

From Lattice QCD to Phenomenology: Nonperturbative renormalization

**Symposium to celebrate
Guido Martinelli's 70th birthday**

Damir Becirevic, Rome 26/09/2022



Guido's main achievements in physics

**HEP
phenomenology**



**Lattice
QCD**

HEP phenomenology

Volume 99B, number 2

PHYSICS LETTERS

12 February 1981

WEAK NON-LEPTONIC DECAYS BEYOND LEADING LOGARITHMS IN QCD

G. ALTARELLI

*Istituto di Fisica dell'Università di Roma, Rome, Italy
and Istituto Nazionale di Fisica Nucleare, Rome, Italy*

G. CURCI

*Scuola Normale Superiore, Pisa, Italy
and Istituto Nazionale di Fisica Nucleare, Rome, Italy*

G. MARTINELLI

CERN, Geneva, Switzerland

and

S. PETRARCA

*Istituto di Fisica dell'Università di Roma, Rome, Italy
and Istituto Nazionale di Fisica Nucleare, Rome, Italy*

Received 17 October 1980

We compute the two-loop anomalous dimensions of the four-fermion operators relevant to weak non-leptonic decays and discuss the physical implications for strange and charm particle decays. In particular, we derive the complete order α_s corrections to the inclusive decay width of a heavy quark.

Nuclear Physics B187 (1981) 461–513
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QCD NON-LEADING CORRECTIONS TO WEAK DECAYS AS AN APPLICATION OF REGULARIZATION BY DIMENSIONAL REDUCTION

G. ALTARELLI

*Istituto di Fisica dell'Università di Roma, and
Istituto Nazionale di Fisica Nucleare, Roma, Italia*

G. CURCI

*Scuola Normale Superiore, Pisa, and
Istituto Nazionale di Fisica Nucleare, Pisa, Italia*

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HEP phenomenology

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Nuclear Physics B347 (1990) 491–536
North-Holland

LEADING AND NEXT-TO-LEADING QCD CORRECTIONS TO ϵ -PARAMETER AND B^0 - \bar{B}^0 MIXING IN THE PRESENCE OF A HEAVY TOP QUARK

Andrzej J. BURAS^{1,2}, Matthias JAMIN¹ and Peter H. WEISZ²

¹Physik Department, Technische Universität München, D-8046 Garching, FRG
²Max-Planck-Institut für Physik und Astrophysik–Werner-Heisenberg-Institut für Physik,
P.O. Box 40 12 12, D-8000 München, FRG

Received 22 March 1990

We present a complete calculation of leading and next-to-leading QCD corrections to the QCD factors η_{2K} and $\bar{\eta}_{2B}$, relevant for the CP-violating ϵ -parameter and B^0 - \bar{B}^0 mixing in the presence of a heavy top quark. We demonstrate explicitly that the resulting η_2 's are gauge as well as renormalization prescription independent. We also show that they do not depend on the infrared structure of the theory. We emphasize that these results only follow after correct factorization of short and long distance contributions, which has not been done properly in many recent QCD analyses present in the literature. We stress however that $\eta_2(x_1)$ with $x_1 = m_t^2/M_W^2$ depends sensitively on the definition of the top quark mass and only the product $\eta_2(x_1)S(x_1)$ ($S(x_1)$ being the function resulting from the lowest order box diagram) is free from this dependence. For $m_t = \bar{m}_t(M_W)$ the corresponding η_2 's decrease strongly with m_t , whereas for $m_t^* = \bar{m}_t(m_t)$ they show only a weak m_t^* -dependence: $0.58 \geq \eta_{2K}^* \geq 0.56$ and $0.88 \geq \bar{\eta}_{2B}^* \geq 0.84$ for $60 \text{ GeV} \leq m_t^* \leq 300 \text{ GeV}$ and $\Lambda_{\overline{MS}} = 200 \text{ MeV}$. A critical discussion of the existing literature on QCD calculations in the presence of a large m_t is presented.

Pre - Lattice QCD

- Hadronic matrix elements computed by means of quark models
- Hadronic matrix elements computed by means of QCDSRs
- Systematic uncertainties impossible to assess
- Precision determination of the CKM couplings and looking for New Physics impossible

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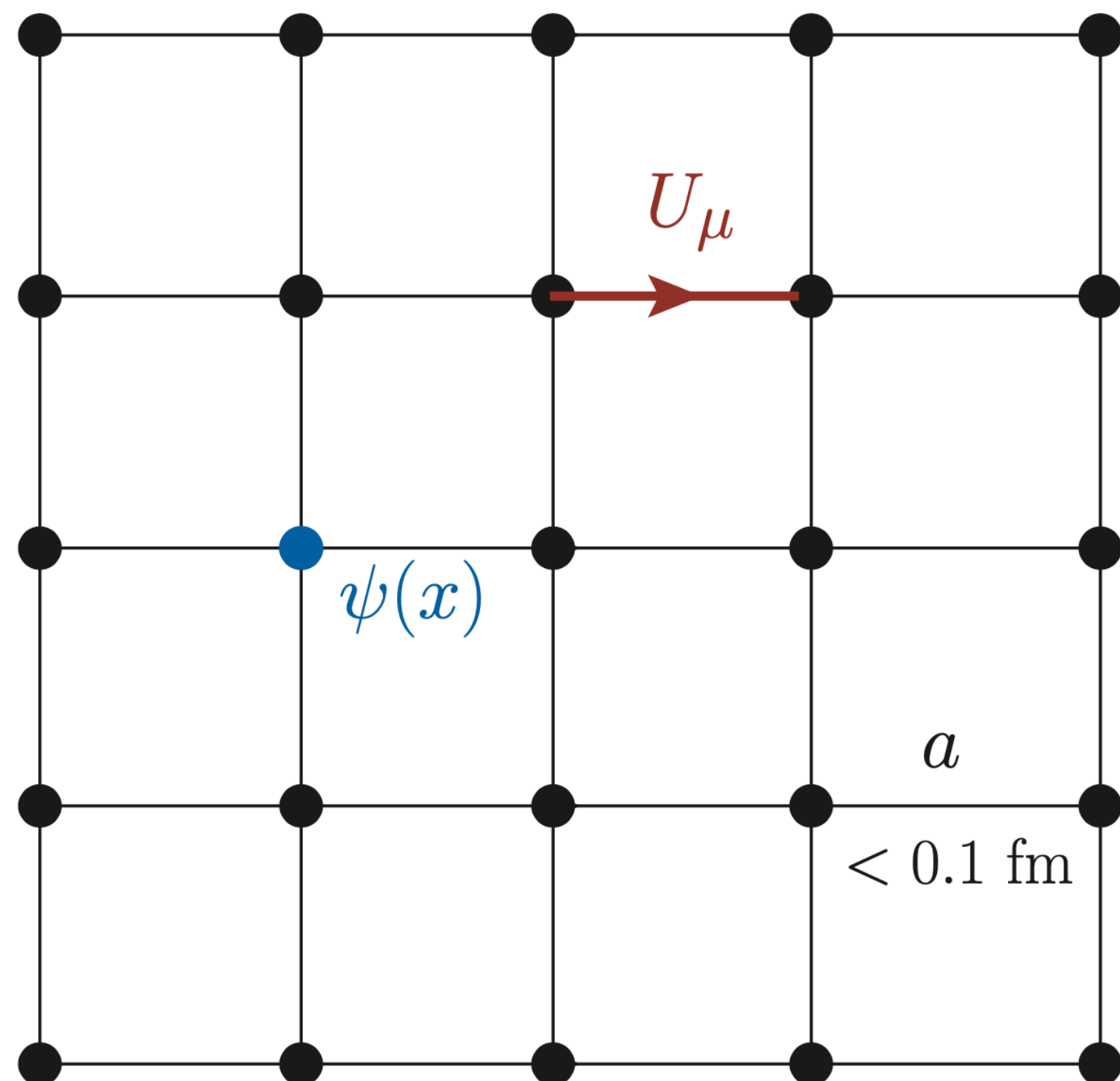
Confining potential and PGB dynamics at odds

