

GUIDO AND THE MATRIX

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Only one paper together

**COMPUTATION OF THE RELATION BETWEEN THE QUARK MASSES
IN LATTICE GAUGE THEORIES AND ON THE CONTINUUM**

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A lot of fun in Frascati

Why The



Humans live inside The Matrix

Maybe not...

- But perhaps space and time lives inside a Matrix

Sorry to QCD phenomenologists

but I will leave $N=3$ and go all the way to $N=\infty$

Twisted Matrix Models (G-A, Okawa 1982)

Translation
Group



Automorphisms
in SU(N) Group

$$\mathbf{U}(n + \hat{\mu})$$

$$\Gamma_\mu \Gamma_\nu = z_{\mu\nu} \Gamma_\nu \Gamma_\mu$$

$$\Gamma_\mu \hat{\Gamma}(p) \Gamma_\mu^\dagger = e^{ip_\mu} \hat{\Gamma}(p)$$

$$\Gamma_\mu \mathbf{U} \Gamma_\mu^\dagger$$

$$z_{\mu\nu} = e^{i2\pi n_{\mu\nu}/N}$$

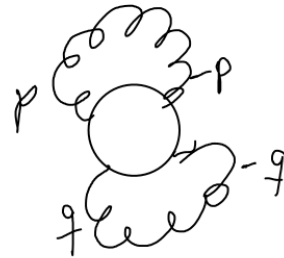
$$\mathbf{U} = \sum_p \hat{U}(p) \hat{\Gamma}(p)$$

At large N these matrix models are equivalent to ordinary field theories at infinite volume

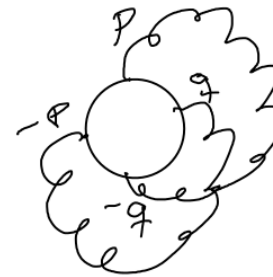
Non-perturbative proof of equivalence based on Schwinger-Dyson equations (EK)

Perturbative proof of equivalence (G-A Okawa) based on:

- Propagators are like in a finite lattice with volume = N^2
- Non-planar graphs suppressed by rapidly oscillating phases



PLANAR



NON-PLANAR

Simplest example: 2d Principal Chiral Model (G-A, Okawa 1984)

$$S = \frac{N}{\lambda} \int d^2x \operatorname{Tr} \left(\partial_\mu \mathbf{U}(x) \partial_\mu \mathbf{U}^\dagger(x) \right) \xrightarrow{\text{lattice}} S = -\frac{N}{\lambda_L} \sum_n \operatorname{Tr}(\mathbf{U}(n) \mathbf{U}^\dagger(n + \hat{\mu})) + \text{h.c.}$$

