Machine learning for Quantum Noise Discrimination

Machine Learning at GGI

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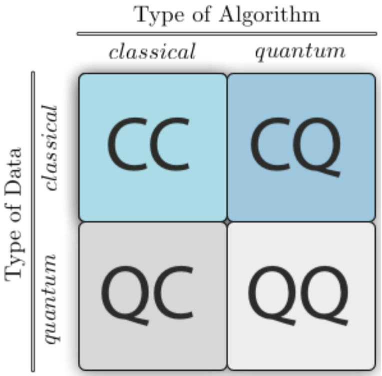
Quantum Driving And
Bio-complexity

UNIVERSITÀ
DEGLI STUDI
FIRENZE

September 5, 2022

Quantum Machine Learning



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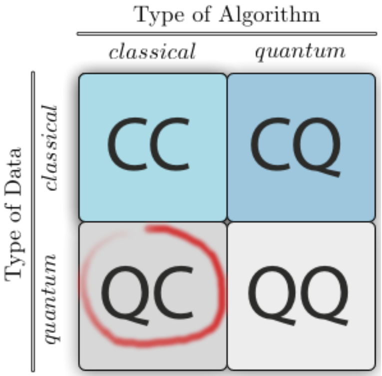

Introduction

Paper in review: Stefano Martina, Stefano Gherardini and Filippo Caruso. Machine learning classification of non-Markovian noise disturbing quantum dynamics

- ✔ We consider a Quantum Random Walker on a complex graph
  - perturbed by noise
- ✔ Discriminate quantum noise, by measuring only walker populations
  - Support Vector Machines
  - Neural Networks and Recurrent Neural Networks
- ✔ The dynamic parameters are crucial to the classification capacity
  - short evolution time / high frequency ⟶ easy
  - long evolution time / low frequency ⟶ hard
- ✔ Over 90% accuracy in classification between
  - two IID noises
  - two coloured noises
  - one IID VS one coloured noises

easier

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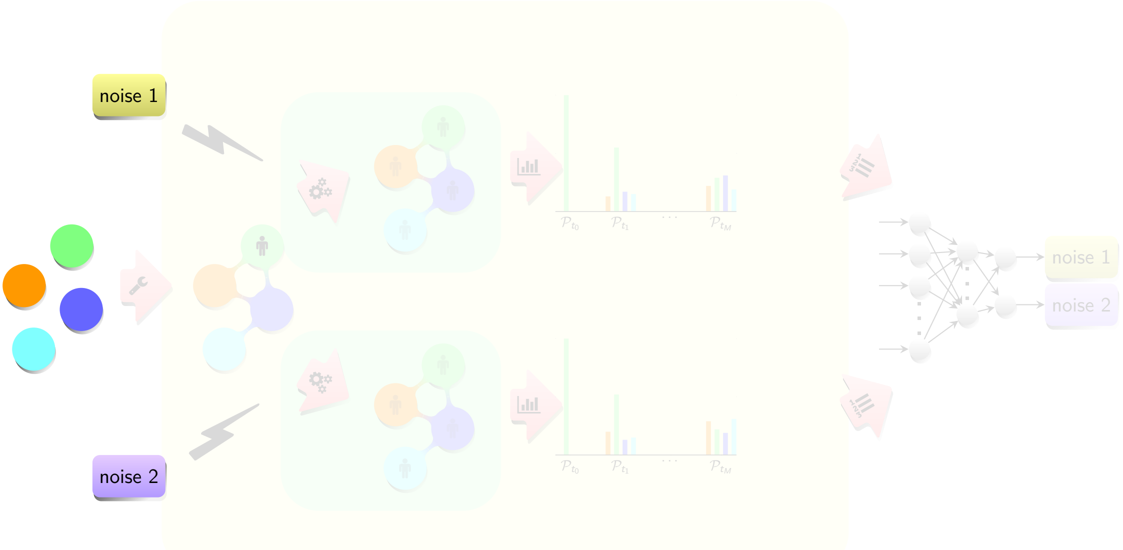
Setting definition



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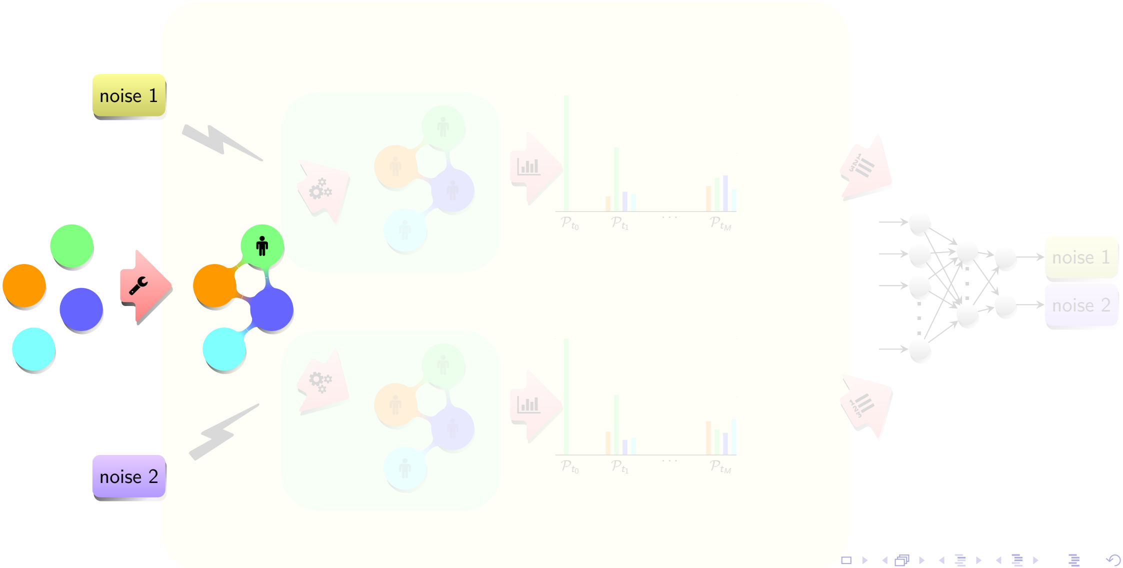

