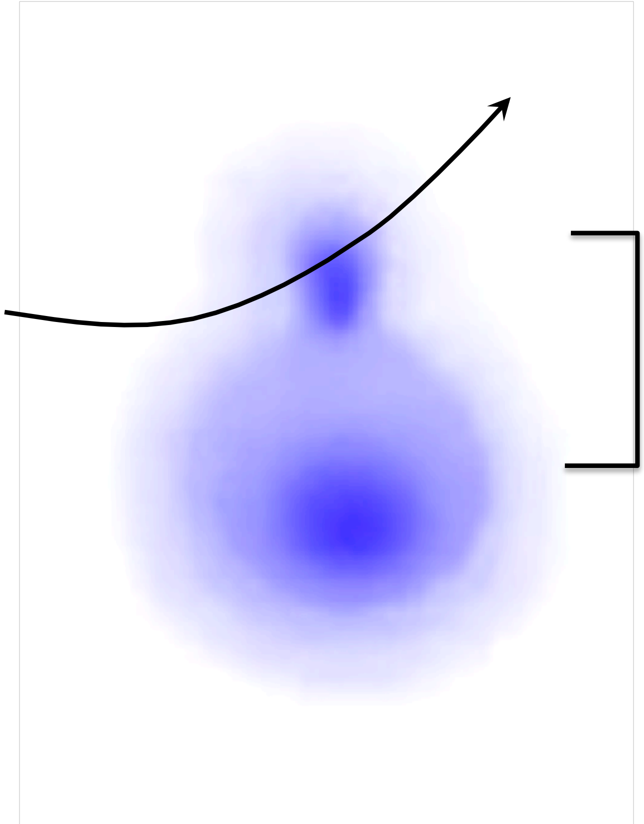
nuclear interaction

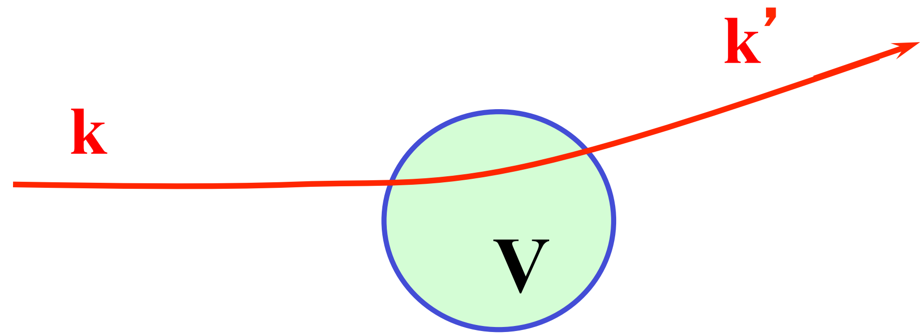


U_{opt}

Horribly phenomenological

Scattering theory (simpler at high energies)

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + U\right]\Psi = E\Psi$$



Partial wave expansion:

$$u_l(r) \xrightarrow[r\to\infty]{} \frac{i}{2}\left\{H_l^{(-)}(kr) - S_l H_l^{(+)}(kr)\right\}$$

Incoming wave

Outgoing wave

"Survival" amplitude (S-matrix)

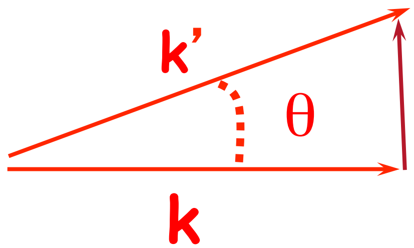
$$S_l = e^{2i\delta_l}$$ (δ_l = Phase shift)

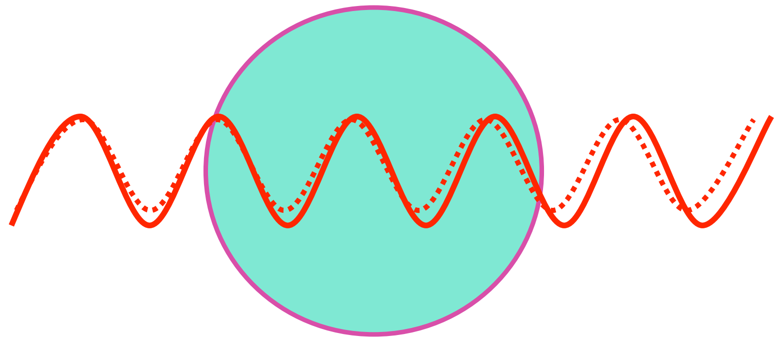
$|S_l|^2$ = "Survival" probability ≤ 1

E_{lab} > 50 MeV/nucleon

Eikonal Waves

$\Delta E \ll E, \quad \theta \ll 1$ radian , $\quad |\Delta\psi/\psi|_{\Delta r=\lambda} \ll 1$

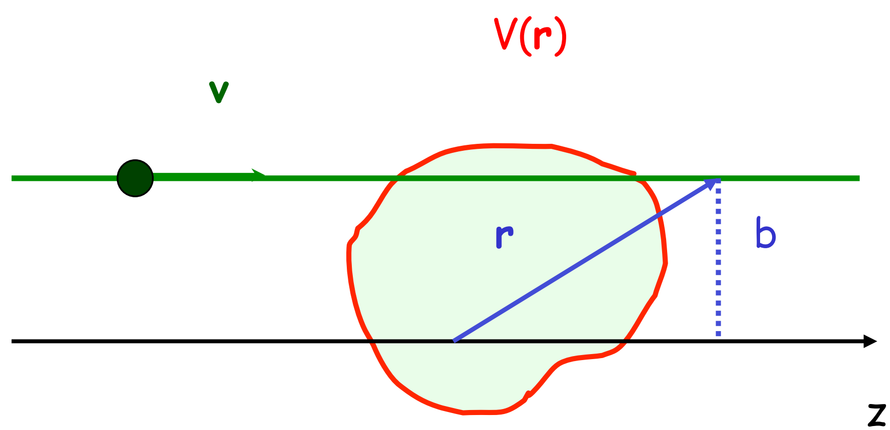




$$\Psi(\mathbf{r}) = S(\mathbf{b},z)\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$S(\mathbf{b},z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{z} U(\mathbf{r}')\,dz'\right\}$$

$$\mathbf{r}' = (\mathbf{b}, z')$$



$z \longrightarrow \infty$ after the collision:

$$\Psi(\mathbf{r}) = S(\mathbf{b})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$S(\mathbf{b}) = e^{i\chi(\mathbf{b})}$$
$$= \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{\infty} U(\mathbf{r}')\,dz'\right\}$$

Eikonal waves (reactions)

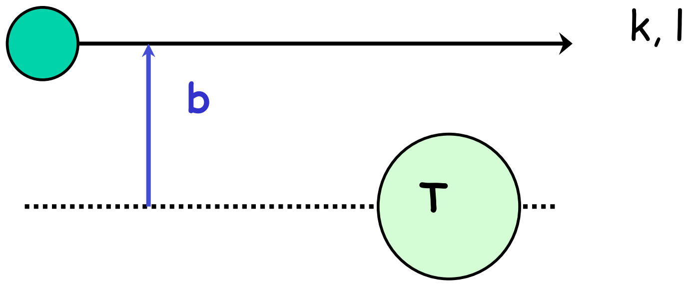
Harmonic oscillator (structure)

Pearls of quantum mechanics

Eikonal Waves: Applications
(sometimes called "Glauber theory")

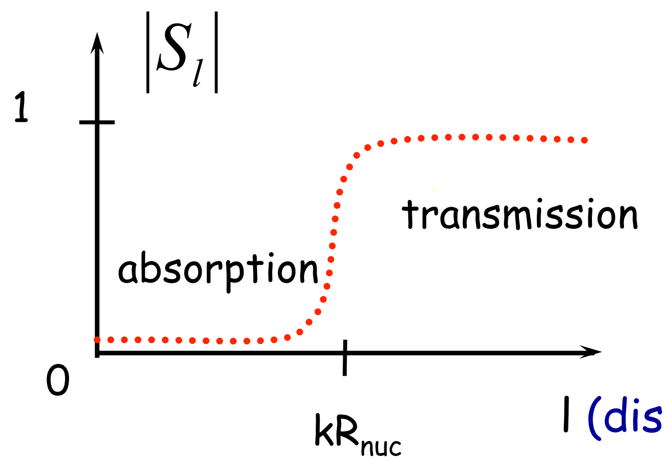
Roy Glauber 2005 Nobel prize
(another "Glauber theory")

S-matrices ("Survival" Amplitudes)

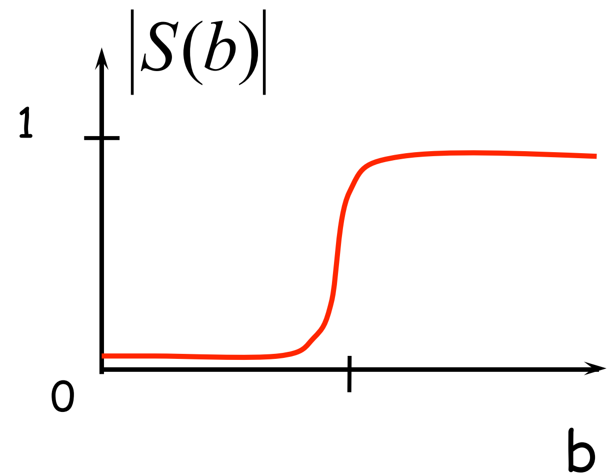


b = impact parameter

l = kb (actually l + 1/2 = kb)



⇒ eikonal



Ex: Elastic Scattering

$$f(\theta) = \frac{i}{k}\sum_l (l+\tfrac{1}{2})(1-S_l)P_l(\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

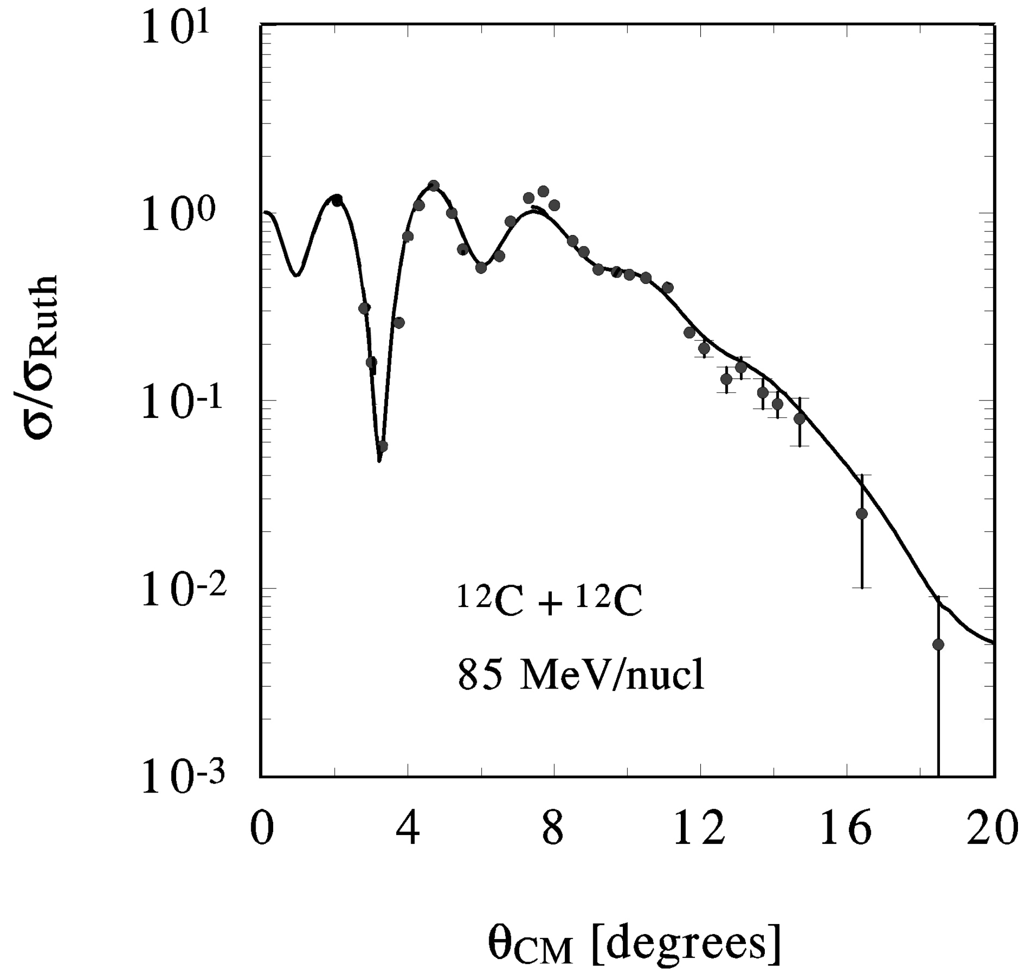
$$f(\theta) = ik\int db\, b\, J_0(kb)\{1-S(b)\}$$

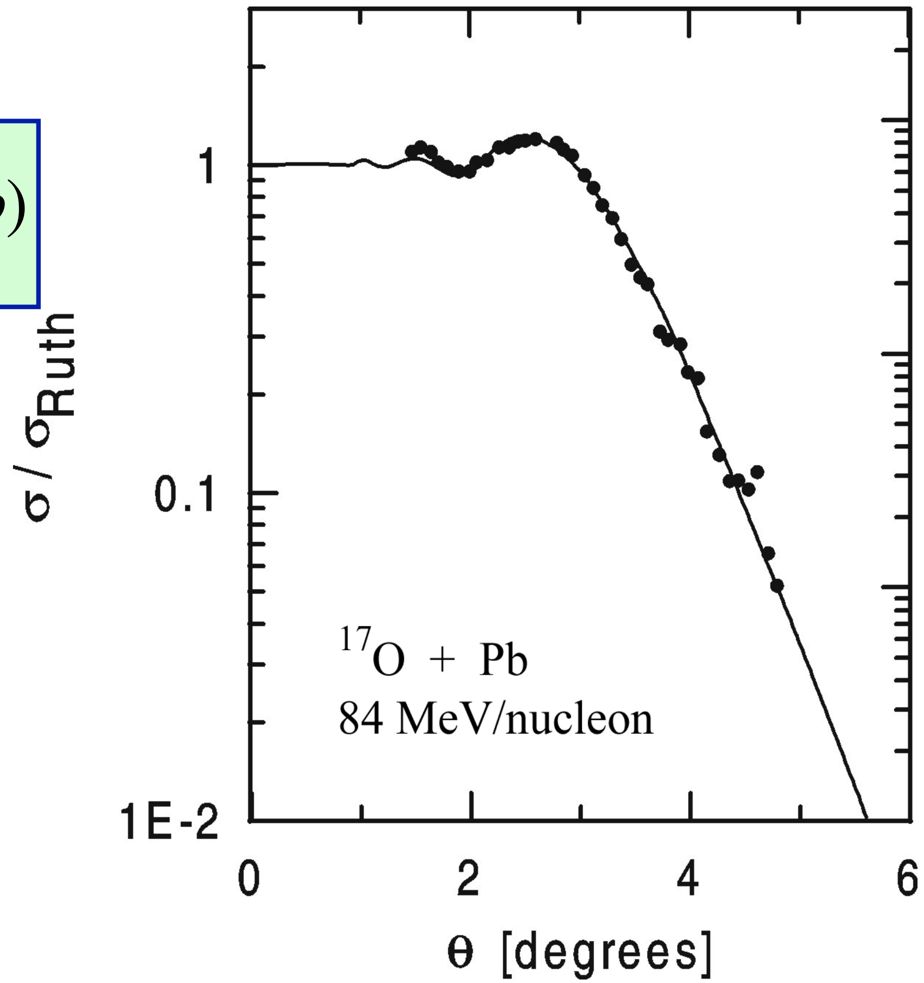
Ex: elastic scattering

$$\chi_{AB}^{(N)}(b) = \frac{1}{k_{nn}} \int_0^\infty dq\, q\, \tilde{\rho}_A(q) \tilde{\rho}_B(q) f_{nn}(q) J_0(qb)$$

$$f_{nn}(q) = \frac{k_{nn}}{4\pi} \sigma_{nn} \left(i + \alpha_{nn}\right) e^{-\beta_{nn} q^2}$$

(from nn scattering)





