Non-parametric analysis of the Hubble Diagram with Neural Networks

GGI ML workshop – 25/08/22

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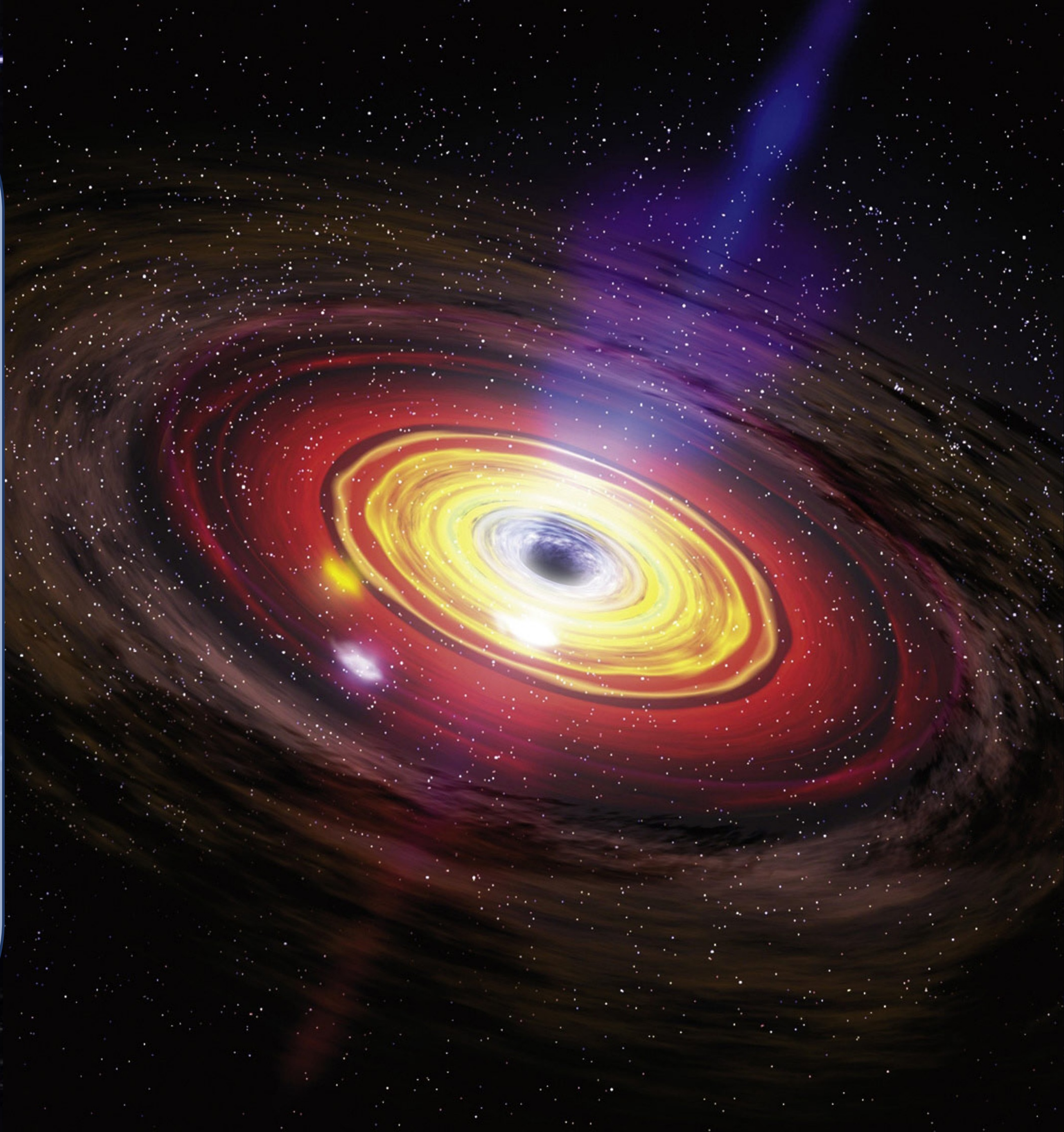
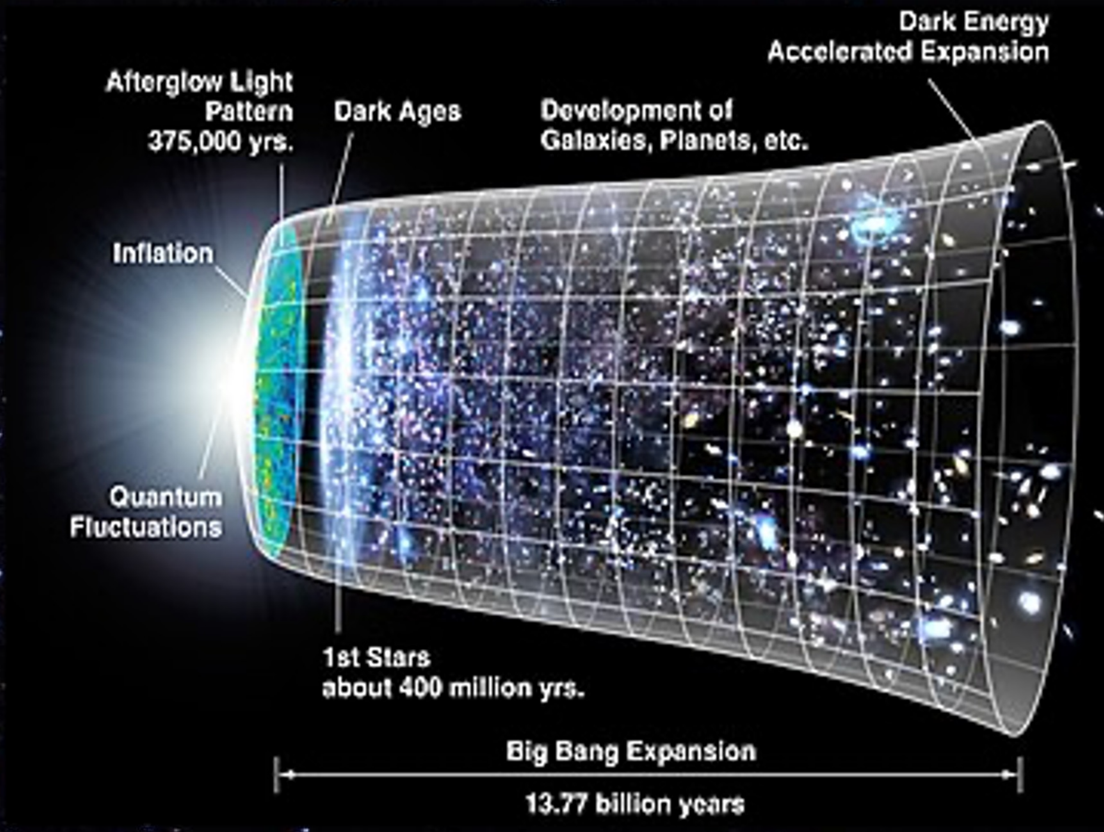


Cosmology: the study of the large scale structure of the universe.

How did the Universe origin? What are its components and its physical properties? How does it change with time?

