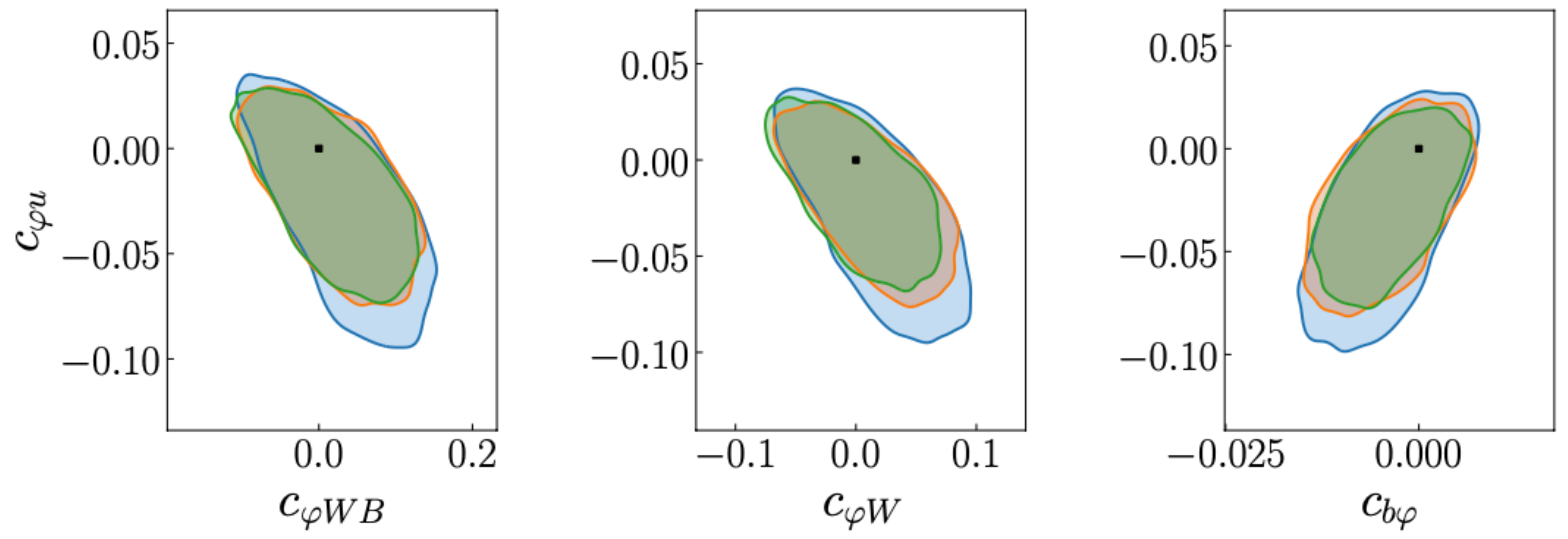

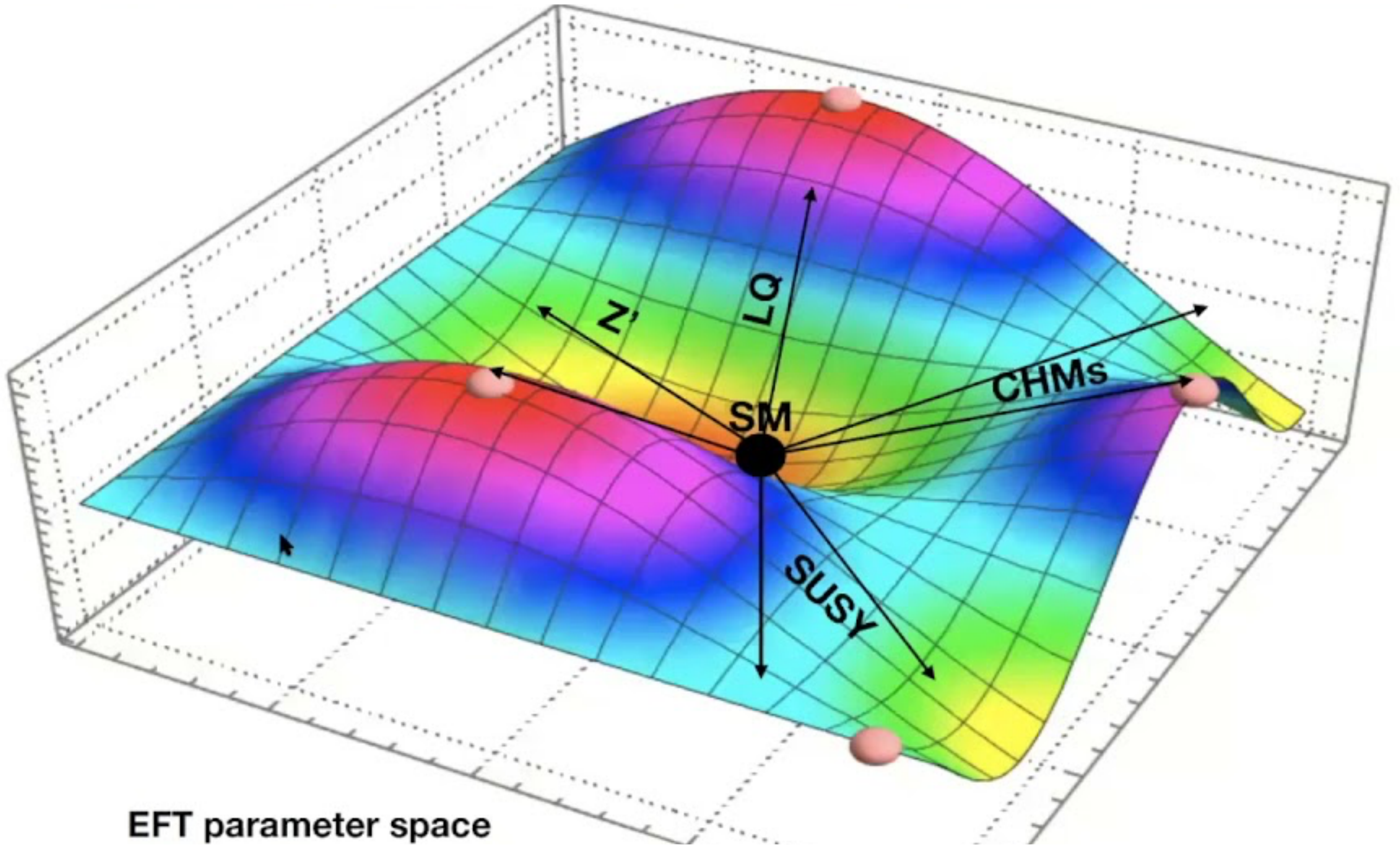

Image credit: V. Sanz

Towards an optimal global SMEFT fit

Machine Learning at GGI
Florence, September 2022

Jaco ter Hoeve
VU Amsterdam & Theory Group, Nikhef

Raquel Gomez-Ambrosio, Maeve Madigan, Juan Rojo, Veronica Sanz

Nikhef

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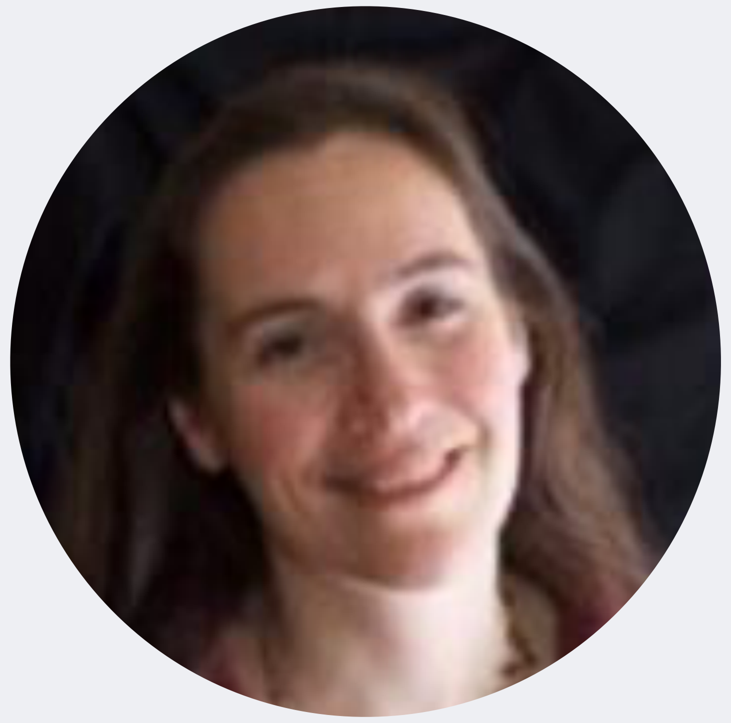


Veronica Sanz

US UNIVERSITY OF SUSSEX

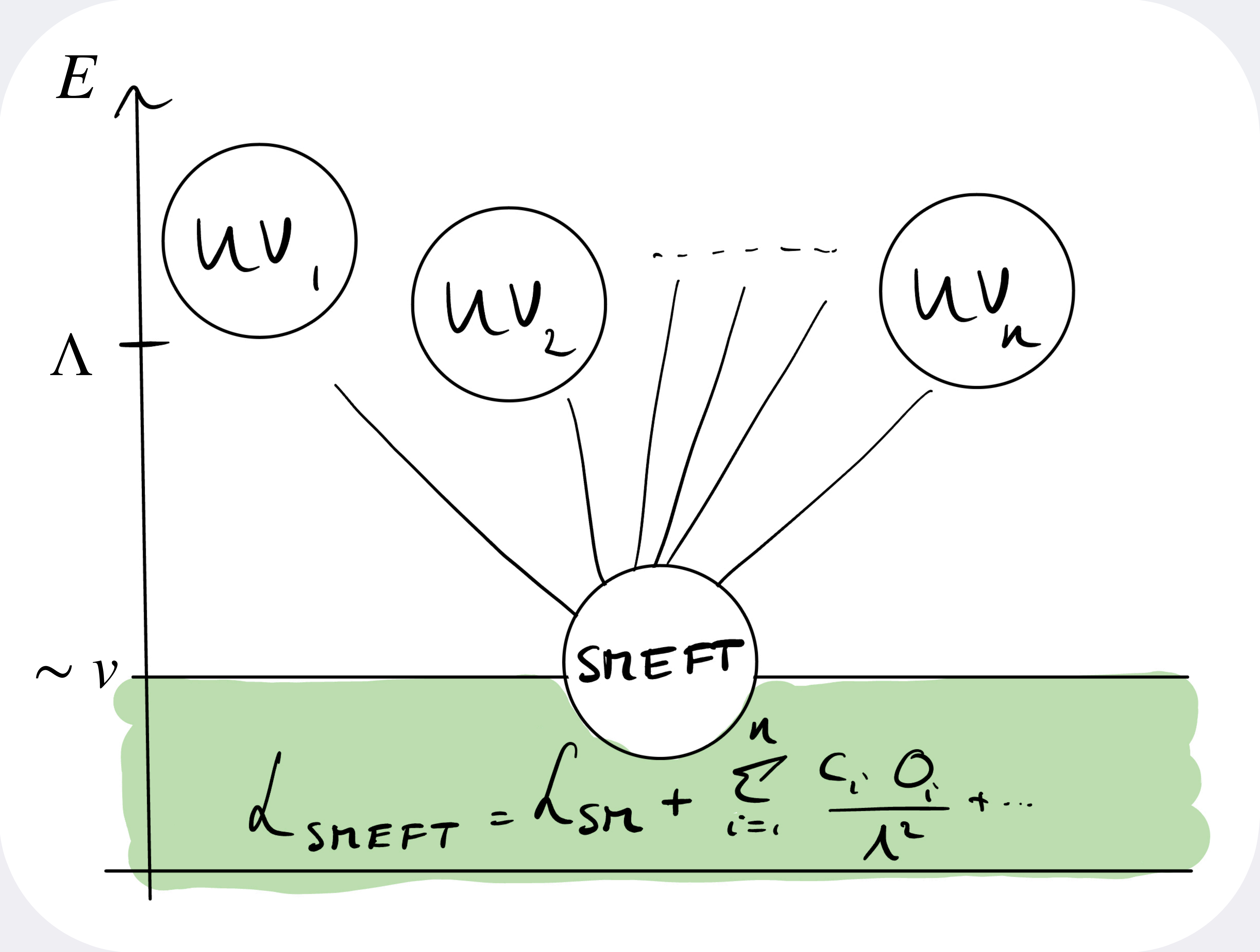
Jaco ter Hoeve

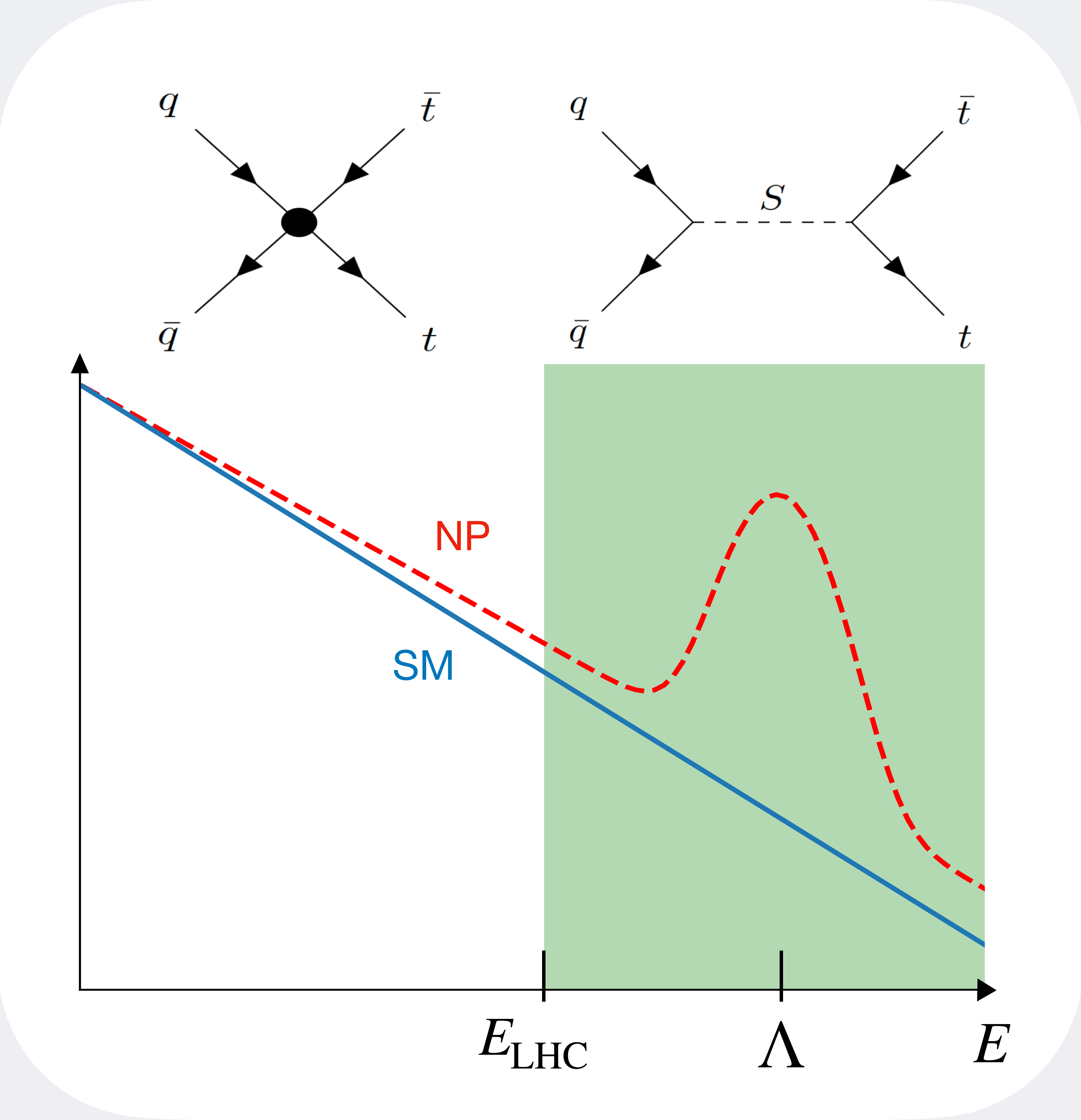
Nikhef

VU VRIJE UNIVERSITEIT AMSTERDAM

The Standard Model as an EFT

Effective Field Theories have become a **main tool** to search for new physics in a “model independent way”

