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The Big Bang Models

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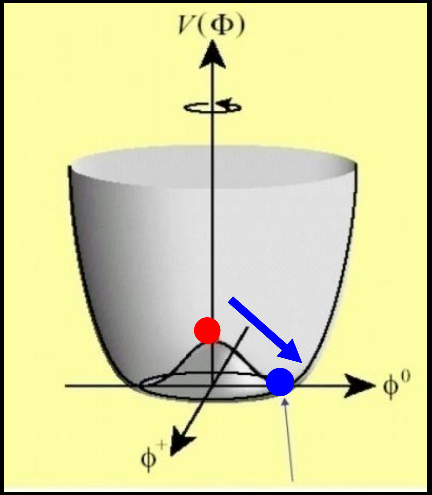
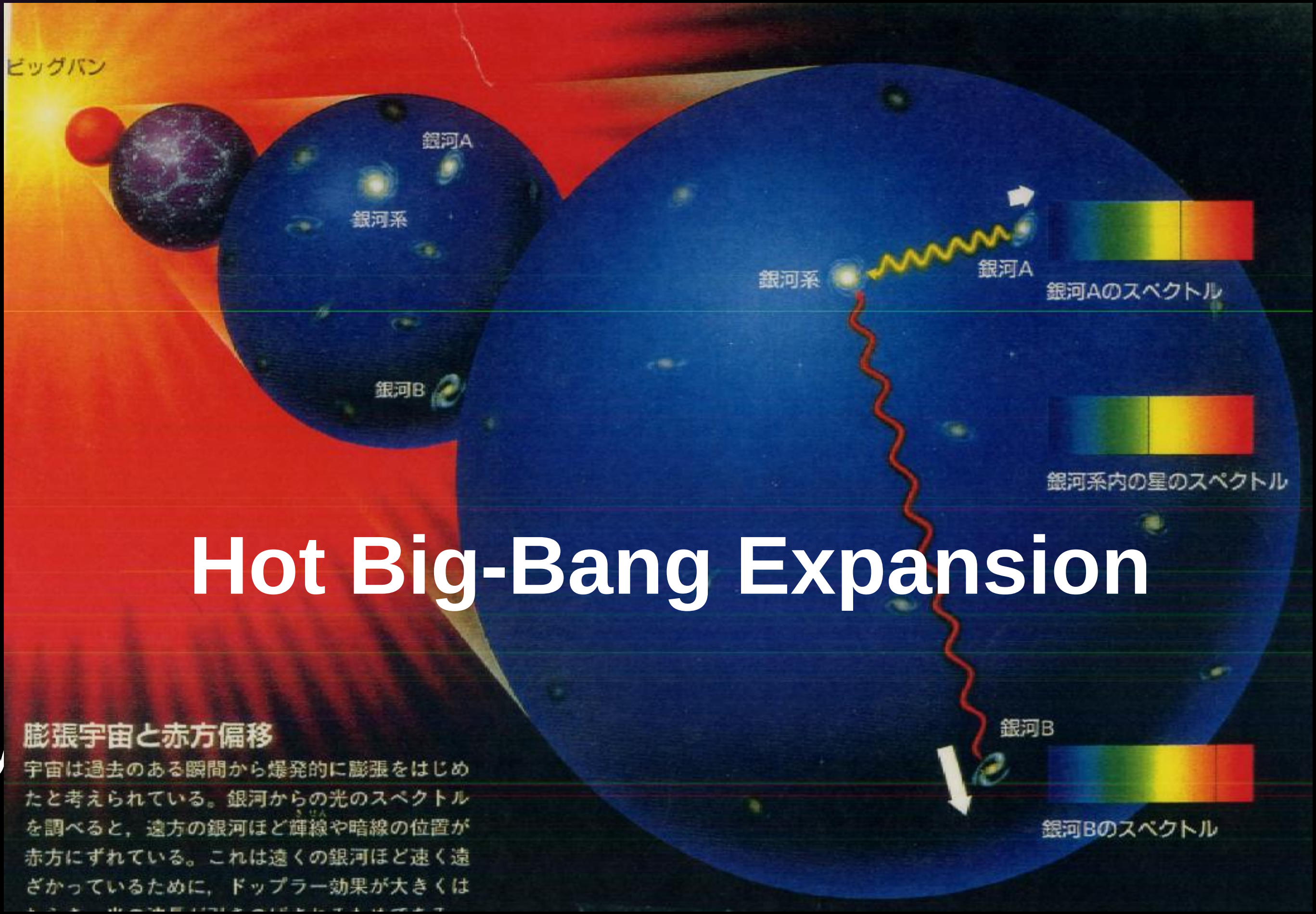
How was the beginning of the Universe?

Quantum gravity or brane world cosmology with extra dimension predicts “quantum fluctuations” of the space-time in bubbly baby Universe.



Spontaneous Symmetry Breaking



膨張宇宙と赤方偏移

宇宙は過去のある瞬間から爆発的に膨張をはじめたと考えられている。銀河からの光のスペクトルを調べると，遠方の銀河ほど輝線や暗線の位置が赤方にずれている。これは遠くの銀河ほど速く遠ざかっているために，ドップラー効果が大きくはたらき，

Hot Big-Bang Expansion

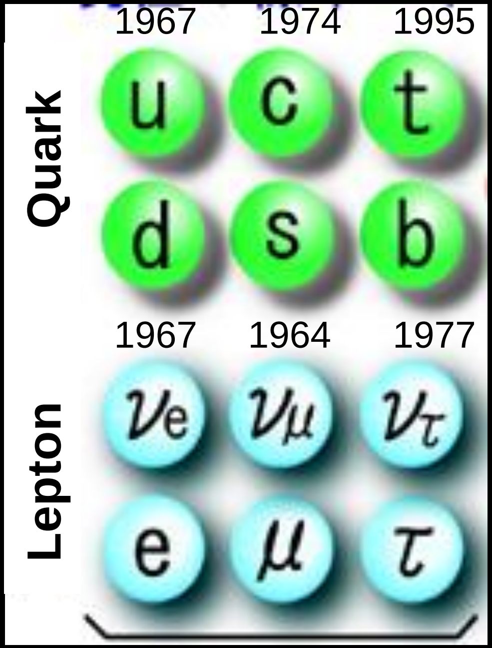
How was the beginning of the Universe?

Quantum gravity or brane world cosmology with extra dimension predicts "quantum fluctuations" of the space-time in bubbly baby Universe.

Higgs Particles

To be found ?

Matter Particles



3 generation

Gauge Particles

+G

Unification ?

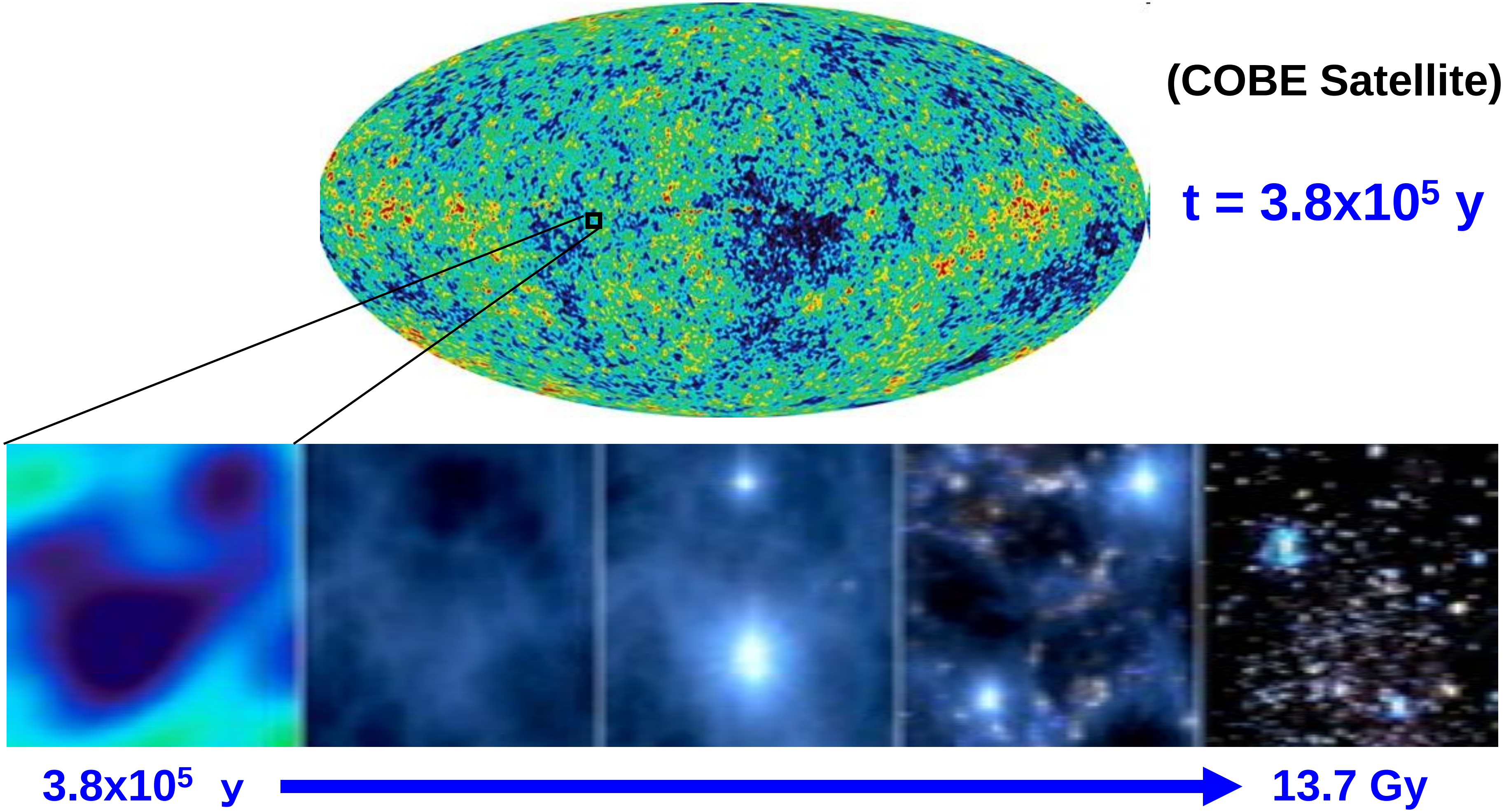
?

U(1) x SU(2) x SU(3)

The first 10^{-39} sec !

Standard Model of Elementary Particle Field

Primordial fluctuations in hot Big-Bang Universe was discovered by Smoot and Mathar in 1992.


(COBE Satellite)

$t = 3.8x10^5$ y

Assuming 73% dark energy (DE) and 23% dark matter (DM), computer simulation of cosmic structure formation best explains the observed structure! **What is the nature and origin of DE and DM ?**

Pie Chart of Cosmic Mystery

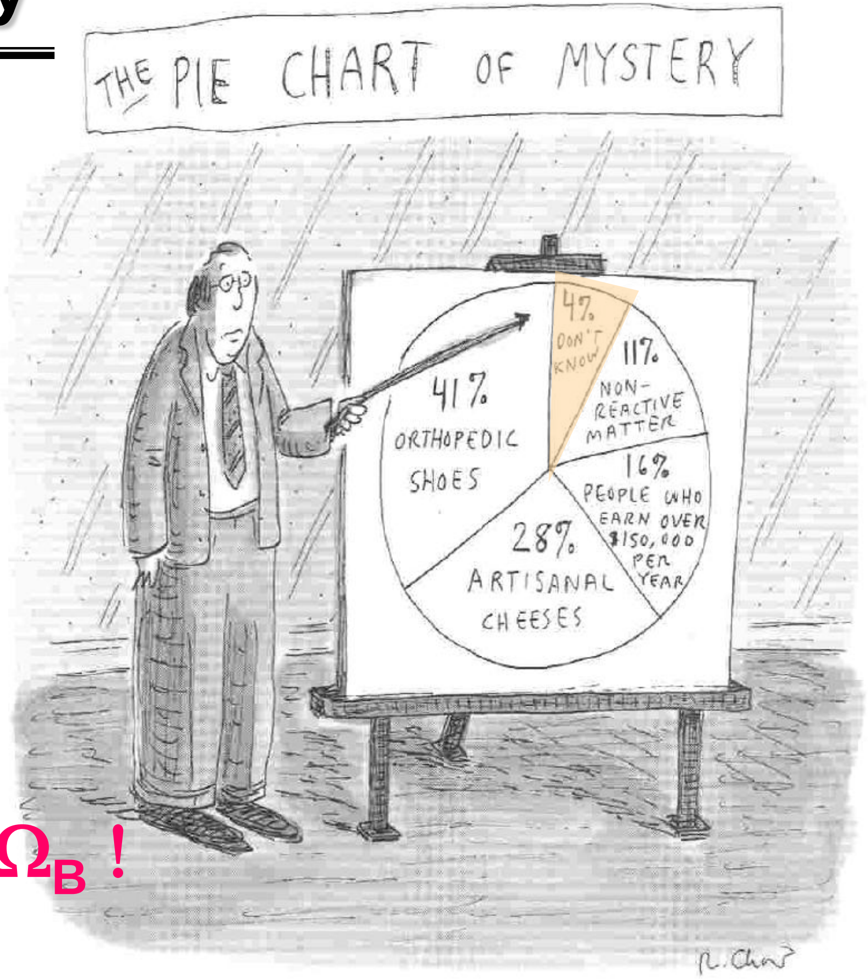
Ordinary matter makes up
a small fraction of mass/energy.

Dark matter and dark energy
dominate.

Ω_Λ ?

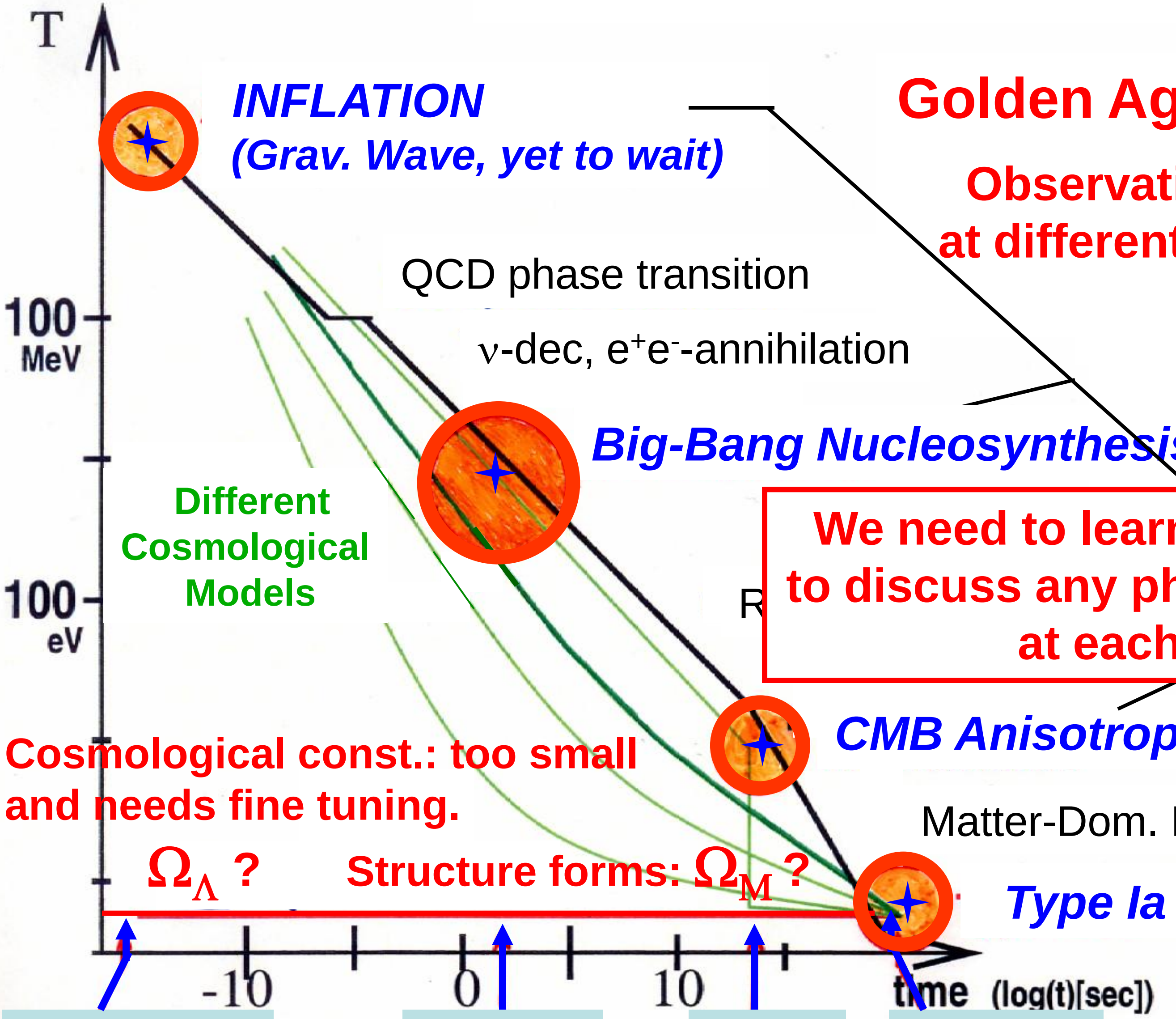
Ω_B !

(HDM: Ω_ν>0.003) Ω_{CDM} ?



What is the dark component of the Universe!

Thermal History of the Universe



Golden Age of Cosmology

Observational Signatures at different cosmic epochs !

INFLATION
(Grav. Wave, yet to wait)

QCD phase transition

ν-dec, e^+e^--annihilation

Big-Bang Nucleosynthesis

Different Cosmological Models

We need to learn the cosmology to discuss any physical processes at each epoch!

CMB Anisotropies

Matter-Dom. Era

Type Ia SNe, Matter Power

Cosmological const.: too small and needs fine tuning.

Ω_Λ ?

Structure forms: Ω_M ?

10^{-43} sec

3 min

10^5 y

1-10 Gy

OUTLINE

Universe is likely to be flat and accelerating!

$$\Omega_B + \Omega_{CDM} + \Omega_\Lambda = 1$$

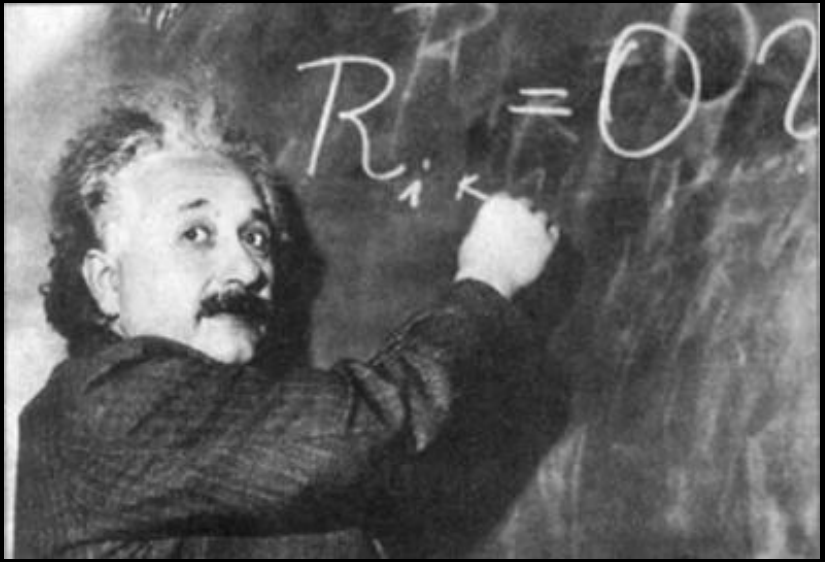
- **What is the CDM, $\Omega_{CDM} = 0.23$?**
  Relic SUSY particles?
- **What is DARK ENERGY, $\Omega_\Lambda = 0.73$?**
  Mass-Energy Flow in Extra-Dimensional Cosmology !

LECTURE

- **CMB Anisotropies ($t \sim 3.8 \times 10^5$ y) constrain cosmic evolution from RD – Last Photon Scatt. – MD – Λ-dominated Universe.**
- **Redshift-magnitude relation of the Type Ia SNe ($t \sim$ 1-10 Gy) constrains turn over from Cosmic Deceleration – Acceleration.**
- **Big-Bang Nucleosynthesis ($t \sim$ 3 min) constrains as a CANDLE of dark side of the Universe.**

SCIENTIFIC GOAL is to elucidate the tight coupling between the frontline of cosmology and nuclear astrophysics.

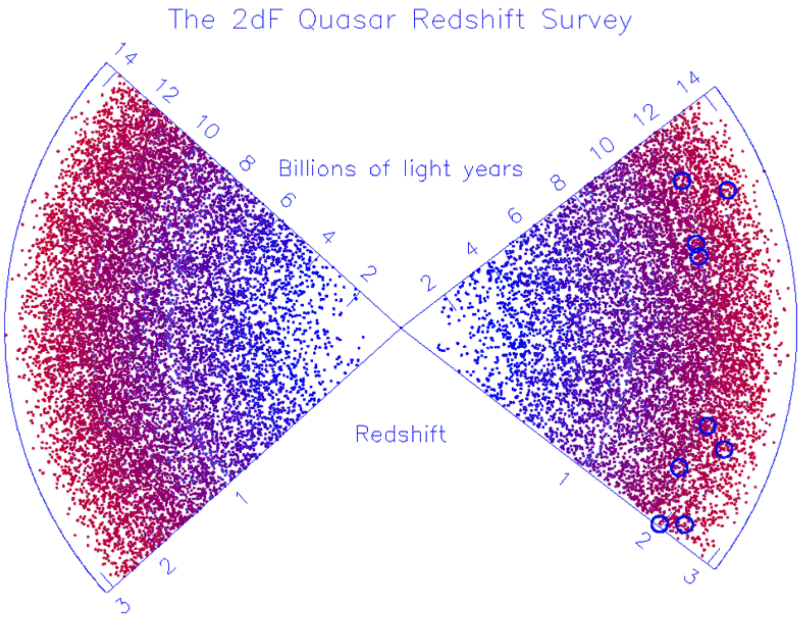
1. Standard Cosmology



Albert Einstein 1915

Standard Big-Bang Cosmology

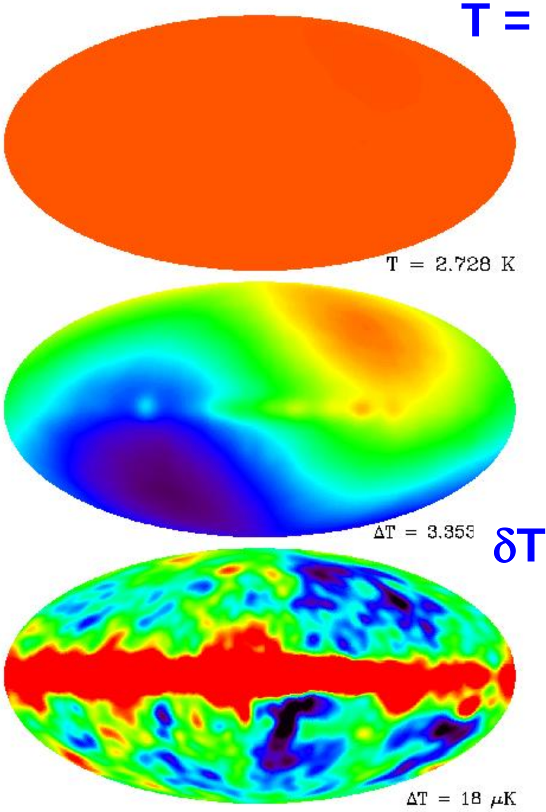
The Universe is homogeneous and isotropic in a large enough scale.



