Coulomb Dissociation for Nuclear Astrophysics

C. A. Bertulani (Texas A&M-Commerce)

HW1 - Why are we here?

HW1 - Why are we here?

N. Alahari

HW2 - Why are we here?

HW2 - Why are we here?

If you want to learn about Coulomb dissociation

HW2 - Why are we here?

If you want to learn about Coulomb dissociation

Talk to: Tohru Motobayashi

HW3 - Why are we here?

HW3 - Why are we here?

If you want to know where Gaddafi is

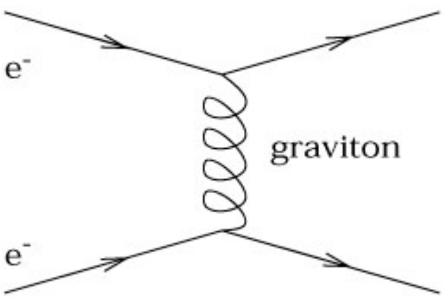
HW3 - Why are we here?

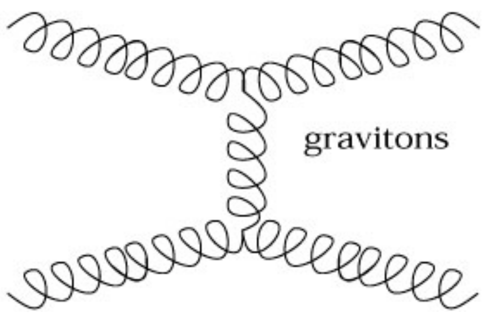
If you want to know where Gaddafi is

Talk to the Libyanese intelligence

Gravity

Classical: $$V = \frac{c}{r}$$



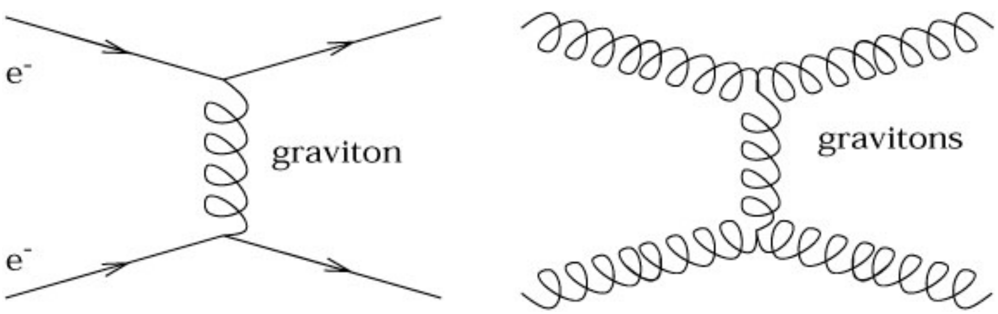


Quantum: $$m_{graviton} = 0$$

Non-linear

Gravity

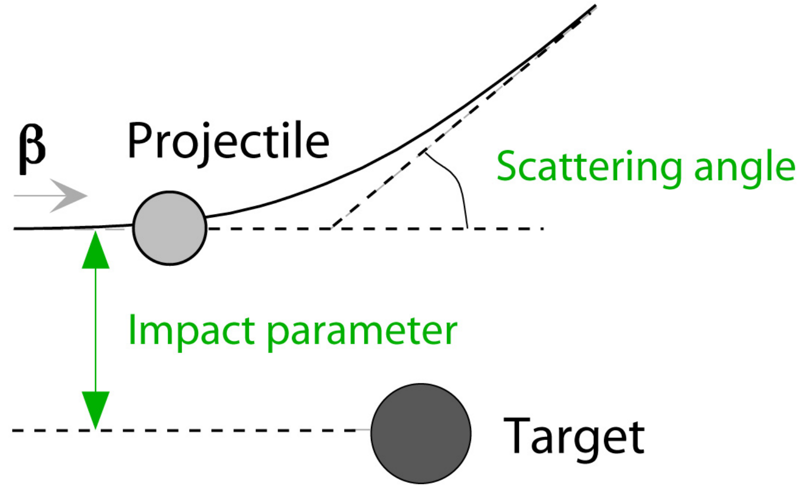
Classical: $V = \frac{c}{r}$



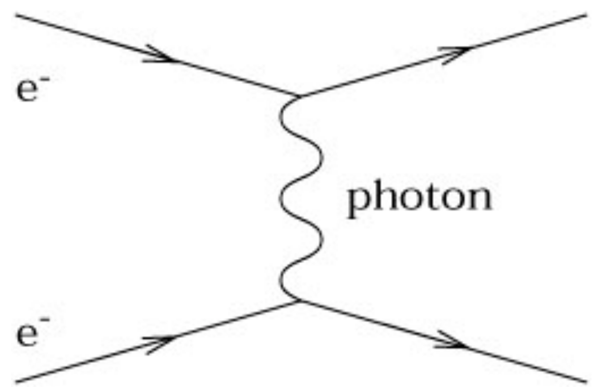
Quantum: $m_{graviton} = 0$

Non-linear

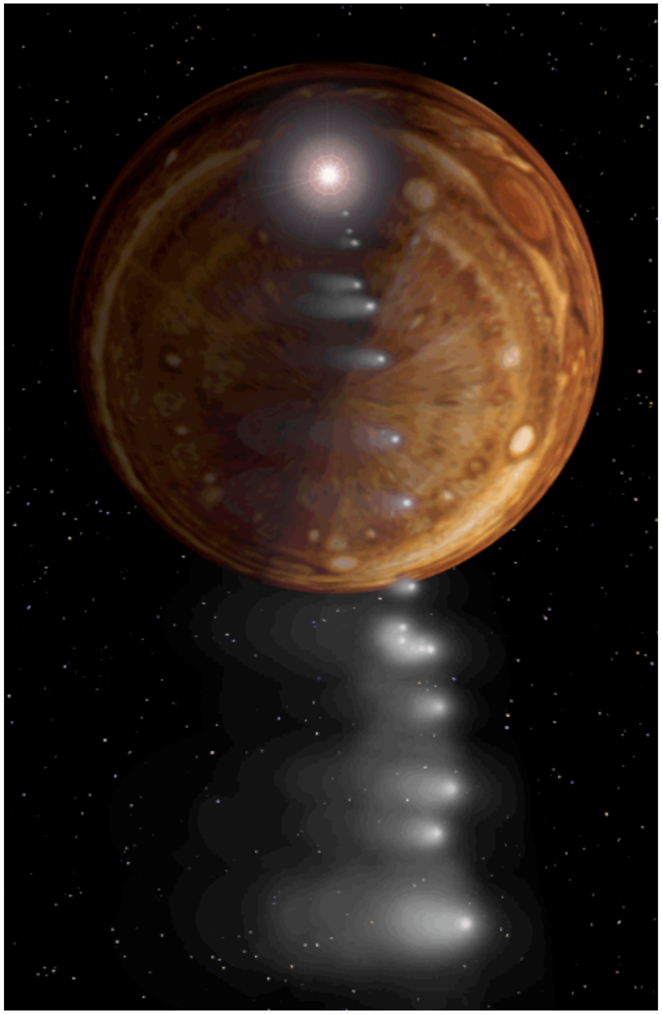
Coulomb



Classical: $V = \frac{c'}{r}$

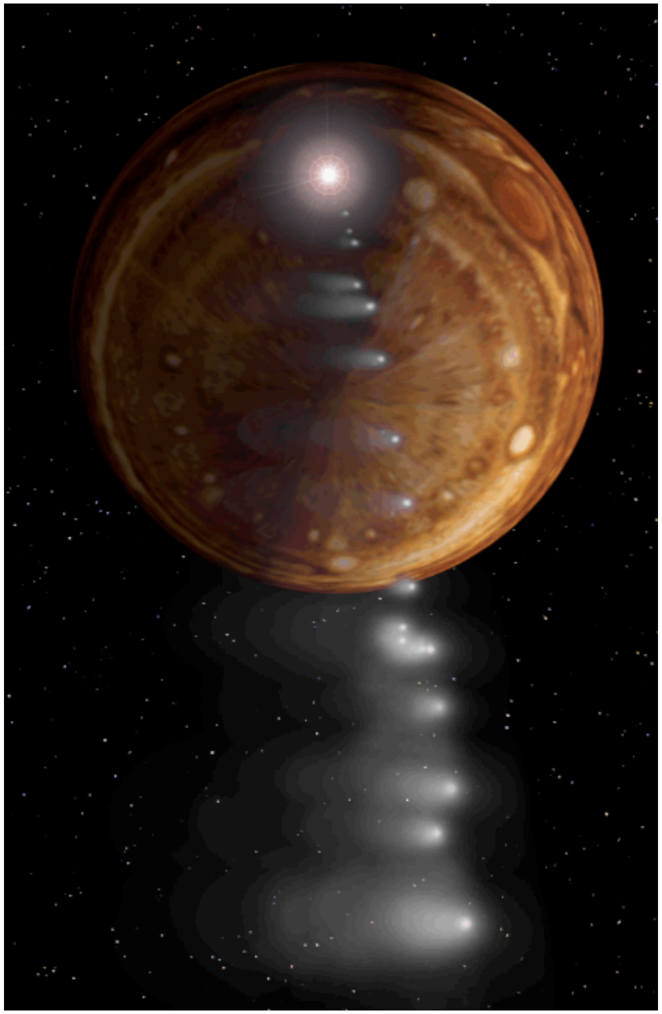


Quantum: $m_{photon} = 0$



Shoemaker-Levi comet
break into many pieces

Classical and complicated



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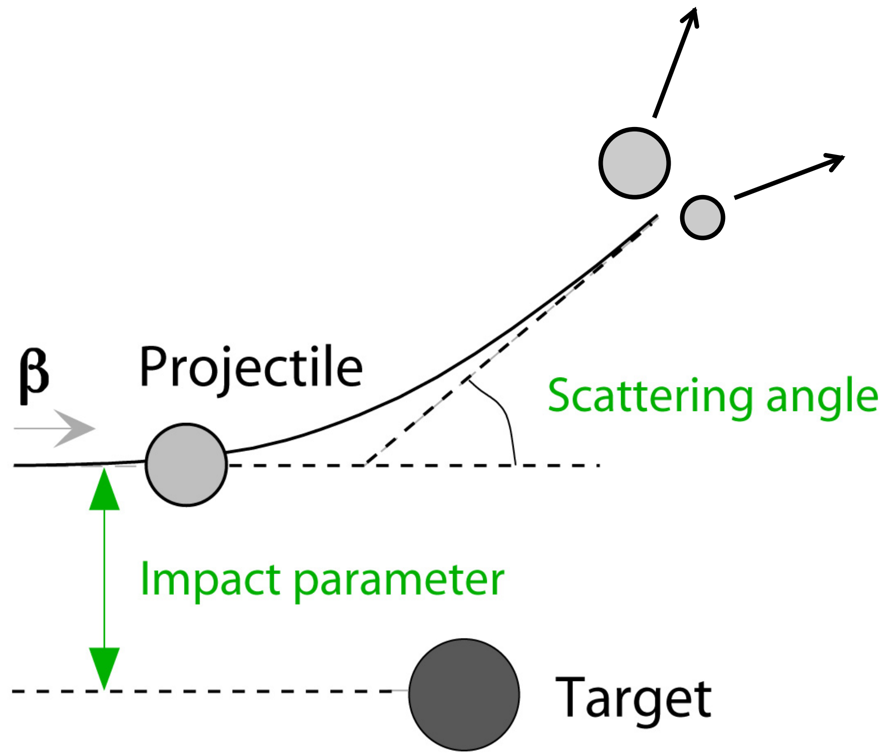
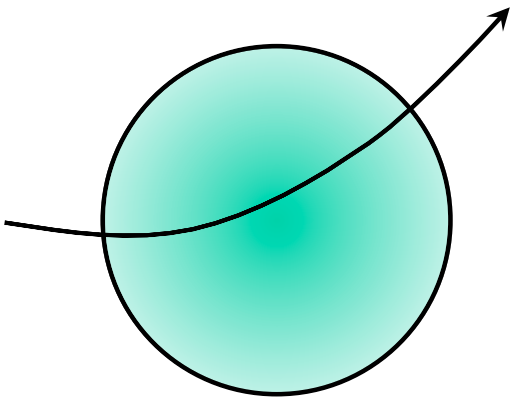


Coulomb breakup much simpler - only few pieces

Quantum and simple

Electron scattering



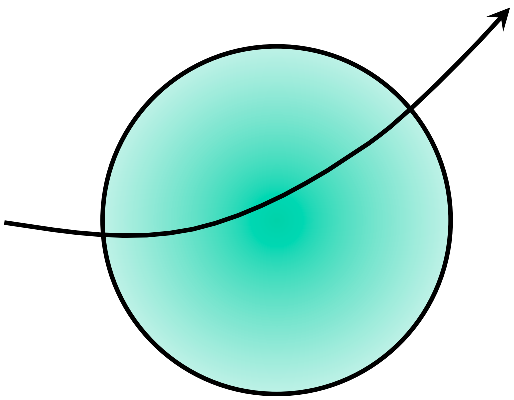
$$\frac{d\sigma}{dEd\Omega}(\Delta\mathbf{p},\ \Delta E)$$

Probes EM matrix elements s function of Δp and ΔE

$$\frac{d\sigma}{dEd\Omega}\left(|\Delta\mathbf{p}| = \frac{\Delta E}{\hbar c} \sim 0,\ \Delta E\right) \sim \sigma_{\gamma}$$

Same matrix elements as real photon

Electron scattering



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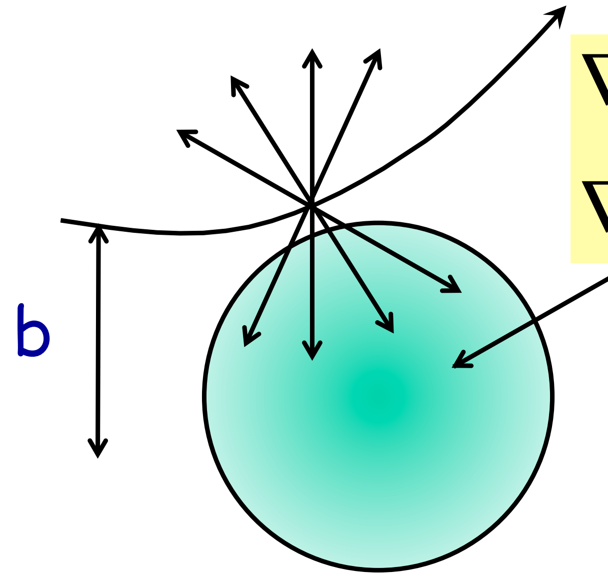
Coulomb scattering



$$\nabla \cdot \mathbf{E}(t) = 0$$

$$\nabla \cdot \mathbf{B}(t) = 0$$

$$\frac{d\sigma}{dEd\Omega}(\Delta E) \sim \sigma_\gamma$$

Always probes same matrix elements as real photon

Swiss candidate's platform: PowerPoint

By **Moni Basu**, CNN

September 17, 2011 -- Updated 1513 GMT (2313 HKT)

Mathias Poehm believes PowerPoint presentations dilute the point, dull the speech and make people less persuasive.