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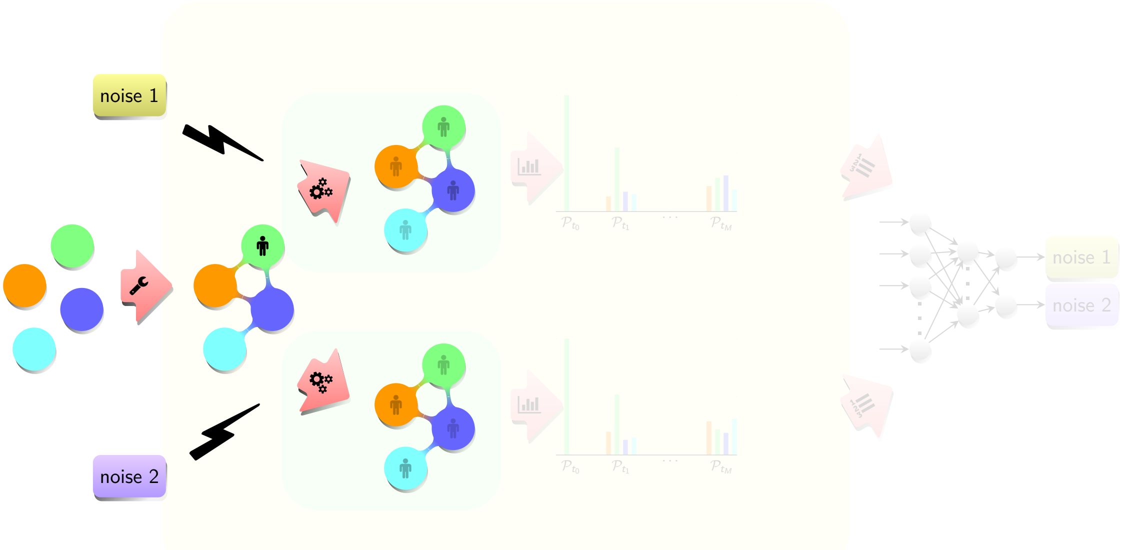

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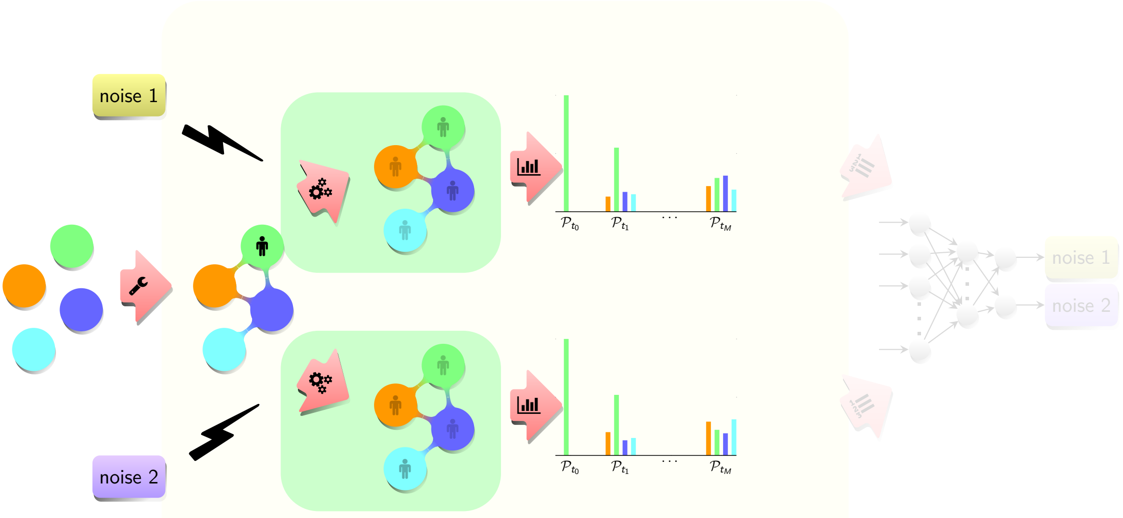

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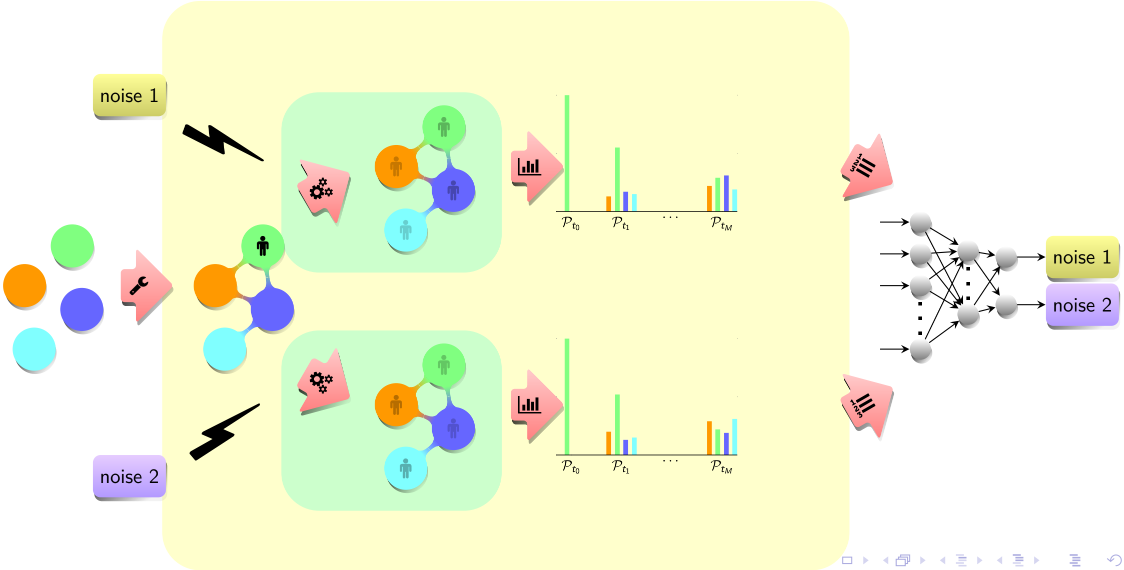
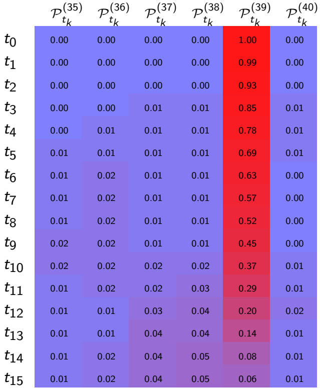


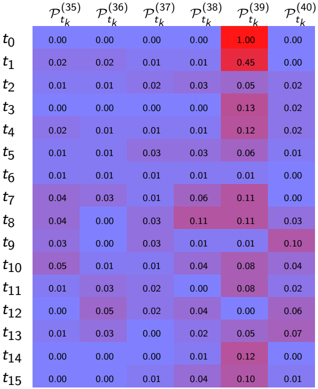
Example

- ✔ Example of populations with $t_{15} = 0.1$



	$\mathcal{P}_{t_k}^{(35)}$	$\mathcal{P}_{t_k}^{(36)}$	$\mathcal{P}_{t_k}^{(37)}$	$\mathcal{P}_{t_k}^{(38)}$	$\mathcal{P}_{t_k}^{(39)}$	$\mathcal{P}_{t_k}^{(40)}$
t_0	0.00	0.00	0.00	0.00	1.00	0.00
t_1	0.00	0.00	0.00	0.00	0.99	0.00
t_2	0.00	0.00	0.00	0.00	0.93	0.00
t_3	0.00	0.00	0.01	0.01	0.85	0.01
t_4	0.00	0.01	0.01	0.01	0.78	0.01
t_5	0.01	0.01	0.01	0.01	0.69	0.01
t_6	0.01	0.02	0.01	0.01	0.63	0.00
t_7	0.01	0.02	0.01	0.01	0.57	0.00
t_8	0.01	0.02	0.01	0.01	0.52	0.00
t_9	0.02	0.02	0.01	0.01	0.45	0.00
t_{10}	0.02	0.02	0.02	0.02	0.37	0.01
t_{11}	0.01	0.02	0.02	0.03	0.29	0.01
t_{12}	0.01	0.01	0.03	0.04	0.20	0.02
t_{13}	0.01	0.01	0.04	0.04	0.14	0.01
t_{14}	0.01	0.02	0.04	0.05	0.08	0.01
t_{15}	0.01	0.02	0.04	0.05	0.06	0.01