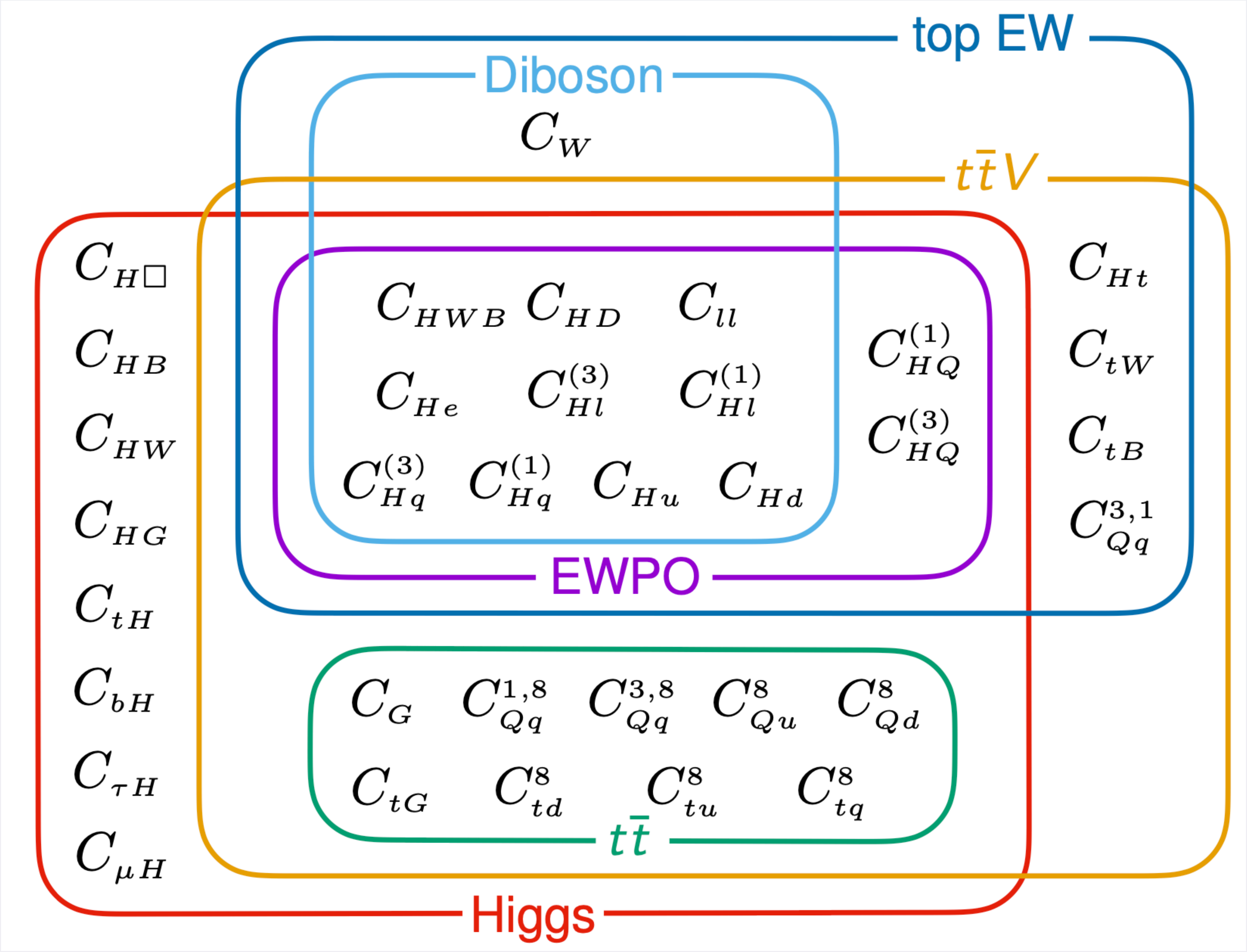




The Standard Model as an EFT

Extend the SM Lagrangian by higher dimensional operators

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i^{N_{d5}} \frac{c_i}{\Lambda}\mathcal{O}_i^{(5)} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_i^{N_{d7}} \frac{c_i}{\Lambda^3}\mathcal{O}_i^{(7)} + \sum_i^{N_{d8}} \frac{b_i}{\Lambda^4}\mathcal{O}_i^{(8)} + \ldots$$



[Fitmaker, 2012.02779]

59 (2499) for one (three) flavour generations

The Standard Model as an EFT

Extend the SM Lagrangian by higher dimensional operators

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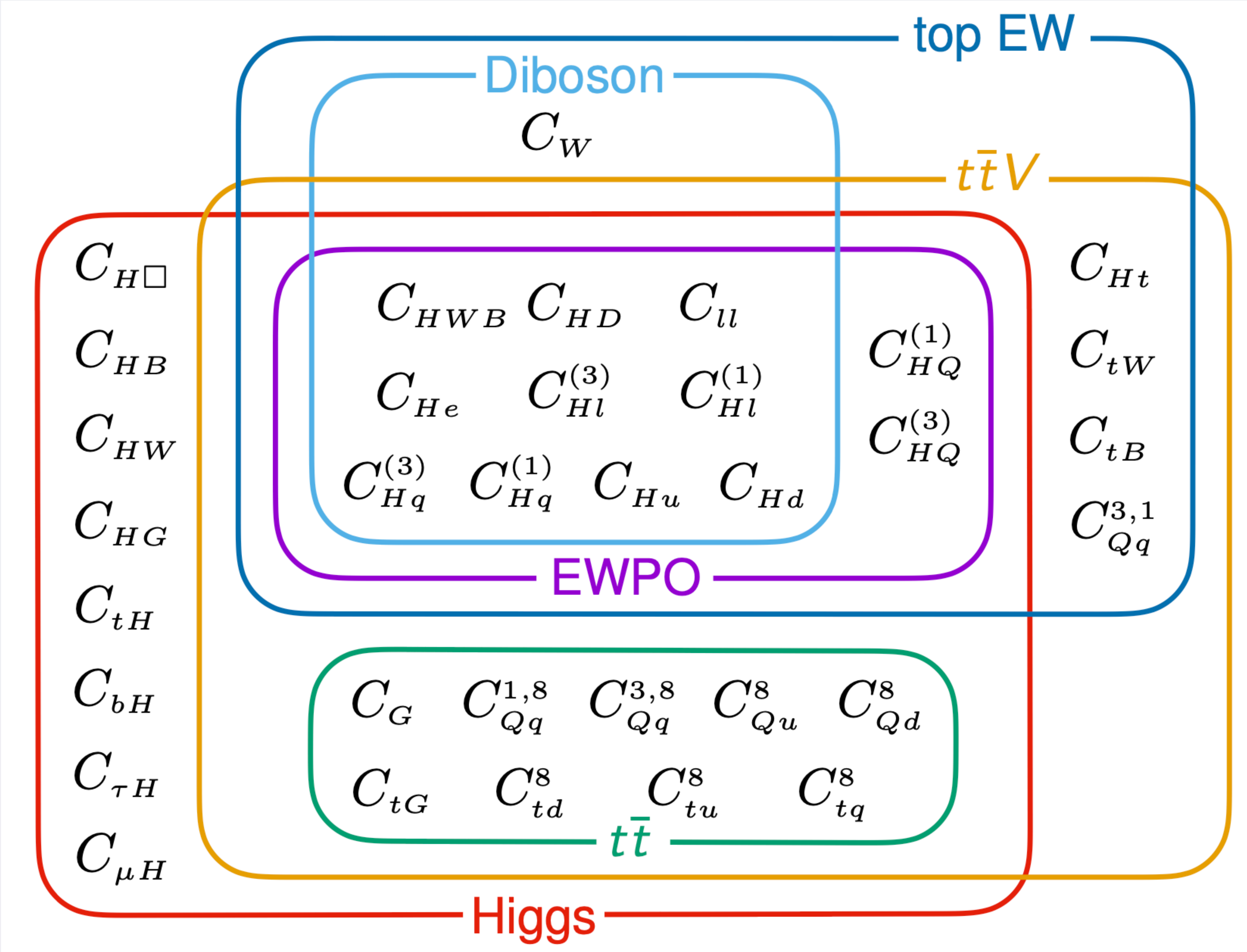


[Fitmaker, 2012.02779]

59 (2499) for one (three) flavour generations

The Standard Model as an EFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i^{N_{d5}} \frac{c_i}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i^{N_{d7}} \frac{c_i}{\Lambda^3} \mathcal{O}_i^{(7)} + \sum_i^{N_{d8}} \frac{b_i}{\Lambda^4} \mathcal{O}_i^{(8)} + \ldots$$

- **Systematic parameterisation** of the theory space in the vicinity of the SM
- **Low energy limit** of generic UV-complete theories at high energies
- Assumes the **SM field content and symmetries**
- **Complete basis** at any given mass dimension
- Fully **renormalizable** QFT
- Can be **matched to any BSM model** that reduces to the SM at low energies

The Standard Model as an EFT

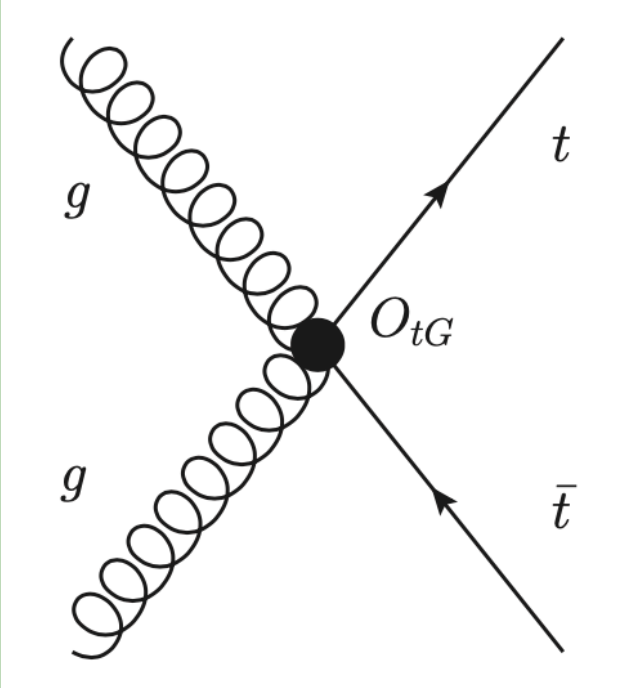
Example: From operator to modified cross section

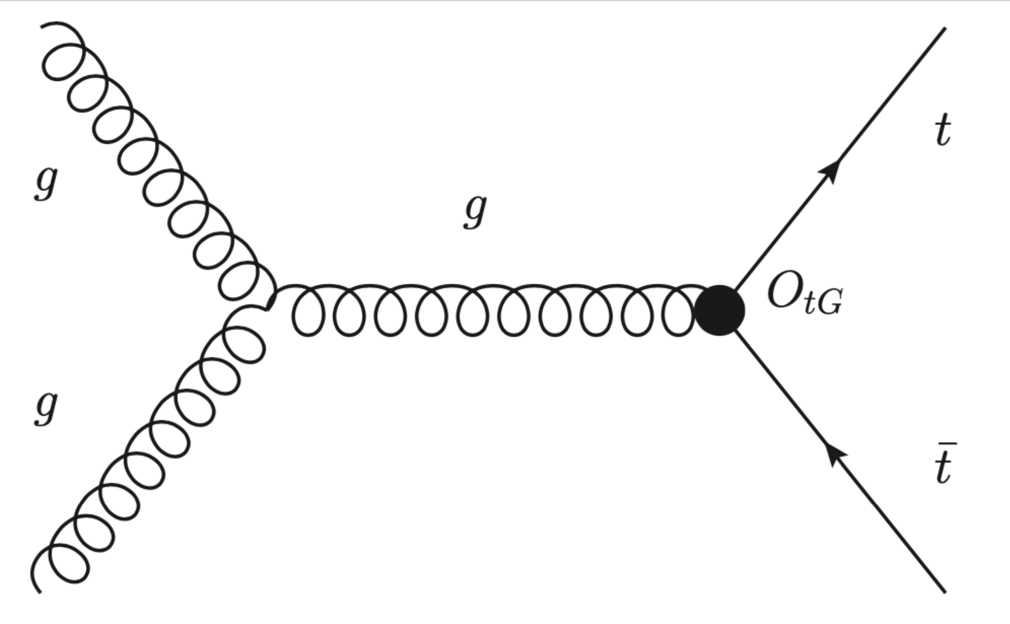
$$\mathcal{O}_{tG} = ig_s \left(\bar{Q}\tau^{\mu\nu}T_A t\right)\tilde{\varphi}G^A_{\mu\nu}$$

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_s f^{ABC}G^B_\mu G^C_\nu$$

$$\bar{Q} = (t_L \quad b_L)$$

$$\varphi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}$$





$$\sigma = \sigma_{\text{SM}} \times \left(1 + \frac{c_{tG}}{\Lambda^2}a + \frac{c_{tG}^2}{\Lambda^4}b\right)$$

Linear EFT corrections: interference SM-EFT$_{d6}$

Quadratic EFT corrections: EFT$_{d6}$-EFT$_{d6}$

The Standard Model as an EFT

from the SMEFT **Lagrangian** ....

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i^{N_{d8}} \frac{c_i}{\Lambda^4} \mathcal{O}_i^{(8)} + \ldots$$

SM

EFT$_{\text{d6}}$

EFT$_{\text{d8}}$

to **cross-sections**....

Linear EFT corrections:
interference SM-EFT$_{\text{d6}}$

Quadratic EFT corrections:
EFT$_{\text{d6}}$-EFT$_{\text{d6}}$

$$\sigma_{\text{SMEFT}} \approx \sigma_{\text{SM}} \times \left(1 + \sum_i^{N_{d6}} \frac{c_i}{\Lambda^2} \kappa_i + \sum_{i,j}^{N_{d6}} \frac{c_i c_j}{\Lambda^4} \tilde{\kappa}_{ij} \right)$$

Evaluate at (N)NLO QCD + NLO EW

Evaluate at NLO QCD with SMEFT@NLO

The Standard Model as an EFT

to **cross-sections**….

