tilde{M}_{\beta(\epsilon)} \\
\cup & \cup & \cup & \cup & \cup \\
M_{\beta(-\epsilon)} \arrow[r, "\cong"] & M_{\beta(-\epsilon)\to\beta(0)} & M_{\beta(0)} & M_{\beta(\epsilon)\to\beta(0)} \arrow[r, "\cong"] & M_{\beta(\epsilon)} \\
s \arrow[r, mapsto] & s \arrow[r, mapsto] & \hat{h}(s) = \hat{h}(s') & s' \arrow[l, mapsto] \arrow[r, mapsto] & s'
\end{tikzcd}
$$

wavefunction collapse   randomness appears

(i) As s enters $M_{\beta(0)}$, s is projected onto $M_{\beta(0)}$ by the internal metric h.

(ii) As $h(s)$ exits $M_{\beta(0)}$, the reverse occurs: a unit vector s' is chosen so that

$$h(s')\,|h(s)| = h(s)\,|h(s')|\,, \qquad \text{or equivalently} \;\; h(\boldsymbol{s})\cdot\boldsymbol{s}' \geq 0.$$

▷ This simplifies to $\hat{h}(s) = \hat{h}(s')$ whenever $h(s)$ and $h(s')$ are nonzero.
▷ If $h(s) = 0$, then s' is unconstrained.

If $h(s) \neq 0$, then the probability that s' is chosen is given by what we call the *Kochen-Specker conditional probability*:

$$p(s'|h(s)) = \tfrac{1}{\pi}\hat{h}(s)^a s'_a = \tfrac{1}{\pi}\hat{h}(\boldsymbol{s})\cdot\boldsymbol{s}'.$$

$$
\begin{tikzcd}
\tilde{M}_{\beta(-\epsilon)} & \tilde{M}_{\beta(0)} \arrow[dr, "h"] & \tilde{M}_{\beta(0)} & \tilde{M}_{\beta(0)} \arrow[dl, "h"'] & \tilde{M}_{\beta(\epsilon)} \\
\cup & \cup & \cup & \cup & \cup \\
M_{\beta(-\epsilon)} \arrow[r, "\cong"] & M_{\beta(-\epsilon)\to\beta(0)} & M_{\beta(0)} & M_{\beta(\epsilon)\to\beta(0)} \arrow[r, "\cong"] & M_{\beta(\epsilon)} \\
s \arrow[r, mapsto] & s \arrow[r, mapsto, "\text{wavefunction collapse}"'] & \hat{h}(s) = \hat{h}(s') & s' \arrow[l, mapsto, "\text{randomness appears}"] \arrow[r, mapsto] & s'
\end{tikzcd}
$$

If $h(s)$ points to, say, the north pole in the unit sphere (in the pointon's rest frame), then s' will be some vector in the northern hemisphere, with it being more likely that it is pointing to the north pole than to the equator.

We will show that this gives a realist non-deterministic model of spin, without physical spin superposition.
In particular, spin superposition is epistemic: a spin wavefunction represents our knowledge about a state, but is not an actual physical thing.

Suppose s exits a 1-dim'l tangent space M_p at $p \in \tilde{M}$,
is parallel transported along β for some time,
and then enters another 1-dim'l tangent space M_q at q.
We say the spin s of the pointon is *prepared* at p and *measured* at q.

The vector $\hat{h}_{\beta(t)}(s)$ sits in the unit sphere parallel transported along β,

$$S^2_{\beta(t)} \subset \operatorname{span}\{e_1, e_2, e_3\} \subset \tilde{M}_{\beta(t)}.$$

The sphere corresponds to the spin Hilbert space $\mathcal{H} = \mathbb{C}\,|\uparrow\rangle \oplus \mathbb{C}\,|\downarrow\rangle$ by

$$\boldsymbol{w} = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta) \;\longleftrightarrow\; |\boldsymbol{w}\rangle = \cos(\theta/2)\,|\uparrow\rangle + e^{i\varphi}\sin(\theta/2)\,|\downarrow\rangle\,,$$

with $0 \le \theta \le \pi$ and $0 \le \varphi < 2\pi$.

