36 Years of Lattice QCD with Guido

From 15 Quenched Configurations to Precision Flavour Physics

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Electroweak, Strong and New Interactions - Guido@70
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Rome, September 26th 2022

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A Lattice Calculation of the Second Moment of the Pion's Distribution Amplitude

G. Martinelli (CERN), Christopher T. Sachrajda (CERN) (Jan, 1987)

Published in: *Phys.Lett.B* 190 (1987) 151-156

DOI cite 84 citatio

Pion Structure Functions From Lattice {QCD}

G. Martinelli (CERN), Christopher T. Sachrajda (CERN) (Jun, 1987)

Published in: *Phys.Lett.B* 196 (1987) 184-190

DOI cite 87 citatio

The Kaon B Parameter and K- pi and K- pi pi Transition Amplitudes on the Lattice

M.B. Gavela (Madrid, Autonoma U.), L. Maiani (Rome U. and INFN, Rome), S. Petrarca (Rome U. and INFN, Rome), F. Rapuano (Rome U. and INFN, Rome), G. Martinelli (CERN) et al. (Nov, 1987)

Published in: *Nucl.Phys.B* 306 (1988) 677

DOI cite 140 citatio

A Lattice Calculation of the Pion's Form-Factor and Structure Function

G. Martinelli (CERN), Christopher T. Sachrajda (Southampton U.) (Nov, 1987)

Published in: *Nucl.Phys.B* 306 (1988) 865-889

DOI cite 161 citatio

A Lattice Study of Nucleon Structure

G. Martinelli (CERN), Christopher T. Sachrajda (Southampton U.) (May, 1988)

Published in: *Nucl.Phys.B* 316 (1989) 355-372

DOI cite 164 citatio

A Lattice Computation of Proton Decay Amplitudes

M.B. Gavela (Madrid, Autonoma U.), S.F. King (Southampton U.), Christopher T. Sachrajda (Southampton U.), G.

Virtual photon emission in leptonic decays of charged pseudoscalar mesons

#1

G. Gagliardi (INFN, Rome3), V. Lubicz (Rome III U. and INFN, Rome3), G. Martinelli (Rome U. and INFN, Rome), F. Mazzetti (Rome III U. and INFN, Rome3), C.T. Sachrajda (Southampton U.) et al. (Feb 8, 2022)

Published in: *Phys.Rev.D* 105 (2022) 11, 114507 • e-Print: 2202.03833 [hep-lat]

pdf DOI cite 4 citations

Comparison of lattice QCD+QED predictions for radiative leptonic decays of light mesons with experimental data

#2

R. Frezzotti (Rome U., Tor Vergata and INFN, Rome2), M. Garofalo (Rome III U. and INFN, Rome3 and Bonn U., HISKP), V. Lubicz (Rome III U. and INFN, Rome3), G. Martinelli (Rome U. and INFN, Rome), C.T. Sachrajda (Southampton U.) et al. (Dec 3, 2020)

Published in: *Phys.Rev.D* 103 (2021) 5, 053005 • e-Print: 2012.02120 [hep-ph]

pdf DOI cite 15 citations

First lattice calculation of radiative leptonic decay rates of pseudoscalar mesons

#3

A. Desiderio (Rome U., Tor Vergata and INFN, Rome), R. Frezzotti (Rome U., Tor Vergata and INFN, Rome), M. Garofalo (Rome III U. and INFN, Rome3), D. Giusti (Regensburg U. and INFN, Rome3), M. Hansen (U. Southern Denmark, Odense, DIAS) et al. (Jun 9, 2020)

Published in: *Phys.Rev.D* 103 (2021) 1, 014502 • e-Print: 2006.05358 [hep-lat]

pdf DOI cite 27 citations

Light-meson leptonic decay rates in lattice QCD+QED

#4

M. Di Carlo (INFN, Rome and Rome U.), D. Giusti (INFN, Rome3 and Rome III U.), V. Lubicz (INFN, Rome3 and Rome III U.), G. Martinelli (INFN, Rome and Rome U.), C.T. Sachrajda (Southampton U.) et al. (Apr 18, 2019)

Published in: *Phys.Rev.D* 100 (2019) 3, 034514 • e-Print: 1904.08731 [hep-lat]

pdf DOI cite 66 citations

First lattice calculation of the QED corrections to leptonic decay rates

#5

D. Giusti (INFN, Rome3 and Rome III U.), V. Lubicz (INFN, Rome3 and Rome III U.), G. Martinelli (INFN, Rome and Rome U.), C.T. Sachrajda (Southampton U.), F. Sanfilippo (INFN, Rome3) et al. (Nov 17, 2017)

Published in: *Phys.Rev.Lett.* 120 (2018) 7, 072001 • e-Print: 1711.06537 [hep-lat]

pdf DOI cite 72 citations

Earliest GM & CTS Publications

Latest GM & CTS Publications

Selected Highlights

1. Early computations of hadronic matrix elements

The early (1986-1990) computations of matrix elements in which we had to figure out the basic procedures; understand the limitations and exploit the constraints of the cubic symmetry and perform lattice perturbation theory for the renormalisation constants.

2. Renormalons and non-perturbative subtraction of UV power divergences.

(i) *Non-perturbative subtractions in the Heavy Quark Effective Theory*

L.Maiani, G.Martinelli and CTS, Nucl. Phys. B368 (1992) 281

(ii) *On the difficulty of computing higher-twist corrections*

G.Martinelli and CTS, Nucl. Phys. B478 (1996) 660

3. *A Lattice computation of the decay constant of the B-meson.*

C.R.Allton, V.Lubicz, L.Maiani, G.Martinelli and CTS,

Nucl. Phys. B349, (1991) 598

3. Improvement

The Improvement of hadronic matrix elements in lattice QCD

G.Heatlie, G.Martinelli, C.Pittori, G.C.Rossi and CTS.
Nucl.Phys. B352 (1991) 266

4. Non-perturbative renormalization

A general method for the non-perturbative renormlization of lattice operators

G.Martinelli, C.Pittori, CTS, M.Testa and A.Vladikas,
Nucl. Phys. B445 (1995) 81

5. Non-leptonic kaon decays and finite-volume corrections

(i) *New lattice approaches to the* $\Delta I = 1/2$ *rule*

C.Dawson, G.Martinelli, G.C.Rossi, CTS, S.Sharpe, M.Talevi and M.Testa, Nucl. Phys. B514 (1998) 313

(ii) $K \to \pi\pi$ *decays in a finite volume*

C.J.D.Lin, G.Martinelli, CTS and M.Testa, Nucl. Phys. B619 (2001) 467

(iii) Effects of finite volume on the $K_L - K_S$ *mass difference*

N.H.Christ, X.Feng, G.Martinelli and CTS, Phys. Rev. D91 (2015) 114510

"At the workshop - Guido Martinelli (left) and Chris Sachrajda contemplate power subtractions for non-leptonic kaon decays."

(Cern Courier - Reporting on the 2000 Ringberg Workshop on Current Theoretical Problems in Lattice Field Theory.)