(CNN) -- Taxes, health care, jobs. These are issues that are center stage in U.S. elections. But a parliamentary candidate in Switzerland has a slightly different platform: PowerPoint.

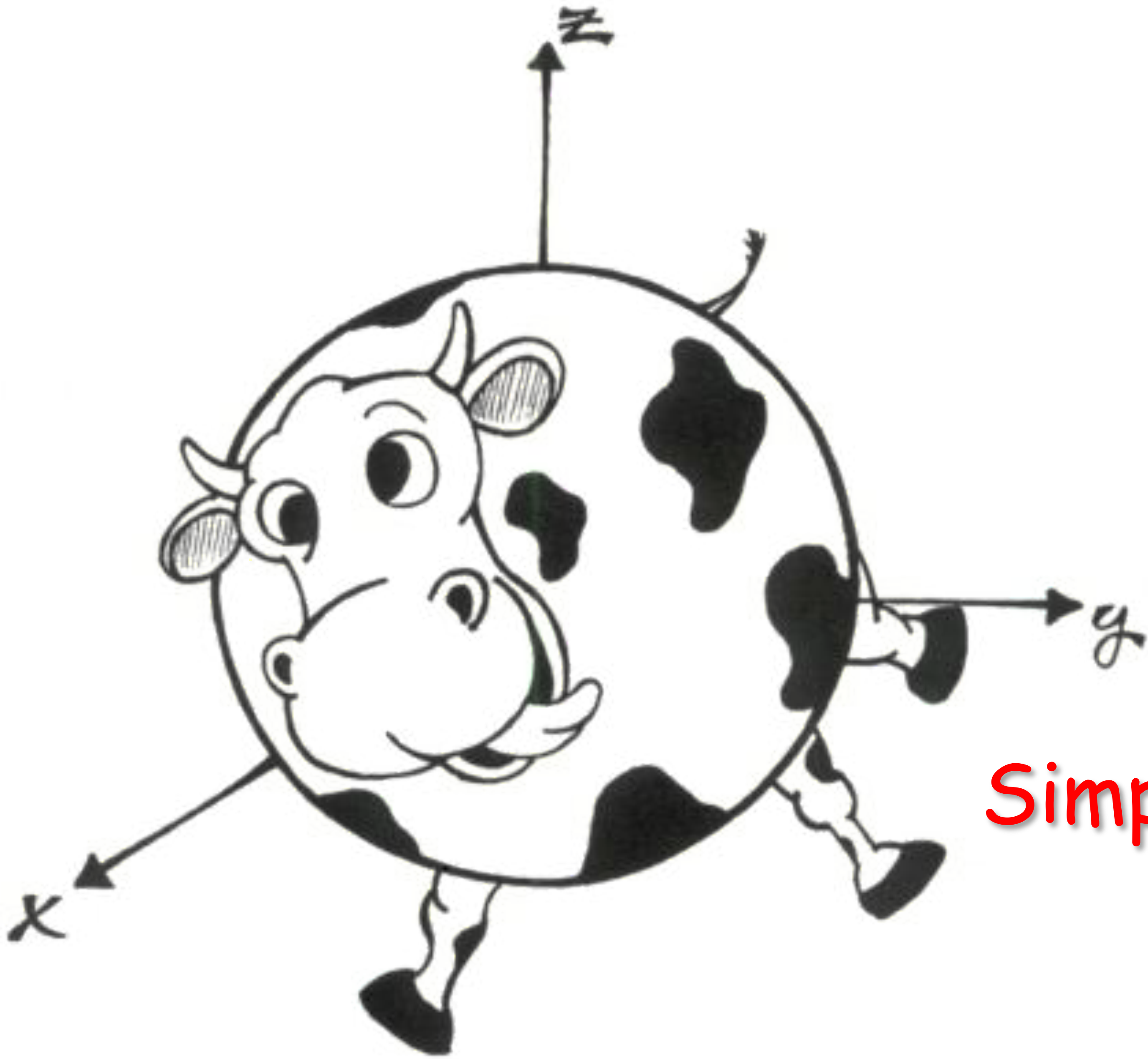
Come again? Yes, we're talking about the computer program that has become the tool of choice for public speakers of all varieties -- such as politicians, businessmen, and educators.

STORY HIGHLIGHTS

- Forget taxes or jobs. Mathias Poehm is rallying against PowerPoint
- The Swiss public speaking coach thinks the program dulls speech

Matthias Poehm would rather see it all stopped. No more discussion points. No more, "Next slide, please." No more droopy eyes tired of following along.

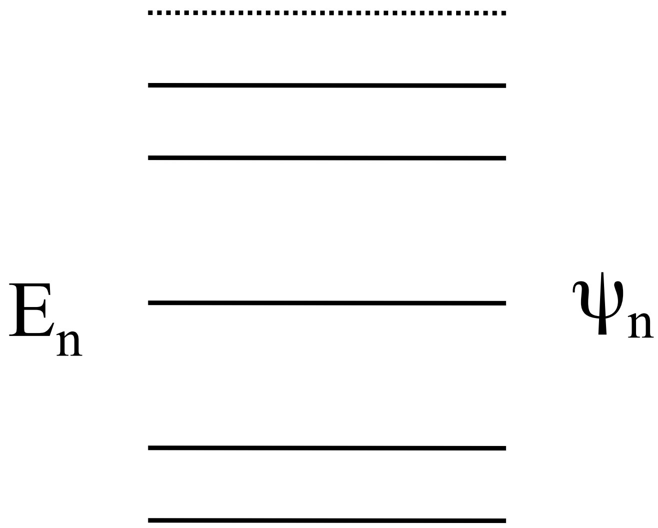
The Swiss public speaking coach believes PowerPoint presentations dilute the point, dull the speech and in the end, make people less persuasive. They are also a huge waste of money, Poehm says.

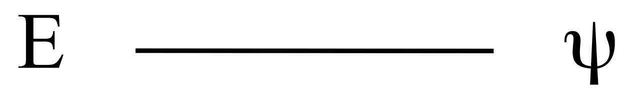


Simple theory

Figure credit: SPS @ Berkeley

t.d. coupled-channels





$H = H_0 + V$ solution

$$\psi = \sum_n a_n(t)\psi_n\, e^{-iE_n t/\hbar}$$

H_0 spectrum: $H_0\psi_n = E_n\psi_n$

$$H\psi = i\hbar\frac{\partial\psi}{\partial t}$$

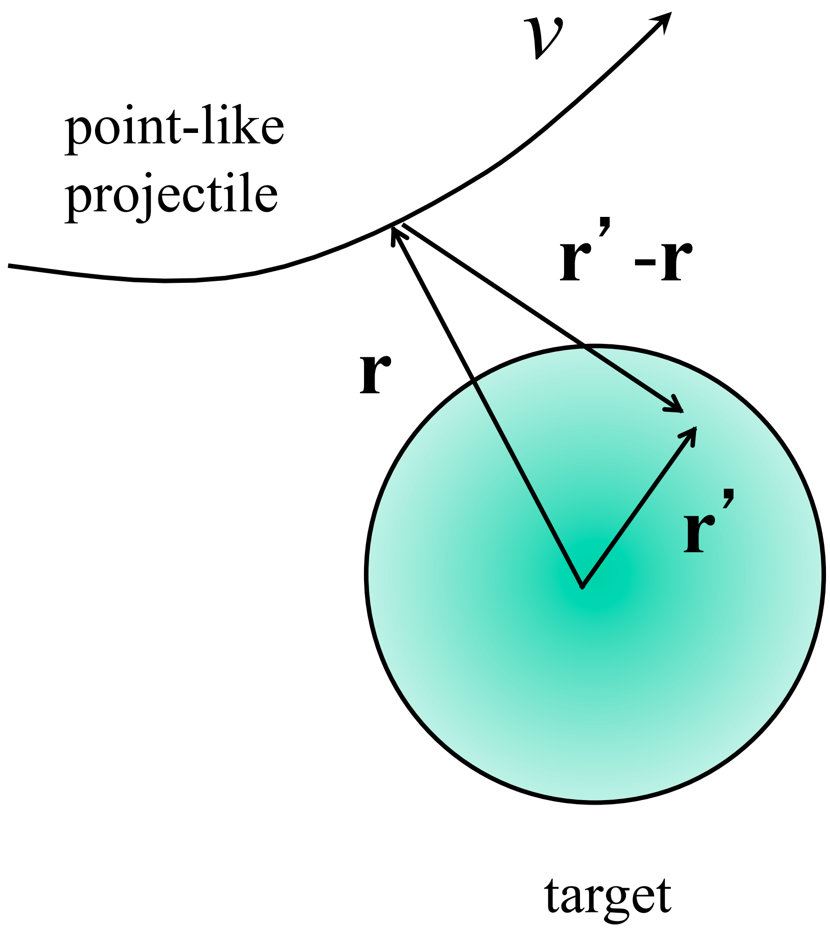
⇒

$$\frac{da_k}{dt} = -\frac{i}{\hbar}\sum_n a_n(t)V_{kn}(t)e^{i(E_k-E_n)t/\hbar}$$

$$V_{kn}(t) = \int \psi_k^* V(t)\,\psi_n\, d^3r$$

1st order: $a_n \sim \delta_{n0}$ ⇒

$$a_k = -\frac{i}{\hbar}\int dt\, V_{k0}(t)\,e^{i(E_k-E_0)t/\hbar}$$



Multipole expansion

$$V_C(r,r') = Z_p e \int \frac{\rho(\mathbf{r}')}{|r-r'|} d^3r'$$

$$= \frac{Z_p e}{r} + \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^3} + \frac{1}{2}\frac{Q_{ij} r_i r_j}{r^5} + \cdots$$

$$\mathbf{p} = \int \mathbf{r}' \rho(\mathbf{r}') d^3r'$$ (dipole)

$$Q_{ij} = \int \left(3r'_i\, r'_j - r'^2 \delta_{ij}\right)\rho(\mathbf{r}') d^3r'$$

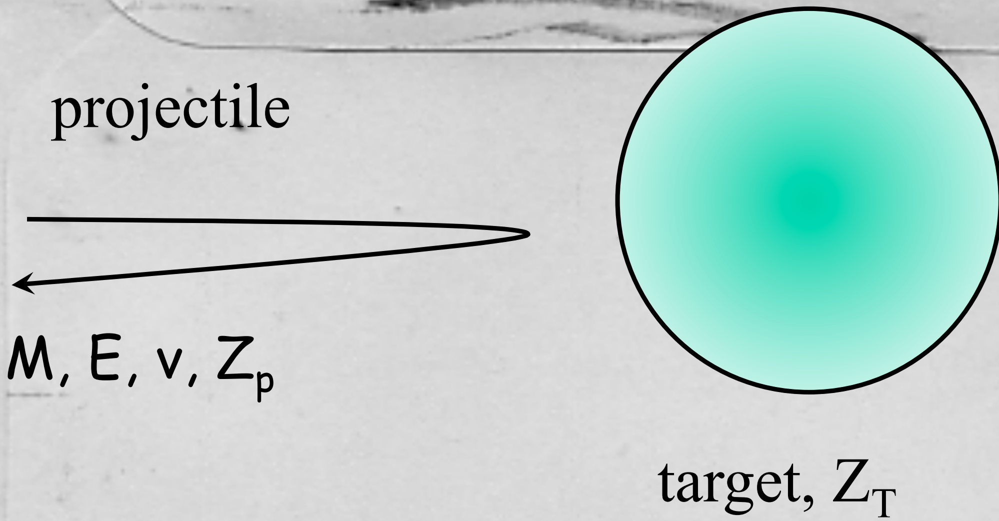
(Quadrupole)

Semiclassical method: r = r(t)

Validity: $\eta = \dfrac{\text{distance of closest approach}}{\text{wavelength}} = \dfrac{Z_1 Z_2 e^2}{\hbar v} >> 1$

Back of envelope: central collision

Kipling's WHO'S WHO
In Executives and Professionals
375 Commack Rd., Suite 204
Deer Park, NY 11729



$$V_C(t) = \frac{1}{2}\frac{Z_p e^2 Q_{fi}}{r^3(t)}$$

$$Q_{fi} = \int \psi_f^*(\mathbf{r}')\left(3z'^2 - r'^2\right)\psi_i(\mathbf{r}')\, d^3 r'$$

excitation amplitude:

$$a_{fi} = \frac{Z_p e^2 Q_{fi}}{2i\hbar}\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{r^3(t)}dt = \frac{4 Q_{fi} E^2}{3 Z_p Z_T^2 e^2 \hbar \mathrm{v}}$$