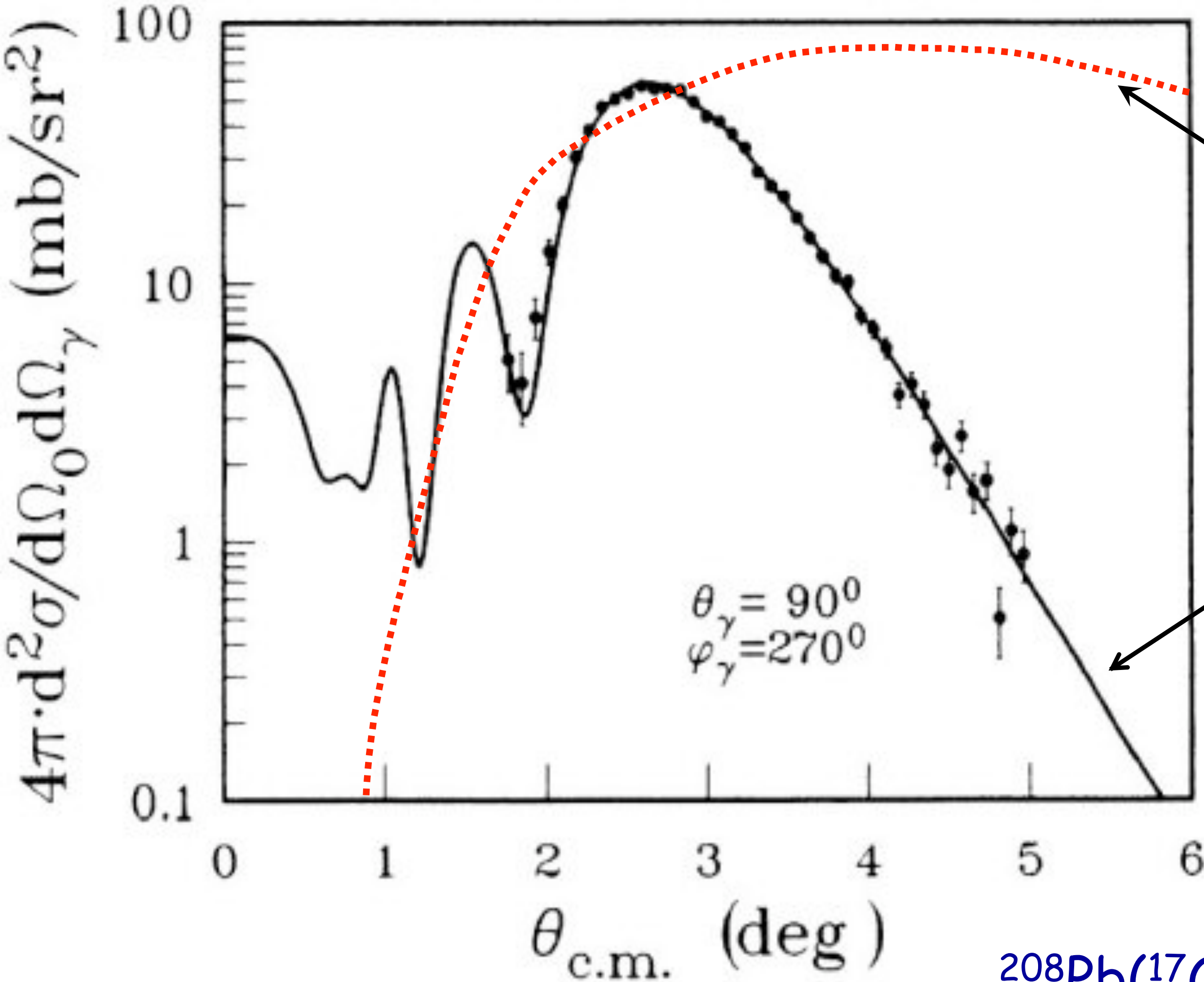
solid curves: Glauber

Nuclear diffraction in Coulomb excitation

$$f_{inel}^{C}(\theta) \approx \int d^3r\, d^3r' \, \Psi_{\mathbf{k}'}^{(-)*}(\mathbf{r})\, \varphi_f(\mathbf{r}')\, V_C(\mathbf{r},\mathbf{r}')\, \Psi_{\mathbf{k}}^{(+)}(\mathbf{r})\, \varphi_i(\mathbf{r}')$$

DWBA

eikonal waves



semiclassical

eikonal

Excitation of GDR
followed by γ-decay

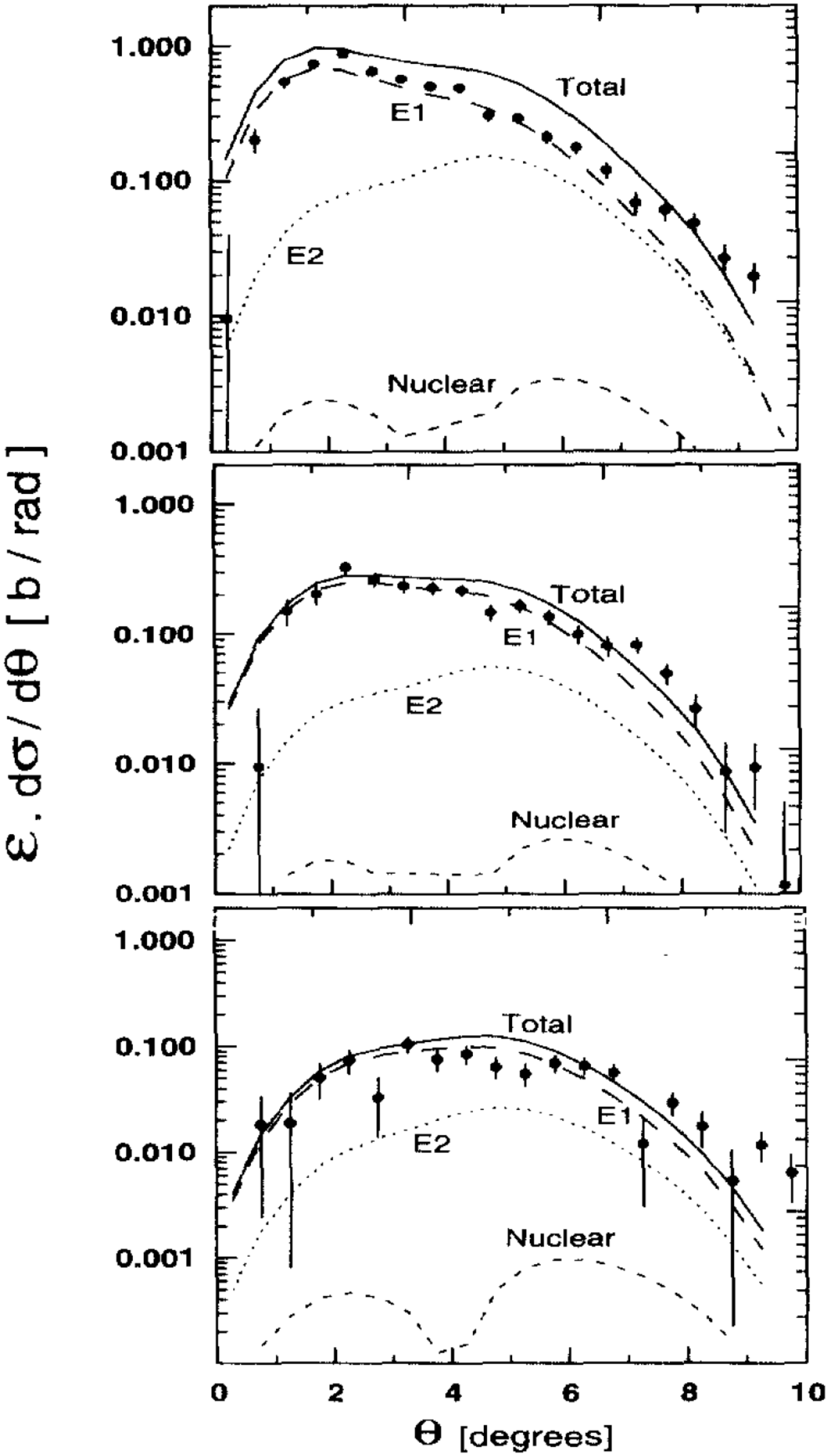
^{208}Pb(^{17}O,^{17}O' γ) at 84 MeV/nucleon
for fixed angle θ_γ=90^o and φ_γ= 270^o

Coulomb excitation + Nuclear excitation

$$f^{C}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\varphi_f(\mathbf{r}') V_C(\mathbf{r},\mathbf{r}') \Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\varphi_i(\mathbf{r}')$$

$$f^{N}_{inel}(\theta) \approx \int d^3r\, d^3r'\, \Psi^{(-)*}_{\mathbf{k}'}(\mathbf{r})\varphi_f(\mathbf{r}') U_N(\mathbf{r},\mathbf{r}') \Psi^{(+)}_{\mathbf{k}}(\mathbf{r})\varphi_i(\mathbf{r}')$$

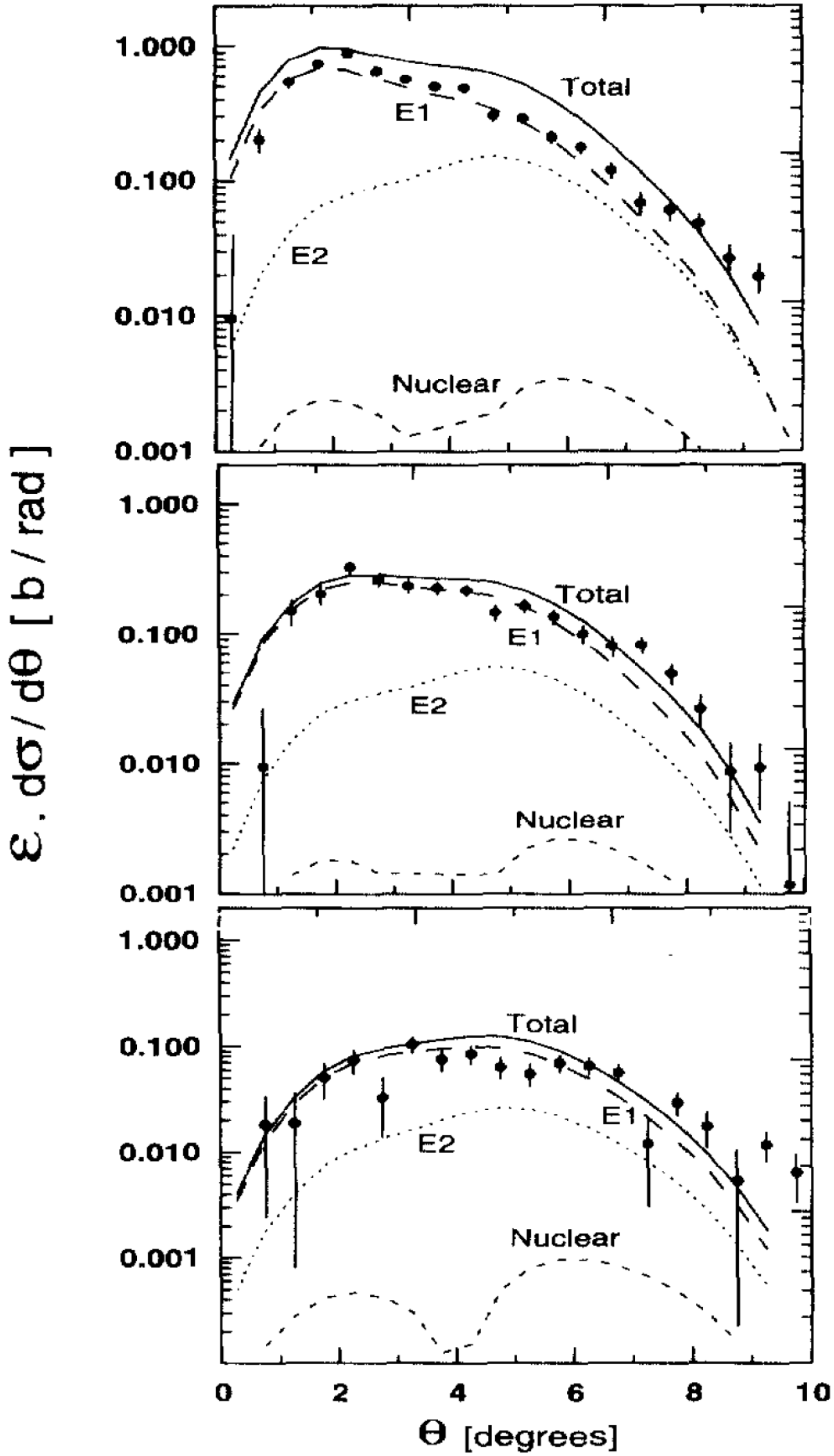
$$\frac{d\sigma}{d\Omega} = \left| f^{N}_{inel}(\theta) + f^{C}_{inel}(\theta) \right|^2$$



$^{8}B + Pb \rightarrow {}^{7}Be + p + Pb$
50 MeV/nucleon

Relevant for $^{7}Be(p,\gamma)^{8}B$ (Sun)

Data: Kikuchi et al, PLB 391, 261 (1997)
Calc: Bertulani, Gai, NPA 636, 227 (1998)

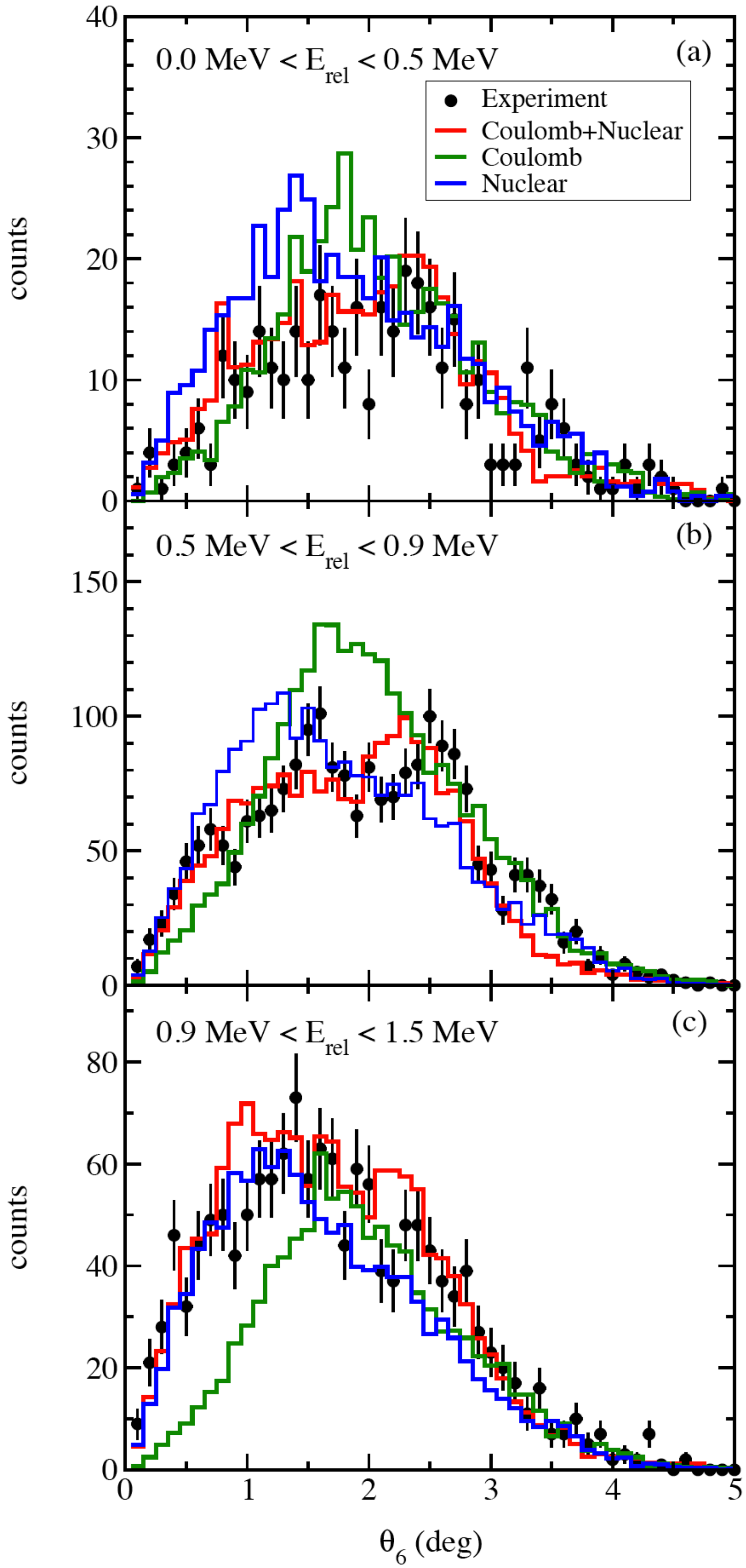


$^{8}B + Pb \rightarrow {}^{7}Be + p + Pb$
50 MeV/nucleon

Relevant for $^{7}Be(p,\gamma)^{8}B$ (Sun)

Data: Kikuchi et al, PLB 391, 261 (1997)
Calc: Bertulani, Gai, NPA 636, 227 (1998)

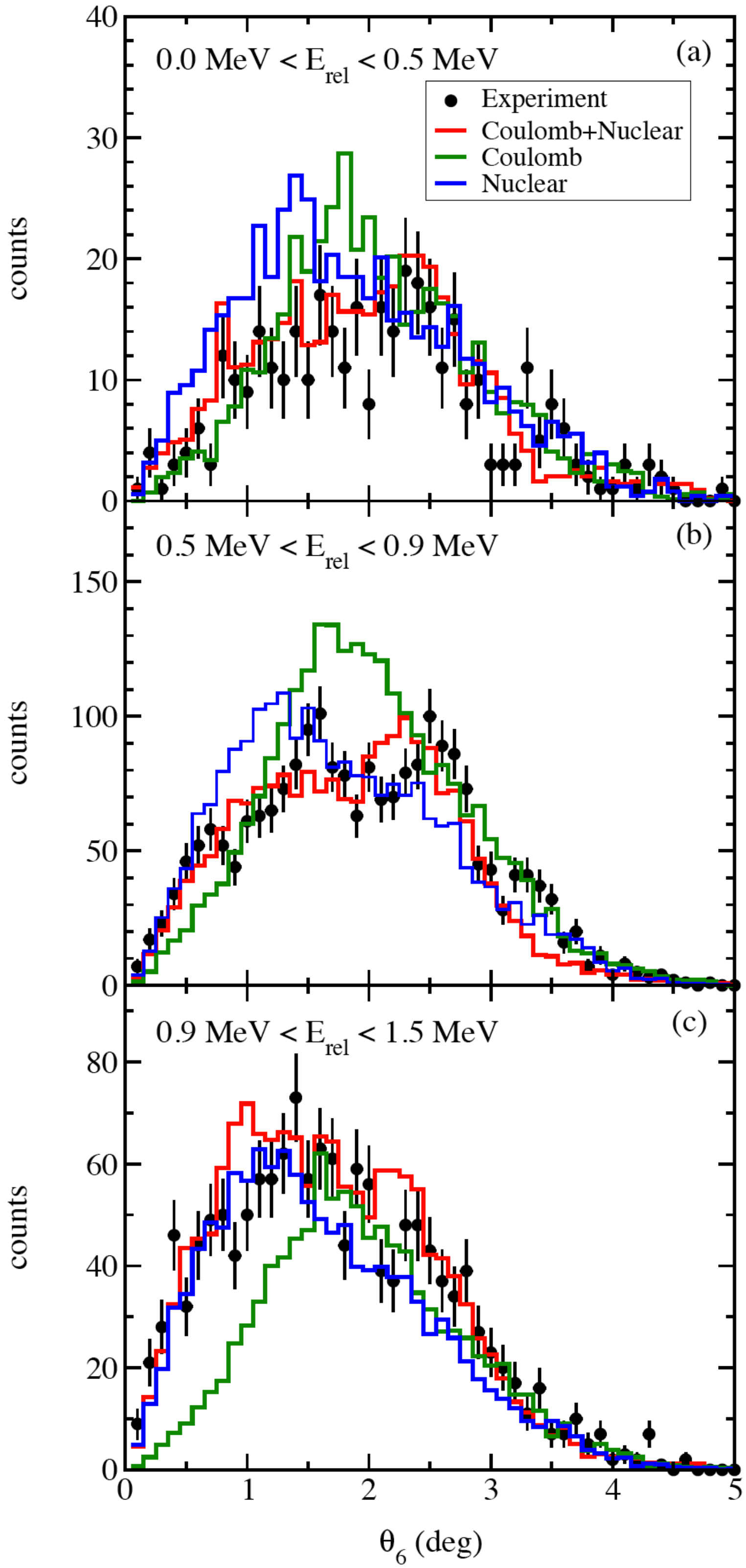
Good wins!