2dF Quasar (Matter) Distribution:

Homogeneous

T = 2.728 K



$\delta T/T < 10^{-5}$

Sky Maps of CMB:

Isotropic

Robertson-Walker Metric: HOMOGENEOUS & ISOTROPIC

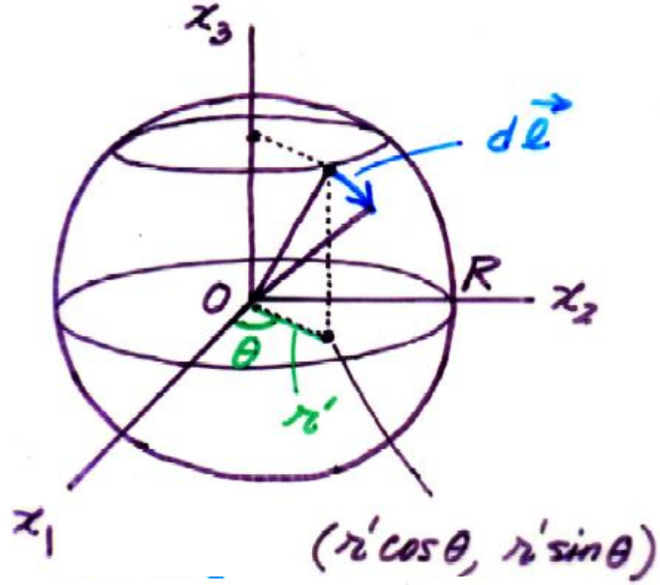
$$ds^2 \equiv -g_{\mu\nu}dx^\mu dx^\nu = \overset{c\equiv 1}{dt^2} - 2\overset{0}{g_{0i}}\,dx^0\cancel{dx^i} - g_{ij}\,dx^i dx^j = dt^2 - d\vec{\ell}^{\,2}$$

Positively curved S2, embedded in 3D Euclidean space

Radius "R"

↓

Scale Factor "a(t)"



$$R^2 = x_1^2 + x_2^2 + x_3^2$$

$$0 = x_1 dx_1 + x_2 dx_2 + x_3 dx_3$$

$$d\vec{\ell}^{\,2} = dx_1^2 + dx_2^2 + dx_3^2$$

$$= dx_1^2 + dx_2^2 + \frac{(x_1dx_1 + x_2dx_2)^2}{R^2 - (x_1^2 + x_2^2)}$$

$$= r'^2 d\theta^2 + \frac{R^2 dr'^2}{R^2 - r'^2}$$

$$\equiv R^2\left(r^2 d\theta^2 + \frac{dr^2}{1 - r^2}\right)$$

$$0 \leqq r \equiv \frac{r'}{R} \leqq 1$$

Positively and Negatively curved S2

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2 d\theta^2\right\}$$

$k = \pm 1,\ 0$ (Flat)

Real World is S3

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right\}$$

General Relativity

(1) Weak Gravity (should include Newtonian Gravity),

(2) Covariance

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi\, G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

$$R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu} = \partial_{\lambda}\Gamma^{\lambda}_{\mu\nu} - \partial_{\nu}\Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\eta\lambda}\Gamma^{\eta}_{\mu\nu} - \Gamma^{\lambda}_{\eta\nu}\Gamma^{\eta}_{\mu\lambda}$$

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\beta} \left\{ \partial_{\nu} g_{\beta\mu} + \partial_{\mu} g_{\beta\nu} - \partial_{\beta} g_{\mu\nu} \right\}$$

$$g_{\mu\nu} = \begin{bmatrix} -1 & & & \\ & \dfrac{a^2(t)}{1-kr^2} & & \\ & & a^2(t)r^2 & \\ & & & a^2(t)r^2\sin^2\theta \end{bmatrix}$$

$$T^{\mu}{}_{\nu} = \begin{bmatrix} -\rho & & & \\ & p & & \\ & & p & \\ & & & p \end{bmatrix}$$

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi\, G T^{\mu\nu} + \Lambda g^{\mu\nu}$$

time-time component:

Dynamical Eq. of Motion

$$\boxed{\dot{a}^2 + k = 8\pi G/3\, \rho a^2 + \Lambda/3\, a^2} \qquad (1)$$

space-space component:

$$2a\dot{a} + \dot{a}^2 + k = -8\pi G p a^2 + \Lambda a^2 \qquad (2)$$

$\frac{\mathbf{d}}{\mathbf{dt}}$[ax(1)] & (2):

3rd Law of Thermodynamic
(Energy Conservation)

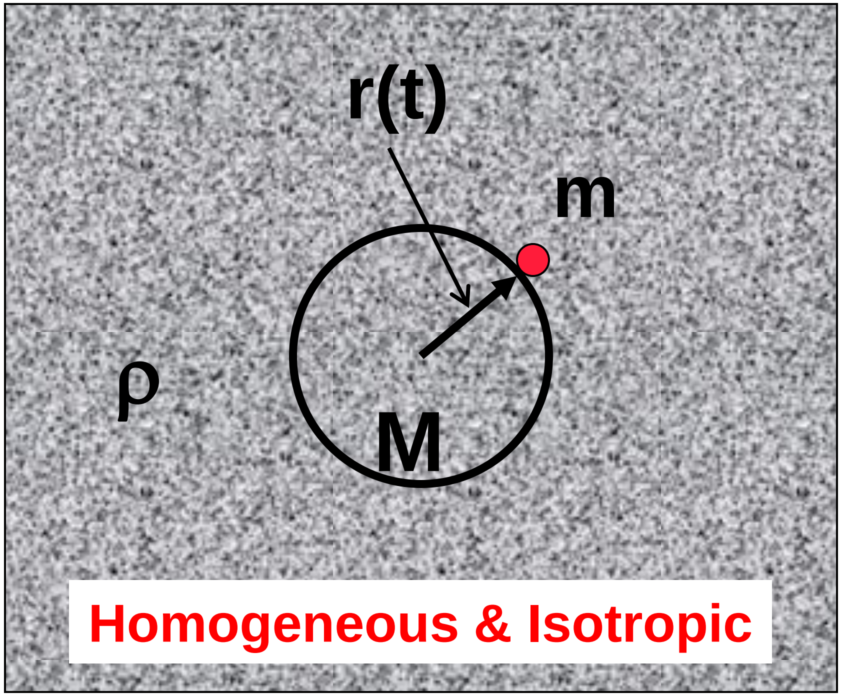
$$\boxed{\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}a^3 = 0} \qquad (3)$$

EOS (Equation of State)

$$\boxed{p = \rho/3 \text{ (rel)}} \qquad (4)$$

Newtonian Equation

Birkoff's Theorem:
Gravity due to mass interior to an arbitrary sphere.



$M = 4/3\pi\rho r^3$

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r}$$

$$\frac{1}{2}mv^2 = \frac{Gm[(4/3)\pi\rho r^3]}{r} + E$$

$$\Big\downarrow \; \times\, 1/2mr^2$$

$$\left(\frac{v}{r}\right)^2 = \frac{8}{3}\pi G\rho + \frac{2E}{mr^2}$$

Einstein Equation

$$G^{00} = 8\pi\, GT^{00} + \Lambda g^{00}$$

Friedmann Eq.

Cosmological Consant.

$$H^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

$$H^2 = H_0^2\left(\frac{\Omega_\gamma}{a^4} + \frac{\Omega_M}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda\right)$$

$T \propto a^{-1}$ $\quad \Omega \longrightarrow 1$ at $t = t_0$, $a_0 = 1$

Newtonian Equation

Hubble parameter

$H = v/r$

$-k = E/m$

$$\left(\frac{v}{r}\right)^2 = \frac{8}{3}\pi G\rho + \frac{E}{mr^2}$$

$\Omega_\alpha = \rho_\alpha/\rho_C$ $\quad \boxed{\rho_C = 3H_0^2/8\pi G}$

a = r = scale factor

Deceleration parameter

$$q_0 = -(d^2r/dt^2)/rH^2 = [\Omega_{CDM}/2 - \Omega_\Lambda]$$

0.3 **0.7**

$$\Omega_{CDM}/2 < \Omega_\Lambda$$

acceleration !

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8}{3}\pi G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}} \quad \boxed{\frac{d}{dt}(\rho a^3)+p\frac{d}{dt}a^3 = 0} \quad \boxed{\text{EOS}}$$

$\Lambda = 0$:

1) k = 0 $\left(\frac{\dot{a}}{a}\right)^2 \propto a^{-3}$ $(\rho \propto m a^{-3})$ $a^{1/2}da \propto dt$ $\boxed{a \propto t^{2/3}}$

2) k =-1 < 0 $\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G/3\ m a^{-3} + a^{-2} \rightarrow a^{-2}$ (for large a)

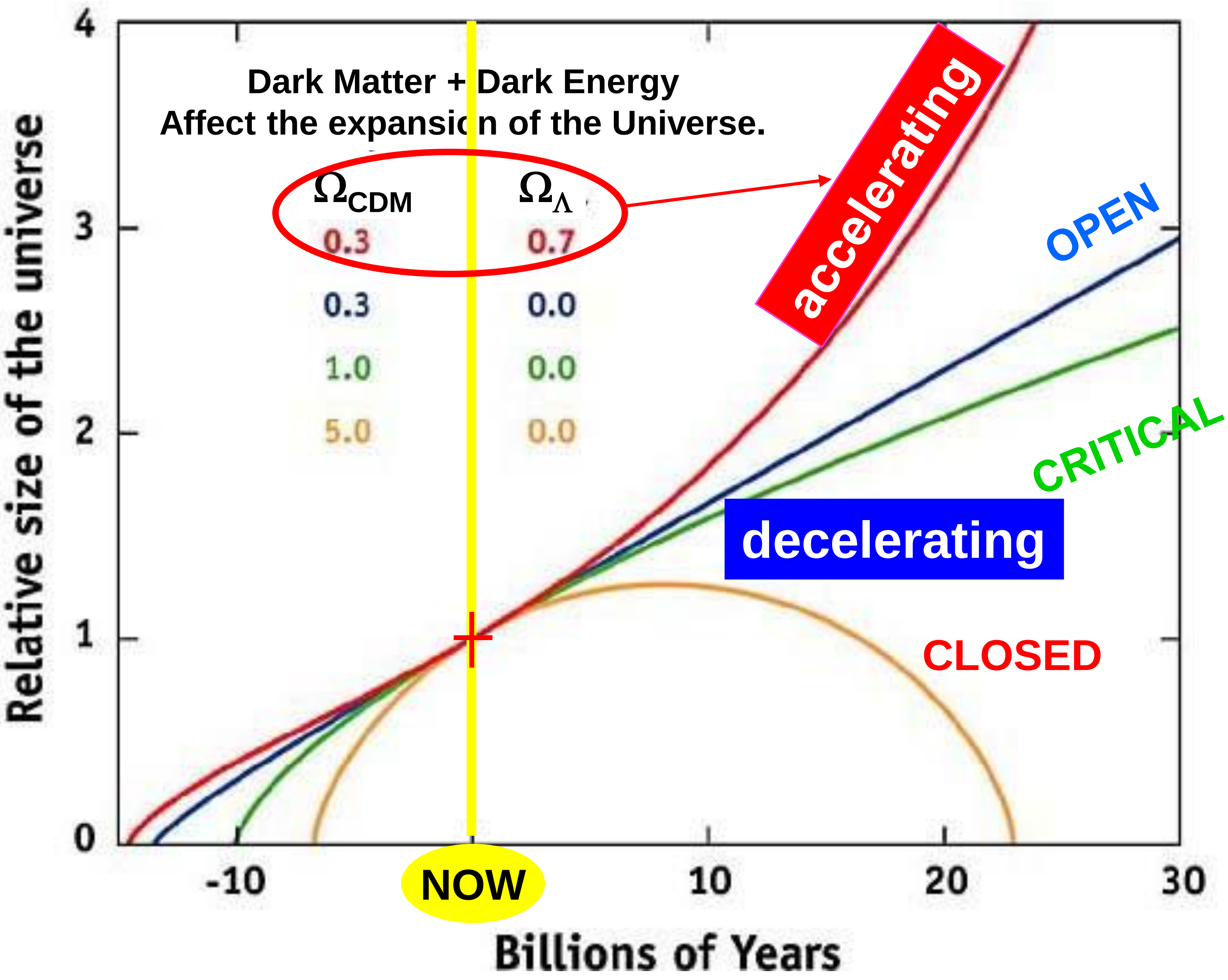
$da \propto dt$ $\boxed{a \propto t}$

3) k =+1 > 0 $\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G/3\ m a^{-3} - a^{-2} \rightarrow$ $\boxed{\begin{array}{l} 0 \ \ (\text{at some } a = a_s) \\ \& \ \ \text{bounce later.} \end{array}}$

$\Lambda > 0$ and dominates:

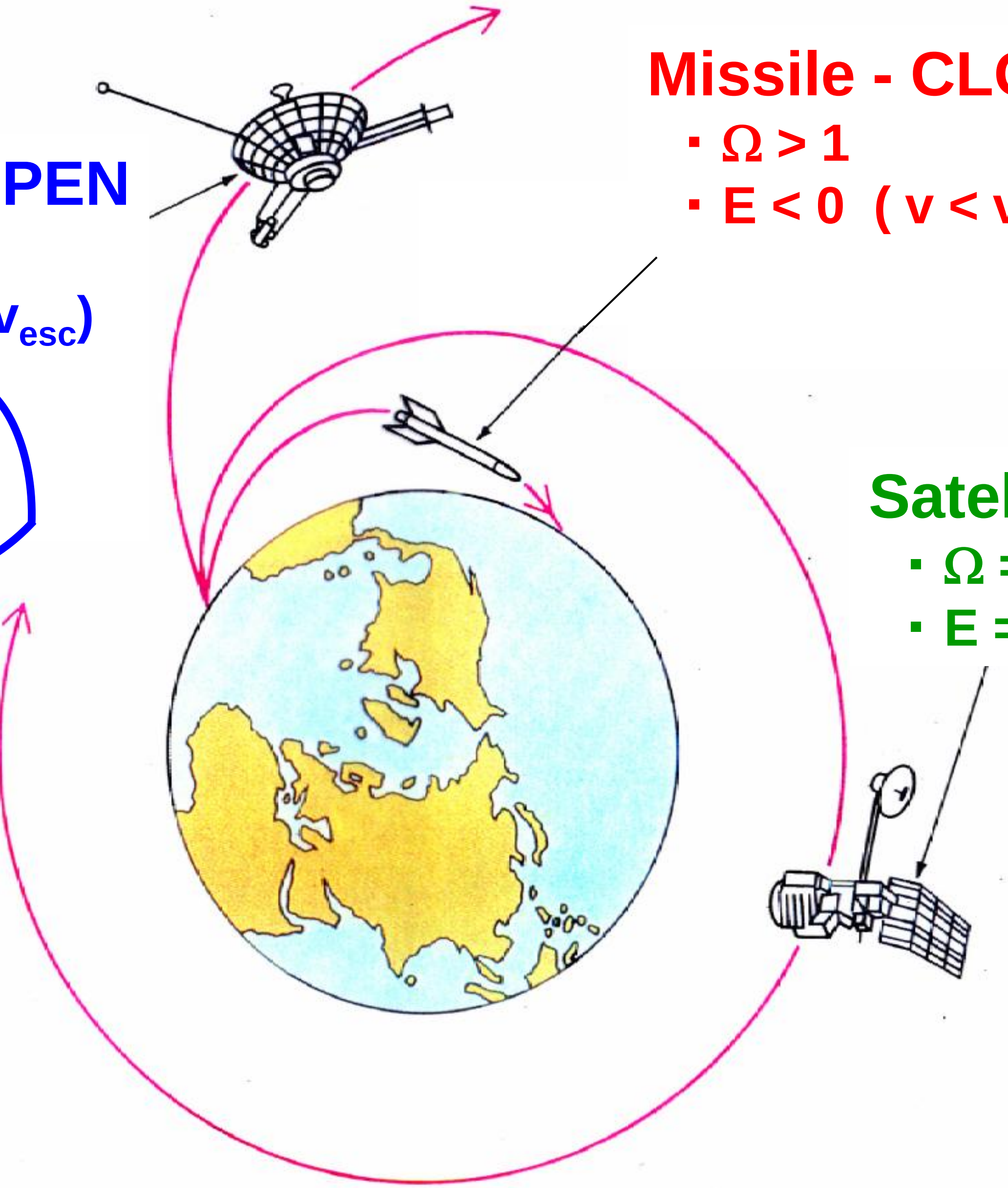
$\left(\frac{\dot{a}}{a}\right)^2 = \Lambda/3$ $a^{-1}da \propto dt$ $\boxed{a \propto \exp[(\Lambda/3)^{1/2}t]}$

Cosmic Expansion



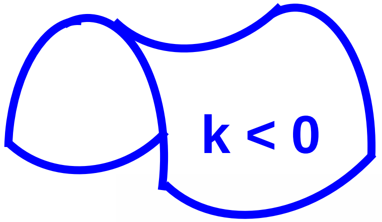
Dark Matter + Dark Energy
Affect the expansion of the Universe.

Newtonian Orbits: OPEN or CLOSED ?



Explorer - OPEN

- $\Omega < 1$
- $E > 0 \ \ (v > v_{esc})$



Missile - CLOSED

- $\Omega > 1$
- $E < 0 \ \ (v < v_{esc})$

k > 0

Satellite -MARGINAL

- $\Omega = 1$
- $E = 0 \ \ (v = v_{esc})$

k = 0

Escaping Velocity

v_{esc} = 11.2 km/s

How old is the Universe?

Photon last scatter
4x10^5 year

Accelerating expansion
Due to Dark Energy

Dark Age

Inflation

Quantum
fluctuation

WMAP

1st star
4 million year

Birth of galaxies & stars

Exercise No. 1: Calculate the cosmic age!

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = H_0^2\left\{\frac{\Omega_\gamma}{a^4}+\frac{\Omega_M}{a^3}+\frac{\Omega_k}{a^2}+\Omega_\Lambda\right\}$$

Cosmic age = Expansion time

$$t_U = \int_0^{t_U} dt = \int_0^{a(t_U)} \underset{\dot a^{-1}}{\underset{\parallel}{\left(\frac{dt}{da}\right)}} da$$

$$\frac{a}{a_0} = \frac{1}{1+z}$$

Z = redshift

$$t_U = \frac{1}{H_0}\int_0^{z\to\infty}\frac{dz}{(1+z)^2\sqrt{F(z)}}$$

$$F(z) = 1 + \cancel{\Omega_\gamma z(z+2)} + \Omega_M z - \Omega_\Lambda\left(1-\frac{1}{(1+z)^2}\right)$$

Simplest case: $\Omega_\gamma << \Omega_M = 1$, and $\Omega_\Lambda = 0$

H_0 = 70 km/s/Mpc

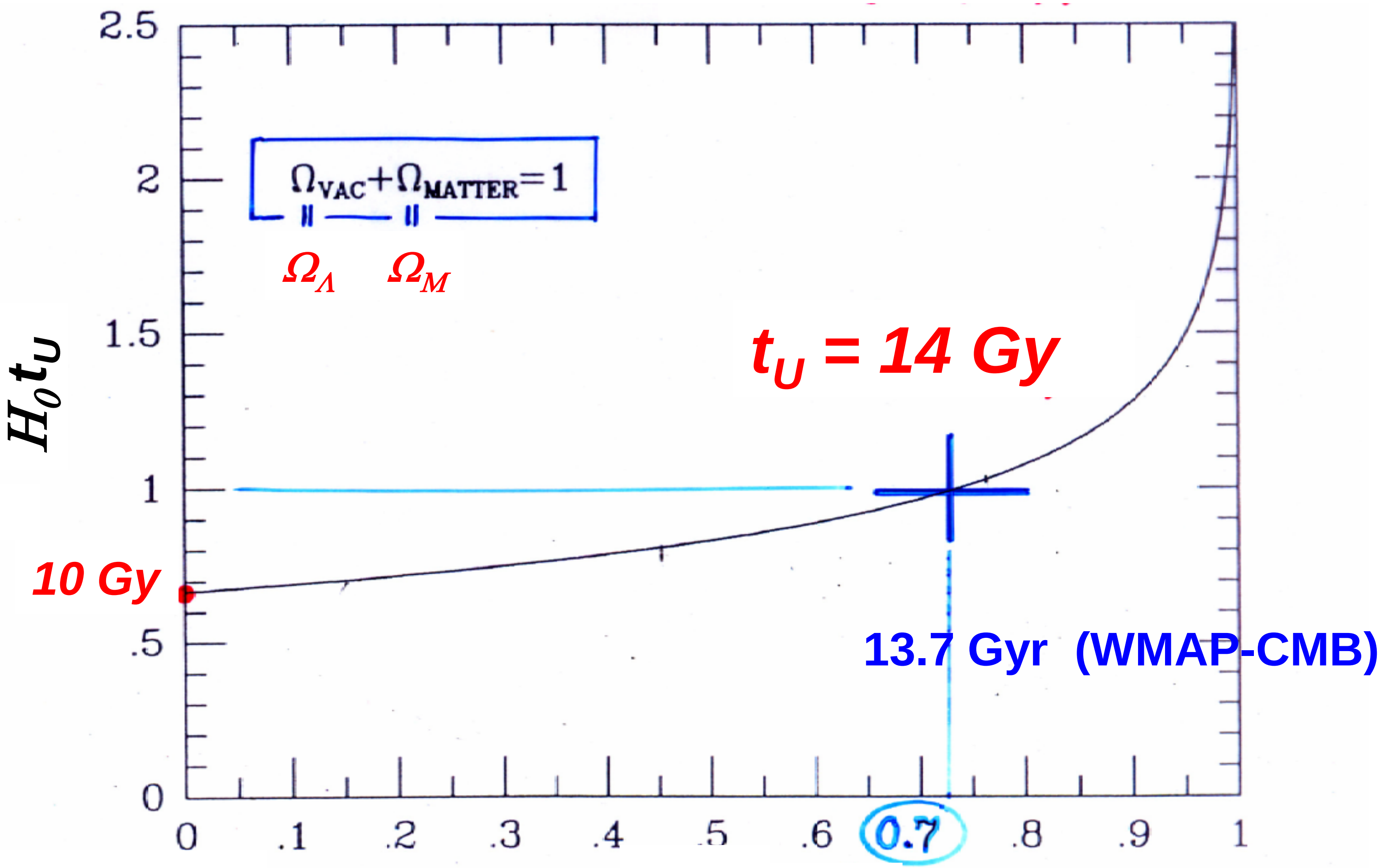
$$t_U = \frac{1}{H_0}\int_0^{\infty}(1+z)^{-5/2}\,dz = \frac{2}{3}H_0^{-1} \longrightarrow$$

10 Gy

$\Omega_\Lambda > 0$ can make cosmic age even LONGER!

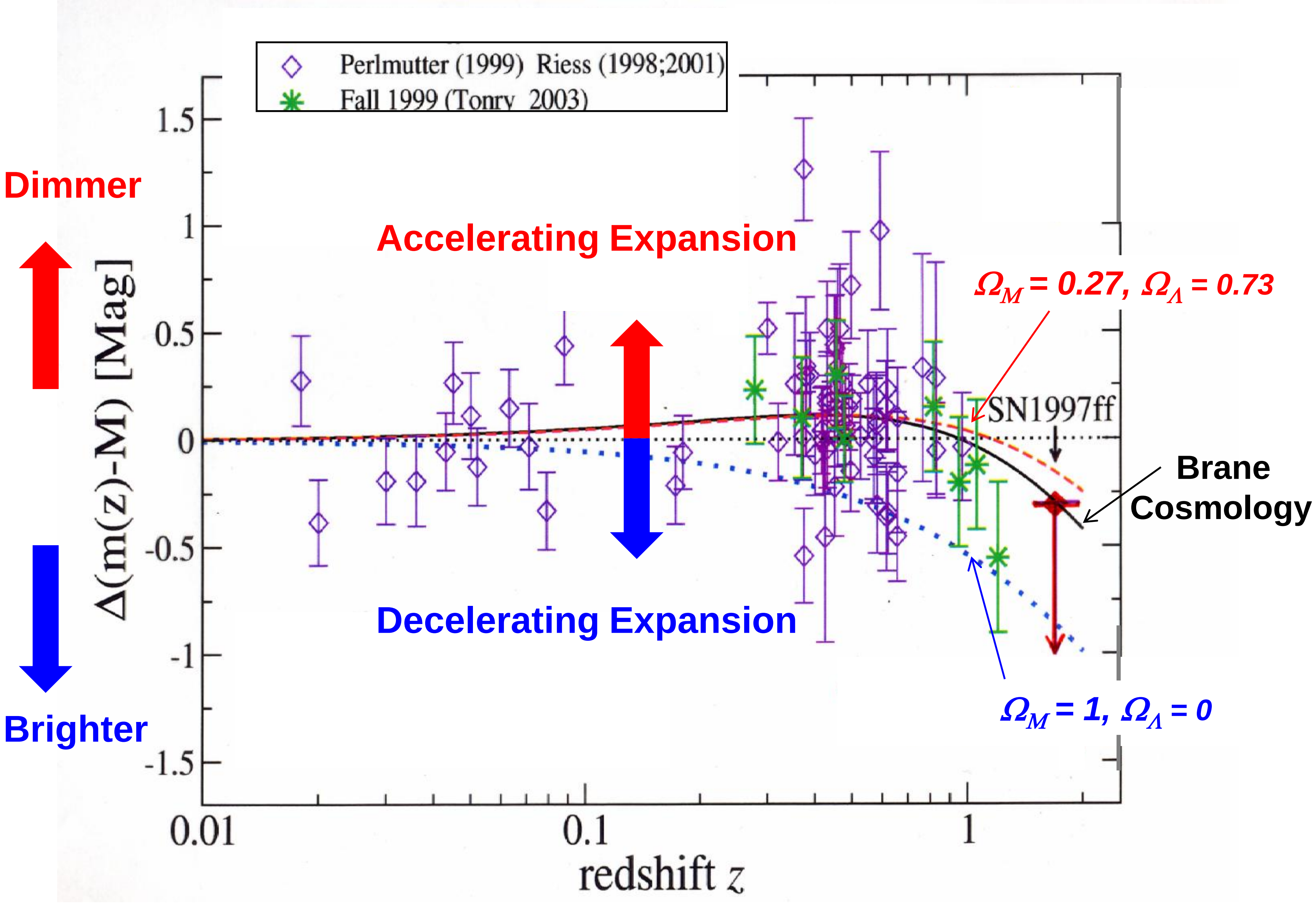
$$\underline{H_0 t_U} = \int \frac{dz}{(1+z)^2 \sqrt{1 + \Omega_M z - \Omega_\Lambda \left(1 - \frac{1}{(1+z)^2}\right)}}$$

H_0^{-1} has dimension of time!



H_0 = 70 km/s/Mpc

Type Ia Supernova Redshift-Magnitude Relation



Dimmer

Brighter

Accelerating Expansion

Decelerating Expansion

Ω_M = 0.27, Ω_Λ = 0.73

Ω_M = 1, Ω_Λ = 0

Brane Cosmology

Exercise No. 2: Calculate cosmological parameter dependence of the redshift-magnitude relation!

apparent magnitude *Absolute Magnitude*

$$m(z) - M = 5 \log D_L + 25 \quad : \quad D_L = \text{Luminosity Distance}$$

$$\boxed{\Delta(m(z) - M) = 5 \log D_L(\Omega_\gamma, \Omega_M, \Omega_\Lambda, \Omega_k) \Big/ D_L(0,0,0,\Omega_k = 1)}$$

$$D_L = (1+z)\frac{1}{\sqrt{|k|}} \left\{ \begin{array}{ll} \sin & (k>0) \\ 1 & (k=0) \\ \sinh & (k<0) \end{array} \right\} \sqrt{|k|} \boxed{\int_0^r \frac{dr}{\sqrt{1-kr^2}}}$$

$$\boxed{\int_0^r \frac{dr}{\sqrt{1-kr^2}}} = \int_0^z dz \left[\Omega_\gamma (1+z)^4 + \Omega_M (1+z)^3 + \underset{\uparrow}{\Omega_k} (1+z)^2 + \Omega_\Lambda \right]^{-1/2}$$

$$\Omega_k = 1 - (\Omega_\gamma + \Omega_M + \Omega_\Lambda)$$

Exercise No. 3: When did the cosmic expansion turn over from deceleration to acceleration?