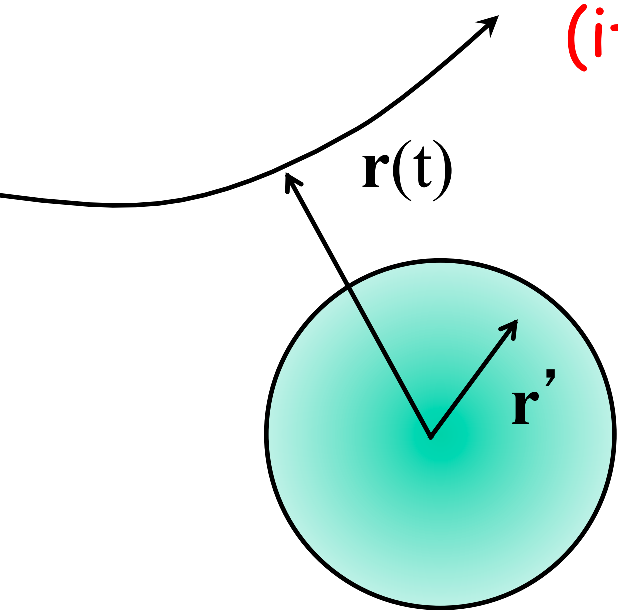
Cross section:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=180^0} = \left.\frac{d\sigma_R}{d\Omega}\right|_{\theta=180^0} \times \left|a_{fi}\right|^2 = \frac{ME\left|Q_{fi}\right|^2}{18\hbar^2 Z_T^2}$$

HW4: Obtain equation for a_{fi} above. X-section does not depend on Z_p! Why is it larger for heavier projectiles?

better theory

General multipole expansion



(if r > r')

$$\frac{1}{|\mathbf{r}(t)-\mathbf{r}'|}=\sum_{L,M}\frac{4\pi}{2L+1}\frac{r'}{r^{L+1}(t)}Y_{LM}\left(\hat{\mathbf{r}}(t)\right)Y_M^*\left(\hat{\mathbf{r}}'\right)$$

Calculate a_{fi} and average over spins:

$$w_{fi}=\frac{1}{2J_i+1}\sum_{M_iM_f}\left|a_{fi}\right|^2$$

Cross section:

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_R}{d\Omega}\cdot w_{fi}=\sum_{L>0}\frac{d\sigma_L}{d\Omega}$$

$$\frac{d\sigma_L}{d\Omega}\sim Z_P^2\,B(EL)\left|I_L\left(\omega_{fi}\right)\right|^2$$

orbital integral

$$I_L(\omega)=\int_{-\infty}^{\infty}dt\,\frac{1}{r^{L+1}(t)}Y_{LM}\left(\hat{\mathbf{r}}(t)\right)e^{i\omega t}$$

$$\omega_{fi}=\frac{E_f-E_i}{\hbar}$$

$$B(EL)\sim\left|\int r^L\delta\rho_{fi}\,d^3r\right|^2$$

reduced matrix element

Virtual photon numbers

$$\frac{d\sigma_L}{d\Omega} \sim Z_P^2 B(EL) \left| I_L(\omega_{fi}) \right|^2$$

⇒

$$\frac{d\sigma_L}{d\Omega} = \int \frac{dE_\gamma}{E_\gamma} \frac{dn_L}{d\Omega}(E_\gamma, \theta) \sigma_L^\gamma(E_\gamma)$$

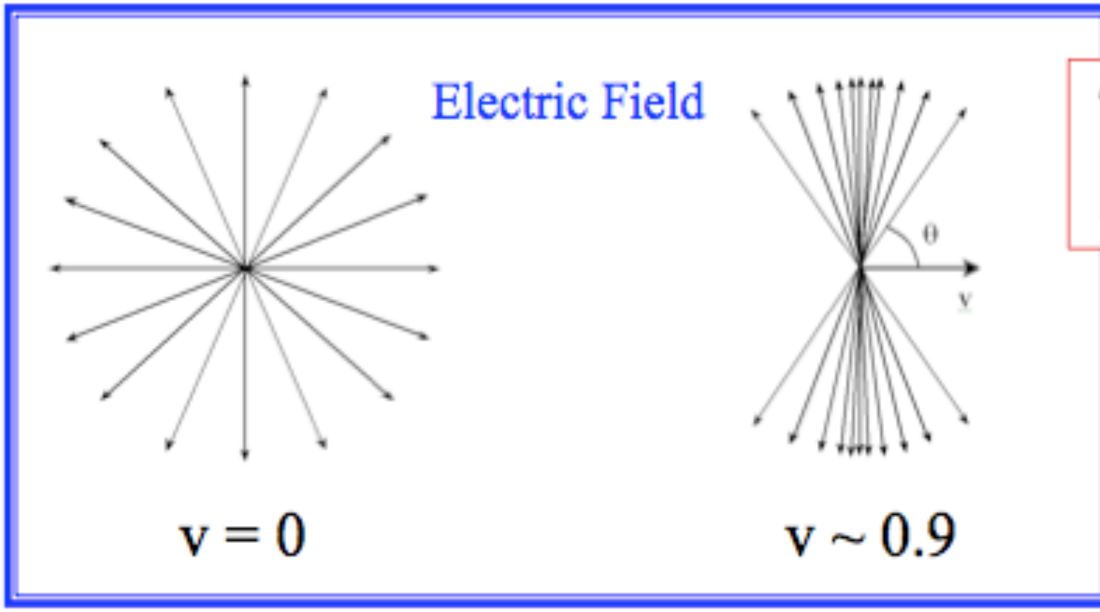
photonuclear X-section: $\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$ $\quad E_\gamma = E_f - E_i$

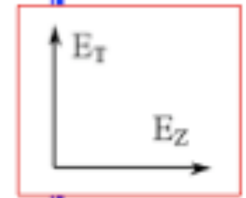
virtual photon numbers: $\frac{dn_L}{d\Omega} \sim Z_P^2 \left| I_L(\omega_{fi}, \theta) \right|^2$

impact parameter dependence: $n_L(E_\gamma, b) \equiv \frac{dn_L}{2\pi b\, db} \sim \sin^4(\theta/2) \frac{dn_L}{d\Omega}$

Magnetic excitations:

more complicated (involves currents, spins), but straight-forward.

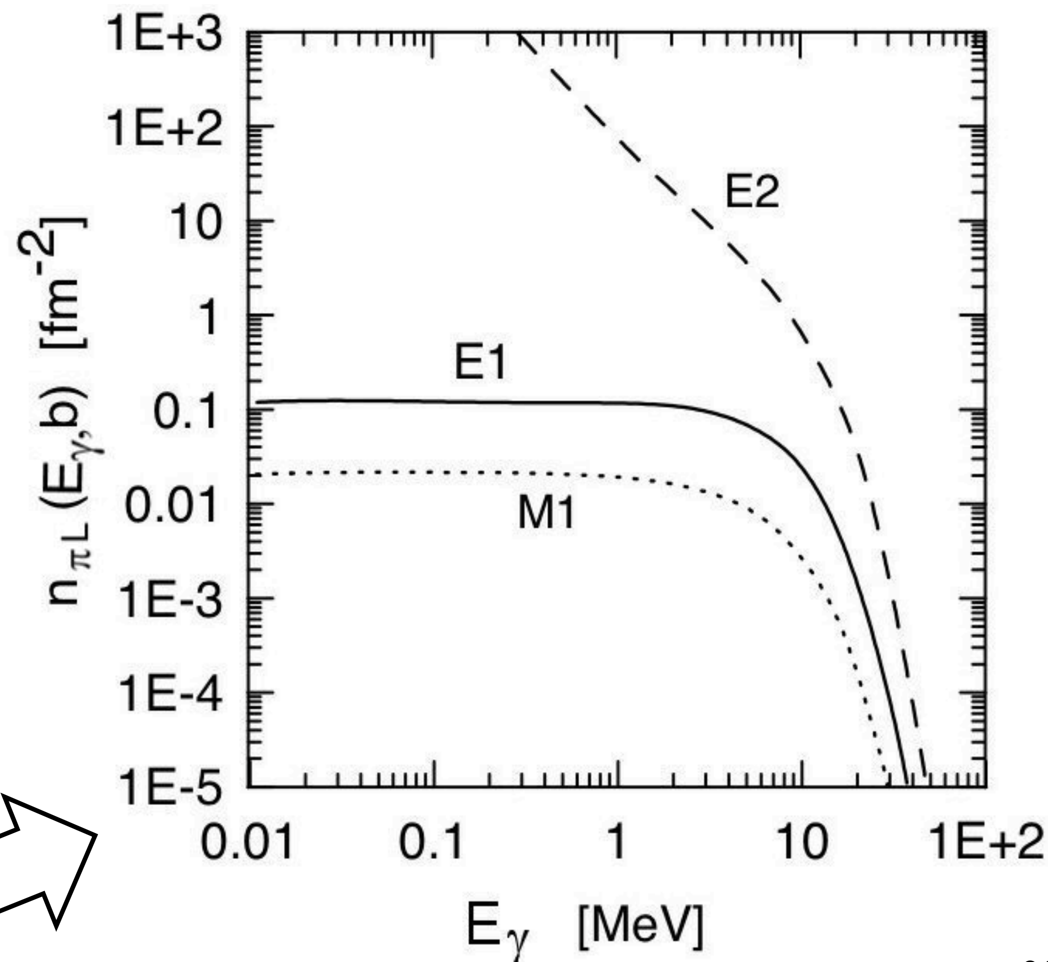




low energy scattering:

$$n_{E2} >> n_{E1} >> n_{M1} = \frac{v^2}{c^2} n_{E1}$$

Virtual photons "seen" by a Pb target due to the passage of an O projectile at 100 MeV/nucleon and b = 15 fm



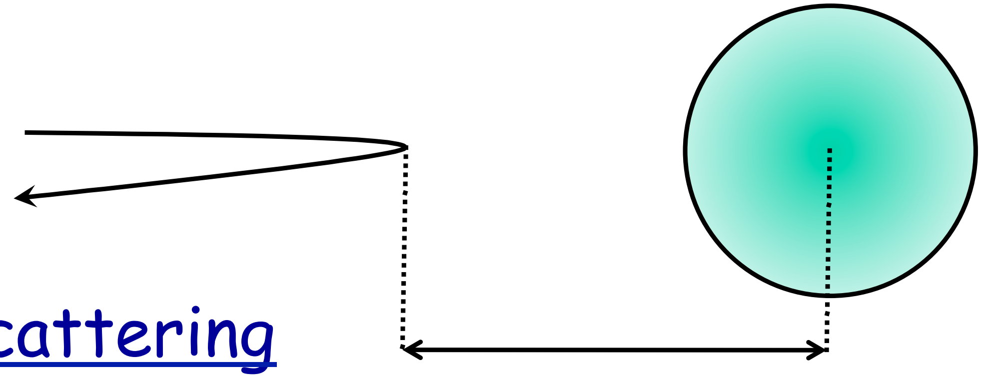

Now, relax and enjoy!

Adiabacity

⇒ Maximum effective excitation energy
Maximum effective impact parameter.

$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

orbital integral



low energy scattering

(1/2) distance of closest approach

$$a = \frac{Z_P Z_T e^2}{2E_{c.m.}}$$

if $|t| > t_{exc} \sim \frac{1}{\omega}$ then $e^{i\omega t}$ oscillates too fast: I_L small

if $|t| > t_{coll} \sim \frac{a}{\mathrm{v}}$ then $\frac{1}{r^{L+1}}$ too large: I_L small

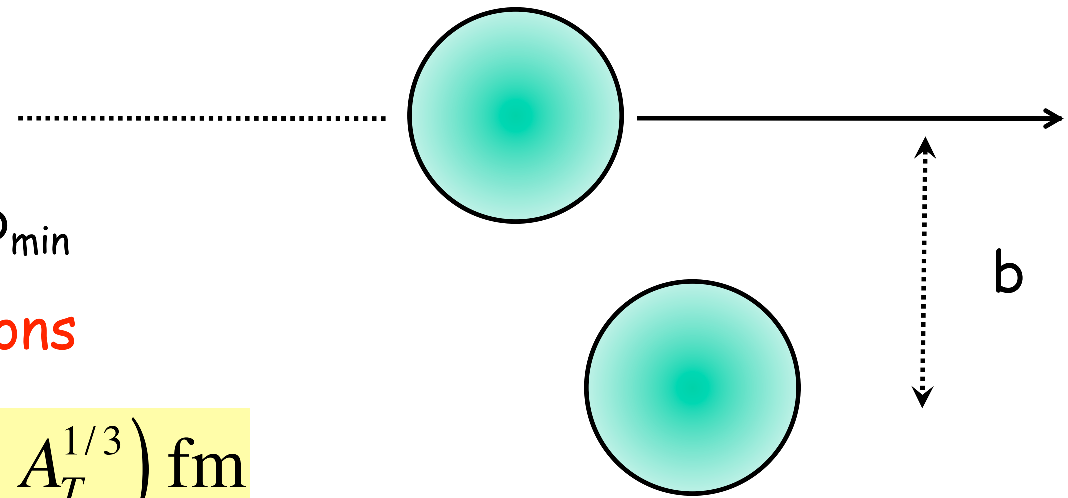
excitation possible if $\frac{t_{coll}}{t_{exc}} \lesssim 1$ ⇒

$$\zeta = \frac{a\omega}{\mathrm{v}} \lesssim 1$$

adiabacity parameter

$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}\left(\hat{\mathbf{r}}(t)\right) e^{i\omega t}$$ orbital integral

high energy collisions



Closest approach distance = b_{min}

b < b_{min} → nuclear interactions

$$b_{mim} \sim R_P + R_T \sim 1.2\left(A_P^{1/3} + A_T^{1/3}\right) \text{fm}$$

$$t_{coll} \sim \frac{R}{\gamma v}$$ (γ due to contraction)