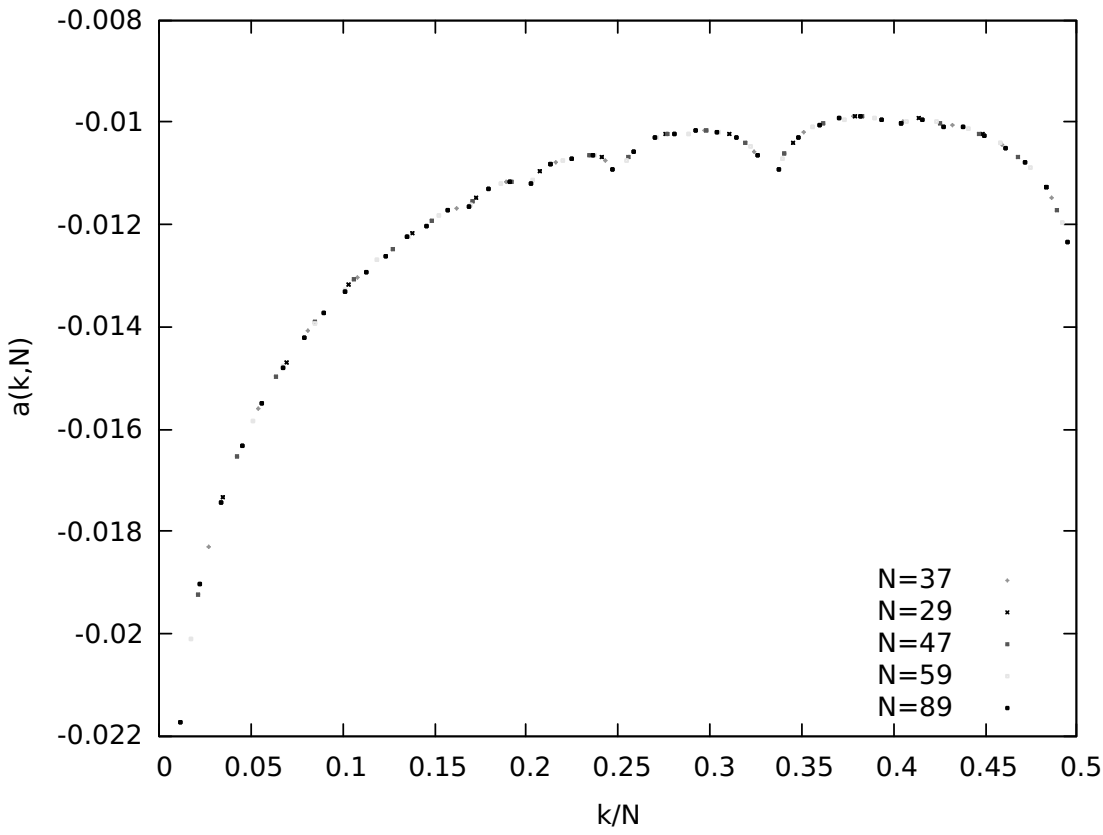
TRPCM $S = -\frac{N}{\lambda_L} \sum_\mu \operatorname{Tr} \left(\mathbf{U} \Gamma_\mu \mathbf{U}^\dagger \Gamma_\mu^\dagger \right) + \text{h.c.}$ $\Gamma_0 \Gamma_1 = \exp\{2\pi i k / N\} \Gamma_1 \Gamma_0$

Test in Perturbation Theory (G-A, Ishikawa, Ji, Okawa 2022)

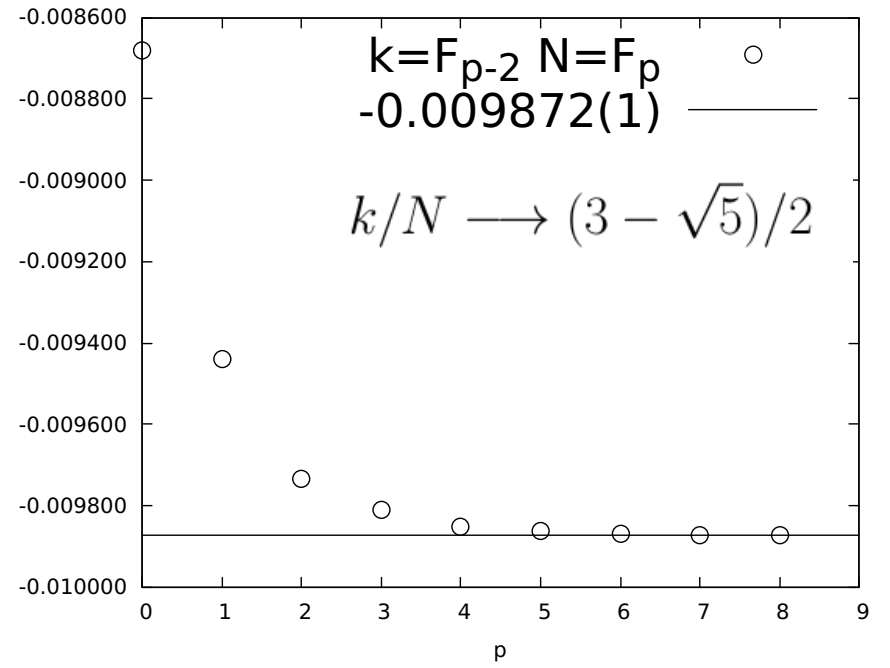
PCM $E = 1 - \frac{\lambda N^2 - 1}{8 N^2} + \lambda^2 \left(-\frac{1}{256} \left(1 - \frac{1}{N^2}\right) \left(1 - \frac{2}{N^2}\right) \right) + \dots$

TRPCM $E = 1 - \frac{\lambda N^2 - 1}{8 N^2} + \lambda^2 \left(-\frac{1}{256} \left(1 - \frac{1}{N^2}\right) \left(1 - \frac{3}{N^2}\right) + \frac{a(k, N)}{N^2} \right) + \dots$