QED corrections to Weak Decay Amplitudes - Motivation

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
m_{ud}[MeV]	3.1.4	3.410(43)	[6, 7]	3.381(40)	[8–12]		
m_s[MeV]	3.1.4	93.40(57)	[6, 7, 13, 14]	92.2(1.0)	[8–11, 15]		
m_s/m_{ud}	3.1.5	27.23(10)	[7, 16, 17]	27.42(12)	[8–10, 15, 18]		
m_u[MeV]	3.1.6	2.14(8)	[6, 19]	2.27(9)	[20]		
m_d[MeV]	3.1.6	4.70(5)	[6, 19]	4.67(9)	[20]		
m_u/m_d	3.1.6	0.465(24)	[19, 21]	0.485(19)	[20]		
$\overline{m}_c$(3 GeV)[GeV]	3.2.2	0.988(11)	[6, 7, 14, 22, 23]	0.992(5)	[11, 24–26]		
m_c/m_s	3.2.3	11.768(34)	[6, 7, 14]	11.82(16)	[24, 27]		
$\overline{m}_b(\overline{m}_b)$[GeV]	3.3	4.203(11)	[6, 28–31]	4.171(20)	[11]		
$f_+(0)$	4.3	0.9698(17)	[32, 33]	0.9677(27)	[34, 35]	0.9560(57)(62)	[36]
$f_{K^\pm}/f_{\pi^\pm}$	4.3	1.1932(21)	[16, 37–39]	1.1917(37)	[8, 40–44]	1.205(18)	[45]
$f_{\pi^\pm}$[MeV]	4.6			130.2(8)	[8, 40, 41]		
$f_{K^\pm}$[MeV]	4.6	155.7(3)	[17, 37, 38]	155.7(7)	[8, 40, 41]	157.5(2.4)	[45]
Re(A_2)[GeV]	6.2			$1.50(4)(14)\times 10^{-8}$	[46]		
Im(A_2)[GeV]	6.2			$-8.34(1.03)\times 10^{-13}$	[46]		
$\hat{B}_K$	6.3	0.717(18)(16)	[47]	0.7625(97)	[8, 48–50]	0.727(22)(12)	[51]
B_2	6.4	0.46(1)(3)	[47]	0.502(14)	[50, 52]	0.47(2)(1)	[51]
B_3	6.4	0.79(2)(5)	[47]	0.766(32)	[50, 52]	0.78(4)(2)	[51]
B_4	6.4	0.78(2)(4)	[47]	0.926(19)	[50, 52]	0.76(2)(2)	[51]
B_5	6.4	0.49(3)(3)	[47]	0.720(38)	[50, 52]	0.58(2)(2)	[51]

Table 1: Summary of the main results of this review concerning quark masses, light-meson decay constants, and hadronic kaon-decay and kaon-mixing parameters. These are grouped in terms of N_f, the number of dynamical quark flavours in lattice simulations. Quark masses are given in the $\overline{\rm MS}$ scheme at running scale $\mu = 2\,$GeV or as indicated. BSM bag parameters $B_{2,3,4,5}$ are given in the $\overline{\rm MS}$ scheme at scale $\mu = 3\,$GeV. Further specifications of the quantities are given in the quoted sections. Results for $N_f = 2$ quark masses are unchanged since FLAG 16 [3], and are not included here. For each result we list the references that enter the FLAG average or estimate, and we stress again the importance of quoting these original works when referring to FLAG results. From the entries in this column one can also read off the number of results that enter our averages for each quantity. We emphasize that these numbers only give a very rough indication of how thoroughly the quantity in question has been explored on the lattice and recommend consulting the detailed tables and figures in the relevant section for more significant information and for explanations on the source of the quoted errors.

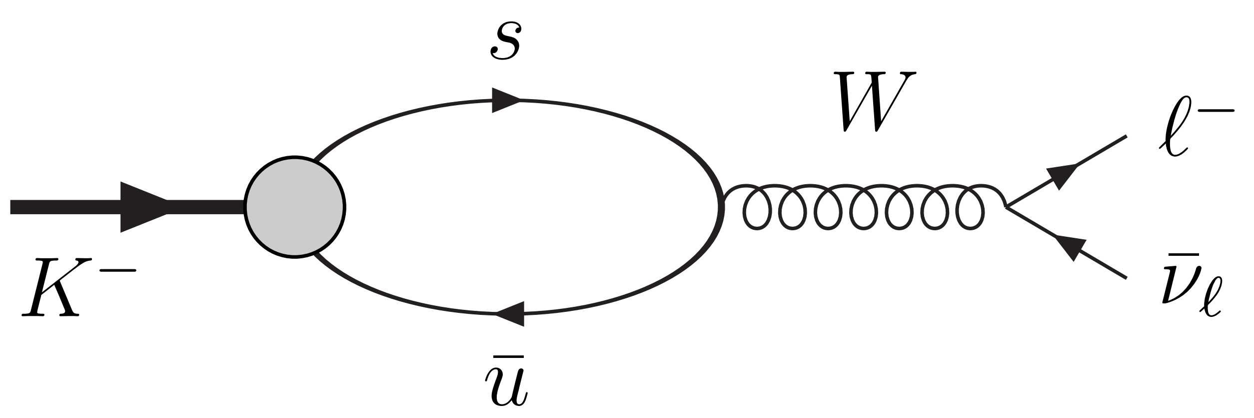
FLAG Review 2021, Y.Aoki et al., arXiv:2111.09849

- Lattice QCD results for some physical quantities are now so precise (sub percent) that QED corrections need to be included to make further progress.

- I shall use

$$f_K = 155.7(3)\ \text{MeV}$$

to illustrate our calculations.



$$\langle 0|A_\mu|K(p)\rangle = f_K p_\mu\,, \qquad r_\ell = m_\ell/m_K$$

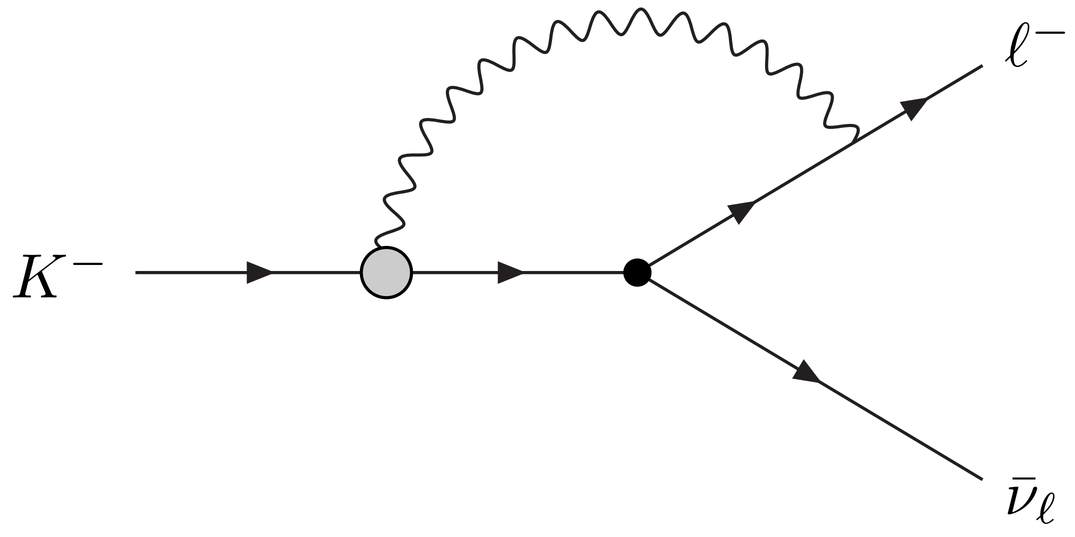
$$\Gamma^{(0)} = \frac{G_F^2|V_{us}|^2 f_K^2}{8\pi} m_K^3 r_\ell^2 \left(1 - r_\ell^2\right)^2$$

Computing QED Corrections to Weak Decay Amplitudes - The Framework

QED Corrections to Hadronic Processes in Lattice QCD
N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino and M.Testa
arXiv:1502.00257

- Our aim is to calculate Γ including $O(\alpha_{\text{em}})$ corrections.

- f_K no longer contains all the QCD effects.



- Calculating electromagnetic corrections to decay amplitudes has the major complication, not present in computations of the spectrum, **the presence of infrared divergences**

- This implies that when studying such processes , the physical observable must include soft photons in the final state.

F.Bloch and A.Nordsieck, PR 52 (1937) 54

$$\Gamma(K^- \to \ell^-\bar{\nu}_\ell(\gamma)) = \Gamma(K^- \to \ell^-\bar{\nu}_\ell) + \Gamma(K^- \to \ell^-\bar{\nu}_\ell\gamma) \equiv \Gamma_0 + \Gamma_1 .$$

- The generic question is how best to combine this understanding with lattice calculations of non-perturbative hadronic effects.