Study of the *expansion history* of the Universe

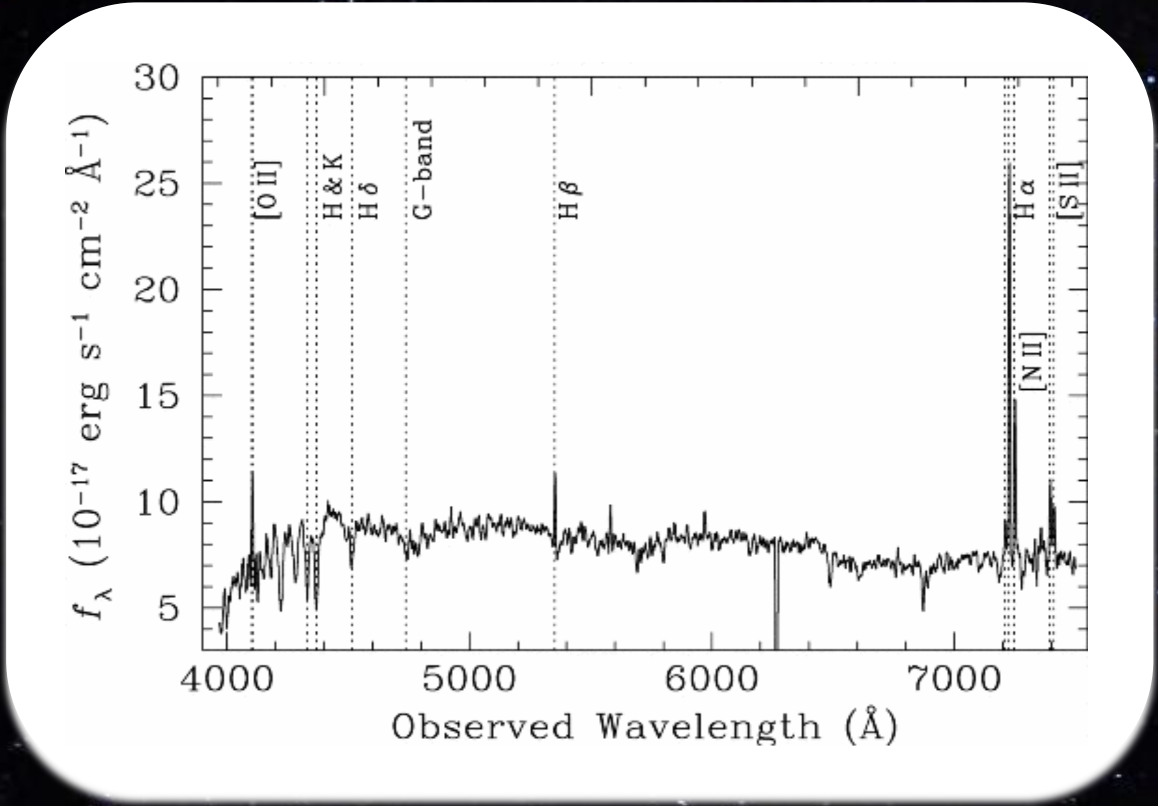


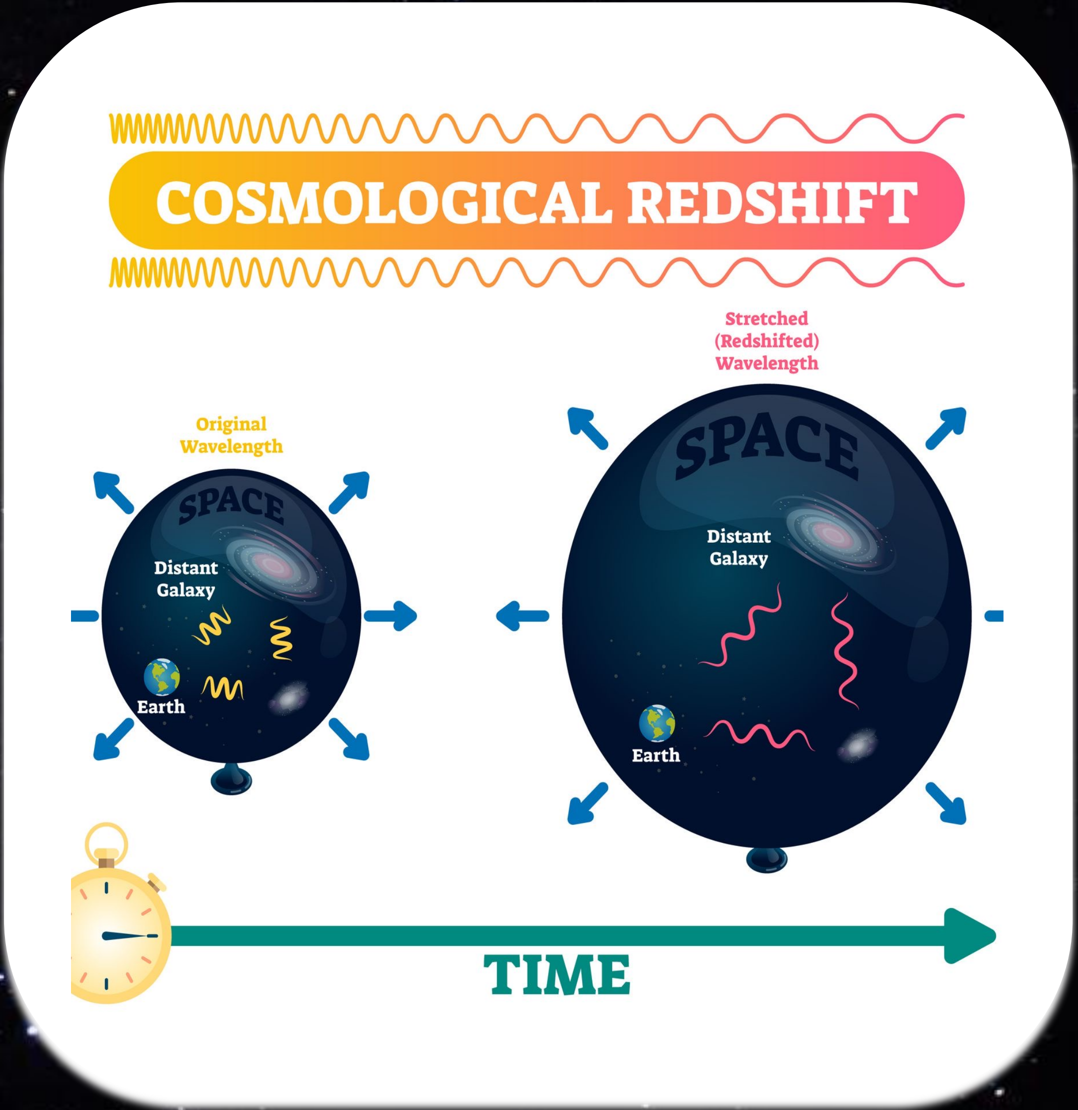
Assumptions:

- The “cosmological principle”: we are no special observer
- The Universe is homogenous, isotropic and it is expanding

How do we know it is expanding?

Cosmological redshift (z): all the galaxies are receding away from us!





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It can be described with the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric

$$\boxed{\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{a(t)^2}{c^2}\left[\mathrm{d}r^2 + \mathcal{R}^2 \sin^2\left(\frac{r}{\mathcal{R}}\right)\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2\right)\right]}$$

$a(t)$: “scale factor” - $a(t) = \frac{1}{1+z}$

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- The cosmological principle: we are no special observer
- The Universe is homogenous, isotropic and it is expanding
- Einstein’s field equations are true

$$\boxed{\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{a(t)^2}{c^2}\left[\mathrm{d}r^2 + \mathcal{R}^2\sin^2\left(\frac{r}{\mathcal{R}}\right)\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]}$$

+

Einstein's field equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^2}T_{\mu\nu} + \Lambda g_{\mu\nu}}$$

=

$$\boxed{\dot{a}^2(t) = \frac{8\pi G\rho}{3}a^2(t) - \frac{c^2}{\mathcal{R}^2} + \frac{1}{3}\Lambda a^2(t)} \quad \boxed{\ddot{a}(t) = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a(t) + \frac{1}{3}\Lambda a(t)}$$

Friedmann's equations

$$\dot{a}^2(t) = \frac{8\pi G\rho}{3}a^2(t) - \frac{c^2}{\mathcal{R}^2} + \frac{1}{3}\Lambda a^2(t)$$

$$\ddot{a}(t) = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right)a(t) + \frac{1}{3}\Lambda a(t)$$

Universe components

EoS $p = w\rho c^2 \rightarrow \rho(t) = \rho_0 a^{-3(1+w)}$

- Matter : $p = 0 \rightarrow w = 0 \rightarrow \rho \sim a^{-3}$
- Radiation : $p = \frac{1}{3}\rho c^2 \rightarrow w = \frac{1}{3} \rightarrow \rho \sim a^{-4}$
- Dark Energy : repulsive force