Relation of parameters to QCD

*Do not trash non-lattice QCD methods though!
They are still the best tools we have when working
out the pheno of higher excited states, modern
spectroscopy...*

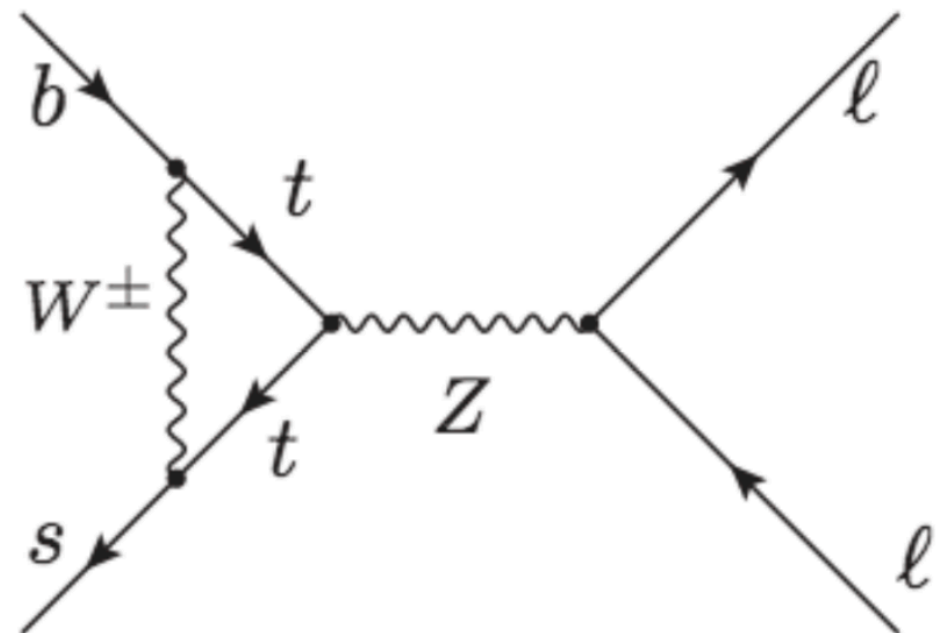
Lattice QCD

$$\mathcal{L}_{\text{QCD}}(x) = \frac{1}{2g^2} \text{tr} \{F_{\mu\nu}(x)F_{\mu\nu}(x)\} + \sum_{i=1}^{N_f} \bar{\psi}_i(x) (\not{D} + m_i) \psi_i(x)$$



- *Ab initio* means that no parameter apart from gauge coupling and quark masses is used to compute a hadronic quantity
- Discretize QCD on 4D-hypercubic lattice with PBC
- Snapshots of the QCD vacuum fluctuations through MC simulations
- Physics from Green functions — (Euclidean) → Correlation functions
- Lattice spacing is UV cutoff: regulator

Lattice QCD



$$if_P m_P = Z_A \langle 0 | A_0(0) | P \rangle$$

$P \in \{\pi, K, D, D_s, B, B_s, B_c\}$

Exp : $\mathcal{B}(B_s \rightarrow \mu\mu) = (2.85 \pm 0.33) \times 10^{-9}$
 12%

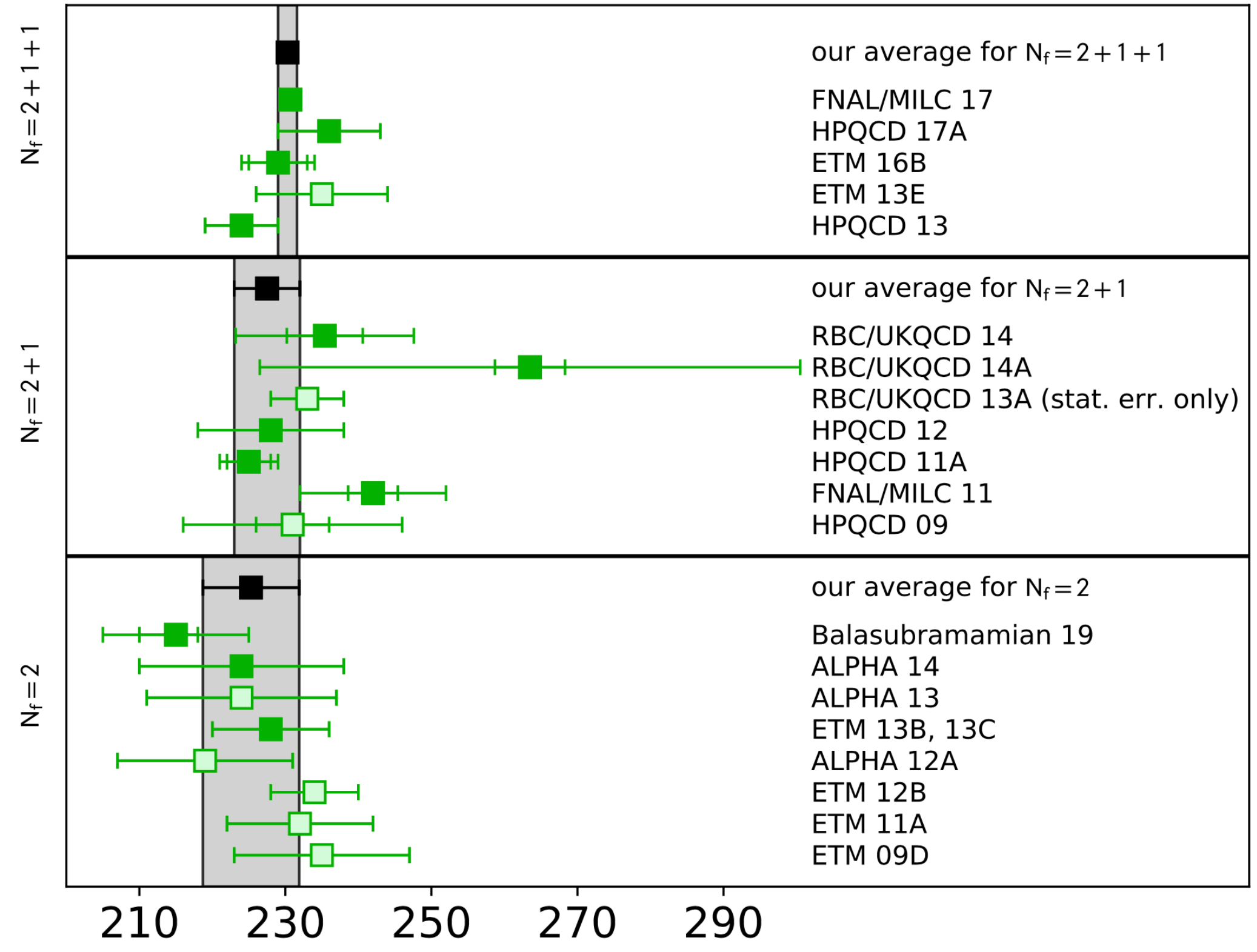
SM : $\mathcal{B}(B_s \rightarrow \mu\mu) = (3.66 \pm 0.14) \times 10^{-9}$
 4%