- Our proposal is to separate $\Gamma_0 + \Gamma_1$ into terms each of which is infrared convergent

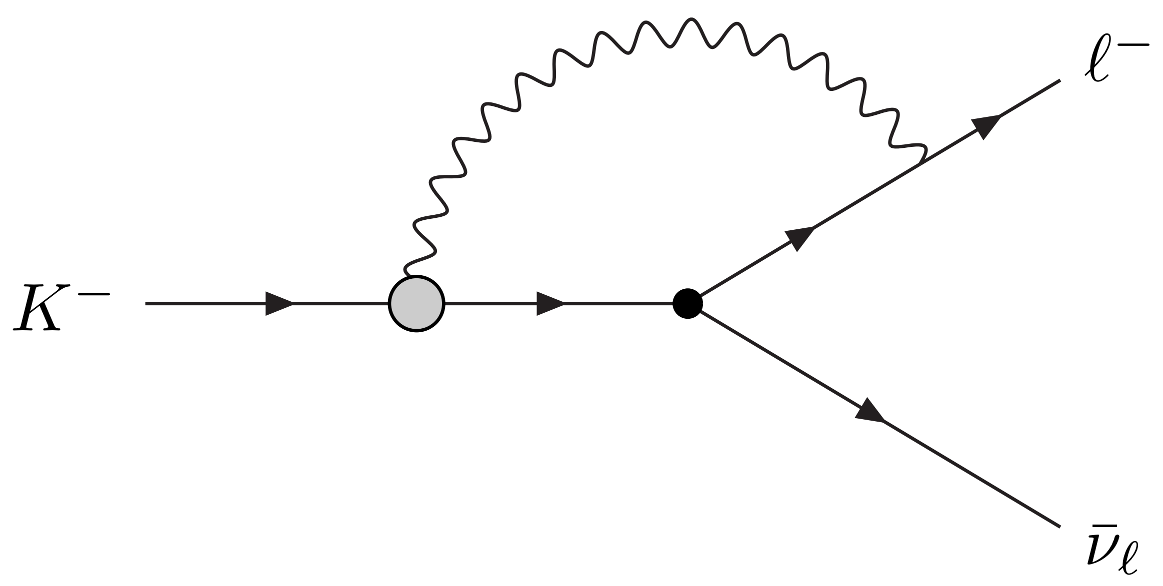
$$\Gamma(\Delta E_\gamma) = \Gamma_0 + \Gamma_1(\Delta E_\gamma) = \Gamma_0 + \int_0^{2\Delta E_\gamma/m_P} dx_\gamma \, \frac{d\Gamma_1}{dx_\gamma}$$

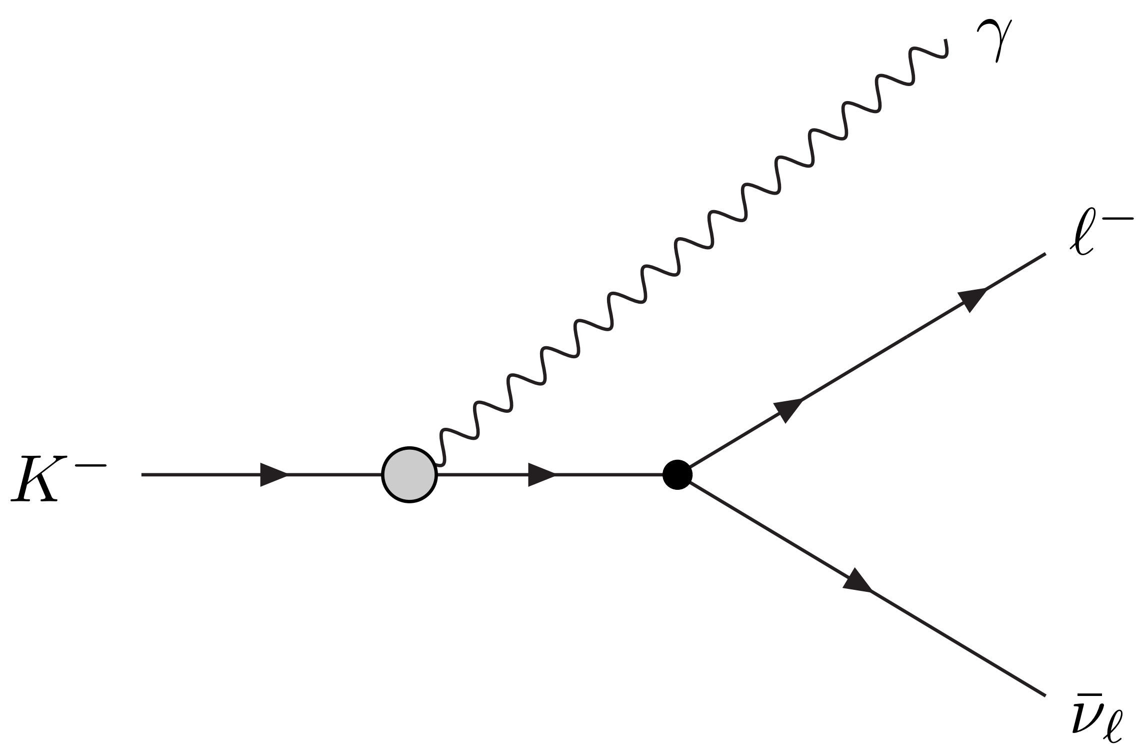
$$= \lim_{L\to\infty}\left[\Gamma_0(L) - \Gamma_0^{\rm pt}(L)\right] + \lim_{\mu_\gamma\to 0}\left[\Gamma_0^{\rm pt}(\mu_\gamma) + \Gamma_1^{\rm pt}(\Delta E_\gamma, \mu_\gamma)\right] + \Gamma_1^{\rm SD}(\Delta E_\gamma) + \Gamma_1^{\rm INT}(\Delta E_\gamma)\,.$$

- $x_\gamma = 2E_\gamma/m_K$ in the rest frame of the kaon

- pt = "point like", SD = "Structure Dependent" and "INT" is the interference between pt and SD

- "pt" contributions can be calculated in perturbation theory, whereas $\Gamma_0(L)$ and (for large ΔE_γ) $\Gamma_1^{\rm SD}$ and $\Gamma_1^{\rm INT}$ need to be computed non perturbatively.





Issues not discussed here

- When including QED, questions such as "**What is QCD?**" or equivalently "**How large are the electromagnetic corrections?**" are convention dependent due to the electromagnetic shift in the quark masses.

Light-meson leptonic decay rates in lattice QCD+QED
M.Di Carlo, D Giusti, V.Lubicz, G.Martinelli, CTS, F.Sanfilippo , S.Simula and N.Tantalo, arXiv:1904.08731

- Definition of G_F at $O(\alpha_{\rm em})$. This must be consistent with the procedure being used.

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}\left[1 - \frac{8m_e^2}{m_\mu^2}\right]\left[1 + \frac{\alpha_{\rm em}}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right]$$