  In ΛCDM: $\rho_{DE} = cost \rightarrow w = -1$

Other models:

- wCDM : $w < 0$ but different from -1
- CPL: $w = w_0 + w_a \frac{z}{1+z}$

$$\dot{a}^2(t) = \frac{8\pi G\rho}{3}a^2(t) - \frac{c^2}{\mathcal{R}^2} + \frac{1}{3}\Lambda a^2(t)$$

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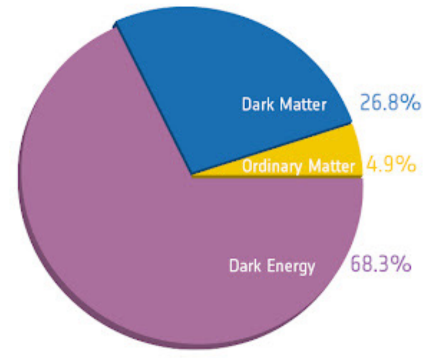
ΛCDM:



$\Omega_M = 0.3, \Omega_\Lambda = 0.7, w = -1$,
Geometrically flat Universe

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ΛCDM:

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- Good at reproducing many observational evidences
- It needs a small number of free parameters
- There are some tensions, like the H_0 (current expansion rate) one:

$H_0 = 67.4 \pm 0.05$ (prediction) vs

$H_0 = 74.0 \pm 1.4$ (local measurement)

$$\dot{a}^2(t) = \frac{8\pi G\rho}{3}a^2(t) - \frac{c^2}{\mathcal{R}^2} + \frac{1}{3}\Lambda a^2(t)$$

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By observing the expansion history, we can understand the physical properties of the Universe

We don't directly observe derivatives of a(t) but we can observe quantities that depend on it

One way to do so is through the luminosity distance – redshfit relation («Hubble diagram»)

$$F = \frac{L}{4\pi d_L^2}$$

$$d_L = \frac{c\,(1+z)}{H_0} \int_0^z dz' \frac{H_0}{H\,(z')}$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

$$H\,(z) = H_0\,\sqrt{\Omega_M\,(1+z)^3 + (1-\Omega_M)\,e^{3\int_0^z \frac{1+w(z')}{1+z'}dz'}}$$

We need «standard candles» !

⬇

Supernovae Ia

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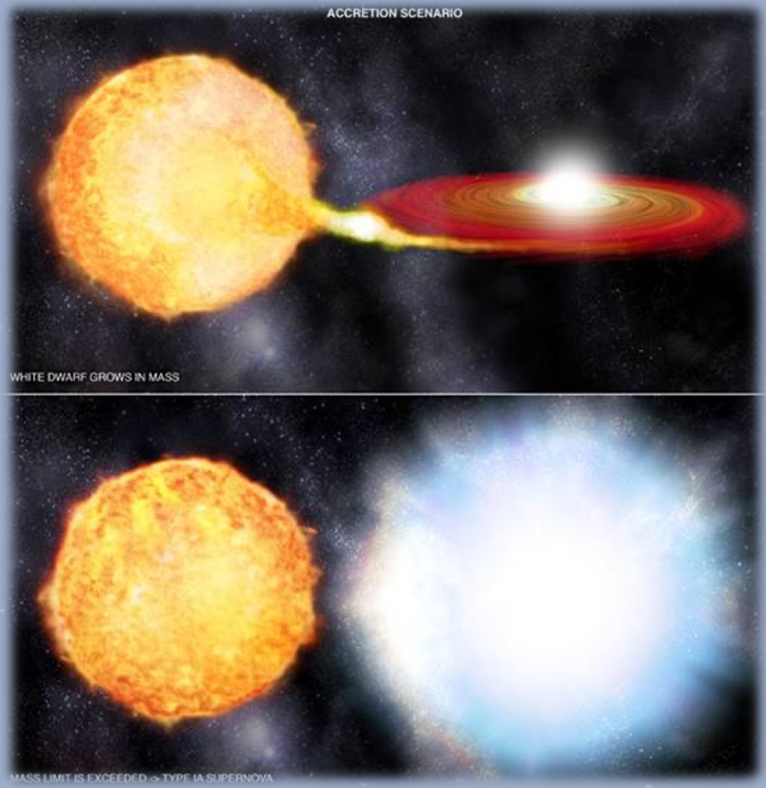

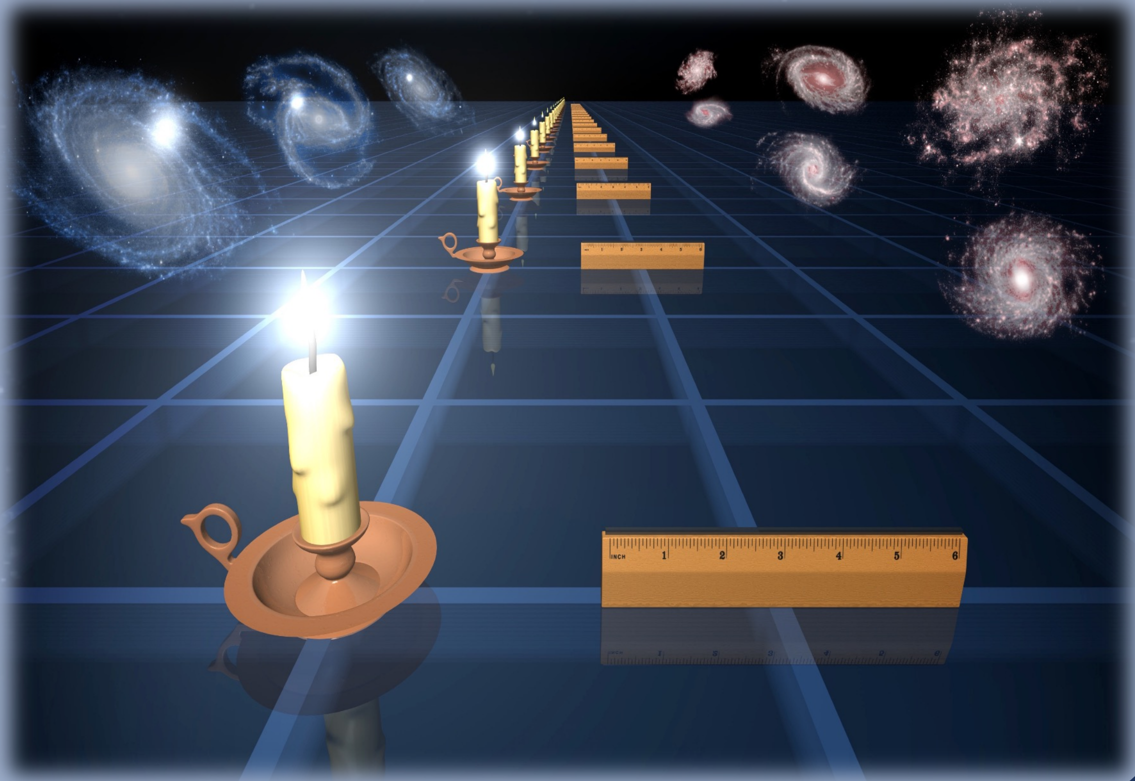

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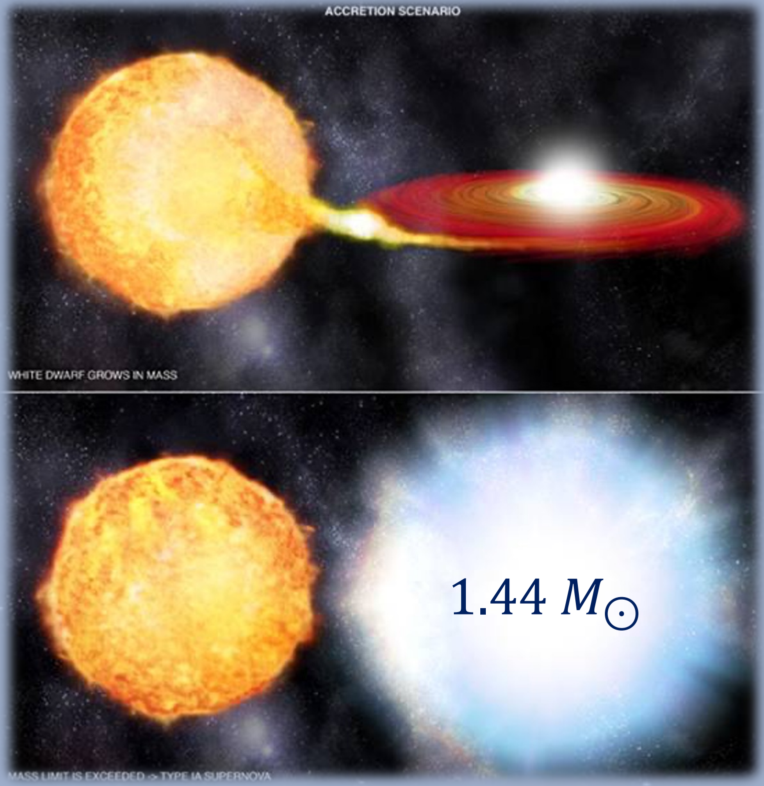