^{6}Li + Pb → α + d + Pb
26 MeV/nucleon

Relevant for BBN

Data: Hammache et al., PRC 82 (2010) 065803
Calc: Stefan Typel



^{6}Li + Pb → α + d + Pb
26 MeV/nucleon

Relevant for BBN

Data: Hammache et al., PRC 82 (2010) 065803
Calc: Stefan Typel

Evil wins!

Rolling back. We forgot lots of things!

Things that make us all unhappy, but give us jobs.

Rutherford is (was) wrong

Aguiar, Aleixo, Bertulani, PRC 42, 2180 (1990)

$$L = L^{LO} + L^{NLO} + L^{N^2LO} + \cdots$$

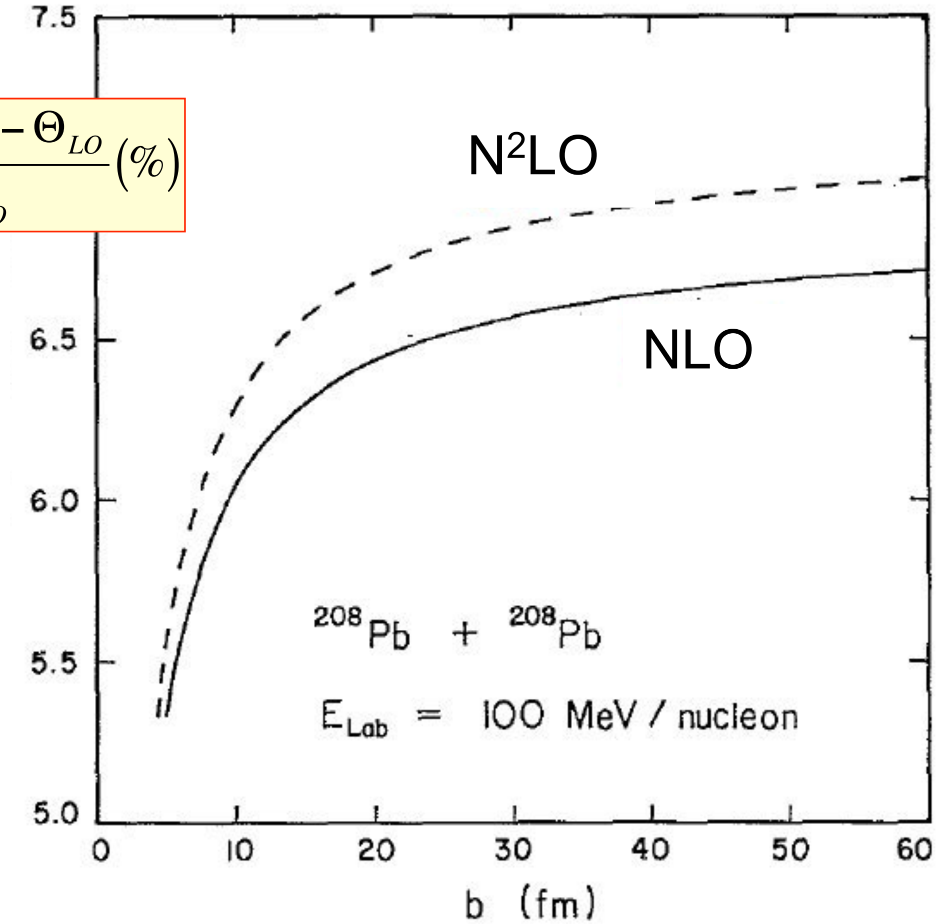
$$L^{LO} = \frac{1}{2}\mu c^2 \left(\frac{v}{c}\right)^2 - \frac{Z_1 Z_2 e^2}{r}$$

$$L^{NLO} = \frac{\mu^4 c^2}{8}\left[\frac{1}{m_1^3} - \frac{1}{m_2^3}\right]\left(\frac{v}{c}\right)^4 - \frac{\mu^2 Z_1 Z_2 e^2}{2 m_1 m_2 r}\left[\left(\frac{v}{c}\right)^2 + \left(\frac{\mathbf{v}\cdot\mathbf{r}}{cr}\right)^2\right]$$

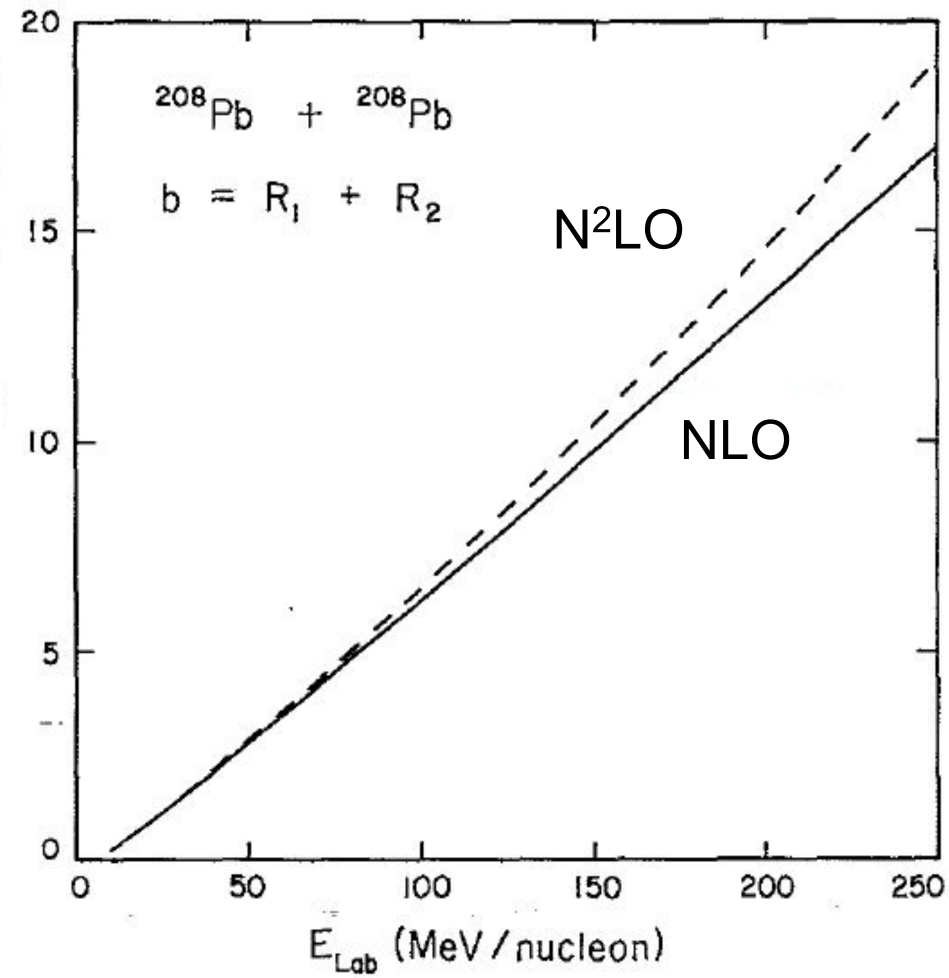
$$L^{N^2LO} = \frac{\mu c^2}{512}\left(\frac{v}{c}\right)^6 + \frac{Z_1 Z_2 e^2}{16 r}$$

$$\times\left[\frac{1}{8}\left\{\left(\frac{v}{c}\right)^4 - 3\left(\frac{v_r}{c}\right)^4 + 2\left(\frac{v_r v}{c}\right)^2\right\} + \frac{Z_1 Z_2 e^2}{\mu c^2 r}\left\{3\left(\frac{v_r}{c}\right)^2 - \left(\frac{v}{c}\right)^2\right\} + \frac{4 Z_1^2 Z_2^2 e^4}{\mu^2 c^4 r^2}\right]$$

$$\Rightarrow \frac{d\sigma}{d\Omega}$$

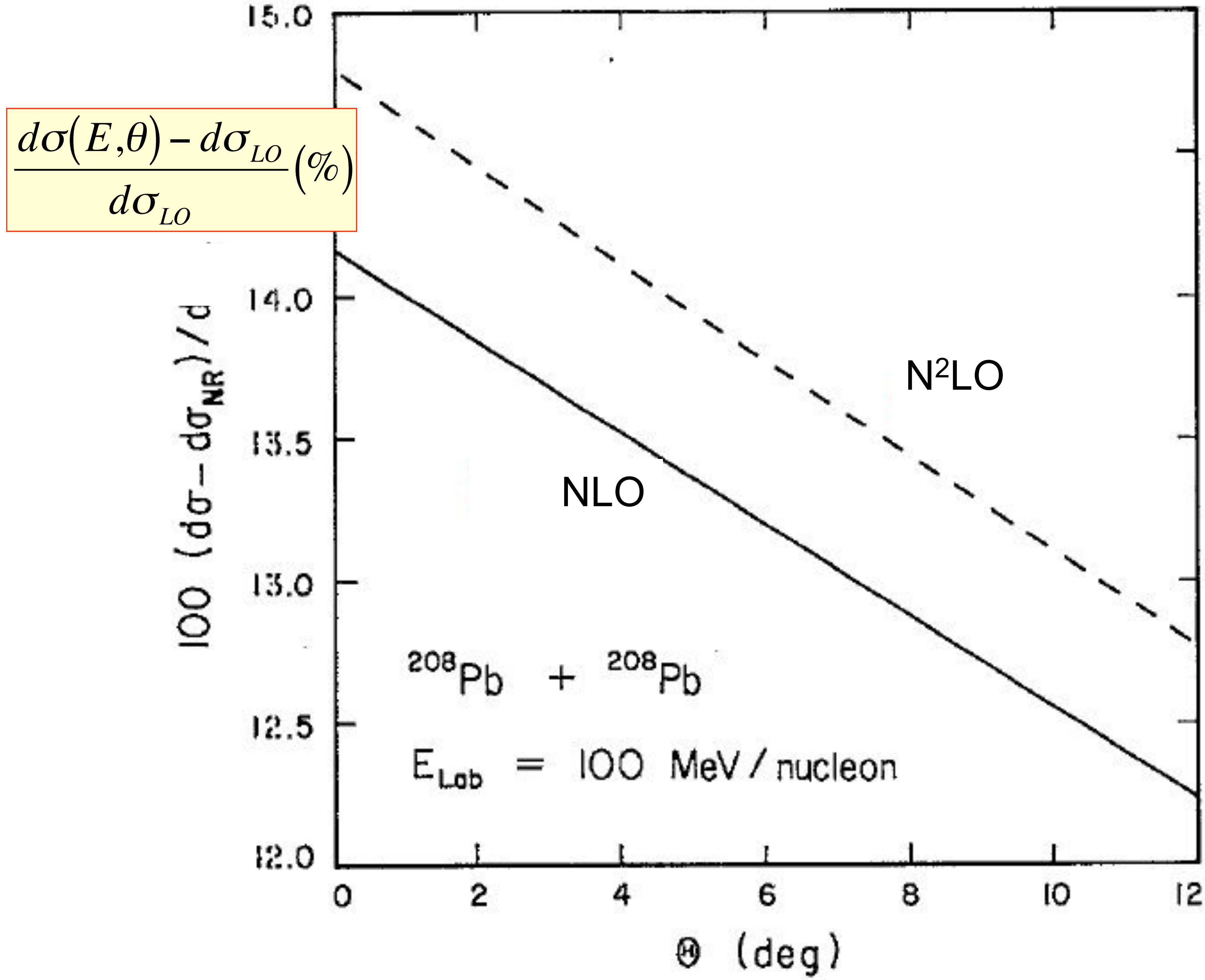


Deviations from Rutherford



important for elastic scattering: experimental data often reported as

$$\frac{d\sigma_{elast}}{d\sigma_{Ruth}}$$


$\frac{d\sigma(E,\theta) - d\sigma_{LO}}{d\sigma_{LO}}(\%)$
100 (dσ − dσ_NR) / d
15.0
14.0
13.5
13.0
12.5
12.0
0
2
4
6
8
10
12
Θ (deg)
N²LO
NLO
²⁰⁸Pb + ²⁰⁸Pb
E_Lab = 100 MeV / nucleon

So what?

Coulomb excitation: orbital integrals with retardation

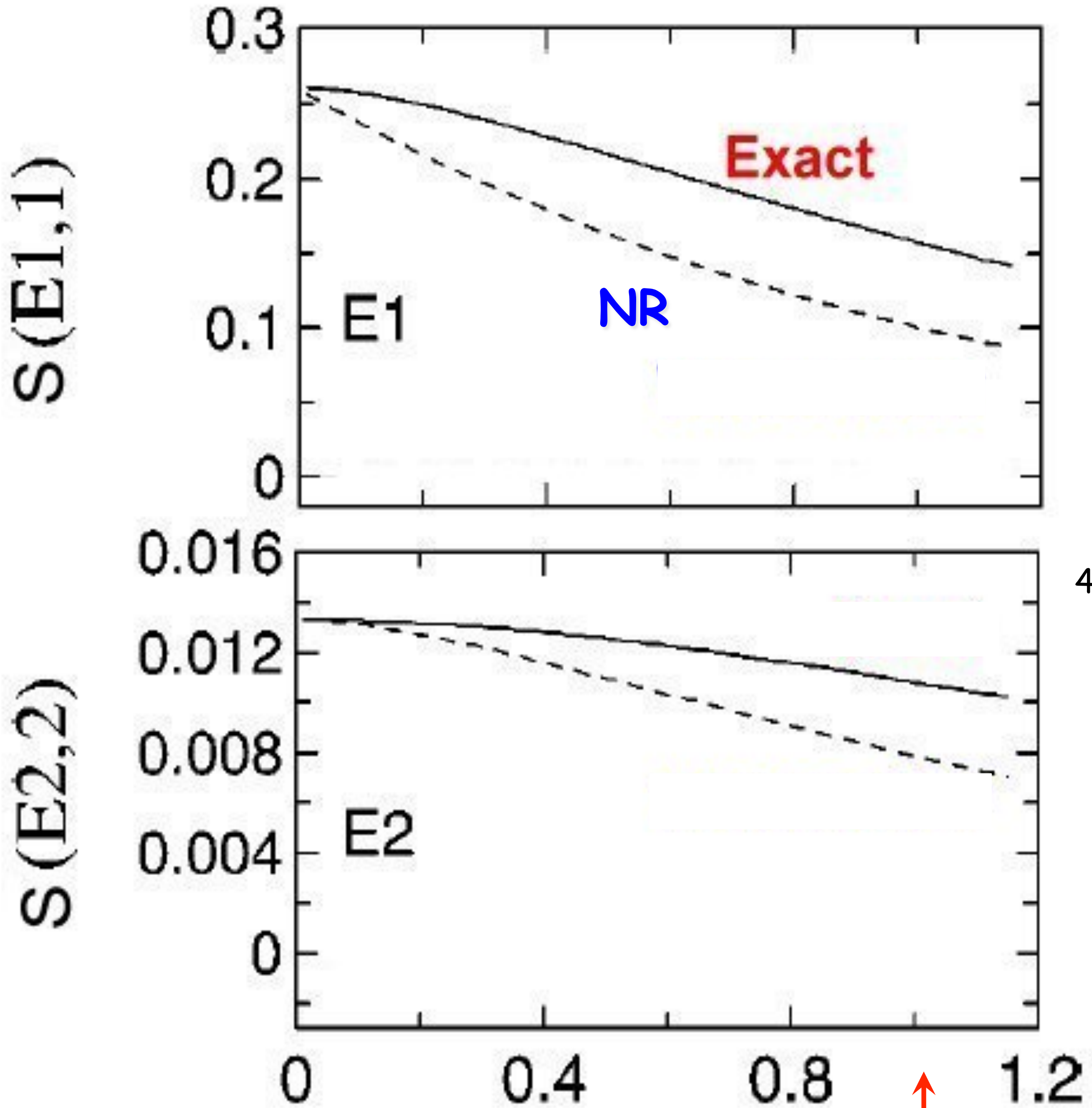
Aleixo, Bertulani, NPA 505, 448 (1989)

