Applying DNN to extract nuclear matter properties from neutron stars

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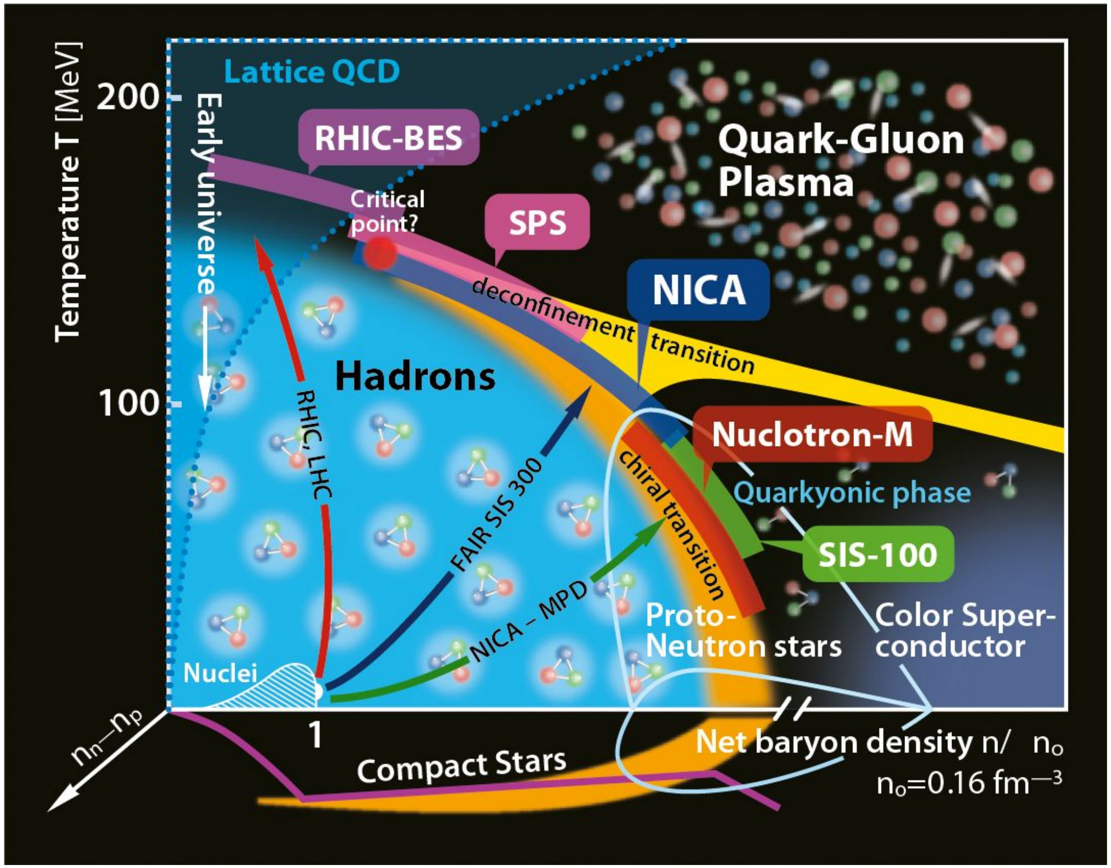
1290 FACULDADE DE CIÊNCIAS E TECNOLOGIA UNIVERSIDADE D COIMBRA

CFisUC

FCT Fundação para a Ciência e a Tecnologia

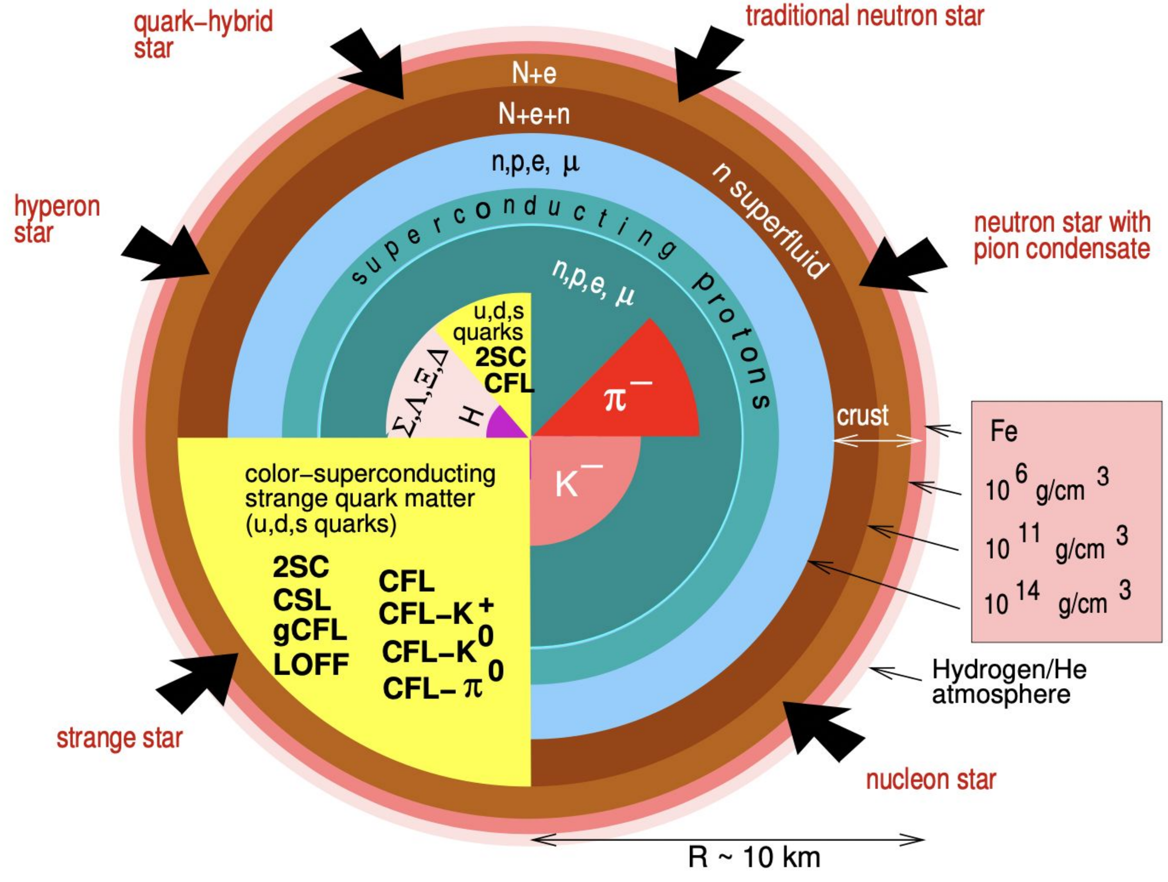
Phase Diagram of QCD



- The different QCD matter phases are still unknown
- Lattice QCD simulations cannot be performed at finite chemical potential
- **Neutron stars can help us in understanding the low T and dense region**