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = H_0^2\left\{\frac{\Omega_\gamma}{a^4}+\frac{\Omega_M}{a^3}+\frac{\Omega_k}{a^2}+\Omega_\Lambda\right\}$$

Deceleration parameter : $q \equiv -\left(\frac{\ddot a\, a}{\dot a^2}\right)$

$$q = -1 + \frac{\frac{2}{a^2} + \cancel{\left(\frac{4}{a^4}-\frac{2}{a^2}\right)\Omega_\gamma} + \left(\frac{3}{a^3}-\frac{2}{a^2}\right)\Omega_M - \frac{2}{a^2}\Omega_\Lambda}{2\left[\frac{1}{a^2}+\cancel{\left(\frac{1}{a^4}-\frac{1}{a^2}\right)\Omega_\gamma}+\left(\frac{1}{a^3}-\frac{1}{a^2}\right)\Omega_M+\left(1-\frac{1}{a^2}\right)\Omega_\Lambda\right]}$$

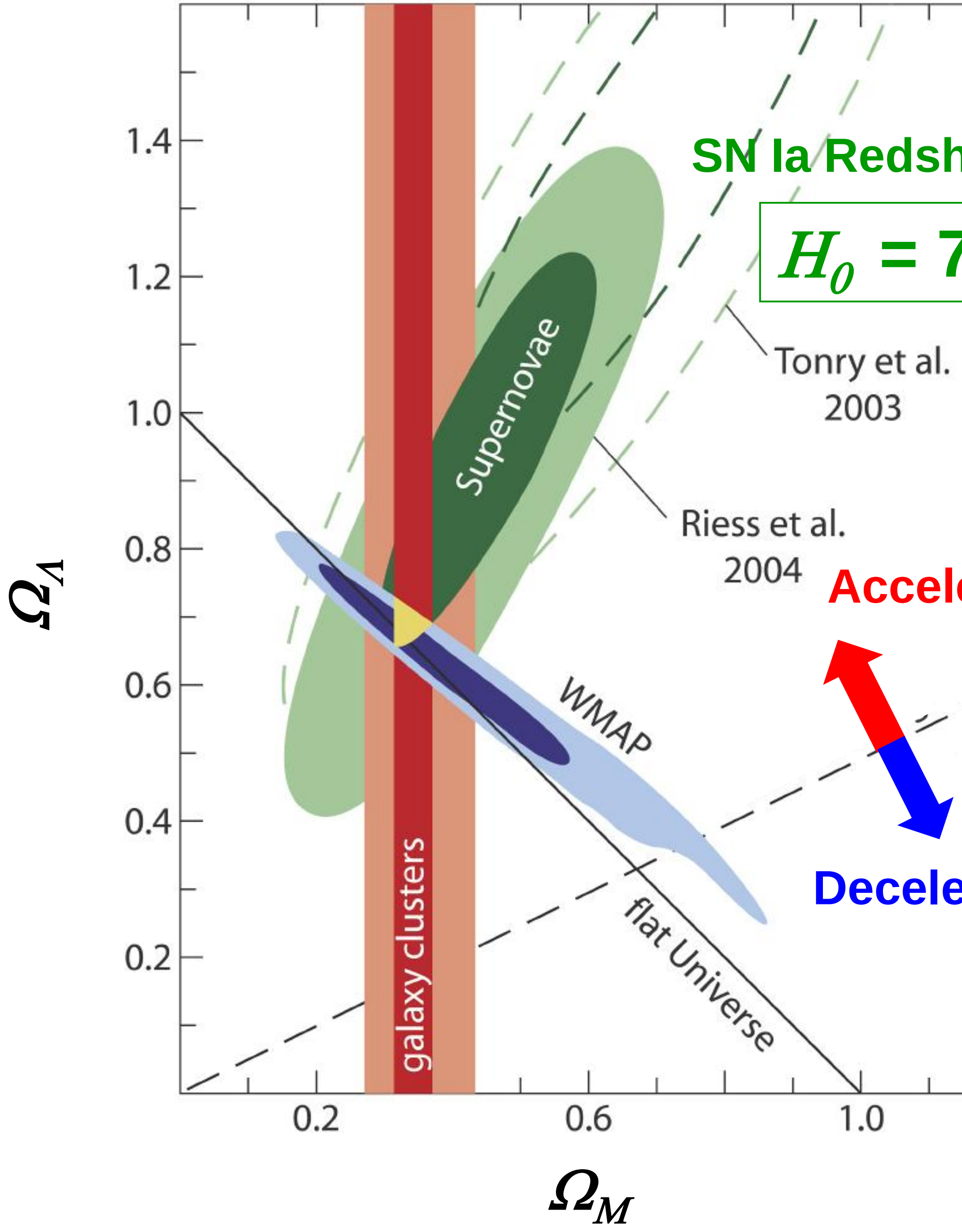
$\Omega_\gamma << \Omega_M = 0.27$, and $\Omega_\Lambda = 0.73$

$$\boxed{\therefore\ q = 0 \longrightarrow z \approx 0{,}8}$$

$a \rightarrow a_0 = 1$

Exercise No. 4: How is the present expansion?

$$q_0 = -1 + \frac{2+2\cancel{\Omega_\gamma}+\Omega_M-2\Omega_\Lambda}{2} = \cancel{\Omega_\gamma} + \boxed{\frac{\Omega_M}{2}-\Omega_\Lambda \gtreqless 0\ ?}$$



SN Ia Redshift-Magnnitude Rel.

H_0 = 72 km/s/Mpc

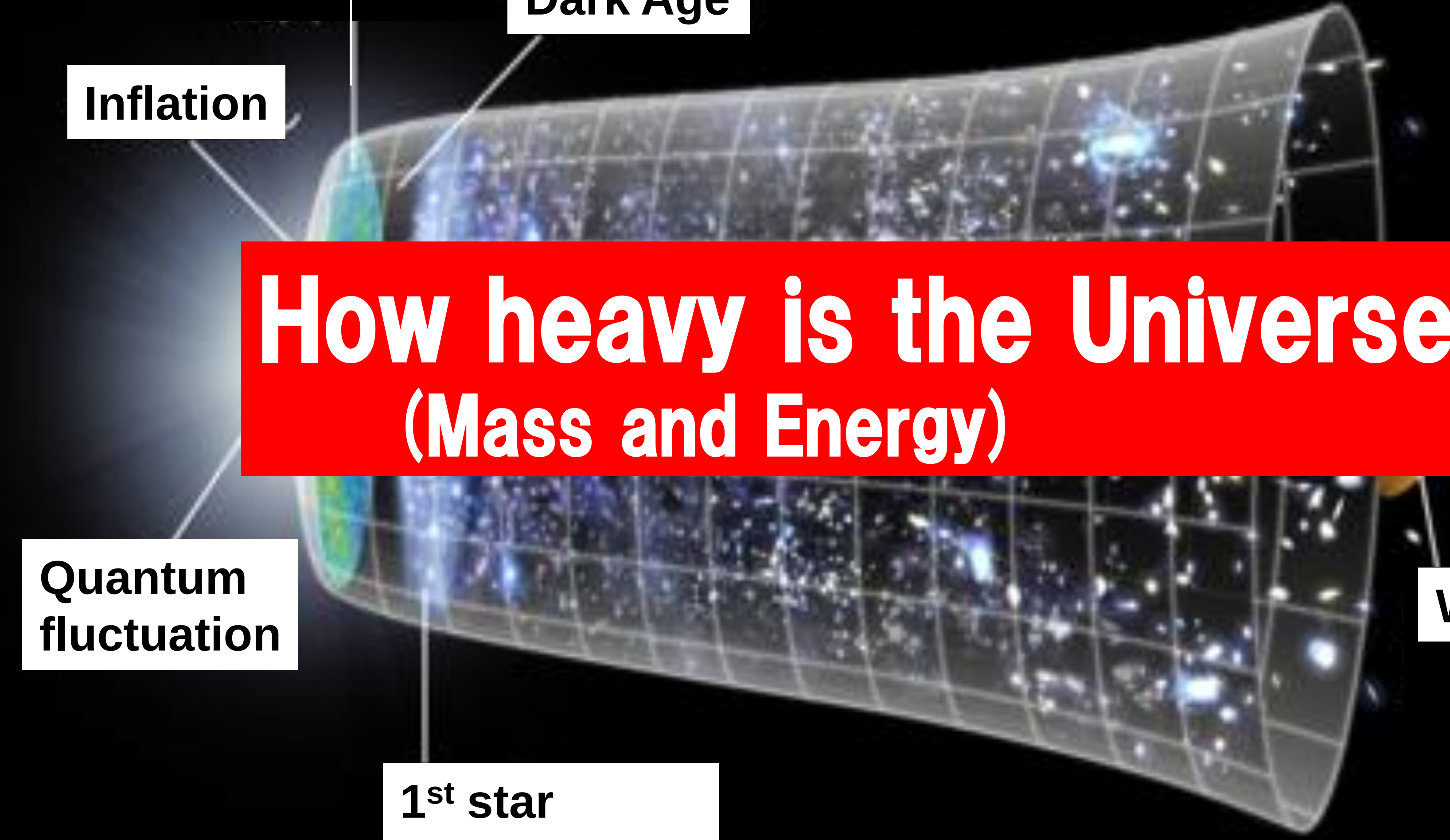
Accelerating

Decelerating

$\Omega_M/2 - \Omega_\Lambda = 0$

How heavy is the Universe?

(Mass and Energy)



Photon last scatter
$4\mathrm{x}10^5$ year

Accelerating expansion
Due to Dark Energy

Dark Age

Inflation

Quantum
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WMAP

1st star
4 million year

Birth of galaxies & stars

Cosmic Microwave Background Anisotropies

Two-point (direction) Correlation Function:

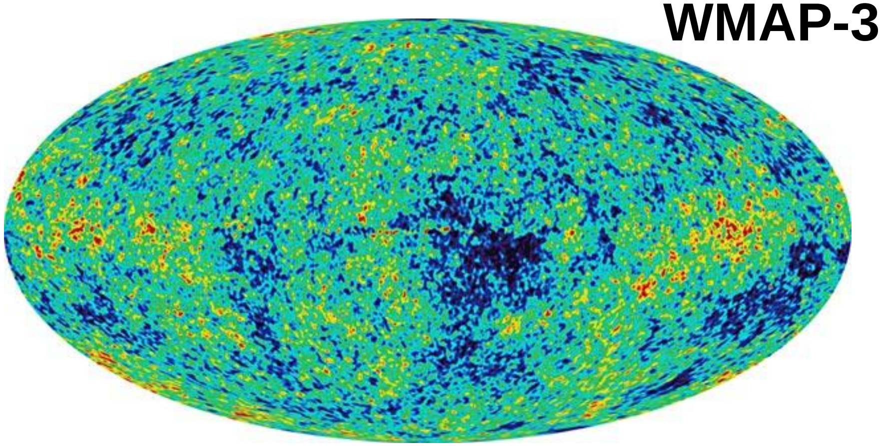
$$\xi = \left< \delta T/T(\mathbf{n}) \cdot \delta T/T(\mathbf{n} + \boldsymbol{\theta}) \right>$$

Temperature Fluctuations, expanded in terms of spherical harmonics:

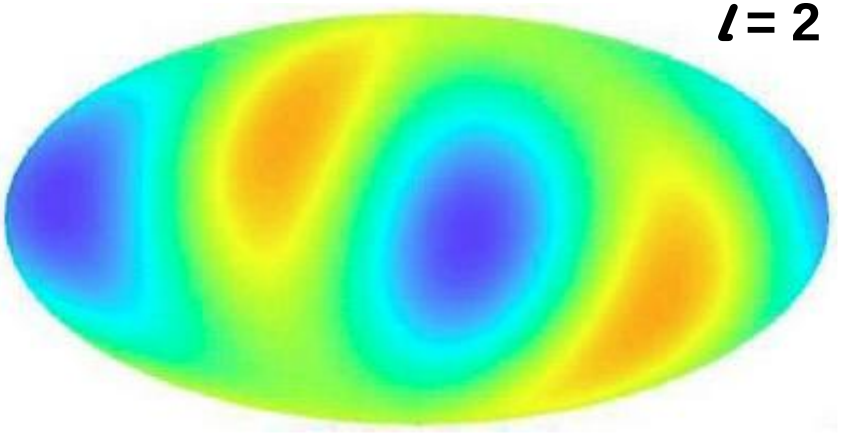
$$\frac{\delta T}{T} = \sum_{l} \sum_{m} a_{lm} Y_{lm}(\theta, \phi)$$

$$C_l \equiv \left\langle \left| a_{lm} \right|^2 \right\rangle$$

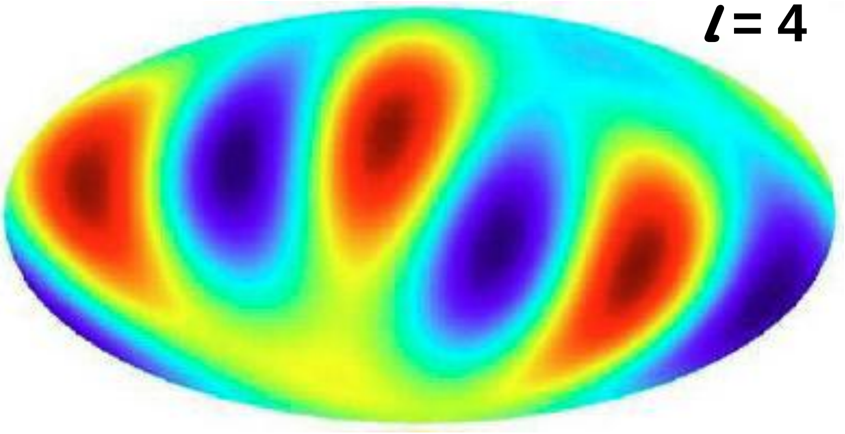
WMAP-3



$l = 2$

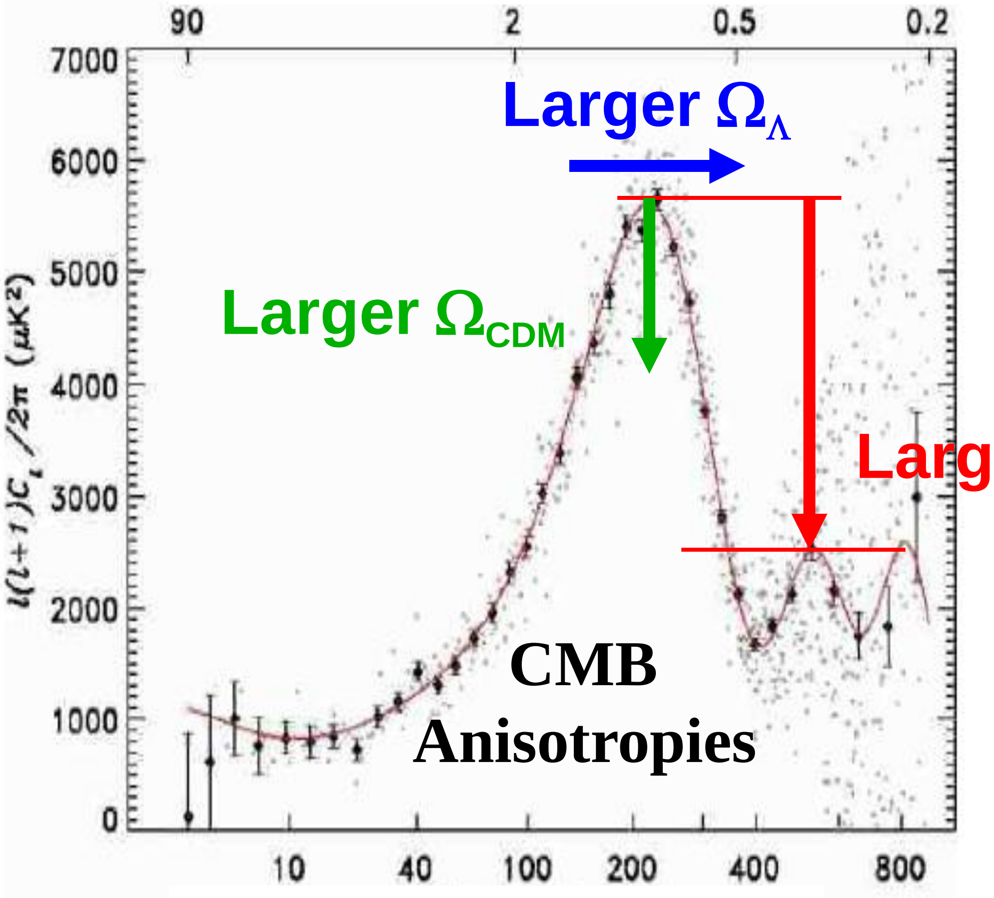


$l = 4$



Cosmological Parameter Dependence

Angle (deg) $\theta \longrightarrow$ smaller



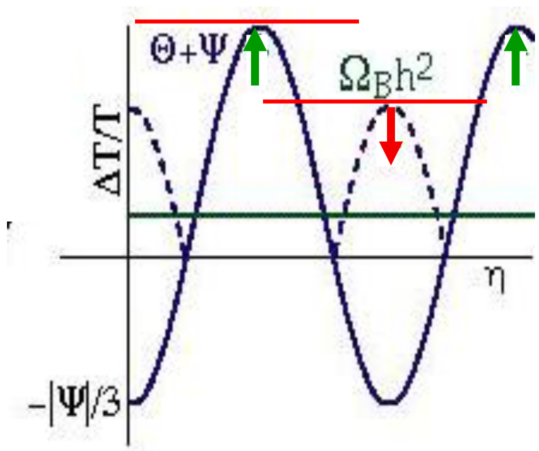
Larger Ω_Λ

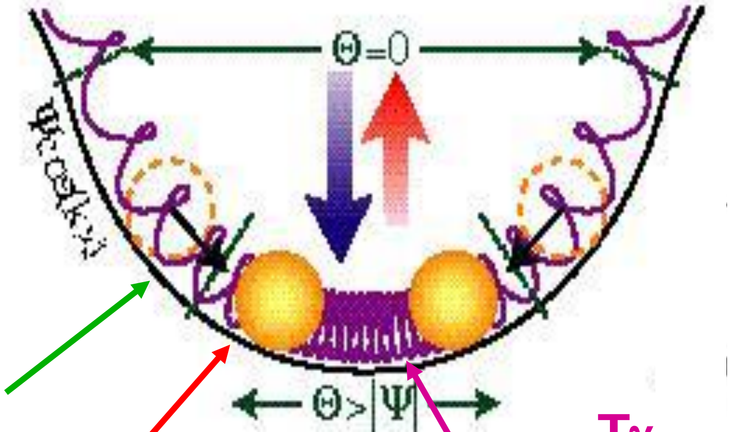
Larger Ω_{CDM}

Larger Ω_B

CMB Anisotropies

Multipole $l \sim \pi/\theta$



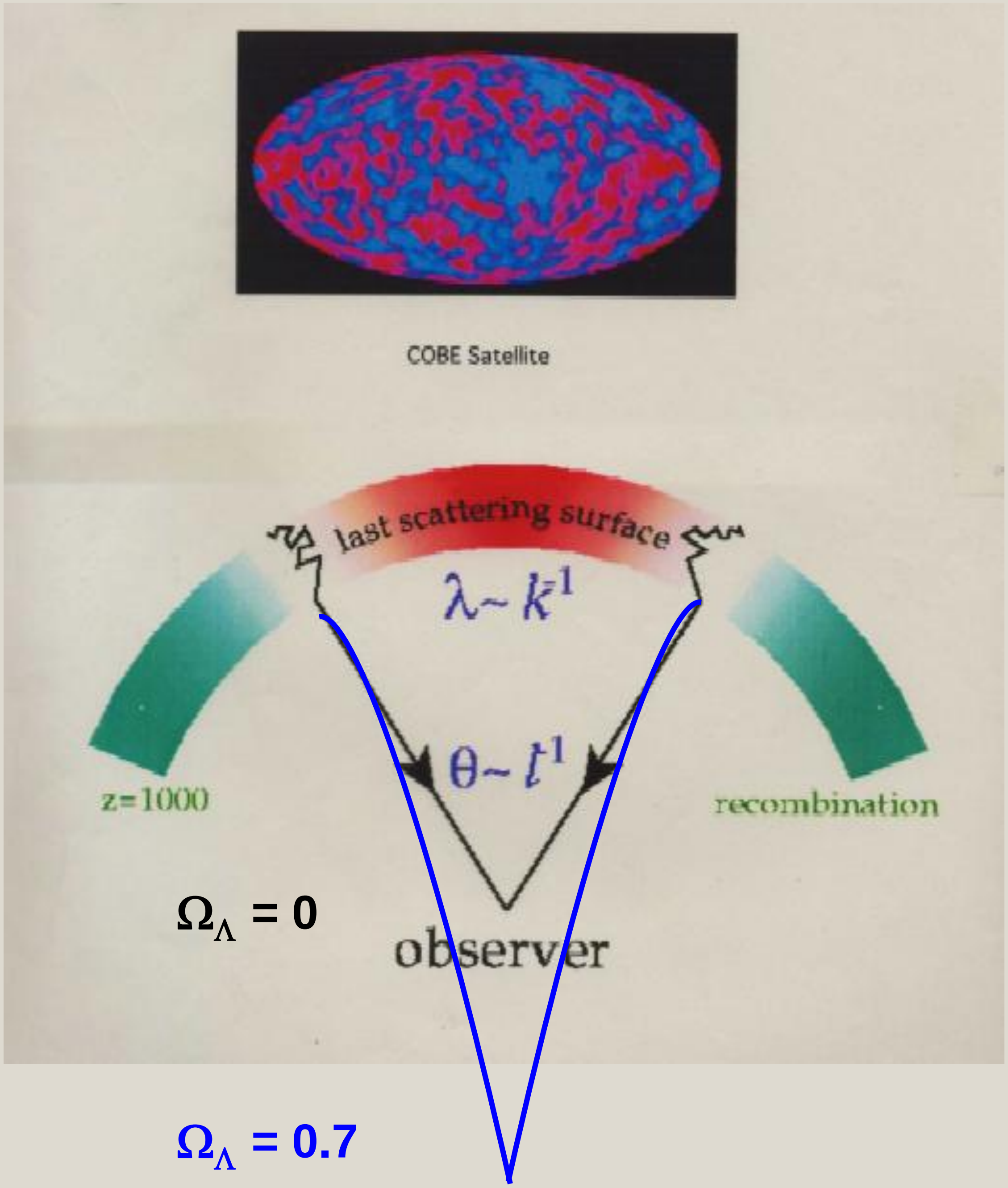


Dark Matter potential Ω_{CDM}

Baryon Mass Ω_B

$T\gamma$

Photon Pressure



$\Omega_\Lambda = 0$

$\Omega_\Lambda = 0.7$

Larger Ω_Λ

↓

Universal expansion becomes faster !

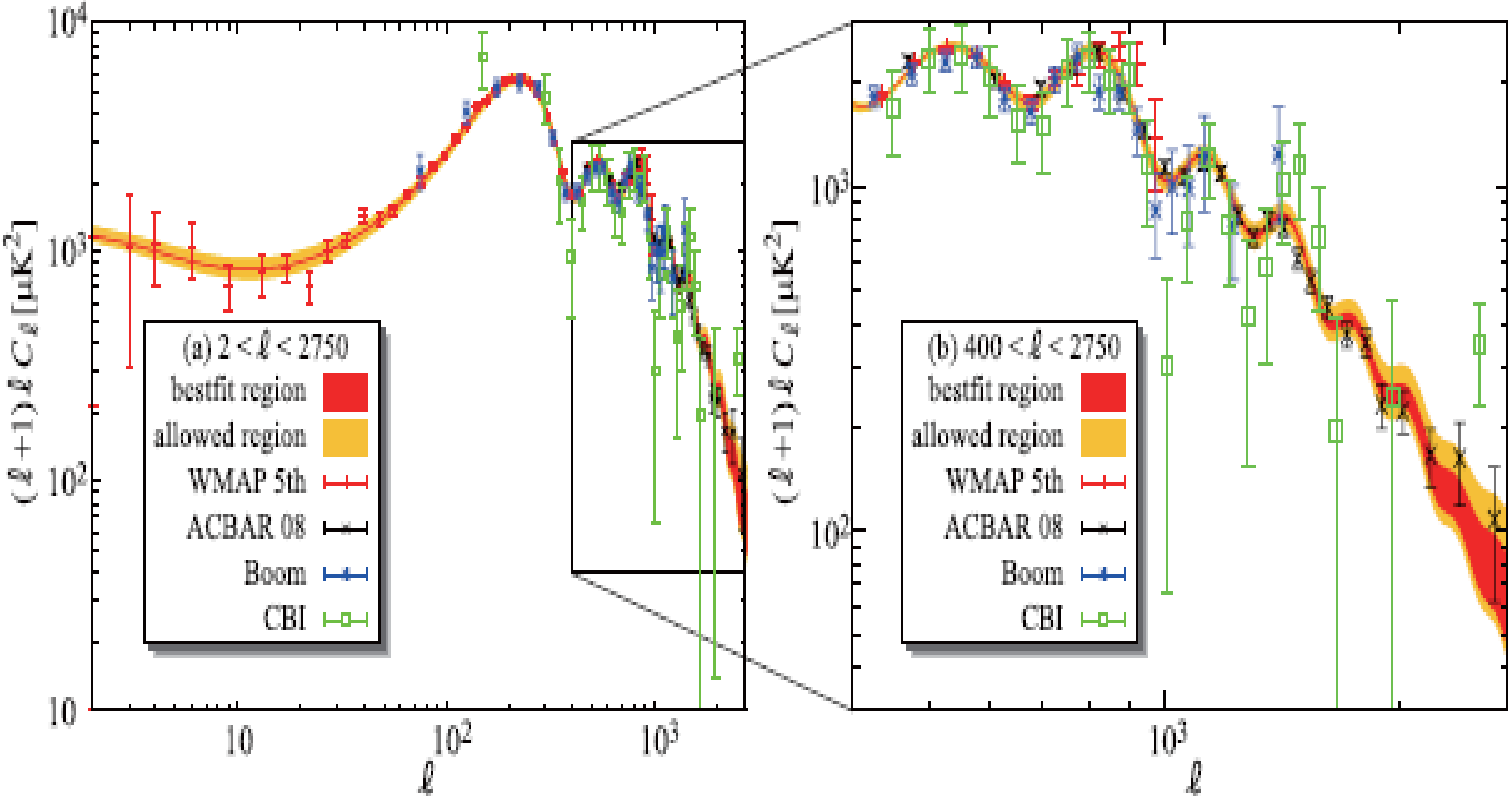
↓

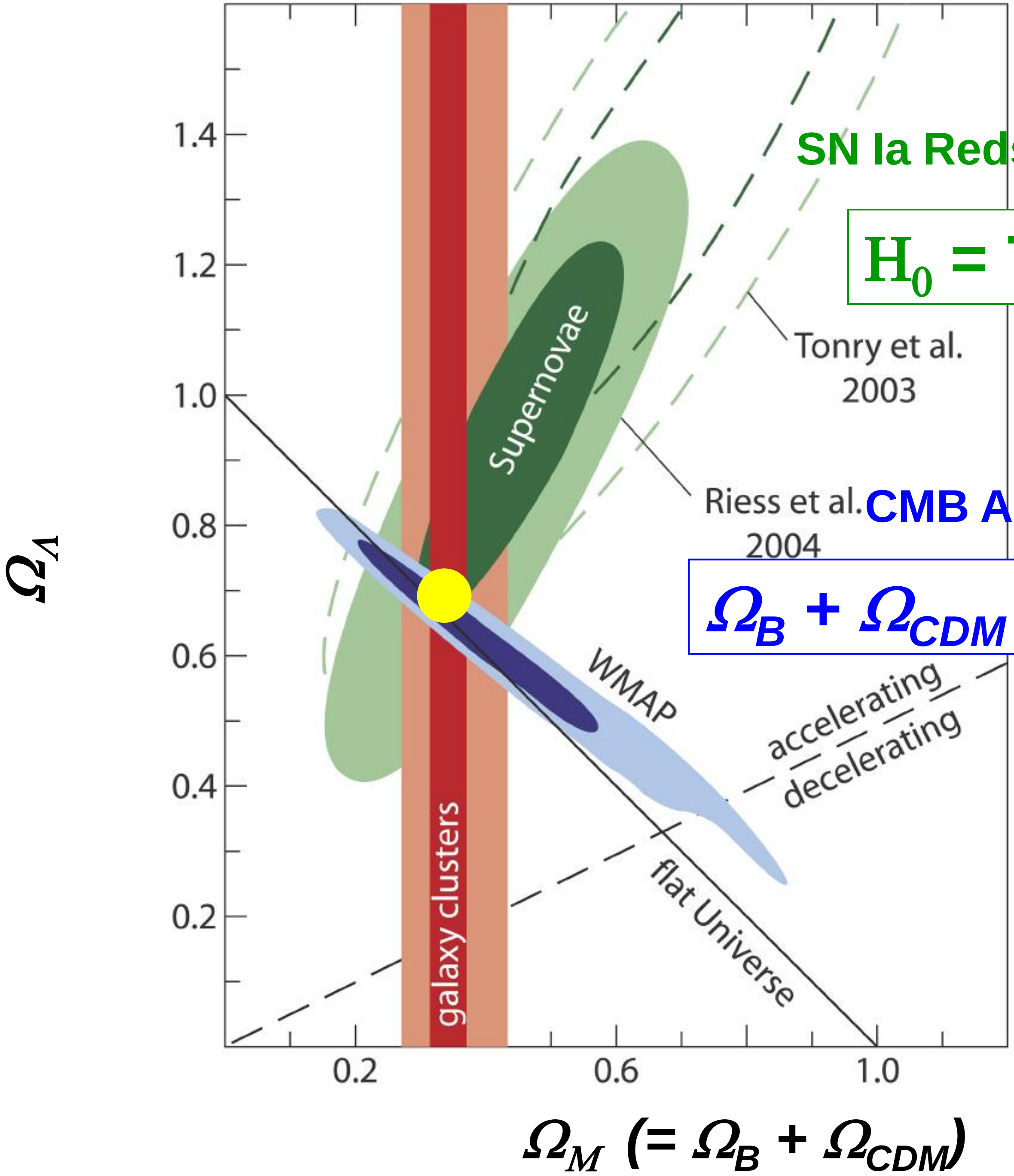
Physical fluctuation length scale $\lambda \sim k^{-1}$ looks smaller in smaller angular scale $\theta \sim l^{-1}$ for observer !