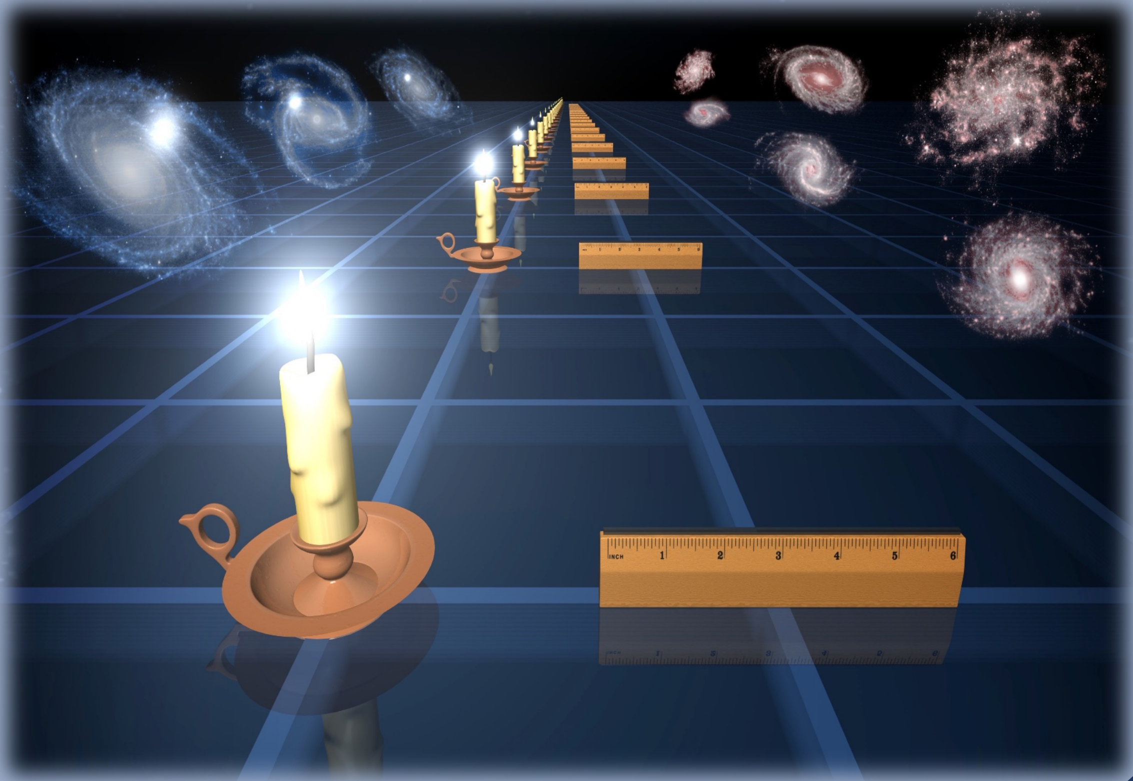

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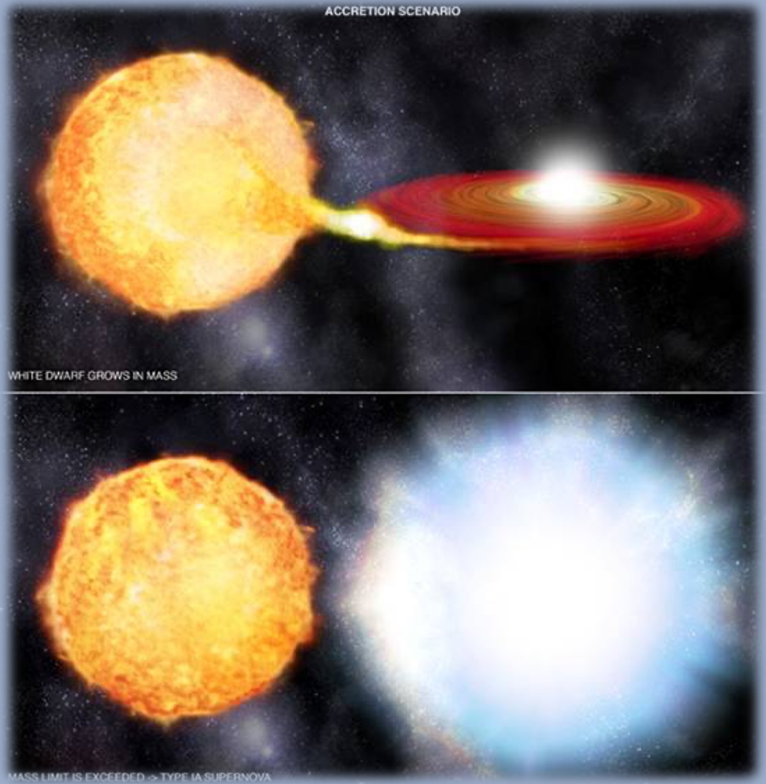