- ✔ Example of populations with $t_{15} = 1$



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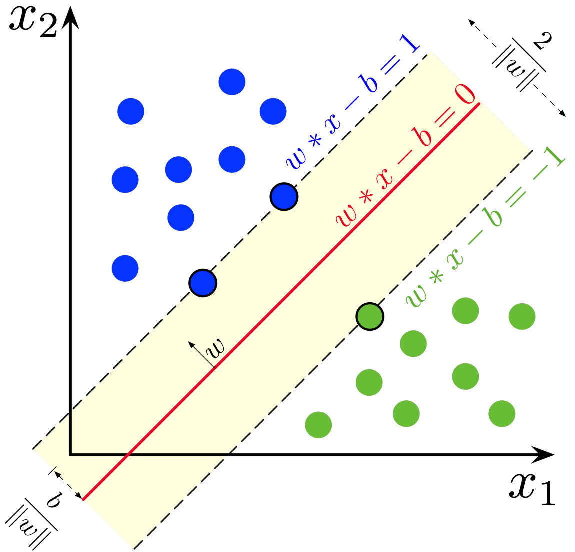
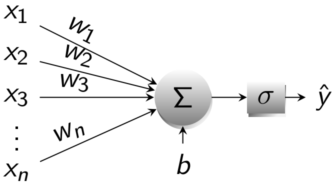
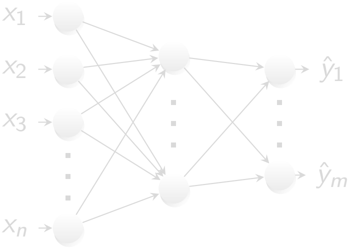
t_0	0.00	0.00	0.00	0.00	1.00	0.00
t_1	0.02	0.02	0.01	0.01	0.45	0.00
t_2	0.01	0.01	0.02	0.03	0.05	0.02
t_3	0.00	0.00	0.00	0.00	0.13	0.02
t_4	0.02	0.01	0.01	0.01	0.12	0.02
t_5	0.01	0.01	0.03	0.03	0.06	0.01
t_6	0.01	0.01	0.01	0.01	0.01	0.00
t_7	0.04	0.03	0.01	0.06	0.11	0.00
t_8	0.04	0.00	0.03	0.11	0.11	0.03
t_9	0.03	0.00	0.03	0.01	0.01	0.10
t_{10}	0.05	0.01	0.01	0.04	0.08	0.04
t_{11}	0.01	0.03	0.02	0.00	0.08	0.02
t_{12}	0.00	0.05	0.02	0.04	0.00	0.06
t_{13}	0.01	0.03	0.00	0.02	0.05	0.07
t_{14}	0.00	0.00	0.00	0.01	0.12	0.00
t_{15}	0.00	0.00	0.01	0.04	0.10	0.01

Support Vector Machines



Multi Layer Perceptron





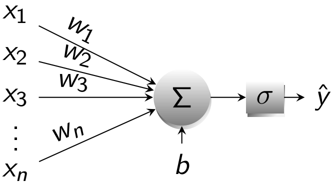
$$\hat{y} \equiv \sigma(\mathbf{w}^T \cdot \mathbf{x} + b)$$

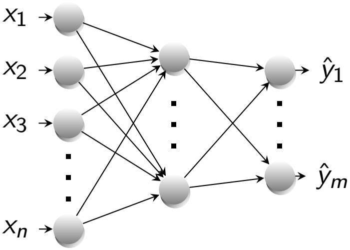
$$\mathbf{h}[0] \equiv \mathbf{x}$$

$$\mathbf{h}[l] \equiv \sigma\left(W[l]^T \cdot \mathbf{h}[l-1] + \mathbf{b}[l]\right)$$

$$\hat{\mathbf{y}} \equiv \mathbf{h}[L]$$

Multi Layer Perceptron





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Loss

Categorical Cross Entropy

$$\ell(\hat{\mathbf{y}}, \mathbf{y}) = -\sum_{j=1}^{O} y^{(j)} \log \hat{y}^{(j)}$$

- ✔ Common used in classification tasks
- ✔ Measure distance between two probability distributions
  - $\hat{\mathbf{y}}$ needs to be a probability distributions
  - obtained with softmax function:

$$\sigma^{(i)}(\mathbf{z}) = \frac{e^{z^{(i)}}}{\sum_{j=1}^{O} e^{z^{(j)}}}$$

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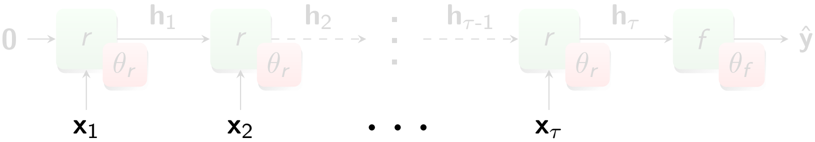
Unidirectional Recurrent Neural Network (RNN)

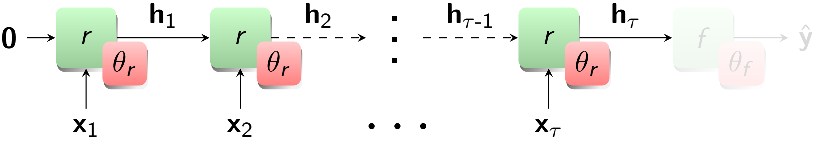


$$\mathbf{h}_t = r(\mathbf{x}_t, \mathbf{h}_{t-1};\, \theta_r)$$

$$\hat{\mathbf{y}} = f(\mathbf{h}_{\mathcal{T}};\, \theta_f)$$

- Used on sequential data
- Processed iteratively by non-linear function r
  - r parametrized with shared set of weights θ_r
  - $\mathbf{h}_t$ sort of memory
- $\mathbf{h}_{\mathcal{T}}$ representation of all the sequence
  - In classification $\mathbf{h}_{\mathcal{T}}$ can be processed by MLP f

