I.S. SFT-MI - INFN, 14 July 2022

Statistical Field Theory

low-dimensional systems, integrable models and applications
studio di sistemi con molti gradi di liberta` con metodi di teoria dei campi

2021: Cosenza (1.5+1.5), Firenze (8+3), Genova (2+2), Milano (3.5+4), Pisa (4+2), Torino (2.4), Trieste (7+25).
Coord. naz. Giuseppe Mussardo

Statistical properties of quantum field theories (out of equilibrium)
Entanglement in quantum extended systems
Topological phases of matter in D=2,3
Conformal invariance, phase transitions and universality classes
Low D QFT, integrable models and breaking of integrability

SFT-MILANO
July 2022

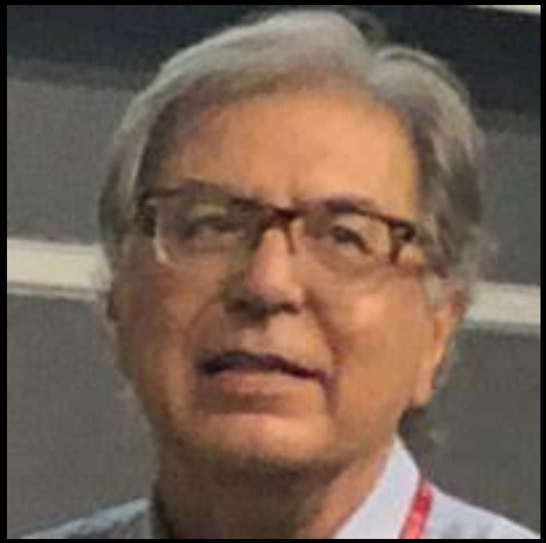
Sergio Caracciolo (PO 100%)
Luca Guido Molinari (PA 50%)
Mario Pernici (Primo Ric INFN 100%)
Marco Gherardi (RTDB 100%)
Carlo Alberto Mantica (37 100%)

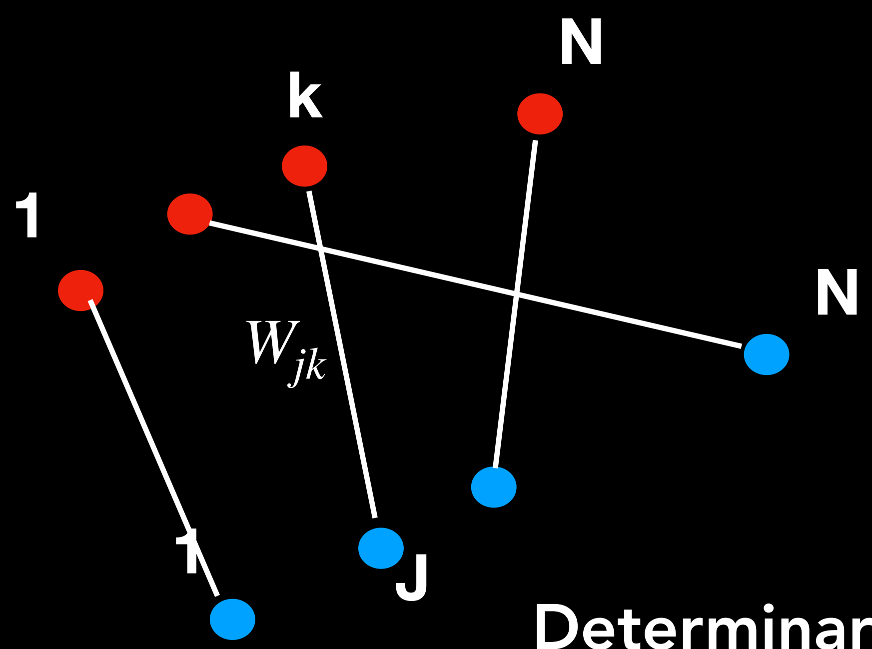
Vittorio Erba (34, 31 giu 2022, now: EPFL Lausanne)
Matteo Cardella (28 feb 22)
Mauro Pastore (33, now: Saclay)
Riccardo Fabbricatore (33, now: Skolkovo RU)
Andrea Di Gioacchino (32, now: Marie Curie ENS)
Enrico Malatesta (31, now: Ric. Bocconi)