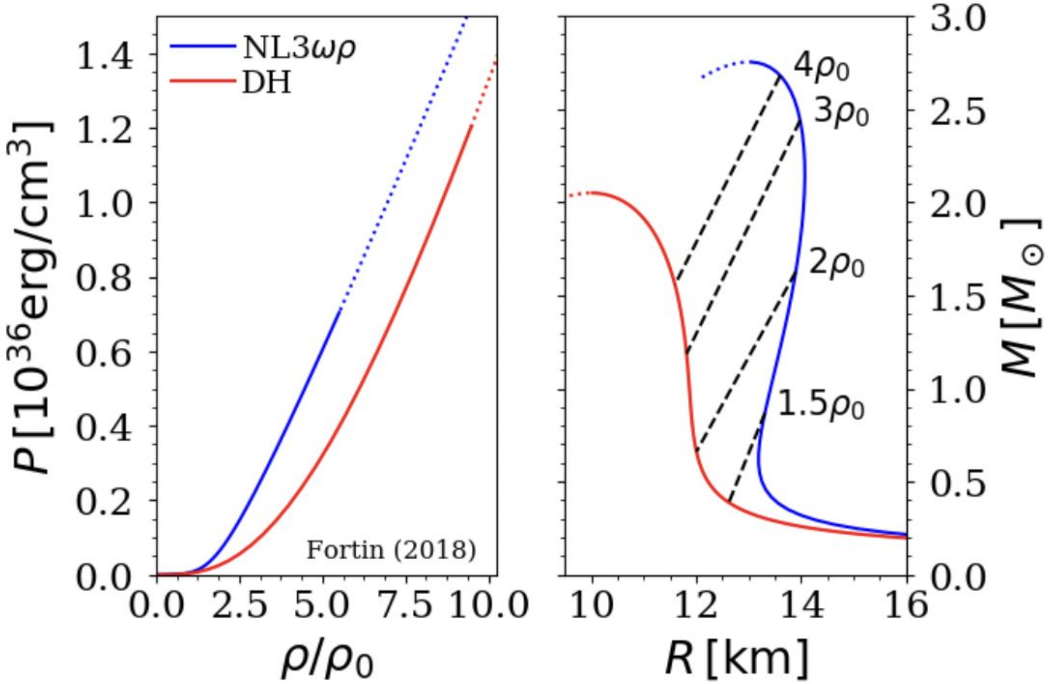
Several hypothesis for neutron star matter

- However, as the QCD solution is still unknown, there are several possible scenarios for NS matter



From the EOS to the mass-radius relation

- We need a microscopic model of nuclear matter: **P(n) and E(n)**
- Neutron star matter is cold matter (T=0) in beta equilibrium
- Solve **Tolman–Oppenheimer–Volkoff equation**: M(R) sequence