Unidirectional RNN



$$\mathbf{h}_t = r(\mathbf{x}_t, \mathbf{h}_{t-1};\, \theta_r)$$

$$\hat{\mathbf{y}} = f(\mathbf{h}_\tau;\, \theta_f)$$

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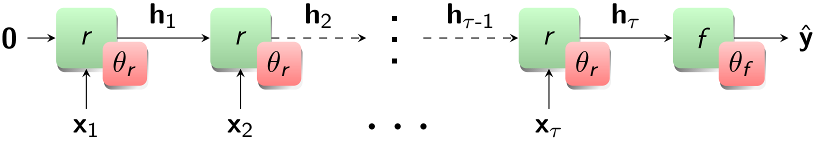

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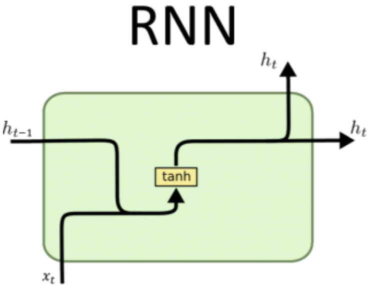
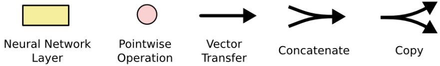
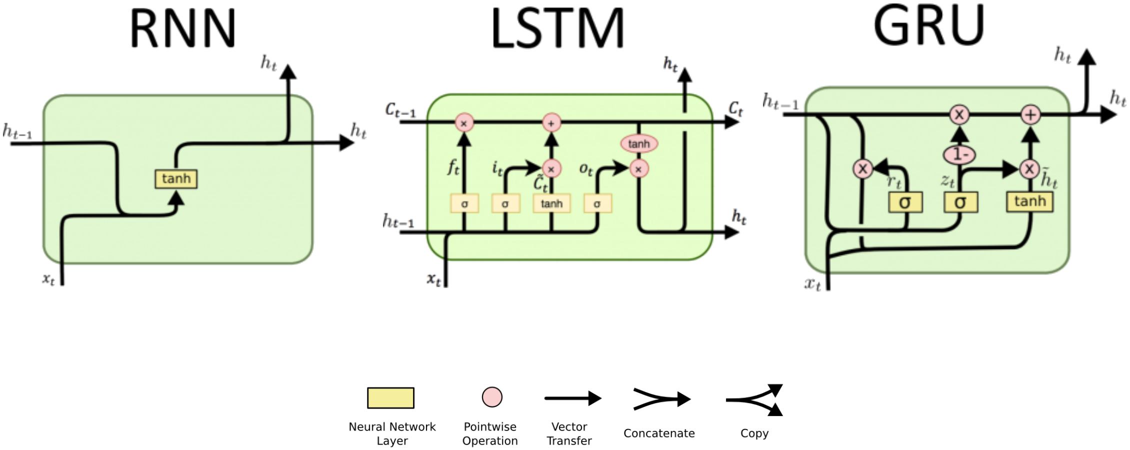
GRU/LSTM

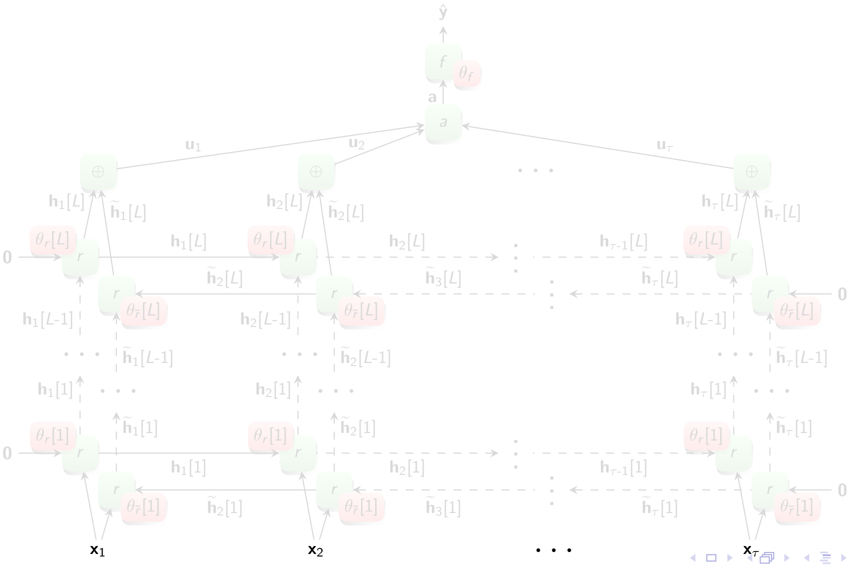

RNN
h_t
h_{t-1}
tanh
h_t
x_t


Neural Network Layer
Pointwise Operation
Vector Transfer
Concatenate
Copy

GRU/LSTM


RNN
h_{t-1}
tanh
h_t
h_t
x_t
LSTM
C_{t-1}
C_t
h_t
×
+
tanh
f_t
i_t
$\tilde{C}_t$
o_t
σ
σ
tanh
σ
h_{t-1}
h_t
x_t
GRU
h_t
h_{t-1}
h_t
X
+
1-
r_t
z_t
$\tilde{h}_t$
σ
σ
tanh
x_t
Neural Network Layer
Pointwise Operation
Vector Transfer
Concatenate
Copy

Bidirectional RNN with aggregation



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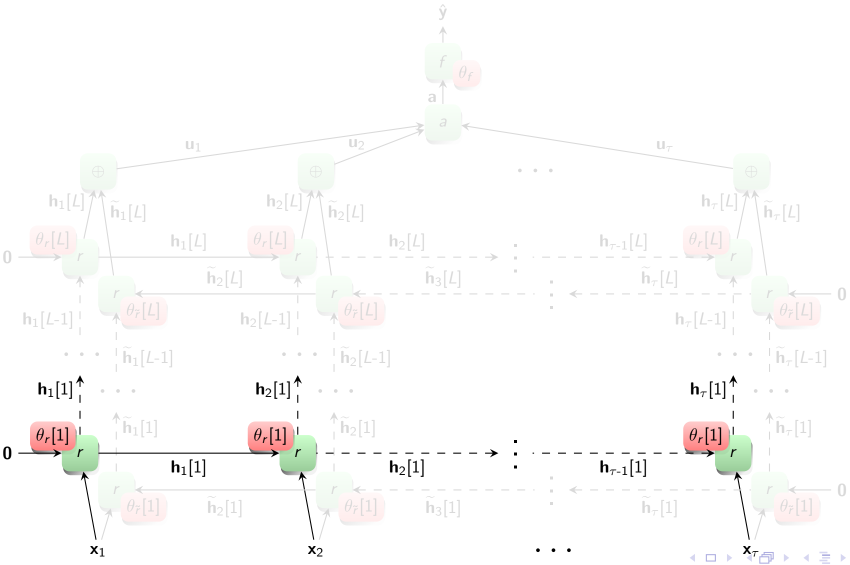