$$O = \frac{1}{\Lambda^2} C_{ij} \bar{Q}_i \gamma^\mu Q_j H^\dagger D_\mu H$$

C_{ij}	1	$V_{ti} V_{tj}^*$
$B_s \rightarrow \mu^+ \mu^-$	$> 10 \text{ TeV}$	$> 2.5 \text{ TeV}$
$K \rightarrow \pi \nu \bar{\nu}$	$> 100 \text{ TeV}$	$> 1.8 \text{ TeV}$

FLAG2021

f_{B_s} [MeV]



Lattice QCD

$$\underbrace{if_P m_P}_{\text{cont}} = Z_A \underbrace{\langle 0 | A_0(0) | P \rangle}_{\text{lattice}}$$

$$A_\mu = \bar{q} \gamma_\mu \gamma_5 q'$$

$$P \in \{\pi, K, D, D_s, B, B_s, B_c\}$$

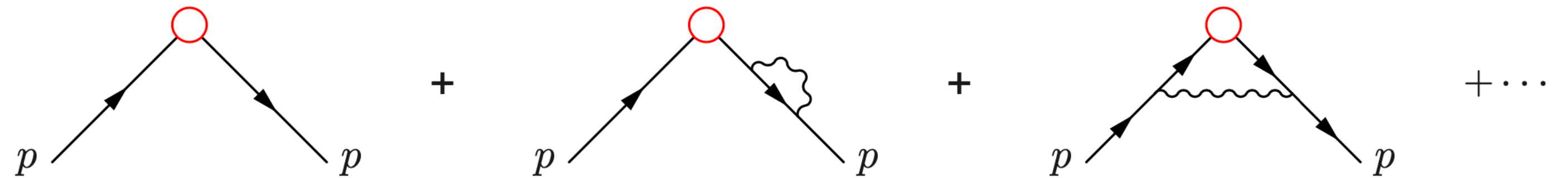
Lattice QCD

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- Wilson regulator breaks ChS
- Z_A encodes the physics of $E > 1/a$
- Can be computed perturbatively

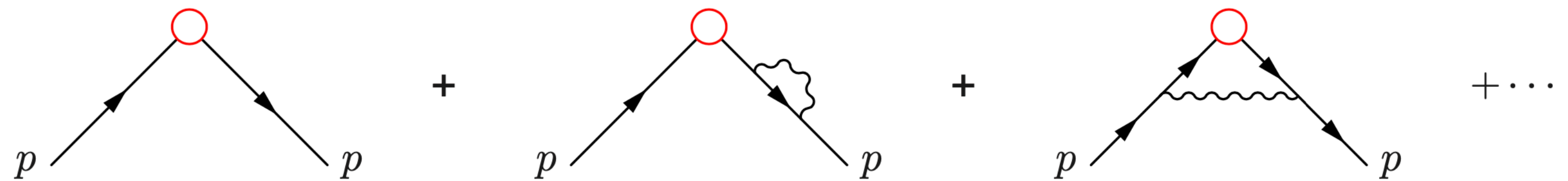


Lattice QCD

$$\underbrace{if_P m_P}_{\text{cont}} = Z_A \underbrace{\langle 0 | A_0(0) | P \rangle}_{\text{lattice}}$$

$$A_\mu = \bar{q} \gamma_\mu \gamma_5 q'$$

$$P \in \{\pi, K, D, D_s, B, B_s, B_c\}$$



$$Z_O(a\mu, g_0^2(a)) = 1 + C_1(a\mu)g_0^2(a) + C_2(a\mu)g_0^4(a) + \dots$$



Physics Letters B
Volume 123, Issue 6, 14 April 1983, Pages 433-436



The connection between local operators on the lattice and in the continuum and its relation to meson decay constants

G. Martinelli, Zhang Yi-Cheng ^{a, b}

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Abstract

We have computed, at first order in perturbation theory, the relation between lattice and continuum local operators used in Monte Carlo simulations to evaluate the meson decay constants. These corrections, although with the correct sign, are too small to compensate the discrepancy between existing lattice calculations and experimental values.

Lattice QCD

$$\underbrace{if_P m_P}_{\text{cont}} = Z_A \underbrace{\langle 0 | A_0(0) | P \rangle}_{\text{lattice}}$$

$$A_\mu = \bar{q} \gamma_\mu \gamma_5 q'$$

$$P \in \{\pi, K, D, D_s, B, B_s, B_c\}$$

$$Z_O(a\mu, g_0^2(a)) = 1 + C_1(a\mu)g_0^2(a) + C_2(a\mu)g_0^4(a) + \dots$$

- which coupling to use (boosting)
- tadpoles are large
- need a nonperturbative determination



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Nonperturbative renormalization

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CHIRAL SYMMETRY ON THE LATTICE WITH WILSON FERMIONS*

Marco BOCHICCHIO and Luciano MAIANI

Dipartimento di Fisica, Università “La Sapienza”, Roma INFN-Sezione di Roma, Italy

Guido MARTINELLI

Laboratori Nazionali dell’INFN, Frascati, Italy

Giancarlo ROSSI

Dipartimento di Fisica, Università di Tor Vergata, Roma INFN-Sezione di Roma, Italy

Massimo TESTA

INFN-Sezione di Roma, Italy

Received 18 April 1985
(Revised 13 June 1985)

The chiral properties of the continuum limit of lattice QCD with Wilson fermions are studied. We show that a partially conserved axial current can be defined, satisfying the usual current algebra requirements.

A proper definition of the chiral symmetry order parameter, $\langle 0 | \bar{\psi} \psi | 0 \rangle$, is given, and the chiral properties of composite operators are investigated. The implications of our analysis to the lattice determination of non-leptonic weak amplitudes are also discussed.