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ Lorentz γ-factor

Excitation possible if $$\xi = \frac{\omega R}{\gamma v} \lesssim 1$$ adiabacity parameter

Energy budget

$$\zeta = \frac{E_\gamma a}{\hbar v}$$

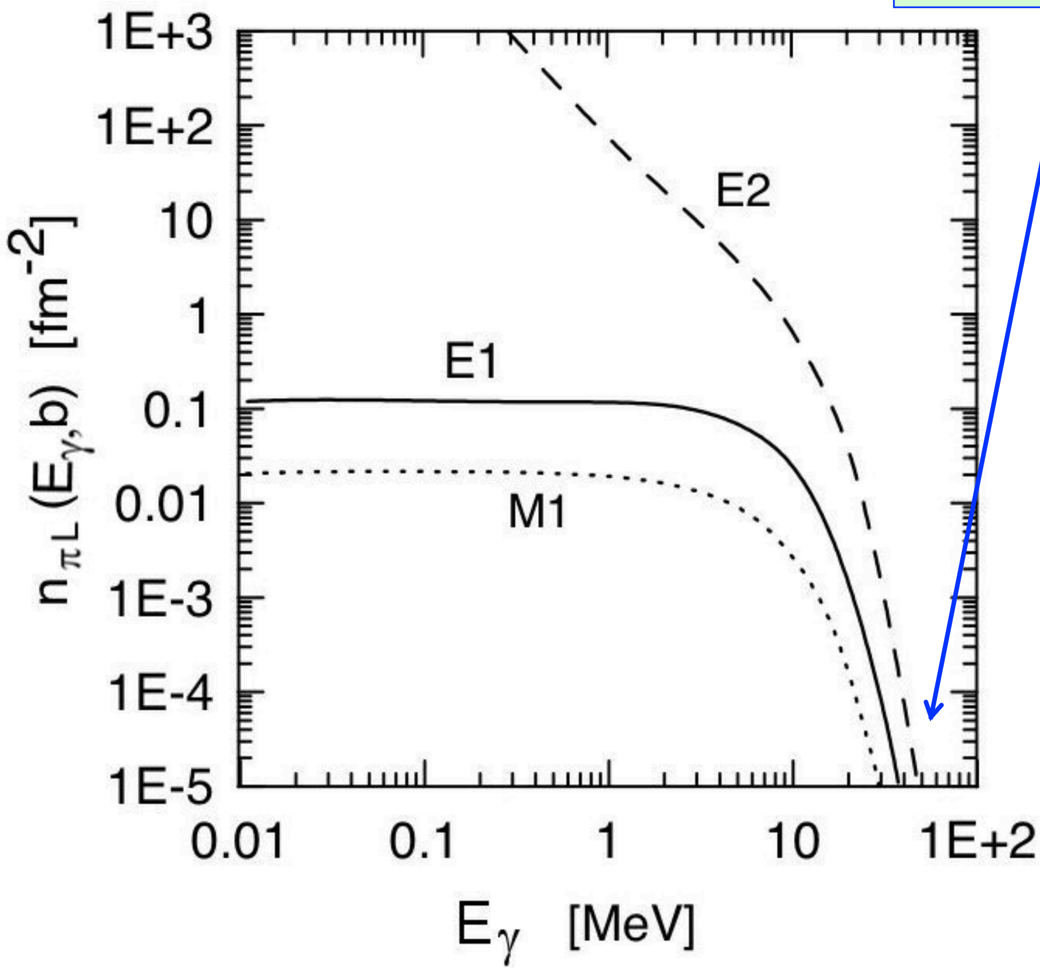
low energy collisions

$$\xi = \frac{E_\gamma R}{\gamma \hbar v}$$

high energy collisions

$$a,\ b_{min} \sim 20\ \text{fm}$$

$$E_\gamma \tilde{<} \frac{200\,\text{MeV. fm}\,\gamma}{20\ \text{fm}} = 10\,\text{MeV.}\,\gamma$$



- small γ' s: giant resonances
- large γ' s: giant resonances,
  quasi-deuteron,
  deltas,
  mesons (ex: J/ψ)

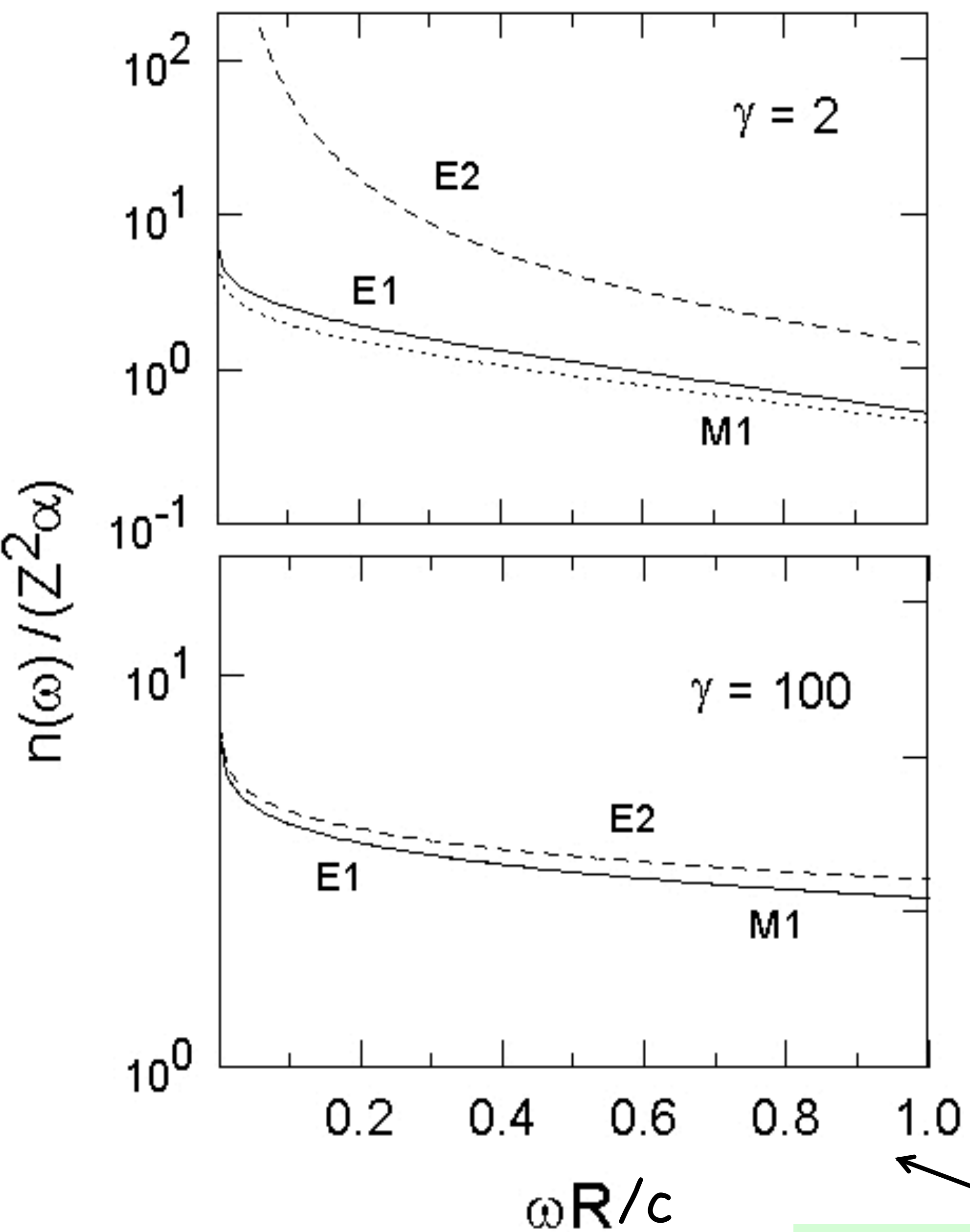
Multipolarity budget

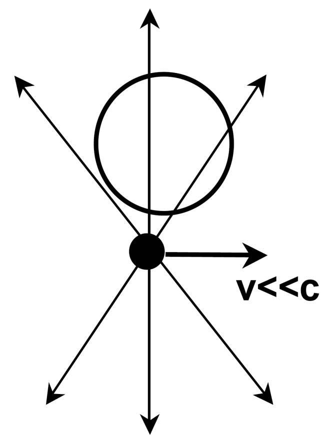
orbital integral

$$I_L(\omega) = \int_{-\infty}^{\infty} dt \frac{1}{r^{L+1}(t)} Y_{LM}(\hat{\mathbf{r}}(t)) e^{i\omega t}$$

ω large, $e^{i\omega t}$ oscillates fast: I_L small

$$n_L(E_\gamma, b) \sim \left| I_L(\omega_{fi}, \theta) \right|^2$$ also small

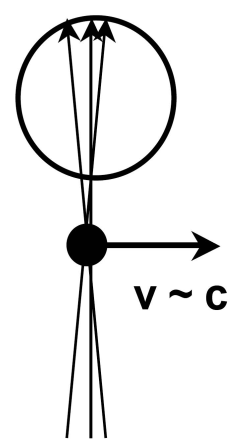




low-energy (tidal)

$$n_{E2} >> n_{E1} >> n_{M1} = \frac{v^2}{c^2} n_{E1}$$

high-energy (contraction)



$$n_{E2} \sim n_{E1} \sim n_{M1}$$

Low-energies: multipolarities of virtual photons have different weights

High energies: multipolarities have same weight

$$n_L(\omega) = 2\pi \int db\, b\, n_L(\omega, b)$$

Are we there yet?

Almost.

Virtual

$$\frac{d\sigma}{db} = \int \sum_L \frac{dE_\gamma}{E_\gamma} n_L\left(E_\gamma, b\right) \sigma_L^\gamma\left(E_\gamma\right)$$

Coulomb excitation: virtual photons

Each part (multipolarity) of a real photon has a different weight n_L

High-energy:

$$n_{E2} \sim n_{E1} \sim n_{M1}$$

$$\frac{d\sigma}{db} = \int \frac{dE_\gamma}{E_\gamma} n\left(E_\gamma, b\right) \sigma^\gamma\left(E_\gamma\right)$$

Real photons

$$\sigma^\gamma\left(E_\gamma\right) = \sum_L \sigma_L^\gamma\left(E_\gamma\right)$$

Real photons

All parts (multipolarities) have the same weight

Coulomb excitation for a fixed energy E_γ probes the same physics as a real photon.

But each E_γ has a different weigth.

Z_P^2 makes number of photons large.

Bertulani, Baur, Phys. Rep. 163, 299 (1988)

Almost there.

Nuclear response to multipolarities

$$\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$$

$$B(EL) \sim \left| \int r^L \delta\rho_{fi} \, d^3 r \right|^2$$

Estimate

$$\delta\rho_{fi} = \psi_f^* \psi_i$$

$$\psi_f \sim \psi_i \sim \frac{1}{\sqrt{R^3}}, \quad \text{if r < R,} \quad \text{0 otherwise}$$

$$B(EL) \sim R^{2L} \Rightarrow \sigma_L^\gamma \sim (kR)^{2L}$$

$$k = \frac{E_\gamma}{\hbar c}$$

$$\frac{\sigma_{L+1}}{\sigma_L} \sim (kR)^2 << 1 \text{ for low lying states}$$

ready.

Applications

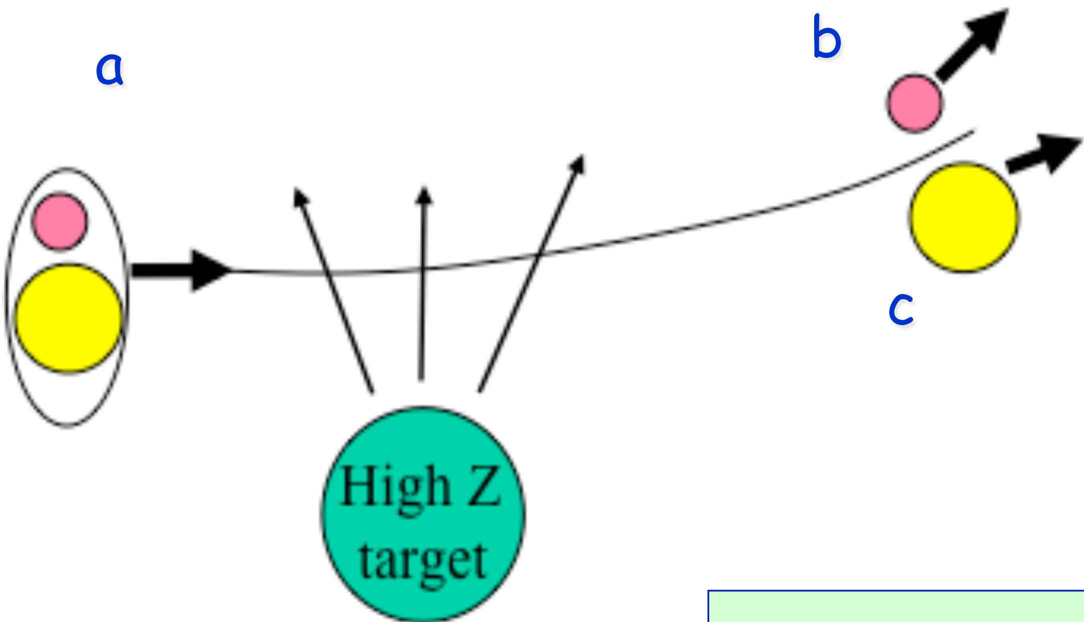
- Coulomb dissociation for astrophysics

  + bonus

Coulomb dissociation method

Baur, Bertulani, Rebel
NPA 458 (1986) 188

$$\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma}\sum_l \frac{dn_l(E_\gamma,\Omega)}{dE_\gamma d\Omega}\,\sigma_{\gamma+\mathrm{a}\rightarrow \mathrm{b}+\mathrm{c}}(E_\gamma)$$