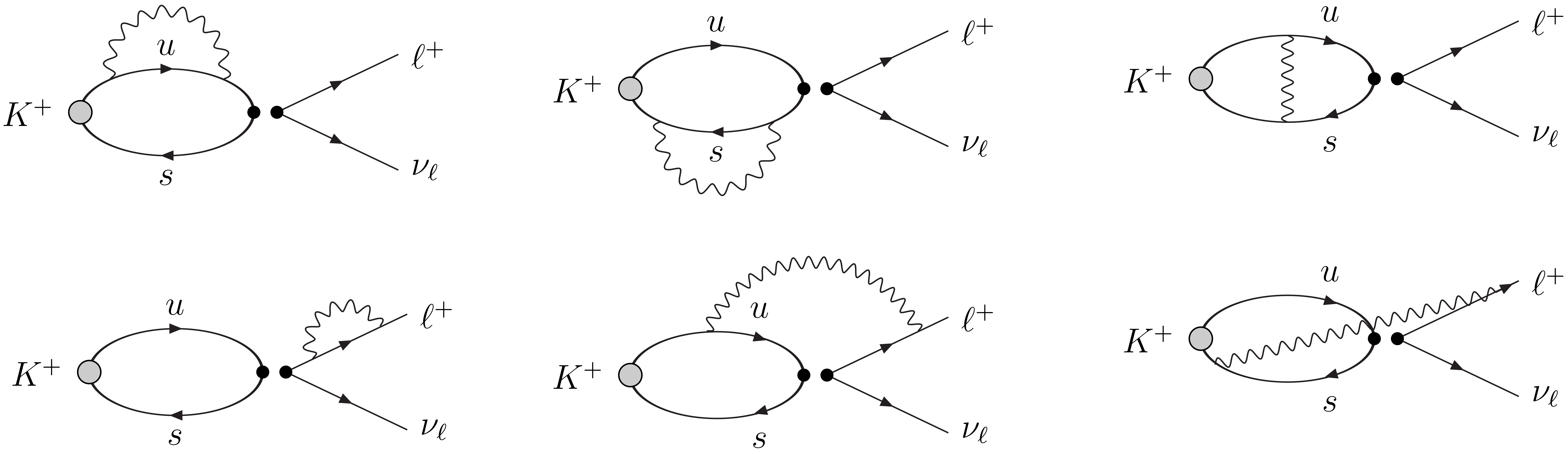
- Renormalization of the lattice operators including $O(\alpha_{\rm em})$ effects.

Non-perturbative renormalization in QCD+QED and its application to weak decays
M.Di Carlo, D Giusti, V.Lubicz, G.Martinelli, CTS, F.Sanfilippo , S.Simula and N.Tantalo, arXiv:1911.00938

- Perturbative evaluation of $\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E_\gamma)$.

QED Corrections to Hadronic Processes in Lattice QCD
N.Carrasco, V.Lubicz, G.Martinelli, CTS, N.Tantalo, C.Tarantino and M.Testa, arXiv:1502.00257

- Evaluation of the diagrams.



+ disconnected diagrams + real photon emission

Finite-Volume Corrections

Finite-Volume QED corrections to decay amplitudes in lattice QCD
V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula and N.Tantalo, arXiv:1611.08497

- The photon is massless $\Rightarrow$ difficulties in a finite volume.

- We have implemented the framework in QED_L in which $A_\mu(\vec{k}=0,k_4)=0$ for all k_4.
  M.Hayakawa and S.Uno arXiv:0804.2044

  - Transfer matrix exists but locality is broken
  - $L\to\infty$ limit should be taken first

- Evaluation of FV effects is based on the Poisson Summation formula, e.g. in one dimension

$$\frac{1}{L}\sum_{n=-\infty}^{\infty} f(p_n^2)=\int_{-\infty}^{\infty}\frac{dp}{2\pi}\,f(p^2)+\sum_{n\neq 0}\int_{-\infty}^{\infty}\frac{dp}{2\pi}\,f(p^2)\,e^{inpL}\,.$$

  - For decay constants, form factors etc the FV effects fall exponentially, typically $\propto \exp[-cm_\pi L]$.
  - This is not the case when $f(p^2)$ has a singularity.

- In the presence of a photon, if the integrand/summand $\rightarrow \frac{1}{(k^2)^{\frac{n}{2}}}$ as $k \rightarrow 0$ then we have the scaling law:

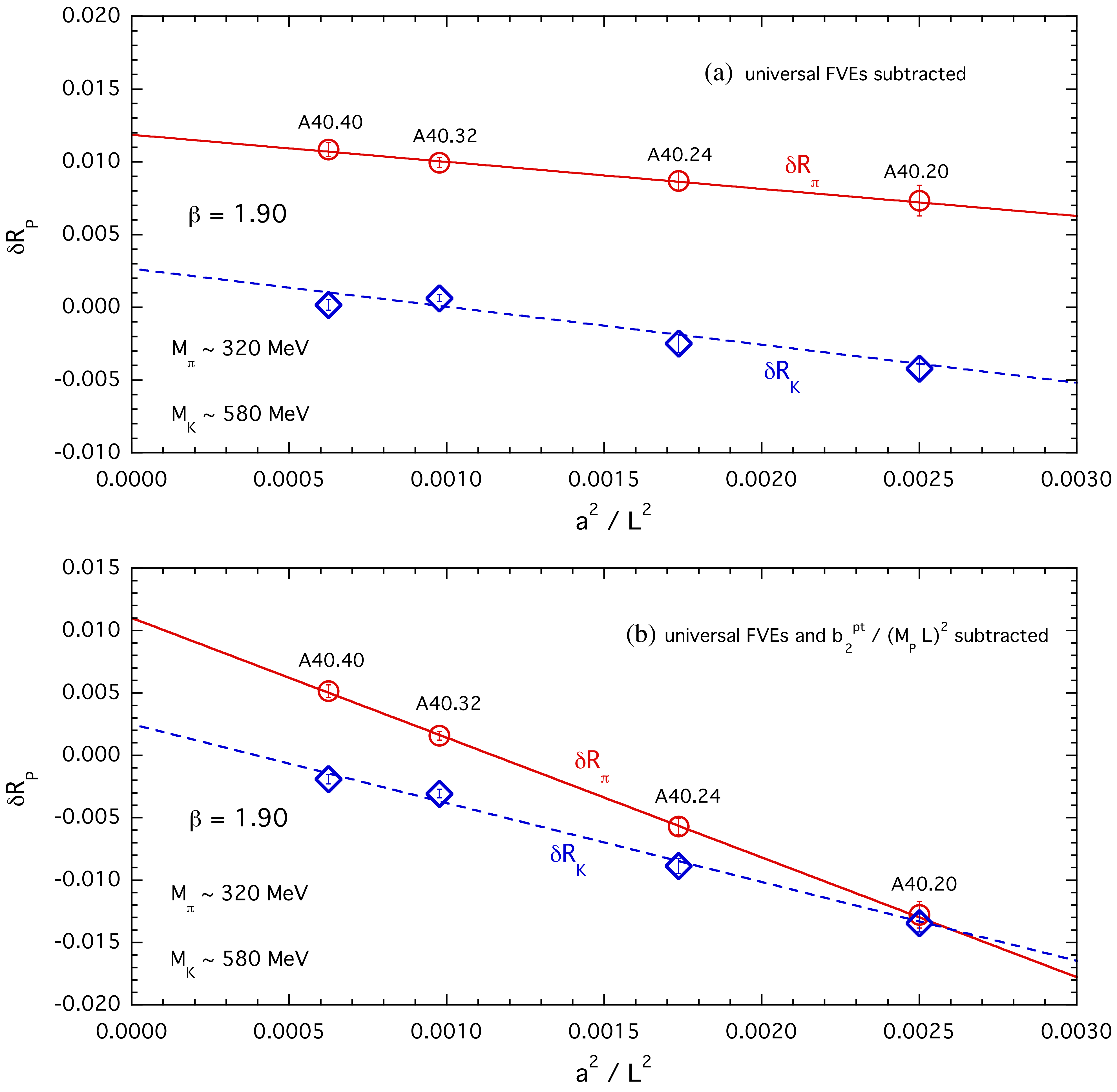
$$\xi' = \int \frac{dk_0}{(2\pi)} \left(\frac{1}{L^3} \sum_{\vec{k} \neq 0} - \int \frac{d^3k}{(2\pi)^3} \right) \frac{1}{(k^2)^{\frac{n}{2}}} = O\left(\frac{1}{L^{4-n}}\right)$$

- For the spectrum $n = 3$ and the leading FV corrections are $O(1/L)$.
- For decay amplitudes $n = 4$ and we have the form: $\Gamma_0^{\mathrm{pt}}(L) = C_0(r_\ell) + \tilde{C}_0(r_\ell)\log\left(m_K L\right) + \frac{C_1(r_\ell)}{m_K L} + \ldots$

where $r_\ell = m_\ell / m_K$.

- The exhibited L-dependent terms are *universal*, i.e. independent of the structure of the meson!
    - We have evaluated these coefficients.