Linear EFT corrections:
interference SM-EFT$_{d6}$

Quadratic EFT corrections:
EFT$_{d6}$-EFT$_{d6}$

$$\sigma_{\rm SMEFT} \approx \sigma_{\rm SM} \times \left(1 + \sum_{i}^{N_{d6}} \frac{c_i}{\Lambda^2} \kappa_i + \sum_{i,j}^{N_{d6}} \frac{c_i c_j}{\Lambda^4} \tilde{\kappa}_{ij} \right)$$

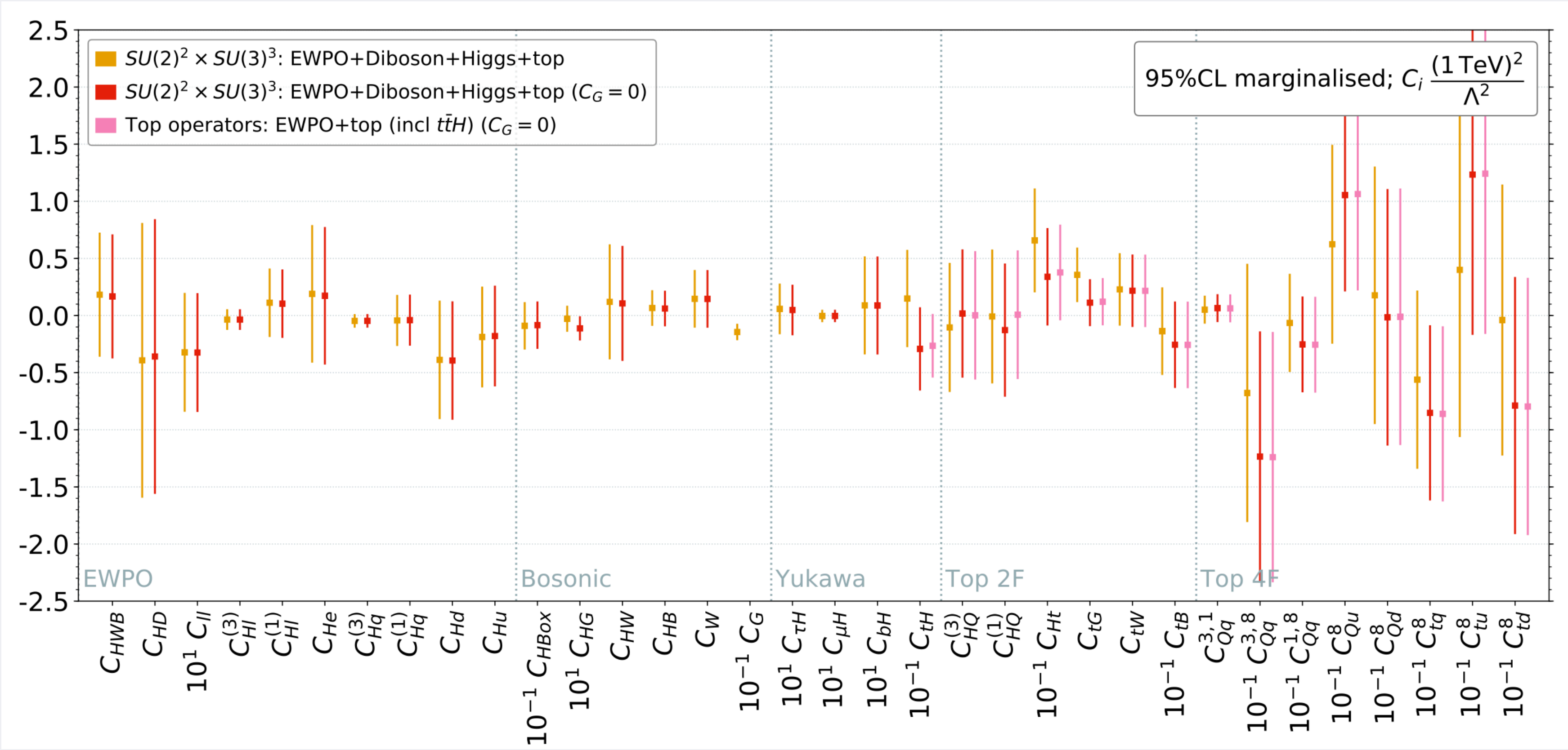
to **constraints** on the EFT parameters

$$\chi^2(\mathbf{c}) = \frac{1}{n_{\rm dat}} \sum_{i,j=1}^{n_{\rm dat}} \left(\sigma_{i,{\rm SMEFT}}(\mathbf{c}) - \sigma_{i,{\rm exp}} \right) \left({\rm cov}^{-1} \right)_{ij} \left(\sigma_{j,{\rm SMEFT}}(\mathbf{c}) - \sigma_{j,{\rm exp}} \right)$$

Multi-Gaussian log likelihood optimisation problem

Global EFT analyses

Several groups have presented global EFT analyses combining data from **many different** processes (top, Higgs, diboson,…), yielding **sensitivity to many directions** in the EFT parameter space



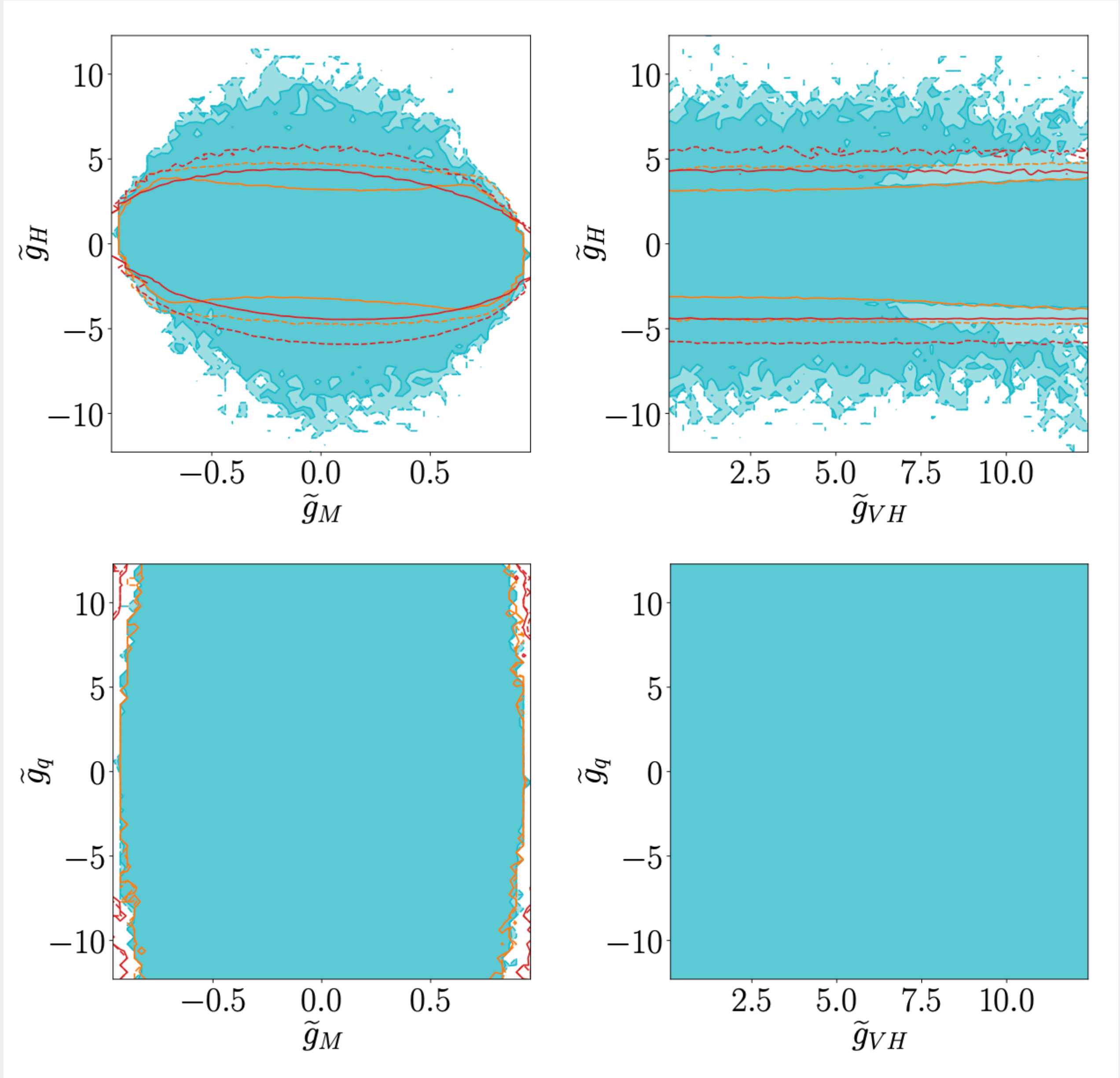
Fitmaker

[2012.02779]

SMEFiT

[2105.00006]

Combines 317 cross section measurements
to constrain 50 EFT parameters



SFitter

[2108.01094]

None of these measurements have been **optimised for EFT studies**, can one do better?

From binned to unbinned

- Most EFT measurements are presented in terms of **multi-Gaussian likelihoods**

Number of bins

Number of observed events

$$\mathscr{L}(\boldsymbol{n};\boldsymbol{\nu}(\boldsymbol{c})) = \prod_{i=1}^{N_b} \exp\left[-\frac{1}{2}\frac{(n_i-\nu_i(\boldsymbol{c}))^2}{\nu_i(\boldsymbol{c})}\right]$$

Number of expected events

Only statistical uncertainties here
No cross-correlations

Q1: What is the optimal number of bins?

Q2: How much information does one gain/lose by measuring the cross-section in additional kinematic variables?

- In the tails of the EFT distributions, the # of events can be small (i.e. < 30), and one must use the Poissonian likelihood:

$$\mathscr{L}(\boldsymbol{n};\boldsymbol{\nu}(\boldsymbol{c})) = \prod_{i=1}^{N_b} \frac{\nu_i^{n_i}(\boldsymbol{c})}{n_i!} e^{-\nu_i(\boldsymbol{c})}$$

From binned to unbinned

<u>**Main goal**</u>: construct unbinned observables from ML and assess their relevance for **global EFT fits** by comparing their impact to those of "traditional binned" observables

Unbinned extended likelihood

# of expected events

# of observed events

$$\mathscr{L}(\boldsymbol{c}) = \frac{\nu_{\text{tot}}^{N_{\text{ev}}}(\boldsymbol{c})}{N_{\text{ev}}\,!} e^{-\nu_{\text{tot}}(\boldsymbol{c})} \prod_{i=1}^{N_{\text{ev}}} f_\sigma\left(\boldsymbol{x}_i, \boldsymbol{c}\right)$$

Probability distribution in the final state kinematics

$$f_\sigma(\boldsymbol{x}, \boldsymbol{c}) = \frac{1}{\sigma(X)} \frac{d\sigma(\boldsymbol{x}, \boldsymbol{c})}{d\boldsymbol{x}}$$

Fully differential cross-section

e.g. in $ZH \to \ell^+\ell^- b\bar{b}$ $\quad \boldsymbol{x} = (p_T^Z, p_T^b, \Delta R_{b\bar{b}}, \Delta\phi_{l,b}, \ldots)$ **"Features"**

By construction, this unbinned likelihood **contains all the information** from the observed events

From binned to unbinned

- For n_{eft} EFT coefficients c_i, we can express the EFT cross-section as

$$f_\sigma(\boldsymbol{x}, \boldsymbol{c}) = f_\sigma(\boldsymbol{x}, \boldsymbol{0}) + \sum_{j=1}^{n_{\text{eft}}} f_\sigma^{(j)}(\boldsymbol{x}) c_j + \sum_{j=1}^{n_{\text{eft}}} \sum_{k \geq j}^{n_{\text{eft}}} f_\sigma^{(j,k)}(\boldsymbol{x}) c_j c_k$$

- For the purpose of limit setting, it is sufficient to consider its ratio to some reference point (e.g. the SM)

$$r_\sigma(\boldsymbol{x}, \boldsymbol{c}) = \frac{f_\sigma(\boldsymbol{x}, \boldsymbol{c})}{f_\sigma(\boldsymbol{x}, \boldsymbol{0})} = 1 + \sum_{j=1}^{n_{\text{eft}}} r_\sigma^{(j)}(\boldsymbol{x}) c_j + \sum_{j=1}^{n_{\text{eft}}} \sum_{k \geq j}^{n_{\text{eft}}} r_\sigma^{(j,k)}(\boldsymbol{x}) c_j c_k$$

- Extend the global EFT likelihood by the unbinned likelihood to assess their relevance (avoid **double counting**):

$$\mathscr{L}(\boldsymbol{c}) = \prod_{k=1}^{N_{\mathscr{D}}} \mathscr{L}_k(\boldsymbol{c}) = \prod_{k=1}^{N_{\mathscr{D}}^{(\text{ub})}} \mathscr{L}_k^{(\text{ub})}(\boldsymbol{c}) \prod_{j=1}^{N_{\mathscr{D}}^{(\text{bp})}} \mathscr{L}_j^{(\text{bp})}(\boldsymbol{c}) \prod_{\ell=1}^{N_{\mathscr{D}}^{(\text{bg})}} \mathscr{L}_\ell^{(\text{bg})}(\boldsymbol{c})$$

Cross section ML parametrisation

- We thus need to parameterise the dependence of the distribution ratio on the kinematics

$$r_\sigma(\boldsymbol{x}, \boldsymbol{c}) = \frac{f_\sigma(\boldsymbol{x}, \boldsymbol{c})}{f_\sigma(\boldsymbol{x}, \boldsymbol{0})} = 1 + \sum_{j=1}^{n_{\rm eft}} r_\sigma^{(j)}(\boldsymbol{x}) c_j + \sum_{j=1}^{n_{\rm eft}} \sum_{k \geq j}^{n_{\rm eft}} r_\sigma^{(j,k)}(\boldsymbol{x}) c_j c_k$$

- Adopt NN as **universal unbiased interpolants** as a proxy of the likelihood ratio that is fast to evaluate

- The structure of the EFT cross section makes it possible to **parallelise** the training to any arbitrary number of EFT parameters

related work by Chen at al 2007.10356, Tito d'Agnolo et al 1912.12155, Brehmer et al 1805.00013 + many others

Cross section ML parametrisation

- We thus need to parameterise the dependence of the distribution ratio on the kinematics

$$r_\sigma(\boldsymbol{x}, \boldsymbol{c}) = \frac{f_\sigma(\boldsymbol{x}, \boldsymbol{c})}{f_\sigma(\boldsymbol{x}, \boldsymbol{0})} = 1 + \sum_{j=1}^{n_{\text{eft}}} r_\sigma^{(j)}(\boldsymbol{x}) c_j + \sum_{j=1}^{n_{\text{eft}}} \sum_{k \geq j}^{n_{\text{eft}}} r_\sigma^{(j,k)}(\boldsymbol{x}) c_j c_k$$

- Use the "Likelihood ratio" trick to obtain a decision boundary that is 1-1 with $r_\sigma(\boldsymbol{x}, \boldsymbol{c})$

$$L[g(\boldsymbol{x}, \boldsymbol{c})] = -\int dx \frac{d\sigma(\boldsymbol{x}, \boldsymbol{c})}{dx} \log(1 - g(\boldsymbol{x}, \boldsymbol{c})) - \int dx \frac{d\sigma(\boldsymbol{x}, \boldsymbol{0})}{dx} \log g(\boldsymbol{x}, \boldsymbol{0})$$

$g \to 0$ for EFT

$g \to 1$ for SM

$$\frac{\delta L}{\delta g} = 0 \implies g^*(\boldsymbol{x}, \boldsymbol{c}) = \left(1 + \frac{d\sigma(\boldsymbol{x}, \boldsymbol{c})}{dx} \Big/ \frac{d\sigma(\boldsymbol{x}, \boldsymbol{0})}{dx}\right)^{-1} = \frac{1}{1 + r_\sigma(\boldsymbol{x}, \boldsymbol{c})}$$

