Another look at hierarchies

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Guido has a very high place in my HIERARCHY of :

- Physicists
- Friends

I am very honoured to have been invited to speak at this event despite the fact that we have not co-authored a single paper

When did our paths cross?

- In the CERN Theory Group (1989-1996)
- An INFN Selection Committee (late 90's)
- My five years at Sapienza (2000-2005)
- The ERC Grant Damesyfla (2010-2016)

And now some physics

(unfinished unpublished work with G. Dall'Agata)

Motivation:

two big unanswered long-standing questions

- Stability of D=4 ~Minkowski background
- Dynamical generation of scale hierarchies

can be quantitively addressed in D>4 local susy broken classically by compactification (Scherk-Schwarz, fluxes)

First step:

compute V_1 as function of those background fields that are classically undetermined (no-scale moduli)

V_1 in the REDUCED theory

- Finite # of d.o.f.: those massless for unbroken susy
- $V_{1,red}$ controlled by Str M^n and generically divergent

N=0: Str $M^0 \neq 0$ $(n_B \neq n_F)$ => quartic divergence

N=1: Str $M^2 \neq 0$ => quadratic divergence

N=4: Str $M^4 \neq 0$ => logarithmic divergence

N=8: Str M^4 = Str $M^6 = 0$ => finite $V_{1,red} < 0$

=> no locally stable Mink or dS vacuum

Disappointing result, but reduced theory does not capture the full physics of the compactified theory!

V_1 in the COMPACTIFIED theory

- Infinite towers of KK modes (and additional string modes, exponentially decoupling at large volume)
- Non-local susy breaking in the extra dimensions => V_1 is automatically finite for N>0 [Rohm 1984]

Strategy

Focus on those Scherk-Schwarz susy breaking models that do not require localized defects

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keep maximal control on the effective theory

Two versions of Scherk-Schwarz

- Explicit breaking of d>4 susy by twisted boundary conditions w.r.t. an internal global R-symmetry
- Spontaneous breaking of d>4 susy where twisted b.c. equivalent to VEV of internal spin connection

=> quantized "geometrical fluxes"

Step-by-step understanding:

(i) Start first with D=5 pure sugra on the circle S^1 to understand better how finite V_1 relates with $V_{1,red}$

(ii) Generalize to truly spontaneous breaking (d≥7) e.g. to d=11 sugra as in Scherk-Schwarz 1979

Scherk-Schwarz of N>0 on S^1

$$V_1 = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \sum_{n=-\infty}^{+\infty} \sum_{\alpha} (-1)^{2J_\alpha}\,(2J_\alpha+1)\,\log\left(p^2 + m_{n,\alpha}^2\right)$$

$$m_{n,\alpha}^2 = \frac{(n+s_\alpha)^2}{R^2}$$

n is KK level, R is S^1 radius, s_α are Scherk-Schwarz shifts

(neglecting for simplicity susy-preserving masses)

Adapting [Delgado-Pomarol-Quiros 1999], V_1 is finite:

$$V_1 = -\frac{3}{128\,\pi^6\,R^4} \sum_\alpha (-1)^{2J_\alpha}\,(2J_\alpha+1)\,\left[\mathrm{Li}_5(e^{-2\pi\, i\, s_\alpha}) + \mathrm{Li}_5(e^{2\pi\, i\, s_\alpha})\right]$$

$\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} x^k/k^n$ are the polylogarithm functions and $\mathrm{Li}_5(1) = \zeta(5) \simeq 1.037$.