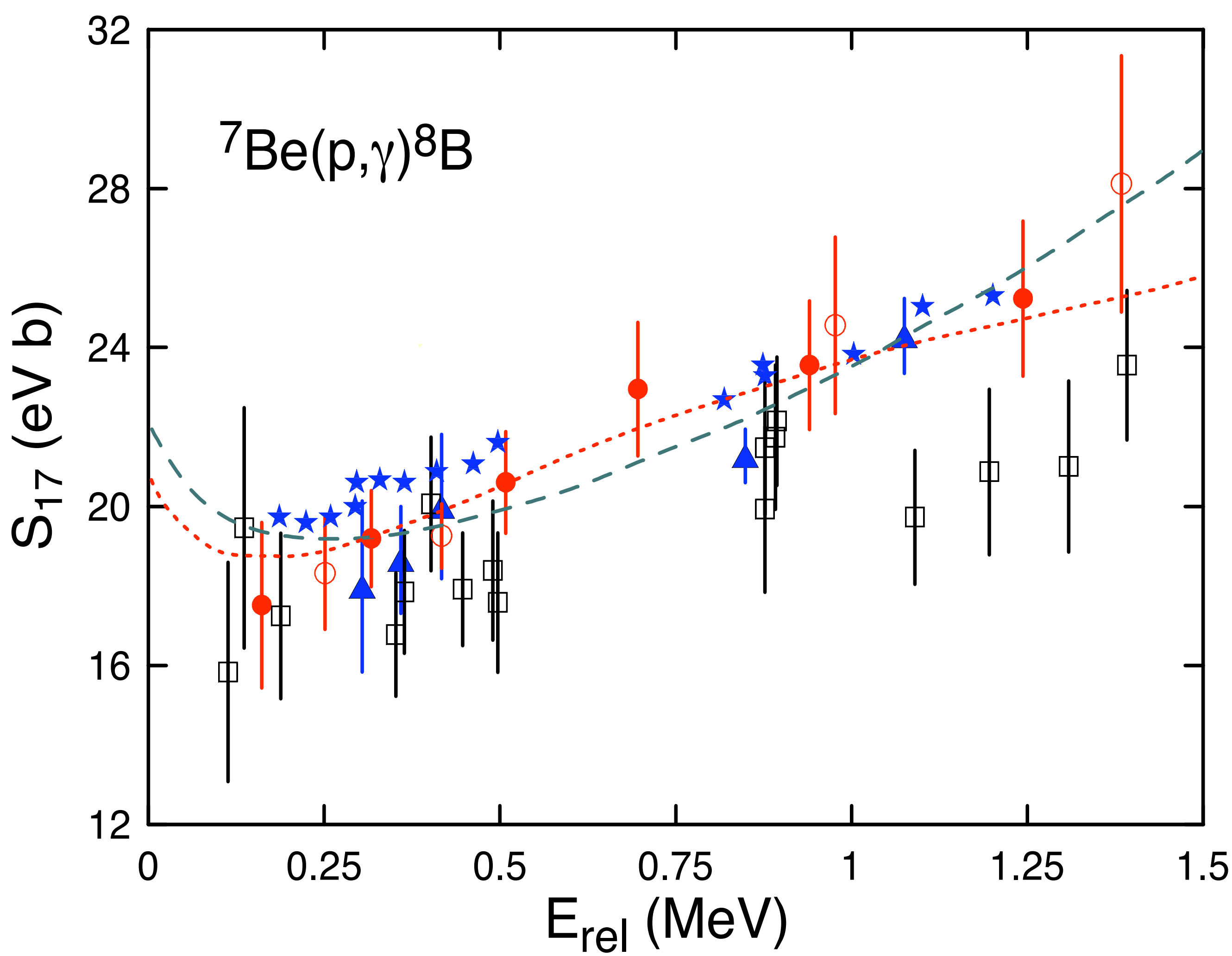
Theory

detailed balance

$$\sigma_{b+c\rightarrow a+\gamma} = \frac{2(2j_a+1)}{(2j_b+1)(2j_c+1)}\frac{k_{bc}^2}{k_\gamma^2}\sigma_{\gamma+a\rightarrow b+c}$$

Applications to radiative capture (n,γ) and (p,γ) reactions in nuclear astrophysics.

Ex: $^7Be(p,\gamma)^8B$



Solar neutrino problem is due to ν-oscillations

But this reaction needs to be known more accurately

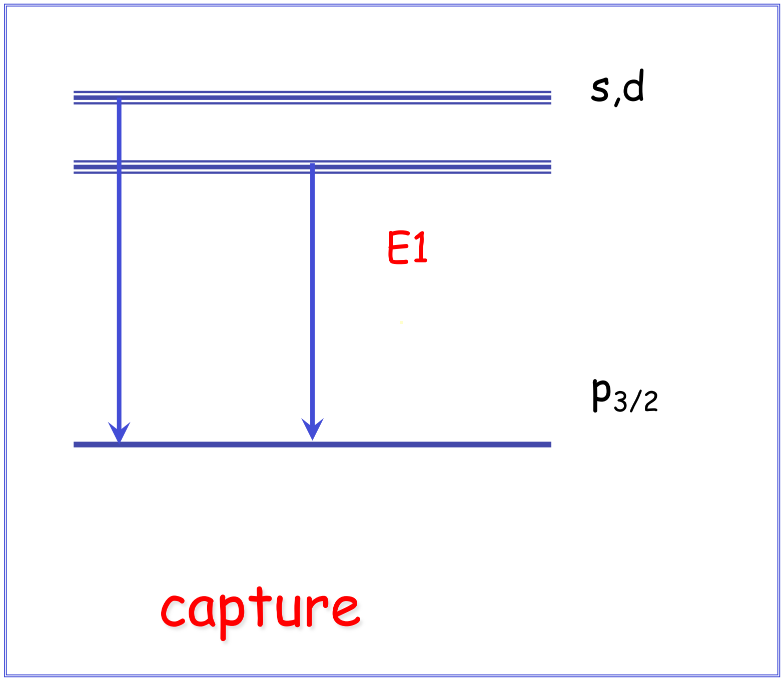
- J. Bahcall

$S_{17}(0)$ = 20.8 (0.7) expt
(1.4) theor
eV b

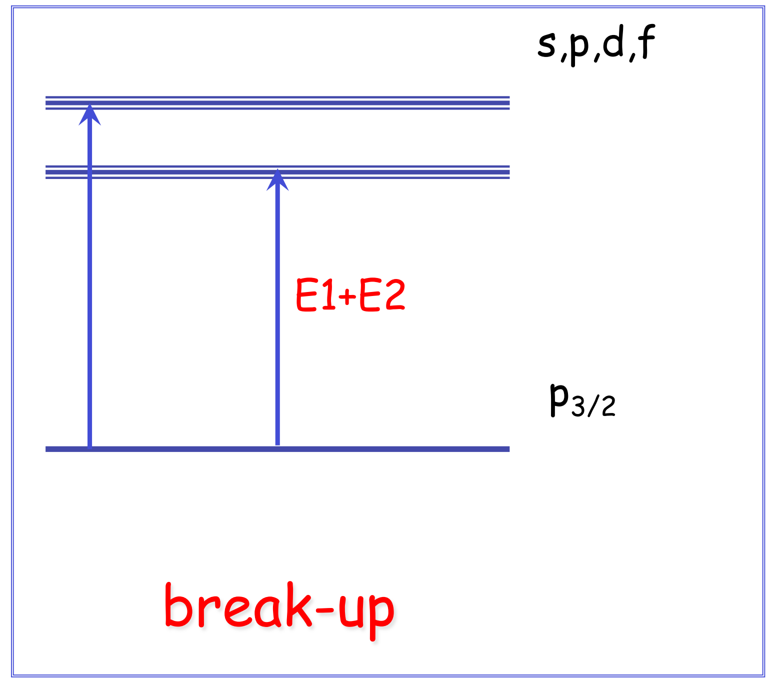
Adelberger et al
RMP 83, 195 (2011)

Life is hard

THIS occurs in stars



THIS is obtained in lab

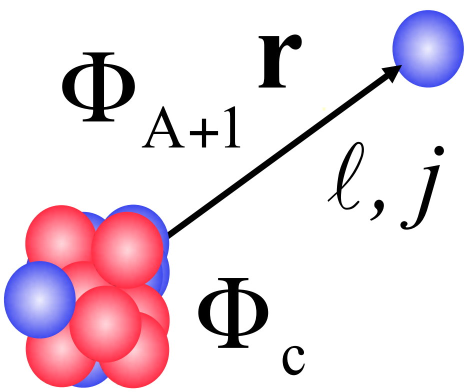


and other issues later

Forget it. Let us have some fun.

Spectroscopic factors

Nucleon removal from Φ_{A+1} will leave mass A residue in the ground or an excited state - amplitude for finding nucleon with sp quantum numbers ℓ, j, about core state Φ_c in Φ_{A+1} is



$$O^{c}_{\ell j}(\mathbf{r}) = \langle \mathbf{r}, \Phi_c \,|\, \Phi_{A+1} \rangle, \quad S_N = E_{A+1} - E_c$$

overlap integral

$$\int d^3r \; |O^{c}_{\ell j}(\mathbf{r})|^2 = C^2 S(\ell j)$$

Spectroscopic factor: occupancy of the state

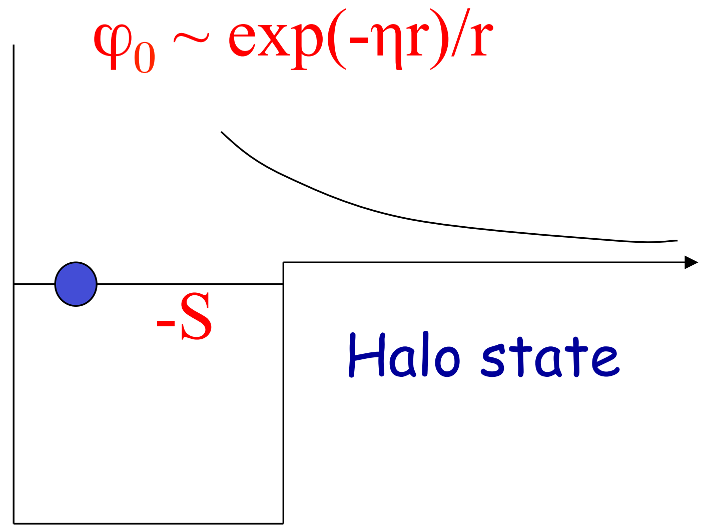
Usual to write

$$O^{c}_{\ell j}(\mathbf{r}) = \sqrt{C^2 S(\ell j)} \; \phi_0(\mathbf{r}) \; ; \qquad \int d^3r \, |\phi_0(\mathbf{r})|^2 = 1$$

Breakup estimates

$$\frac{d\sigma_C}{dE_x} \approx n_{EL}(E_\gamma) \times \mathrm{C}^2\mathrm{S} \left| \langle \psi_{\mathbf{k}} \| r^L Y_L \| \phi_0 \rangle \right|^2 \frac{d^3k}{(2\pi)^3}$$

$$\frac{dB(EL)}{dE_\gamma}$$



Estimate

$$\psi_{\mathbf{k}} \approx e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$\phi_0 \approx \frac{1}{r} e^{-\eta r}$$

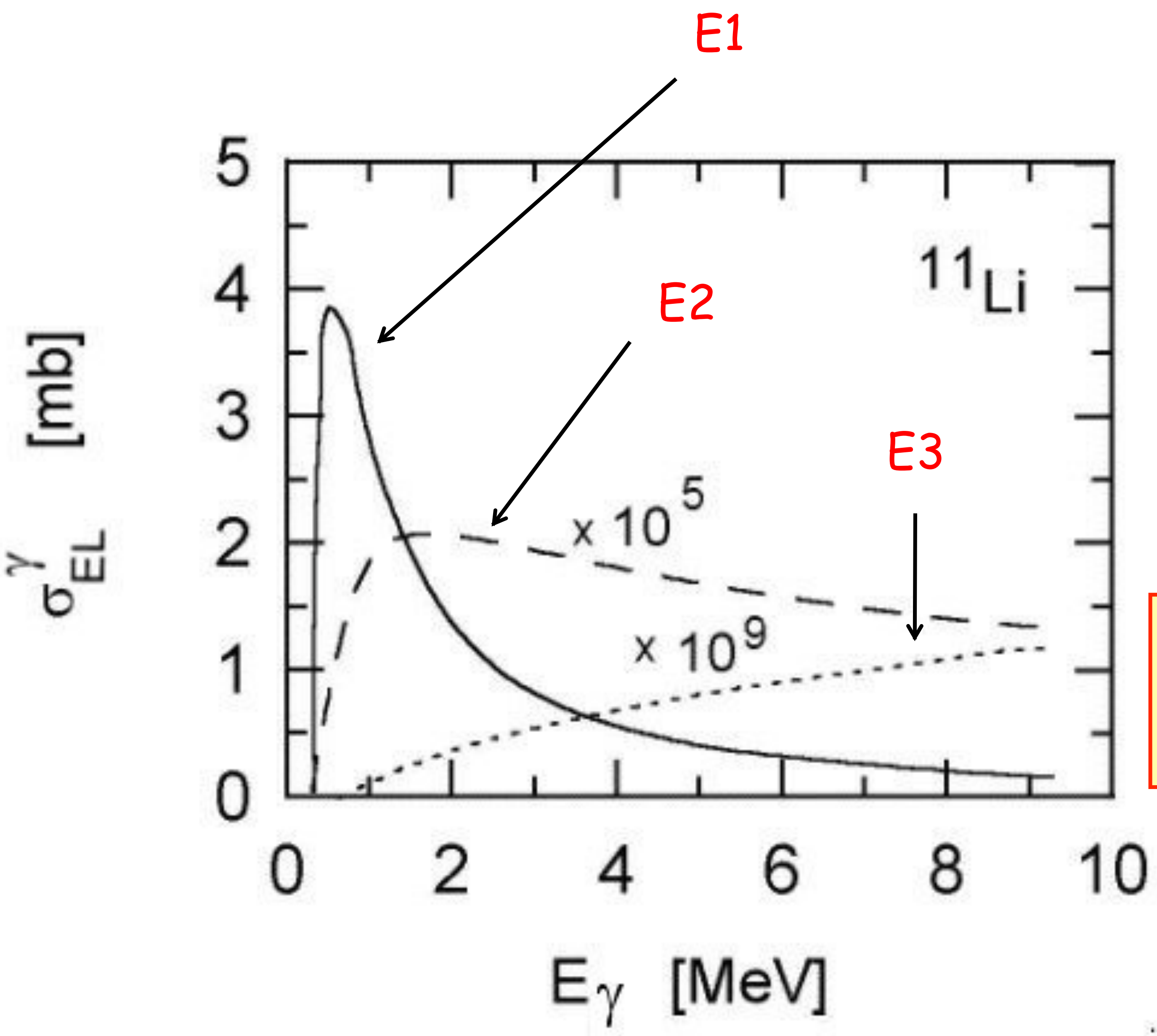
$$\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}\, E_{rel}^{L+1/2}}{(E_{rel} + S)^{2L+2}}$$