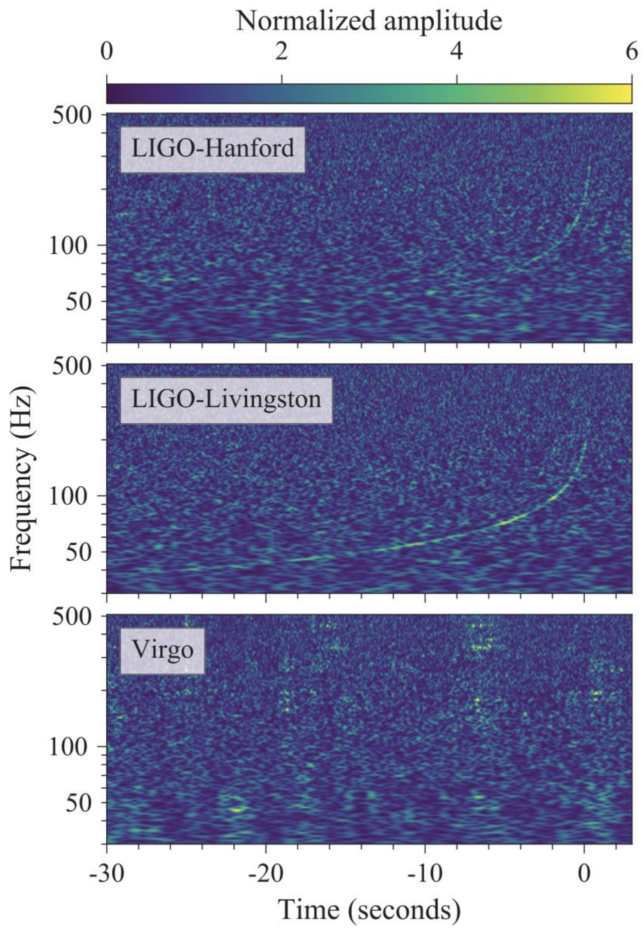


Observational constraints: neutron stars M/R

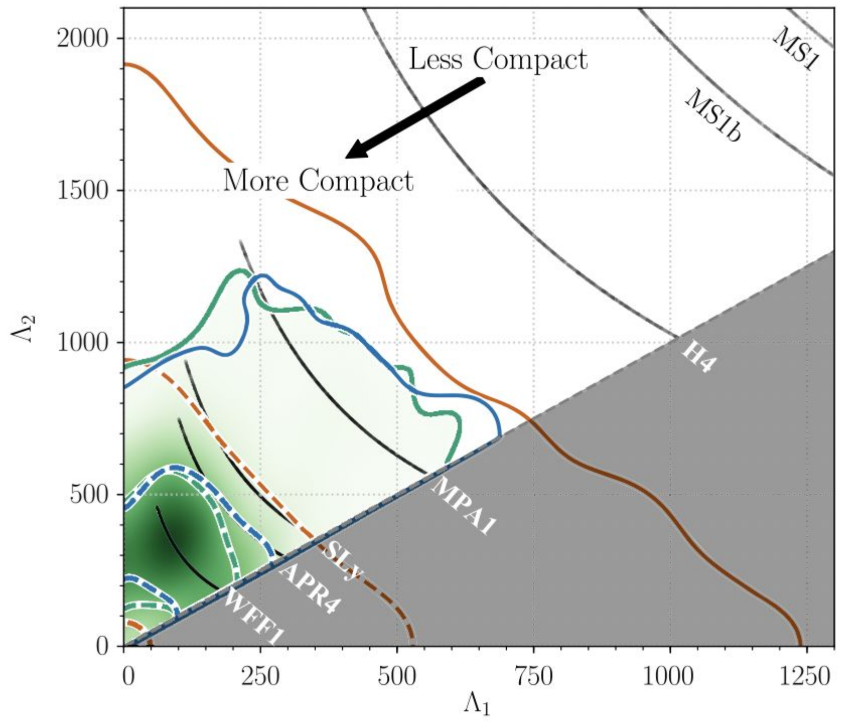
- **Highest NS mass measured**
  - $M = 1.908 \pm 0.016 M_\odot$ (J1614 - 2230)
    [Z. Arzoumanian et al., Astrophys. J., Suppl. Ser. 235, 37 (2018)]
  - $M = 2.08 \pm 0.07 M_\odot$ (J0740 + 6620)
    [E. Fonseca et al., Astrophys. J. Lett. 915, L12 (2021)]
  - $M = 2.13 \pm 0.04 M_\odot$ (J1810+171)
    [R. Romani, et. al., Astrophys. J. Lett., 908, L46]
- **NS masses and radii**
  - $M = 1.34^{+0.15}_{-0.16} M_\odot$ and $R = 12.71^{+1.14}_{-1.19}$ km (J0030 + 0451)
    [T. E. Riley et al., Astrophys. J. Lett. 887, L21 (2019)]
  - $M = 1.44^{+0.15}_{-0.14} M_\odot$ and $R = 13.02^{+1.24}_{-1.06}$ km (J0030 + 0451)
    [M. C. Miller et al., Astrophys. J. Lett. 887, L24 (2019)]
  - $M = 2.072^{+0.067}_{-0.066}$ $\mathrm{M}_\odot$ and $R = 12.39^{+1.30}_{-0.98}$ km (PSR J0740+6620)
    [T. E. Riley, et. al., Astrophys.J.Lett. 918 (2021) 2, L27]

Any reliable nuclear model must satisfy these constraints

GW constraints: BNS inspiral (GW170817)



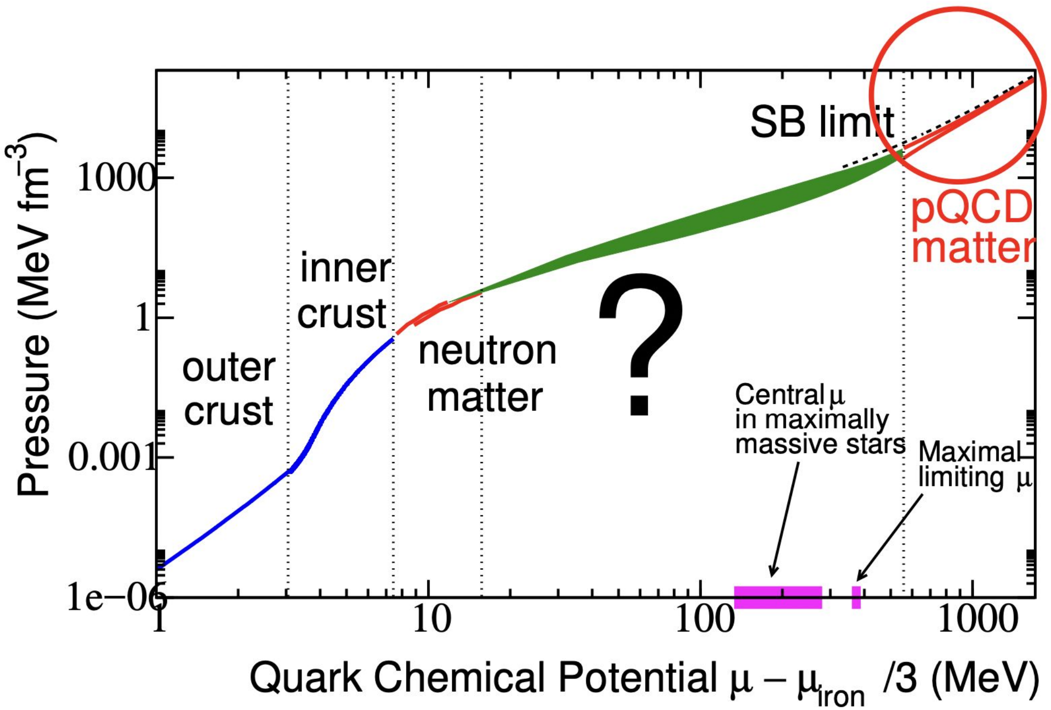
Λ (tidal deformability) quantifies how deformed is the NS when subject to an external tidal field.



PRL 119, 161101 (2017), PRL 121, 161101 (2018)

Allows to restrict the set of EOS of NS matter

High density constraints: perturbative QCD



Kurkela ApJ 789, 2014

- Perturbative QCD at zero temperature
- Good convergente at around $\mu_B = 2.6\ \mathrm{GeV}$

Kurkela et. al. PRD81 2010

Nuclear matter properties

- Nuclear matter properties are usually characterized by

$$\frac{E_{nuc}}{A}(n,\delta) = \frac{E_{SNM}}{A}(n) + E_{sym}(n)\,\delta^2$$

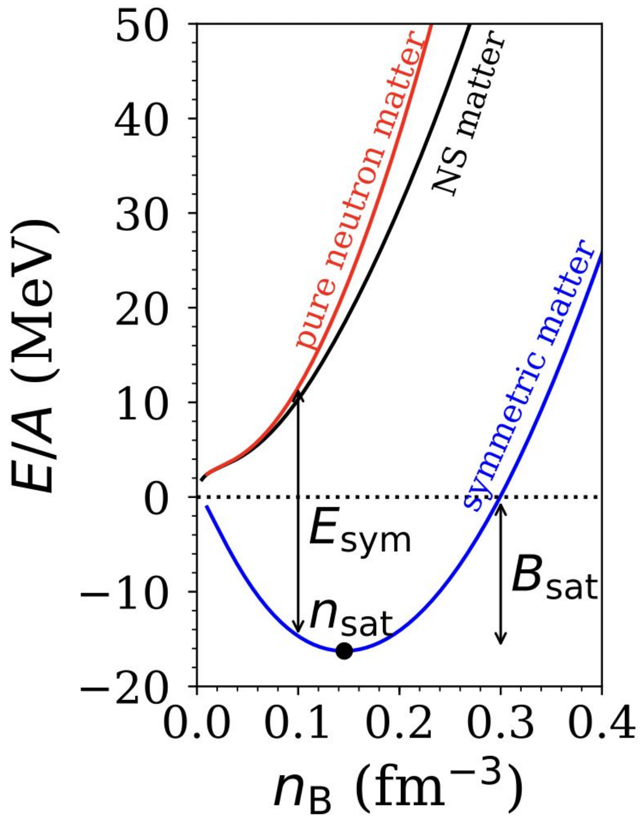
- $n = n_n + n_p$ is the baryonic density
- $\delta = (n_n - n_p)/(n_n + n_p)$ is the asymmetry