Fit to CMB-Temperature Fluctuation Anisotropies

D. Yamazaki, K. Ichiki, T. Kajino, G. J. Mathews

PR D77, 043005 (2009); PR D81 (2010), 023008; PR D81 (2010), 103519.





SN Ia Redshift-Magnnitude Rel.

$$H_0 = 72 \text{ km/s/Mpc}$$

CMB Anisotropies

$$\Omega_B + \Omega_{CDM} + \Omega_\Lambda = 1$$

Flat Universe:
Because $\Omega_k = 0$

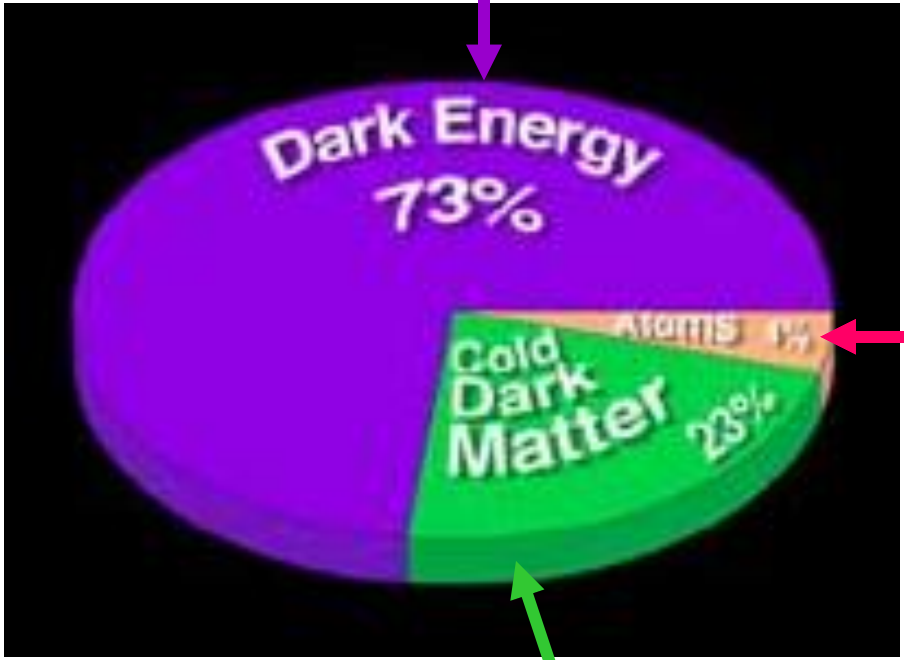
Pie Chart of Cosmic Mystery

$t = 3 \times 10^5$ yr

Ordinary matter makes up a small fraction of mass/energy.

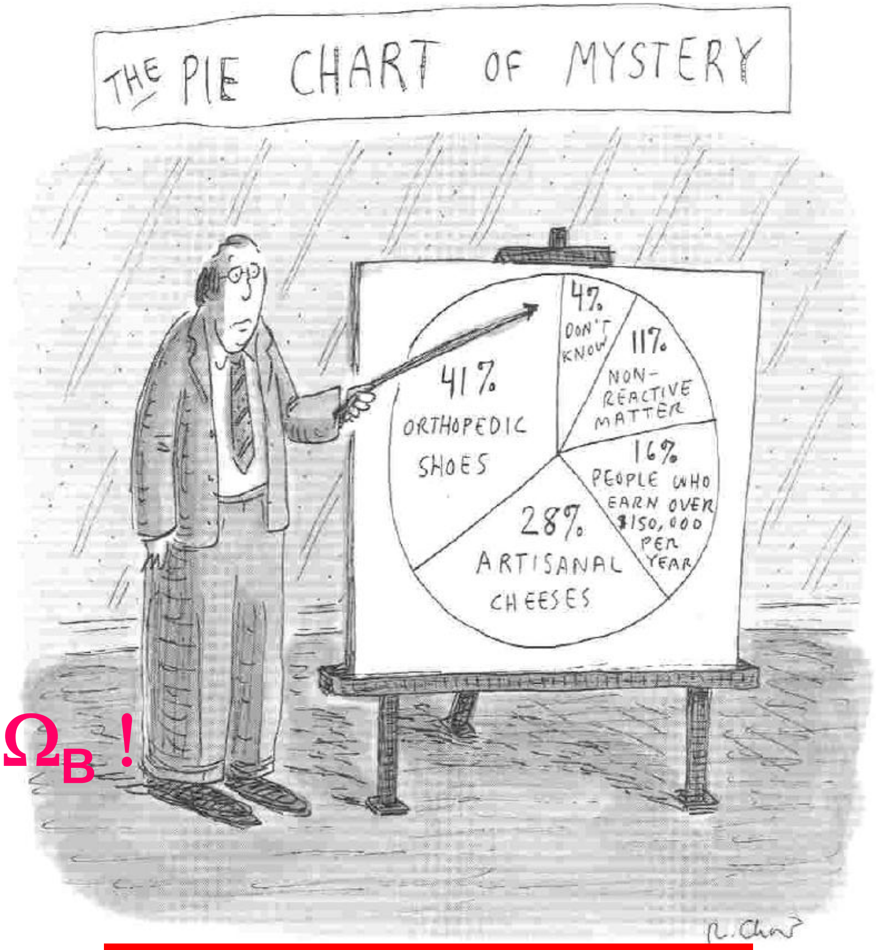
Dark matter and dark energy dominate.

Ω_Λ ???



Ω_B !

Ω_{CDM} ??



Scientists hate “unknown stuff”!

What is Dark Matter?

Photon last scatter
4×10^5 year

Accelerating expansion
Due to Dark Energy

Dark Age

Inflation

Quantum
fluctuation

WMAP

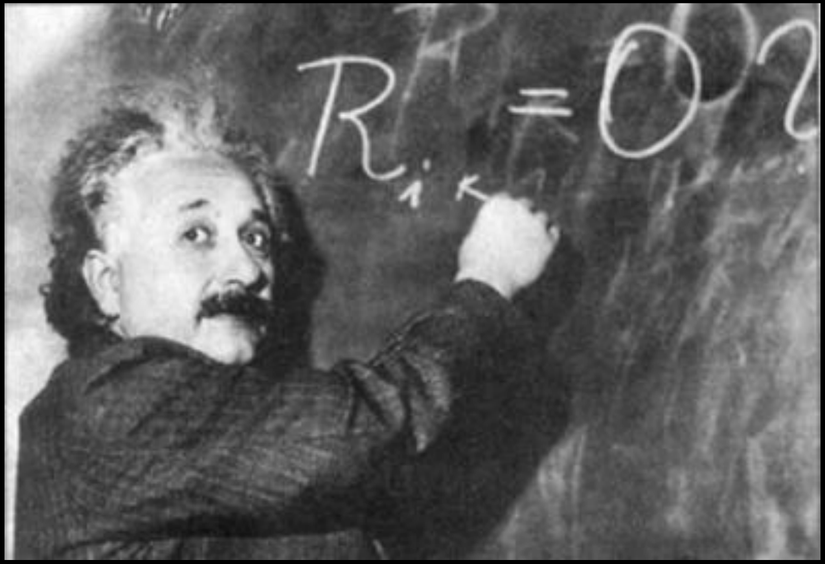
1st star
4 million year

Birth of galaxies & stars

2. Big-Bang Nucleosynthesis

Einstein Cosmology
1915

Big-Bang Nucleosynthesis
1948



Albert Einstein

George Gamow

George Gamow's predictions in 1948

If the Universe began from the Fire Ball of hot Big-Bang and then expanded, then:

1. We can detect today the Cosmic Background Radiation of T = 5 K !

   → 2.7K CBR was discovered by Penzias & Wilson (1965)

   → CMB anisotropies by Smoot & Mathar (1992)

2. In the hot Big-Bang Universe were created almost all atomic nuclides !

   → ^{4}He & ^{7}Li, discovered by astronomer (1980')

Big-Bang Nucleosynthesis !

The Power of BBN is that the Physics is Accessible

Thermodynamic Equilibrium of Particles and Nuclei

$$n_i(p)dp = \frac{1}{2\pi^2} g_i \, p^2 \left[\exp\left(\frac{E_i(p)-\mu_i}{kT}\right) \pm 1\right]^{-1} dp$$

$$\rho_i = \int p \, [n_i(p) + n_{\bar{i}}(p)] \, dp$$

$$\rho_\gamma = \frac{\pi^2}{15}(kT_\gamma)^4 \quad , \quad \rho_{\nu_i} = \frac{7}{8}\frac{\pi^2}{15}(kT_\nu)^4$$

$$\rho = \rho_\gamma + \rho_{\nu_i} + \rho_i = \frac{\pi^2}{30} g_{eff}(kT)^4$$

$$g_{\mathrm{eff}}(T) = \sum_{\mathrm{bose}} g_{\mathrm{bose}} + \frac{7}{8}\sum_{\mathrm{fermi}} g_{\mathrm{fermi}}$$

Cosmic Expansion

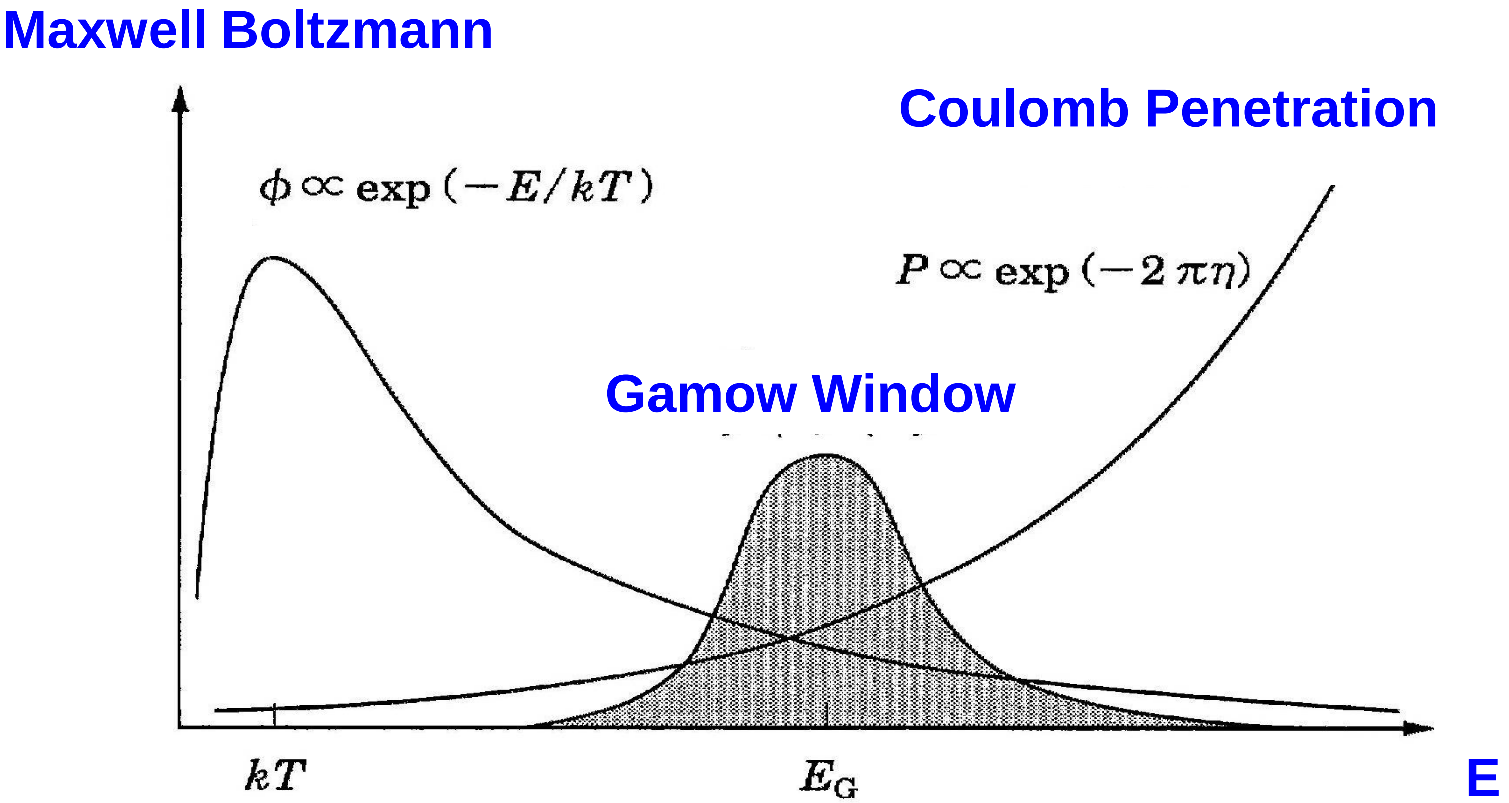
$$H^2(t) = \left(\frac{1}{R}\frac{dR}{dt}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{R^2}$$

Nuclear Reactions

$$\frac{dY_i}{dt} = \sum_{ijk} N_i \left(\frac{Y_l^{N_l} Y_k^{N_k}}{N_l! N_k!} \langle n_k \sigma_{lk} v \rangle - \frac{Y_i^{N_i} Y_j^{N_j}}{N_i! N_k!} \langle n_j \sigma_{ij} v \rangle \right)$$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi \mu} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int S(E) \exp(-2\pi\eta) \exp(-E/kT) dE$$

$$\sigma(E) = \frac{S(E)}{E} \exp(-2\pi\eta)$$



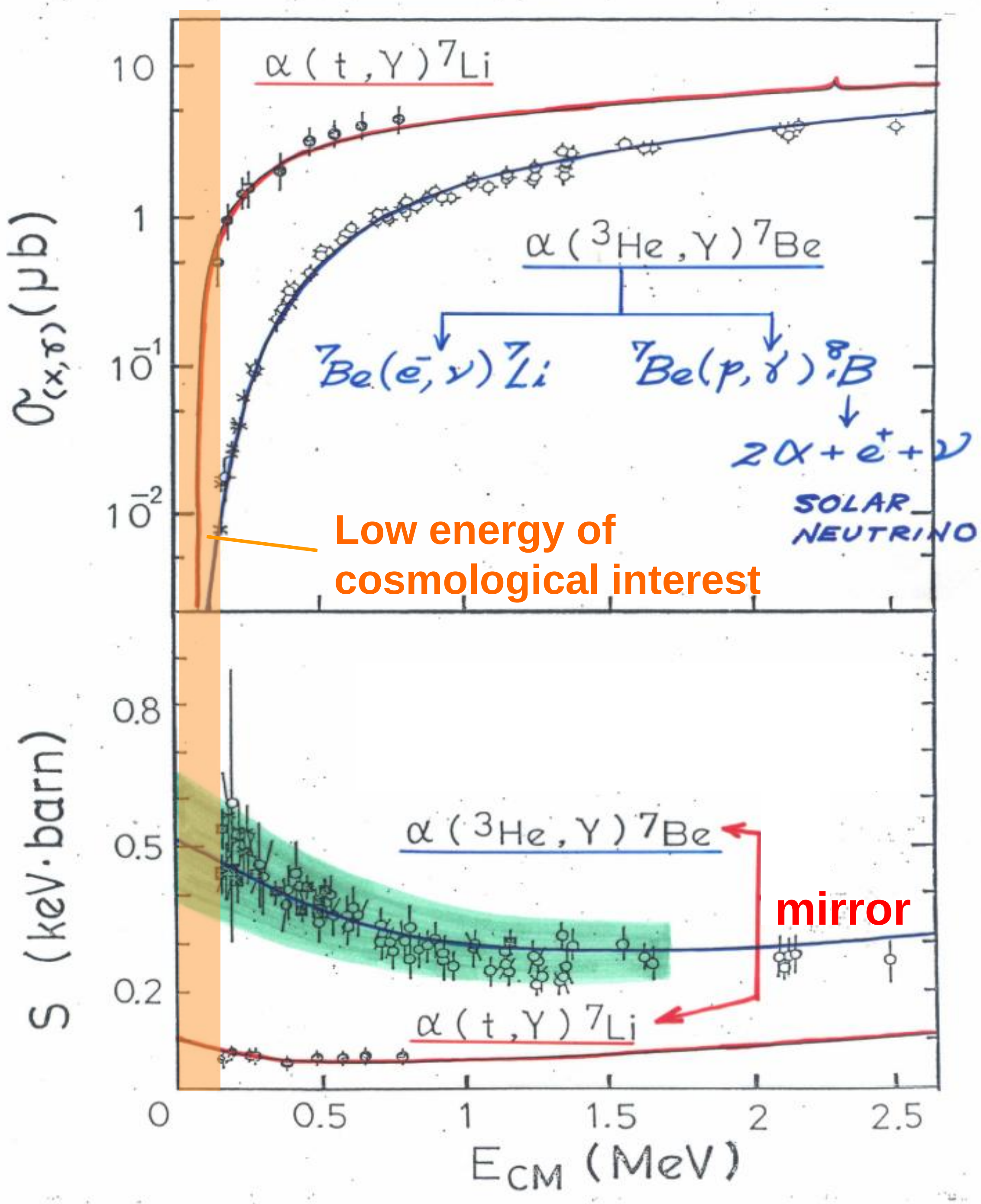
Maxwell Boltzmann

Coulomb Penetration

Gamow Window

E

$$E_G = \left(\frac{\mu}{2} \right)^{1/3} \left(\frac{\pi Z_A Z_a e^2 kT}{\hbar} \right)^{2/3}$$



Low energy of cosmological interest

mirror

Cross section

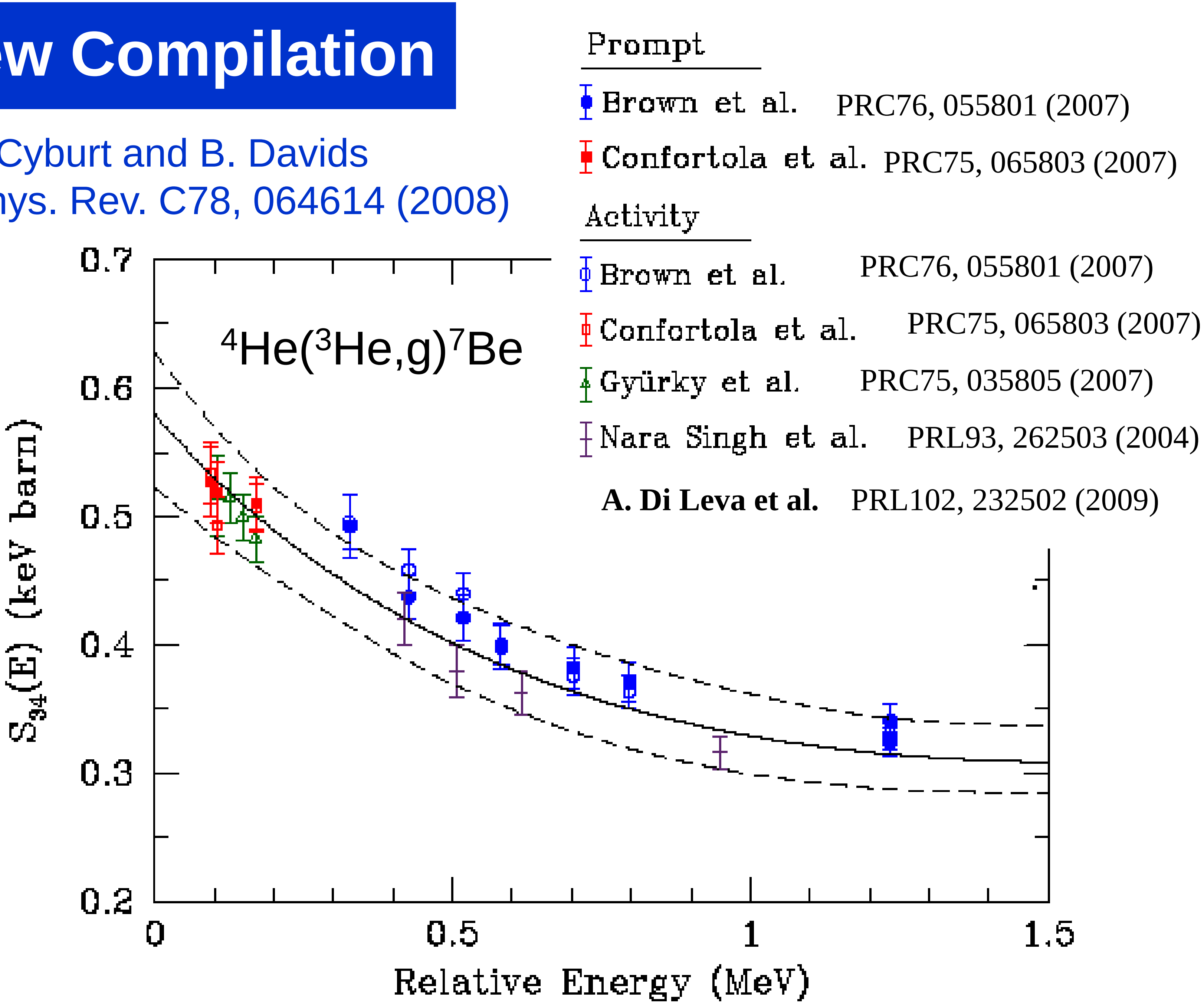
decreases exponentially due to the Coulomb barrier, which makes laboratory experiment extremely difficult at astrophysical low energies!