- There are no conserved chiral currents with Wilson quarks
- Chiral symmetry is to be restored in the continuum
- Use continuum chiral Ward identities and impose them as renormalization conditions at fixed lattice spacing: renormalized theory has the proper chiral symmetry

Nonperturbative renormalization

$$2\rho = \frac{\sum_{\vec{x}} \nabla_0 \langle A_0(x) P(0) \rangle}{\sum_{\vec{x}} \langle P(x) P(0) \rangle}$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) V_0(y) A_0(0) \rangle = \frac{1}{Z_V} \sum_{\vec{y}} \langle A_0(y) A_0(0) \rangle$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) V_k(y) A_k(0) \rangle = -\frac{Z_V}{Z_A^2} \sum_{\vec{y}} \langle V_k(y) V_k(0) \rangle + \frac{1}{Z_V} \sum_{\vec{y}} \langle A_k(y) A_k(0) \rangle$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) S(y) P(0) \rangle = \frac{Z_P}{Z_A Z_S} \sum_{\vec{y}} \langle P(y) P(0) \rangle + \frac{Z_S}{Z_A Z_P} \sum_{\vec{y}} \langle S(y) S(0) \rangle$$

Nonperturbative renormalization

$$2\rho = \frac{\sum_{\vec{x}} \nabla_0 \langle A_0(x) P(0) \rangle}{\sum_{\vec{x}} \langle P(x) P(0) \rangle}$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) V_0(y) A_0(0) \rangle = \frac{1}{Z_V} \sum_{\vec{y}} \langle A_0(y) A_0(0) \rangle$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) V_k(y) A_k(0) \rangle = -\frac{Z_V}{Z_A^2} \sum_{\vec{y}} \langle V_k(y) V_k(0) \rangle + \frac{1}{Z_V} \sum_{\vec{y}} \langle A_k(y) A_k(0) \rangle$$

$$2\rho \sum_x \sum_{\vec{y}} \langle P(x) S(y) P(0) \rangle = \frac{Z_P}{Z_A Z_S} \sum_{\vec{y}} \langle P(y) P(0) \rangle + \frac{Z_S}{Z_A Z_P} \sum_{\vec{y}} \langle S(y) S(0) \rangle$$

- $Z_V, Z_A, Z_P/Z_S$ at fixed a
- Fully nonperturbatively
- Gauge independent
- What about the bilinear quark operators with non-zero anomalous dimension?
- What about the four-quark operators?

Nonperturbative renormalization



ELSEVIER

Nuclear Physics B 445 (1995) 81–105

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PHYSICS B

A general method for non-perturbative renormalization of lattice operators

G. Martinelli^{a,b}, C. Pittori^c, C.T. Sachrajda^d, M. Testa^a, A. Vladikas^e

^a *Dipartimento di Fisica, Università degli Studi di Roma "La Sapienza" and INFN, Sezione di Roma, P. le A. Moro 2, 00185 Rome, Italy*

^b *Theory Division, CERN, 1211 Geneva 23, Switzerland*

^c *LPTHE, Université de Paris Sud, Centre d'Orsay, 91405 Orsay, France*

^d *Department of Physics, University of Southampton, Southampton SO17 1BJ, UK*

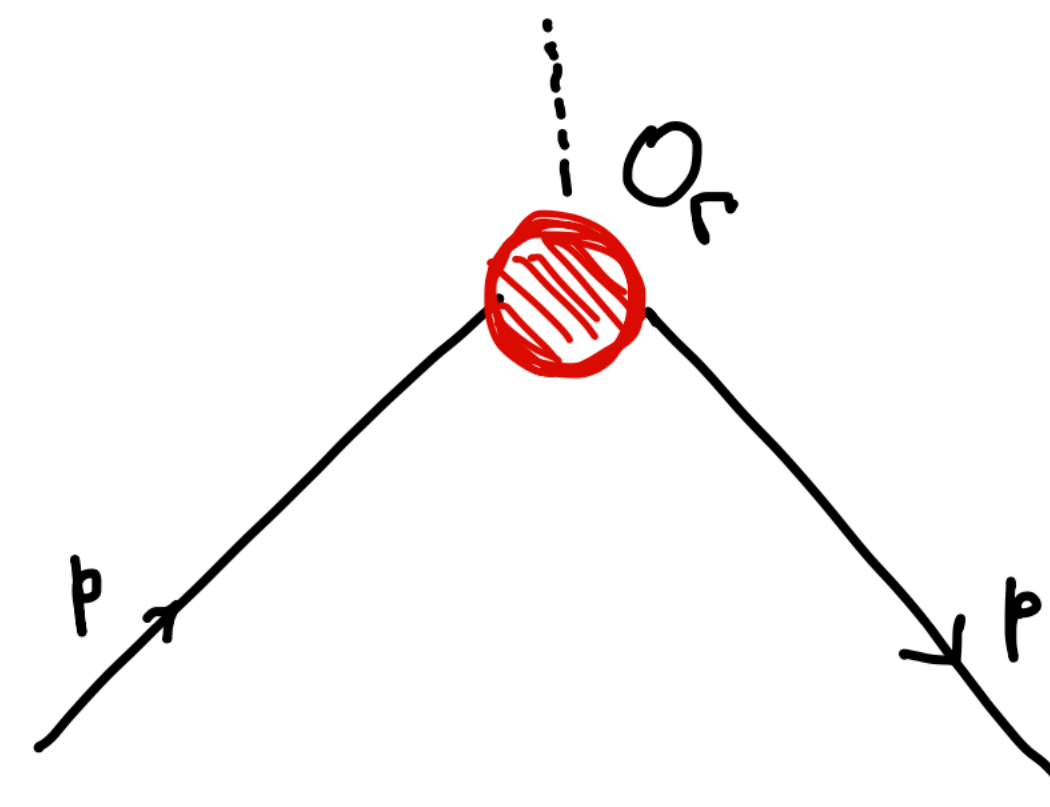
^e *Dipartimento di Fisica, Università di Roma "Tor Vergata", and INFN, Sezione di Roma II, Via della Ricerca Scientifica 1, 00133 Rome, Italy*

Received 14 November 1994; revised 7 March 1995; accepted 13 March 1995

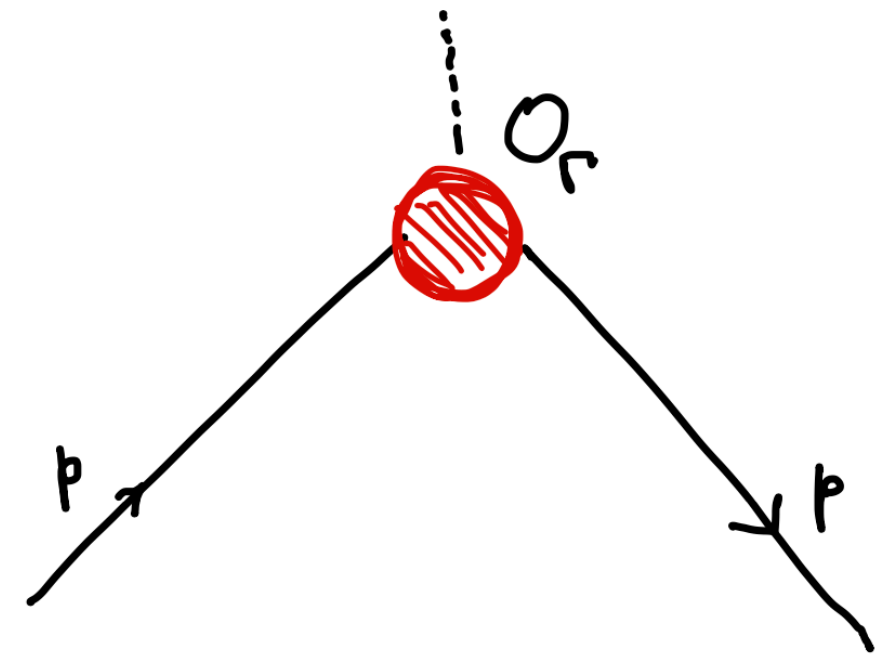
Abstract

We propose a non-perturbative method for computing the renormalization constants of generic composite operators. This method is intended to reduce some systematic errors, which are present when one tries to obtain physical predictions from the matrix elements of lattice operators. We also present the results of a calculation of the renormalization constants of several two-fermion operators, obtained, with our method, by numerical simulation of QCD, on a $16^3 \times 32$ lattice, at $\beta = 6.0$. The results of this simulation are encouraging, and further applications to four-fermion operators and to the heavy quark effective theory are proposed.