Cross section ML parametrisation

- Generate **Monte Carlo data** with single EFT parameter activated + SM baseline

$$\mathcal{D}_{\mathrm{eft}}(\boldsymbol{c} = (0,\ldots,0,c_j^{(\mathrm{tr})},0,\ldots,0))$$

to extract the coefficient function accompanying $c_j^{(\mathrm{tr})}$ at the **linear level**

$$r_\sigma(\boldsymbol{x}, c_j^{(\mathrm{tr})}) = 1 + c_j^{(\mathrm{tr})} \cdot \mathrm{NN}^{(j)}(\boldsymbol{x}) \qquad \mathrm{NN}^{(j)}(\boldsymbol{x}) \rightarrow r_\sigma^{(j)}(\boldsymbol{x})$$

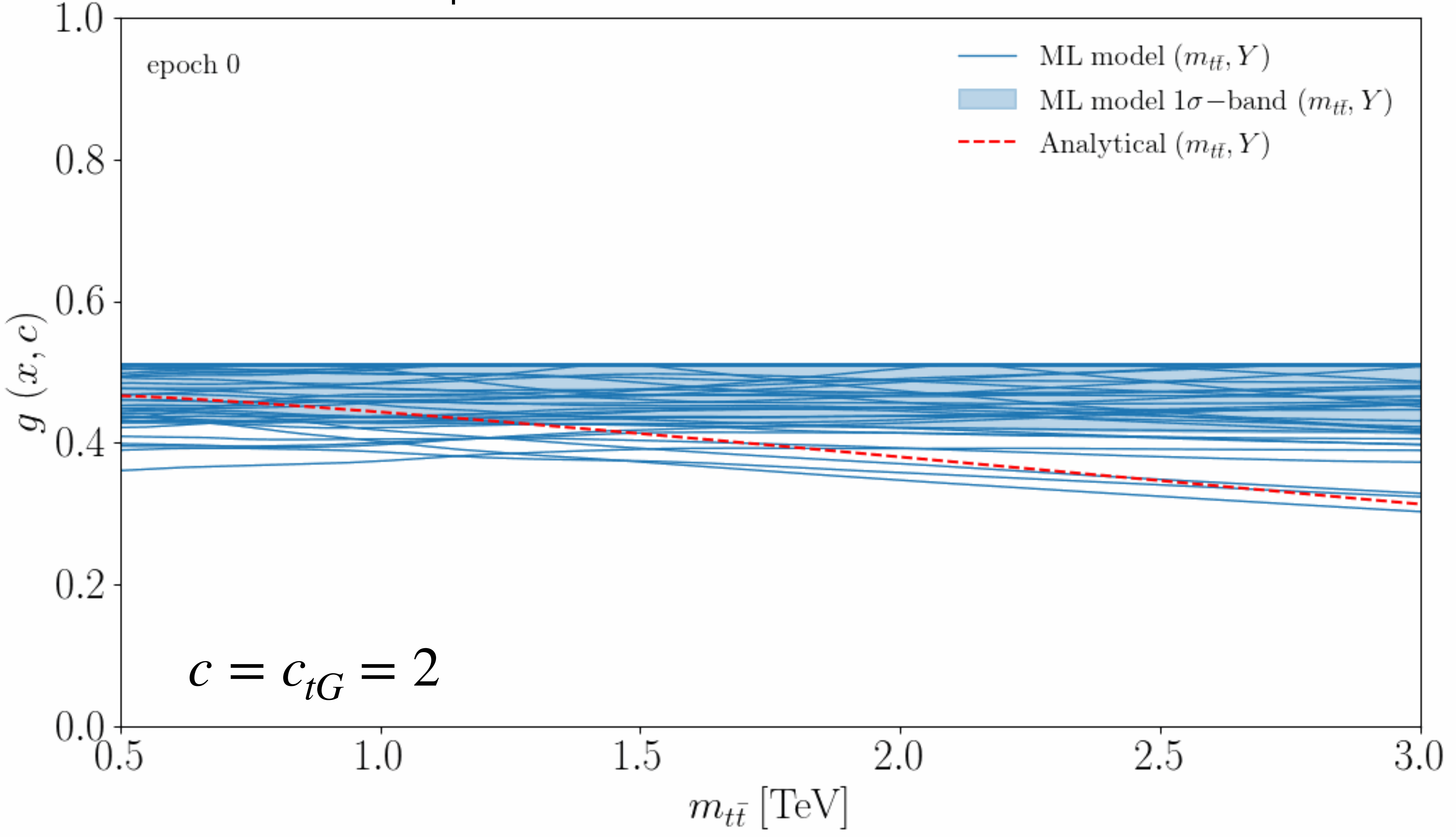
- The same logic holds at the **quadratic level**

$$\mathcal{D}_{\mathrm{eft}}(\boldsymbol{c} = (0,\ldots,0,c_j^{(\mathrm{tr})},0,\ldots,0,c_k^{(\mathrm{tr})},0,\ldots,0)) \qquad r_\sigma(\boldsymbol{x}, c_j^{(\mathrm{tr})}) = 1 + c_j^{(\mathrm{tr})}c_k^{(\mathrm{tr})} \cdot \mathrm{NN}^{(j,k)}(\boldsymbol{x})$$

Neural network training

Model uncertainties are estimated by means of the **Monte Carlo replica method:** train a collection of 50 NN instances on independent MC datasets

Animations are static in the pdf



Training settings

- LO QCD MC dataset (100K)
- Mini-batch gradient descent
- Training/Validation sets (80/20)
- Runtime ~20 min per core (1 replica/core)
- 3 hidden layers (100 units)
- ReLU activation functions

The MC replica method allows us to propagate model and methodological uncertainties to the space of EFT parameters! **(NNPDF approach)**

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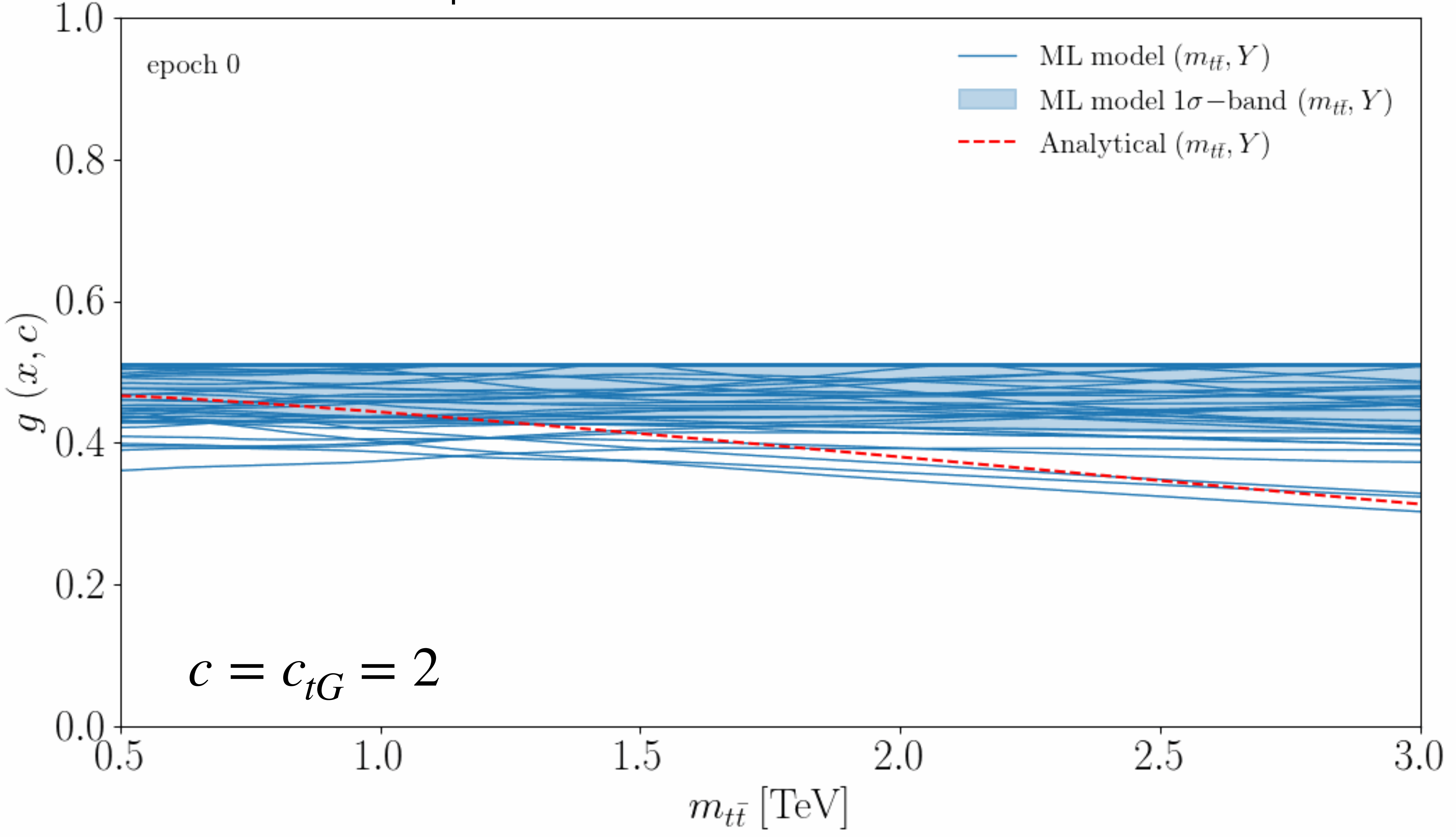

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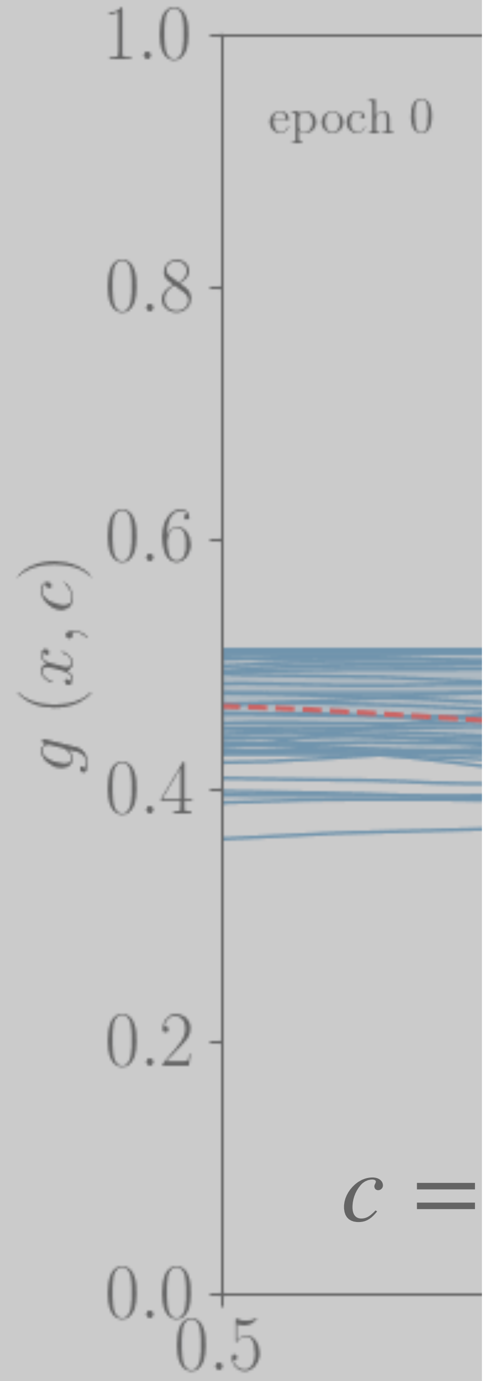


Replicas

NNPDF Approach: Monte Carlo sample (empirical distribution)

Naïve idea: **uncertainty comes from data**, let's propagate the data distribution back through the *regularized inverse function*:

$$f_i(x) = \arg\min_f (\mathcal{L}(D_i)) = \mathcal{NN}(x \mid D_i)$$

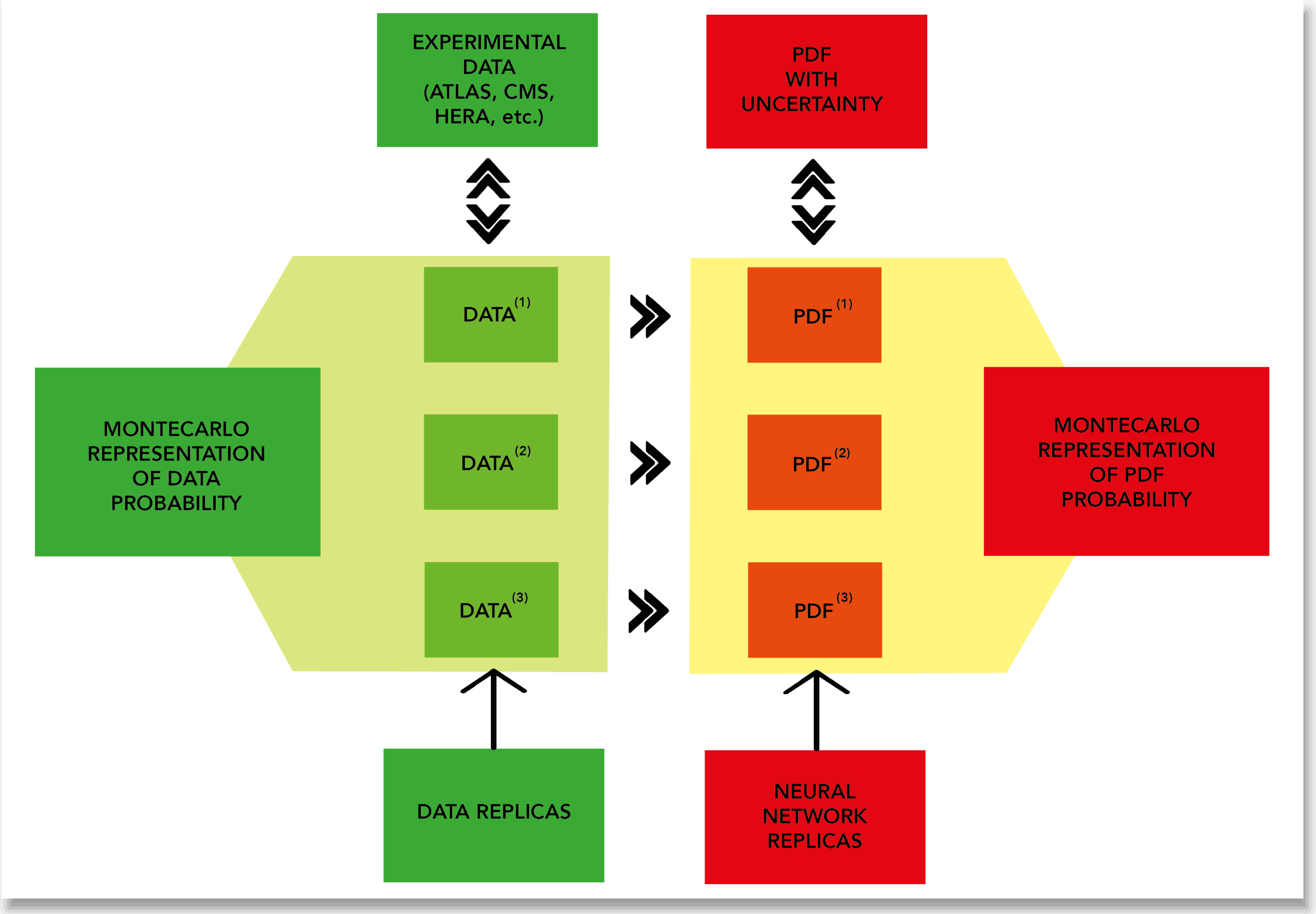
Loss contains the χ^2, but χ^2 is not the end of the story.