Astrophysical S-factor

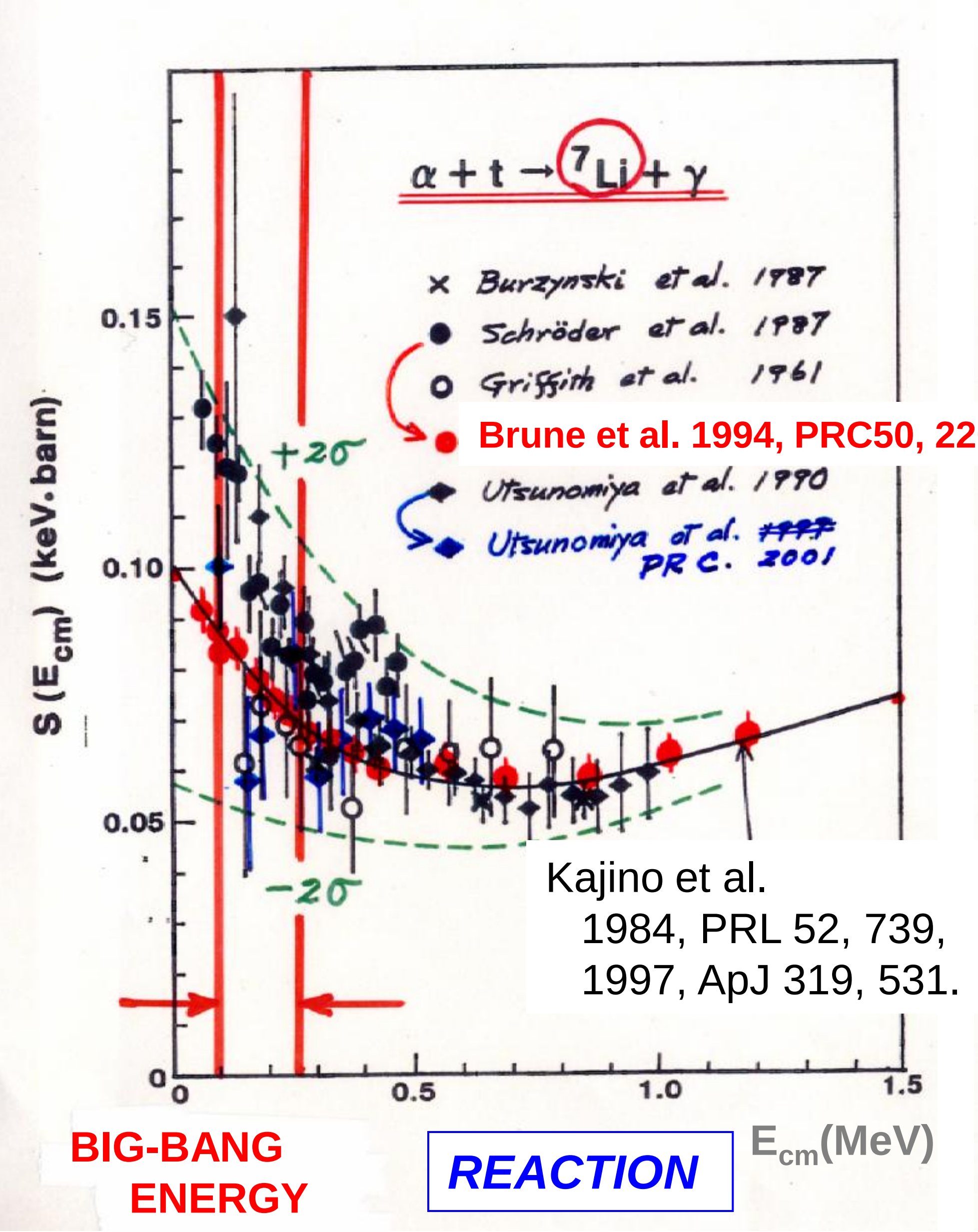
$$S(E) = E\,\sigma(E)\,\exp(2\pi\eta)$$

New Compilation

R.H. Cyburt and B. Davids
Phys. Rev. C78, 064614 (2008)



$\alpha(^3He,\gamma)^7Be$ -- Mirror Conjugate -- $\alpha(^3H,\gamma)^7Li$



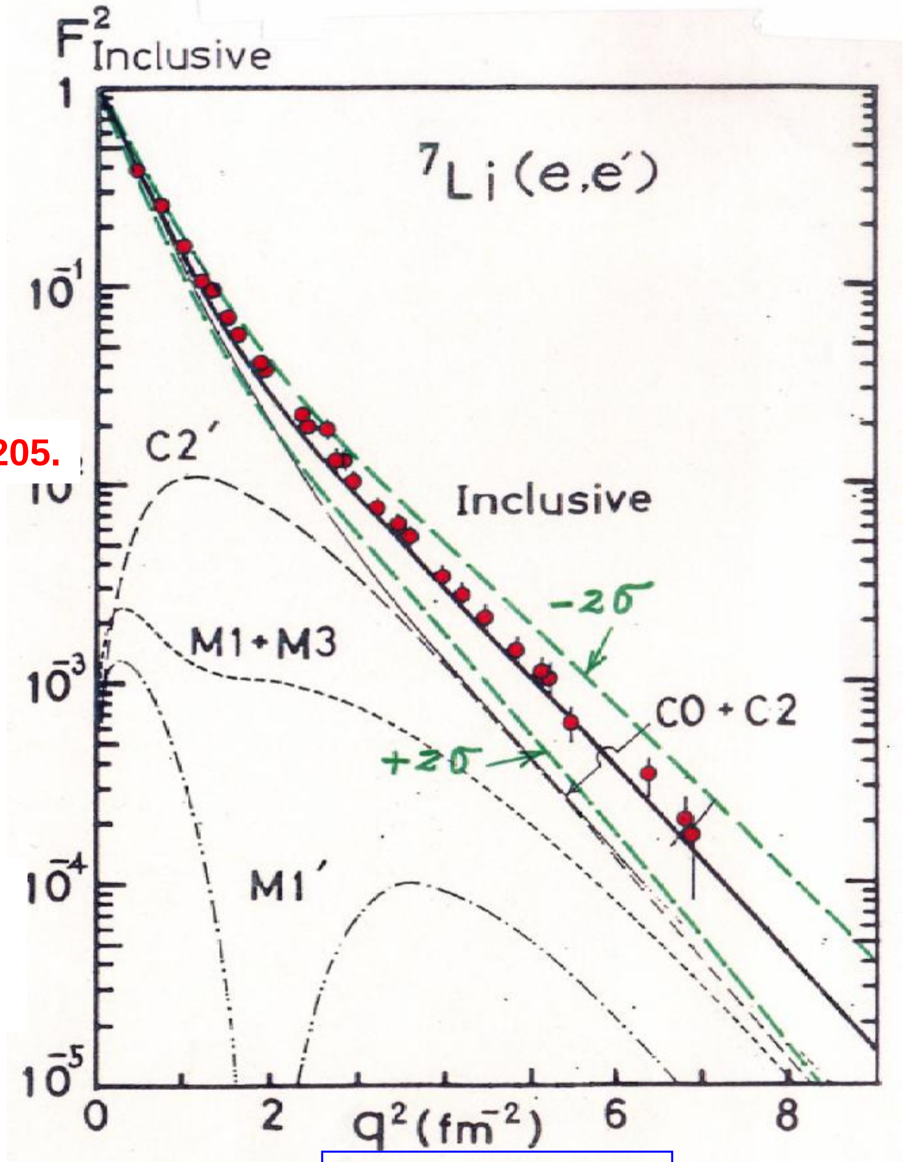
Brune et al. 1994, PRC50, 2205.

Kajino et al.
1984, PRL 52, 739,
1997, ApJ 319, 531.

E_{cm}(MeV)

BIG-BANG
ENERGY

REACTION



STRUCTURE

DIRECT Reaction Cross Section in the limit of E → 0 :

$$\sigma_{fi}(E) = \frac{2\pi}{\hbar}\left|\langle\psi_f|H'|\psi_i(E)\rangle\right|^2\rho_f \text{ — } (E+Q)^3$$

$$\psi_i(\vec{k},\vec{r}) = \frac{1}{\sqrt{v}}\frac{e^{i(\sigma_\ell+\delta_J)}}{kr}\left[F_\ell(k,r)\cos\delta_J + G_\ell(k,r)\sin\delta_J\right]Y_0^{(\ell)}(\hat{r})$$

Hard-Core Scattering Approximation :

$$\psi_i(\vec{k},\vec{r}) \approx \frac{1}{\sqrt{v}}\frac{e^{i(\sigma_\ell+\delta_J)}}{kr}\underset{\substack{\downarrow\\1\\(E\to 0)}}{\cos\delta_J}\left[F_\ell(k,r) - \underbrace{\frac{F_\ell(k,r_c)}{G_\ell(k,r_c)}}_{\tan\delta_J}G_\ell(k,r)\right]Y_0^{(\ell)}(\hat{r})$$

Energy dependence, arises from regular Coulomb function $F_\ell(k,r)$:

WKB approximation for s-wave:

$$F_0(k,r) \approx \frac{\sqrt{kr}}{2(2\eta kr)^{1/4}}\exp(-\pi\eta)\exp\left[2\sqrt{2\eta kr} - \frac{(kr)^2}{3\sqrt{2\eta kr}}\right]$$

E !

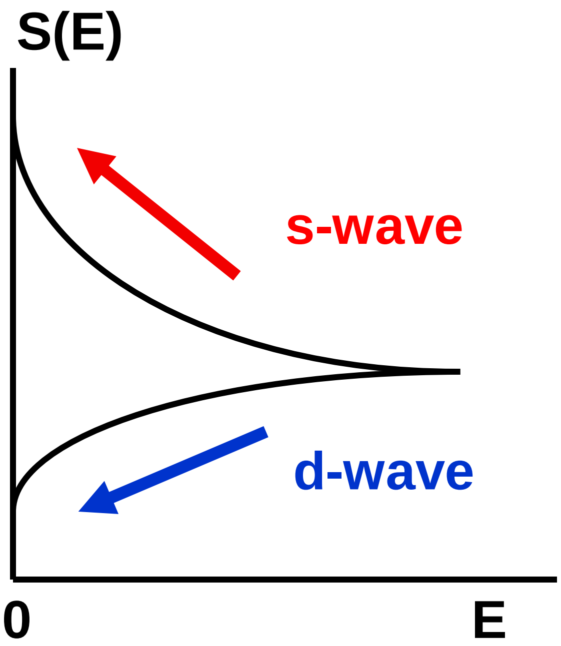
E-indep.

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v}$$

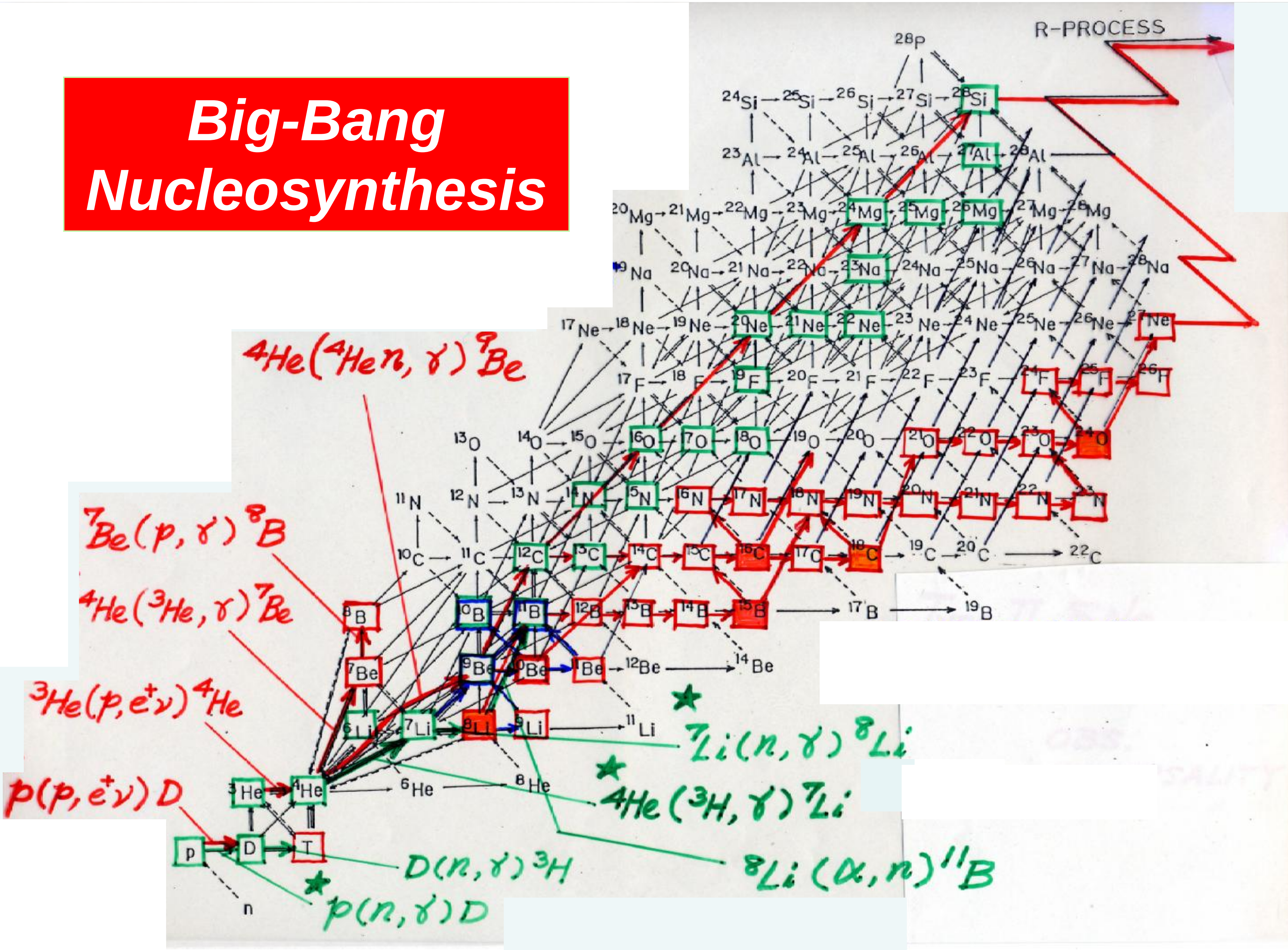
$$\sigma_{fi} \approx \frac{\exp(-2\pi\eta)}{E}\underbrace{\left|\langle\psi_f|H'|\exp\left[-\frac{4\mu r^2}{3\hbar^2\sqrt{2\eta kr}}E\right]\tilde{\psi}_i\rangle\right|^2\rho_f}_{S(E)}$$

$\pi\bar{\lambda}^2{}_{\times}\exp(-2\pi\eta)$

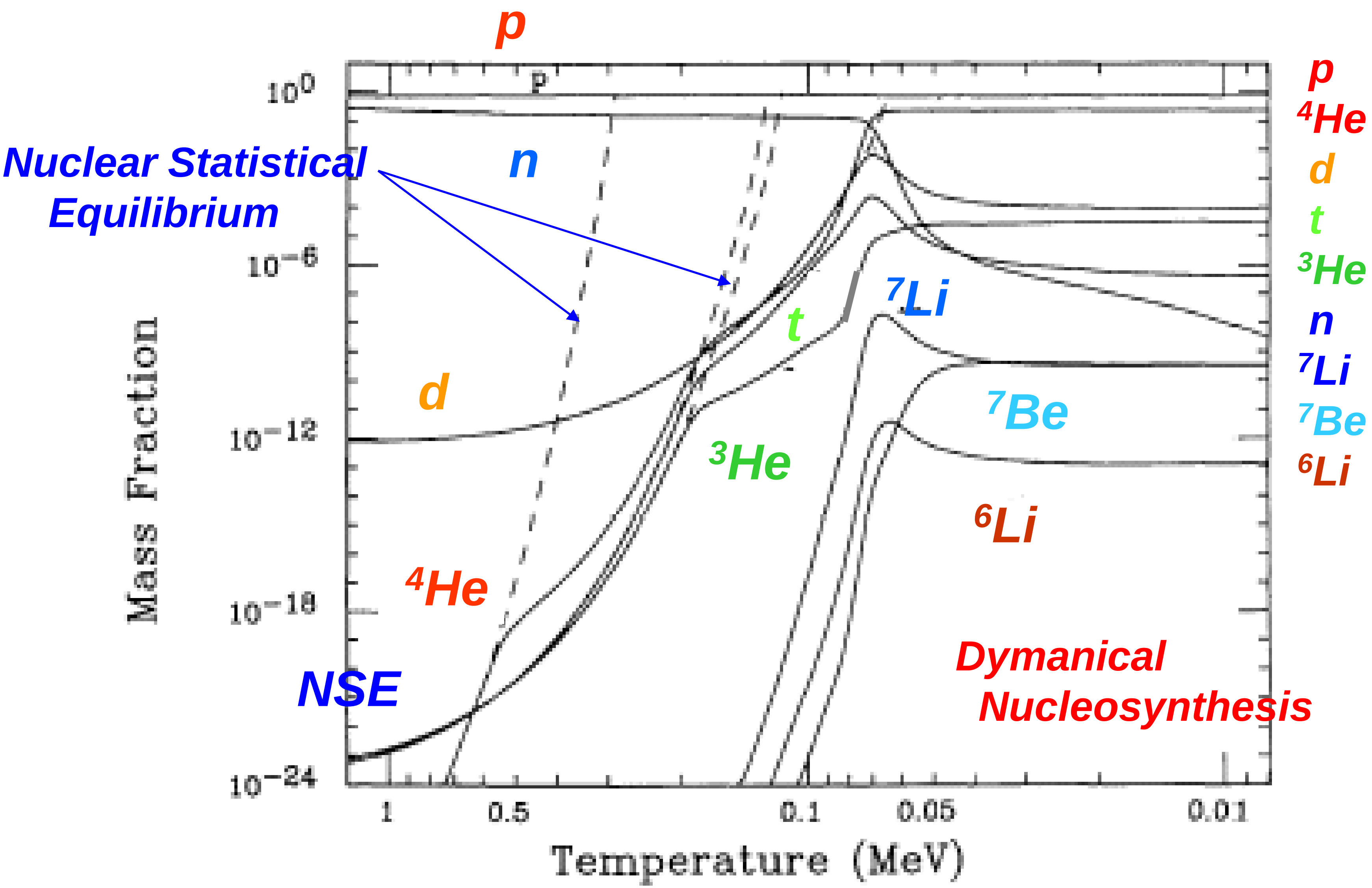
$\bar{\lambda}^2 = 1/k^2 \propto E^{-1}$



Big-Bang Nucleosynthesis



Evolution of Abundances



Nuclear Statistical Equilibrium

Dymanical Nucleosynthesis

Big-Bang Nucleosynthesis
D, ^{3}He, ^{4}He and ^{7}Li

Y_p - Extragalactic HII Regions

0.240 ≤Y_p≤ 0.244

Izotov & Thuan (2003)

0.240 ≤Y_p≤ 0.258

Olive & Skillman (2004)

D - QSO absorption systems

2.4x10^{-5}≤D/H≤ 3.2x10^{-5}

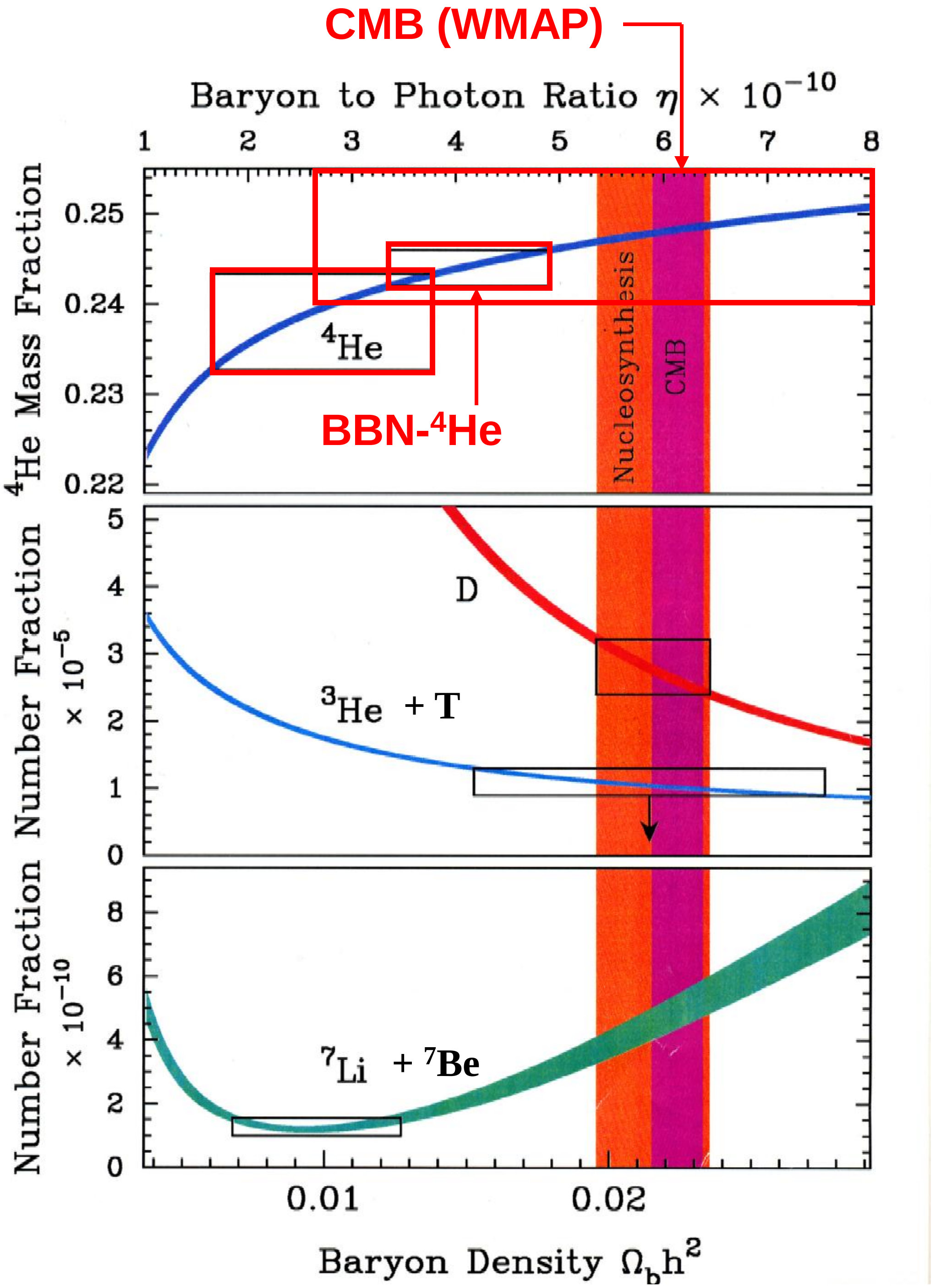
Kirkman et al. (2003)

WMAP

^{7}Li - Halo Stars

0.91x^{-10}≤^{7}Li/H≤ 1.91 x10^{-10}

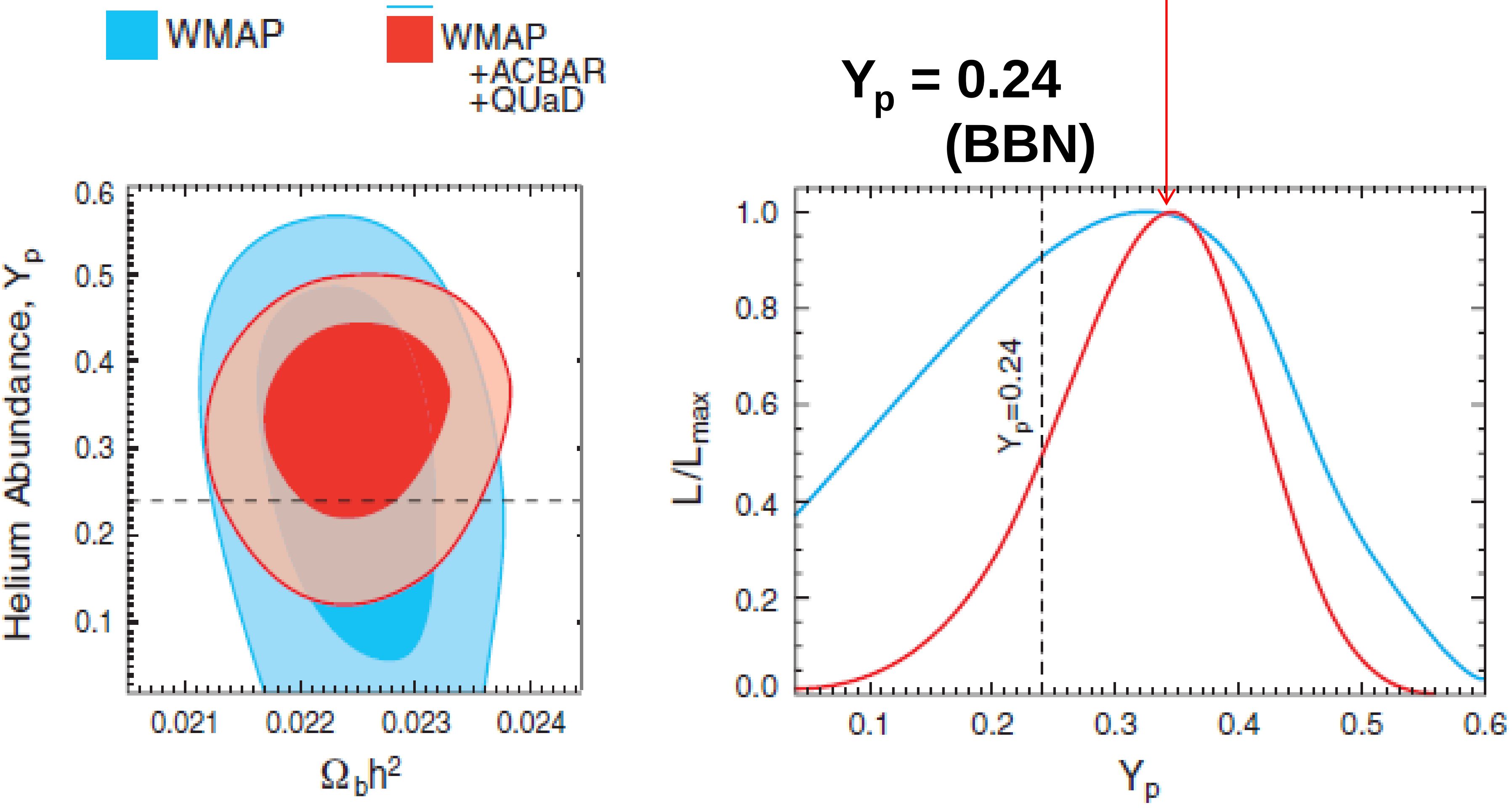
Ryan et al. (2000)



WMAP 7yr data: E. Komatsu et al.,

The Astrophysical Journal Supplement Series, 192:18 (47pp), 2011 February

Combined data — $Y_p = 0.345$

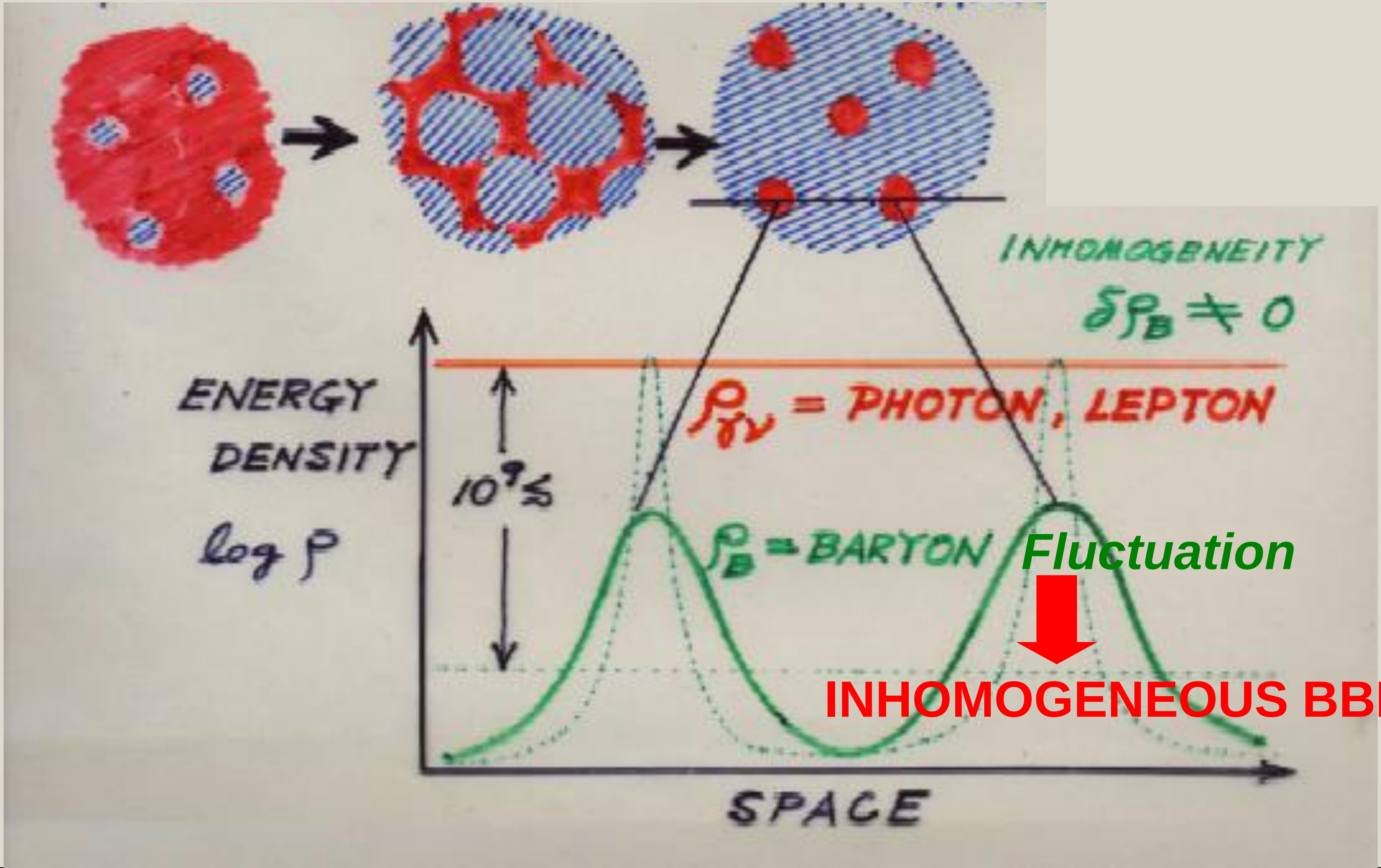


$Y_p = 0.24$
(BBN)