- RI-MOM
- Correlator of renormalized operators do not depend on regularization up to cutoff effects: RI
- Momentum subtraction scheme, just like in continuum: MOM



Nonperturbative renormalization



- Fixed gauge (Landau)

$$\widehat{S}(p) = \int d^4x e^{-ip \cdot x} \langle \widehat{q}(x) \widehat{q}(0) \rangle$$

$$G_\Gamma(p, p') = \int d^4x d^4y e^{-ip \cdot x + ip' \cdot y} \langle \widehat{q}(x) O_\Gamma(0) \widehat{q}(y) \rangle$$

$$\Lambda_\Gamma(p) = \widehat{S}(p)^{-1} G_\Gamma(p, p) \widehat{S}(p)^{-1}$$

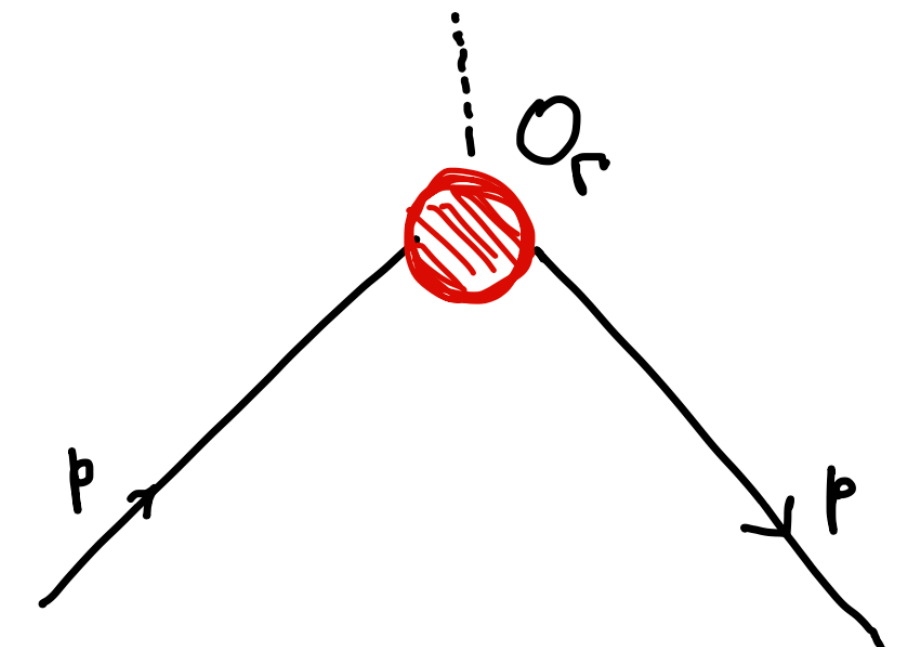
$$\frac{i}{12} \text{Tr} \left[\frac{\not{p} \widehat{S}(p)^{-1}}{p^2} \right]_{p^2=\mu^2} = Z_q \frac{i}{12} \text{Tr} \left[\frac{\not{p} S(p)^{-1}}{p^2} \right]_{p^2=\mu^2} = 1$$

$$Z_\Gamma \Gamma_\Gamma(p) \Big|_{p^2=\mu^2} \equiv Z_\Gamma \text{Tr} [\Lambda_\Gamma(p) P_\Gamma] \Big|_{p^2=\mu^2} = 1$$

$$\text{Tr} [\Gamma P_\Gamma] = 1$$

$$\Lambda_{\text{QCD}} \ll \mu \ll \pi/a$$

Nonperturbative renormalization



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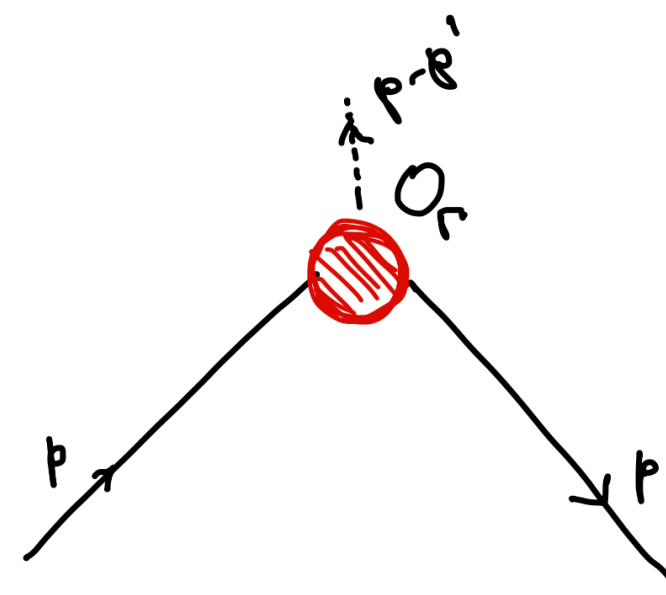
$$Z_\Gamma \Gamma_\Gamma(p) \Big|_{p^2=\mu^2} \equiv Z_\Gamma \text{Tr} [\Lambda_\Gamma(p) P_\Gamma] \Big|_{p^2=\mu^2} = 1$$

- IR problems appear when choosing $\Gamma = \gamma_5$: pion with $q^2=0$
- Either fit it away by working with several light quark masses or use mass non-degenerate vertices

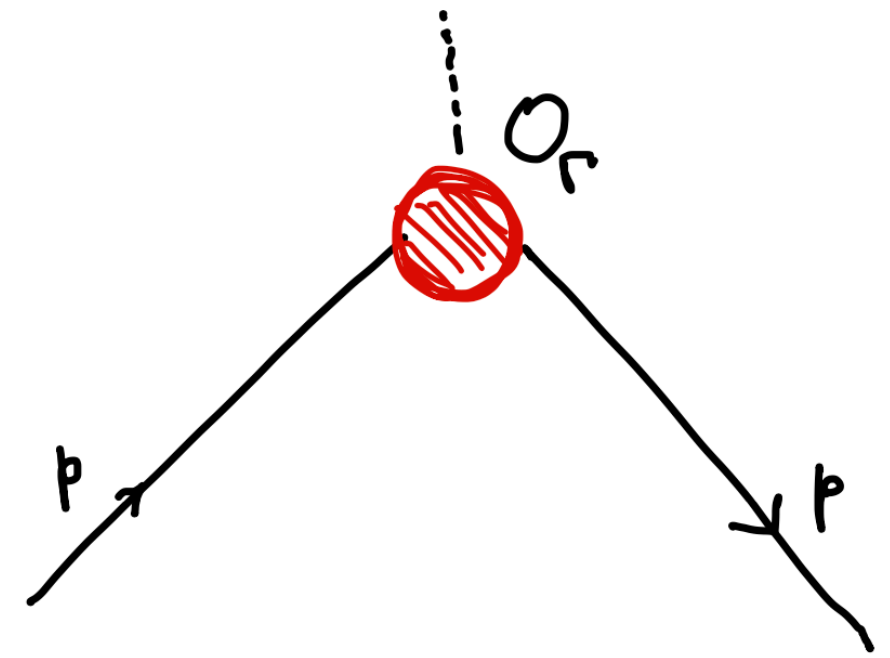
$$\Gamma_P(p^2, m) = A(p^2) + \frac{B(p^2)}{m} + C(p^2) m$$

$$\Gamma_P^{\text{SUB}}(p^2, m_1, m_2) = \frac{m_1 \Gamma_P(p^2, m_1) - m_2 \Gamma_P(p^2, m_2)}{m_1 - m_2}$$