$$E_{rel} = E_\gamma - S$$

$$S = \frac{\hbar^2 \eta^2}{2\mu}$$

Separation energy of fragments with reduced mass μ

Electric response

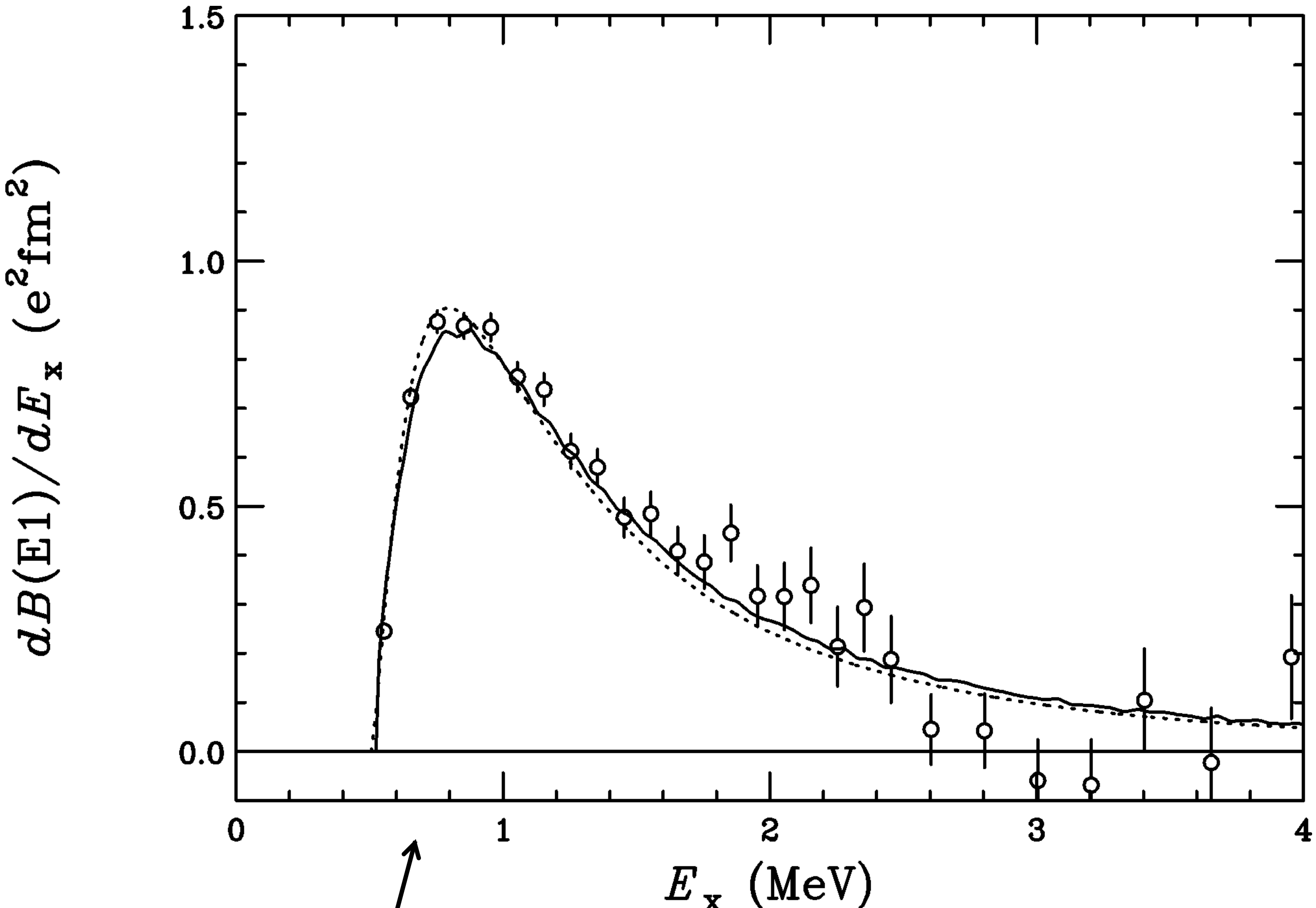


$$\sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL)$$

$$\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}\,E_{rel}^{L+1/2}}{\left(E_{rel}+S\right)^{2L+2}}$$

Bertulani, Sustich
PRC46, 2340 (1992)

Ex: breakup of ^{11}Be



Data: Nakamura *et al.*, PLB 331, 296(1994)

$$B(E1) = 1.05 \pm 0.06\ e^2 \mathrm{fm}^2$$
$$(3.29 \pm 0.06 \text{ W.u})$$

$$S = 0.54 \text{ MeV}$$

$$C^2 S \sim 1$$

peak at $E_x = \frac{8}{5} S = 0.76$ MeV

$$\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}(E_x - S)^{3/2}}{E_x^{\ 4}}$$

Life is hard

$$\mathrm{I}_{sp} = \langle \psi_{\mathbf{k}} \| r^1 Y_1 \| \phi_0 \rangle \approx \frac{k^2}{(k^2+\eta^2)^2}\left[\cos\delta + \sin\delta \frac{\eta(\eta^2+3k^2)}{2k^3}\right]$$

$$\approx \frac{E_{rel}}{(S+E_{rel})^2}\left[1+\left(\frac{\mu}{2\hbar^2}\right)\frac{\sqrt{S}(S+3E_{rel})}{-1/a+(\mu E_{rel}/\hbar^2)r_{eff}}\right]$$

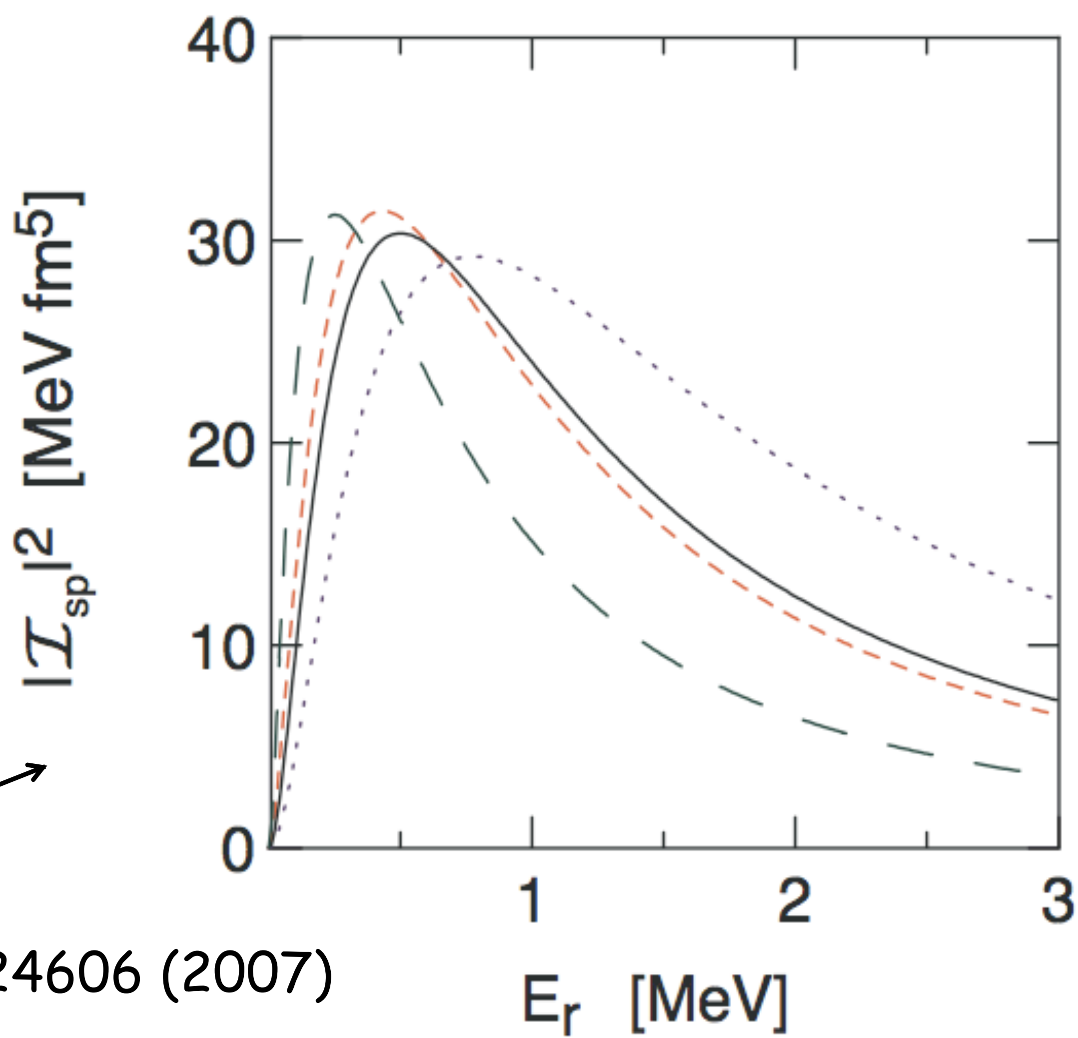
Final state interactions

δ = scattering phase shift

a = scattering length

r_{eff} = effective range

Strongly ddependent on final state interactions (phase-shifts)



Bertulani, PRC 75, 024606 (2007)


Life is complicated

Sum-rules

$$S = \sum_f \left(E_f - E_i\right)\left|\langle f|\hat{O}|i\rangle\right|^2$$

$$\hat{O} \equiv \hat{O}(z)$$

$$\hat{H} = \frac{\hat{p}_z^2}{2m} + V(z)$$

$$\Rightarrow \quad S = \frac{1}{2}\left\langle i\left|\left[[H,\hat{O}],\hat{O}\right]\right|i\right\rangle$$

$$\hat{O} = f(z) \quad \Rightarrow \quad S = \frac{\hbar^2}{2m}\left\langle i\left|\left|\frac{df}{dz}\right|^2\right|i\right\rangle$$

Example: electric dipole operator

$$\hat{O} = e\,z \quad \Rightarrow \quad S = \frac{\hbar^2 e^2}{2m}$$

independent of V !

A-particles:

$$\hat{H} = \sum_b \left[\frac{\hat{p}_b^2}{2m_b} + V(z_b)\right]$$

$$\hat{O} = \sum_a e_a z_a \quad \Rightarrow \quad S = \sum_a \frac{\hbar^2 e_a^2}{2m_a}$$

Effective charges, c.m. motion

$$d_z = \sum_a e_a z_a = \sum_a e_a r_a Y_{10}(\hat{\mathbf{r}}_a)$$

$$z_a \rightarrow z_a - R_z$$

$$R_z = \sum_a \frac{z_a}{A} \quad \text{(center of mass)}$$

$$d_z = \sum_a e_a (z_a - R_z) = e \sum_p z_p - \frac{Ze}{A}\left(\sum_p z_p + \sum_n z_n \right)$$