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{elast} \times P_{exc}$$

$$= \sum_{\pi\lambda\mu} \left|S(\pi\lambda\mu)\right|^2 \left|M_{fi}(\pi\lambda,-\mu)\right|^2$$

$$M_{fi}(\pi\lambda\mu) = \langle f \left| \text{EM Operator}(\lambda\mu) \right| i \rangle$$

$$S(\pi\lambda\mu) = \text{orbital integrals}$$



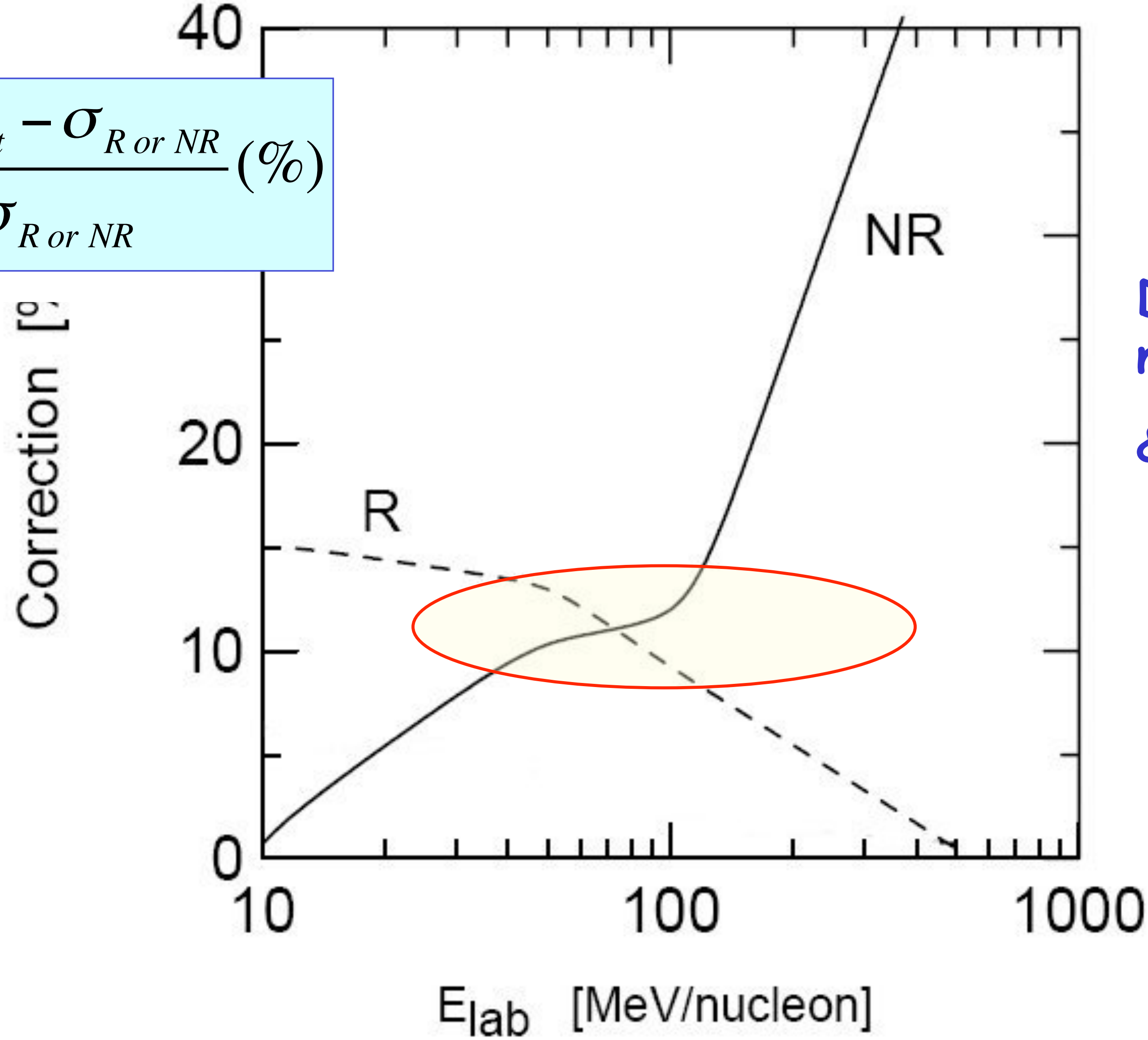
$$\xi = \frac{E_x b}{\gamma \hbar v}$$

Deviations from non-relativistic

^{40}S (100 MeV/nucleon) + Au

Corrections important large b's, large E_x's

$$\frac{\sigma_{exact} - \sigma_{R\ or\ NR}}{\sigma_{R\ or\ NR}} (\%)$$



Deviations from non-relativistic

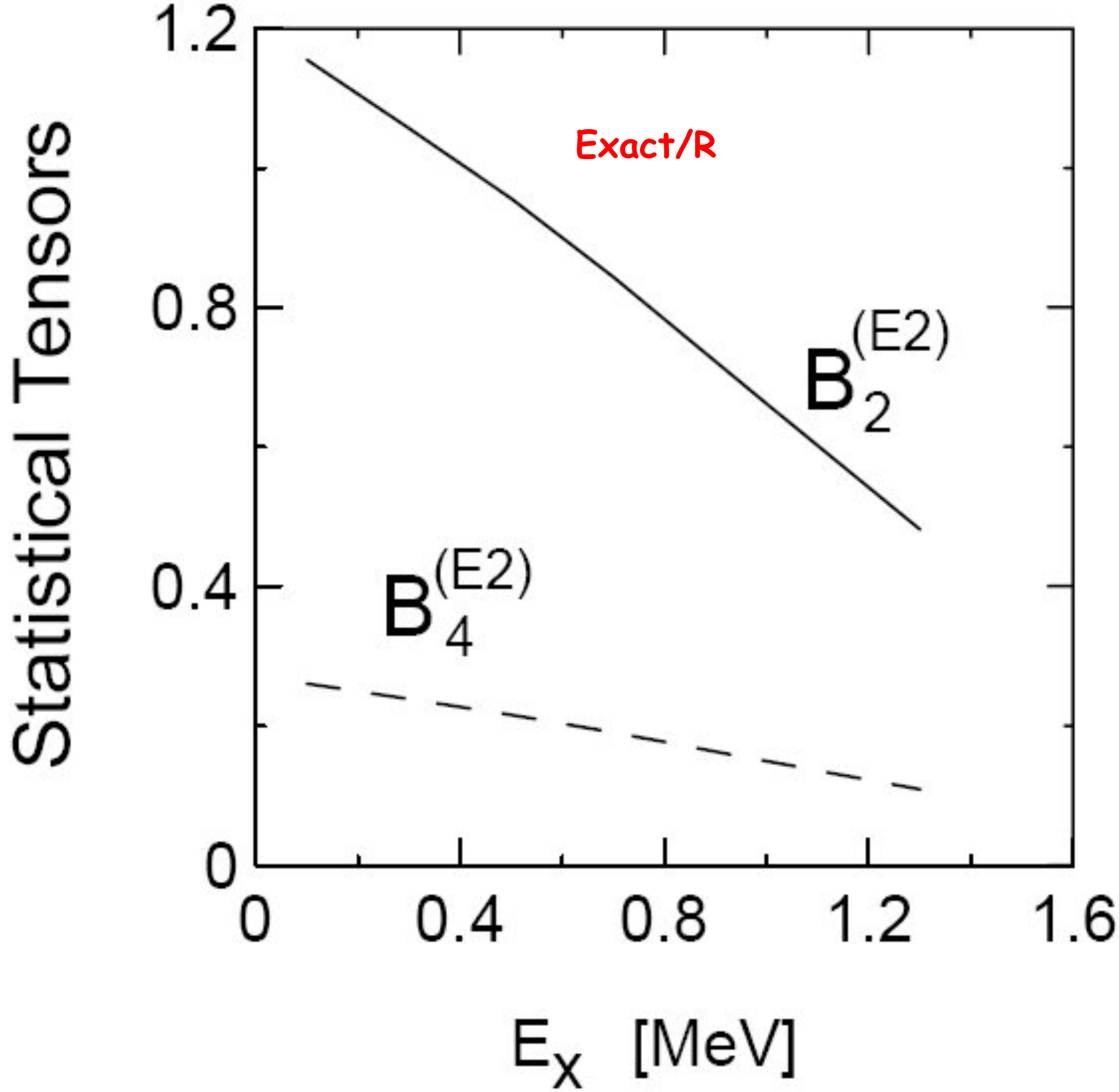
& from relativistic

^{40}S (100 MeV/nucleon) + Au

E_x = 0.89 MeV

De-excitation by γ-ray emission

$$W_\gamma(\theta_\gamma) = 1 + \sum_{\kappa=2,4} B_\kappa Q_\kappa(E_\gamma)\, P_\kappa(\cos\theta_\gamma)$$



Deviations from from relativistic theory

^{38}S (100 MeV/nucleon) + Au

We all know that:

- Relativitiy obviously important at GANIL, GSI, MSU and RIKEN
  (we are talking dynamics)

- 'Rather' easy to include for Coulomb interaction
  (transformation properties of E/M fields well known)

How about nuclear interaction?

- Transformation properties of nucleus-nucleus potentials not exactly known

- Solution has to be based on QFT (QM + relativity)

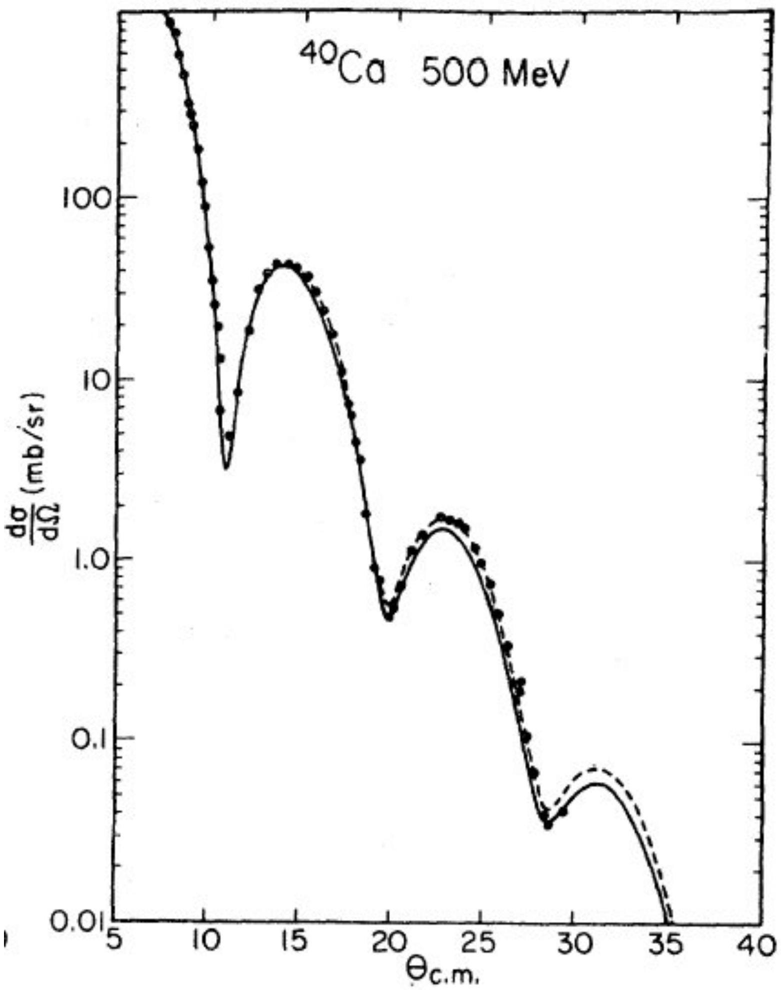
- Can we save our DWBA, CC, or CDCC knowledge for something practical?

You won't find answers in a book.

Clue: Proton-nucleus scattering at intermediate energies

- meson exchange, two-nucleon interaction
- mean field approximation, U_0 (ω exchange), U_S (2π exchange)

$$\left[E - V_C - U_0 - \beta\left(mc^2 + U_S\right)\right]\Psi = -i\hbar c\alpha\cdot\nabla\Psi$$



non-relativistic reduction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{cent} + \left(\frac{\hbar}{2mc}\right)^2 \frac{1}{r}\frac{d}{dr}U_{SO}\,\sigma\cdot\mathbf{L}\right]\phi = E\phi$$

$$U_{cent} = m^*\left(U_0 + U_S\right) + \cdots$$

$$m^* = 1 - \frac{U_0 - U_S}{2mc^2} + \cdots$$

$$U_{SO} = U_0 - U_S + \cdots$$

Arnold, Clark, PLB 84, 46 (1979)

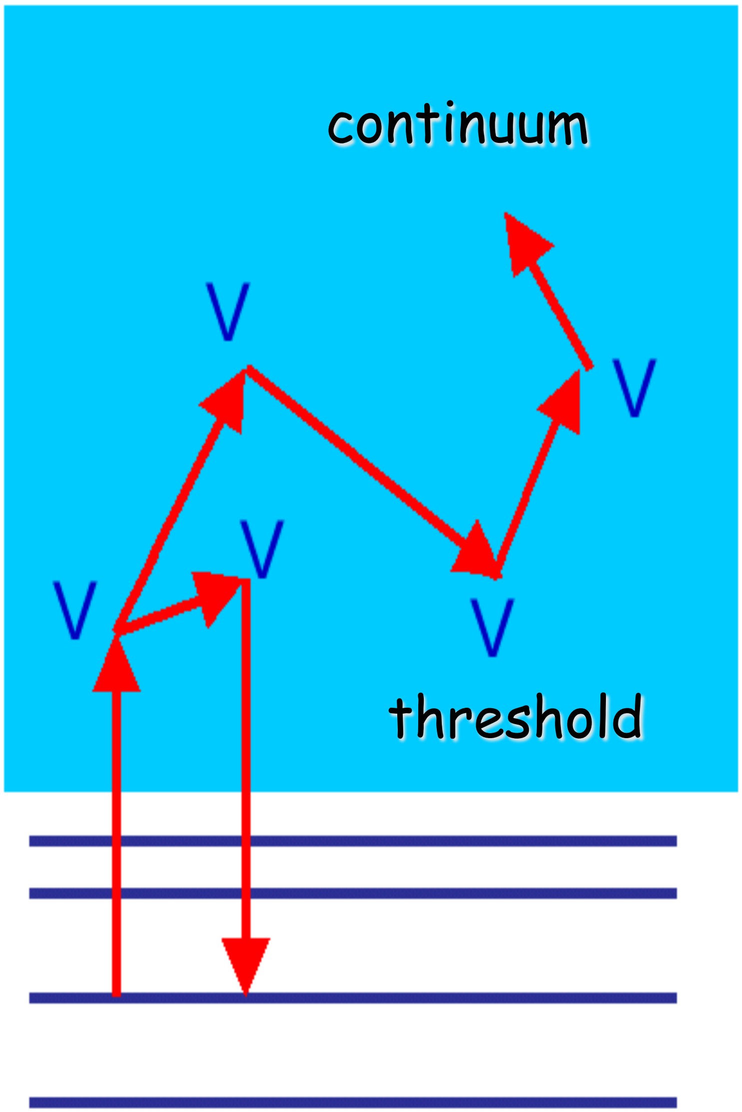
Continuum (CDCC)

$$\left|\varphi_b\right\rangle = e^{-iE_b t/\hbar}\left|E_b, J_b M_b\right\rangle$$

$$\left|\varphi_{jJM}^{(c)}\right\rangle = e^{-iE_j t/\hbar}\int \Gamma_j(E)\left|E, JM\right\rangle dE$$

$$\int \Gamma_i(E)\Gamma_j(E)\, dE = \delta_{ij}$$

continuum discretization



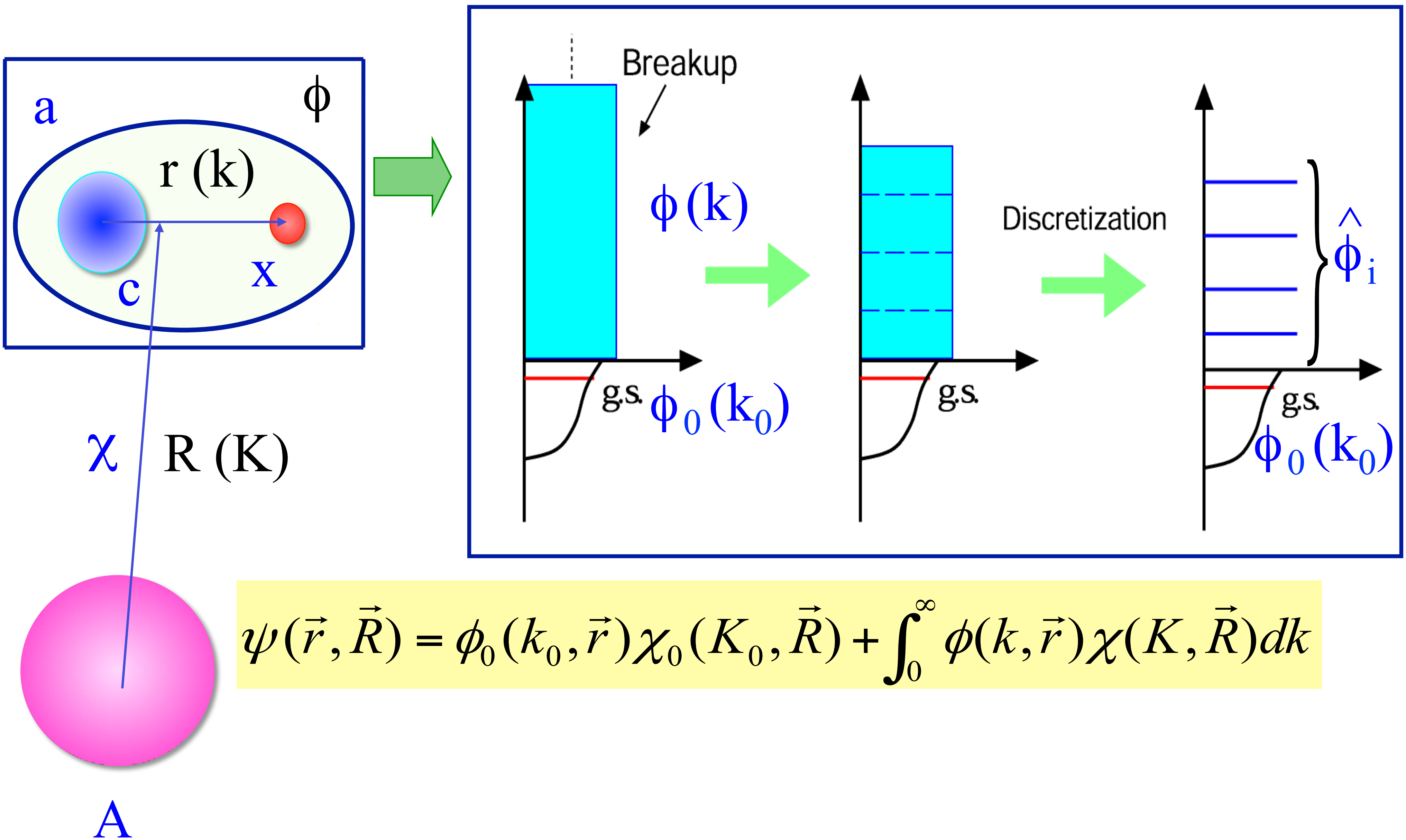
$$V_{\alpha\beta}(\mathbf{R}) = \left\langle \phi_\alpha(\mathbf{r})\middle| U(\mathbf{R},\mathbf{r}) \middle| \phi_\beta(\mathbf{r})\right\rangle$$