Very simple (possibly expensive[a]) usage: in order to obtain the **distribution of predictions**, apply your theoretical calculation to the element of the sample:

$$\mathcal{P}red\mathcal{S} = \{\mathcal{F}(f) \;\; \forall f \in \mathcal{P}ost\mathcal{S}\}$$

[a] To address this compressed sets are distributed along the main release. There is also a program to obtain specialized minimal sets for dedicated studies:

https://github.com/scarrazza/smpdf



Alessandro Candido's talk from Tuesday 11

tings

set (100K)

nt descent

n sets (80/20)

per core (1

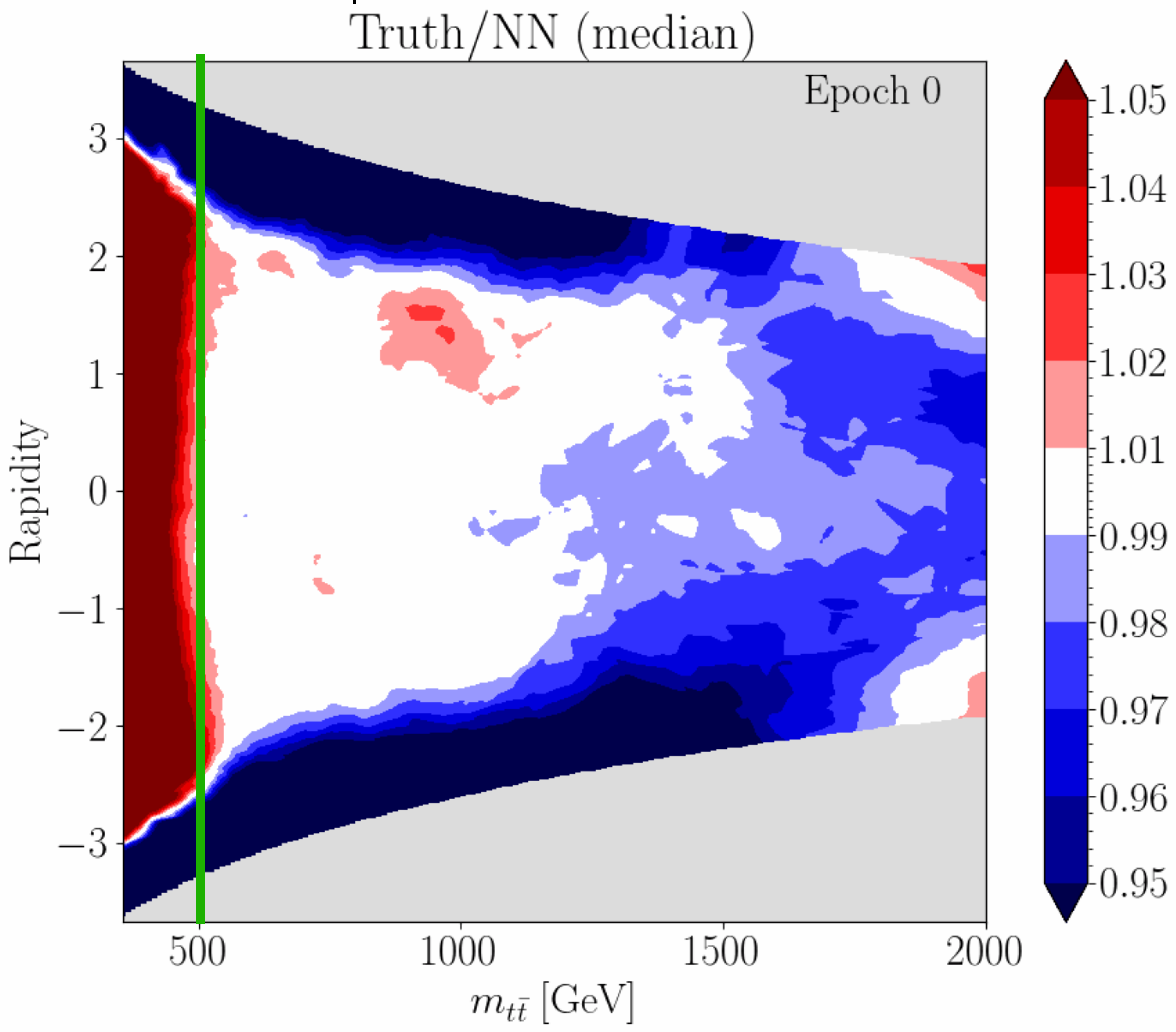
00 units)

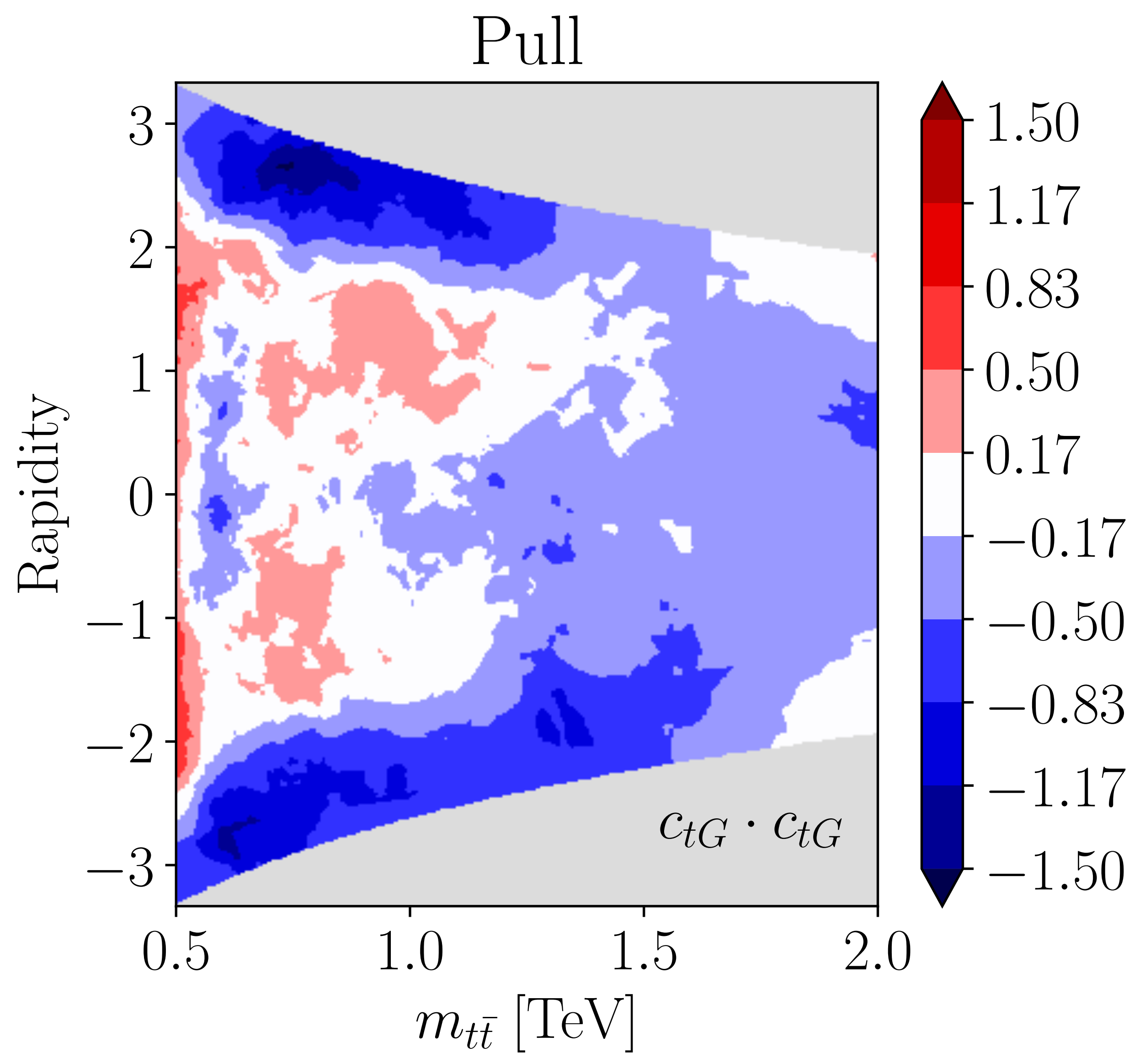
unctions

The MC replica method allows us to propagate model and methodological uncertainties to the space of EFT parameters! **(NNPDF approach)**

Neural network training

Animations are static in the pdf





Neural network training

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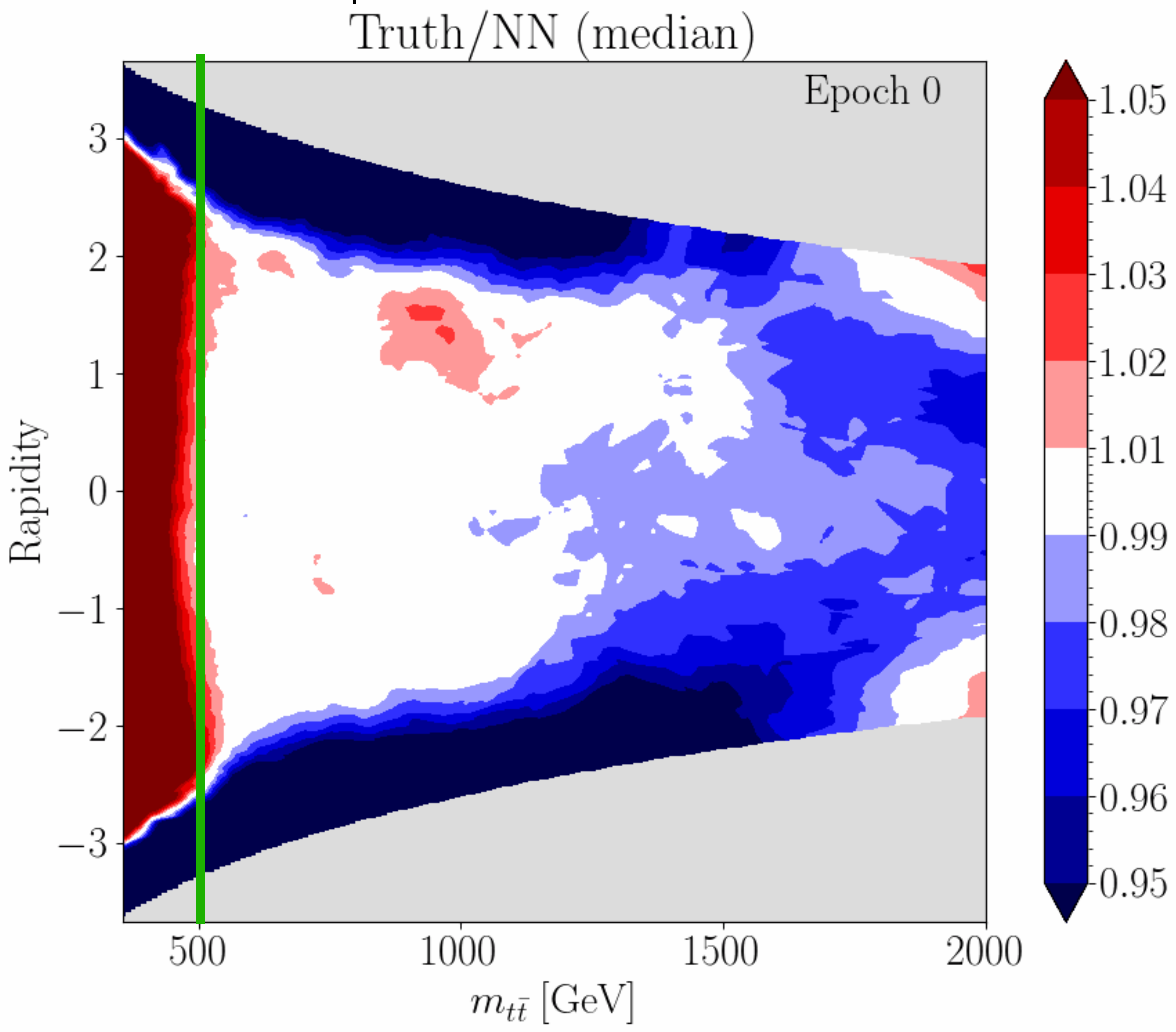
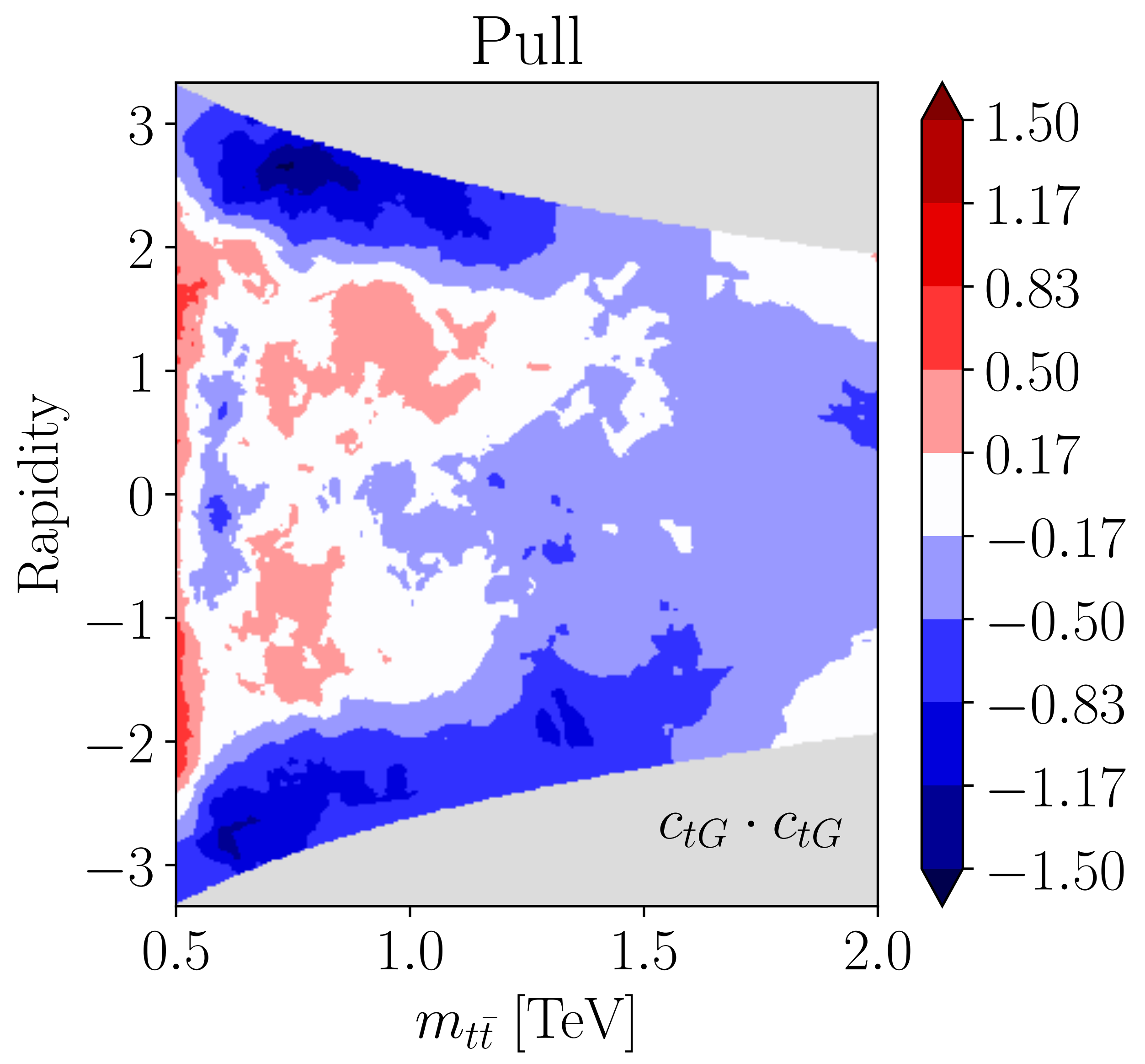
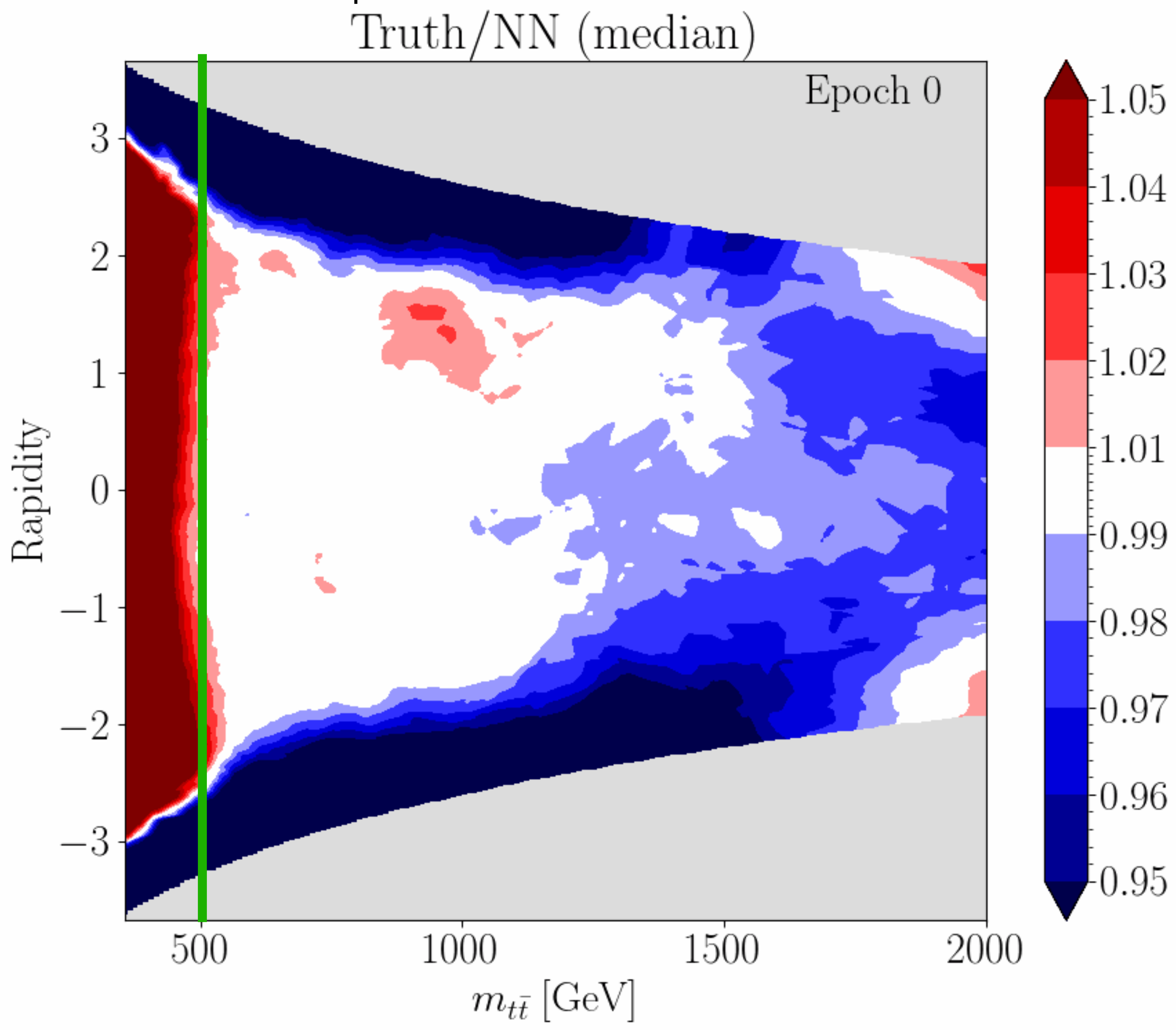