Inhomogeneous Cosmology

due to Cosmic Phase Transition

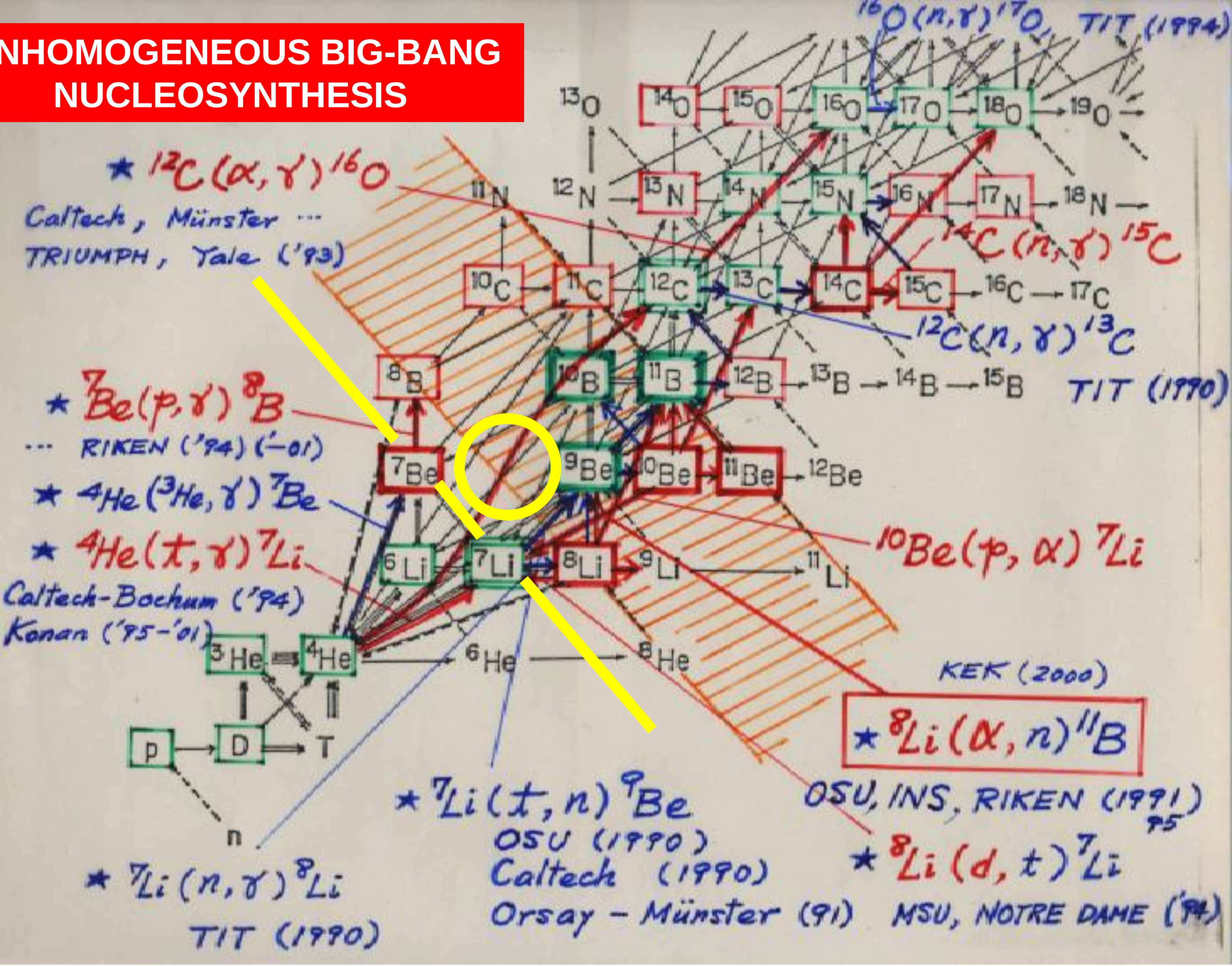
High ρ-T phase **Low ρ-T hadron phase**



Fluctuation

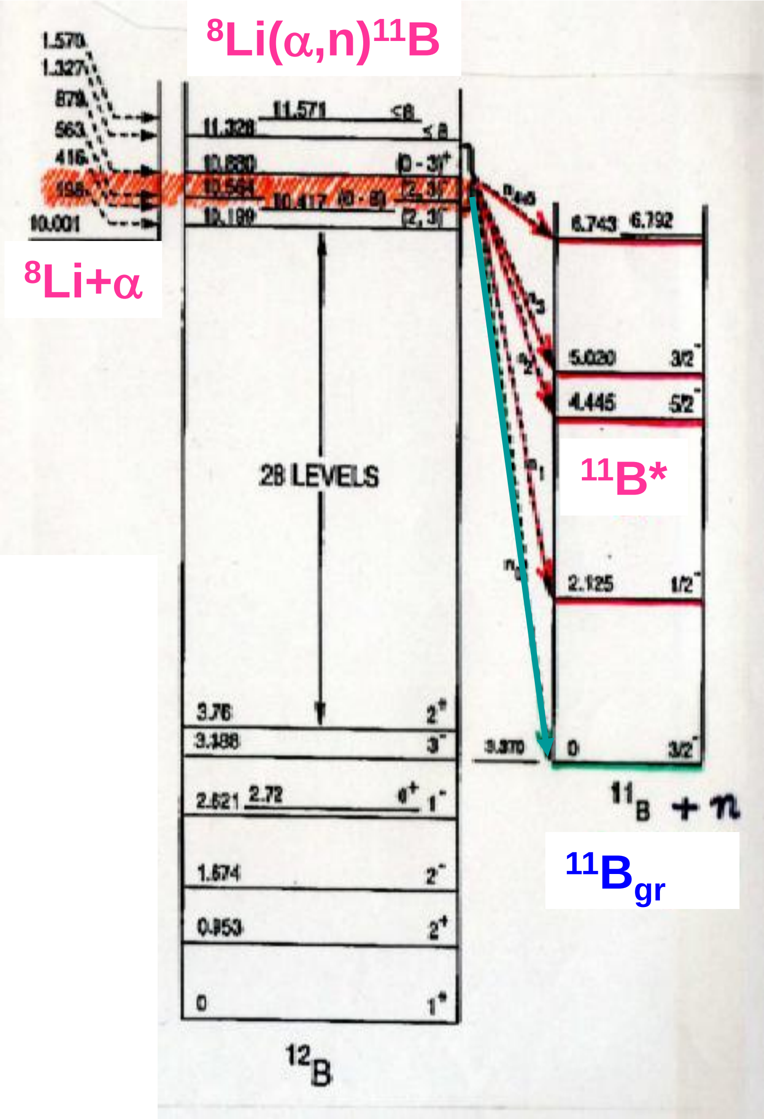
INHOMOGENEOUS BBN

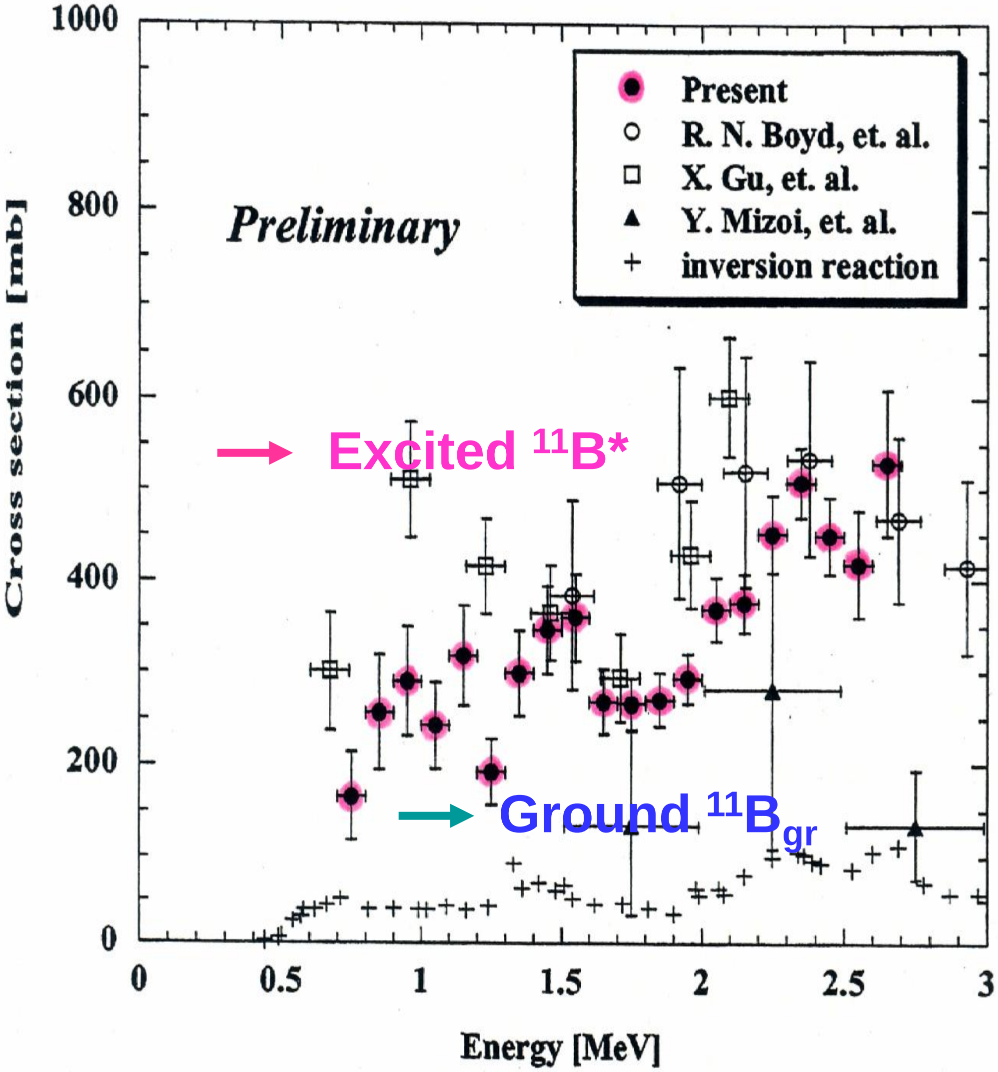
INHOMOGENEOUS BIG-BANG NUCLEOSYNTHESIS



$^{8}Li(\alpha,n)^{11}B$

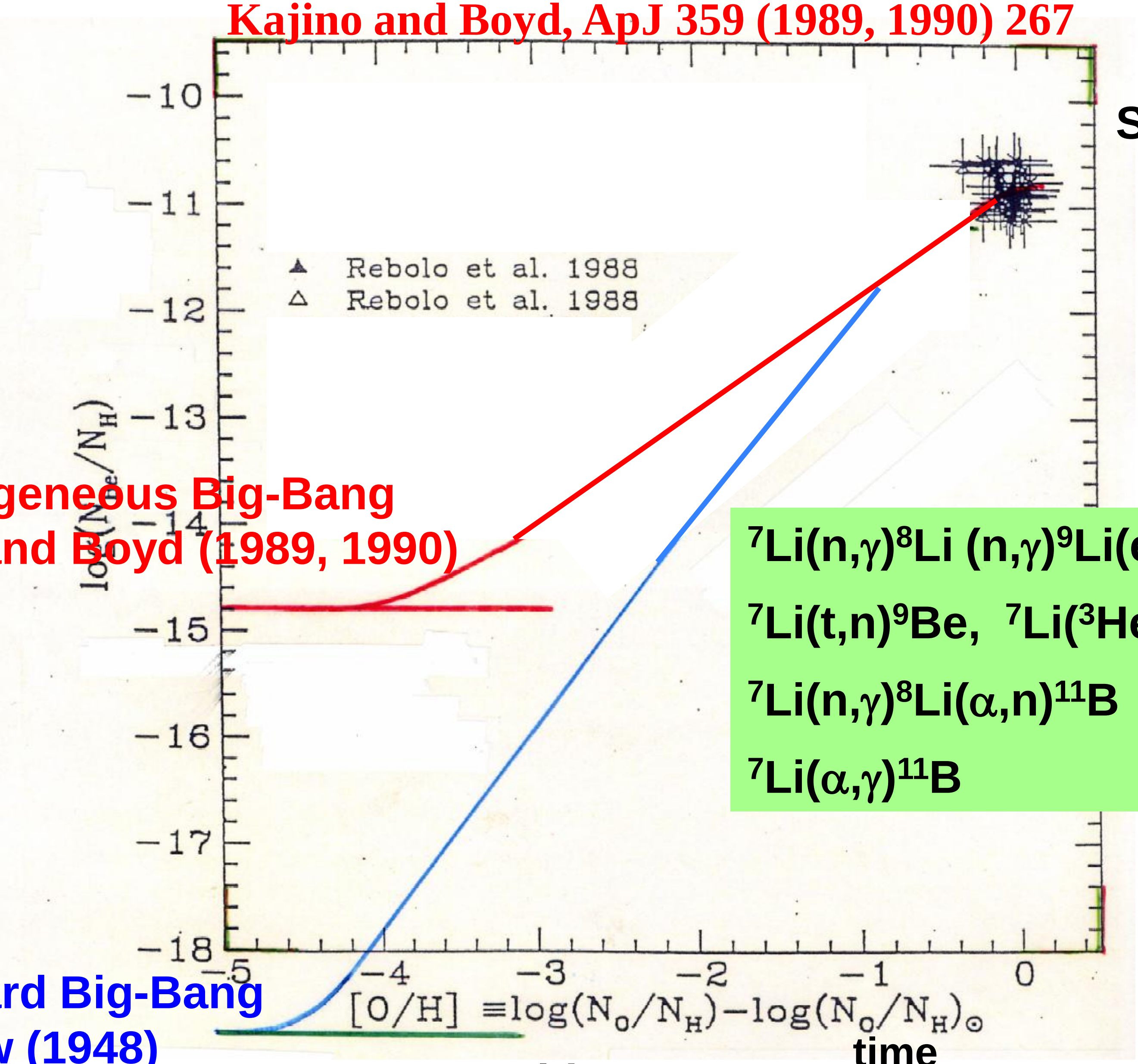
H. Ishiyama et al. AIP Conf. Proc. 704 (2004) 453.
T. Hashimoto et al. Phys. Lett. B 674 (2009) 276.





INHOMOGENEOUS BIG-BANG NUCLEOSYNTHESIS

Kajino and Boyd, ApJ 359 (1989, 1990) 267



$\frac{{}^{9}\mathrm{Be}}{\mathrm{H}}$

Solar Neighbor

Inhomogeneous Big-Bang
Kajino and Boyd (1989, 1990)

Standard Big-Bang
Gamow (1948)

$^{7}Li(n,\gamma)^{8}Li\ (n,\gamma)^{9}Li(e^{-}\nu)^{9}Be$,

$^{7}Li(t,n)^{9}Be$, $^{7}Li(^{3}He,p)^{9}Be$

$^{7}Li(n,\gamma)^{8}Li(\alpha,n)^{11}B$

$^{7}Li(\alpha,\gamma)^{11}B$

Old stars **time** → **Young stars**

Big-Bang Nucleosynthesis D, ^{3}He, ^{4}He and ^{7}Li

Y_p - Extragalactic HII Regions

$0.240 \leq Y_p \leq 0.244$

Izotov & Thuan (2003)

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Olive & Skillman (2004)

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$2.4\text{x}10^{-5} \leq \text{D/H} \leq 3.2\text{x}10^{-5}$

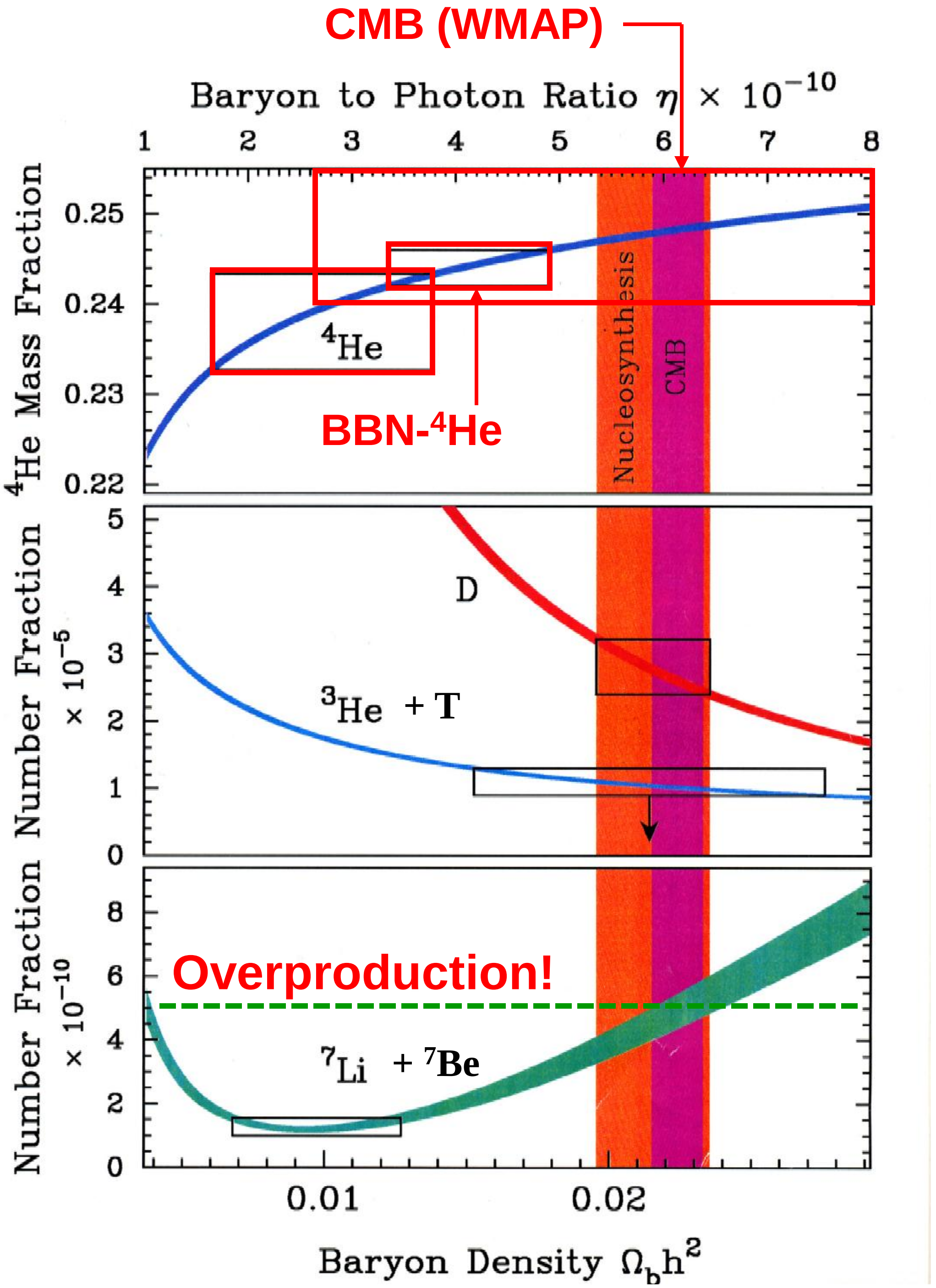
Kirkman et al. (2003)

WMAP

^{7}Li – Halo Stars

$0.9\text{x}10^{-10} \leq {}^7\text{Li/H} \leq 1.91\ \text{x}10^{-10}$

Ryan et al. (2000 –)



CMB (WMAP)

BBN-^{4}He

Overproduction!

What is Dark Matter?

Photon last scatter
4×10^5 year

Accelerating expansion
Due to Dark Energy

Dark Age

Inflation

Quantum
fluctuation

WMAP

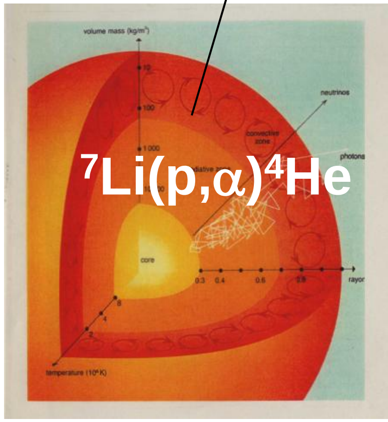
1st star
4 million year

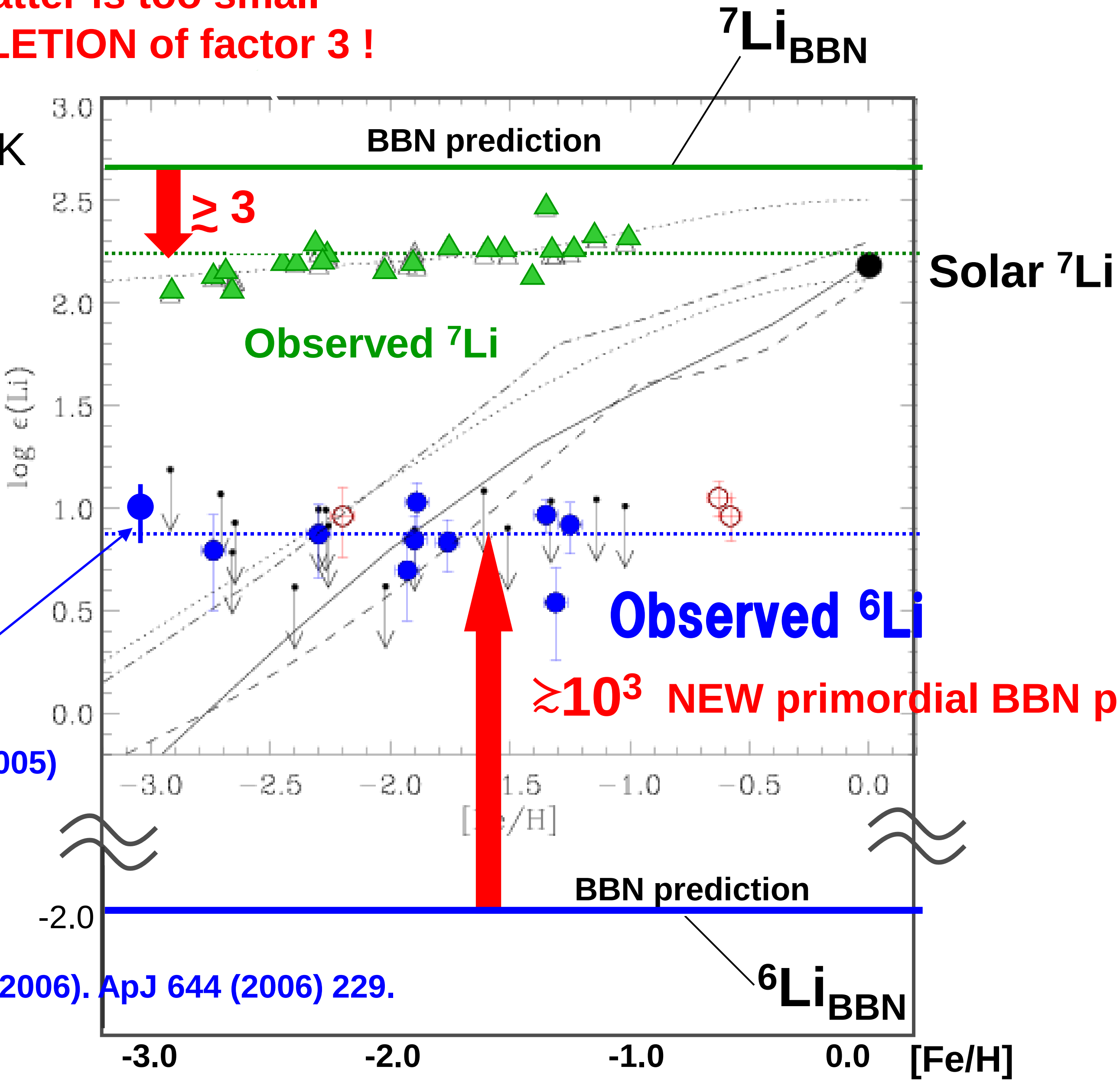
Birth of galaxies & stars

Plateau like HIGH ^{6,7}Li ABUNDANCE --- primordial ?

Abundance scatter is too small
to accept DEPLETION of factor 3 !

T = 1~3x10^6 K





≳10^3 NEW primordial BBN process ?

SUBARU OBS.
Inoue, Aoki et al (2005)

Asplund et al. (2006). ApJ 644 (2006) 229.

How to solve HIGH ^{6}Li primordial abundance?