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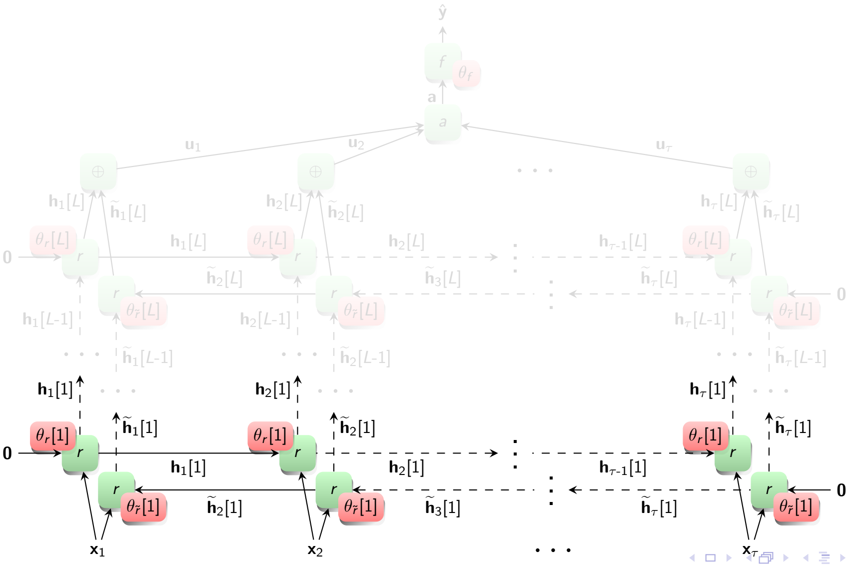

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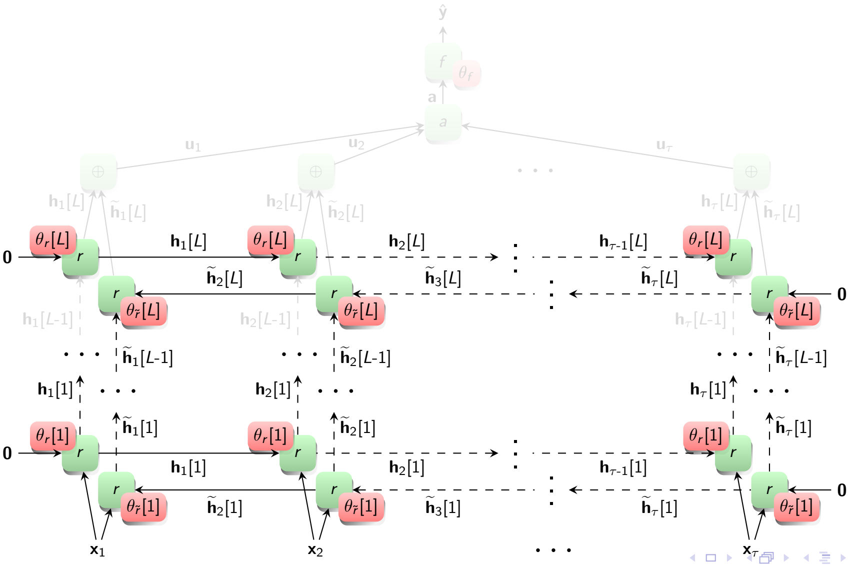

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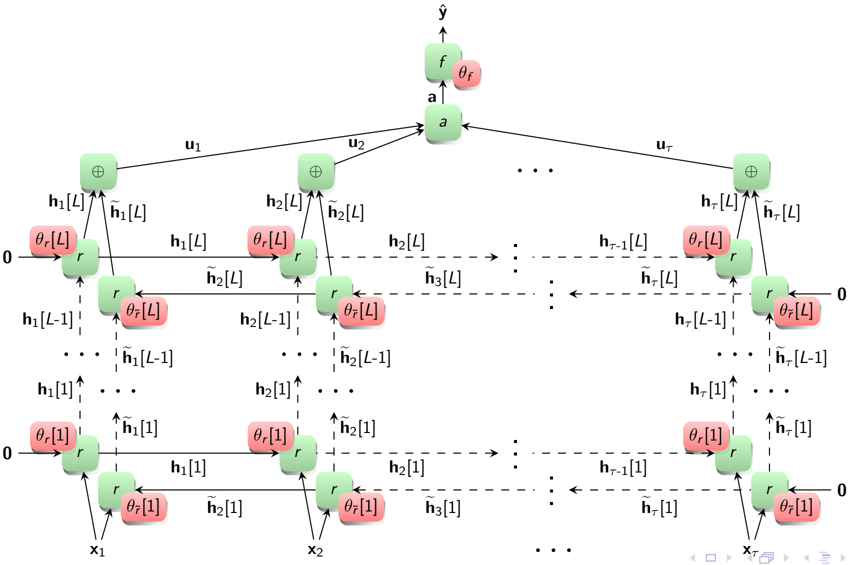

Aggregation

Standard

$$\mathbf{a} = \mathbf{h}_t[L] \oplus \widetilde{\mathbf{h}}_1[L]$$

Max pooling

$$\mathbf{u}_t = \mathbf{h}_t[L] \oplus \widetilde{\mathbf{h}}_t[L]$$

$$\mathbf{a}^{(j)} = \max_t \mathbf{u}_t^{(j)}$$

Attention mechanism

$$\mathbf{u}_t = \mathbf{h}_t[L] \oplus \widetilde{\mathbf{h}}_t[L]$$

$$\mathbf{v}_t = \tanh\left(\mathbf{W}^T \cdot \mathbf{u}_t + \mathbf{b}\right)$$

$$\alpha_t = \frac{e^{\langle \mathbf{v}_t, \mathbf{c} \rangle}}{\sum_{j=1}^{T} e^{\langle \mathbf{v}_j, \mathbf{c} \rangle}}$$

$$\mathbf{a} = \sum_{t=1}^{T} \alpha_t \mathbf{u}_t$$

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Results

		$t_{15} = 0.1$			$t_{15} = 1$		
		IID	**NM**	**VS**	**IID**	**NM**	**VS**
$\mathcal{P}_{t_{15}}$	**SVM**	97.0	82.3	96.5	50.3	51.2	49.5
	MLP	96.9	80.7	96.6	49.5	50.7	50.2
$\mathcal{P}_{t_0}, \dots, \mathcal{P}_{t_{15}}$	**SVM**	96.4	80.1	96.3	73.6	61.9	75.0
	GRU	96.5	91.5	96.7	90.5	73.3	88.2
	LSTM	96.8	90.4	96.4	88.6	70.3	86.3
	bi-GRU	96.6	92.2	96.6	91.0	74.6	90.6
	bi-LSTM	96.7	89.7	96.5	90.8	70.6	87.2
	bi-GRU-att	97.0	91.6	96.1	90.9	73.4	87.9
	bi-LSTM-att	96.9	87.9	96.3	89.0	71.6	87.4
	bi-GRU-max	96.6	92.6	96.6	91.8	76.1	90.4
	bi-LSTM-max	96.6	91.4	96.3	91.4	74.9	89.0