We may thus identify $S^2_{\beta(t)}$ with the Bloch sphere, and therefore

$$h_p(s) \in M_p \;\longleftrightarrow\; |\hat{h}_p(s)\rangle \in \mathcal{H} \quad \& \quad h_q(s) \in M_q \;\longleftrightarrow\; \langle\hat{h}_q(s)| \in \mathcal{H}^*.$$

Theorem

Our model is a spacetime geometric realization of the Kochen-Specker model of spin, and therefore reproduces the Born rule for spin wavefunction collapse:

$$p(h_q(s)|h_p(s)) = |\langle\hat{h}_q(s)|\hat{h}_p(s)\rangle|^2.$$

Photons and polarization

If v is null, then $[v]_{ab} = g_{ab} - v_a v_b$ does not project out v.

Lemma

Let v be a null vector. Any minimal orthogonal projection $[v]^a{}_b$ of v is of the form

$$[v]_{ab} := g_{ab} + v_a v'_b + v_b v'_a,$$

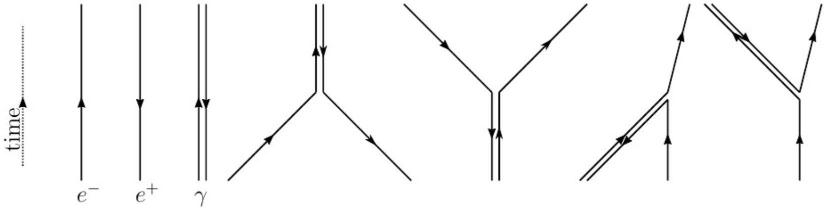
where v' is a null vector satisfying $v^a v'_a = -1$.

The internal 4-velocities of v and v' are equal up to sign,

$$\check{v} = -\check{v}'.$$

Theorem

A pointon with a null geodesic worldline cannot exist in isolation, but must travel with another pointon of opposite charge. The bound state of the two null pointons then necessarily has zero electric charge.



Consider two pointons of opposite charge with coincident worldline $\beta \subset \tilde{M}$.

The pointons form a photon along β, which is *on shell* if β is null, and *off shell* if β is timelike.

The pointons each have an ontic spin vector, denoted $s_{\pm}$.
We impose the following:

(I) The dot product of the spin vectors is nonnegative,

$$\boldsymbol{s}_{-} \cdot \boldsymbol{s}_{+} \geq 0.$$

(II) A point $p \in \beta$ is an endpoint of β (that is, p is an electron-photon vertex) if and only if $\dim M_p = 1$ and

$$h_p(s_{-}) \left| h_p(s_{+}) \right| = h_p(s_{+}) \left| h_p(s_{-}) \right|.$$

This simplifies to $\hat{h}_p(s_{-}) = \hat{h}_p(s_{+})$ if $h_p(s_{-})$ and $h_p(s_{+})$ are nonzero.

Theorem

Suppose there is an electron-photon vertex at $p \in \tilde{M}$. Then

$$\dim M_p = \begin{cases} 1 & \text{if the photon is on shell} \\ 2 & \text{if the photon is off shell} \end{cases}$$

Consequently, full spin/polarization wavefunction collapse can only occur at vertices for which the photon leg is on shell.

Suppose that an on-shell photon meets an electron at a point $p \in \tilde{M}$ in a linear polarizer.

The internal tangent space M_p at p is then 1-dim'l by the theorem.

Thus, the condition $\hat{h}_p(s_-) = \hat{h}_p(s_+)$ determines whether the photon will pass by the electron with no interaction, or interact with the electron at an electron-photon vertex.

- In the first case the photon passes through the polarizer with s_-, s_+ altered by the 1-dim'l tangent space M_p.
- In the second case the photon is 'absorbed' by the electron.