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 - E\right]\chi_\alpha(\mathbf{R}) = -\sum_{\beta=0}^{N} V_{\alpha\beta}(\mathbf{R})\chi_\beta(\mathbf{R})$$

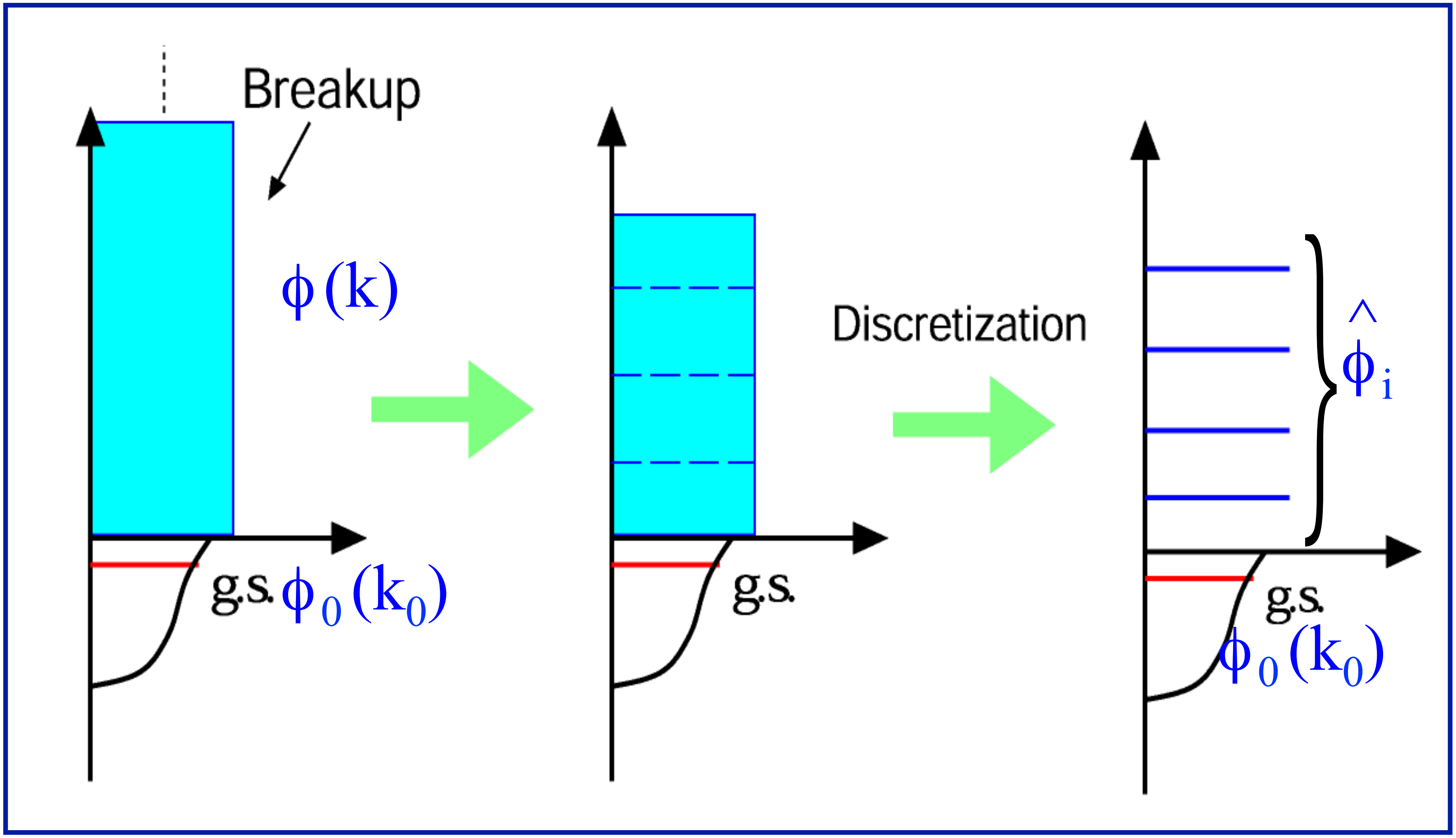
Bertulani, Canto, NPA 539, 163 (1992)

$^{11}Li+^{208}Pb$ (100 MeV/nucleon)

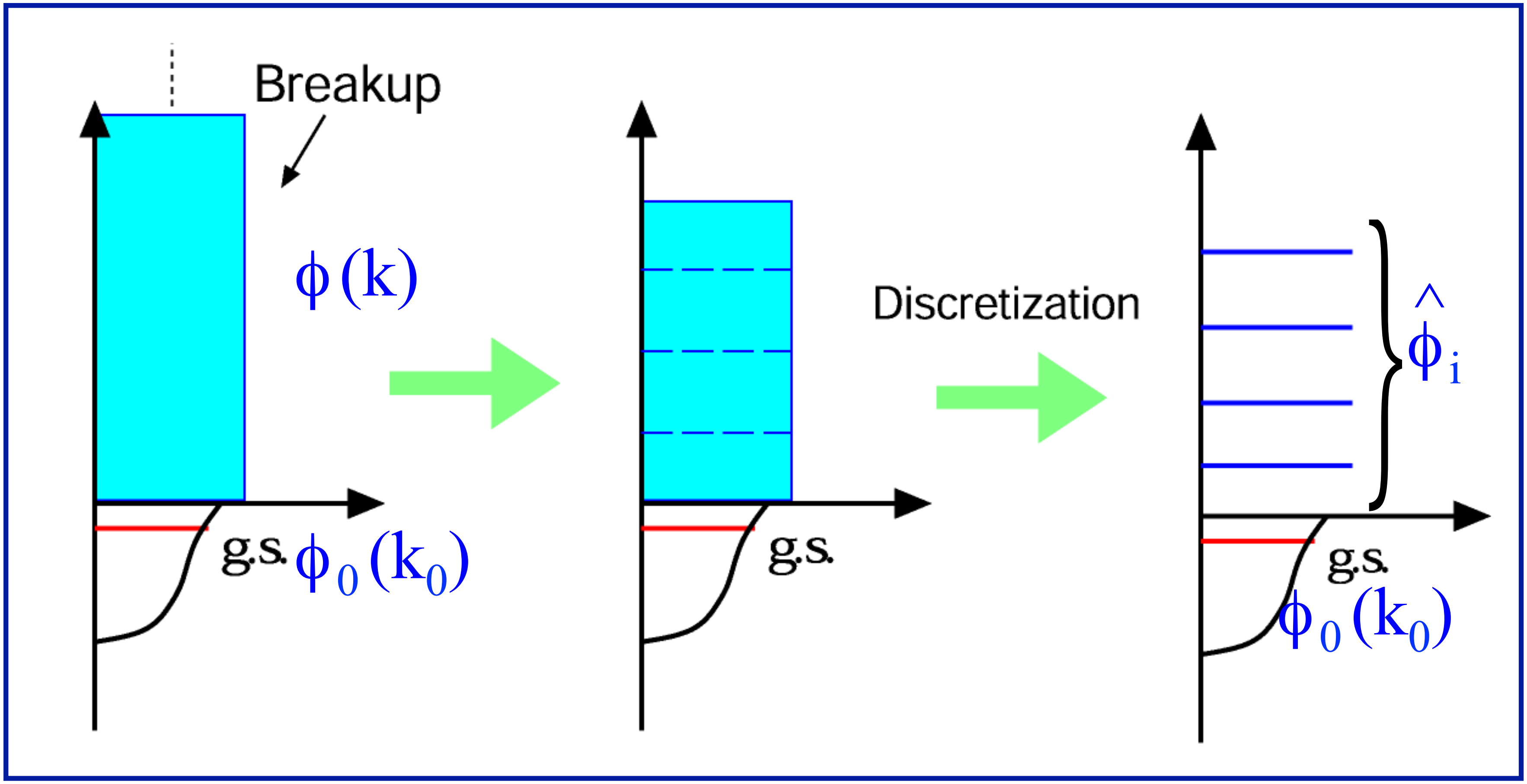
Theory movie in next 5 transparencies (enjoy!)©



$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

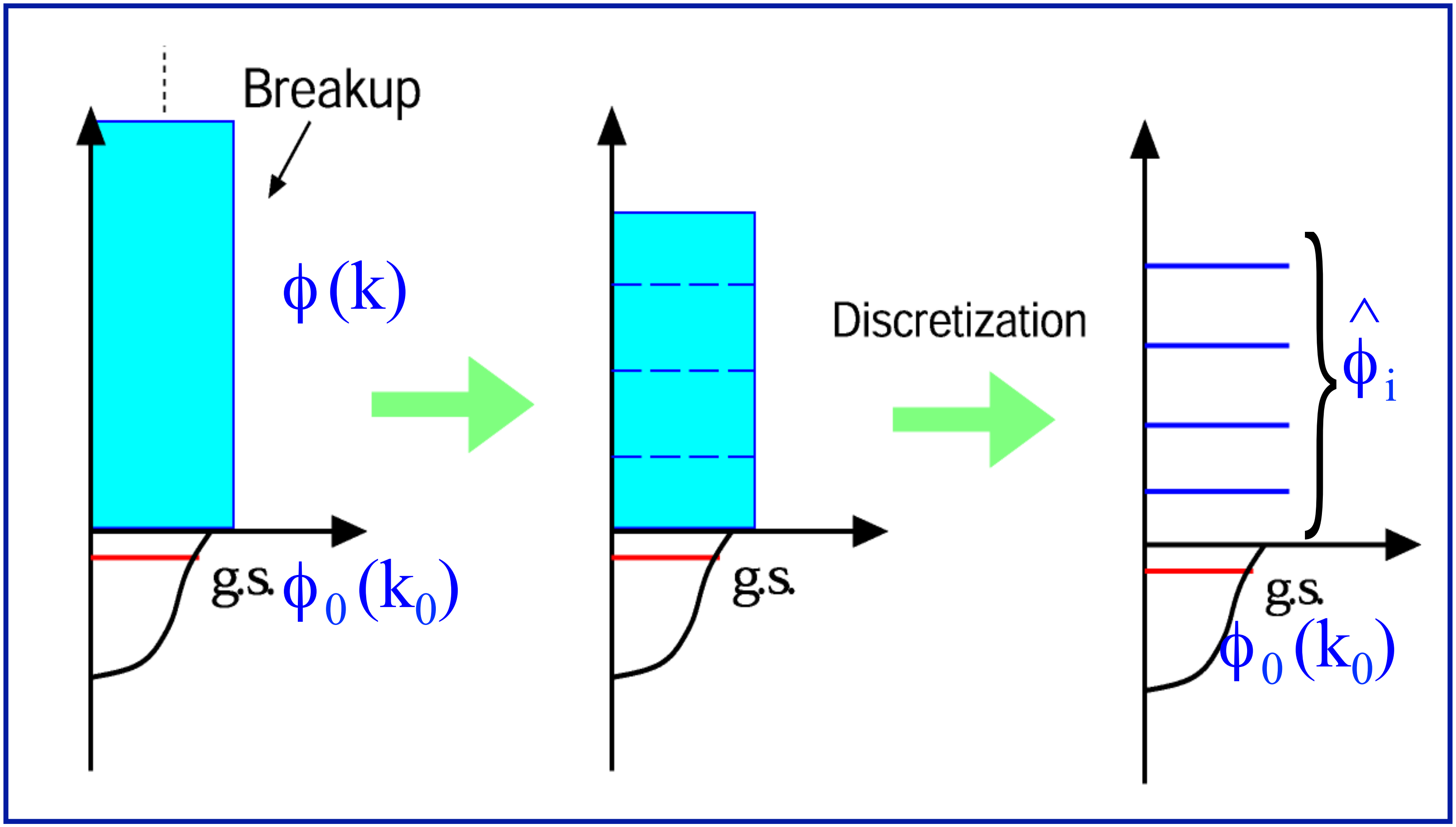


$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$



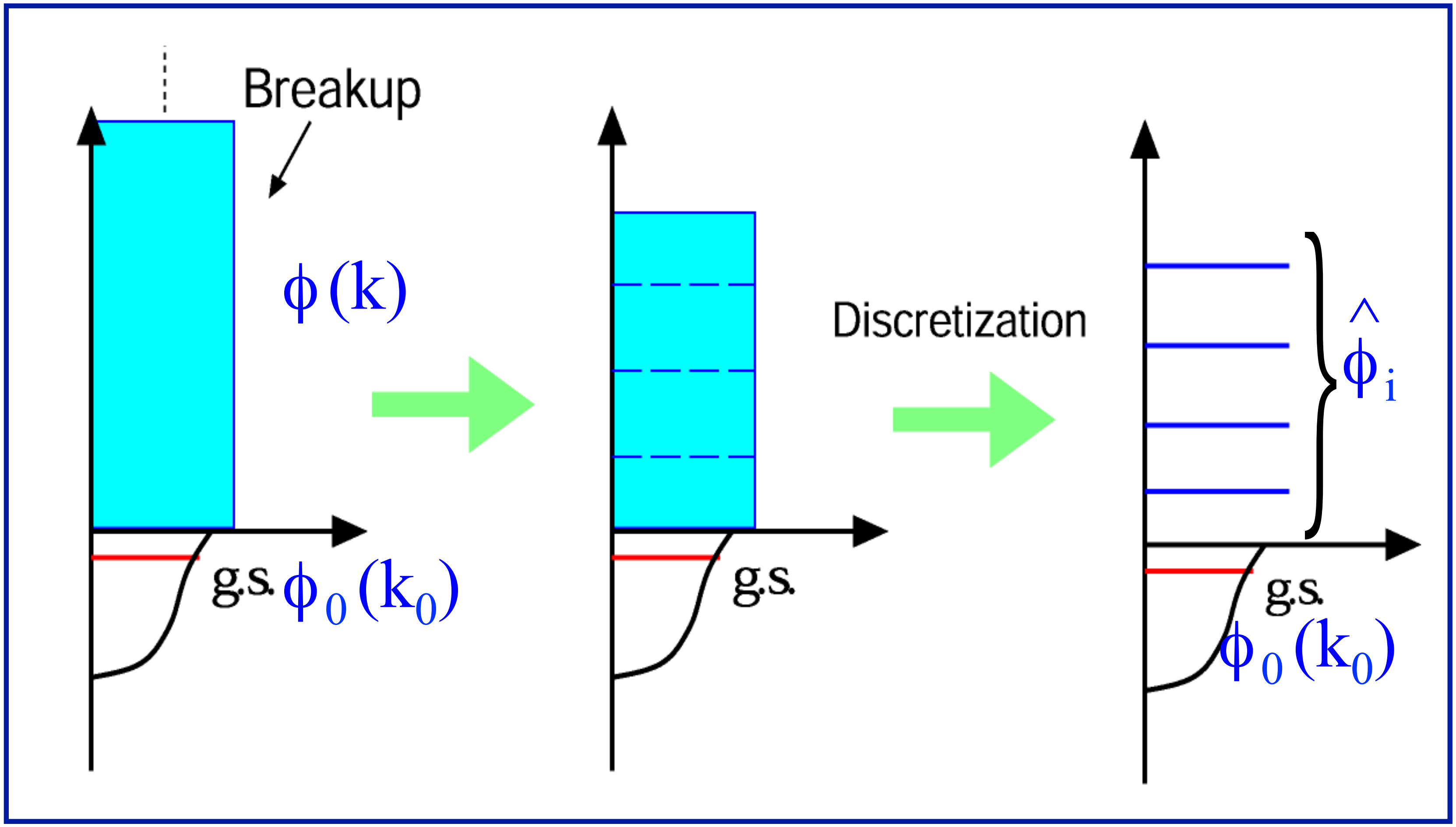
$$\psi(\vec{r},\vec{R}) = \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \int_0^\infty \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



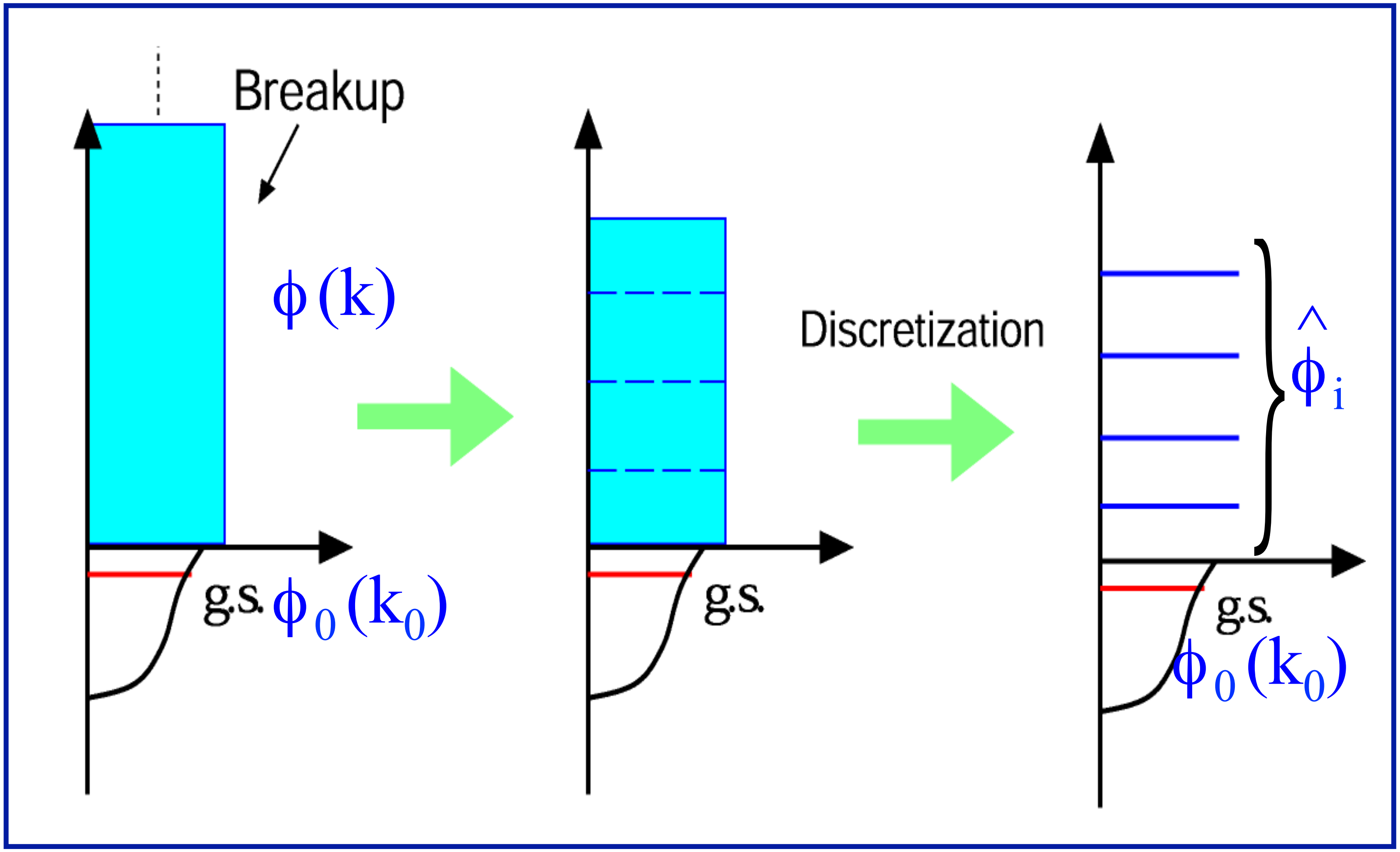
$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{max}} \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})\chi(K,\vec{R})dk$$