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	GRU	96.5	91.5	96.7	90.5	73.3	88.2
	LSTM	96.8	90.4	96.4	88.6	70.3	86.3
	bi-GRU	96.6	92.2	96.6	91.0	74.6	90.6
	bi-LSTM	96.7	89.7	96.5	90.8	70.6	87.2
	bi-GRU-att	97.0	91.6	96.1	90.9	73.4	87.9
	bi-LSTM-att	96.9	87.9	96.3	89.0	71.6	87.4
	bi-GRU-max	96.6	92.6	96.6	91.8	76.1	90.4
	bi-LSTM-max	96.6	91.4	96.3	91.4	74.9	89.0

Results

		$t_{15} = 0.1$			$t_{15} = 1$		
		IID	**NM**	**VS**	**IID**	**NM**	**VS**
$\mathcal{P}_{t_{15}}$	**SVM**	97.0	82.3	96.5	50.3	51.2	49.5
	MLP	96.9	80.7	96.6	49.5	50.7	50.2
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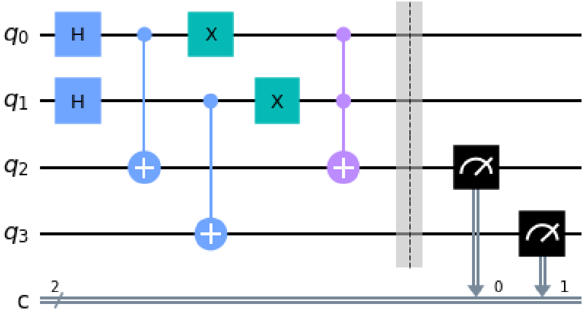

- ✔ We add a temporal dimension
- ✔ The circuit is repeated 3 times
- ✔ The measurements are after each CNOT and Toffoli for a total of 9 steps

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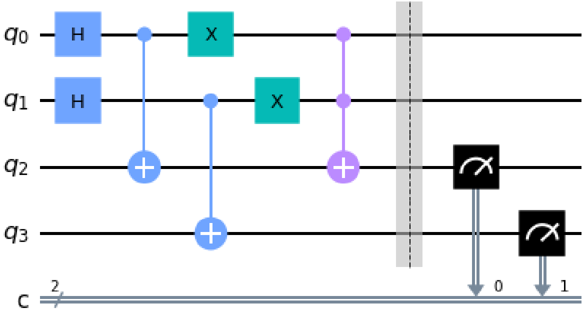

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- ✓ Temporally close executions
- ✓ 7 different IBM NISQ devices
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Binary device classification

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- ✓ For each pair of devices
  - the task is to find the used device from the measurement probabilities
- ✓ Both considering single and incremental steps
- ✓ Trained SVM classifiers
  - 60% – 20% – 20% train, validation and test split
  - kernels linear, RBF and polynomial with degree 2, 3 and 4

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Machines	Athens			Bogota			Casablanca			Lima			Quito			Santiago		
	k	$\alpha(k)$	$\alpha([1,k])$	k	$\alpha(k)$	$\alpha([1,k])$	k	$\alpha(k)$	$\alpha([1,k])$	k	$\alpha(k)$	$\alpha([1,k])$	k	$\alpha(k)$	$\alpha([1,k])$	k	$\alpha(k)$	$\alpha([1,k])$
Bogota	1	0.915	0.915															
	2	0.944	0.975															
	3	0.999	1.000															
	4	0.954	0.999															
	5	0.981	0.999															
	6	0.989	1.000															
	7	0.949	1.000															
	8	0.990	1.000															
	9	0.991	1.000															
Casablanca	1	0.895	0.895	1	0.831	0.831												
	2	0.740	0.921	2	0.932	0.959												
	3	0.968	0.983	3	0.943	0.995												
	4	0.994	0.998	4	0.960	0.999												
	5	0.969	0.999	5	0.889	0.998												
	6	0.988	1.000	6	0.811	1.000												
	7	0.869	1.000	7	0.830	0.999												
	8	0.906	1.000	8	0.818	0.999												
	9	0.927	1.000	9	0.782	1.000												
Lima	1	0.879	0.879	1	0.772	0.772	1	0.724	0.724									
	2	0.762	0.915	2	0.983	0.984	2	0.869	0.887									
	3	0.999	1.000	3	0.989	0.999	3	0.951	0.966									
	4	1.000	1.000	4	0.996	1.000	4	0.829	0.975									
	5	0.999	1.000	5	0.787	1.000	5	0.814	0.993									
	6	0.999	1.000	6	0.996	1.000	6	0.990	0.999									
	7	0.940	1.000	7	0.795	1.000	7	0.882	0.999									
	8	0.784	1.000	8	0.912	1.000	8	0.823	0.999									
	9	0.978	1.000	9	0.950	1.000	9	0.879	0.999									
Quito	1	0.685	0.685	1	0.815	0.815	1	0.834	0.834	1	0.725	0.725						
	2	1.000	1.000	2	1.000	1.000	2	1.000	1.000	2	1.000	1.000						
	3	1.000	1.000	3	0.990	1.000	3	1.000	1.000	3	1.000	1.000						
	4	0.998	1.000	4	1.000	1.000	4	1.000	1.000	4	1.000	1.000						
	5	0.993	1.000	5	0.787	1.000	5	0.881	1.000	5	0.714	1.000						
	6	0.966	1.000	6	0.983	1.000	6	0.979	1.000	6	1.000	1.000						
	7	0.948	1.000	7	0.965	1.000	7	0.940	1.000	7	0.978	1.000						
	8	0.998	1.000	8	0.969	1.000	8	0.959	1.000	8	0.991	1.000						
	9	0.988	1.000	9	0.891	1.000	9	0.864	1.000	9	0.953	1.000						

Multiclass device classification

- ✔ Not considering pairs, but a multiclass setting (one-vs-rest)

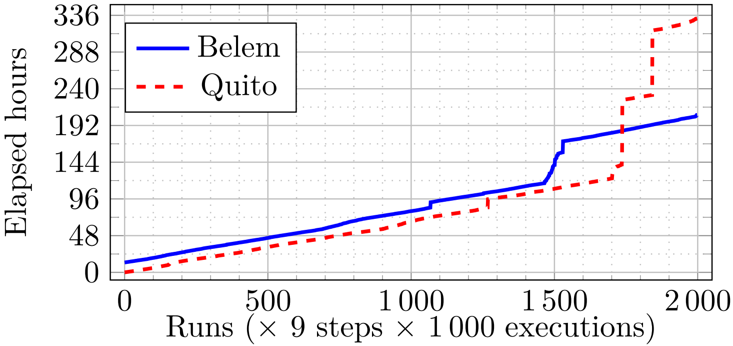
Machines	k	$\alpha(k)$	$\alpha([k-1,k])$	$\alpha([k-2,k])$	$\alpha([k-3,k])$	$\alpha([k-4,k])$	$\alpha([1,k])$
Athens & Bogota & Casablanca & Lima & Quito & Santiago & Yorktown	1	0.529					0.529
	2	0.691	0.850				0.850
	3	0.920	0.975	0.983			0.983
	4	0.896	0.983	0.991	0.992		0.992
	5	0.680	0.955	0.992	0.995	0.995	0.995
	6	0.789	0.946	0.988	0.997	0.998	0.998
	7	0.776	0.941	0.974	0.993	0.998	0.999
	8	0.703	0.911	0.960	0.982	0.994	0.999
	9	0.681	0.871	0.952	0.970	0.986	0.999
Average		0.740	0.929	0.977	0.988	0.994	