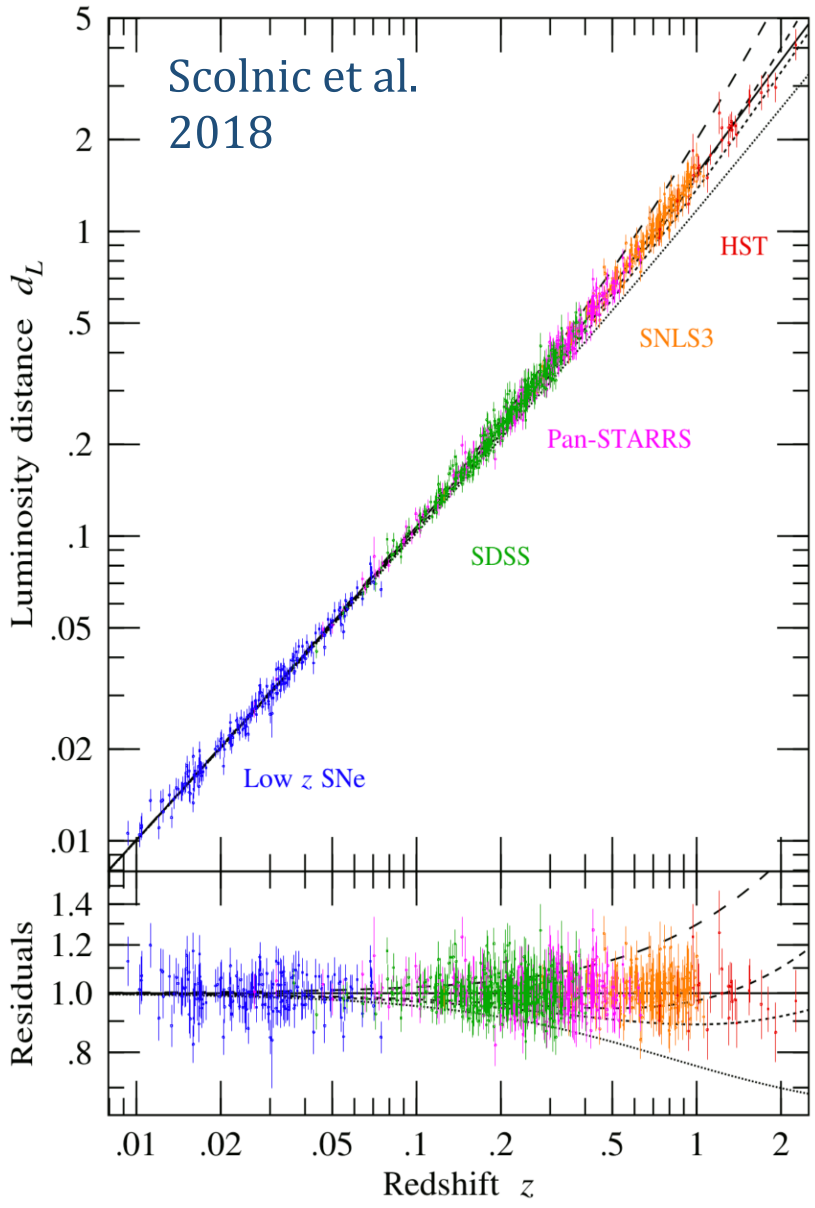

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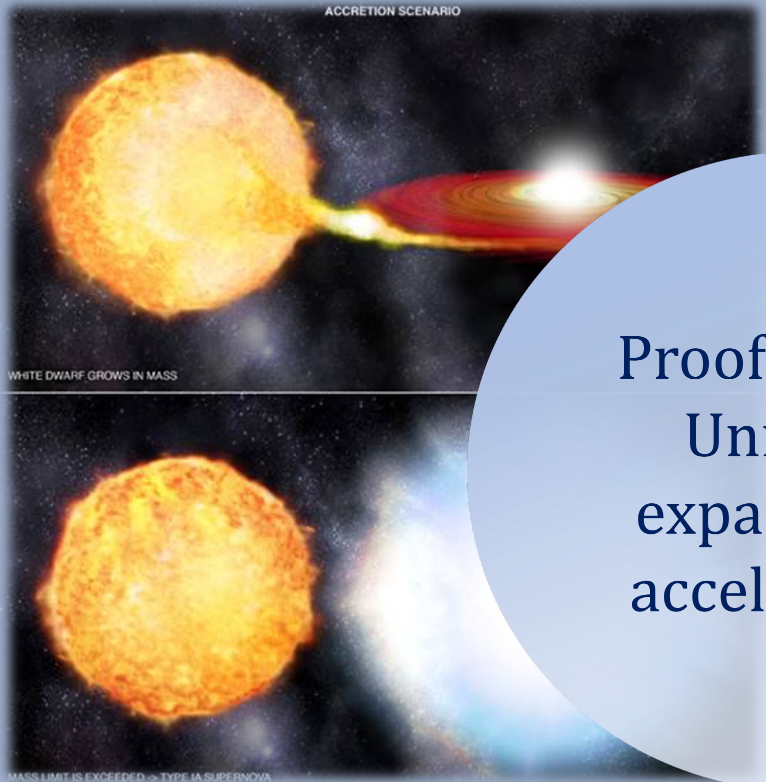
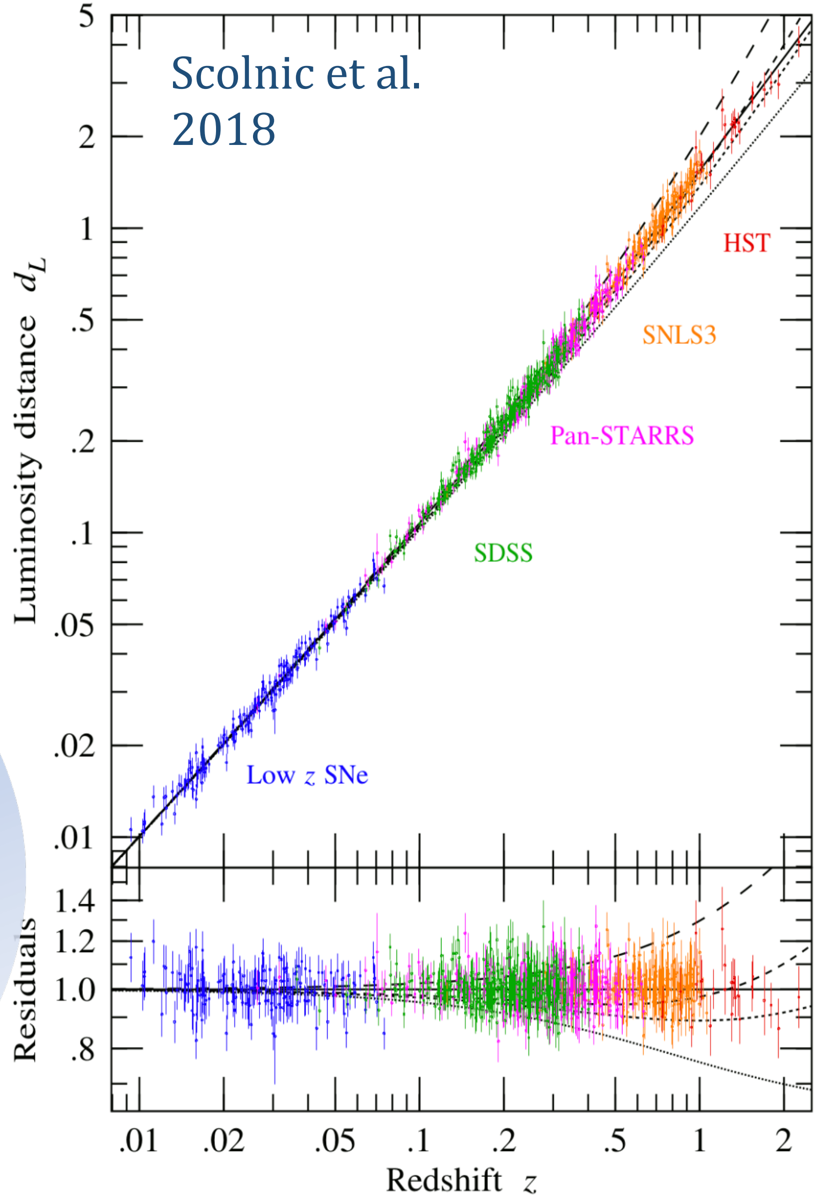