- The leading structure-dependent FV effects in $\Gamma_0 - \Gamma_0^{\mathrm{pt}}$ are of $O(1/L^2)$.



- Finite-volume behaviour of 4-points, obtained at the same value of β and quark masses using ETMC twisted mass ensembles.
  - The universal $O(1/L)$ terms have been subtracted.
  - The leading SD finite-volume terms appear to be of $O(1/L^2)$ as expected.

- However, it has recently been shown that the point-like $O(1/L^3)$ terms are not negligible together with an argument that the SD $O(1/L^2)$ terms are very small.

M.Di Carlo, M.T.Hansen, A.Portelli and N.Hermansson -Truedsson
Phys. Rev. D105 (2022) 074509

To be investigated further

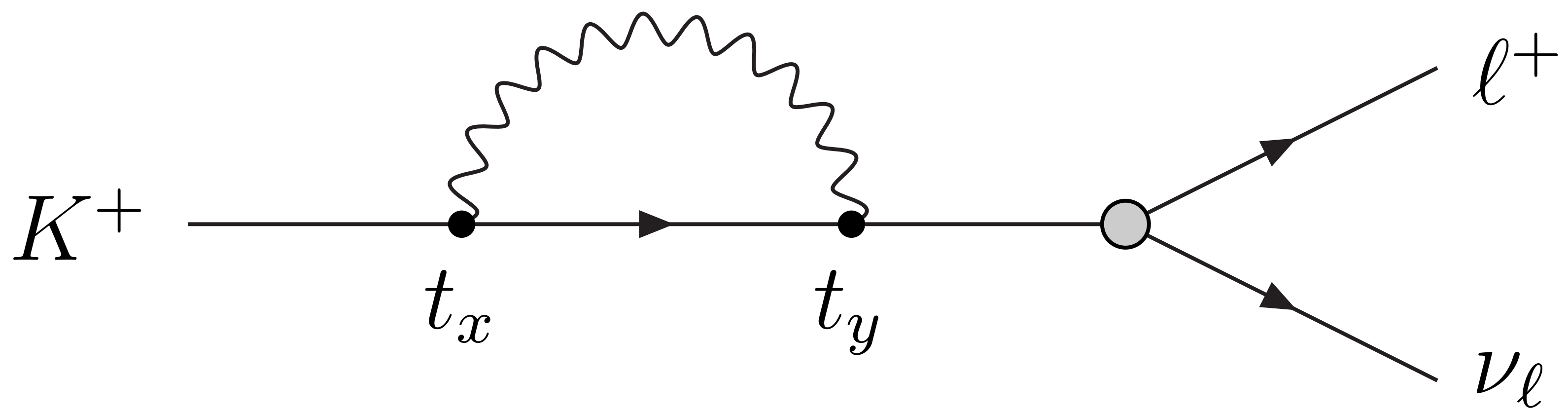
Light-meson leptonic decay rates in lattice QCD+QED
M.Di Carlo, D Giusti, V.Lubicz, G.Martinelli, CTS, F.Sanfilippo , S.Simula and N.Tantalo, arXiv:1904.08731

Infinite-volume reconstruction

- IVR is an idea by Xu Feng and Luchang Jin originally introduced to avoid non-exponential FV effects in calculations of QED corrections to the spectrum. X.Feng and L.Jin, arXiv:1812.09817

- We have been extending the technique to QED corrections to leptonic decay amplitudes.
  N.H.Christ, X.Feng, L.Jin and CTS, PoS **LATTICE2019** (2020), 259
  N.H.Christ, X.Feng, L.Jin, CTS and T.Wang (in preparation)

- For illustration consider the following diagram which contributes both to the electromagnetic mass-shift and to the wave function renormalisation of the kaon:



- For large $|t_y - t_x|$, $|t_y - t_x| > t_s$ say, the only state propagating between the two currents is $|K^+\gamma\rangle$.
- It is therefore sufficient to evaluate the correlation functions with $|t_y - t_x| \le t_s$ and avoid non-exponential FV effects. For example:

$$H_2(\vec{z}, t_z)_{t_z \ge t_s} = \langle K^+(\vec{0})|J^\mu_{\rm em}(z)\, J^\nu_{\rm em}(0)|K^+(\vec{0})\rangle$$

$$= \int \frac{d^3p}{(2\pi)^3} \int d^3z'\, H_2(\vec{z}\,', t_s) e^{-(E_p - m_K)(t_z - t_s)}\, e^{i\vec{p}\cdot(\vec{z}-\vec{z}\,')} \qquad \left(E_p = \sqrt{\vec{p}^{\,2} + m_K^2}\right)$$

QED Corrections to V_{us}

- Writing

$$\frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} = \left| \frac{V_{us}}{V_{ud}} \frac{f_K^{(0)}}{f_\pi^{(0)}} \right|^2 \frac{m_\pi^3}{m_K^3} \left(\frac{m_K^2 - m_\mu^2}{m_\pi^2 - m_\mu^2} \right)^2 (1 + \delta R_{K\pi}),$$

where $m_{K,\pi}$ are the physical masses, using numerous twisted mass ensembles we find

$$\delta R_{K\pi} = -0.0126(14) \qquad \left[\delta R_\pi = +0.0153(19),\ \delta R_K = +0.0024(10)\right]$$

- $f_P^{(0)}$ are the decay constants obtained in iso-symmetric QCD with the renormalized $\overline{\text{MS}}$ masses and coupling equal to those in the full QCD+QED theory extrapolated to infinite volume and to the continuum limit.

  - Using ChPT, $\delta R_\pi = +0.0176(21),\ \delta R_K = +0.0064(24).$ PDG(2018)

QED Corrections to V_{us} (cont.)

- We obtained

$$\left|\frac{V_{us}}{V_{ud}}\right| = 0.23135(46)\,.$$

- Taking $V_{ud} = 0.97420(21)$ (J.Hardy and I.S.Towner, CKM(2016) 028) $\Rightarrow$ $V_{us} = 0.22538(46)$ and with $|V_{ub}| = 0.00413(49)$,

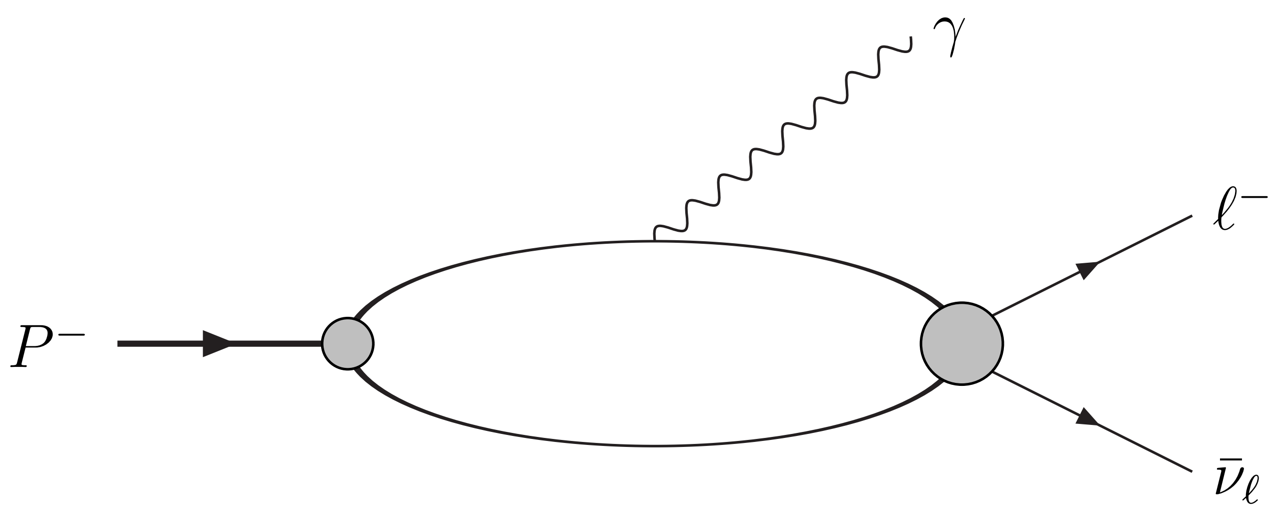
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99988(46)\,.$$