Ellis et al. (1986); Moroi and Kawasaki (1994); Jedamzik PRL 84 (2000) 3248;
Cyburt et al., PRD 67 (2003) 103521; Ellis et al. PLB619 (2005) 30;
Kusakabe, Kajino & Mathews, D74 (2006), 023526, PR D76 (2007) 121302(R);
ApJ 680 (2008) 846

Cosmological Solution

DM = gravitino

1st possibility: SUSY Leptonic "stau" (NLS-particles) :

X^{+-} are bound to ^{4}He, ^{7}Li, ^{7}Be and catalize new BBN:

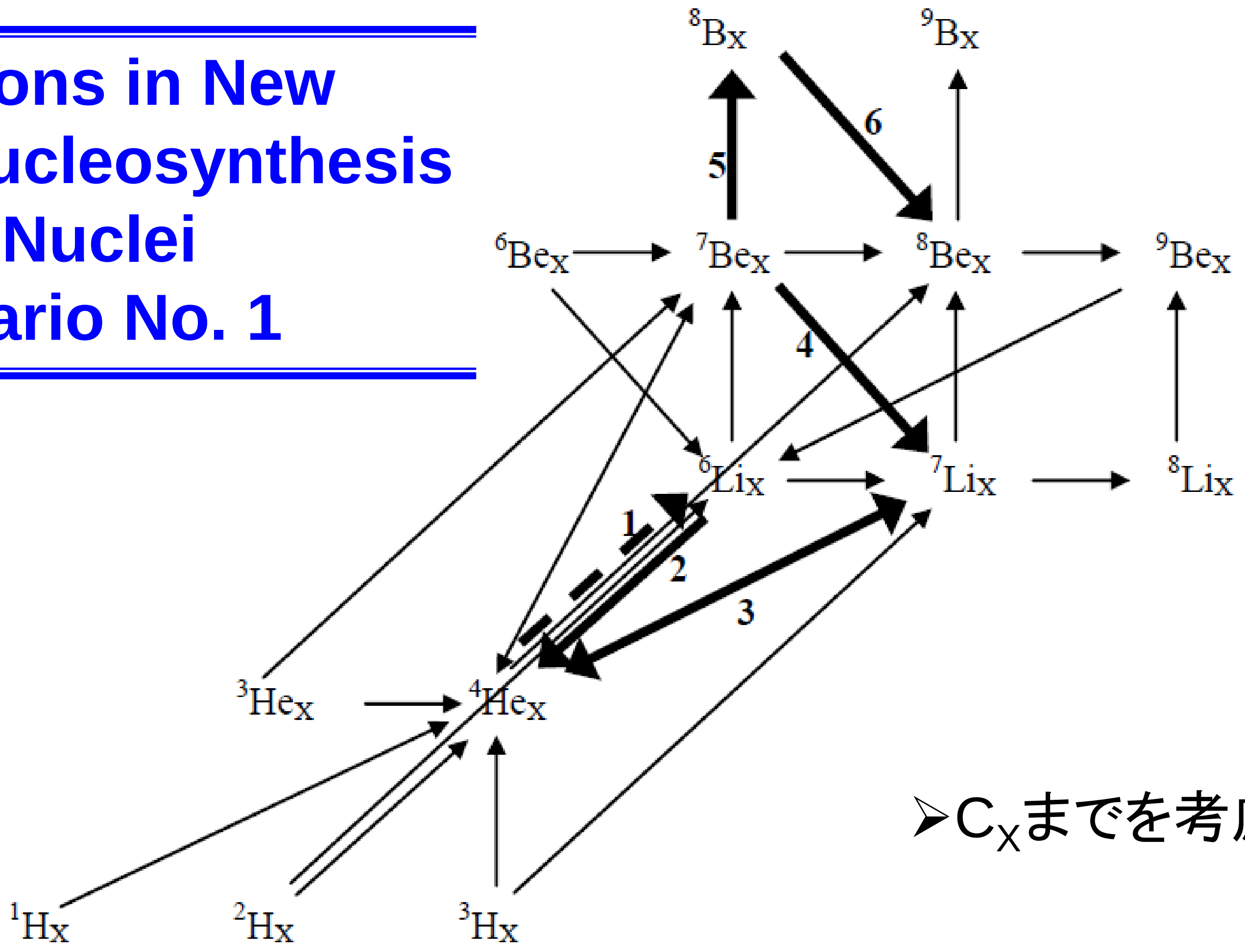
$$^{4}He_{X}(d,X^{-})^{6}Li, \quad ^{7}Be_{X}(p,\gamma)^{8}B_{X}$$

2nd possibility: Decaying relic DM particles : X decay to non-th γ's:

$$^{4}He(\gamma_{NT},n)^{3}He(^{4}He,p)^{6}Li,$$

$$^{4}He(\gamma_{NT},p)^{3}H(^{4}He,n)^{6}Li.$$

Main Reactions in New Big-Bang Nucleosynthesis including X-Nuclei
Scenario No. 1



➢C_Xまでを考慮

Main reactions

1. $^4He_X(d,X^-)^6Li$
2. $^6Li_X(p,^3He\ \alpha)X^-$
3. $^4He_X(t,\gamma)^7Li_X$ & $^7Li_X(p,2\alpha)X^-$
4. $^7Be_X(,X^0)^7Li$
5. $^7Be_X(p,\gamma)^8B_X$
6. $^8B_X(,e^+\nu_e)^8Be_X$

✓X^-再結合: 16
✓X核反応: 59
（含β崩壊: 2）
✓X^-荷電移行: 3
✓X-decay: 11

Scenario No. 2:Theoretical of X decay: X → γ_{NT}

Ellis et al. (1986); Moroi and Kawasaki (1994);Jedamzik PRL 84 (2000) 3248; Cyburt et al., PRD 67 (2003) 103521; Ellis et al. PLB619 (2005) 30; Kusakabe, Kajino & Mathews, D74 (2006), 023526.

Spectrum of non-thermal γ_{NT} $p_\gamma(E_\gamma)$

Primary γ_{NT} interacts with CBRs

- Pair creation ($\gamma\gamma_{bg} \to e^+e^-$)
- Inverse Compton ($e^\pm + \gamma_{bg} \to e^\pm + \gamma$)

Then it degrades its energy by:

- Compton scattering ($\gamma + e^\pm_{bg} \to \gamma + e^\pm$)
- Bethe–Heitler process (γ+nuclus$_{bg} \to e^+ + e^-$+nucleus)
- Photon–photon scattering ($\gamma\gamma_{bg} \to \gamma\gamma$)

Two Parameters

Life time of X τ_X

Number density ∗ Eγ of X

$$\zeta_X = \frac{n_X^0}{n_\gamma^0} E_{\gamma 0}$$

Reaction process

Rate equation

$$\frac{dY_A}{dt} = \sum_P N_A(P)\left(-\frac{Y_A}{N_A(P)!}[A\gamma]_P + \frac{Y_P}{N_P(P)!}[P\gamma]_A\right) \quad + \text{SBBN}$$

$$[A\gamma]_P \equiv \frac{n_\gamma^0 \zeta_X}{\tau_X}\left(\frac{1}{2H_r t}\right)^{3/2} \exp(-t/\tau_X)\int_0^\infty \left(\frac{\tau_X}{E_{\gamma 0} n_X} N_\gamma^{QSE}(E_\gamma)\right)\sigma_{\gamma+A\to P}(E_\gamma)$$

Photon # density

$$N_\gamma^{QSE}(E_\gamma) = \frac{n_X\, p_\gamma(E_\gamma)}{\Gamma_\gamma(E_\gamma)\tau_X} \qquad H_r = \sqrt{\frac{8\pi G \rho_{rad}^0}{3}}$$

Cosmological Solution

#2: Decaying relic DM X

$^4He(\gamma_{NT},n)^3He(^4He,p)^6Li$

$^4He(\gamma_{NT},p)^3H(^4He,n)^6Li$

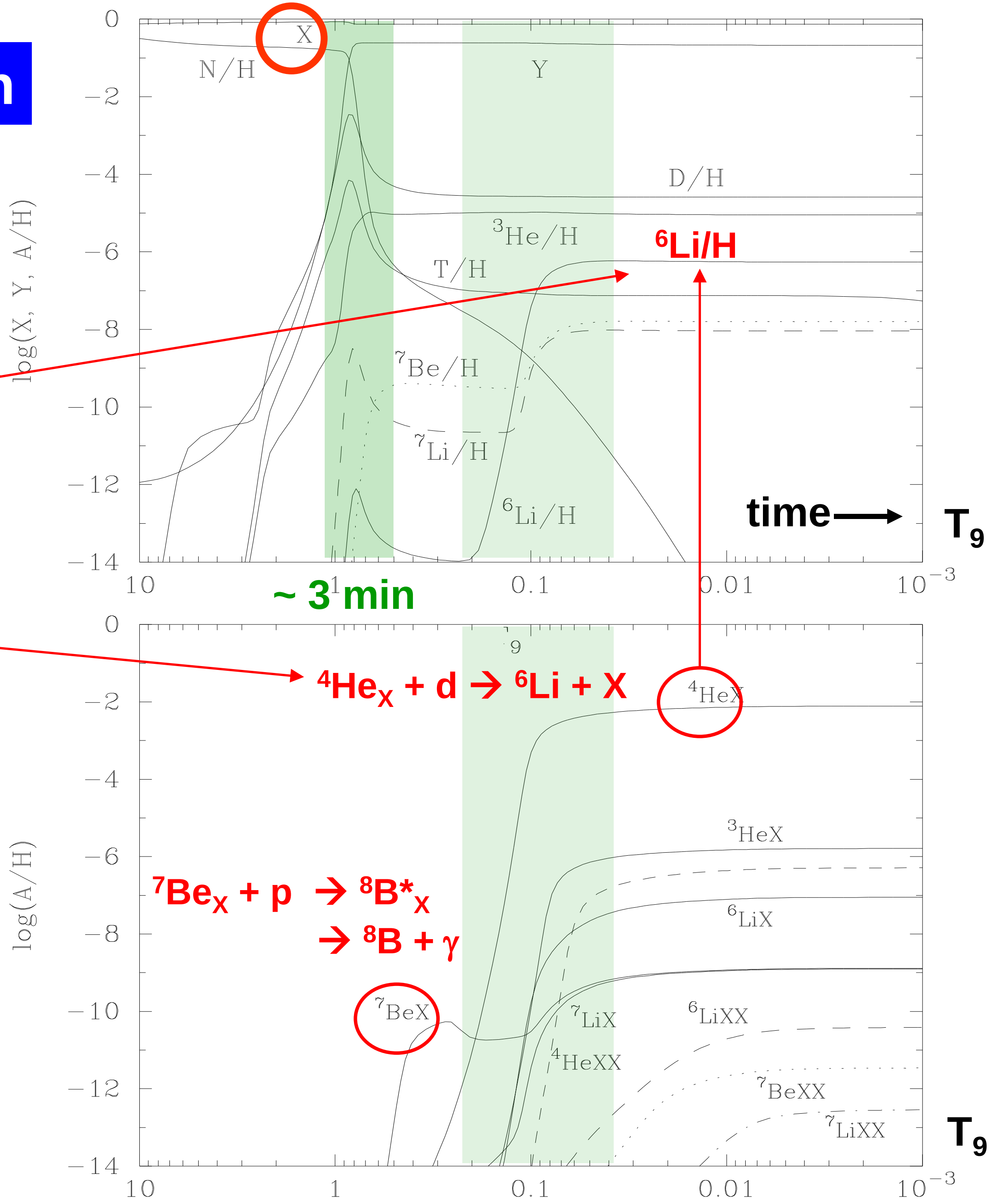
#1: SUSY Leptonic Stau

$^4He_X(d,X^-)^6Li$

$^7Be_X(p,\gamma)^8B_X$

Pospelov (2007)
Hamaguchi et al. (2007)
Bird et al. (2007)

Kusakabe, Kajino, Boyd, Yoshida, and Mathews,
PRD74 (2006), 023526; PRD76 (2007), 121302(R); ApJ 680 (2008), 846; PRD79 (2009) 123513; PRD80 (2009), 103501; PRD (2010), in press.



$^6Li/H$

time⟶ T_9

~ 3 min

$^4He_X + d \rightarrow {}^6Li + X$

$^7Be_X + p \rightarrow {}^8B^*_X$

$\rightarrow {}^8B + \gamma$

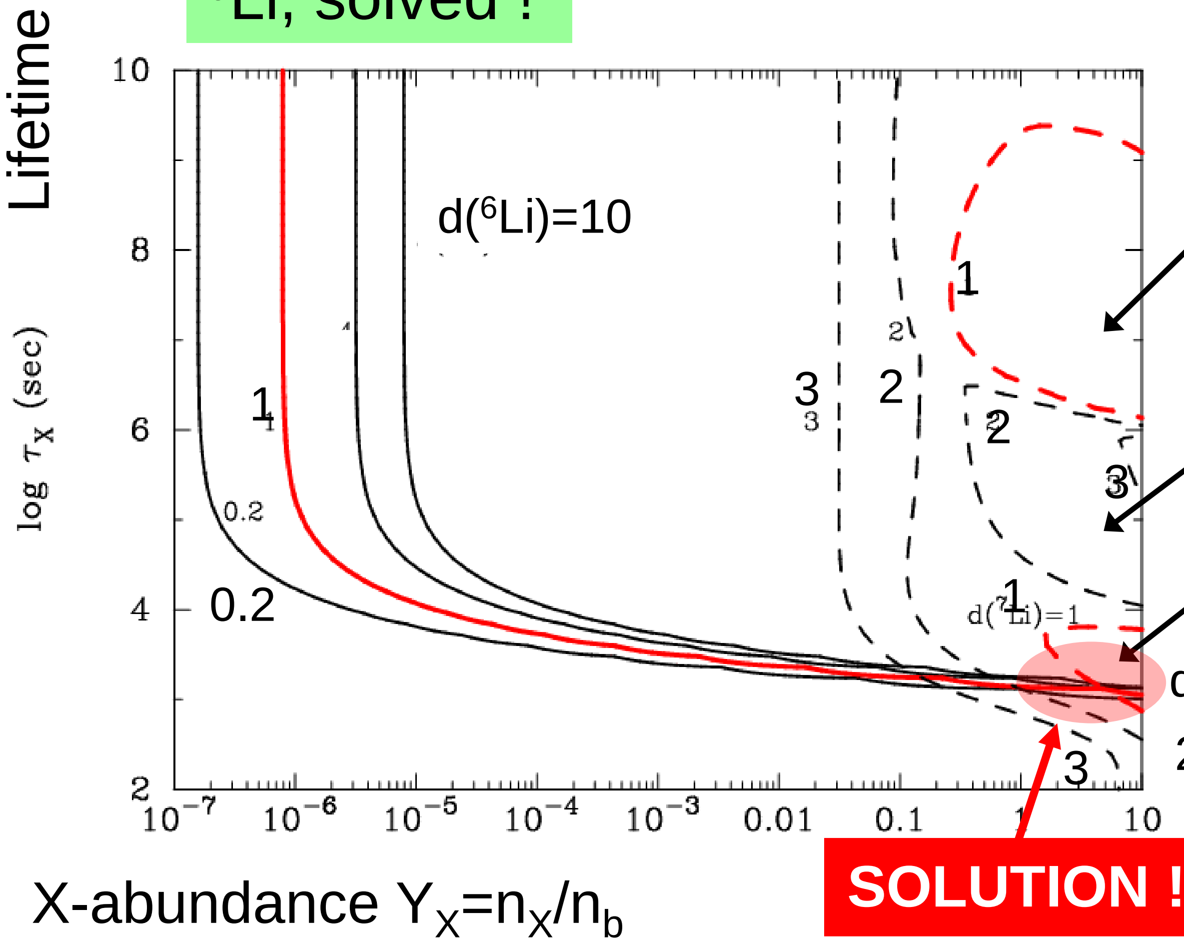
T_9

Cosmological Solution to both ^{6,7}Li problems

Kusakabe, Kajino, Boyd, Yoshida, and Mathews ApJ 680 (2007), 846.

$d(^{A}Li)={}^{A}Li^{Calc}/{}^{A}Li^{Obs}$ $\eta=6.1\times10^{-10}$

^{6}Li, solved !



$^{1}H_X(^{7}Be,X^{-})^{8}B$

$^{4}He_X(t,X^{-})^{7}Li$

$^{4}He_X(^{3}He,X^{-})^{7}Be$

$^{7}Be_X(p,\gamma)^{8}B_X$

^{7}Li, solved !

SOLUTION !

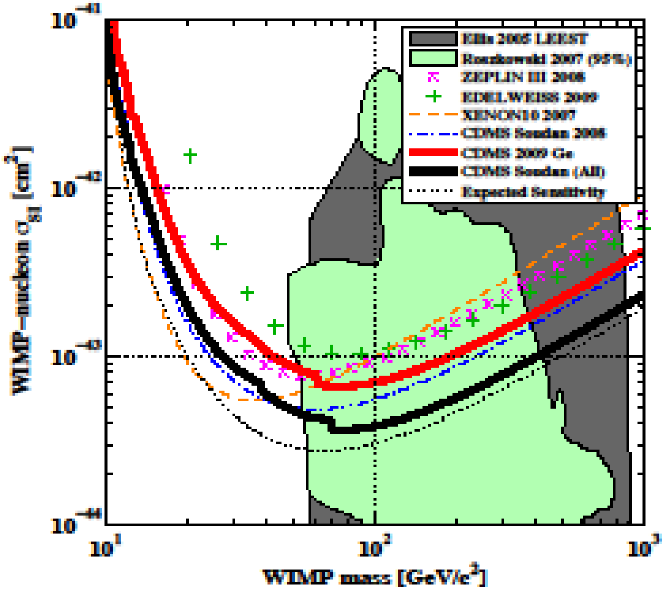
$\Omega_{CDM}=0.2$

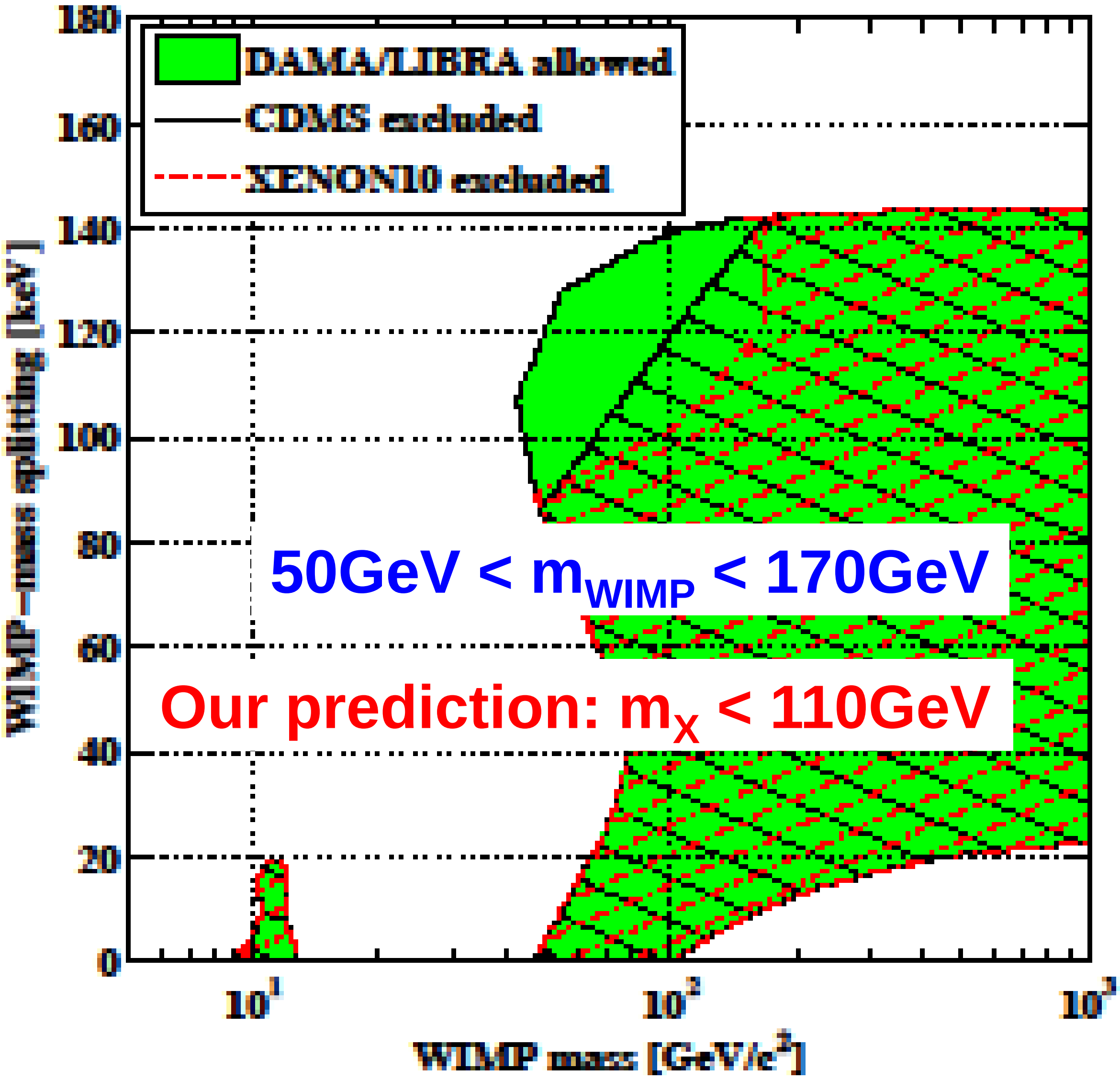
$m_X < 10\sim100$GeV

Recent Result from CDMS II Experiment

Z. Ahmed et al.
(CDMS Collaboration)

arXiv:0912.3592v1





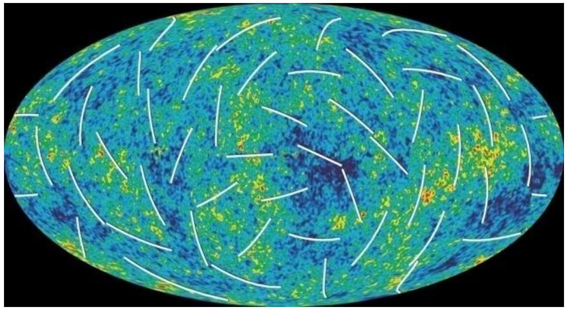
$50\text{GeV} < m_{\text{WIMP}} < 170\text{GeV}$

Our prediction: $m_X < 110\text{GeV}$

Neutrino Mass Constraint from Cosmology

CMB & LSS are strongly affected by:

– integrated Sachs-Wolfe
– neutrino free streaming
– compensation mode of anisotropic stress for neutrinos (π_ν) and cosmic magnetic field (π_B) or extra dimension (π_5).



http://lambda/gsfc.nasa.gov/

Total Neutrino Mass

Cosmology

CMB and LSS constraint from cosmological parameter-fit:

$\Sigma m_\nu < 1.3$ eV (2σ C.L.) $\Longrightarrow$ $\Omega_\nu h^2 < 0.013$

WMAP-5yr, 7yr: Komatsu et al. (2008, 2010)

New constraint: CMB + Magnetic Field + ν-mass :

$\Sigma m_\nu < 0.8$ eV (1σ C.L.) $\Longrightarrow$ $\Omega_\nu h^2 < 0.008$ (1σ)

Yamazaki, Ichiki, Kajino & Mathews, PR D81 (2010), 103519.

Nuclear Physics

0ν-$\beta\beta$: $|\sum U^2_{e\beta} m_\beta| < 1 \sim 6$ eV $\Longrightarrow$ 0.1~0.05 eV !? (future)

Lesgourgues and Pastor (2006)

What is Dark Energy?

Photon last scatter
4×10^5 year

Accelerating expansion
Due to Dark Energy

Dark Age

Inflation

Quantum
fluctuation

WMAP

1st star
4 million year

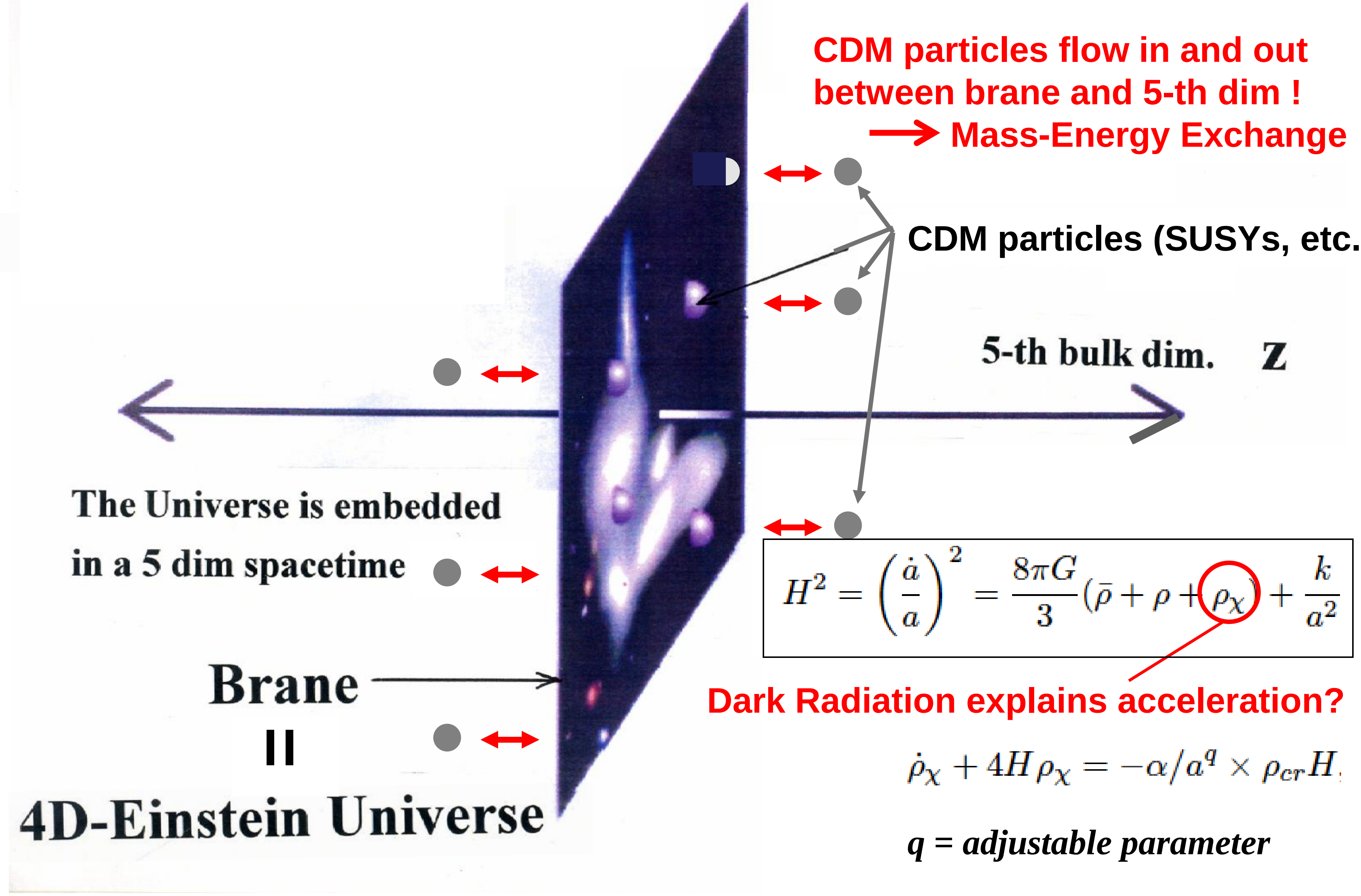
Birth of galaxies & stars

2. Dark Energy in Extra-Dimension Universe

Randall and Sundrum, PRL 83 (1999) proposed brane world cosmology, motivated by 10 dim String Theory.

5D Brane World Cosmology with $\Omega_\Lambda = 0$

—— A Model for Accelerating Universe ——



CDM particles flow in and out between brane and 5-th dim !

→ Mass-Energy Exchange

CDM particles (SUSYs, etc.)

5-th bulk dim. z

The Universe is embedded in a 5 dim spacetime

Brane

||

4D-Einstein Universe

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\bar{\rho} + \rho + \rho_\chi) + \frac{k}{a^2}$$

Dark Radiation explains acceleration?