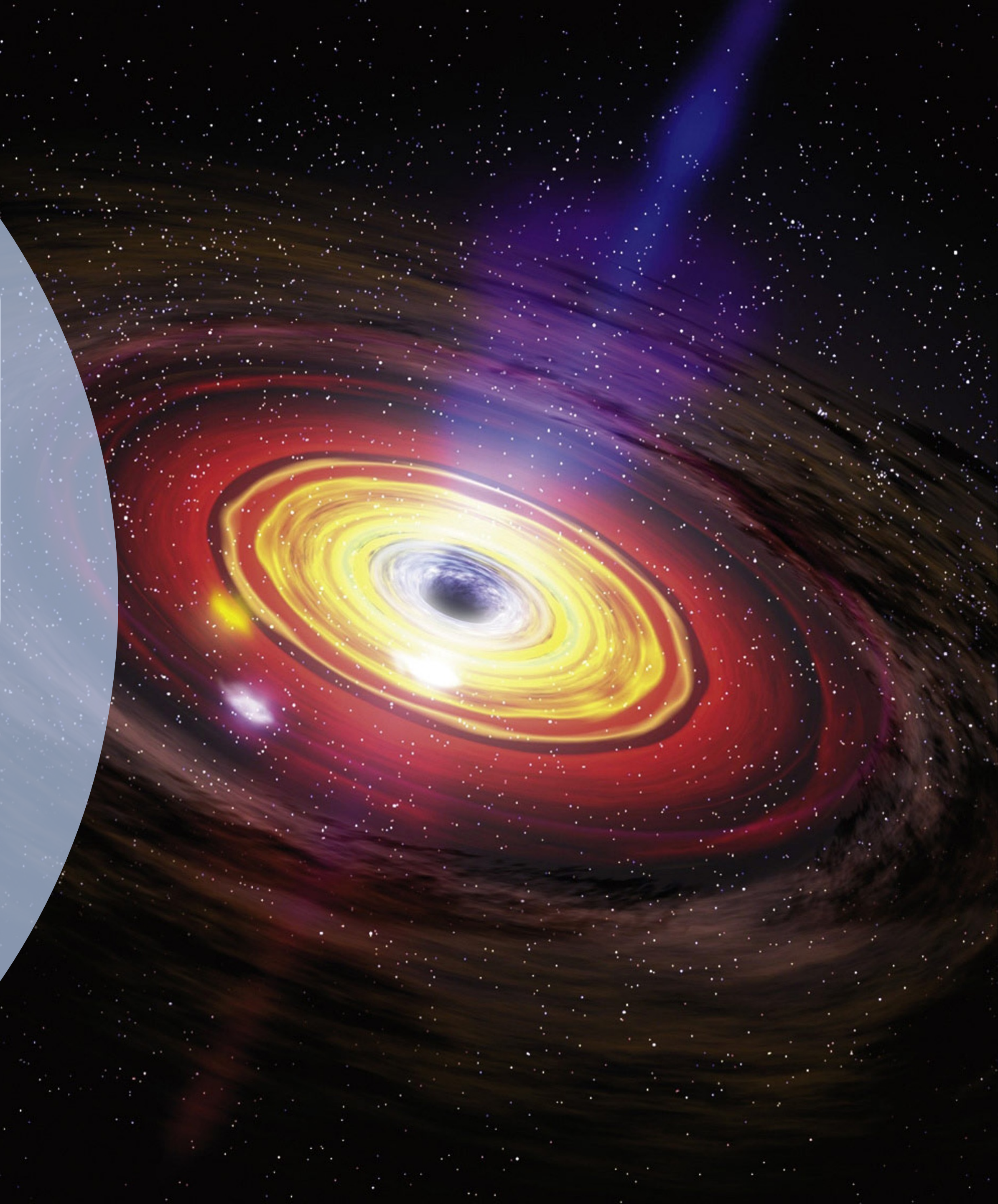
Proof that the Universe expansion is accelerating!



Quasars

Active Galactic Nuclei that are more luminous than the whole host galaxy

- Observed at all redshifts
- Observed up to $z \sim 7.5$ (age of the Universe: less than 1 Gyr)
- Numerous



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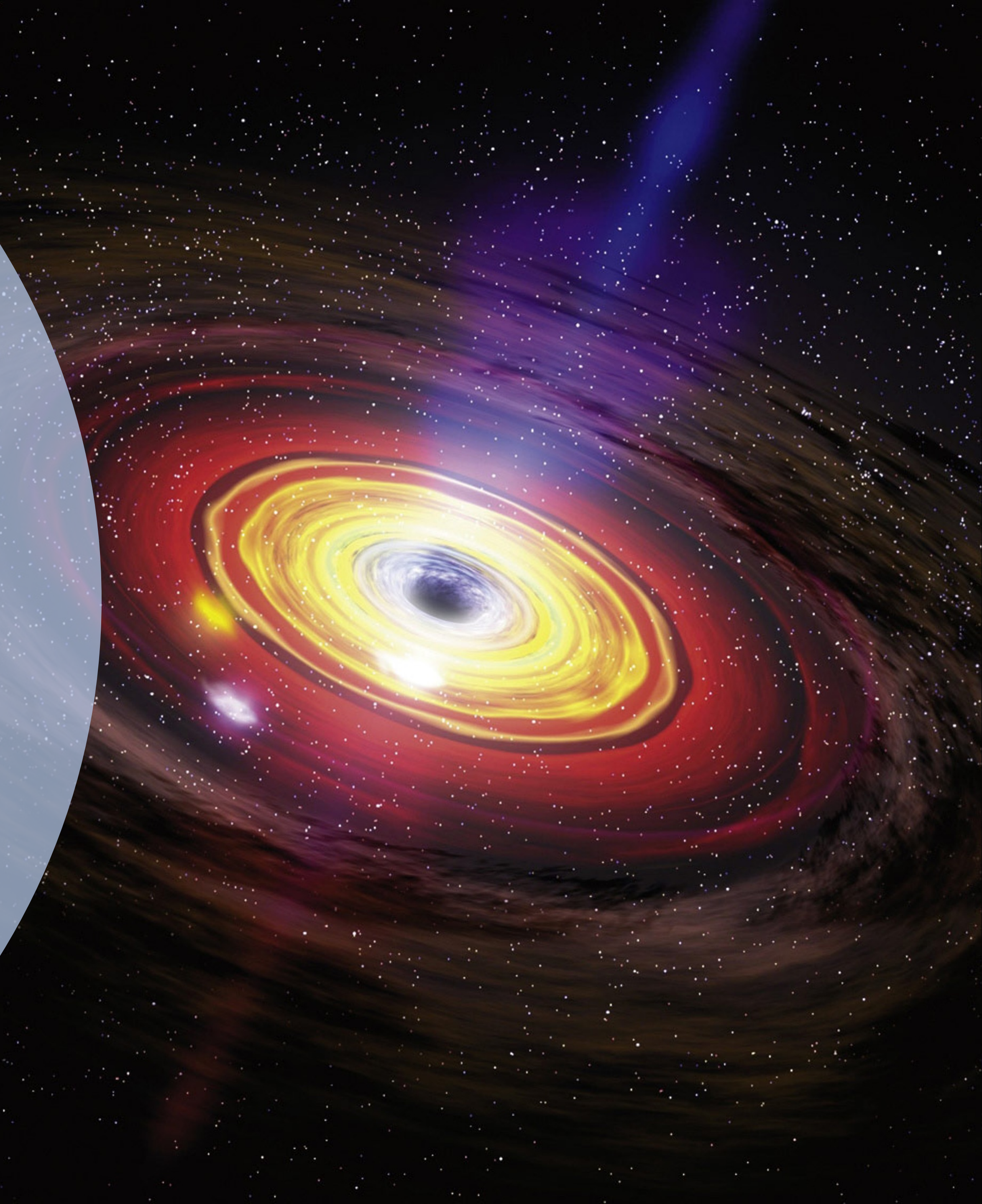
they are not standard candles (bolometric L: 10^{11} - $10^{14} L_{\odot}$)

... but they can be standardized!