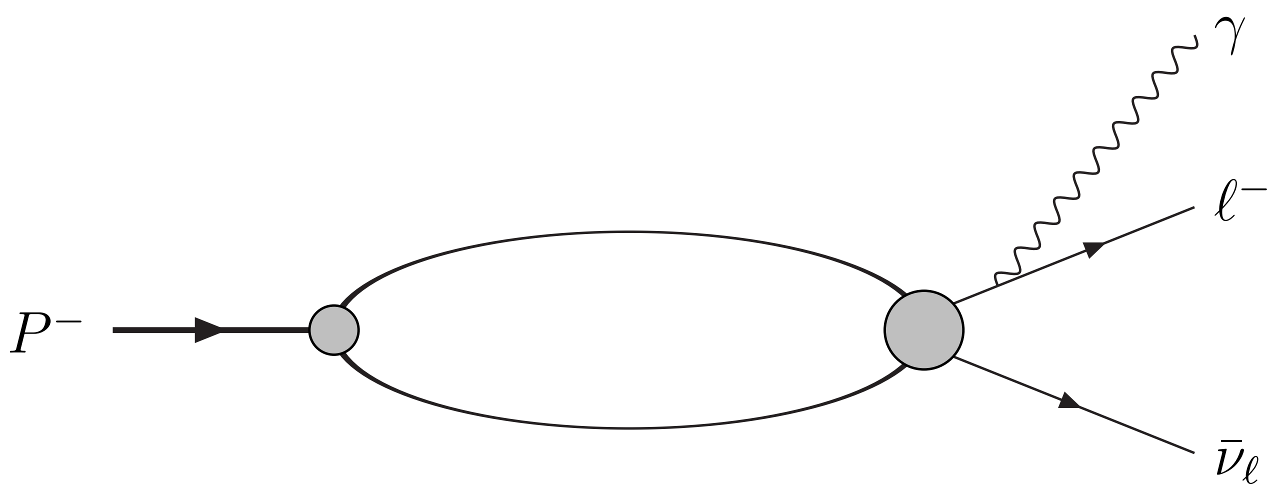
- However, taking $|V_{ud}| = 0.97370(14)$ (C.Y.Seng et al., arXiv:1807.10197), $|V_{us}|$ = 0.22526(46),

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99885(34)\,.$$

- The latest PDG value is $V_{ud} = 0.97373(31)$, which is the average of the 15 most precise determinations and with a more conservative error. (Unitarity within a little more than 1σ.)

$P \to \ell \nu_\ell \gamma$ radiative decays - the form factors





Non-perturbative contribution to $P \to \ell \bar{\nu}_\ell \gamma$ is encoded in:

$$H_W^{\alpha r}(k, \vec{p}) = \epsilon_\mu^r(k)\, H_W^{\alpha\mu}(k, \vec{p}) = \epsilon_\mu^r(k) \int d^4y\, e^{ik\cdot y}\, \mathrm{T}\, \langle 0 |\, j_W^\alpha(0)\, j_{\mathrm{em}}^\mu(y)\, | P(\vec{p}) \rangle$$

$$= \epsilon_\mu^r(k) \Bigg\{ \frac{H_1}{m_P} \left[k^2 g^{\mu\alpha} - k^\mu k^\alpha\right] + \frac{H_2}{m_P} \frac{\left[(p\cdot k - k^2)k^\mu - k^2(p-k)^\mu\right](p-k)^\alpha}{(p-k)^2 - m_P^2}$$

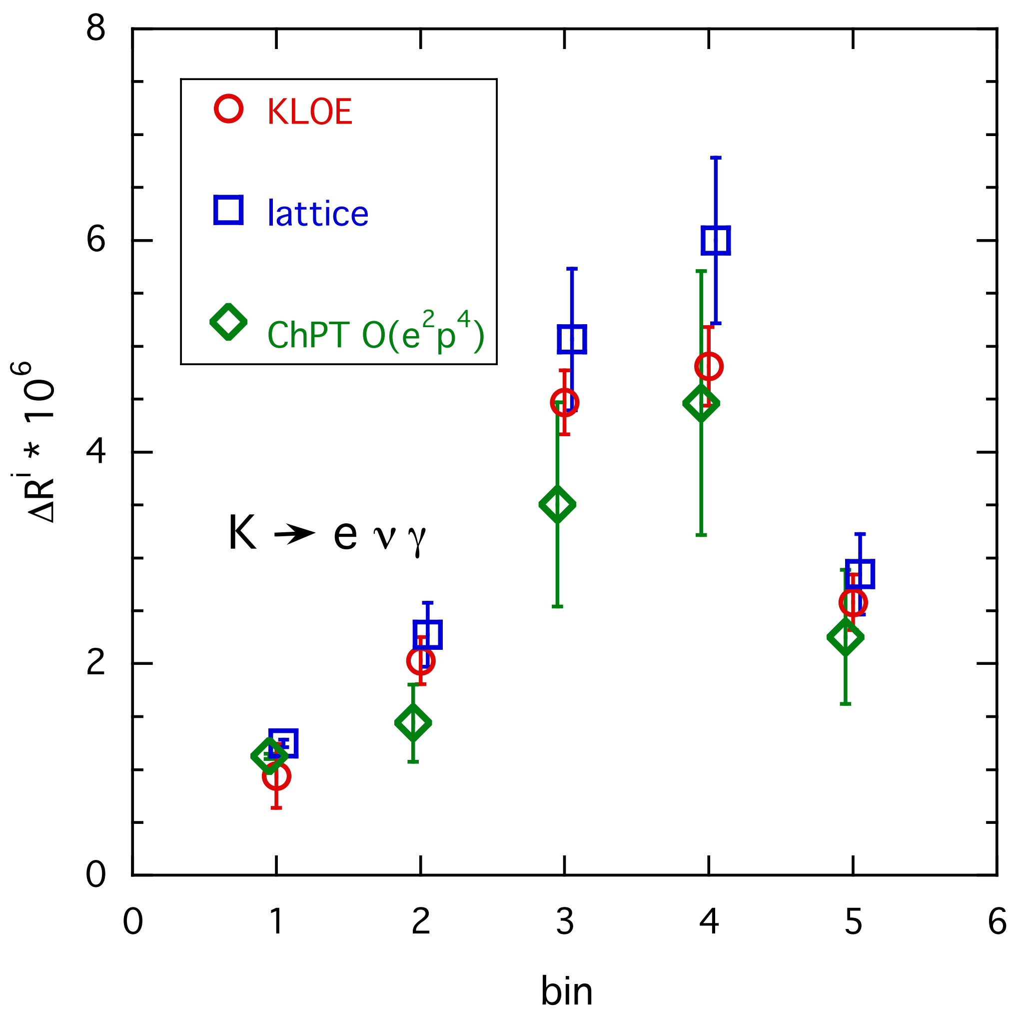
$$-i\frac{F_V}{m_P}\varepsilon^{\mu\alpha\gamma\beta}k_\gamma p_\beta + \frac{F_A}{m_P}\left[(p\cdot k - k^2)g^{\mu\alpha} - (p-k)^\mu k^\alpha\right] + f_P \left[g^{\mu\alpha} - \frac{(2p-k)^\mu(p-k)^\alpha}{(p-k)^2 - m_P^2}\right] \Bigg\}$$