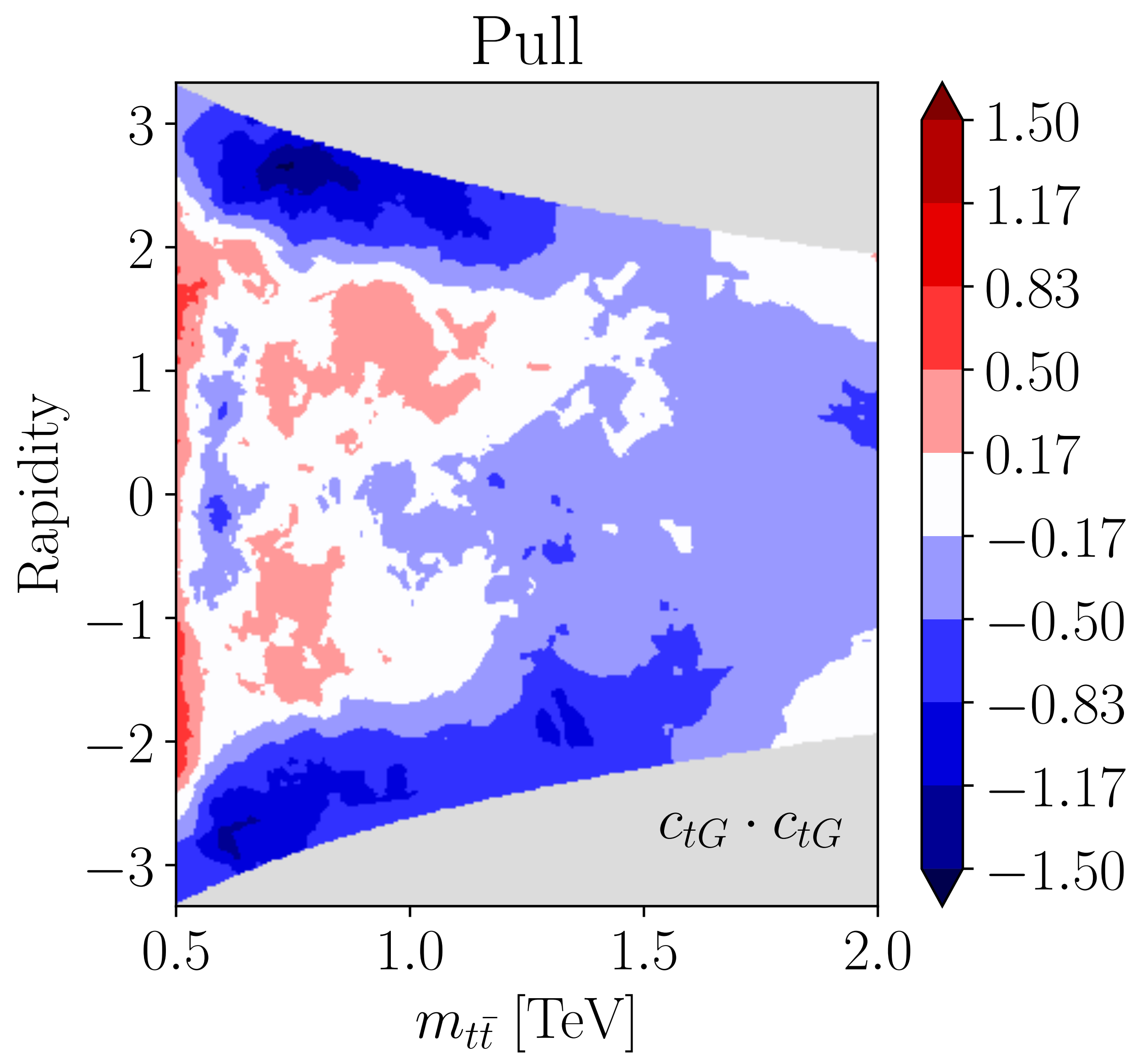


Challenge: correctly describing tails of distributions (low stats)

Neural network training

Animations are static in the pdf





Challenge: correctly describing tails of distributions (low stats)

Cross section ML parametrisation

- Cross section **positivity** can be enforced through either Lagrange multipliers ….

$$L[g] \to L[g] + \lambda \cdot \mathrm{ReLU}\left(\frac{g-1}{g}\right)$$

Non-zero whenever $r_\sigma(\boldsymbol{x}, \boldsymbol{c}) < 0$

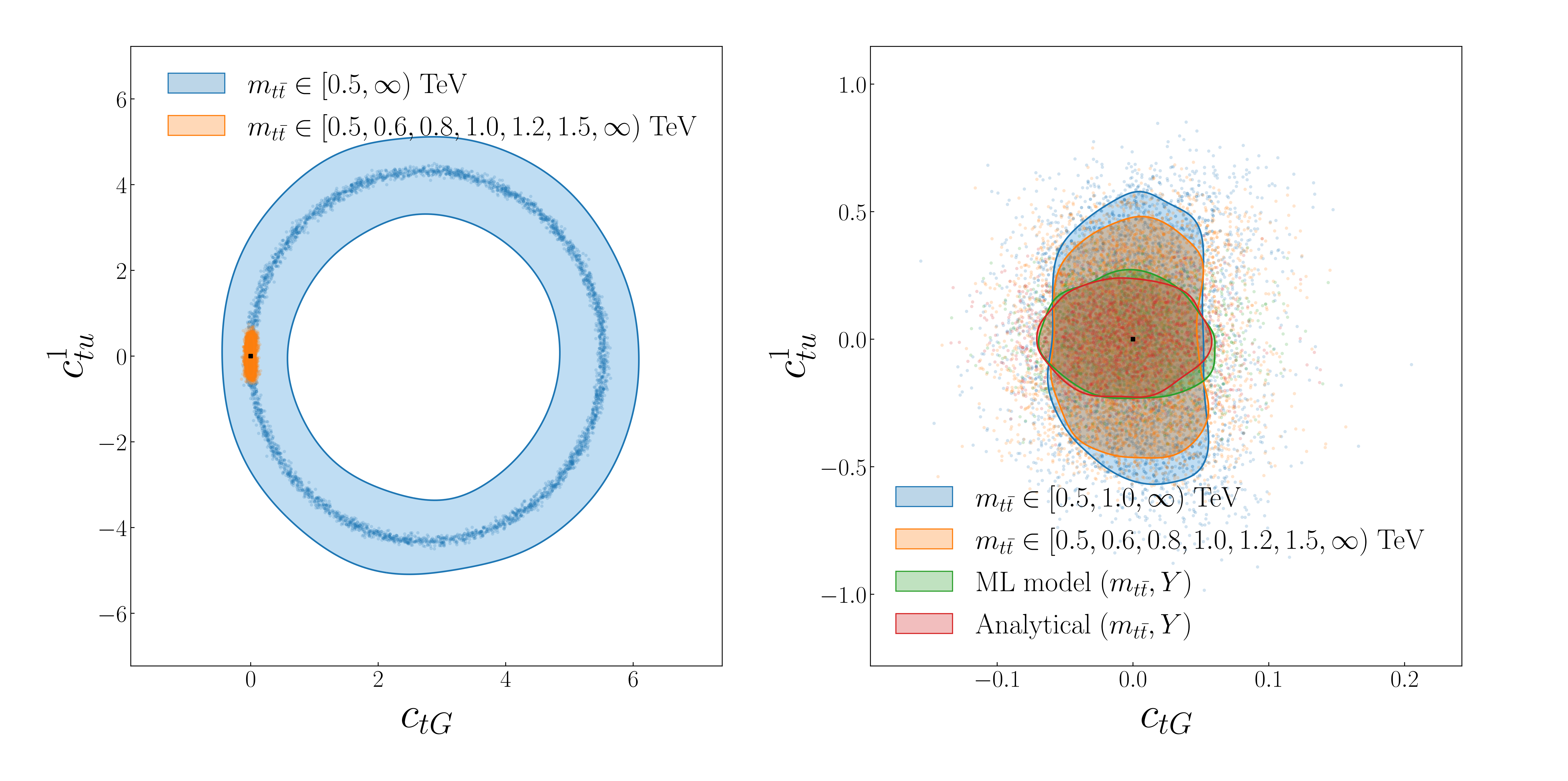
…. or a final ReLU depending on the value of $c_j^{(\mathrm{tr})}$

$$\mathrm{NN}^{(j)}(\boldsymbol{x}) \to \mathrm{NN}^{(j)}(\boldsymbol{x}; c_j^{(\mathrm{tr})}) = \begin{cases} \mathrm{ReLU}(\mathrm{NN}^{(j)}(\boldsymbol{x})) - 1/c_j^{(\mathrm{tr})}, & \text{if } c_j^{(\mathrm{tr})} > 0 \\ -\mathrm{ReLU}(\mathrm{NN}^{(j)}(\boldsymbol{x})) - 1/c_j^{(\mathrm{tr})}, & \text{if } c_j^{(\mathrm{tr})} < 0 \end{cases}$$

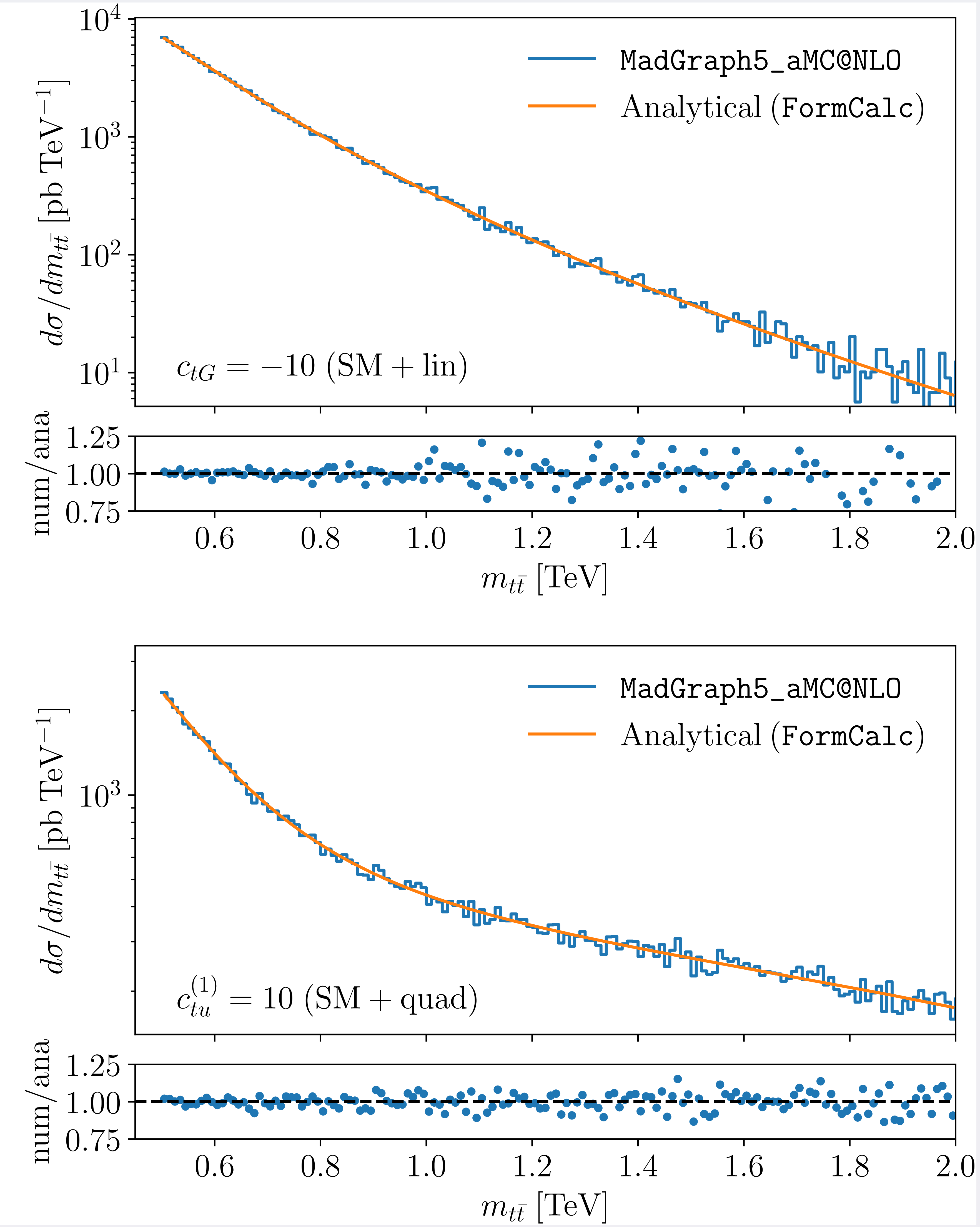
- # of processors scales like $\mathcal{O}(n_{\mathrm{eft}})$, making our approach **suitable** for a large EFT parameter space