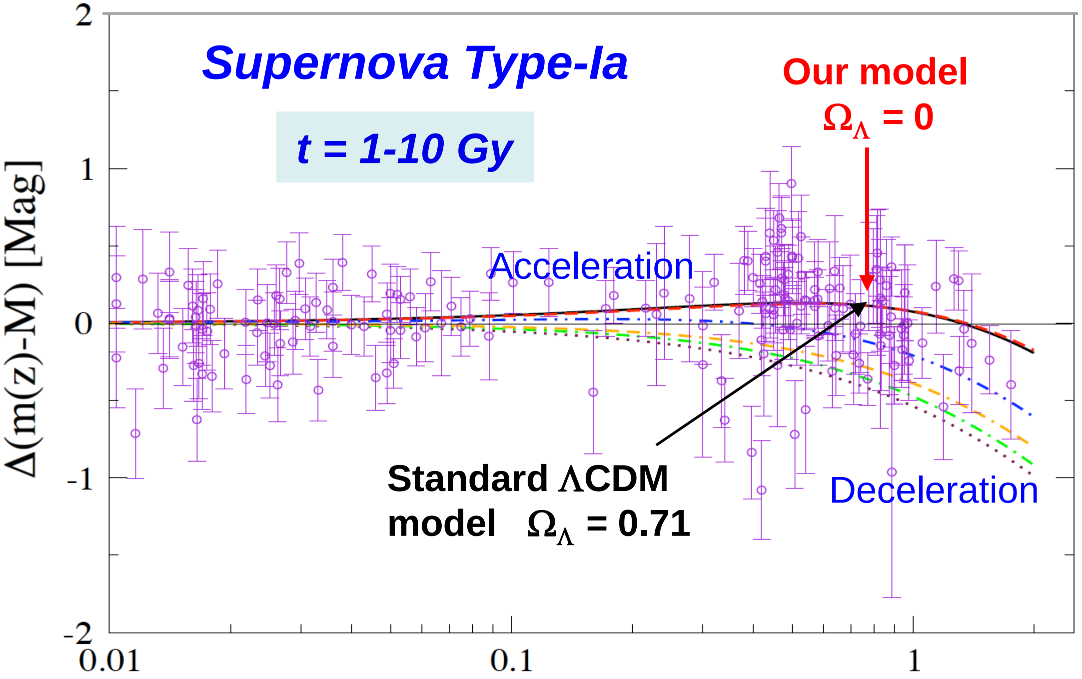
$$\dot{\rho}_\chi + 4H\rho_\chi = -\alpha/a^q \times \rho_{cr} H$$

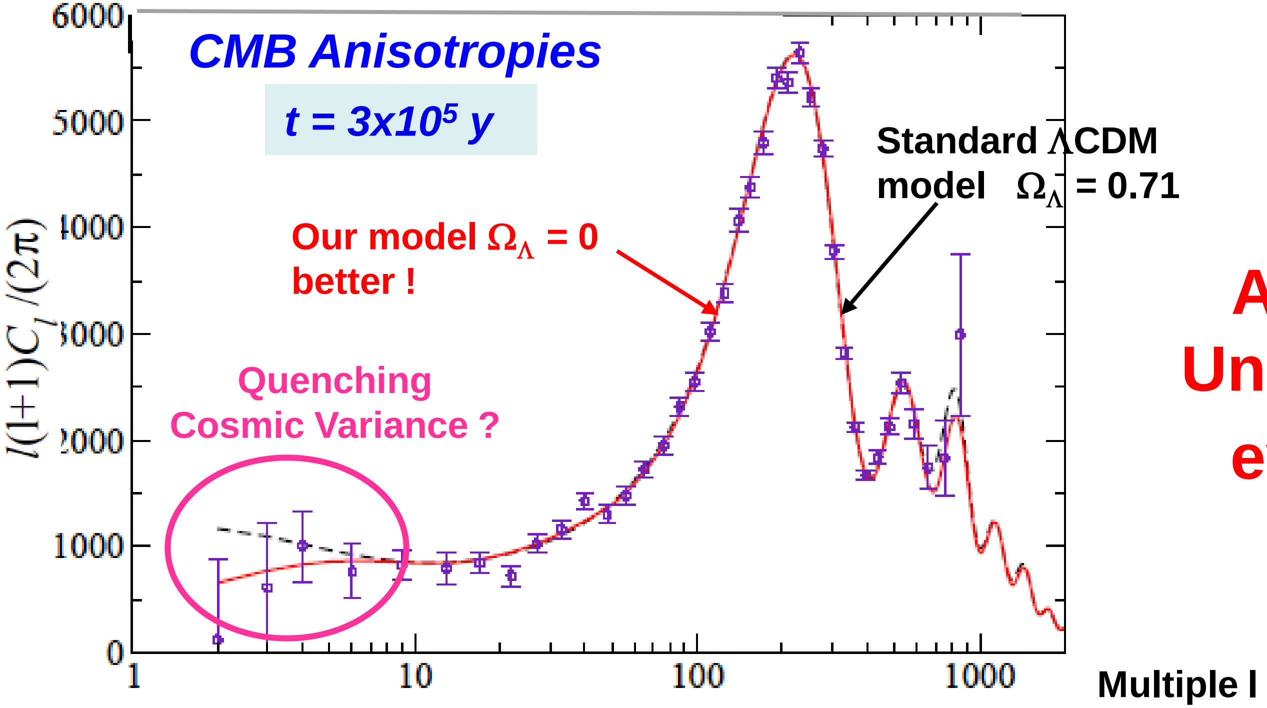
q = adjustable parameter

BRANE WORLD COSMOLOGY
with $\Omega_\Lambda=0$

Standard ΛCDM
with $\Omega_\Lambda \neq 0$

Umezu, Ichiki, Kajino, Mathews,
Nakamura & Yahiro, PRD 73 (2006), 063527





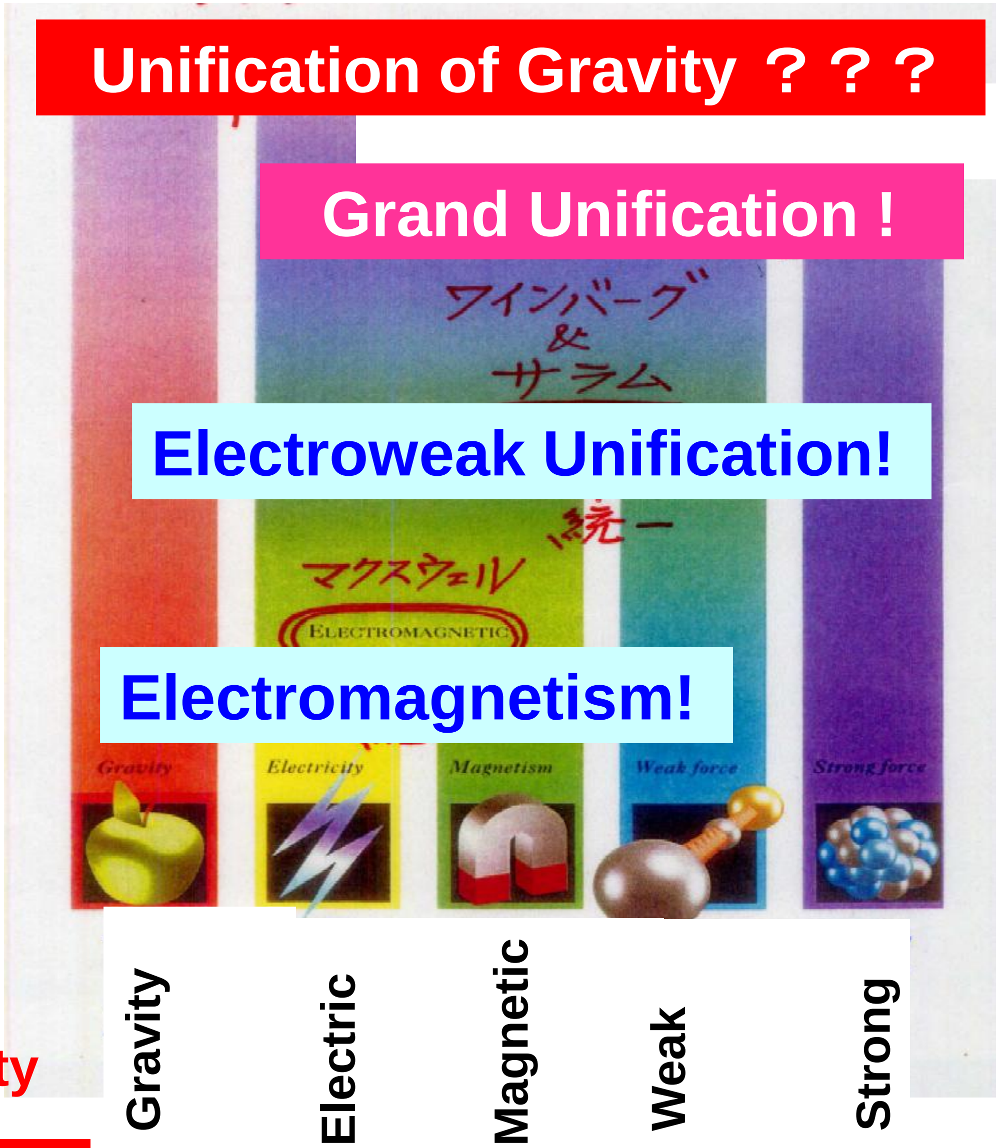
ACCELERATING
Universal expansion
even for Ω_Λ = 0 !

Ultimate Challenge of Modern Science !

:- is to construct Unified Theory of Fundamental Forces, and to resolve the mystery of the beginning and evolution of the Universe!

- ●**Electromagnetism**
  Maxwell (1864)
- ●**Electroweak Unification**
  Weinberg and Salam (1973)
- ●**Grand Unification !**
  Gauge Theory, unfinished !
- ●**Unification of Gravity ???**
  Superstring, SUSY, Supergravity

Need EXTRA DIMENSION ?



Unification of Gravity ？？？

Grand Unification !

Electroweak Unification!

Electromagnetism!

Gravity

Electric

Magnetic

Weak

Strong

Variation of Fundamental Constants

Motivation

- Gravitational const. G could change in cosmic time: G(t) P. Dirac

- **Extra space dimensions** (Kaluza-Klein, Superstring and M-theories). Extra space dimensions is a common feature of theories unifying **gravity** with other interactions. Any change in size of these dimensions would manifest itself in the 3D world as variation of fundamental constants.

- **Scalar fields** . Fundamental constants depend on scalar fields which vary in space and time (variable vacuum dielectric constant ε_0). May be related to “dark energy” and accelerated expansion of the Universe..

Variation of the fundamental coupling const. provides natural explanation of the “fine tunung”.

V. V. Flambaum

Variation of strong coupling const. α

Grand unification models

$$\boxed{\Delta(m/\Lambda_{QCD})/(m/\Lambda_{QCD})=35\Delta\alpha/\alpha}$$

1. Proton mass $M_p=3\Lambda_{QCD}$, measure m_e/M_p
2. Nuclear magnetic moments μ=g eh/4M_pc

   $g=g(m_q/ \Lambda_{QCD})$

3. Nuclear energy levels

→ **Big-Bang Nucleosynthesis**

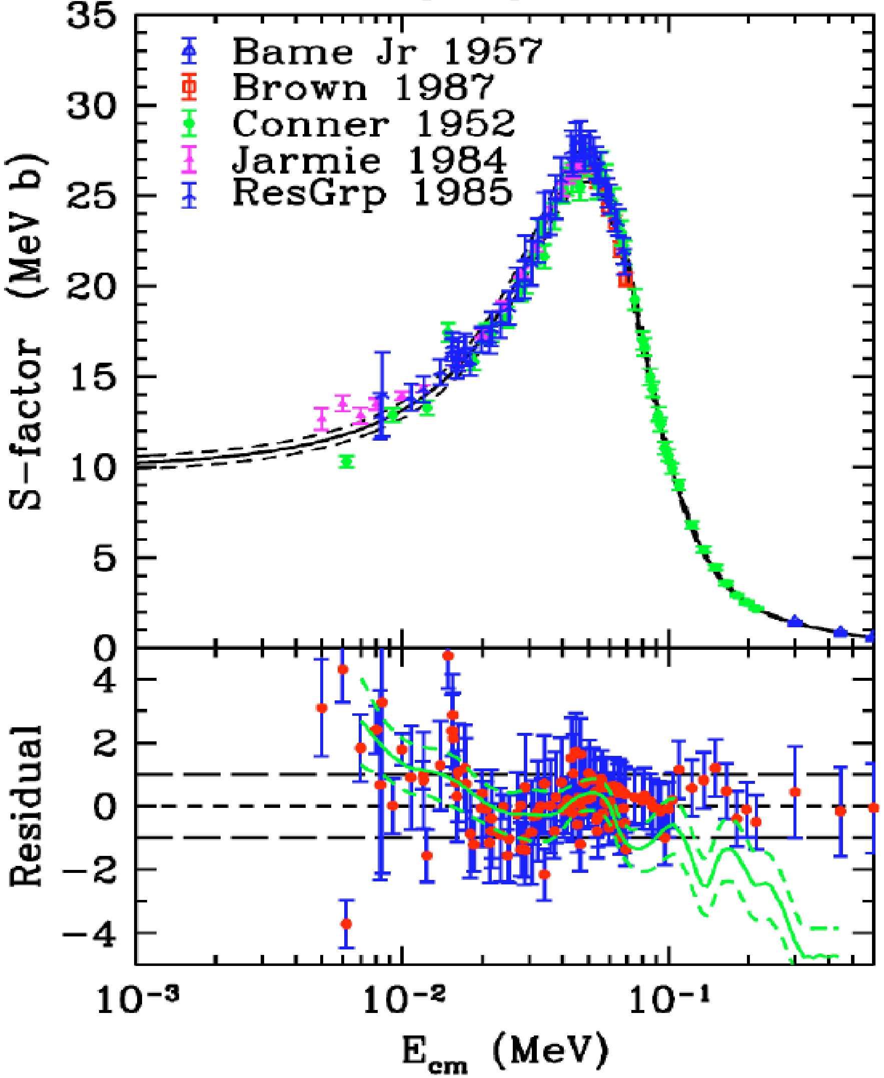
$$\boxed{\delta E(A)/E(A) = K\ \delta(m_q/\Lambda_{QCD})/(m_q/\Lambda_{QCD})}$$

K-values: V.V. Flambaum and R.B. Wiringa, PRC79, 034302 (2009)

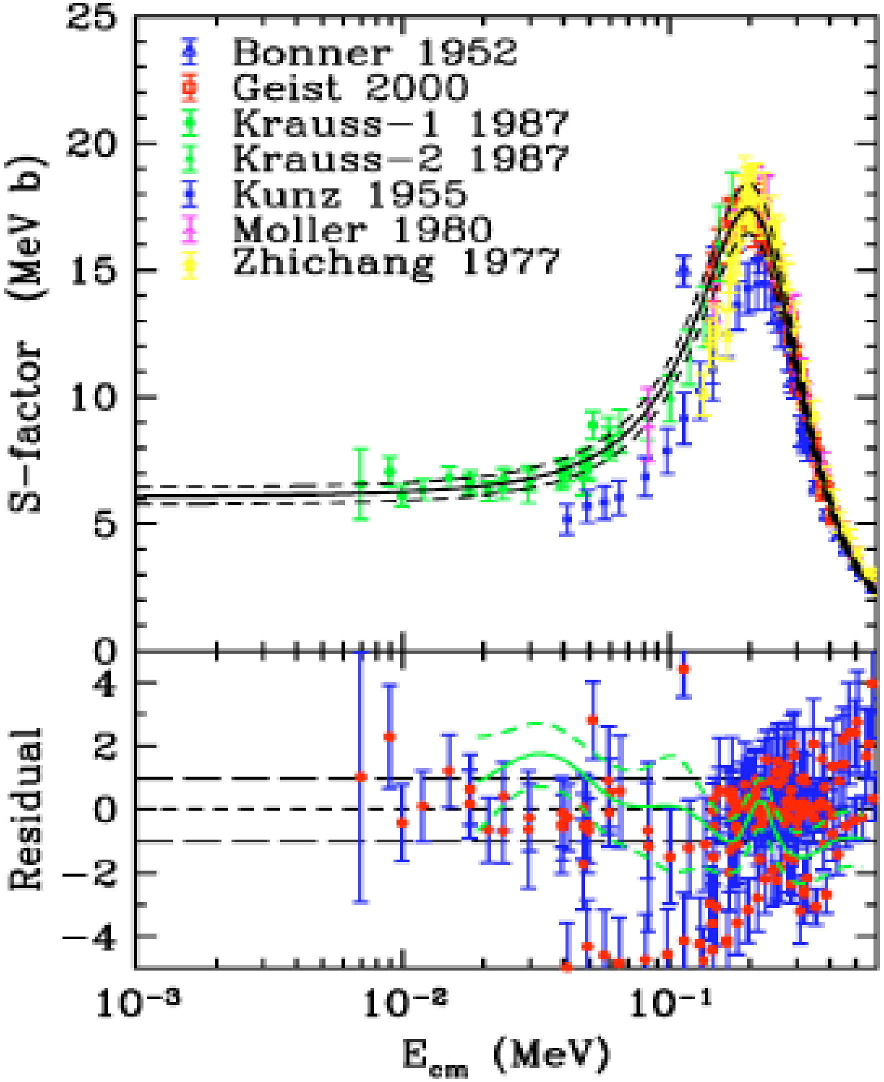
Richard H. Cyburt

PHYSICAL REVIEW D 70, 023505 (2004)

$D(t,n)^4He$



$D(^3He,p)^4He$

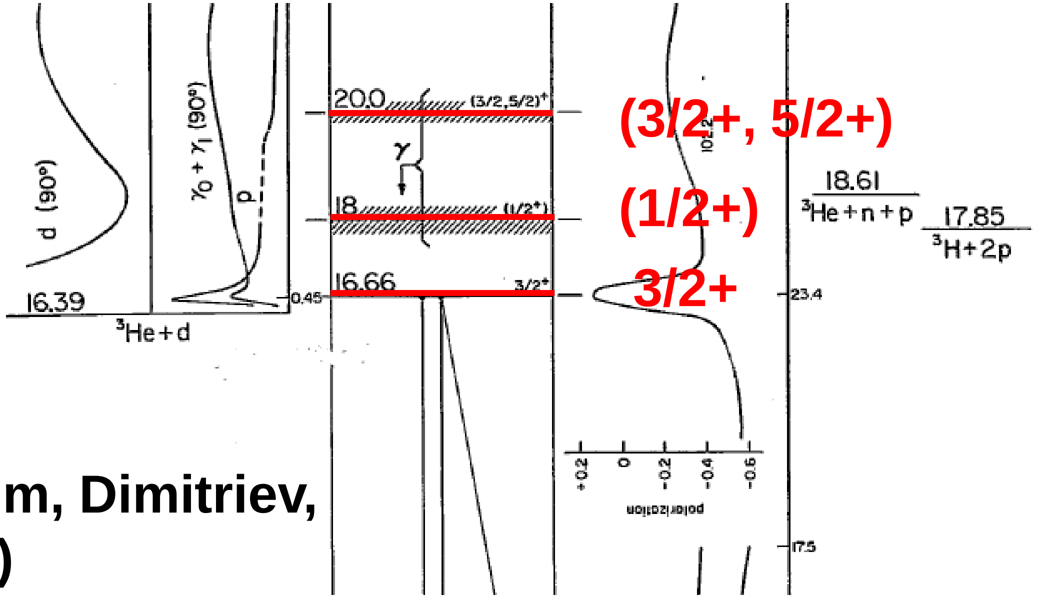


1s-wave

$D + {}^3He$

1+ ½+

Berengut, Flambaum, Dimitriev, PL B683, 114 (2101)



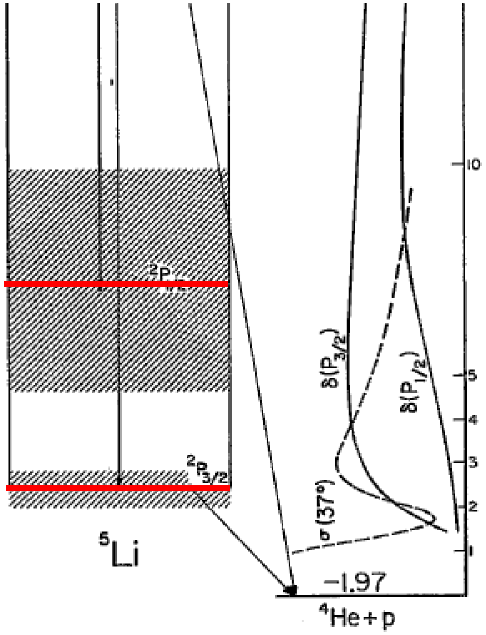
(3/2+, 5/2+)

(1/2+)

3/2+

0d-wave

$D({}^3He,p){}^4He$



P 1/2-

P 3/2-

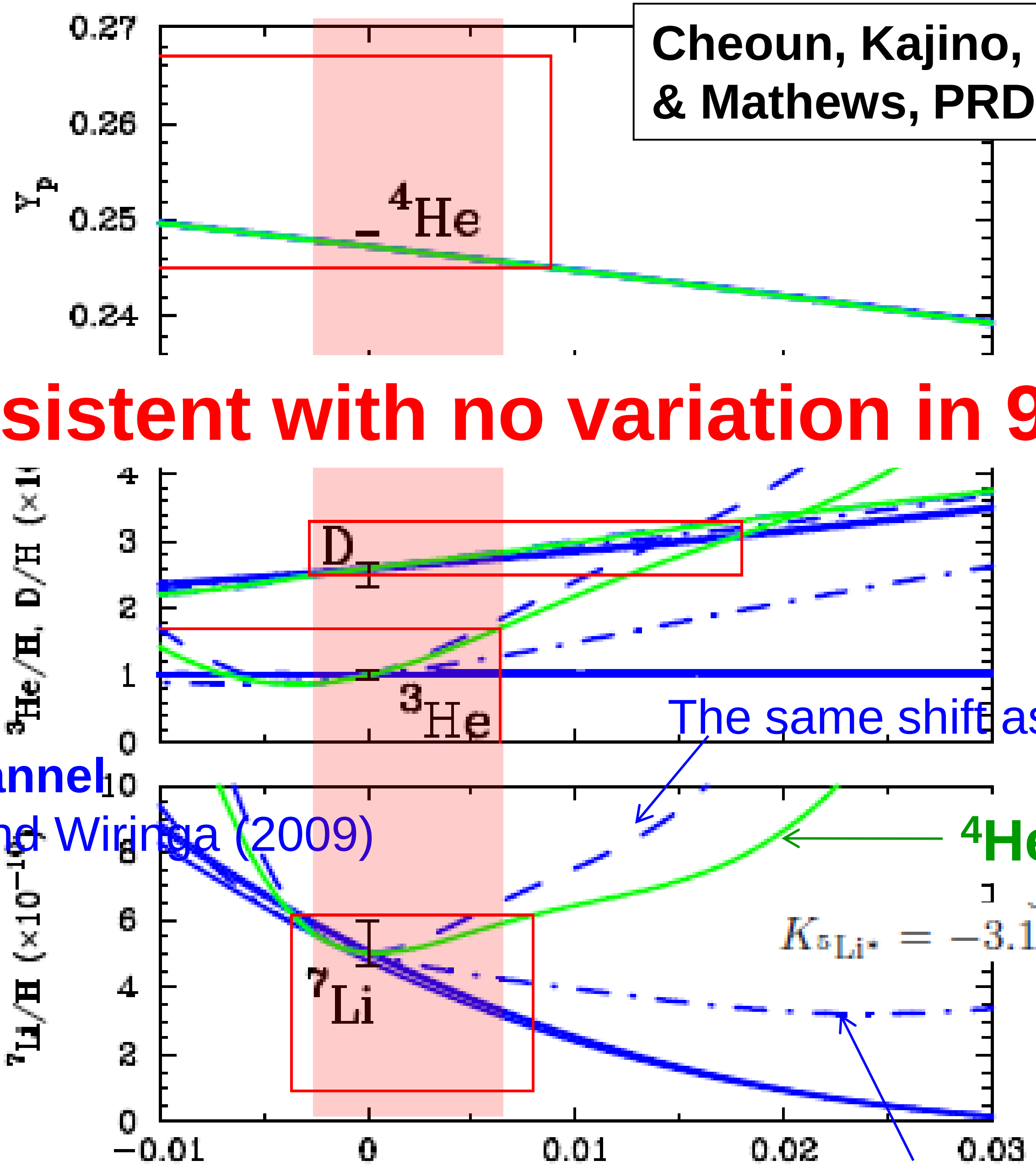
0p-wave

0+ ½+

${}^4He + p$

Cheoun, Kajino, Kusakabe & Mathews, PRD (2011), in press.

Consistent with no variation in 95% C.L. !



^{3}He(t)+D channel

Flambaum and Wiringa (2009)

The same shift as g.s.

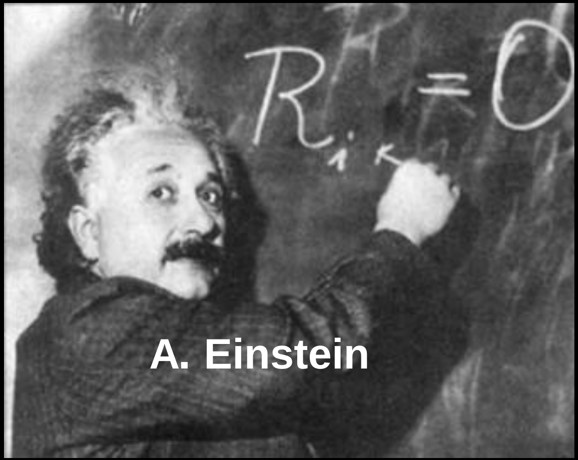
^{4}He+p/n channel

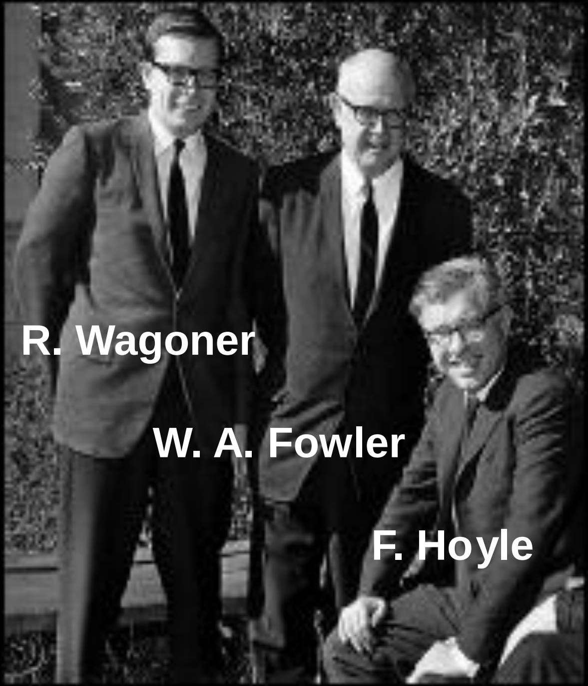
$K_{^5\mathrm{Li}^*} = -3.131,\ K_{^5\mathrm{He}^*} = -2.867$

The same average shift.

$\delta(m_q/\Lambda_{QCD}) / (m_q/\Lambda_{QCD})$

The Universe is the "Laboratory" for the fundamental science!





Catania summer school is a center of the Universe.