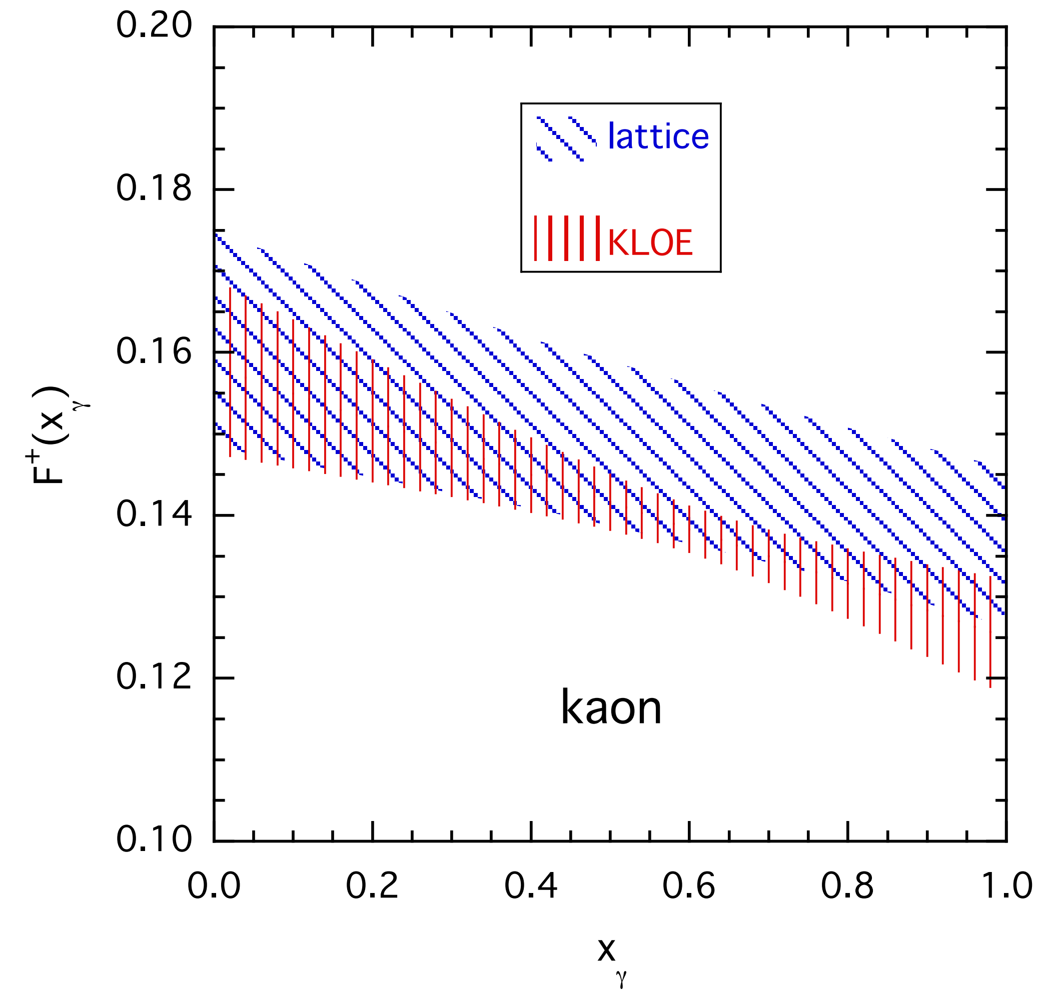
- For decays into a real photon, $k^2 = 0$ and $\varepsilon \cdot k = 0$, only the decay constant f_p and the vector and axial form factors $F_V(x_\gamma)$ and $F_A(x_\gamma)$ are needed to specify the amplitude ($x_\gamma = 2p\cdot k/m_P^2$, $0 < x_\gamma < 1 - m_\ell^2/m_P^2$).

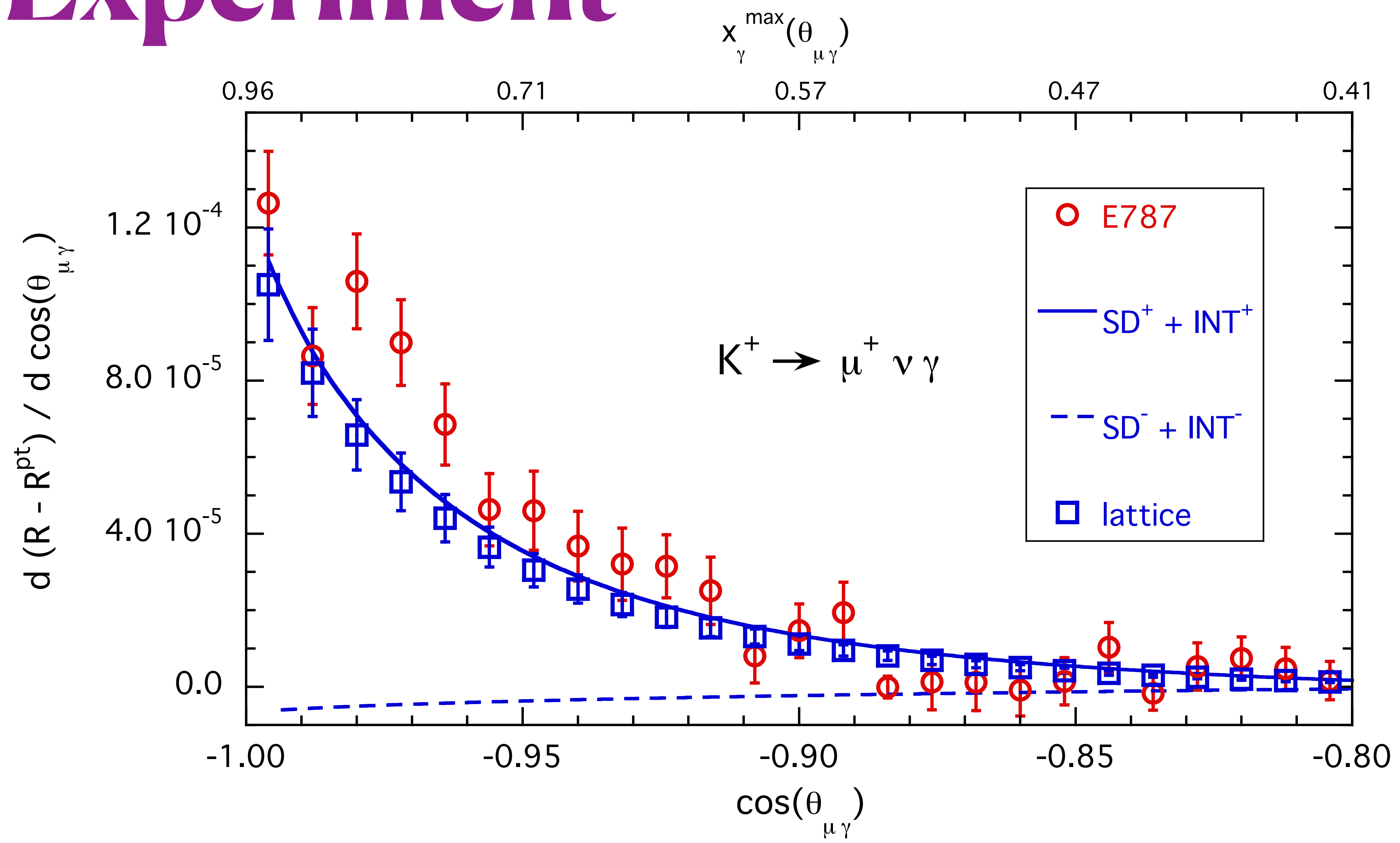
- In phenomenology $F^\pm \equiv F_V \pm F_A$ are more natural combinations.