Truncation and Discretization



$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R})\int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

Truncation and Discretization



$$\psi(\vec{r},\vec{R}) \cong \phi_0(k_0,\vec{r})\chi_0(K_0,\vec{R}) + \sum_{i=1}^{i_{\max}} \chi(K_i,\vec{R}) \int_{k_{i-1}}^{k_i} \phi(k,\vec{r})dk$$

Truncation and Discretization

THE END.

$$\psi^{\mathrm{CDCC}}(\vec{r},\vec{R}) = \sum_{i=0}^{i_{\max}} \hat{\phi}_i(\vec{r})\hat{\chi}_i(K_i,\vec{R})$$

Eikonal CDCC

$$\psi = \sum_j S_j(z,b)\exp\left(ik_j z\right)\phi_{k_j}(\xi)$$

$$S_0 = \exp\left[\frac{1}{i\hbar v}\int_{-\infty}^{z} V\left(r'\right)dz'\right]$$ (ground state)

$$V = V_C + V_N$$

$$i\hbar \mathrm{v}\frac{\partial S_j(z,b)}{\partial z} = \sum_m \langle j|V|m\rangle S_m(z,b)\exp\left[-\left(k_m - k_j\right)z\right]$$ (excited states)

Note similarity with semiclassical t.d. equation with $z = vt$

$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_j a_j(t)V_{kj}(t)e^{i\left(E_k - E_j\right)t/\hbar}$$

Corrections due to energy conservation ($v \neq$ constant) straightforward

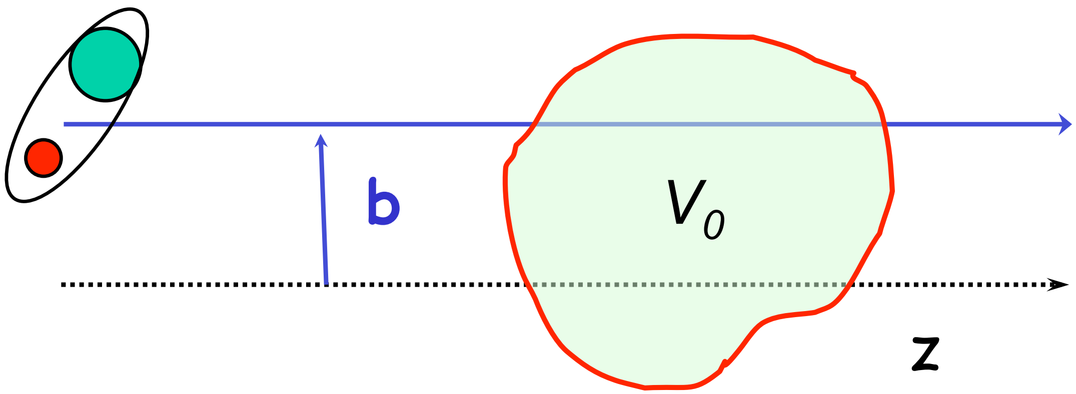
From S_j calculate ψ and observables of interest

Relativistic-CDCC

Bertulani, PRL 94, 072701 (2005)

$$\left[\nabla^2 + k^2 - U\right]\Psi(\mathbf{R},\mathbf{r}) = 0$$

$$U = V_0(2E - V_0)$$



$$\Psi(\mathbf{R},\mathbf{r}) = \sum_\alpha S_\alpha(\mathbf{b},z)\, e^{ik_\alpha z}\, \phi_\alpha(\mathbf{r}),$$

$$\mathbf{R} = (\mathbf{b},z)$$

$$U \approx 2V_0 E$$

$$\nabla^2 S << ik_z\partial_z S$$

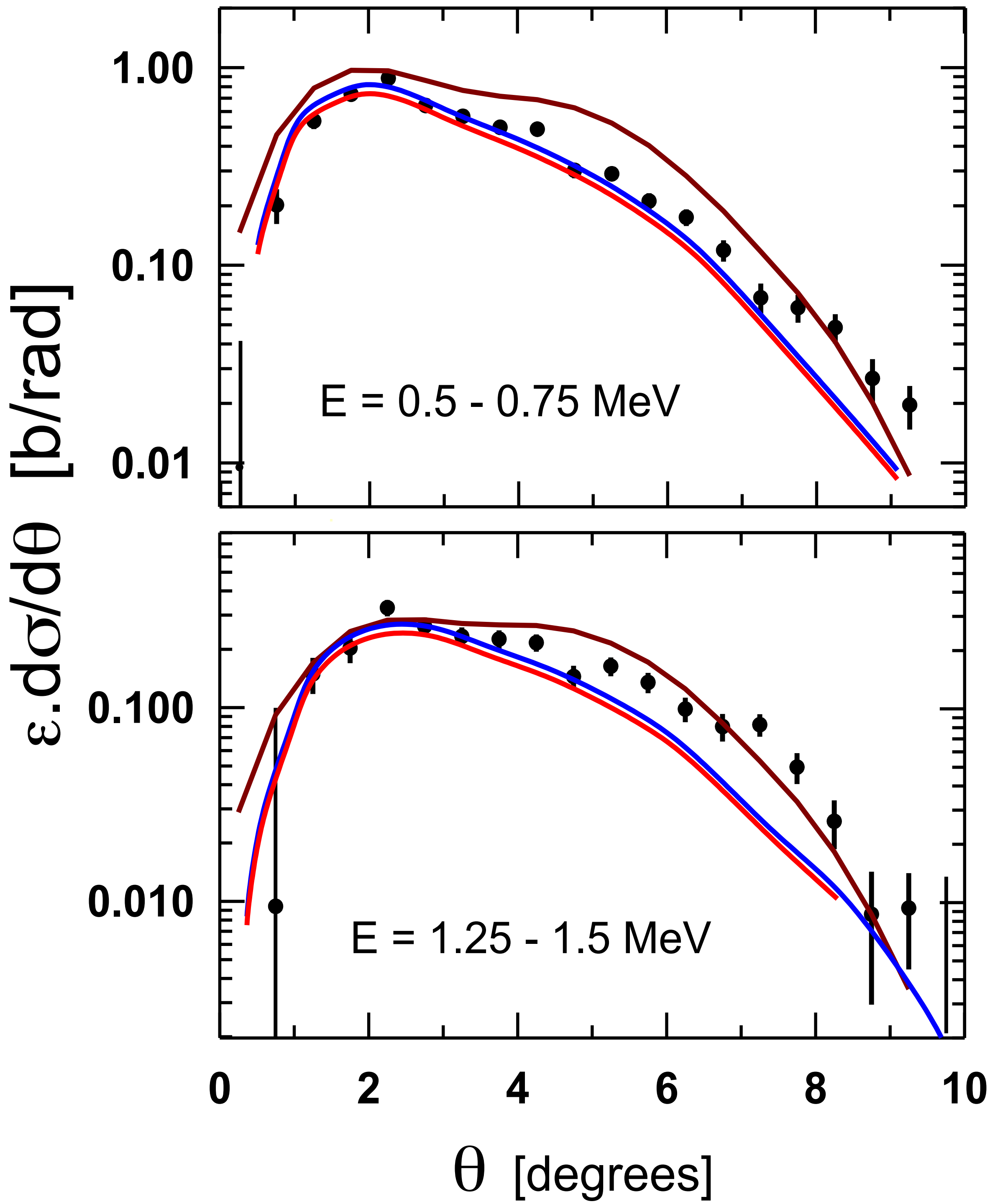
$$iv\partial_z S_\alpha(\mathbf{b},z) = \sum_\beta V_{\alpha\beta}(\mathbf{b},z) S_\beta(\mathbf{b},z)\, e^{i(k_\beta - k_\alpha)z}$$

$$f_\alpha(\mathbf{Q}) = -\frac{ik}{2\pi}\int d\mathbf{b}\; e^{i\mathbf{Q}.\mathbf{b}}\left[S_\alpha(\mathbf{b},z=\infty) - \delta_{\alpha,0}\right]$$

$$\mathbf{Q} = \mathbf{K}'_\perp - \mathbf{K}_\perp \qquad \alpha = jlJM$$

V_0 = time-like component of 4-vector → Relativistic CDCC = Lorentz invariant

Pb(^{8}B,p^{7}Be) at 50 MeV/nucleon



DATA: Kikuchi et al, 1997

LO

Bertulani, Gai, NPA 626, 227 (1998)

All orders

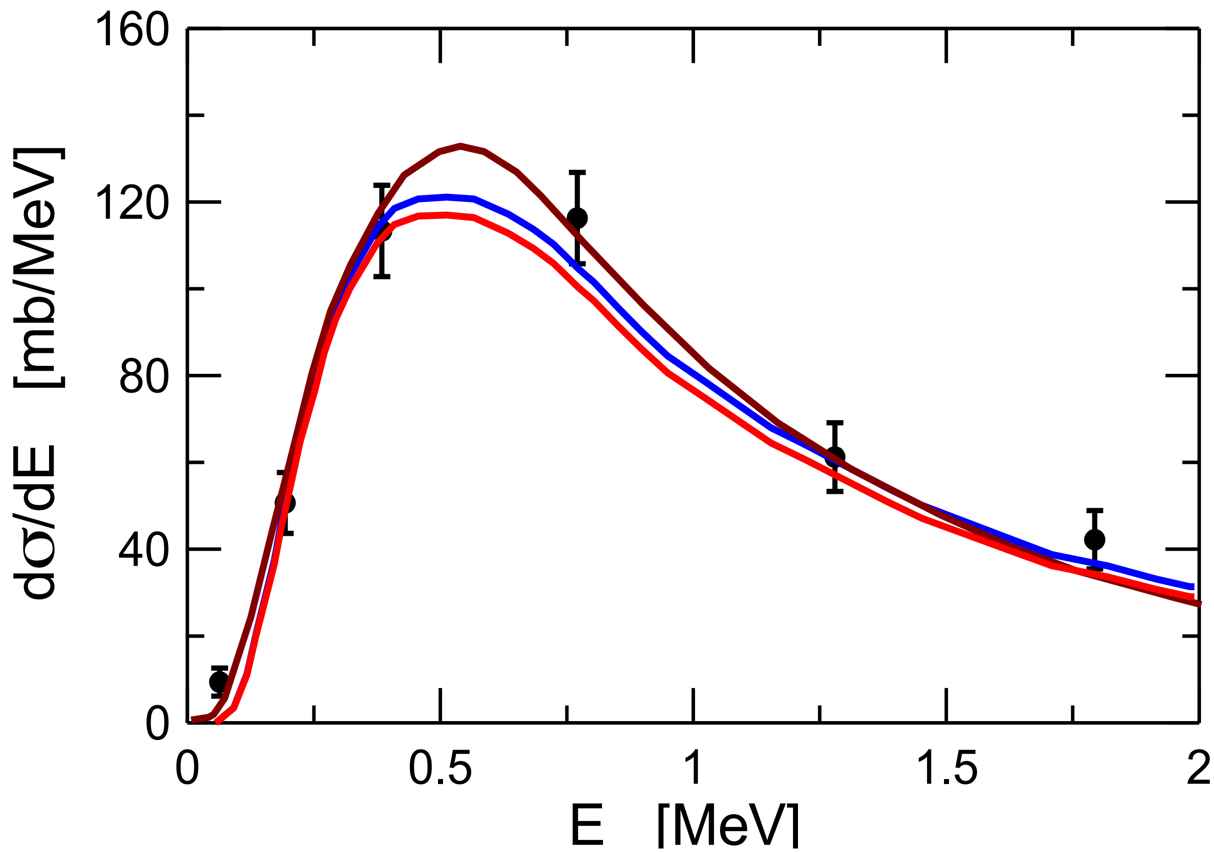
All orders relativistic

Bertulani, PRL 94, 072701 (2005)

V_0 = Coulomb + nuclear with relativistic corrections

5-7% effect

Pb(^{8}B,p^{7}Be) at 83 MeV/nucleon



DATA: Davids et al, 2002

LO

all orders

all orders relativistic

V_0 = Coulomb + nuclear with relativistic corrections

4-10% effect

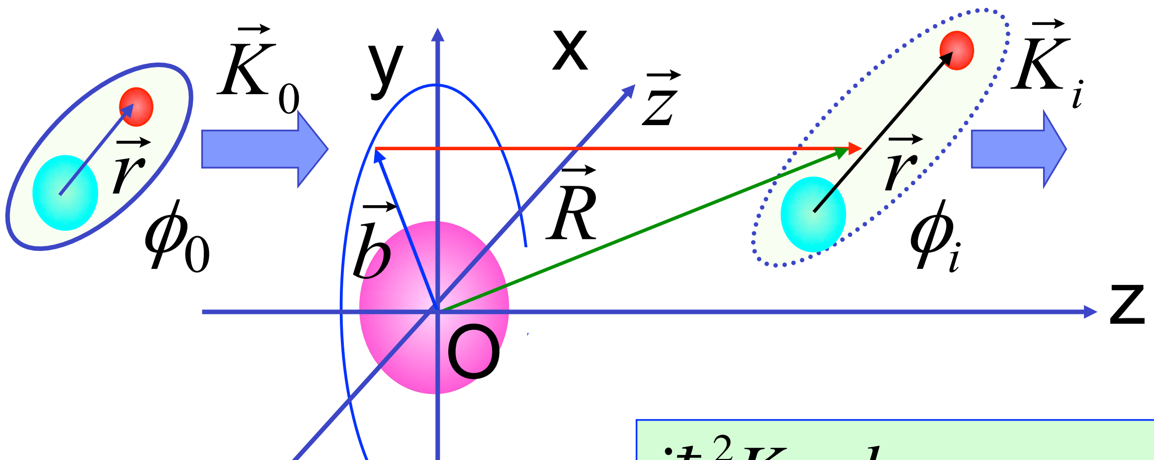
Transition: low to high energies

Eikonal scattering waves $\hat{S}_i(K_i, \vec{R})$

$$\psi^{E-CDCC} = \sum_i \hat{\phi}_i(\vec{r})\ \hat{S}_i(b,z)\exp(i\vec{K}_i \cdot \vec{R})$$

$$K_i = \sqrt{2\mu_R(E-\varepsilon_i)}/\hbar,$$

Energy conservation



- Boundary condition

$$\hat{S}_i(b,z) \xrightarrow{z \to -\infty} \delta_{i,0}$$

$$\Delta \hat{S}_i(b,z) \cong 0$$

$$\frac{i\hbar^2 K_i}{\mu_R}\frac{d}{dz}\hat{S}_i^{(b)}(z) = \sum_{i'} \mathrm{F}_{ii'}^{(b)}(z)\ \hat{S}_{i'}^{(b)}(z) e^{i(K_{i'}-K_i)z}$$

Eikonal scattering amplitude transformed into QM form

$$f_{i,0}^{\mathrm{E}} = \sum_L f_L^{\mathrm{E}} \equiv \sum_L \frac{2\pi}{iK_i}\sqrt{\frac{2L+1}{4\pi}} i^m Y_{Lm}(\Omega)[S_{i,0}^{b(L;i)} - \delta_{i,0}]$$

Hybrid scattering amplitude is given by

$$f_{i,0}^{\mathrm{H}} \equiv \sum_{L=0}^{L_\mathrm{C}} f_L^{\mathrm{Q}} + \sum_{L=L_\mathrm{C}+1}^{L_{\max}} f_L^{\mathrm{E}}$$

Ogata., et al, PRC68, 064609 (2003)

Relativistic CDCC

Form factor of non-rel. E-CDCC

$$F_{c'c}^{(b)}(Z) = \left\langle \Phi_{c'} \middle| U_{1A} + U_{2A} \middle| \Phi_c \right\rangle_{\mathbf{r}} e^{-i(m-m')\phi} = \sum_{\lambda} F_{c'c}^{(b);\lambda}(Z)$$

Lorentz tranform of form factor and coordinates

$$F_{c'c}^{(b);\lambda}(Z) \to f_{\lambda,m'-m} \gamma F_{c'c}^{(b)\lambda}(\gamma Z)$$

$$f_{\lambda,m'-m}^{\mathrm{Coul}} = \begin{cases} 1/\gamma & (\lambda=1,\ m'-m=0) \\ \gamma & (\lambda=2,\ m'-m=\pm 1) \\ 1 & (\text{otherwise}) \end{cases}$$

$$f_{\lambda,m'-m}^{\mathrm{nucl}} = 1$$

Assumptions

- ✓ Point charges for 1, 2 and A
- ✓ Neglecting far-field (r_i > R) contribution
- ✓ Correction to nuclear form factor