Results

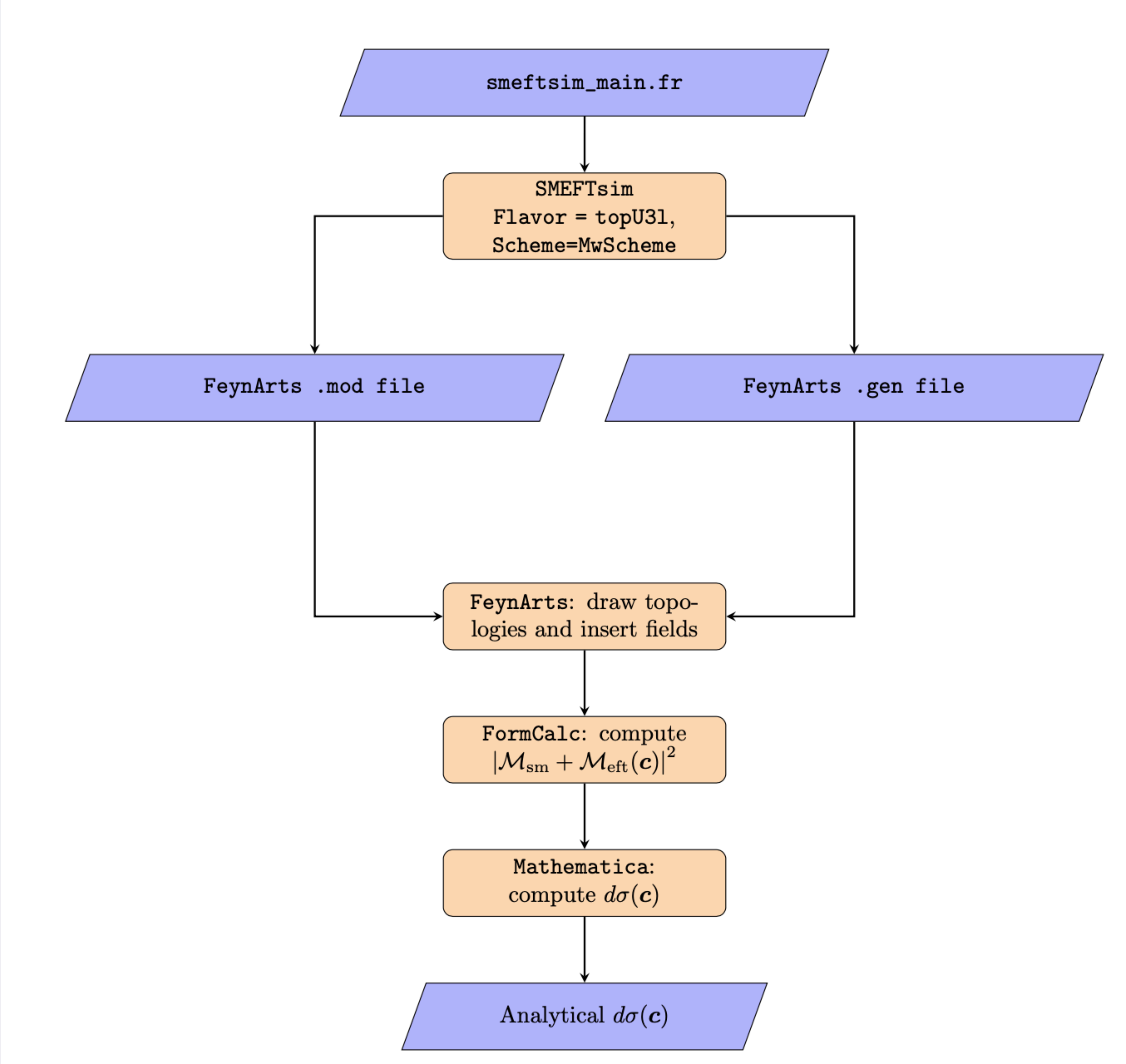
- To **validate** our methodology, we performed a study of top-quark pair production at the parton level and compared the outcome to the **analytical** result



Results



Use FormCalc to obtain analytical expressions T. Hahn [9807565]



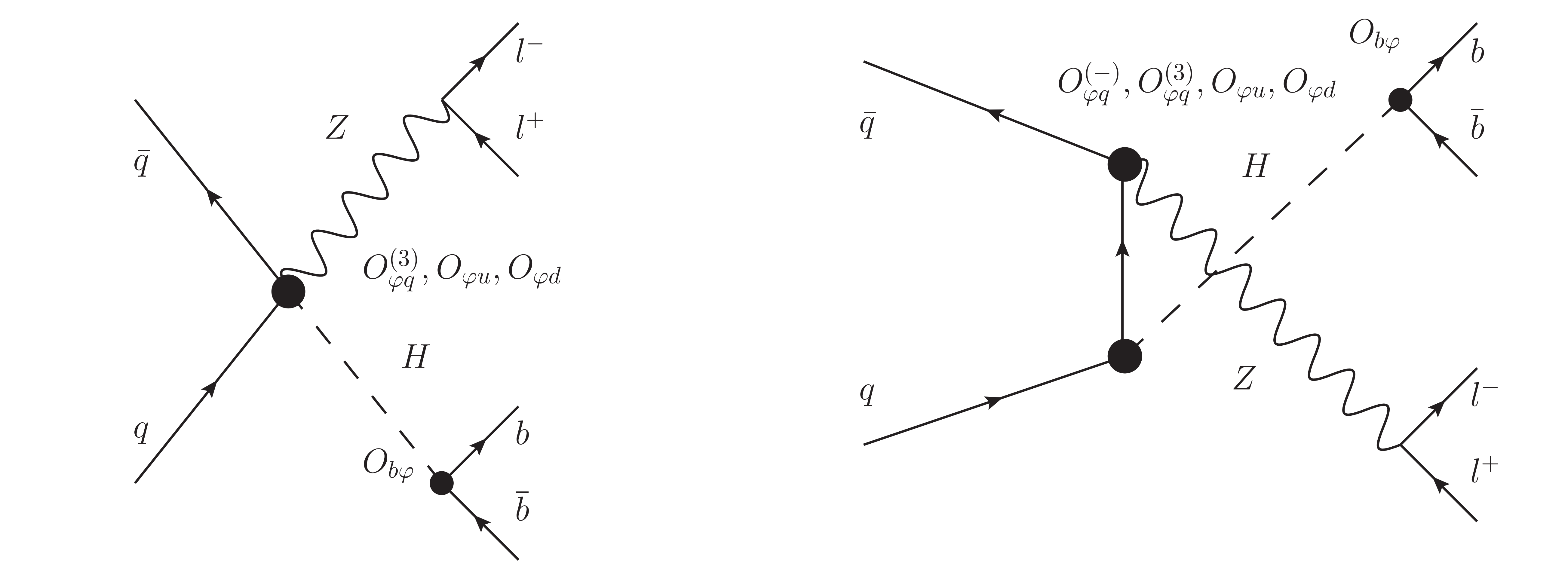
Results

- Next, we consider **Higgs associated production** and **top-quark pair production** (already included in the **SMEFiT global analysis)**
- We make sure to use the same operator basis, flavour assumptions as in SMEFiT to allow for **integration**

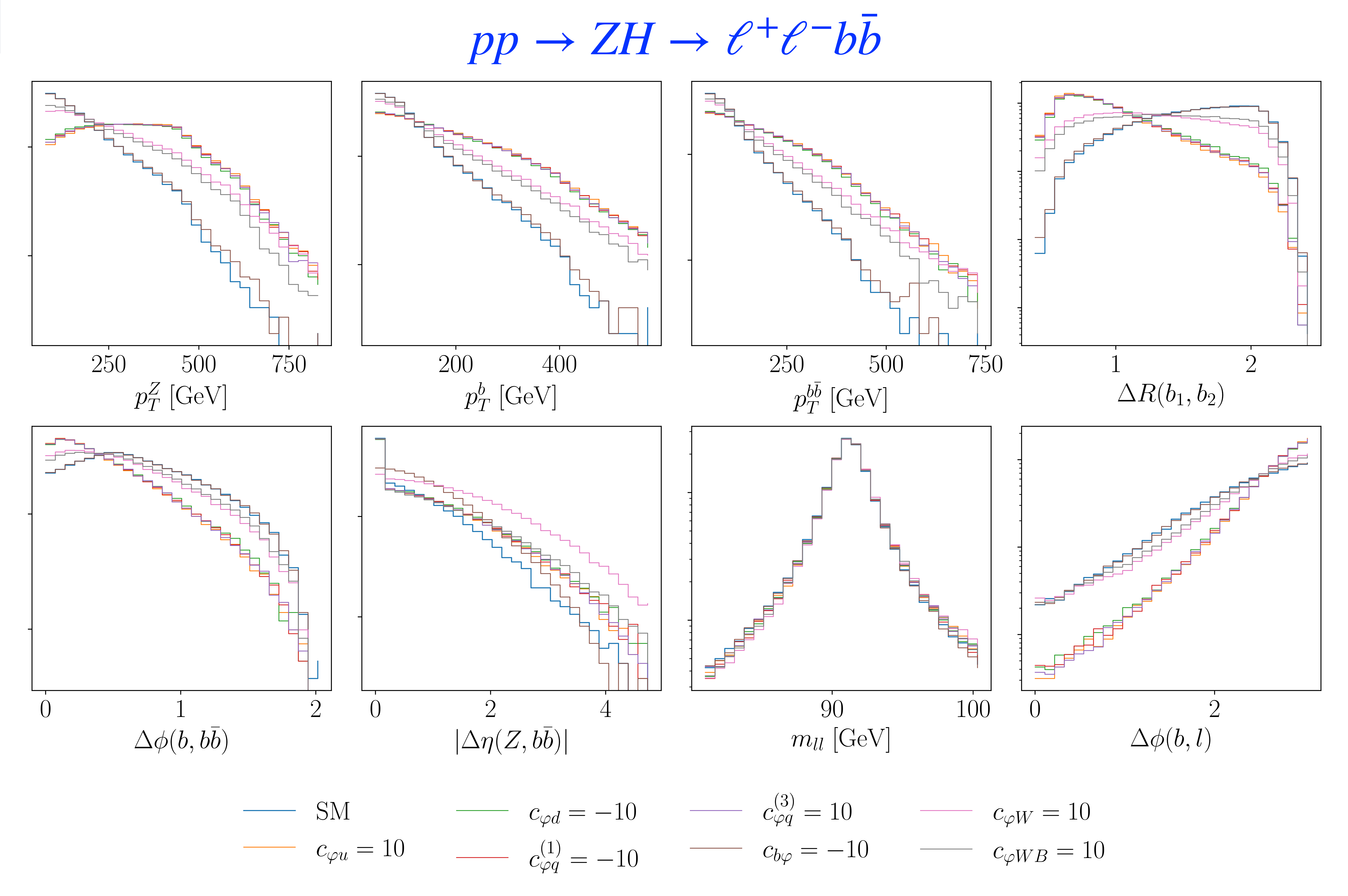
operator	SMEFTsim	SMEFiT	Definition
$\mathcal{O}_{\varphi u}$	cHu	cpui	$\sum_{i=1,2}(\varphi^\dagger i D_\mu \varphi)(\bar{u}_i\gamma^\mu u_i)$
$\mathcal{O}_{\varphi d}$	cHd	cpdi	$\sum_{i=1,2}(\varphi^\dagger i D_\mu \varphi)(\bar{d}_i\gamma^\mu d_i)$
$\mathcal{O}_{\varphi q}^{(1)}$	cHj1	–	$\sum_{i=1,2} i(\varphi^\dagger \overleftrightarrow{D}_\mu \varphi)(\bar{q}_i\gamma^\mu q_i)$
$\mathcal{O}_{\varphi q}^{(3)}$	cHj3	c3pq	$\sum_{i=1,2} i(\varphi^\dagger \overleftrightarrow{D}_\mu \tau_I \varphi)(\bar{q}_i\gamma^\mu \tau^I q_i)$
$\mathcal{O}_{\varphi q}^{(-)}$	cHj1 − cHj3	cpqMi	–
$\mathcal{O}_{b\varphi}$	cbHRe	cbp	$(\varphi^\dagger\varphi)\bar{Q}b\varphi + \text{h.c.}$
$\mathcal{O}_{\varphi W}$	cHW	cpW	$(\varphi^\dagger\varphi)W_I^{\mu\nu}W_{\mu\nu}^I$
$\mathcal{O}_{\varphi WB}$	cHWB	cpWB	$(\varphi^\dagger\tau_I\varphi)B^{\mu\nu}W_{\mu\nu}^I$