Consequently, the condition $\hat{h}_p(s_-) = \hat{h}_p(s_+)$ *determines whether the photon will pass through or be absorbed by the polarizer.*

Theorem

Suppose a photon has spin vectors s_-, s_+, and consider the line

$$\ell := \text{span}\{s_- + s_+\} \subseteq M_{\beta(t)}.$$

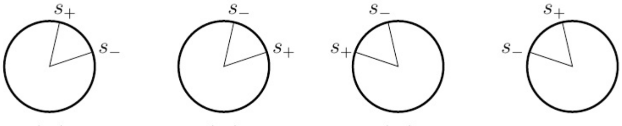
Further suppose the photon meets an electron at $p \in \tilde{M}$ for which M_p is a line. Then an electron-photon vertex will occur at p if and only if the minimal angle $\theta \in [-\pi, \pi]$ between the parallel transport of ℓ at p and the line M_p satisfies

$$|\theta| < \tfrac{\pi}{4}.$$

Let s_-, s_+ be the spin vectors of a photon, and let ϕ be the unique oriented angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$ from s_- to s_+ in the plane $P = \text{span}\{s_-, s_+\}$.
We say the photon has

- *linear polarization* $\ell = \text{span}\{s_- + s_+\}$, and
- *left-* resp. *right-circular polarization* if $\phi > 0$ resp. $\phi < 0$.

A photon thus has, at all times, both a well-defined linear polarization ℓ and circular polarization $\text{sign}(\phi)$, though a measurement of one renders the other random and unknown.



Relation to classical polarization

In terms of classical electromagnetic theory,

- $\phi = 0$ corresponds to linearly polarized light;
- $\phi = \pm\pi/2$ to circularly polarized light; and
- generic values of ϕ to elliptically polarized light.

Relation to quantum polarization

Consider an on-shell photon with null worldline β and 4-velocity $v = e_0 + e_3$.
Denote by $\mathcal{H} := \mathbb{C}\,|e_1\rangle \oplus \mathbb{C}\,|e_2\rangle$ the Hilbert space with basis vectors $|e_i\rangle := |\mathrm{span}\{e_i\}\rangle$; by $\mathbb{P}\mathcal{H}$ its projectivization; and by $[|e_i\rangle]$ the class of $|e_i\rangle$ in $\mathbb{P}\mathcal{H}$.
For $a_1, a_2 \in \mathbb{R}$, set

$$[|a_1 e_1 + a_2 e_2\rangle] = a_1[|e_1\rangle] + a_2[|e_2\rangle] \in \mathbb{P}(\mathbb{R}\,|e_1\rangle \oplus \mathbb{R}\,|e_2\rangle) \subset \mathbb{P}\mathcal{H}.$$

Observe that this linearity is well-defined in the projectivization $\mathbb{P}\mathcal{H}$, though not in $\mathcal{H}$ itself. We may thus map polarization states in our model *noninjectively* (!) to states in $\mathbb{P}\mathcal{H}$:

$$(\ell, \phi) \mapsto [|\ell, \phi\rangle] := [|s_-\rangle + e^{i\phi}\,|s_+\rangle] \in \mathbb{P}\mathcal{H}.$$

Under the identification

$$(\ell, \phi) \mapsto [|\ell, \phi\rangle] := [|s_-\rangle + e^{i\phi}\,|s_+\rangle] \in \mathbb{P}\mathcal{H},$$

we recover the Jones vectors for both linear polarization,

$$|s_- = s_+ = e_1\rangle = |H\rangle$$

$$|s_- = s_+ = e_2\rangle = |V\rangle$$

$$|s_- = s_+ = e_1 + e_2\rangle = \tfrac{1}{\sqrt{2}}\,(|H\rangle + |V\rangle) = |D\rangle$$

$$|s_- = s_+ = e_1 - e_2\rangle = \tfrac{1}{\sqrt{2}}\,(|H\rangle - |V\rangle) = |A\rangle$$

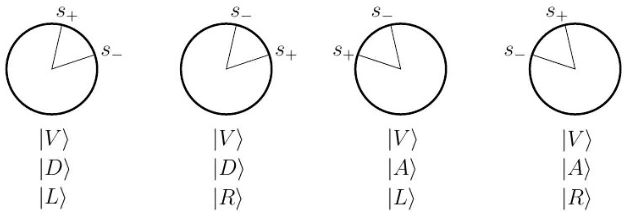
and circular polarization,

$$|s_- = e_1,\ s_+ = e_2\rangle = \tfrac{1}{\sqrt{2}}\,(|H\rangle + i|V\rangle) = |L\rangle$$

$$|s_- = e_1,\ s_+ = -e_2\rangle = \tfrac{1}{\sqrt{2}}\,(|H\rangle - i|V\rangle) = |R\rangle$$