$$\frac{E_{SNM}}{A}(n) = E_{sat} + \frac{K_{sat}}{2}\eta^2 + \frac{J_{sat}}{3!}\eta^3 + \frac{Z_{sat}}{4!}\eta^4$$

$$E_{sym}(n) = E_{sym} + L_{sym}\eta + \frac{K_{sym}}{2}\eta^2 + \frac{J_{sym}}{3!}\eta^3 + \frac{Z_{sym}}{4!}\eta^4$$

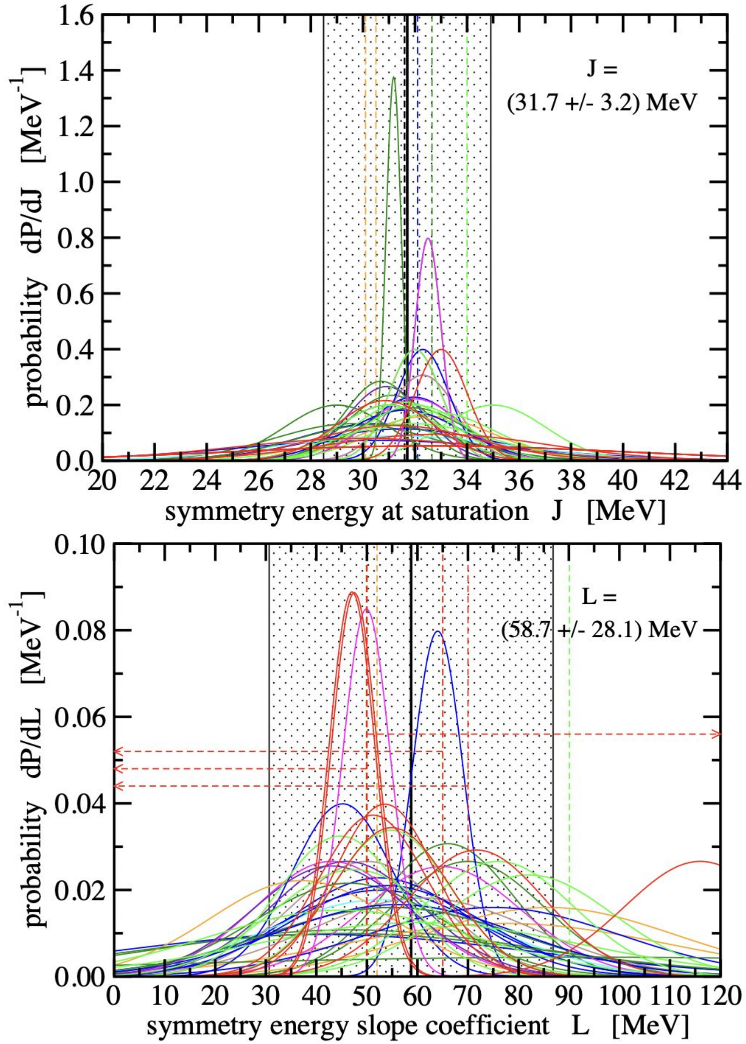
- $\eta = (n - n_0)/(3n_0)$.

[Morgin Fortin]



Constraints on the symmetry energy

- systematic of nuclear masses
- nuclear structure information
- nuclear resonances
- dipole polarizability of nuclei
- heavy-ion collisions
- theoretical calculations
- properties of neutron stars



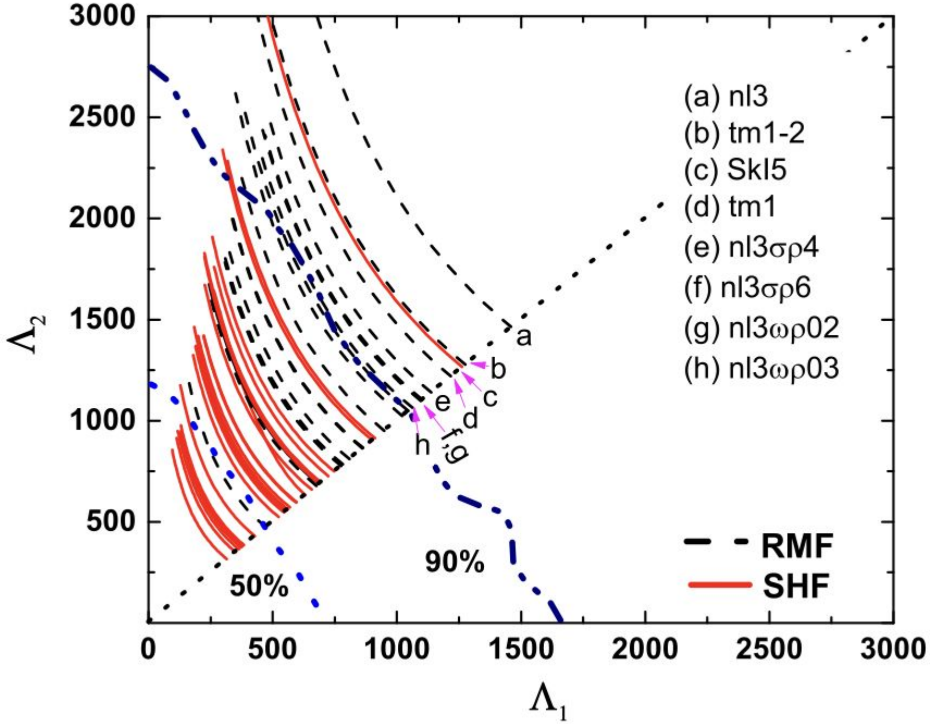
M. Oertel, et. al., Rev. Mod. Phys. 89, 015007