- Or define RI-SMOM $p^2 = p'^2 = (p-p')^2 = \mu^2$



Nonperturbative renormalization



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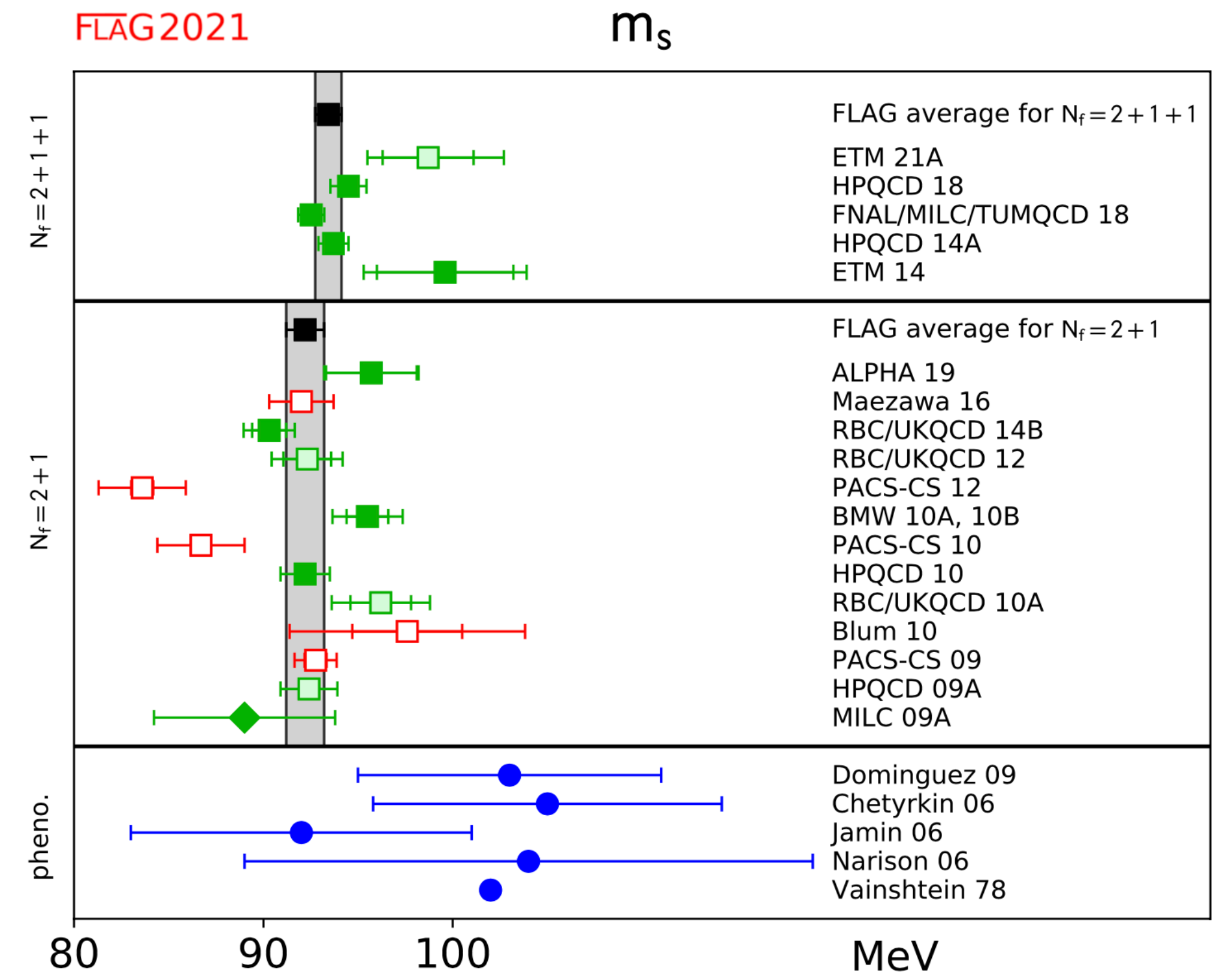
$$Z_\Gamma \Gamma_\Gamma(p) \Big|_{p^2=\mu^2} \equiv Z_\Gamma \text{Tr} [\Lambda_\Gamma(p) P_\Gamma] \Big|_{p^2=\mu^2} = 1$$

- Matching to MS scheme is done perturbatively (many people worked on this)
- We can therefore renormalize quark mass via $Z_S(\mu)$ and/or $Z_P(\mu)$
- This was a huge progress in particle physics thanks to LQCD

$$\text{Tr} [\Gamma P_\Gamma] = 1$$

Nonperturbative renormalization

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$$G_i(x_1, x_2, x_3, x_4) = \langle \widehat{q}_1(x_1) \widehat{q}_2(x_2) \mathcal{O}_i(0) \widehat{q}_3(x_3) \widehat{q}_4(x_4) \rangle$$

$$\Lambda_i(p)_{\alpha' \beta' \gamma' \delta'}^{a' b' c' d'} = \widehat{S}^{-1}(p)_{\alpha' \alpha}^{a' a} \widehat{S}^{-1}(p)_{\gamma' \gamma}^{c' c} G_i(p)_{\alpha \beta \gamma \delta}^{abcd} \widehat{S}^{-1}(p)_{\beta \beta'}^{bb'} \widehat{S}^{-1}(p)_{\delta \delta'}^{dd'}$$

$$Z_{ij}^{\pm} \Gamma_{jk}^{\pm}(p) \Big|_{p^2=\mu^2} \equiv Z_{ij}^{\pm} \text{Tr} \{ \Lambda_j^{\pm}(p) P_k^{\pm} \} \Big|_{p^2=\mu^2} = \delta_{ik}$$

$$Z_{il}^{\pm} (I + \Delta^{\pm})_{lj} \Gamma_{jk}^{\pm}(p) \Big|_{p^2=\mu^2} \equiv Z_{il}^{\pm} (I + \Delta^{\pm})_{lj} \text{Tr} \{ \Lambda_j^{\pm}(p) P_k^{\pm} \} \Big|_{p^2=\mu^2} = \delta_{ik}$$

$$\text{Tr} \{ \Lambda_i^{\pm(0)} P_k^{\pm} \} = \delta_{ik}$$

- Easily extended to four-quark operators, except that the procedure becomes extra-involved due to mixing of all d=6 PC operators.
- Good news: method allows us to compute subtraction constants too!