$$= e_p \sum_p z_p + e_n \sum_n z_n$$

$$e_p = \frac{N}{A} e, \quad e_n = -\frac{Z}{A} e$$

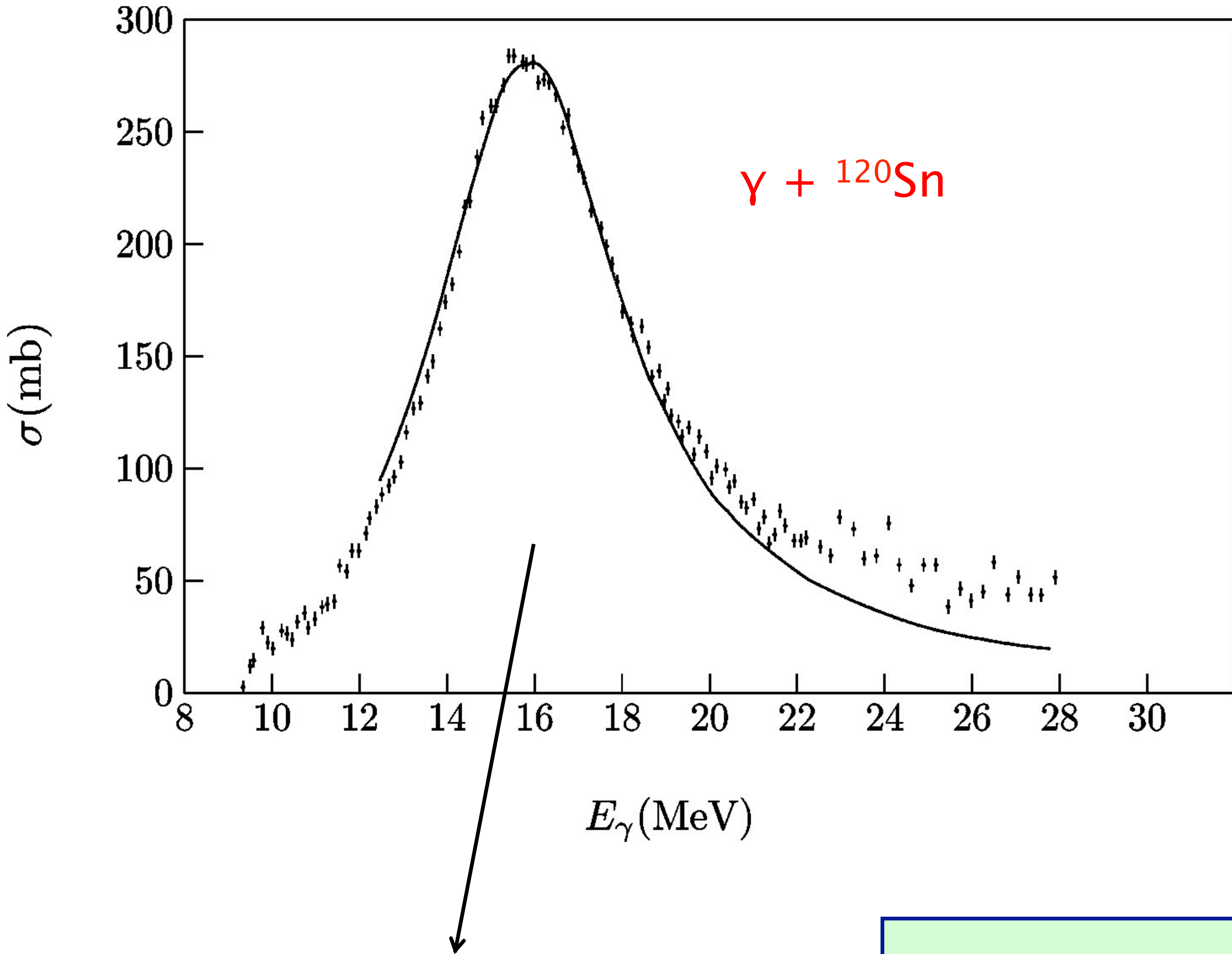
effective charges

$$S = \sum_f E_{fi} \left| d_{fi}^z \right|^2 = \frac{\hbar^2 e^2}{2m_N} \left[Z\left(\frac{N}{A}\right)^2 + N\left(-\frac{Z}{A}\right)^2 \right]$$

$$= \frac{\hbar^2 e^2}{2m_N} \frac{NZ}{A}$$

Thomas-Reiche-Kuhn sum-rule

Nuclear response to photon energies

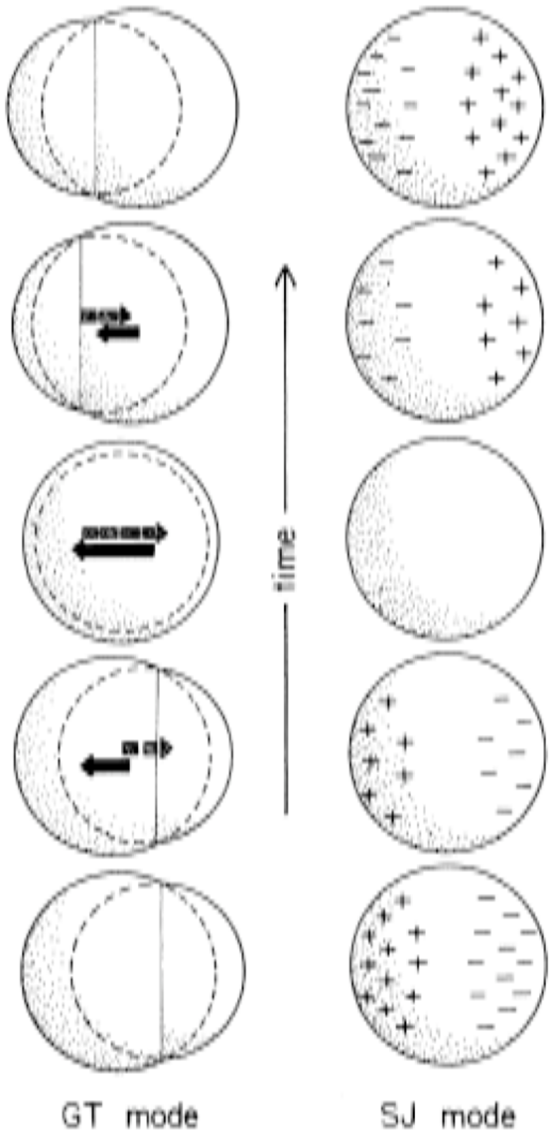


Area ~ 100% TRK sum-rule:

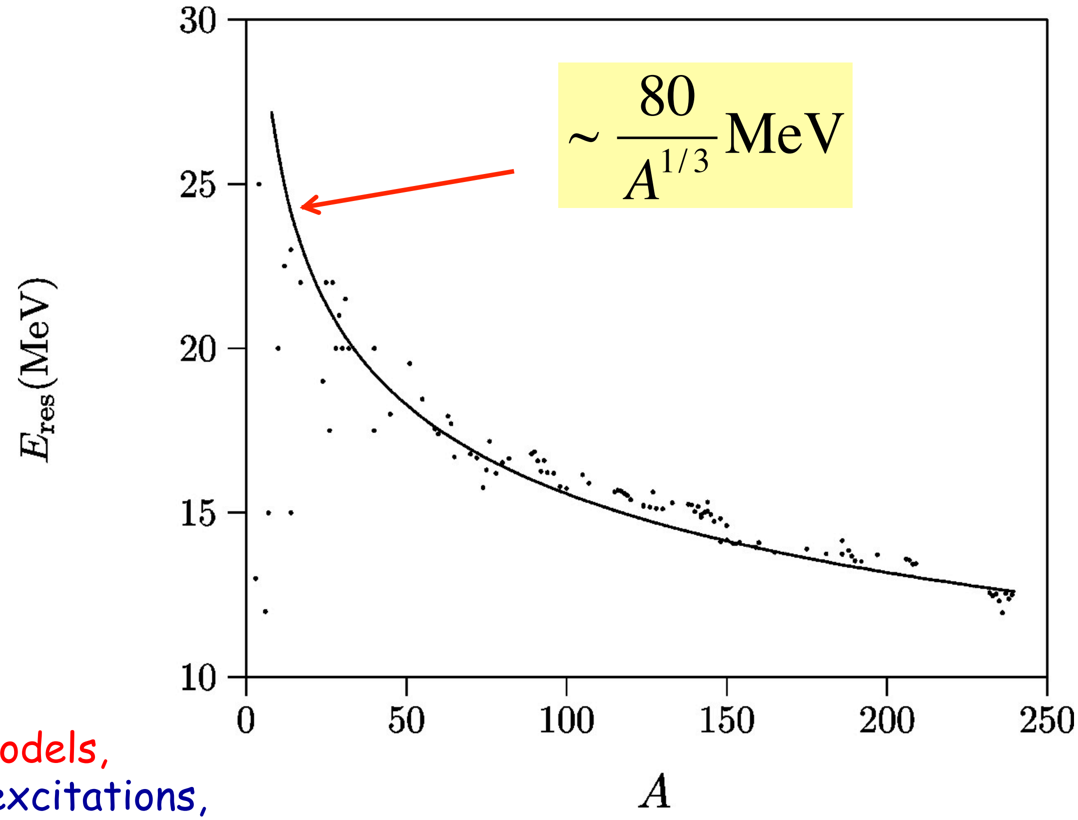
$$\int dE_\gamma \, \sigma^\gamma (E_\gamma) = 2\pi^2 \frac{\hbar e^2}{m_N c} \frac{NZ}{A}$$

Giant resonances

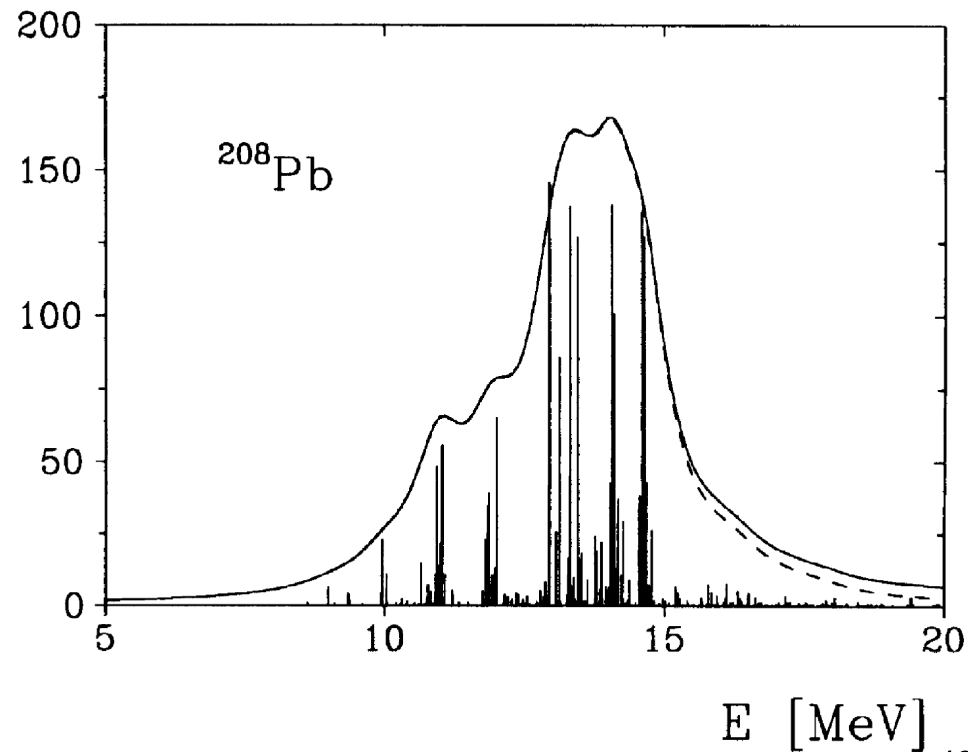
Macroscopic models, liquid drop



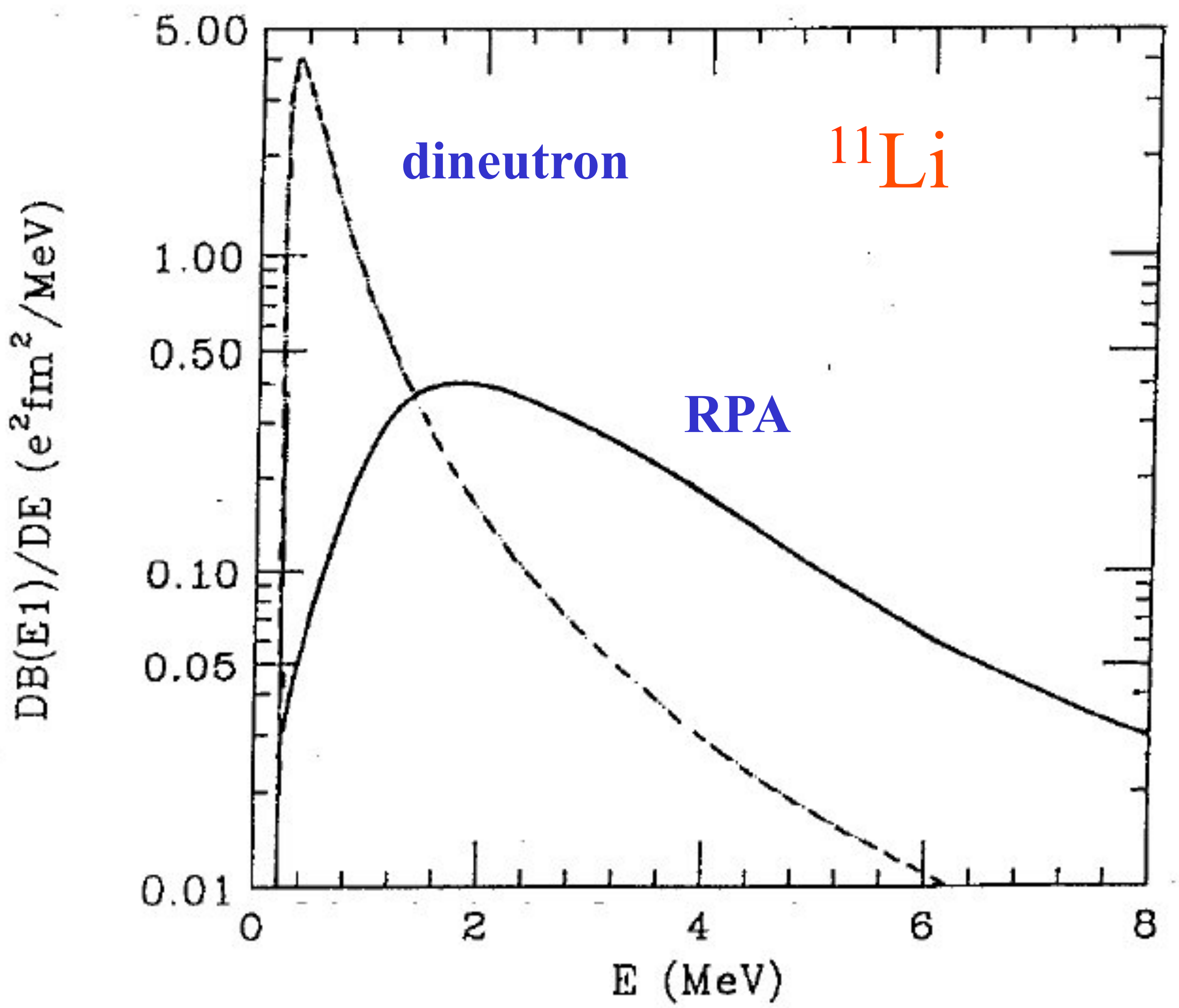
GT: Goldhaber-Teller
SJ: Steinwedel-Jensen