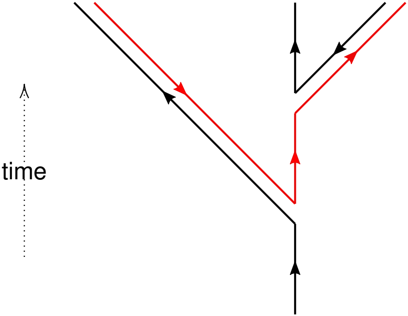
$$|s_- = -e_2,\ s_+ = e_1\rangle = \tfrac{1}{\sqrt{2}}\,(-|V\rangle + i|H\rangle) = -i|L\rangle$$

$$|s_- = e_2,\ s_+ = e_1\rangle = \tfrac{1}{\sqrt{2}}\,(|V\rangle + i|H\rangle) = i|R\rangle$$



Future directions: entanglement

Consider two photons emitted from an electron:



We say the photons are *geometrically entangled* because they share a common pointon worldline (drawn in red).

If each photon encounters a polarizer, then the two polarizers are effectively 'touching', regardless of how far apart they appear in $\tilde{M}$, since the pointon worldlines are single points in spacetime M.

⚠ We expect that geometric entanglement will only approximately reproduce quantum entanglement, and so quantum theory and our model will differ in certain situations.

Future directions: a pointon model of the standard model

We would like to construct a composite model of all standard model particles using only pointons and their pair creation/annihilation, just as we have done for electrons and photons.

Proposition

A choice of time orientation $o_0 \in \{\pm 1\}$ of $\mathbb{R}^{1,3}$, and thus of electric charge of a pointon, corresponds to a unique $\mathfrak{su}(2)_{\mathbb{C}} := \mathfrak{su}(2) \oplus i\mathfrak{su}(2)$ subalgebra of the complexified Lorentz algebra,

$$\mathfrak{so}(1,3)_{\mathbb{C}} \cong \mathfrak{su}(2)_{\mathbb{C}} \oplus \mathfrak{su}(2)_{\mathbb{C}}.$$

Thus, chirality = electric charge, and not spin as is usually assumed:

Corollary

If $\psi \in \mathbb{C}^4$ is an eigenspinor of γ^5 with eigenvalue ± 1, then ψ has electric charge ± 1; otherwise ψ is neutral. In the latter case, the projections

$$\psi^- := \tfrac{1}{2}(1 - \gamma^5)\psi = \begin{bmatrix} * \\ * \\ 0 \\ 0 \end{bmatrix}_{\mathscr{C}} \quad \text{and} \quad \psi^+ := \tfrac{1}{2}(1 + \gamma^5)\psi = \begin{bmatrix} 0 \\ 0 \\ * \\ * \end{bmatrix}_{\mathscr{C}}$$

are spinors with charges -1 and $+1$, respectively.

This significantly changes the meaning of the Dirac Lagrangian...

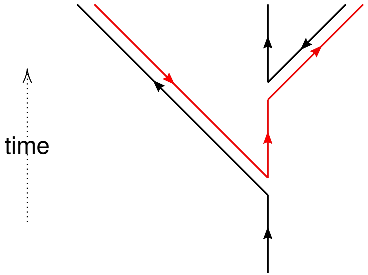
Recall the standard chiral decomposition of the Dirac Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi = i\bar{\psi}^{-}\partial\!\!\!/\psi^{-} - m\bar{\psi}^{-}\psi^{+} + (+ \leftrightarrow -).$$

The mass terms now represent

- couplings between pointons of opposite charge; and
- pair creation/annihilation of pointons of opposite charge.

Recall the diagram where an electron emits two photons:



The photons are bound pairs of pointons of opposite charge, and there are two points of pair creation.

In ongoing work, we are using these couplings to construct a composite model of the standard model.

Thank you!

Spacetime geometry of spin, polarization, and wavefunction collapse
available on my website:
https://charlesbeil.wixsite.com/charlie-beil.