Time classification

- 2 runs of FAST dataset on the same machine but with 24 hours gap

Machine	k	$\alpha(k)$	$\alpha([1,k])$
Casablanca	1	0.882	0.882
	2	0.815	0.917
	3	0.757	0.948
	4	0.974	0.994
	5	0.969	1.000
	6	0.895	0.999
	7	0.917	0.999
	8	0.859	0.999
	9	0.721	0.999

SLOW dataset



- ✔ Temporally delayed executions (at least 2 minutes between each 1000 shots batch)
- ✔ 2 different IBM NISQ devices
- ✔ For each device 2000 sequences of 9 steps
  - each one is a distribution probability obtained running 1000 shots of the circuit

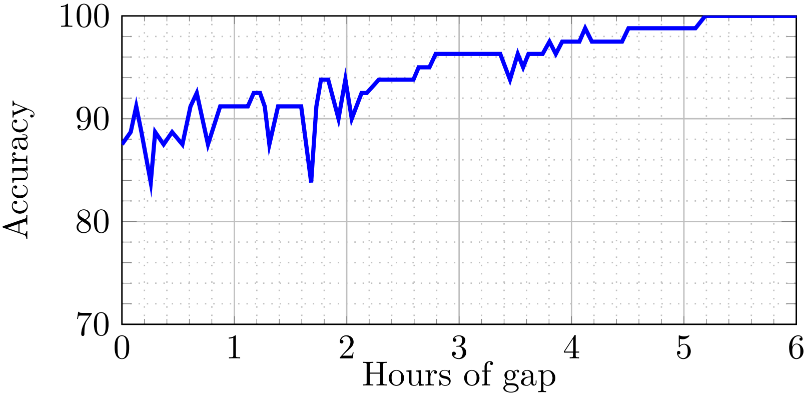
Time window classification

- ✔ in 1 machine of SLOW dataset
- ✔ discriminate between the first window of 200 runs and the subsequent windows

$[1, 200]$ vs		$[201, 400]$	$[401, 600]$	$[601, 800]$	$[801, 1000]$	$[1001, 1200]$	$[1201, 1400]$	$[1401, 1600]$	$[1601, 1800]$	$[1801, 2000]$
Machines	k	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$	$\alpha(k)$
Belem	1	0.838	0.975	0.975	0.950	0.938	0.938	0.750	0.950	0.963
	2	0.812	0.850	0.912	0.875	0.975	0.925	0.800	0.863	0.875
	3	0.688	0.812	0.688	0.738	0.650	0.500	0.738	0.613	0.700
	4	0.738	0.800	0.700	0.750	0.700	0.713	0.863	0.875	0.875
	5	0.662	0.700	0.800	0.800	0.725	0.863	0.762	0.838	0.812
	6	0.700	0.700	0.938	0.950	0.838	0.762	0.800	0.750	0.800
	7	0.675	0.850	0.887	0.975	0.912	0.887	0.713	0.875	0.950
	8	0.775	0.800	0.900	0.912	0.938	0.988	0.787	0.938	0.938
	9	0.750	0.900	0.912	0.988	0.850	0.838	0.787	0.812	0.838
Average		0.738	0.821	0.857	0.882	0.837	0.824	0.778	0.835	0.861

$[1, 200]$ vs		$[201, 400]$	$[401, 600]$	$[601, 800]$	$[801, 1000]$	$[1001, 1200]$	$[1201, 1400]$	$[1401, 1600]$	$[1601, 1800]$	$[1801, 2000]$
Machines	k	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$	$\alpha([1, k])$
Belem	1	0.838	0.975	0.975	0.950	0.938	0.938	0.750	0.950	0.963
	2	0.850	0.963	0.988	0.975	1.000	0.950	0.825	0.988	0.988
	3	0.887	0.975	0.988	0.975	0.988	0.988	0.850	1.000	0.988
	4	0.850	0.950	1.000	0.988	0.988	0.975	0.975	0.988	1.000
	5	0.850	0.963	1.000	0.988	0.988	0.975	0.963	1.000	1.000
	6	0.850	0.988	0.988	0.988	1.000	0.988	0.975	1.000	0.988
	7	0.863	0.988	1.000	0.988	1.000	1.000	0.988	1.000	1.000
	8	0.850	1.000	1.000	0.988	1.000	1.000	0.975	1.000	1.000
	9	0.875	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

- ✓ in same machine as previous slide
- ✓ $\alpha([1,9])$ discriminating the first window of 200 runs from another window sliding in time



Robustness in time

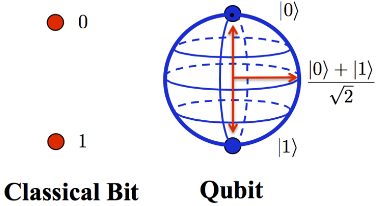
- Both machines of SLOW dataset
- $\alpha([1,9])$ discriminating the used machine
- Train on the window in row index; test on window in column index

	1	2	3	4	5	6	7	8	9	10
1	1.000	1.000	0.995	0.925	0.880	0.865	0.995	1.000	1.000	1.000
2	1.000	1.000	0.995	0.925	0.920	0.910	0.980	1.000	1.000	1.000
3	1.000	1.000	1.000	0.970	0.950	0.950	0.980	1.000	1.000	1.000
4	1.000	0.980	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.980	0.935	0.955	0.995	1.000	0.995	1.000	1.000	1.000	1.000
6	0.995	0.995	0.995	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	1.000	1.000	0.995	0.985	1.000	0.990	1.000	1.000	1.000	1.000
8	1.000	1.000	0.995	0.995	1.000	0.990	0.995	1.000	1.000	1.000
9	1.000	1.000	0.995	0.995	0.970	0.960	1.000	1.000	1.000	1.000
10	1.000	1.000	0.995	0.995	0.995	0.995	0.995	1.000	1.000	1.000

Thank you! Questions?