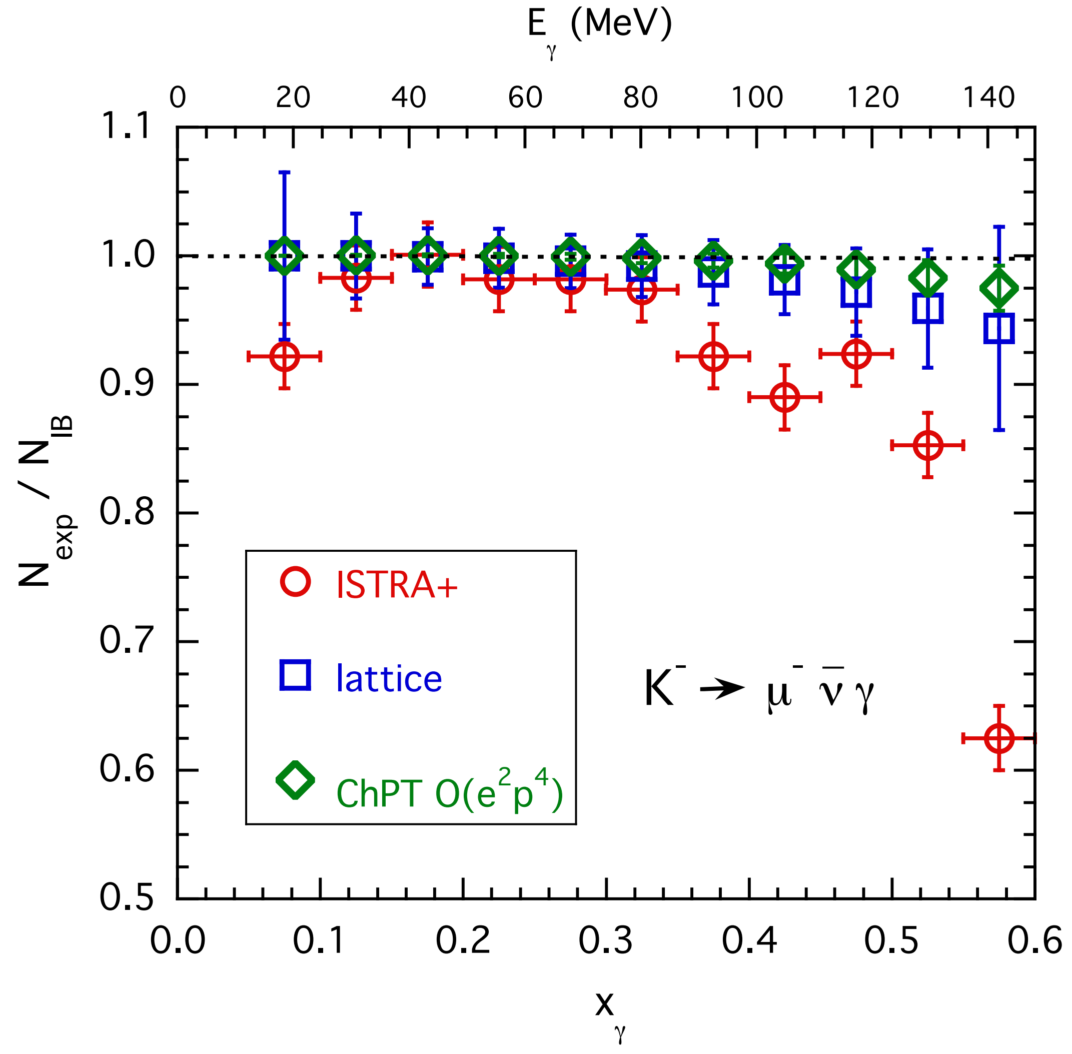
- We have computed $F_V(x_\gamma)$ and $F_A(x_\gamma)$ for $\pi, K, D_{(s)}$ mesons (and $H_{1,2}$ in an exploratory simulation for $K \to \pi \ell \nu_\ell \ell'^+ \ell'^-$ decays).

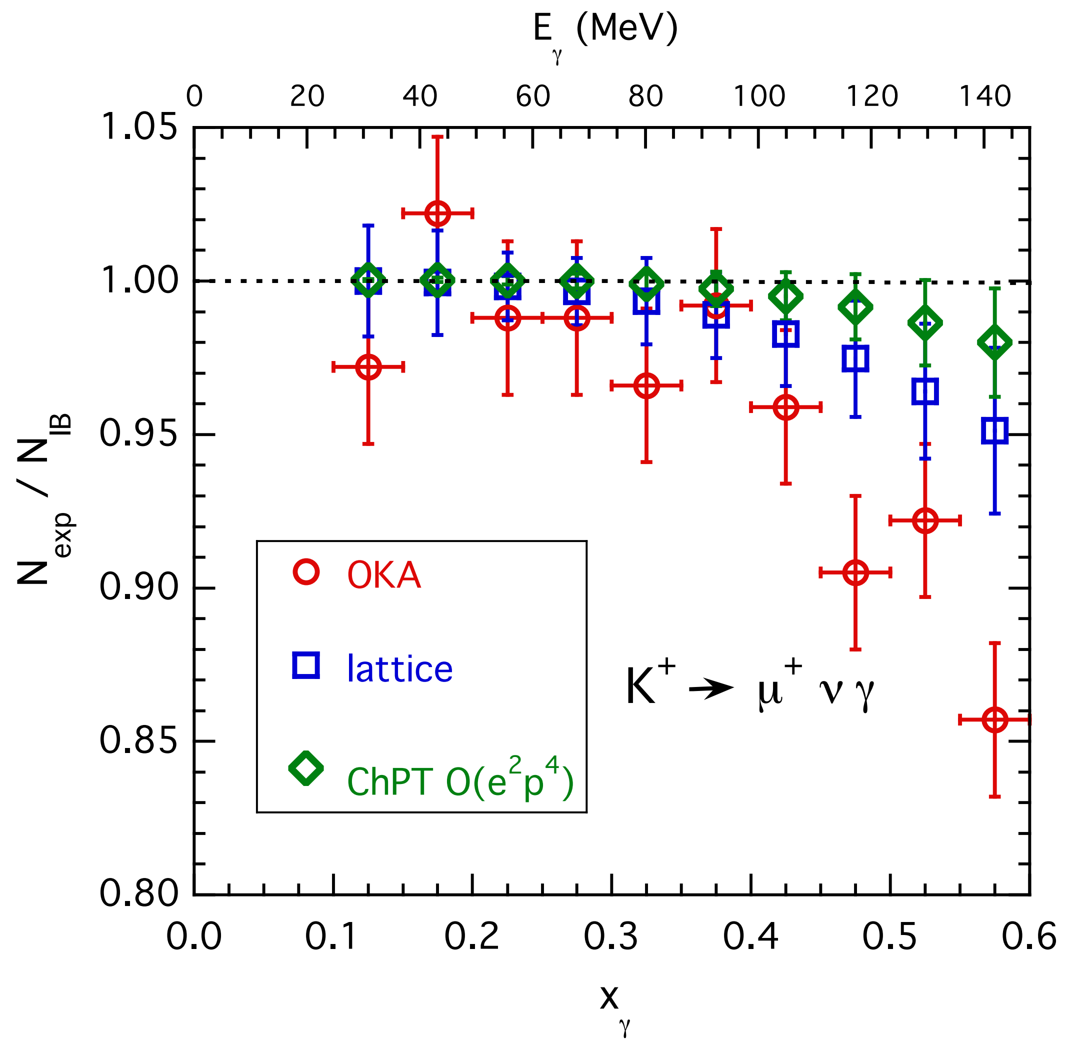
Comparison with Experiment











- Good Agreement with KLOE
- Significant tensions with $K \to \mu\nu_\mu\gamma$ experiments
- Unable to find a set of phenomenological form factors to account for all the data.
- NA62 will soon have the most precise results for $K \to e\nu_e\gamma$ decay rates.
- Is it conceivable that we have LFU-violation here also ?

A.Desiderio, R.Frezzotti, M.Garofalo, D.Giusti, M.Hansen, V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula and N.Tantalo. arXiv:2006.05358
R.Frezzotti, M.Garofalo, V.Lubicz, G.Martinelli, CTS, F.Sanfilippo, S.Simula and N.Tantalo, arXiv:2012.02120

- Thank you Guido for your constant friendship and for an exciting 36 years of collaboration, but

  much still remains to be done !

- I look forward to reporting on progress in 2032.


BILLY WTH
DUCA !
WHAT HAVE YOU DONE?!
I just cut in half the spaghetti
BLASPHEMY
SRGRAFO


Irena and I wish you many more years of health and happiness