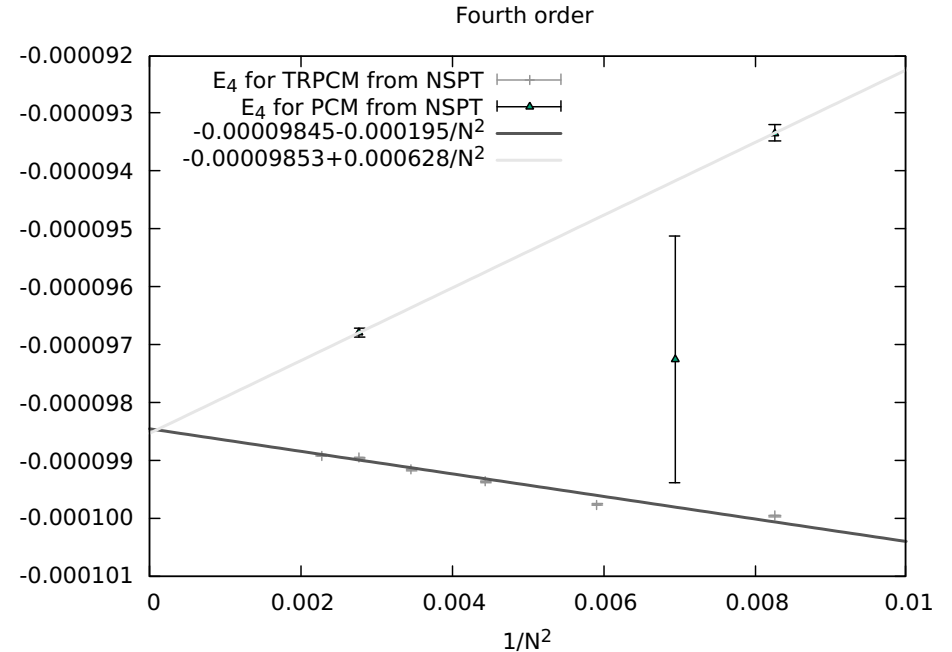
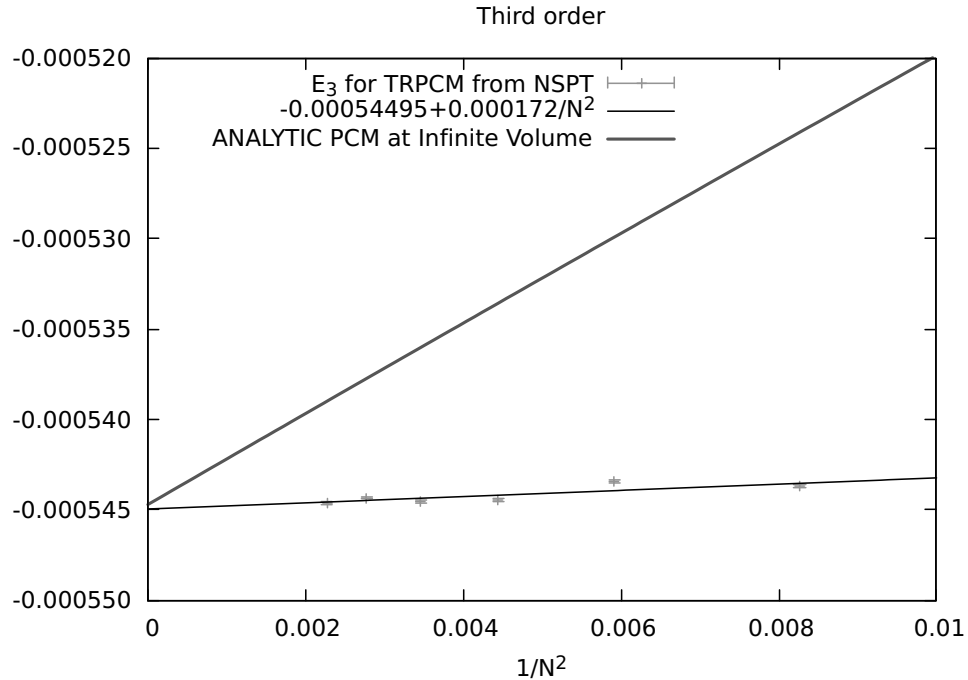
Non-planar graph contribution



Best large N limit approach (GA Chamizo 2016)