Combinatorial Optimization
Statistical mechanics of Machine Learning
Sparse random block matrices
Carbon nanocones and Pascal matrices
Covariant characterisation of spacetimes

OPTIMIZATION AND MATCHING

S. Caracciolo and V. Erba





Matching = biezione rosso-blu
= permutazione $j \to \pi_j$

Determinare π con costo $W_\pi = W_{1\pi_1} \dots W_{N,\pi_N}$ minimo

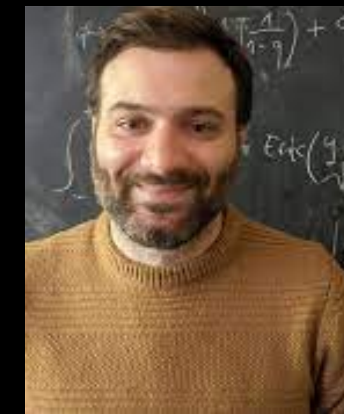
su un manifold: $W_{kj} = d(x_k, x_j)$

D.Benedetto, E.Caglioti, S.Caracciolo, M.D'Achille, G.Sicuro, A.Sportiello, Random assignment problems on 2d manifolds, J. Stat. Phys.183:34 (2021)

S.Caracciolo, V.Erba, A.Sportiello, The number of optimal matchings for Euclidean assignment on the line, J. Stat. Phys. 183:3 (2021)

S.Caracciolo, R.Fabbricatore, M.Gherardi, R.Marino, G.Parisi, G.Sicuro, Criticality and conformality in the random dimer model, PRE 103 (2021) 042127

STATISTICAL MECHANICS OF MACHINE LEARNING



Due modi di fare "**fisica e machine learning**":

1) ML come uno strumento al servizio della fisica.
2) La fisica per studiare il ML come sistema complesso.

Scopo: comprensione teorica dell'efficacia del deep learning; leggi quantitative, principi ``fondamentali"

Metodi: metodi mean-field, teoria dei sistemi disordinati, repliche

Risultati: nuove transizioni di fase di espressività, universalità nelle performance di generalizzazione

$$Z = \int \mathrm{d}^N \mathbf{v} \exp\left\{ -\beta \sum_{\mu=1}^{P} \left[f_{\mathrm{T}}\left(\mathbf{x}^{\mu}\right) - f_{\mathrm{S}}\left(\mathbf{x}^{\mu};\mathbf{v}\right)\right]^2 - \beta\lambda \|\mathbf{v}\|^2 \right\}$$

funzione di partizione (legata al training error a bassa temperatura)

campi dinamici (parametri di training)

temperatura inversa

disordine (training set)

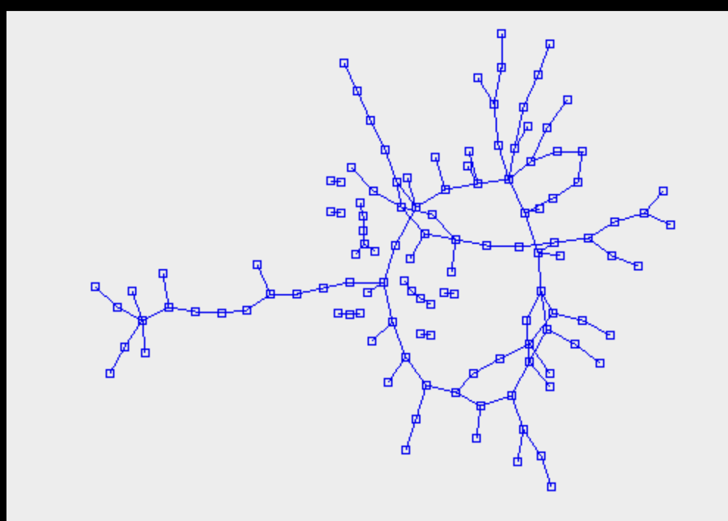
regolarizzazione

S. Ariosto, R. Pacelli, F. Ginelli, M. Gherardi, P. Rotondo, Universal mean field upper bound for the generalisation gap of deep neural networks, Phys.Rev.E 105 (2022)