Luminosity relation

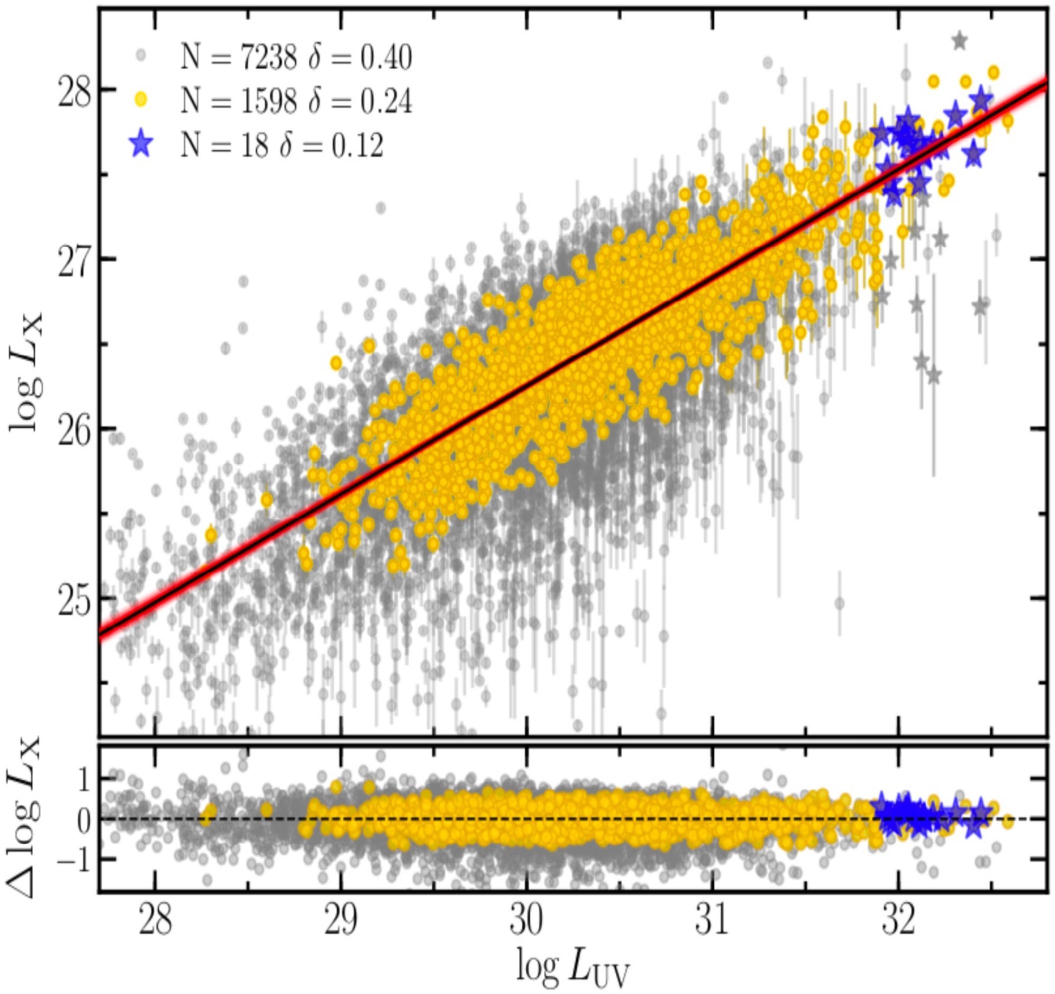
$$\log(L_X) = \gamma \log(L_{UV}) + \beta$$

with

$$F = \frac{L}{4\pi D_L^2}$$

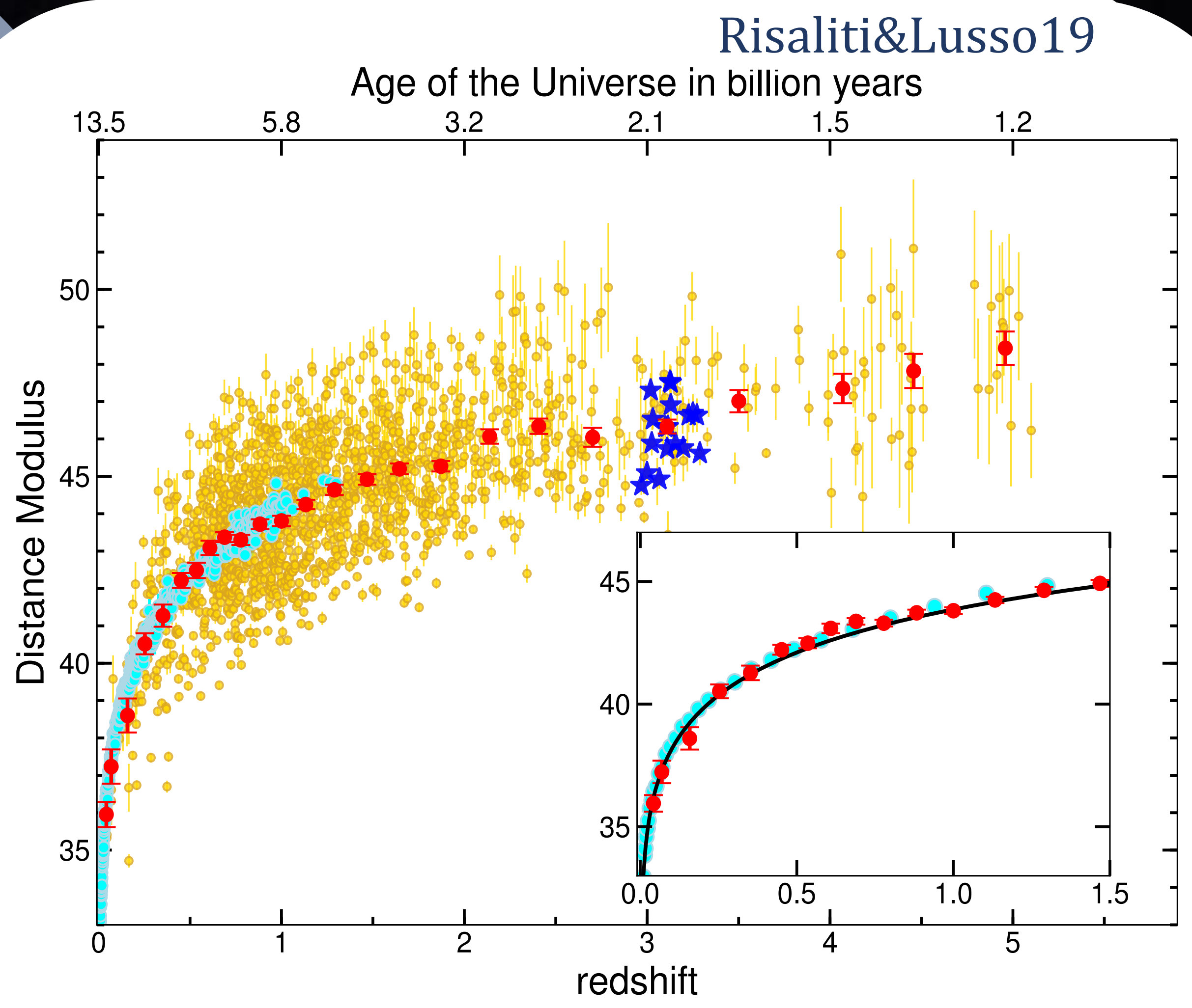
$$\log(D_L) = \frac{1}{2 - 2\gamma} \cdot (\log(f_X) - \gamma \cdot \log(f_{UV})) + \beta'$$