K^0 – \bar{K}^0 mixing with Wilson fermions without subtractions

D. Becirevic ^{a,✉}, Ph. Boucaud ^b, V. Giménez ^c, V. Lubicz ^d, G. Martinelli ^{a, c}, J. Micheli ^b, M. Papinutto ^{b, f}[Show more](#) [+ Add to Mendeley](#) [Share](#) [Cite](#)[https://doi.org/10.1016/S0370-2693\(00\)00797-8](https://doi.org/10.1016/S0370-2693(00)00797-8)[Get rights and content](#)

Abstract

By using suitable Ward identities, we show that it is possible to compute K^0 – \bar{K}^0 mixing without subtracting the terms generated by explicit chiral symmetry breaking present in Wilson-like lattice actions. The accuracy in the determination of the amplitudes is of $O(a)$, which is the best one attainable in the absence of improvement.

Nonperturbative renormalization

$$G_i(x_1, x_2, x_3, x_4) = \langle \hat{q}_1(x_1) \hat{q}_2(x_2) \mathcal{O}_i(0) \hat{q}_3(x_3) \hat{q}_4(x_4) \rangle$$

$$\Lambda_i(p)_{\alpha'\beta'\gamma'\delta'}^{a'b'c'd'} = \hat{S}^{-1}(p)_{\alpha'a}^{a'a} \hat{S}^{-1}(p)_{\gamma'\gamma}^{c'c} G_i(p)_{\alpha\beta\gamma\delta}^{abcd} \hat{S}^{-1}(p)_{\beta\beta'}^{bb'} \hat{S}^{-1}(p)_{\delta\delta'}^{dd'}$$

$$Z_{ij}^{\pm} \Gamma_{jk}^{\pm}(p) \Big|_{p^2=\mu^2} \equiv Z_{ij}^{\pm} \text{Tr} \{ \Lambda_j^{\pm}(p) P_k^{\pm} \} \Big|_{p^2=\mu^2} = \delta_{ik}$$

$$Z_{il}^{\pm} (I + \Delta^{\pm})_{lj} \Gamma_{jk}^{\pm}(p) \Big|_{p^2=\mu^2} \equiv Z_{il}^{\pm} (I + \Delta^{\pm})_{lj} \text{Tr} \{ \Lambda_j^{\pm}(p) P_k^{\pm} \} \Big|_{p^2=\mu^2} = \delta_{ik}$$

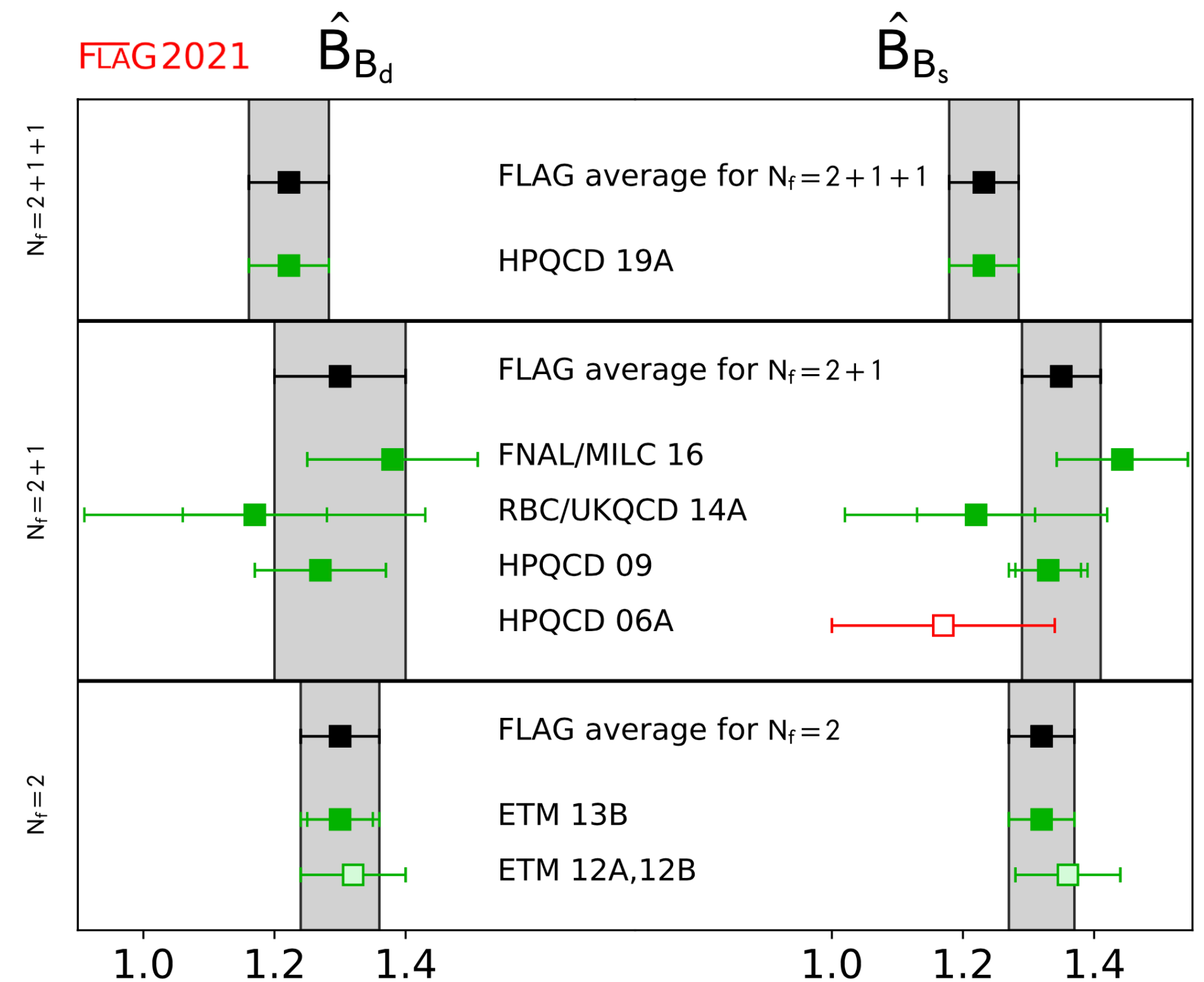
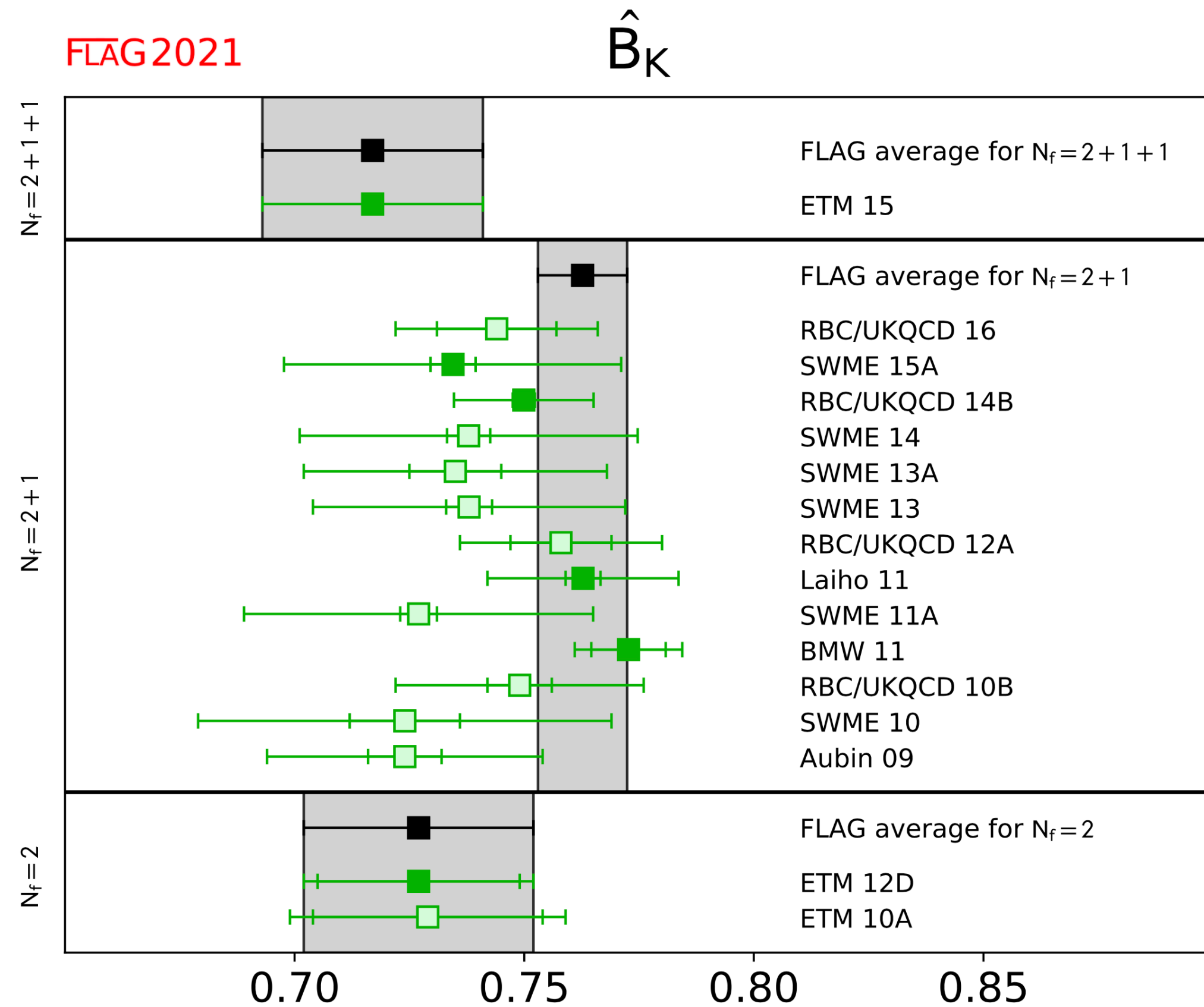
$$\text{Tr} \{ \Lambda_i^{\pm(0)} P_k^{\pm} \} = \delta_{ik}$$