Ogata, Bertulani, PTP 121 (2009), 1399
PTP, 123 (2010) 701

Reaction

$^{208}Pb(^{8}B,\ ^{7}Be+p)$ at 250 A MeV and 100 A MeV

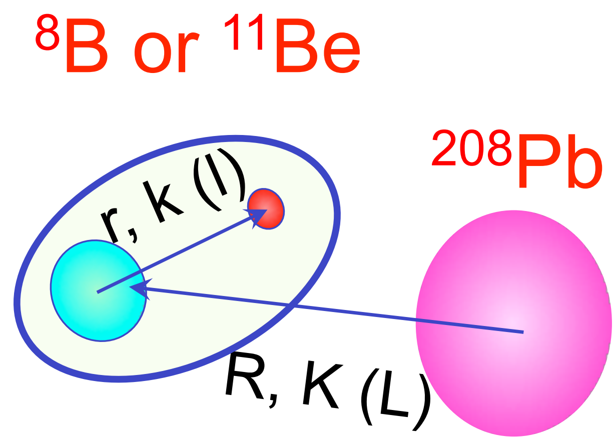
$^{208}Pb(^{11}Be,\ ^{10}Be+n)$ at 250 A MeV and 100 A MeV

Projectile wave function and distorting potential

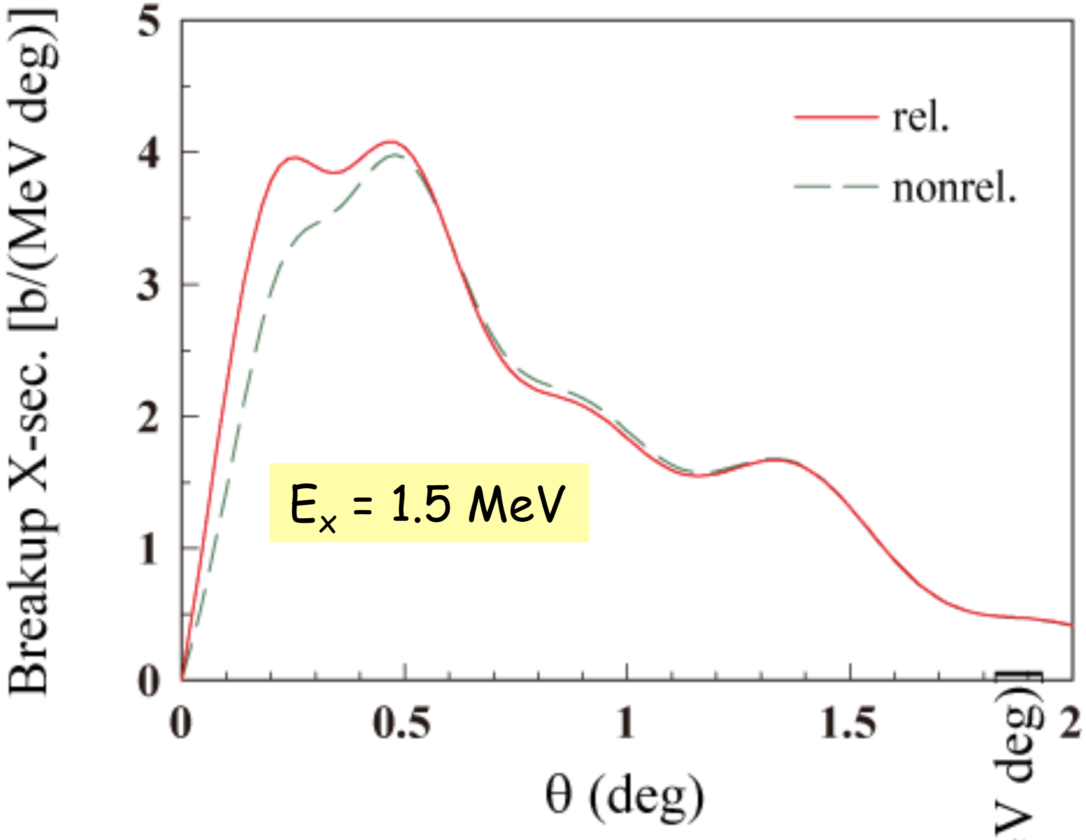
Standard Woods-Saxon

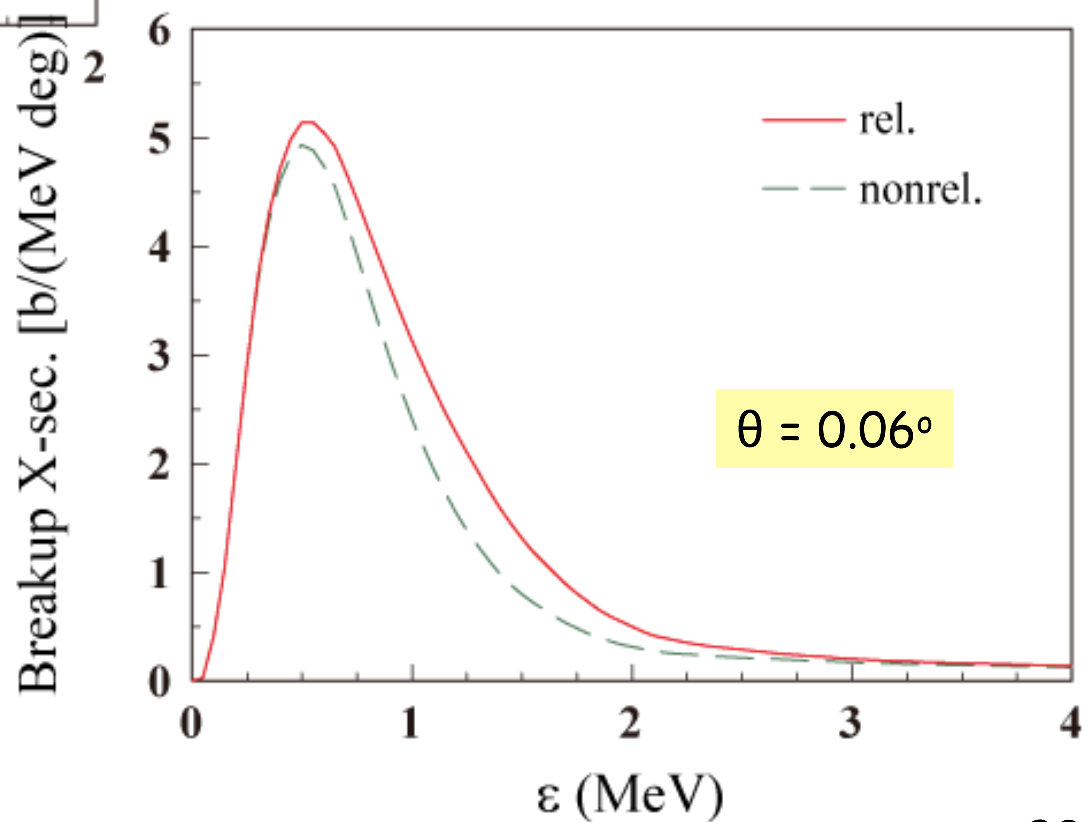
Modelspace

^{8}B	^{11}Be
l_{max}= 3	l_{max}= 3
N_s=20, $N_{p\text{-}d}$=10 , N_f=5	$N_{s,p}$=20, N_d=10 , N_f=5
ε_{max}=10 MeV	ε_{max}=10 MeV
r_{max}= 200 fm	r_{max}= 200 fm
R_{max}= 500 fm	R_{max}= 450 fm
N_{ch} = 138	N_{ch} = 166



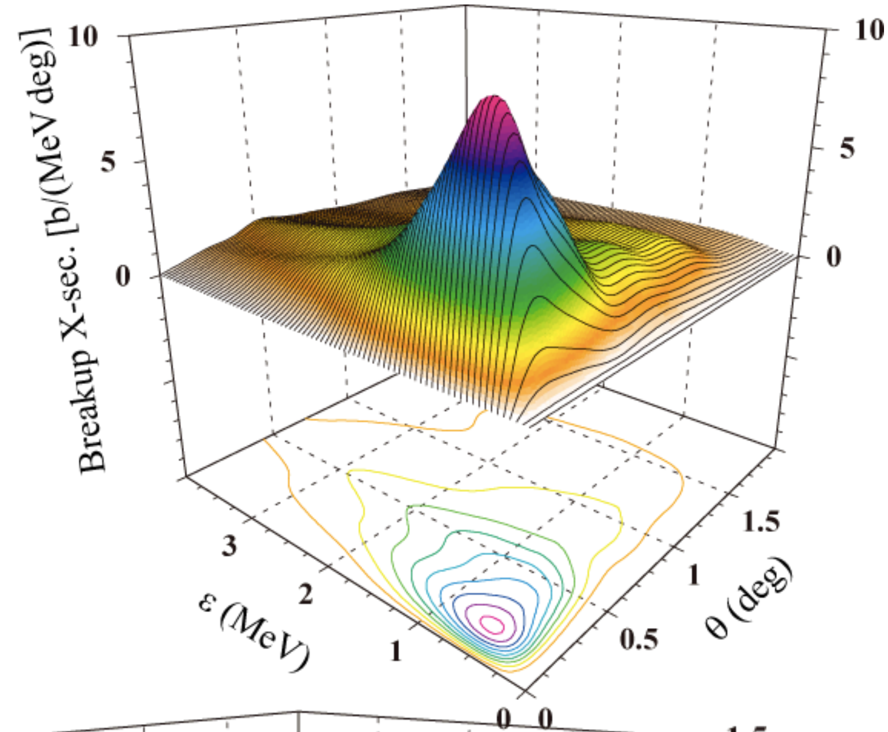
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon



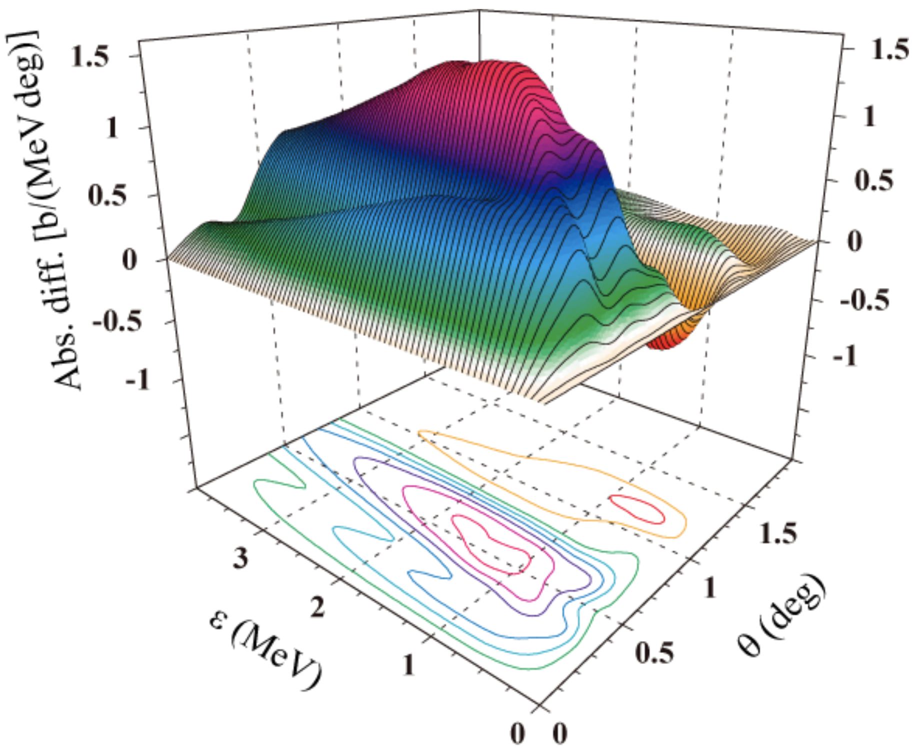


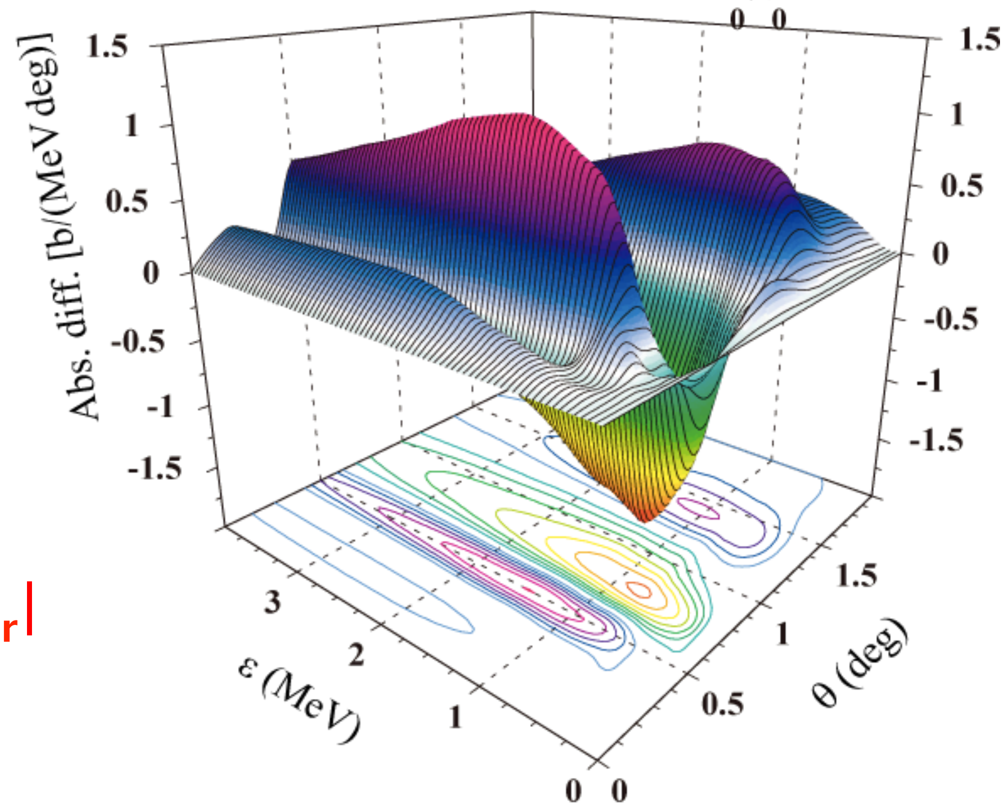
Pb(^{8}B,p^{7}Be) at 250 MeV/nucleon

all orders



$|\sigma_{all} - \sigma_{NR}|$





$|\sigma_{all} - \sigma_{no\text{-}nuclear}|$

Please don't be scared, yet

Please don't be scared, yet

But things can get much worse

Relativistic MF nucleus-nucleus potential

Long, Bertulani, PRC 83, 024907 (2011).

σ, ω, ρ and γ exchange

$$E = \int d^3r \sum_a \bar{\psi}_a \left(-i\gamma\cdot\nabla + M\right)\psi_a$$

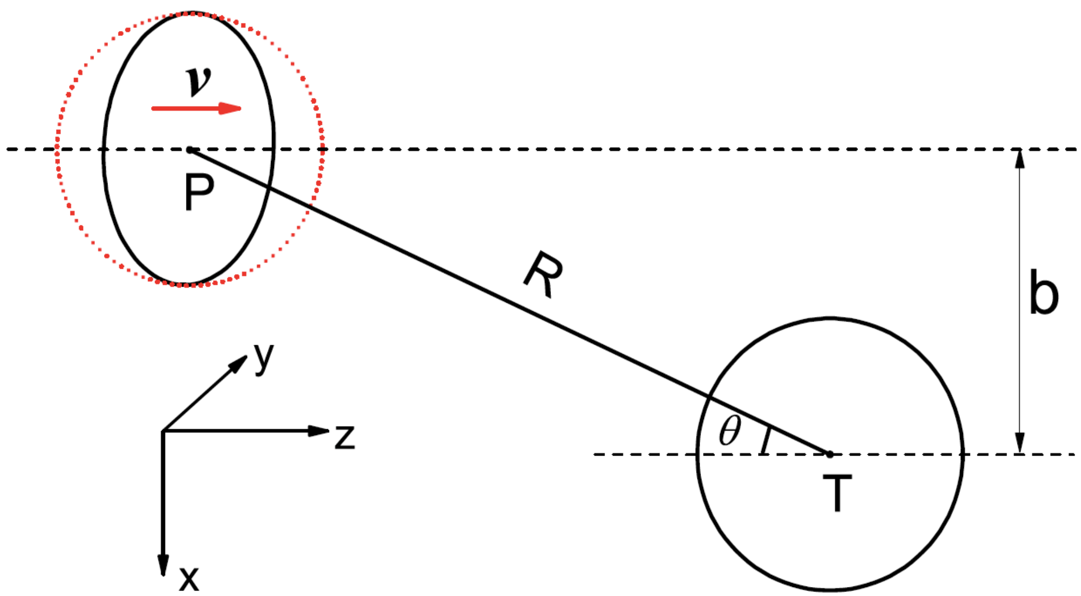
$$+\frac{1}{2}\sum_{\phi=\sigma,\omega,\rho,\gamma}\int d^3r d^3r' \sum_{ab}\bar{\psi}_a(\mathbf{r})\bar{\psi}_b(\mathbf{r}')\Gamma_\phi(\mathbf{r},\mathbf{r}')D_\phi(\mathbf{r}-\mathbf{r}')\psi_a(\mathbf{r})\psi_b(\mathbf{r}')$$

$$\Gamma_\phi(\mathbf{r},\mathbf{r}') = -g_\sigma(\mathbf{r})g_\sigma(\mathbf{r}')$$

$$\Gamma_\omega(\mathbf{r},\mathbf{r}') = -\left(g_\omega\gamma^\mu\right)_{\mathbf{r}}\cdot\left(g_\omega\gamma_\mu\right)_{\mathbf{r}'}$$

$$\Gamma_\rho(\mathbf{r},\mathbf{r}') = -\left(g_\rho\gamma^\mu\vec{\tau}\right)_{\mathbf{r}}\cdot\left(g_\rho\gamma_\mu\vec{\tau}\right)_{\mathbf{r}'}$$

$$\Gamma_\gamma(\mathbf{r},\mathbf{r}') = \frac{e^2}{4}\left[\gamma^\mu(1-\tau_z)\right]_{\mathbf{r}}\cdot\left[\gamma_\mu(1-\tau_z)\right]_{\mathbf{r}'}$$



$$D_\phi = \frac{1}{4\pi}\frac{e^{m_\phi|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

$$D_\gamma = \frac{1}{|\mathbf{r}-\mathbf{r}'|}$$

Lorentz transform

$$x_p = x_t + b, \qquad y_p = y_t$$

$$z_p = \gamma(z_t + R\cos\theta)$$

$$E(A_t, A_p, v) = E(A_t) + E(A_p, \mathrm{v}) + \mathsf{E}(A_t, A_p, \mathrm{v})$$

$$\mathsf{E}(A_t, A_p, \mathrm{v}) = \sum_{\phi=\sigma,\omega,\rho,\gamma} \int d^3r \int d^3r' \sum_{ab} \overline{\psi}_{t,a}(\mathbf{r}) \overline{\psi}_{p,b}(\mathbf{r}') \, \Gamma_\phi(\mathbf{r},\mathbf{r}') \, D_\phi(\mathbf{r}-\mathbf{r}') \, \psi_{t,a}(\mathbf{r}) \psi_{p,b}(\mathbf{r}')$$