$pp \to ZH \to \ell^+\ell^- b\bar{b}$

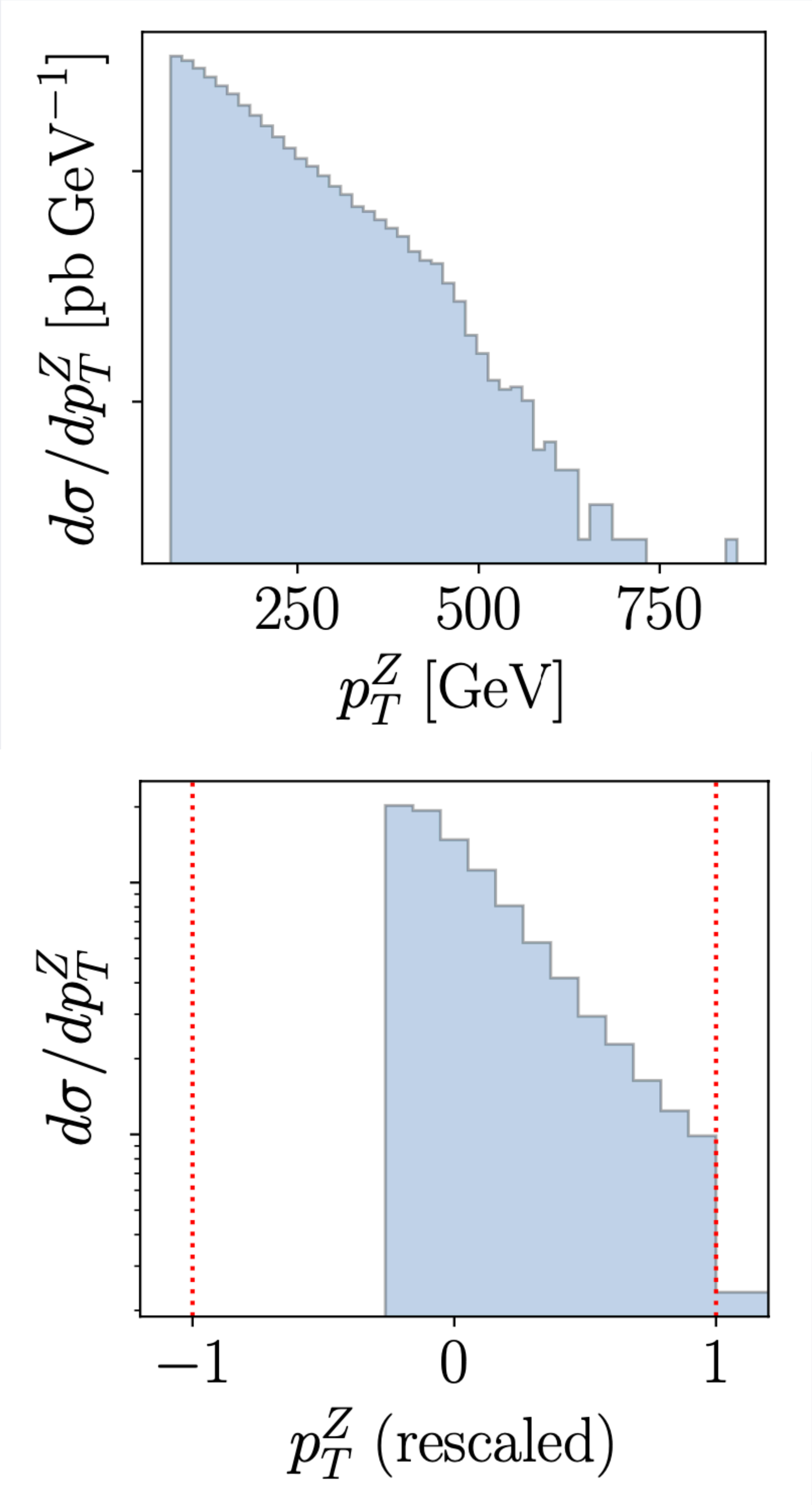
Use realistic **cuts** from arXiv:1708.03299

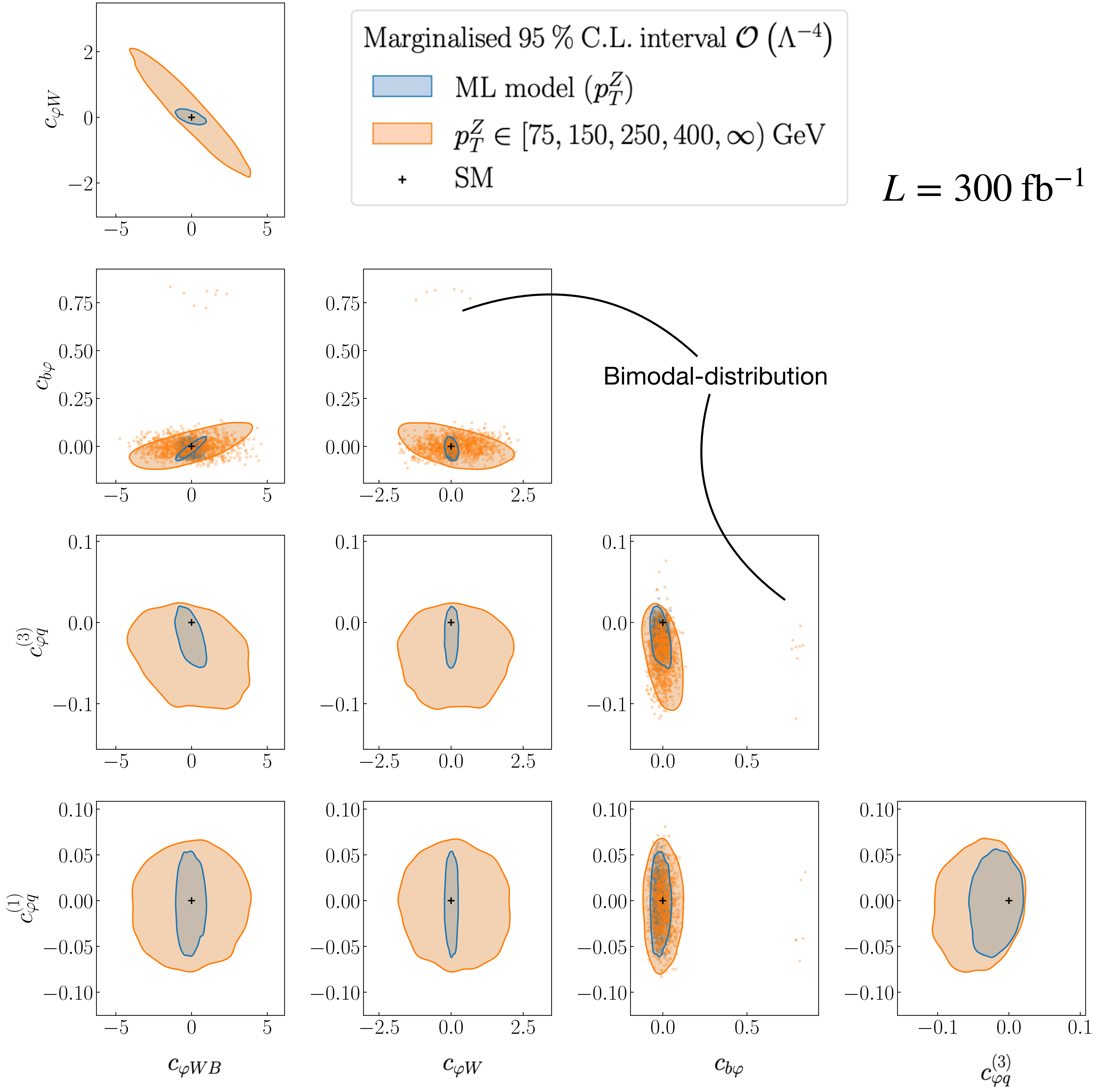


Results



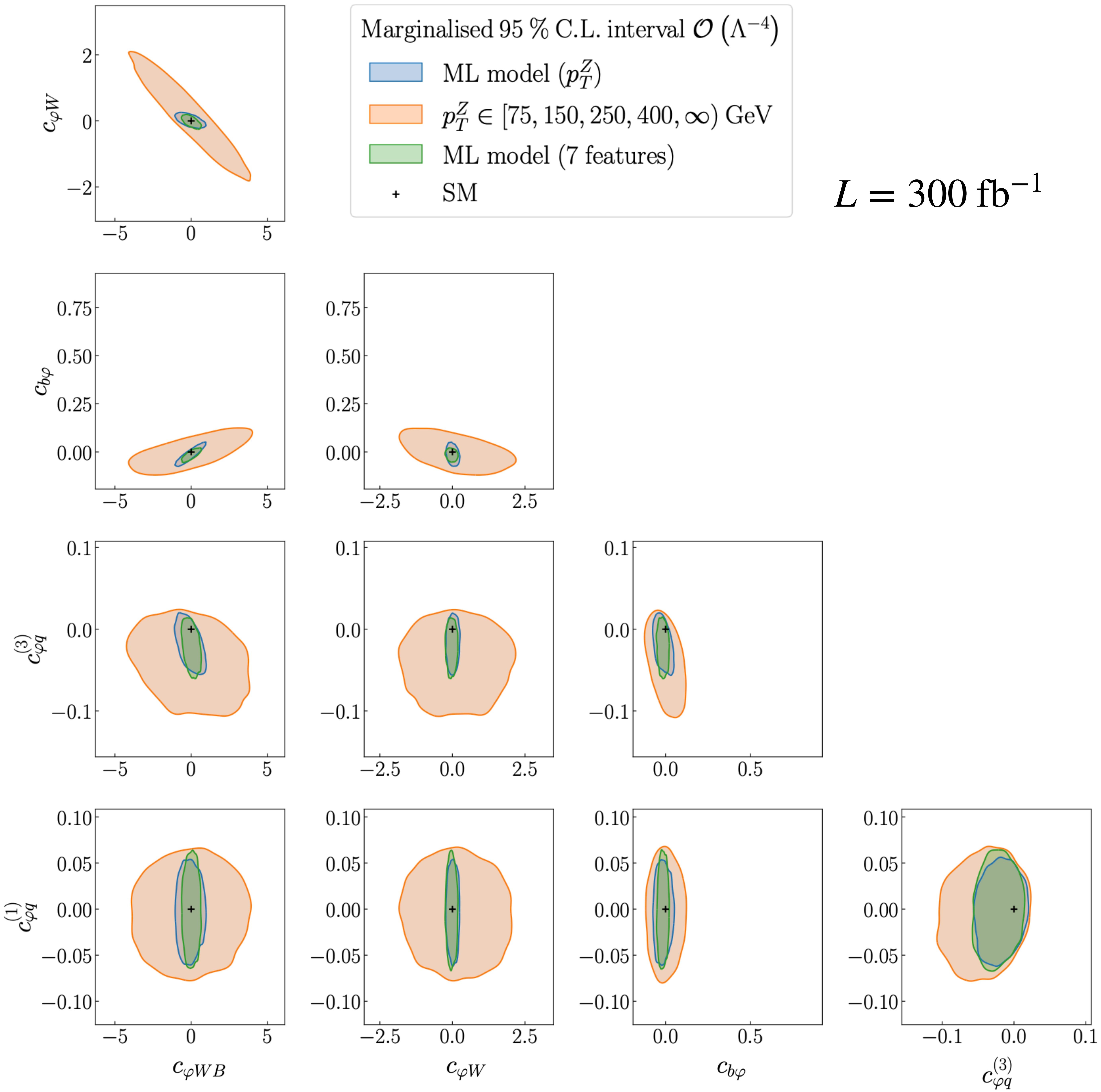
Rescale the features to the same scale





Results

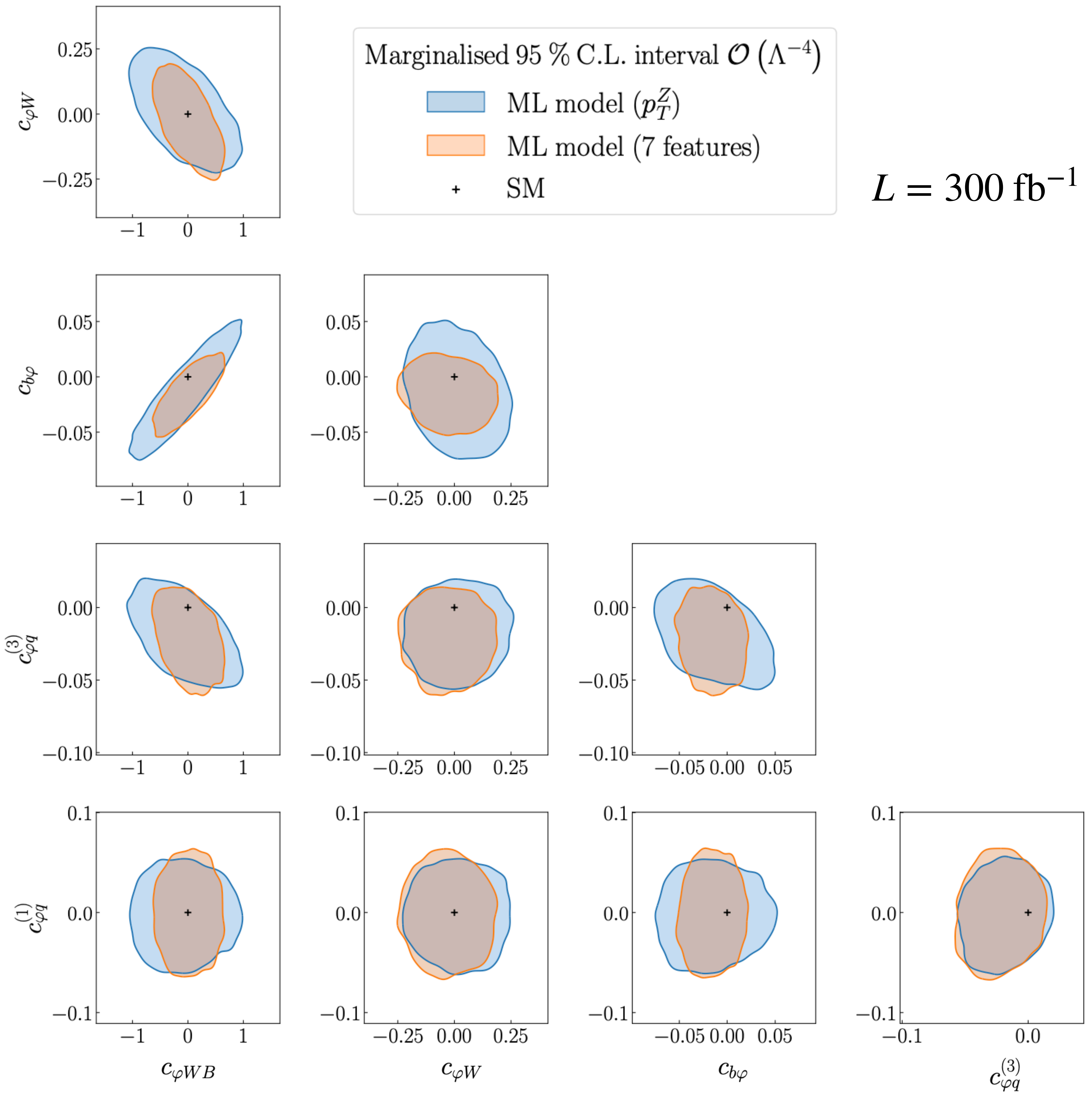
- Assess the **sensitivity** of the STXS binning in p_T^Z by comparing against the unbinned ML model trained on p_T^Z only
- This particular STXS binning is **suboptimal** for EFT studies
- The ML model provides a very **useful benchmark** for optimality studies



$L = 300\ \text{fb}^{-1}$

Results

- Enforce a lower bound on the (relative) statistical uncertainty of $1\,\%$
- Including all features leads to an **enhanced sensitivity** in most cases
- An unbinned multivariate analysis thus pays off, especially for $c_{\varphi WB}$ and $c_{b\varphi}$



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Conclusion and outlook

- The SMEFT provides a model independent framework to search for NP that fully takes into account **correlations** and **connects** new BSM phenomena

- A global fit will profit from unbinned measurements: we are extending SMEFiT based on a **ML likelihood (ratio) parameterisation**

- Our approach scales to an **arbitrary large number of EFT coefficients** and gives a faithful estimate of the **model uncertainties**

- Our tool can be used to determine the **optimal EFT sensitivity** of measurements

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Thank you!