Risaliti&Lusso19



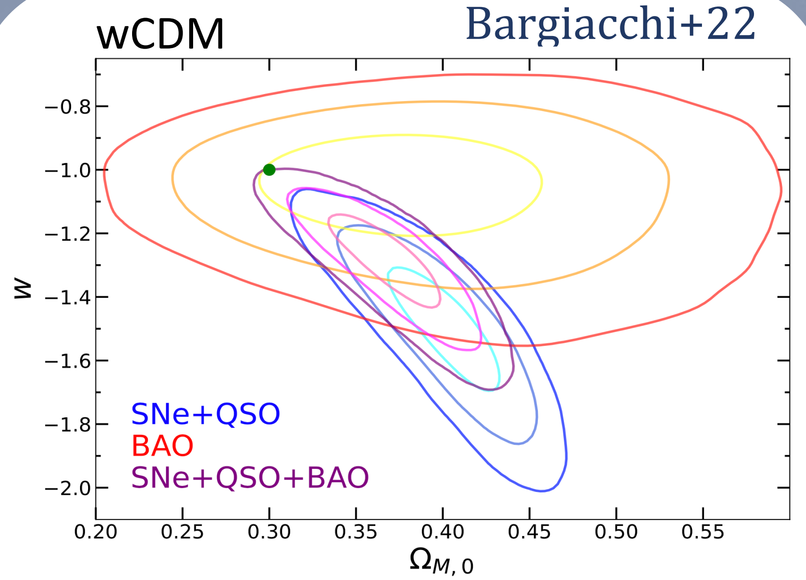
Hubble Diagram of SN + Quasar

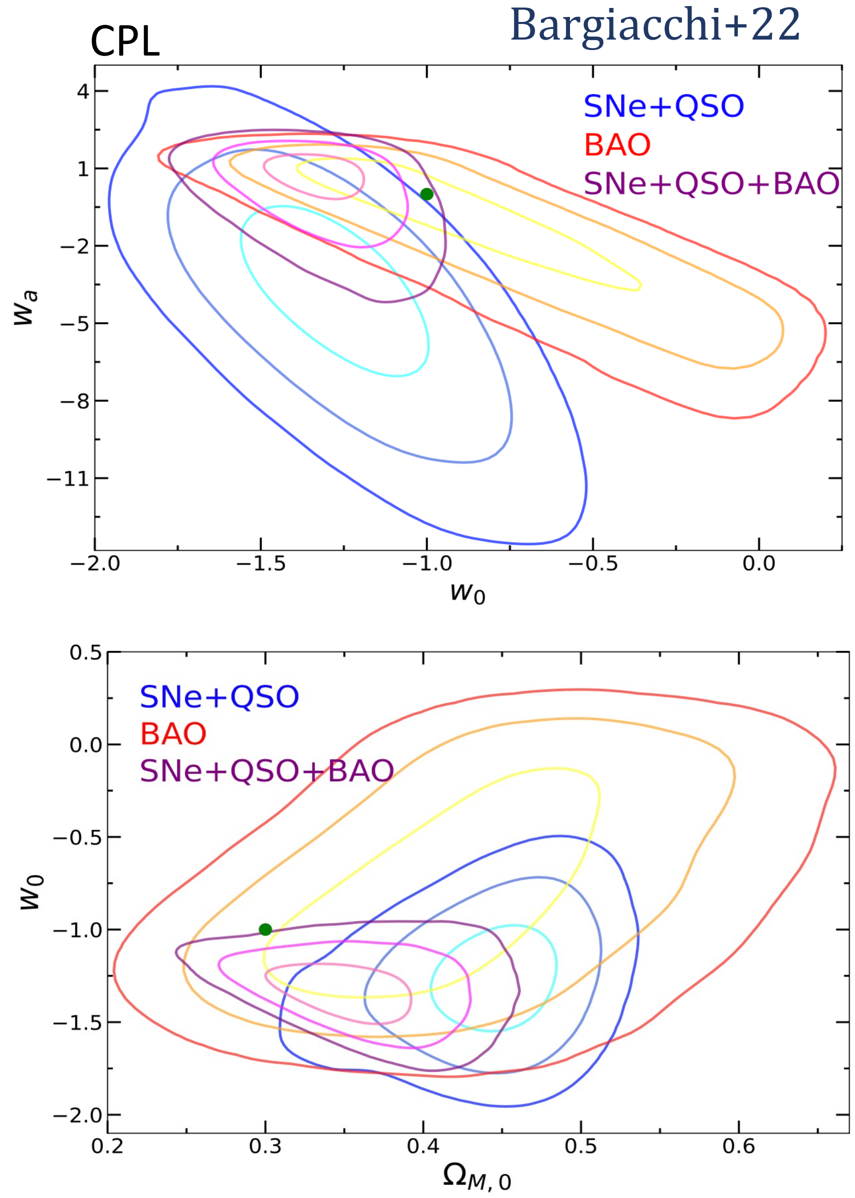
Significant extension to earlier cosmic epochs!



Test of cosmological models

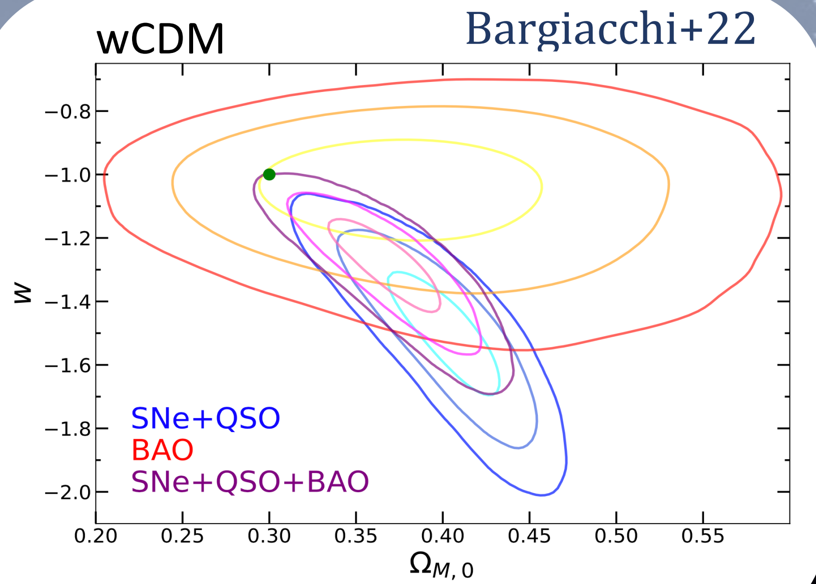
$$d_L = \frac{c\,(1+z)}{H_0}\int_0^z dz'\,\frac{H_0}{H\,(z')}$$

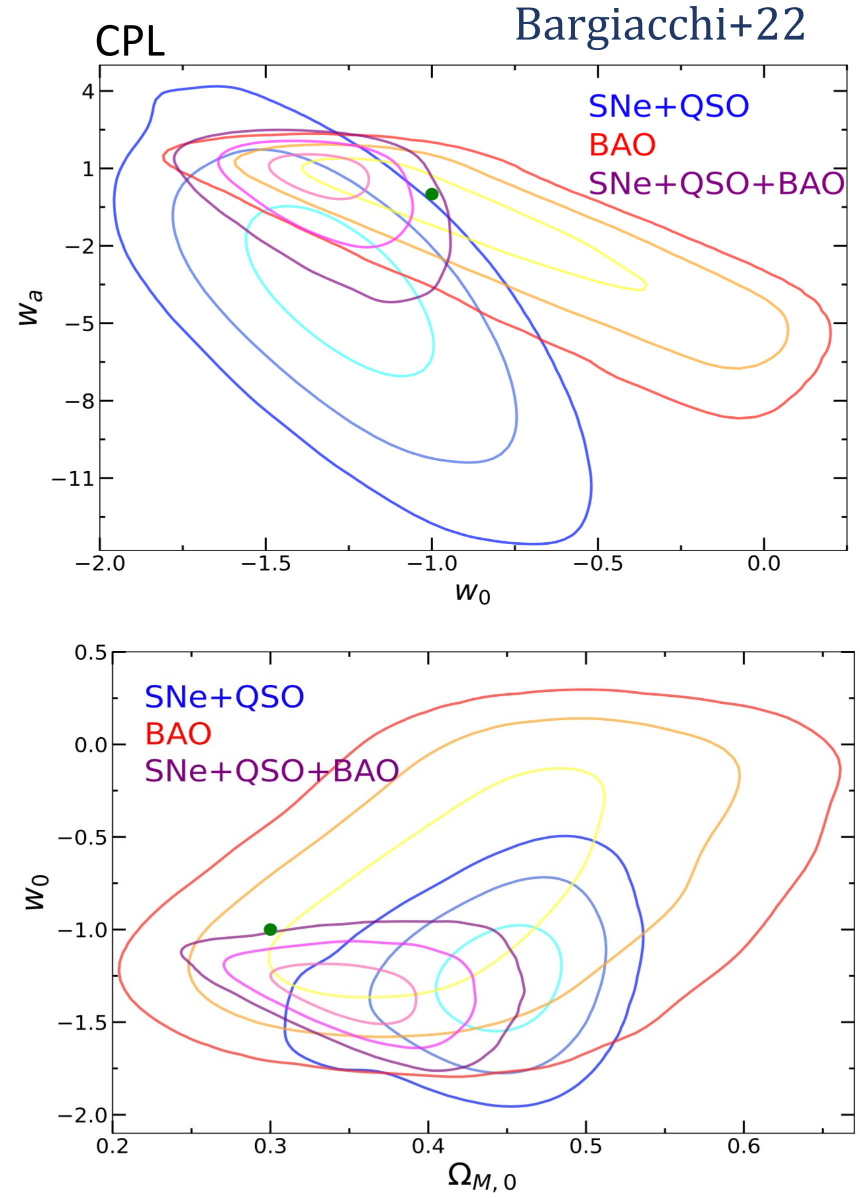




Test of cosmological models

Model-dependent tension!





Test of cosmological models

Model-dependent tension ΛCDM!

We want a model-independent way to test this!

Neural Network Regression