Qubit

- Classic bit can take one value between 0 and 1
- A qubit can take one of infinite values
  - in Hilbert vector space with basis of two elements $|0\rangle$ and $|1\rangle$
- A qubit is in superposition $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$
  - Where amplitudes α and β are complex numbers such that $|\alpha|^2 + |\beta|^2 = 1$



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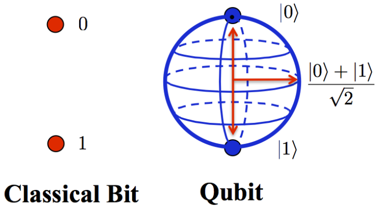

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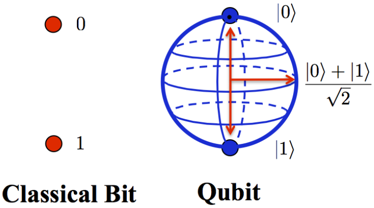

Measure

- ✔ The result of the measure is random
- ✔ When we measure a qubit we obtain a classical bit
- ✔ The measure of $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ is
  - 0 with probability $|\alpha|^2$
  - 1 with probability $|\beta|^2$

Effect of measure

Wavefunction collapse the new state after the measurement will be $|0\rangle$ or $|1\rangle$ depending on the measurement result

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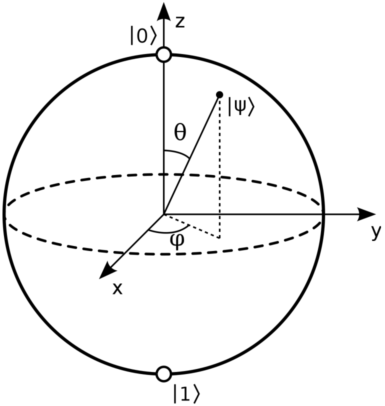
Bloch sphere

- ✔ We can rewrite $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \rightarrow |\psi\rangle = \cos\frac{\theta}{2} |0\rangle + e^{i\varphi} \sin\frac{\theta}{2} |1\rangle$
  - with $0 \leq \theta \leq \pi$ and $0 \leq \varphi < 2\pi$



Quantum gates

- The evolution of a state is given by the Schrödinger equation

$$H(t)\left|\psi(t)\right\rangle = i\hbar\frac{\partial}{\partial t}\left|\psi(t)\right\rangle$$

- In quantum circuits, the operation given by complex unitary matrices, i.e. verifying

$$UU^\dagger = U^\dagger U = I$$

  where $U^\dagger$ is the complex conjugate transpose of U
- Each such matrix is a possible quantum gate in a quantum circuit

Application (for 1-qubit gate)

For $U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $\left|\psi\right\rangle = \alpha\left|0\right\rangle + \beta\left|1\right\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

$$U\left|\psi\right\rangle = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} a\alpha + b\beta \\ c\alpha + d\beta \end{pmatrix} = (a\alpha + b\beta)\left|0\right\rangle + (c\alpha + d\beta)\left|1\right\rangle$$

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Pauli Gates (rotation of π along correspondig axis in Bloch)

X or NOT (Pauli σ_X)

$$X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\alpha\,|0\rangle + \beta\,|1\rangle \;—\boxed{X}—\; \beta\,|0\rangle + \alpha\,|1\rangle$$

Y (Pauli σ_Y)

$$Y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\alpha\,|0\rangle + \beta\,|1\rangle \;—\boxed{Y}—\; -i\beta\,|0\rangle + i\alpha\,|1\rangle$$

Z (Pauli σ_Z)

$$Z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\alpha\,|0\rangle + \beta\,|1\rangle \;—\boxed{Z}—\; \alpha\,|0\rangle - \beta\,|1\rangle$$

I

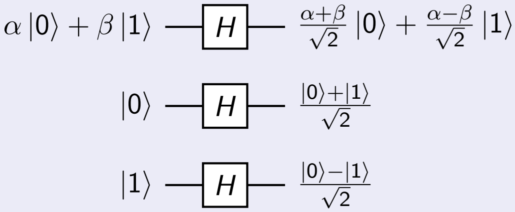
$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\alpha\,|0\rangle + \beta\,|1\rangle \;—\boxed{I}—\; \alpha\,|0\rangle + \beta\,|1\rangle$$

Hadamard gate

H

$$H := \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$



$$\alpha\left|0\right\rangle + \beta\left|1\right\rangle \; \text{---}\boxed{H}\text{---} \; \tfrac{\alpha+\beta}{\sqrt{2}}\left|0\right\rangle + \tfrac{\alpha-\beta}{\sqrt{2}}\left|1\right\rangle$$

$$\left|0\right\rangle \; \text{---}\boxed{H}\text{---} \; \tfrac{\left|0\right\rangle+\left|1\right\rangle}{\sqrt{2}}$$

$$\left|1\right\rangle \; \text{---}\boxed{H}\text{---} \; \tfrac{\left|0\right\rangle-\left|1\right\rangle}{\sqrt{2}}$$

$$\left|+\right\rangle := \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}} \qquad\qquad \left|-\right\rangle := \frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}}$$

2 qubit systems

- ✔ Each qubit can be in state $|0\rangle$ or $|1\rangle$
- ✔ We have 4 possibilities, equivalently ($\otimes$ is Kroneker product)

$$|0\rangle \otimes |0\rangle, \quad |0\rangle \otimes |1\rangle, \quad |1\rangle \otimes |0\rangle, \quad |1\rangle \otimes |1\rangle$$
$$|0\rangle |0\rangle, \quad |0\rangle |1\rangle, \quad |1\rangle |0\rangle, \quad |1\rangle |1\rangle$$
$$|00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle$$

- ✔ We can have superposition

$$|\psi\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle$$

with amplitudes α_{xy} complex numbers such that $\sum_{x,y=0}^{1} |\alpha_{xy}|^2 = 1$

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Measuring 2 qubit systems

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Measuring both qubits

- ✓ 00 with probability $|\alpha_{00}|^2$, new state $|00\rangle$
- ✓ 01 with probability $|\alpha_{01}|^2$, new state $|01\rangle$
- ✓ 10 with probability $|\alpha_{10}|^2$, new state $|10\rangle$
- ✓ 11 with probability $|\alpha_{11}|^2$, new state $|11\rangle$

Measuring only one qubit (the first in this case)

- ✓ 0 with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$, new state $\frac{\alpha_{00}|00\rangle+\alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2+|\alpha_{01}|^2}}$
- ✓ 1 with probability $|\alpha_{10}|^2 + |\alpha_{11}|^2$, new state $\frac{\alpha_{10}|10\rangle+\alpha_{11}|11\rangle}{\sqrt{|\alpha_{10}|^2+|\alpha_{11}|^2}}$

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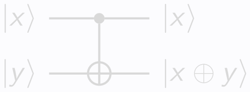
2 qubit gates

- ✔ If A and B are one-qubit gates acting on two different cubits, then on the two qubit $A \otimes B$
- ✔ In general all unitary matrices 4×4

CNOT

$$CNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

for $x, y \in \{0, 1\}$



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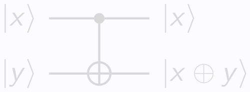

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