M. Gherardi, Solvable models for the linear separability of structured data, Entropy 23 (3) (2021)

SPARSE RANDOM BLOCK-MATRICES

G.M. Cicuta, M. Pernici, Sparse random block matrices: universality,
arXiv:2206.09356
G.M. Cicuta, M. Pernici, Sparse random block matrices,
J. Phys. A: Math. Theor. 55 (2022), 175202



GRAFO DI ERDOS-RENYI
N nodi, ogni coppia connessa con probabilita` p
Componente connessa gigante per p>1/N

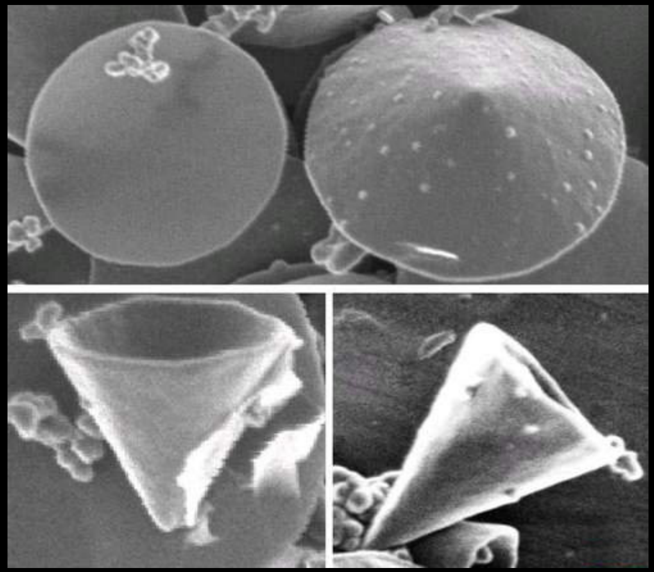
$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2N} \\ \dots & \dots & \dots & \dots \\ \alpha_{N1} & \alpha_{N2} & \dots & \alpha_{NN} \end{bmatrix} \qquad P(\alpha) = \frac{Z}{N}\delta(1-\alpha) + \left[1 - \frac{Z}{N}\right]\delta(\alpha)$$

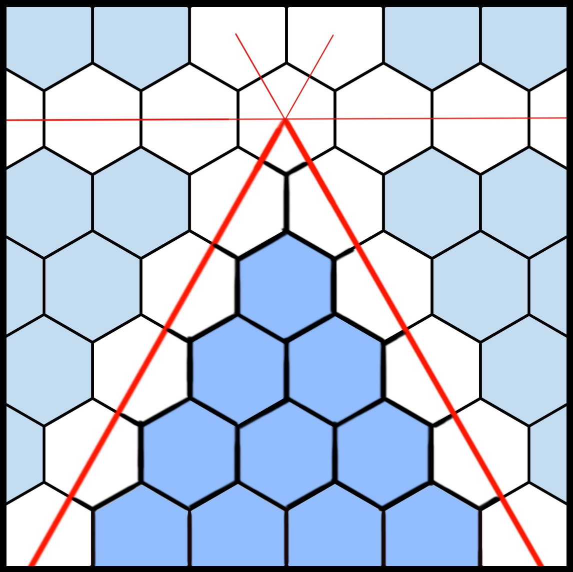
Il grafo descrive la connettivita` tra N comunita`,
ognuna descritta da matrice X_{ii} con interazioni X_{ij}

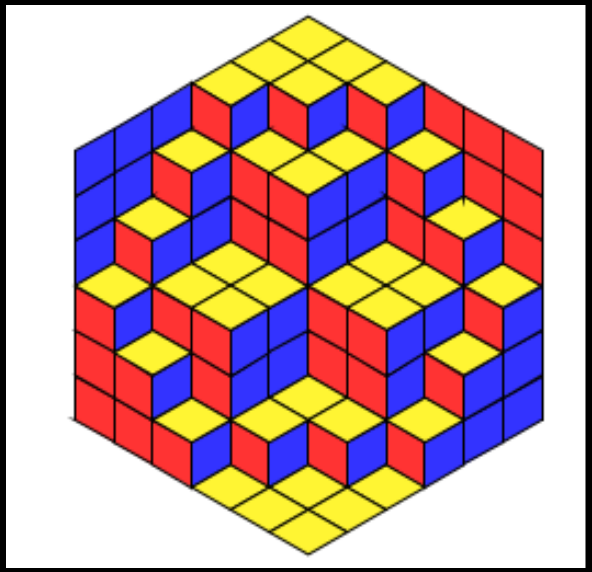
$$\begin{bmatrix} \alpha_{11}X_{11} & \alpha_{12}X_{12} & \dots & \alpha_{1N}X_{1N} \\ \alpha_{21}X_{21} & \alpha_{22}X_{22} & \dots & \alpha_{2N}X_{2N} \\ \dots & \dots & \dots & \dots \\ \alpha_{N1}X_{N1} & \alpha_{N2}X_{N2} & \dots & \alpha_{NN}X_{NN} \end{bmatrix}$$