Uncertainties on nuclear parameters

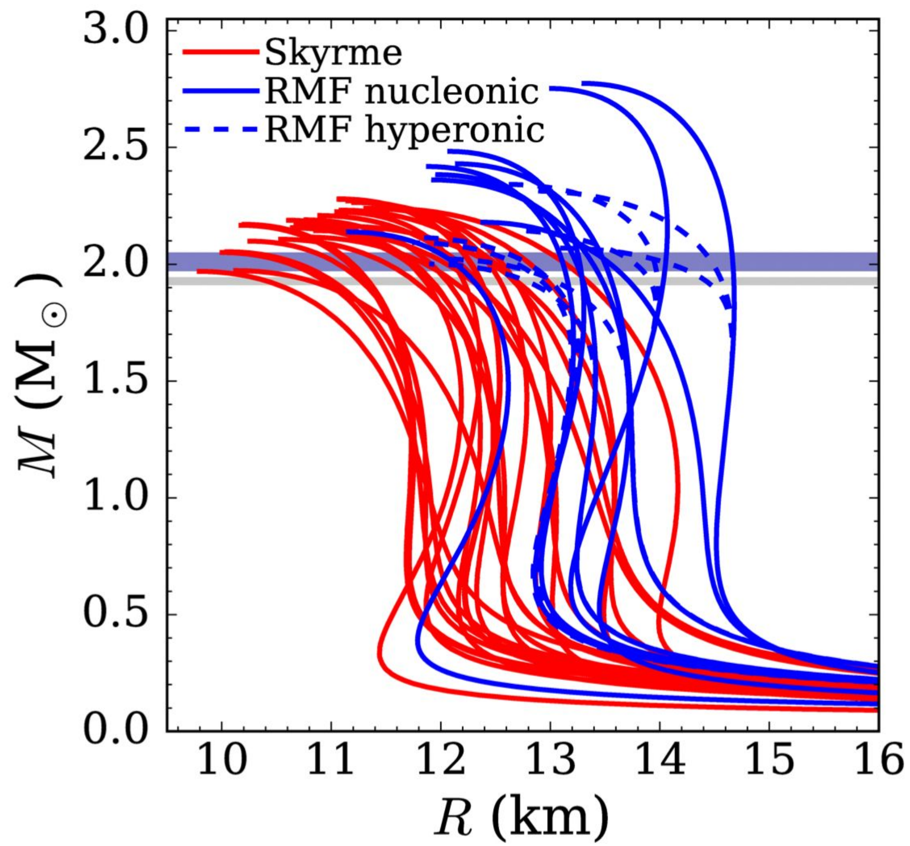
- E_{sat} and n_{sat} known to a few %
- E_{sym} and K_{sat} known within 10%
- L_{sym} known within 50%
- **Other remain almost unknown**

J. Margueron, et. al. *Phys.Rev.C* 97 (2018) 2, 025805

Nuclear models vs. NS observations



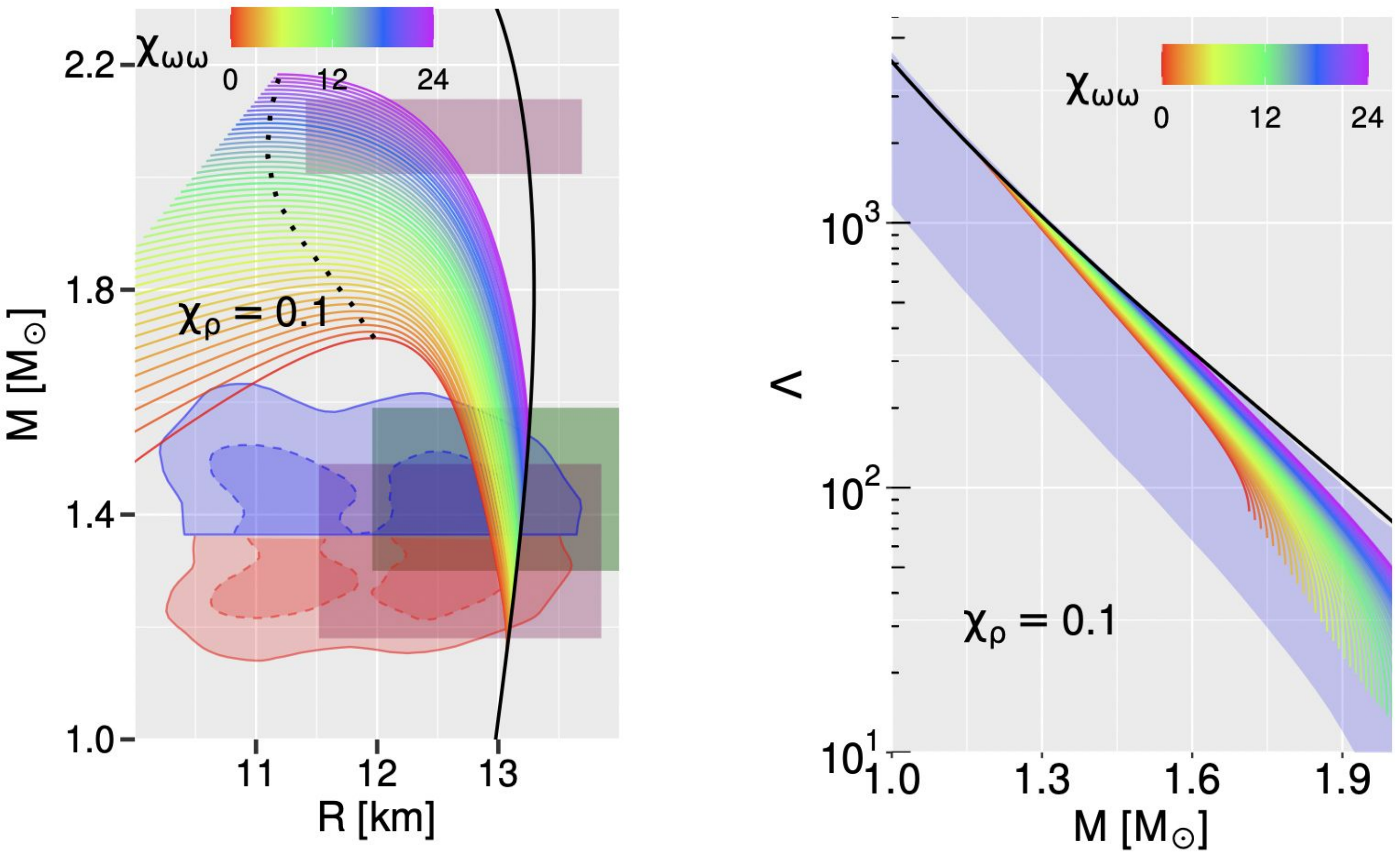
Tuhin Malik, et. al., *Phys.Rev.C* 98 (2018) 3, 035804