Microscopic models, particle-hole excitations, RPA

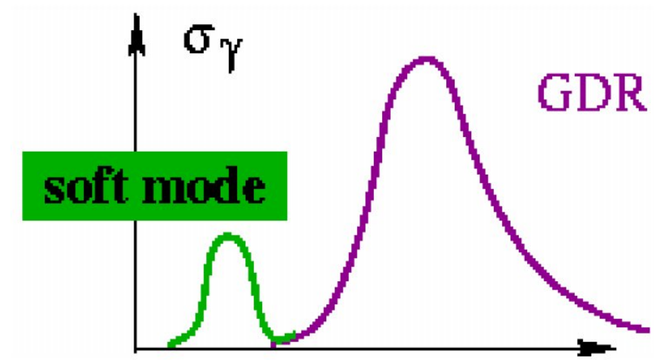


Pigmy resonances

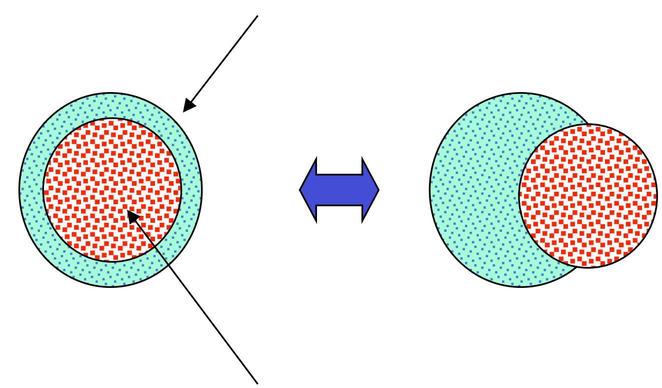


Teruya, Bertulani
PRC 43, 2049 (1991)

RPA + $2n_p$-$2n_h$ excitations



excess neutrons



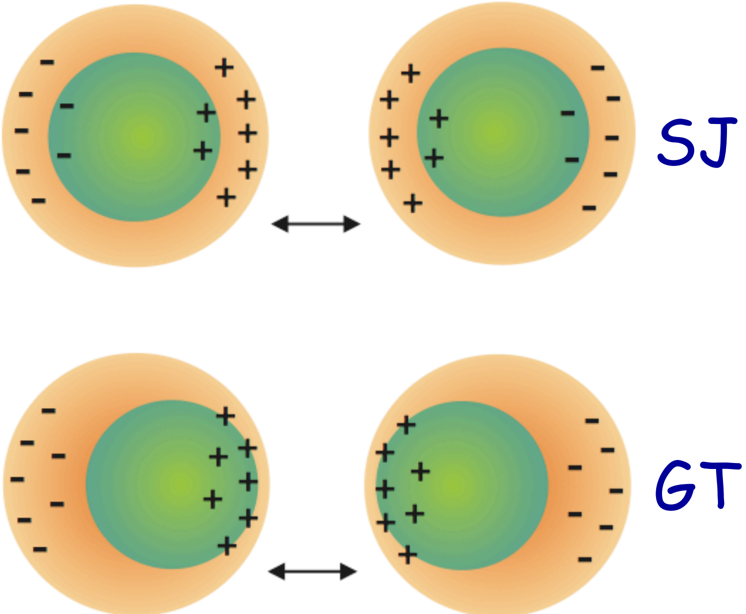
"core" with p and n

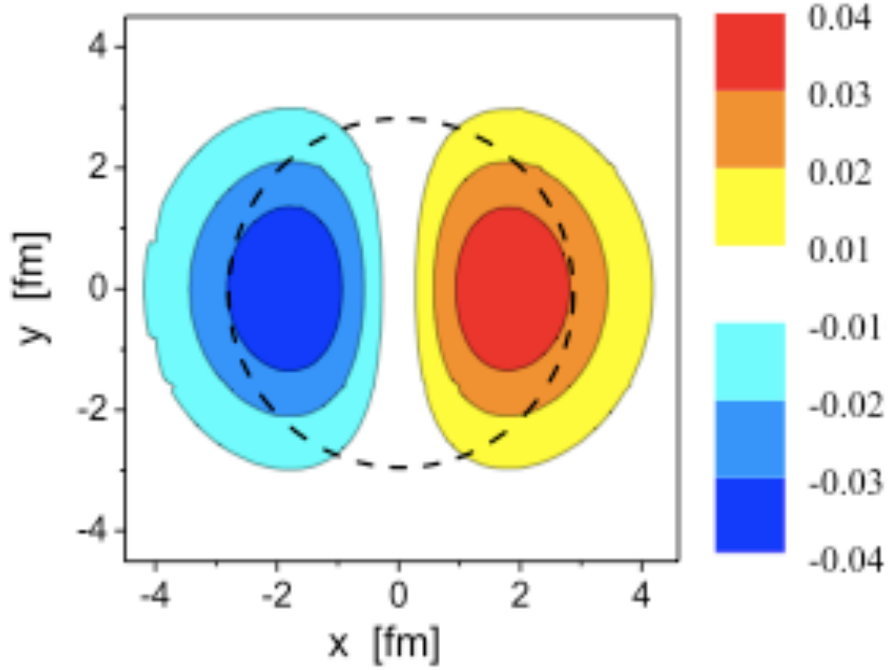
Is there a soft dipole ?

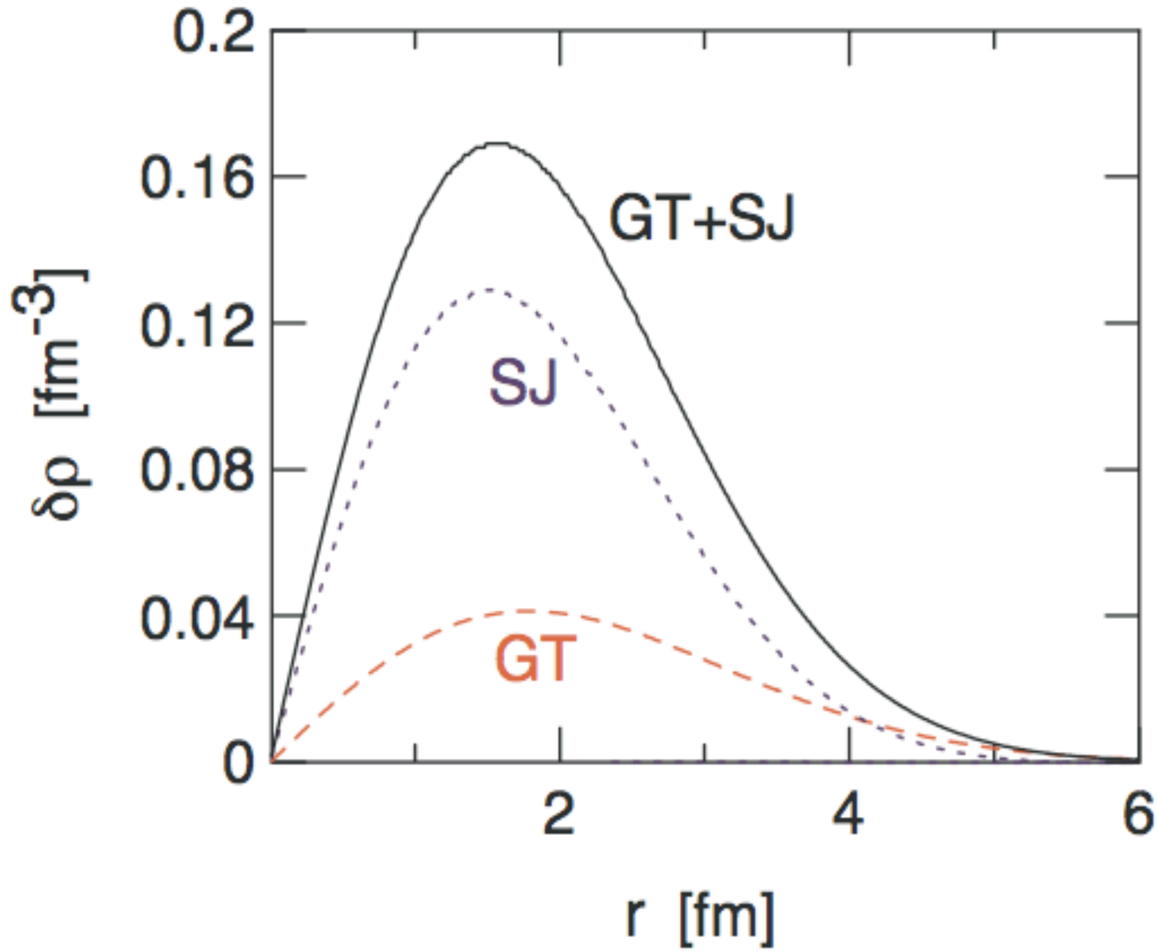
Pigmy resonances

$$B(EL) \sim \left| \int r^L \delta\rho_{fi} \, d^3r \right|^2$$

$$\delta\rho_P(r) \approx Z_{eff}^{GT} \alpha_{GT} \frac{d\rho_0}{dr} + Z_{eff}^{SJ} \alpha_{SJ} j_1(kr) \rho_0(r)$$







Estimates

hydrodynamical model

$$E_{PR} = \left(\frac{3\hbar^2}{2aRm_N A_r} \right)$$

$$A_r = A_c (A - A_c)/A$$

R = nuclear size

a = difuseness

$$\Rightarrow \quad E_{PR} \approx 1 \text{ MeV}$$

Bertulani
PRC 75, 024606 (2007)

$$\Gamma \sim \frac{\hbar \bar{v}_N}{R}$$

$$\bar{v}_N = \frac{3}{4} v_F = \frac{3}{4} \sqrt{\frac{2E_F}{m_N}}$$

$E_F \sim 35$ MeV

$R \sim 5$ fm

$$\Rightarrow \quad \Gamma \sim 6 \text{ MeV}$$

usual GR

pygmy GR

$\bar{v} \sim$ relative velocity of core and halo

$E_F \rightarrow E_P \sim 1$ MeV

$$\Rightarrow \quad \Gamma \sim 1 \text{ MeV}$$

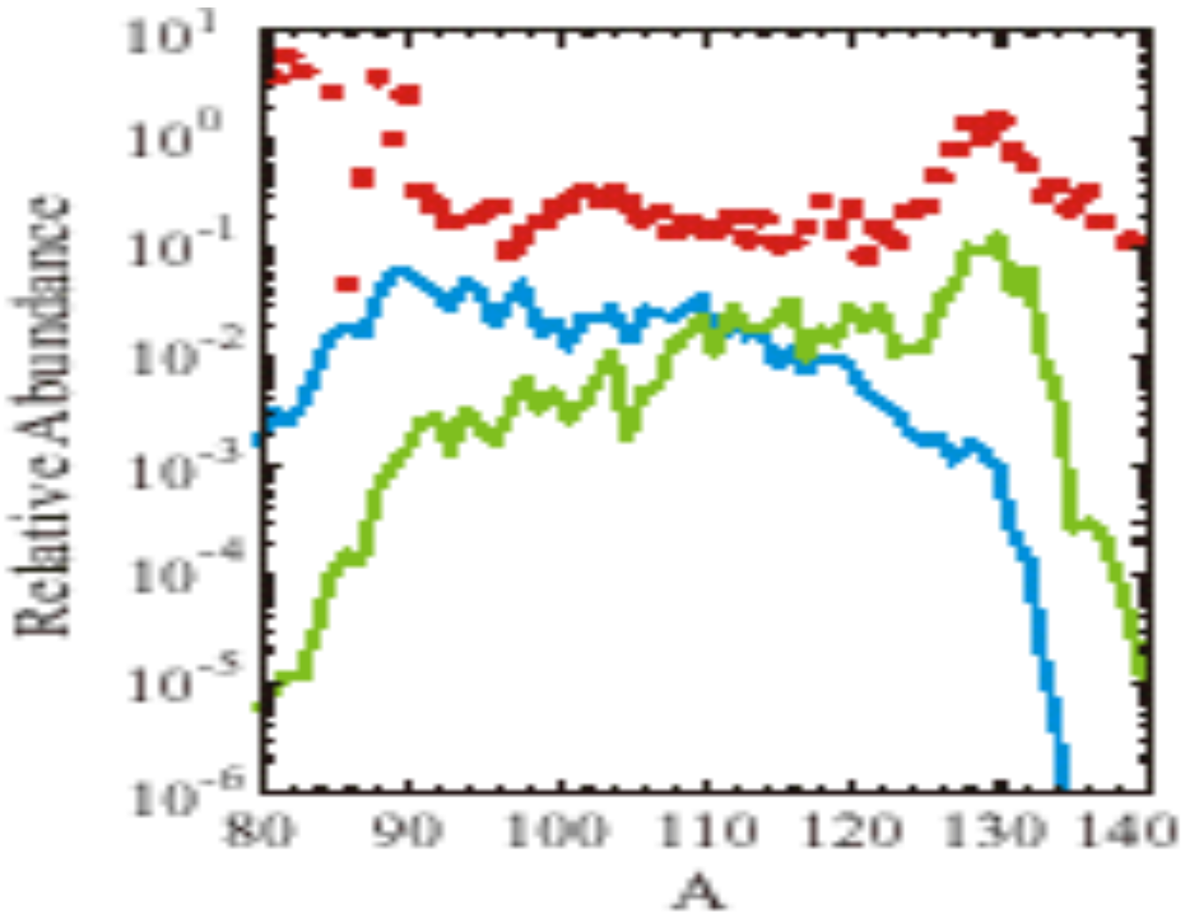
Trouble!

only accurate microscopic models
can resolve pygmy from direct breakup

Relevance for nuclear astrophysics (medium A)

Nucleosynthesis: (γ,n) or (n,γ) cross sections in the r-process

Importance of the "pygmy" states



Red: empirical

Blue: no pygmy

Green: with pygmy

S. Goriely, PLB 2000