- Avoid subtractions by using Ward identity: compute matrix element of PV operator (with axial variation) to get the desired result
- Using maximally twisted QCD avoids subtractions altogether. The overall renormalization constant should still be computed [RI-(S)MOM]

Nonperturbative renormalization

- We can therefore renormalize four-quark $\Delta F=2$ operators via $Z_{O_i}(\mu)$
- RGI is done perturbatively

$$B_K(\mu) = \frac{\langle \bar{K}^0 | Q_R^{\Delta S=2}(\mu) | K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$



Working with Guido...



- Guido **respects students** - discussions stop when a student knocks on the door and wants to ask a question... *Professor is Present*
- Guido has **a rare talent to inspire people and share his contagious passion for physics**: young collaborators are convinced that they are doing something of great importance for science
- Guido is bursting with ideas... **motivating** factor **for his environment**
- Guido has a pronounced **critical attitude** and loves challenging other people's ideas
- Guido spontaneously attracts **collaborators** and **makes them be/give their best**
- Guido is a leader, but also a good friend, generous and caring...

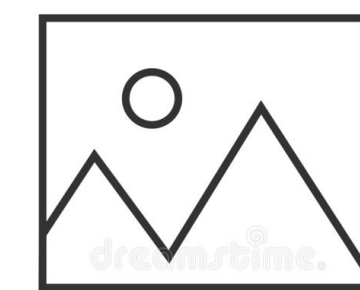
(ex)Young (ex)students,
(ex)postdocs...

Working with Guido

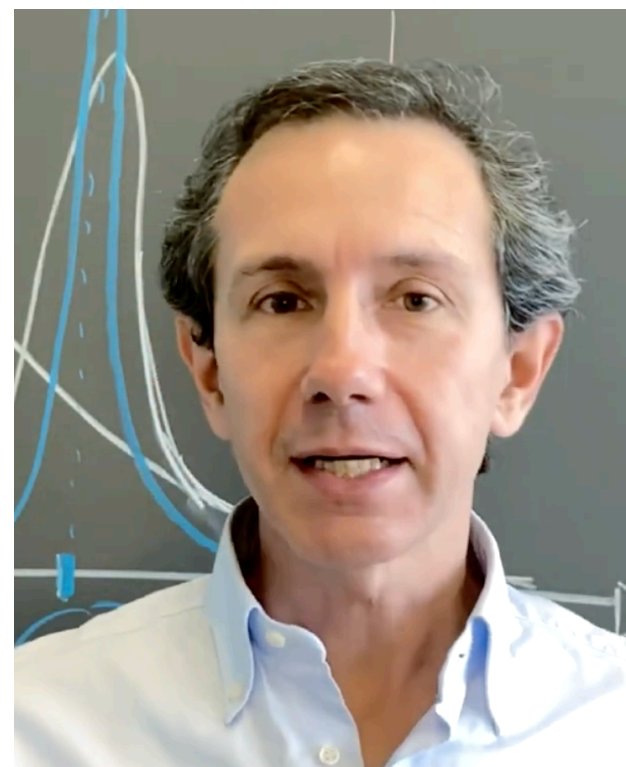
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(not shown here)...



Gimenez



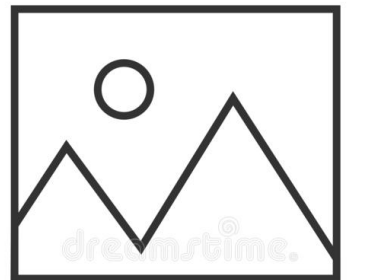
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Collaborators...

Working with Guido

and many more
(not shown here)...



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G. Curci e S. Petrarca

Work in progress...



- Guido is a larger than life person who happens to be a brilliant physicist, but also a caring person - who loves/enjoys life, passionate about art and history, about travels and who is dedicated to his family, his Giovanna/Lorenzo, his Nanie...
- Retired? Not really ;)

Work in progress...



•BUON COMPLEANNO