M. Fortin, et.al., *Phys.Rev.C* 94 (2016) 3, 035804

Different scenarios for NS matter

- **Hybrid stars**: first-order phase transition from hadronic to quark matter



M. Ferreira, et. al., PRD101, 123030 & PRD102, 083030 & PRD103, 123020

EOS parametrization and inference

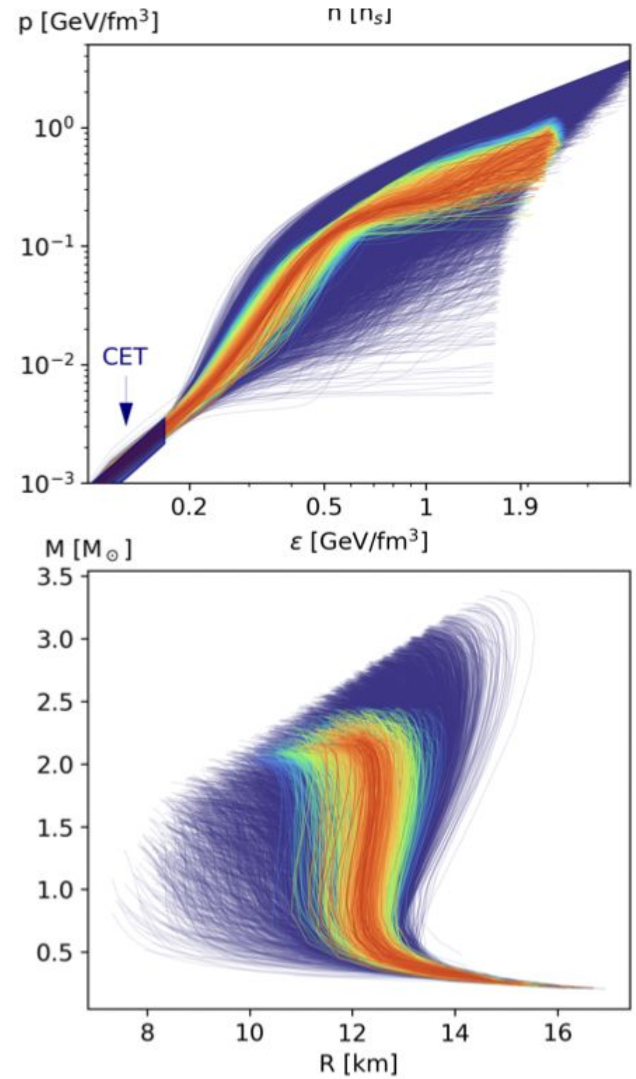
- Parametrize the EOS: Gaussian processes

  Prior: P(EOS)
- Bayesian inference:

  P(EOS|data) = P(data|EOS)P(EOS)/P(data)
- The data corresponds to astrophysical observations and theoretical calculations

The future observations will help to constrain the EOS of NS

Can we determine the nuclear matter properties from the NS EOS?



Tyler Gorda, et. al., arXiv:2204.11877v1

Generating the dataset

- Supervised problem: **y(x)**
  - **X:** energy density of beta-equilibrium matter

$$\mathbf{x_i} = [e(n_1), e(n_2), e(n_3), \cdots, e(n_{41})]$$

$$0.08 \leq n_i/\,\mathrm{fm}^{-3} \leq 0.30$$

[Tovar et. al., *Phys.Rev.D* 104 (2021) 12, 123036]

  - **Y:** Properties of nuclear matter

$$\mathbf{y_i} = (E_{sat}, K_{sat}, J_{sat}, E_{sym}, L_{sym}, K_{sym}, J_{sym}, n_0)$$

	[min, max]	# of points
E_{sat}	$[-16.7, -14.9]$	6
K_{sat}	$[170, 290]$	6
J_{sat}	$[-900, 150]$	6
Z_{sat}	$[-3500, 2500]$	6
E_{sym}	$[26, 38]$	6
L_{sym}	$[15, 105]$	6
K_{sym}	$[-400, 200]$	6
J_{sym}	$[-1200, 1200]$	6
Z_{sym}	$[-3500, 2500]$	6
n_0	$[0.145, 0.166]$	16

Dataset size: 9×10^7 EOS

Deep Neural Network

- **We want a good interpolating model**
- Loss Function (mse)

$$L(\mathbf{w}) = \sum_{i=1}^{M} (\hat{\mathbf{y}}_i(\mathbf{w}) - \mathbf{y}_i)^2$$

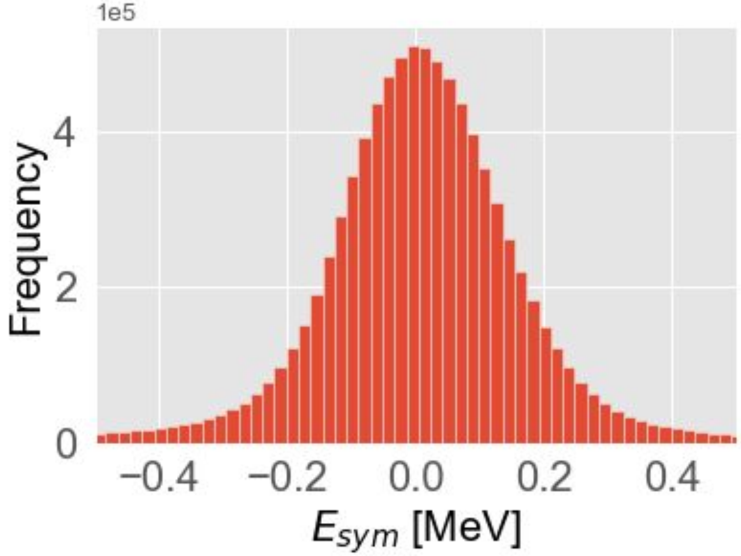
- Adam optimizer: lr =0.001
- Validation/training split of 0.2