α_{ij} = 0,1
X_{ij} = dxd matrix

Momenti della densita` spettrale N,d>>1, Z/d costante







$$H_2 = \left[\begin{array}{c|ccc|ccccc} x+y & 0 & 1 & & & & & & \\ \hline 0 & 0 & 1 & y & 0 & 1 & & & \\ 1 & 1 & 0 & 1 & 0 & & 0 & & \\ & x & 1 & 0 & 0 & & & 1 & \\ \hline & 0 & 0 & 0 & 0 & 1 & & & y \\ & 1 & & & 1 & 0 & 1 & & \\ & & 0 & & & 1 & 0 & 1 & \\ & & & 1 & & & 1 & 0 & 1 \\ & & & & x & & & 1 & 0 \end{array}\right]$$

Matrice di adiacenza di nanocono

$$Q_5 = \left[\begin{array}{ccc|cc} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 6 & 10 & 15 \\ \hline 1 & 4 & 10 & 20 & 35 \\ 1 & 5 & 15 & 35 & 70 \end{array}\right]$$

Matrice di Pascal

Graphene nanocones and Pascal matrices

Una inattesa connessione con famosi modelli combinatorici (partitions, lozenge tilings, dense loops)

$$\det H_n(e^{-i\theta}, e^{i\theta}) = e^{-i(n+1)\theta} \det(Q_n + e^{2i\theta})$$

n	$\theta = 0$	$\pi/6$	$\pi/3$	$\pi/2$	$\pi/4$
2	20	$3^2\sqrt{3}$	7	0	$8\sqrt{2}$
3	132	10^2	42	2^4	70
4	1452	$25^2\sqrt{3}$	429	0	$526\sqrt{2}$
5	26741	140^2	7436	7^4	13167
6	826540	$588^2\sqrt{3}$	218348	0	$280772\sqrt{2}$

L.G. Molinari, arXiv:2206.14428

Covariant characterisation and properties of spacetimes

$$ds^2 = -\, e^{-\beta(x,t)} dt^2 + e^{\alpha(x,t)} g^{\star}_{\mu\nu} dx^\mu dx^\nu$$

$$\nabla_i \tau_j = \kappa g_{ij} + \alpha_i \tau_j + \tau_i \beta_j$$

$$\alpha_j \tau^j = 0, \quad \beta_j \tau^j = 0$$

Perfect scalars & perfect fluid tensors

$$\partial_i S = -\, \dot{S} u_i \qquad\qquad R_{ij} = \frac{1}{3}(R - 4\xi) u_i u_j + \frac{1}{3}(R - \xi) g_{ij}$$

For many extended theories of gravity in RW (or GRW with constant space curvature) background (ex: f(R), string-inspired corrections to Einstein-Hilbert action) the curvature corrections to Einstein field equations have the perfect fluid form, as required by the COSMOLOGICAL PRINCIPLE.

They modify the perfect-fluid parameters of matter source and may correspond to observed effects of unobserved DM (S. Capozziello).

C.A. Mantica, L.G. Molinari, Spherical doubly warped spacetimes for radiating stars and cosmology, arXiv:2204.11617

S. Capozziello, C.A. Mantica, L.G. Molinari, Geometric perfect fluids from extended gravity, Europhys. Lett 13 (2022) 190001.

C.A. Mantica, L.G.Molinari, The Jordan algebras of Riemann, Weyl and curvature compatible tensors, Coll. Math. 167 (2022) 63-72

C.A. Mantica, L.G. Molinari, Doubly torqued vectors and a classification of doubly twisted and Kundt spacetimes, GERG 53 (2021) 48.