Higher order can be studied with NSPT



Rossi Vicari 1994 vs us


Yang-Mills theory in 4D: **TEK Model** (G-A, Okawa 1983)

A 4 matrix model

$$N = \hat{L}^2 \quad |n_{\mu\nu}| = k\hat{L}$$

Simplest test:

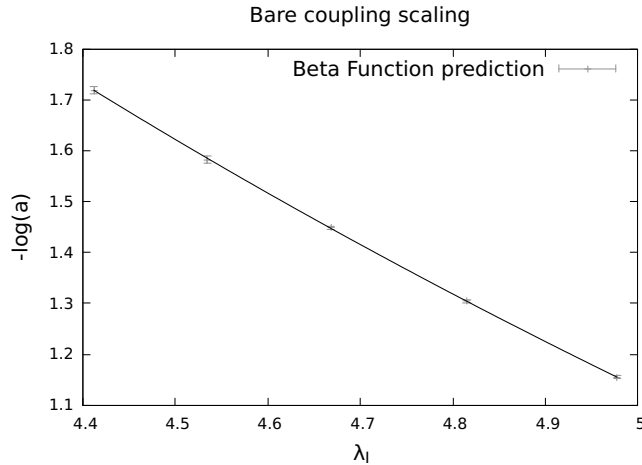
Plaquette
expectation value
at $1/\lambda_L=0.36$

- 
- Extrapolated N=8-16 0.55801(1)
 - Value at N=16(#dof=2 10^7) 0.56170(1)
 - TEK N=289 (#dof=0.8 10^5) 0.55800(1)

(G-A, Okawa 2014)

Also tested in PT both analytically (2017) and with NSPT(2019)

Continuum limit: 4d Yang-Mills theory at $N=\infty$



String Tension (PLB 2013) $\Lambda_{\overline{MS}}/\sqrt{\sigma}$

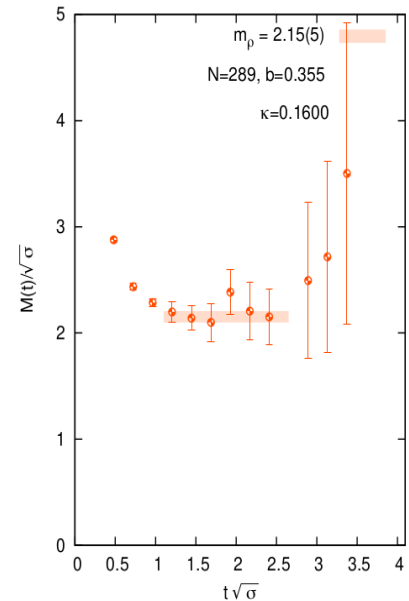
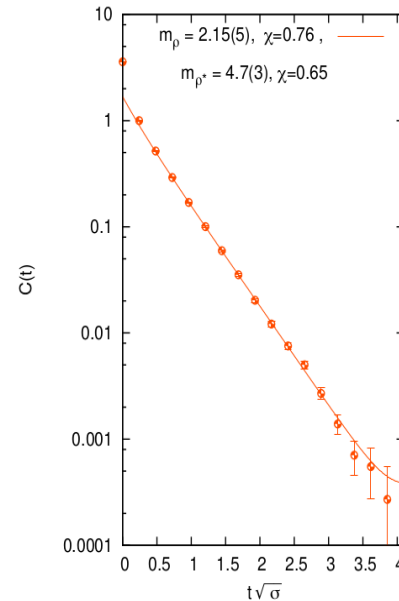
Extrapolated $N=3-8$ 0.525(2)

TEK $N=841$ 0.523(5)

Meson Masses

(Garcia Perez, GA, Okawa JHEP 2021)

$$m_\rho/\sqrt{\sigma} = 1.67(3) + 0.19(1)m_\pi^2/\sigma$$



A wide range of large N theories

- CP_{N-1} field theory
- $SU(\infty)$ Gauge Theory with N_f flavours of adjoint quarks (2013)
- $SU(\infty)$ Gauge Theory with quarks in 2-index representations
- $SU(\infty)$ Gauge Theory in the Veneziano limit
- N=1 SUSY Yang-Mills in 4D (JHEP 2022)
- N=4 SUSY Yang-Mills in 4D

DANGER!! : Trapped in The Matrix

Guido, I hope to enjoy
your friendship for
many more years