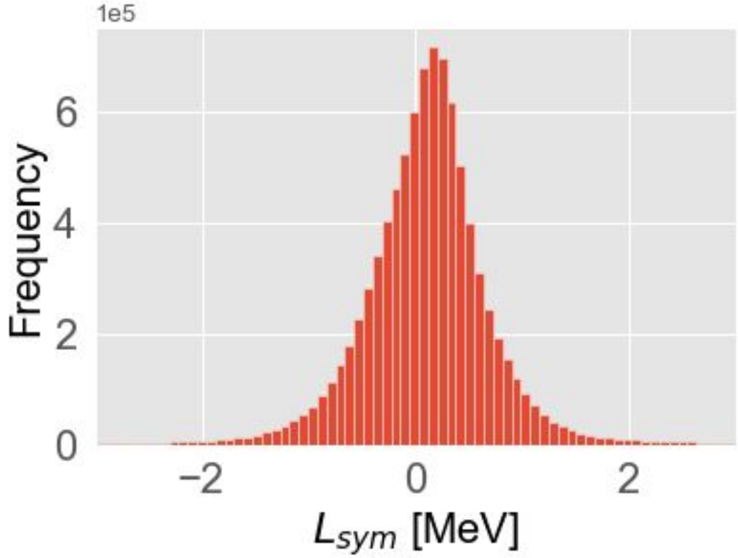
Layer	Activation function	size
0	-	41
1	Sigmoid	80
2	Sigmoid	80
3	Sigmoid	80
4	Sigmoid	40
5	Sigmoid	15
6	Linear	8

Performance on the test set

- Test set size: 9×10^6 EOS

	rho0	esat	ksat	qsat	esym	lsym	ksym	qsym
mean	-0.00017	-0.01678	-5.35536	9.44117	0.00324	0.10386	8.03442	3.56895
std	0.00117	0.11622	16.67010	171.85117	0.25254	1.26632	23.22310	207.55388
10%	-0.00086	-0.10795	-25.20679	-185.83968	-0.18173	-0.59669	-16.39285	-221.66793
90%	0.00052	0.07520	11.41779	197.97955	0.19610	0.76707	35.22026	234.07942





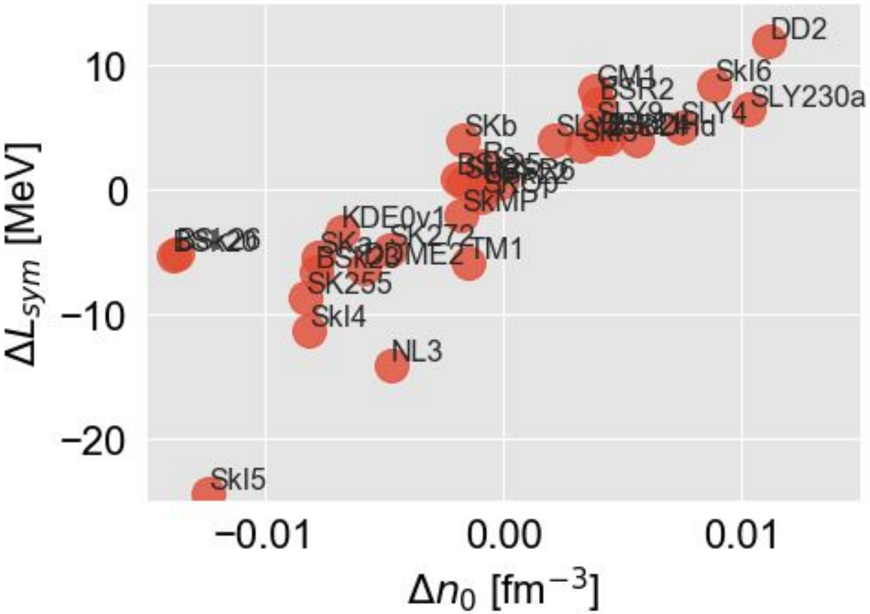
Applying the DNN to set of nuclear models

- Set of 33 EOS built from a relativistic mean field approach and non-relativistic Skyrme interactions

M. Fortin, et. al., Phys.Rev.C 94 (2016) 3, 035804

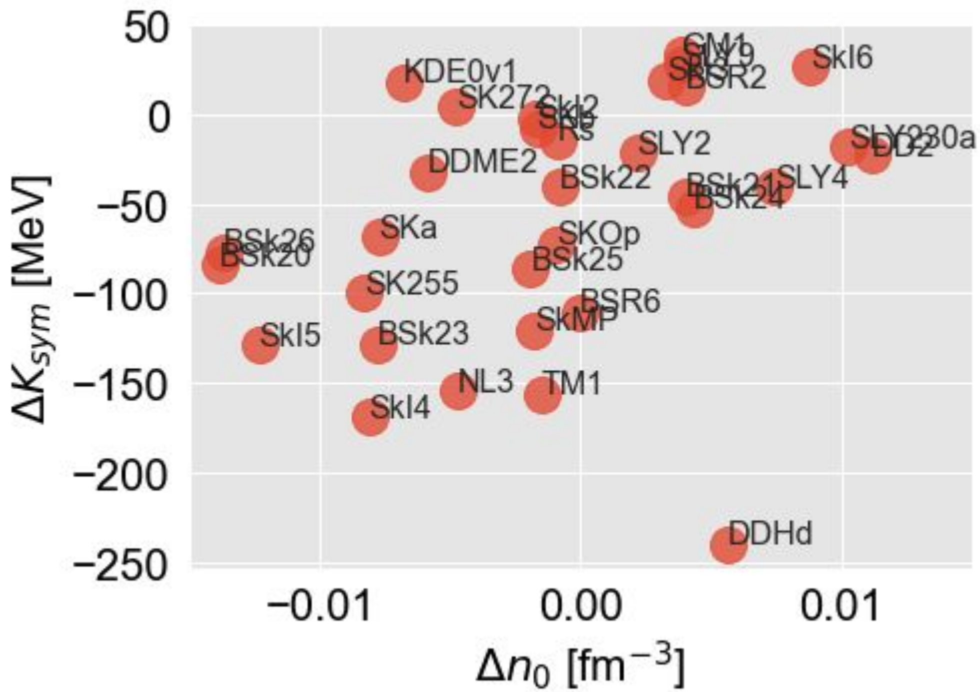
Model	n_s (fm^{-3})	E_s (MeV)	K (MeV)	J (MeV)	L (MeV)	K_{sym} (MeV)
NL3	0.149	-16.2	271.6	37.4	118.9	101.6
NL3$\omega\rho$	0.148	-16.2	271.6	31.7	55.5	-7.6
DDME2	0.152	-16.1	250.9	32.3	51.2	-87.1
GM1	0.154	-16.3	300.7	32.5	94.4	18.1
TM1	0.146	-16.3	281.2	36.9	111.2	33.8
DDHδ	0.153	-16.3	240.3	25.6	48.6	91.4
DD2	0.149	-16.0	242.6	31.7	55.0	-93.2
BSR2	0.149	-16.0	239.9	31.5	62.0	-3.1
BSR6	0.149	-16.1	235.8	35.6	85.7	-49.6