Ex: σ and ω contributions

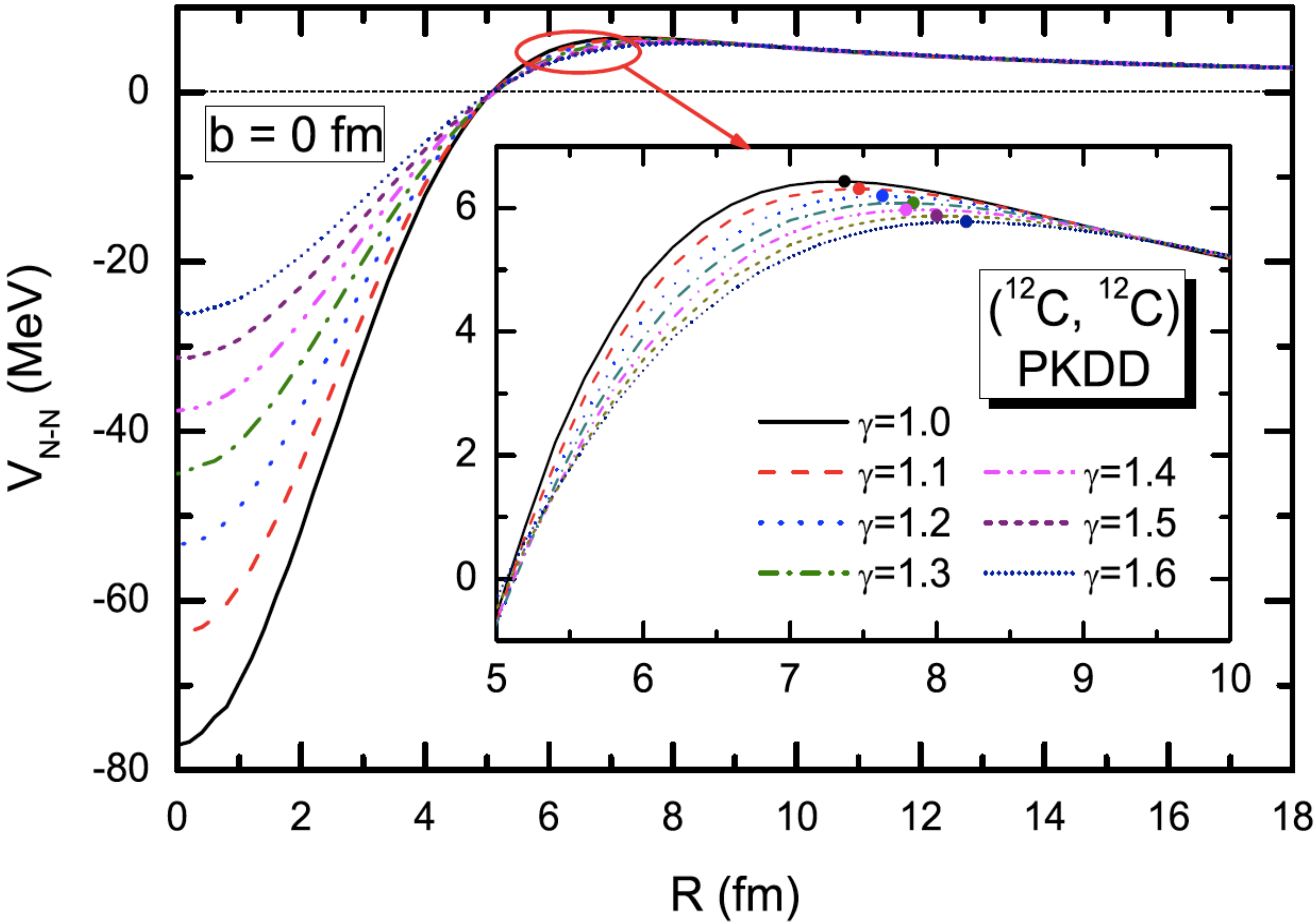
$$\mathsf{E}_\sigma = -\frac{1}{\gamma} \int d^3r_t \int d^3r_p' \, g_\sigma(\mathbf{r}_t) \, \rho_{s,t}(\mathbf{r}_t) \, D_\sigma(\mathbf{r}-\mathbf{r}') \, \rho_{s,p}(\mathbf{r}_p') g_\sigma(\mathbf{r}_p')$$

$$\mathsf{E}_\omega = \int d^3r_t \int d^3r_p' \, g_\omega(\mathbf{r}_t) \, \rho_{b,t}(\mathbf{r}_t) \, D_\omega(\mathbf{r}-\mathbf{r}') \, \rho_{b,p}(\mathbf{r}_p') g_\omega(\mathbf{r}_p')$$

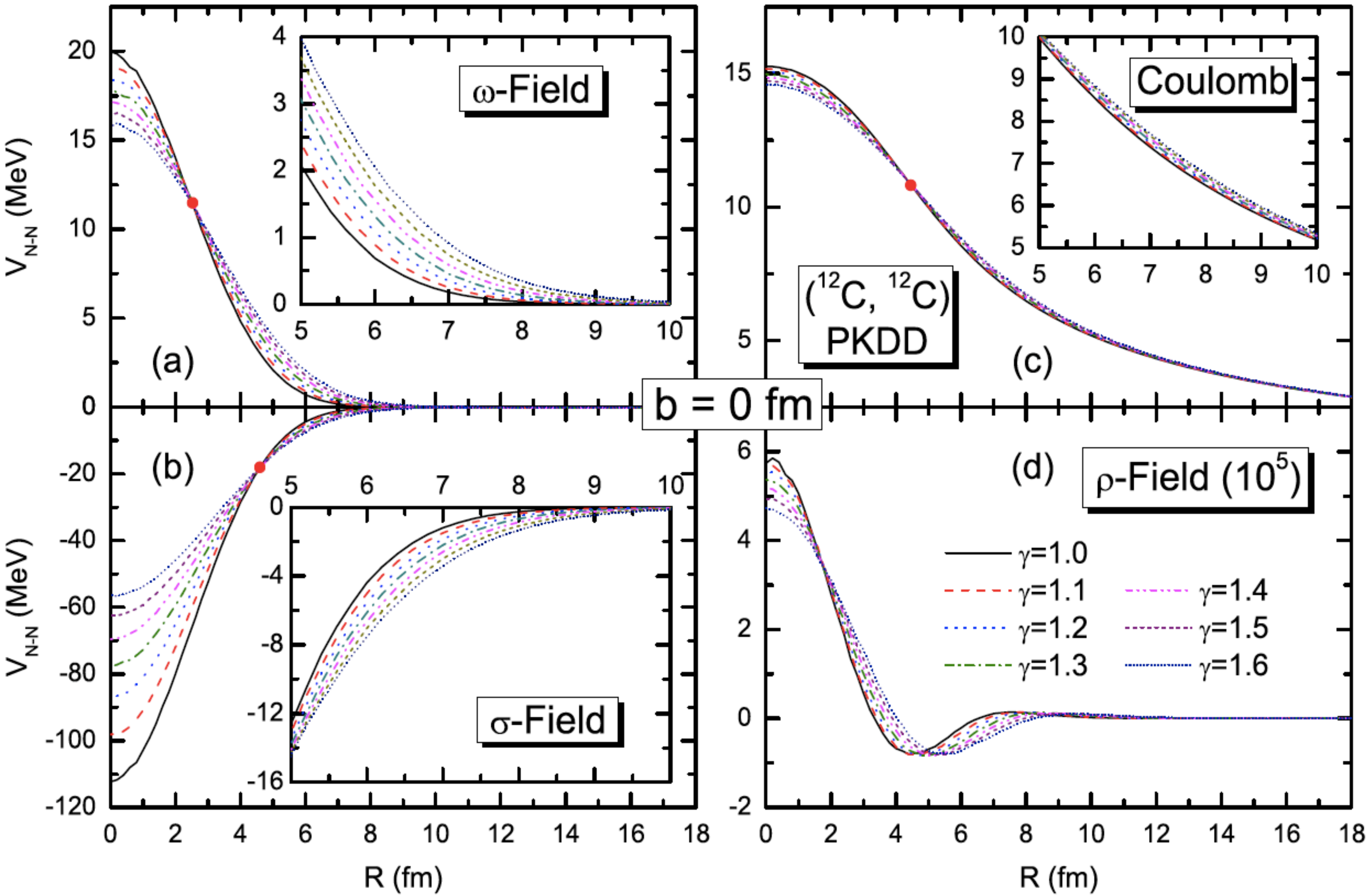
$$\rho_s(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \psi_a(\mathbf{r}), \qquad \rho_b(\mathbf{r}) = \sum_a \overline{\psi}_a(\mathbf{r}) \gamma^0 \psi_a(\mathbf{r})$$

Projectile densities boosted to the target frame

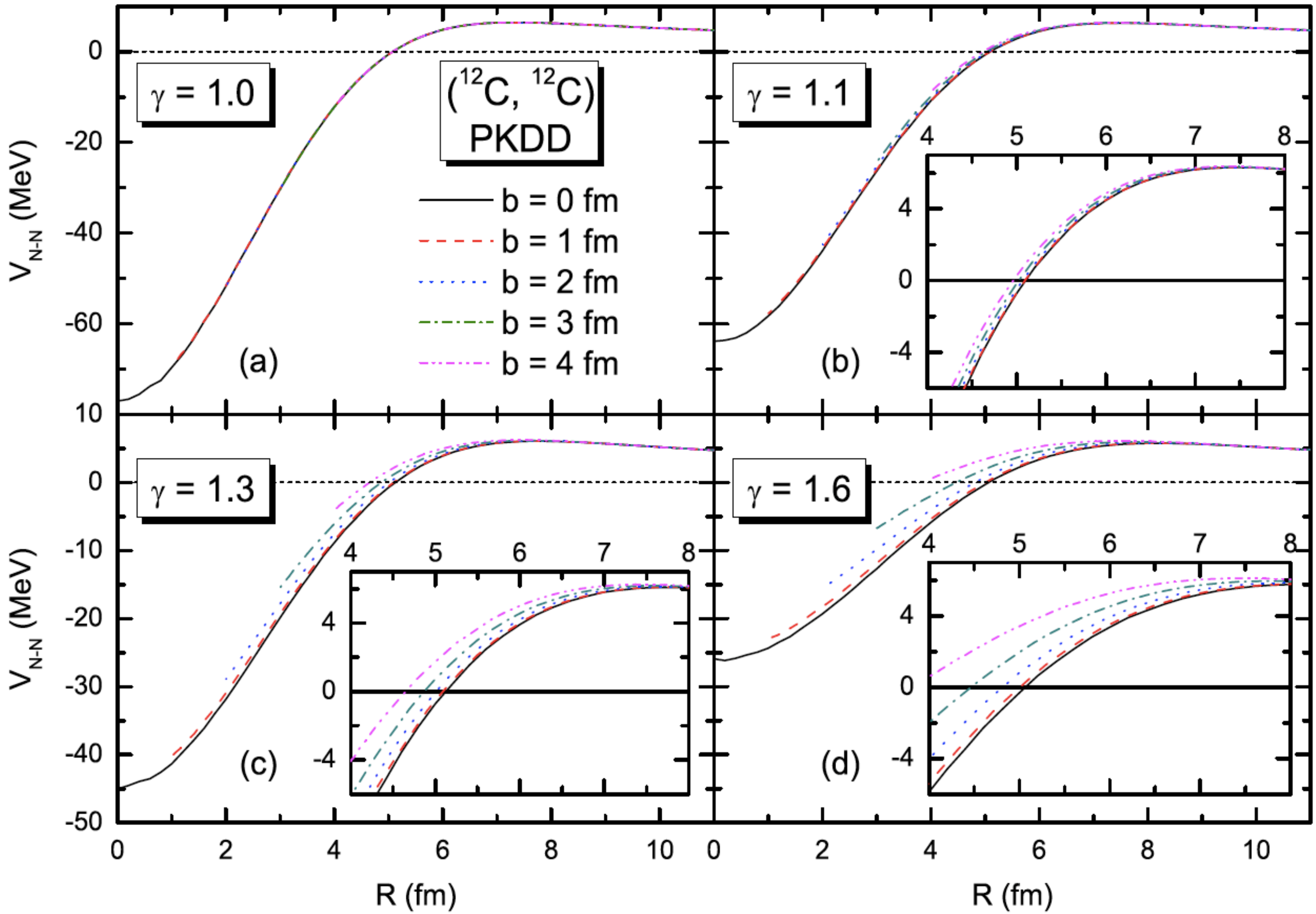
Results for $^{12}C + ^{12}C$

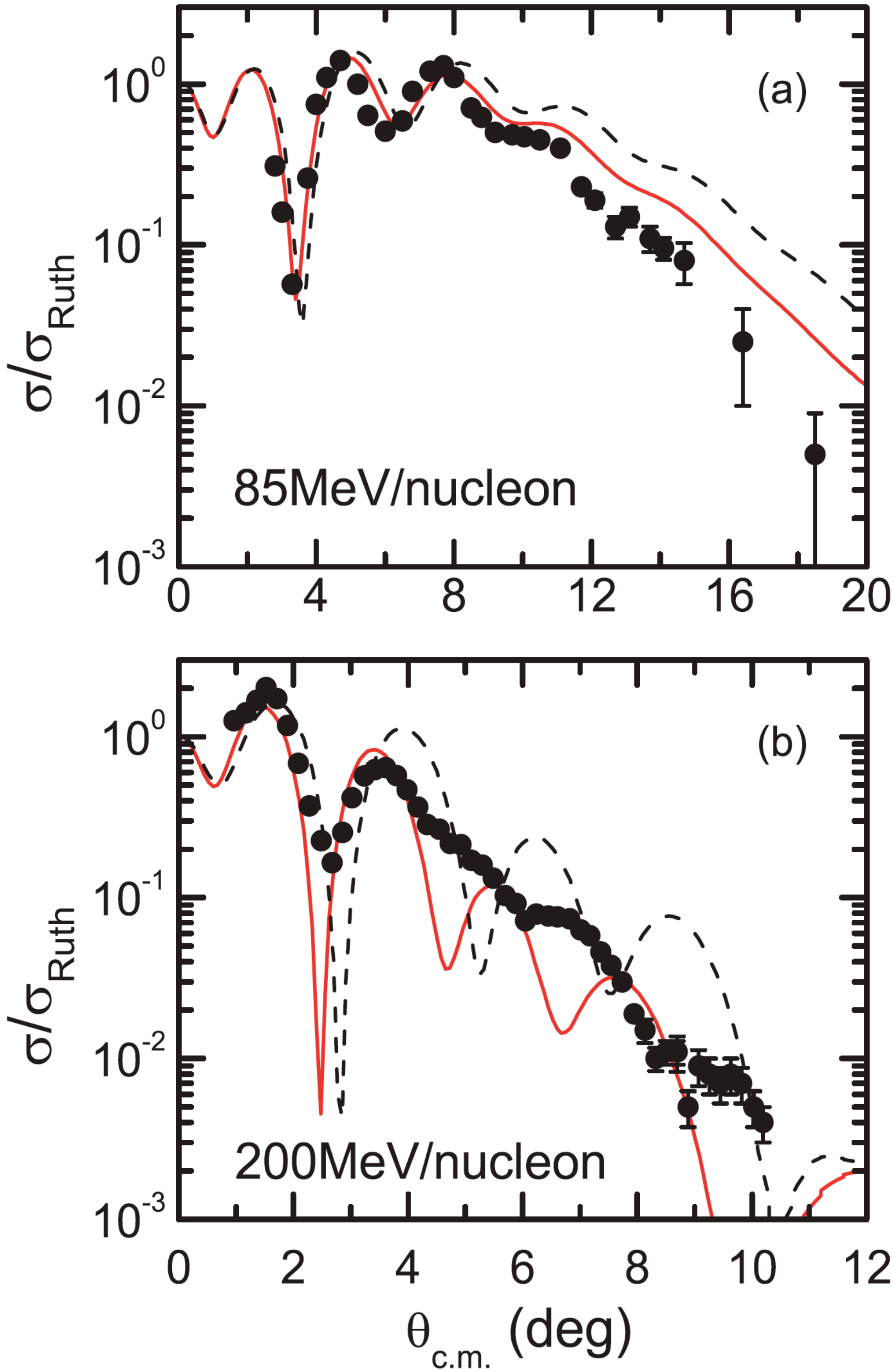


Contribution of different fields



Dependence on energy and impact parameter





$^{12}C + ^{12}C$

Elastic scattering

—— relativistic

- - - - - non-relativistic

Conclusions:

Too many. And my time is out.

But pictures are worth a thousand words*.

Conclusions:

Too many. And my time is out.

But pictures are worth a thousand words*.

* Locker room of FC Lybia

Thank you so much.

The Nuclear Astrophysics group at Catania.