Relating V_1 with $V_{1,red}$ (n=0 only)

$V_1 \to V_{1,red}$ for $|s_\alpha| \ll 1 \leftrightarrow m_{0,\alpha}^2 \ll 1/R^2$
if we cutoff divergent $V_{1,red}$ with $\boldsymbol{\Lambda} \sim 1/R$

$$V_{1,red} = \frac{1}{32\,\pi^2}\,\mathrm{Str}\,\mathcal{M}_0^2\,\Lambda^2 + \frac{1}{64\,\pi^2}\,\mathrm{Str}\,\mathcal{M}_0^4\,\log\frac{\mathcal{M}_0^2}{\Lambda^2}$$

$$\mathrm{Str}\,\mathcal{M}_n^p \equiv \sum_\alpha (-1)^{2J_\alpha}\,(2J_\alpha+1)\,m_{n,\alpha}^p$$

And for $|s_\alpha| \ll 1$:

$$\left[\mathrm{Li}_5(e^{-2\pi\,i\,s_\alpha}) + \mathrm{Li}_5(e^{2\pi\,i\,s_\alpha})\right] = \cancel{2\,\zeta(5)} - 4\pi^2\,\zeta(3)\,s_\alpha^2 + \frac{\pi^4}{3}\left[\frac{25}{3} - 4\,\log(2\pi)\right]\,s_\alpha^4$$

cancels for N>0

$$-\frac{2}{3}\,\pi^4\,s_\alpha^4\,\log s_\alpha^2 + \ldots\; O(s_\alpha^6)$$

Indeed: $\Lambda \simeq 0.6/R$ for quadr div, $\Lambda \simeq 2.7/R$ for log div

Ex 1: twisted pure N=2, d=5 sugra on S^1

- Single twist parameter a, $V_1 < 0$
- $V_{1,\mathrm{min}}$ at fixed R for a=1/2 (mod.1)

$$Str\ \mathcal{M}_0^2 = Str\ \mathcal{M}_n^2 = Str\ \frac{s_\alpha^2}{R^2} = -8\,\frac{a^2}{R^2} < 0$$

Ex 2: twisted pure N=4, d=5 sugra on S^1

- Two twist parameters $a_{1,2}$, $V_1 < 0$
- $V_{1,\mathrm{min}}$ at fixed R for $a_1=a_2=1/2$ (mod.1)

$$Str\ \mathcal{M}_0^4 = Str\ \mathcal{M}_n^4 = Str\ \frac{s_\alpha^4}{R^4} = 72\,\frac{a_1^2\,a_2^2}{R^4} > 0$$

Ex 3: twisted pure N=6, d=5 sugra on S^1

- Three indep twists $a_{1,2,3}$, $V_1 < 0$
- $V_{1,min}$ at fixed R for $a_{1,2,3}$=1/2 (mod.1)

$$Str\ \mathcal{M}_0^6 = Str\ \mathcal{M}_n^6 = Str\ \frac{s_\alpha^6}{R^6} = -1440\,\frac{a_1^2\, a_2^2\, a_3^2}{R^6} < 0$$

Ex 4: twisted pure N=8, d=5 sugra on S^1

- Four ind twists $a_{1,2,3,4}$, $V_1 < 0$
- $V_{1,min}$ at fixed R for $a_1=a_2=a_3=a_4$=1/2 (mod.1)

$$Str\ \mathcal{M}_n^8 = Str\ \frac{s_\alpha^8}{R^8} = 40320\,\frac{a_1^2\, a_2^2\, a_3^2\, a_4^2}{R^8} < 0$$

Lessons from computing V_1 on S^1

- No qualitatively new features wrt $V_{1,red}$
- In particular, $V_1 = -k/R^4$ with $k > 0$
- No quantisation of twist parameters
- Leading contribution repeats at each KK level

Must move to "geometrical fluxes", where:

- Susy breaking spontaneous in the full theory
- Twist parameters quantized
- At least three internal dimensions required

=> KK spectrum much more challenging to get

Technical points to be addressed

- Classify twisted 3-tori with the help of wallpaper groups (discrete subgroups of E_2 that contain two independent translations); combine if possible
- Determine the full KK spectrum: powerful tools developed [Malek-Samtleben 2020] in exceptional field theory (duality-covariant formulation of d>4 supergravity), applied so far mostly to AdS vacua
- Generalise the simple sums on S^1 described today

…work in slow progress…

Thank you Guido!

-for your research

-for your generous contributions to our scientific community

-for your friendship

And best wishes of many more happy years with Nanie and us all