Results: DNN residuals



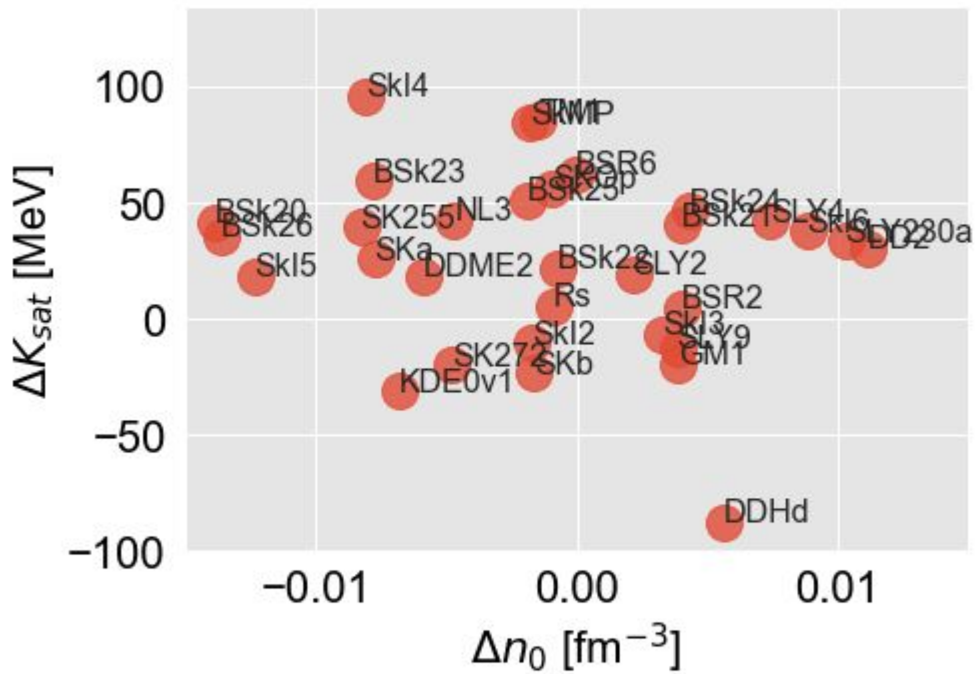
- Prediction errors: $\sigma\left(\Delta L_{sym}\right) = 22.3 \text{ MeV}$
- Increasing the number of n_0 points would increase the L_{sym} accuracy

Results: DNN residuals



$$\sigma\left(\Delta K_{sym}\right) = 85.4 \text{ MeV}$$

Results: DNN residuals



$$\sigma\left(\Delta K_{sat}\right) = 40.19 \text{ MeV}$$

Conclusions

- DNN model maps the NS EOS to the nuclear matter properties
- Fast model to extract the nuclear model parameters from neutron star EOS
- Low order parameters show good accuracy (with real nuclear models)
- Future work: Increase the accuracy my mapping the isoscalar/isovector parts separately.

Uncertainties on nuclear parameters

J. Margueron, et. al. *Phys.Rev.C* 97 (2018) 2, 025805

Model	Ref.	E_{sat} MeV	n_{sat} fm^{-3}	K_{sat} MeV	E_{sym} MeV
El. scatt.	Wang-99 [56]		0.1607	235	
				±15	
LDM	Myers-66 [57]	-15.677	0.136†	295	28.06
LDM	Royer-08 [58]	-15.5704	0.133†		23.45
LSD	Pomorski-03 [59]	-15.492	0.142†		28.82
DM	Myers-77 [60]	-15.96	0.145†	240	36.8
FRDM	Buchinger-01 [61]		0.157		
			±0.004		
INM	Satpathy-99 [62]	-16.108	0.1620	288	
				±20	
DF-Skyrme	Tondeur-86 [63]		0.158		
DF-Skyrme	Klupfel-09 [64]	-15.91	0.1610	222	30.7
		±0.06	±0.0013	±8	±1.4
DF-BSK2	Goriely-02 [65]	-15.79	0.1575	234	28.0
DF-BSK24,	Goriely-15 [66]	-16.045	0.1575	245	30.0
28,29		±0.005	±0.0004		
DF-Skyrme	McDonnell-15 [67]	-15.75	0.160	220	29
		±0.25	±0.005	±20	±1
DF-NLRMF	NL3* [68]	-16.3	0.15	258	38.7
DF-NLRMF	PK [69]	-16.27	0.148	283	37.7
DF-DDRMF	DDME1,2 [70, 71]	-16.17	0.152	247	32.7
		±0.03	±0.00	±3	±0.4
DF-DDRMF	PK [69]	-16.27	0.150	262	36.8
present		-15.8	0.155	230	32
estimation		±0.3	±0.005	±20	± 2

Model	Ref.	Q_{sat} MeV	L_{sym} MeV	K_{sym} MeV	K_{τ} MeV
DF-Skyrme	Berdichevsky-88 [94]	30	0		
DF-Skyrme	Farine-97 [95]	-700			
		± 500			
DF-Skyrme	Alam-14 [31]	-344	65	-23	-322
		± 46	±14	±73	±34
DF-Skyrme	McDonnell-15 [67]		40		
			±20		
DF-NLRMF	NL3* [68]	124	123	106	-690
DF-NLRMF	PK [69]	-25	116	55	-630
DF-DDRMF	DDME1,2 [70, 71]	400	53	-94	-500
		±80	±3	±7	±7
DF-DDRMF	PK [69]	-119	79.5	-50	-491
Correlation	Centelles-09 [96]		70		-425
			± 40		±175
DF-RPA	Carbone-10 [85]		60		
			± 30		
Correlation	Danielewicz-14 [87]		53		
			± 20		
Correlation	Newton-14 [97]		70		
			± 40		
Correlation	Lattimer-14 [98]		53		
			± 20		
GMR	Sagawa-07 [99]				-500
					± 50
GMR	Patel-14 [100]				-550
					± 100
present		300	60	-100	-400
estimation		±400	±15	±100	± 100