Electroweak, Strong and New Interactions:
a symposium to celebrate Guido Martinelli's 70^{th} birthday

Accademia dei Lincei

Rome, September 26^{th}, 2022

Giovanni Ridolfi

Università di Genova and INFN Sezione di Genova, Italy

International School of Physics "Enrico Fermi"

Villa Monastero, Varenna, 26 June - 6 July 1984

So ietà Italiana di Fisica

INTERNATIONAL SCHOOL OF PHYSICS
"ENRICO FERMI"
VILLA MONASTERO - VARENNA -

SUMMER COURSES 1984

26 June - 6 July

"Elementary Particles"

G. Martinelli

EXPERIMENTAL TESTS AND THEORETICAL PREDICTIONS FOR ELECTROWEAK PROCESSES

1) BASIC $SU(2)\times U(1)$: DEFINITION OF THE PARAMETERS OF THE STANDARD MODEL

—o—

2) LOW ENERGY PROCESSES AND THE DETERMINATION OF $\sin^2\theta_W$

—o—

3) RADIATIVE CORRECTIONS TO LOW ENERGY PROCESSES

—o—

4) MEASUREMENTS OF M_W AND M_Z AT THE SPS-COLLIDER AND COMPARISON WITH PREDICTIONS

Z. Phys. C – Particles and Fields 39, 21–37 (1988)

Zeitschrift für Physik C Particles and Fields

Parton densities from deep inelastic scattering to hadronic processes at super collider energies

M. Diemoz[1], F. Ferroni[1], E. Longo[1], G. Martinelli[2]

[1] Dipartimento di Fisica, Università "La Sapienza" di Roma, and I.N.F.N. Sezione di Roma, I-00100 Roma, Italy
[2] CERN, CH-1211 Geneva 23, Switzerland

Received 6 August 1987

A groundbreaking work in many respects:

- **next-to-leading order Altarelli-Parisi evolution**

  [first application of Curci, Furmanski, Petronzio 1980]

- **heavy quark thresholds**

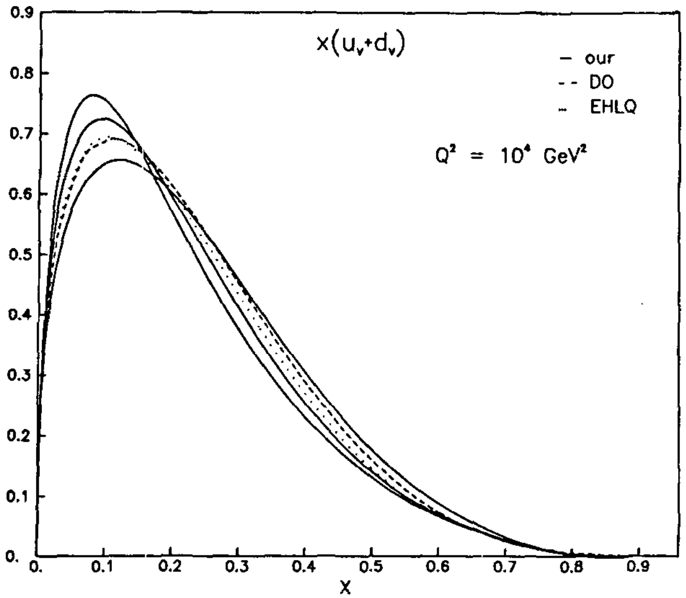
- **an estimate of uncertainties**

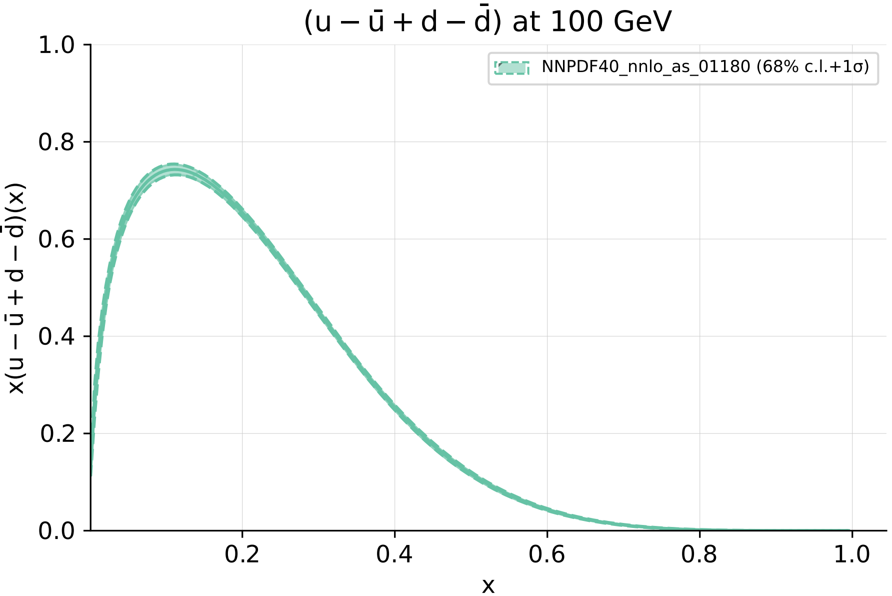
Numerical evolution in the space of Mellin moments and numerical Mellin inversion

More on this later.

DFLM 1987

NNPDF 2022





Left plot: fig. 20 of the original DFLM paper

Right plot: courtesy of Maria Ubiali for the NNPDF collaboration

Sudakov resummation: an overview

Electroweak, Strong and New Interactions:
a symposium to celebrate Guido Martinelli's 70^{th} birthday
Accademia dei Lincei
Rome, September 26^{th}, 2022

Giovanni Ridolfi
Università di Genova and INFN Sezione di Genova, Italy

1. Sudakov logarithms

Generic observable in perturbative QCD:

$$\sigma(Q^2,x)=\sum_n \alpha_s^n(Q^2)\sigma_n(Q^2,x); \qquad 0\le x\le 1$$

with x defined so that the Born kinematics (soft emission, or threshold, limit) corresponds to $x=1$.

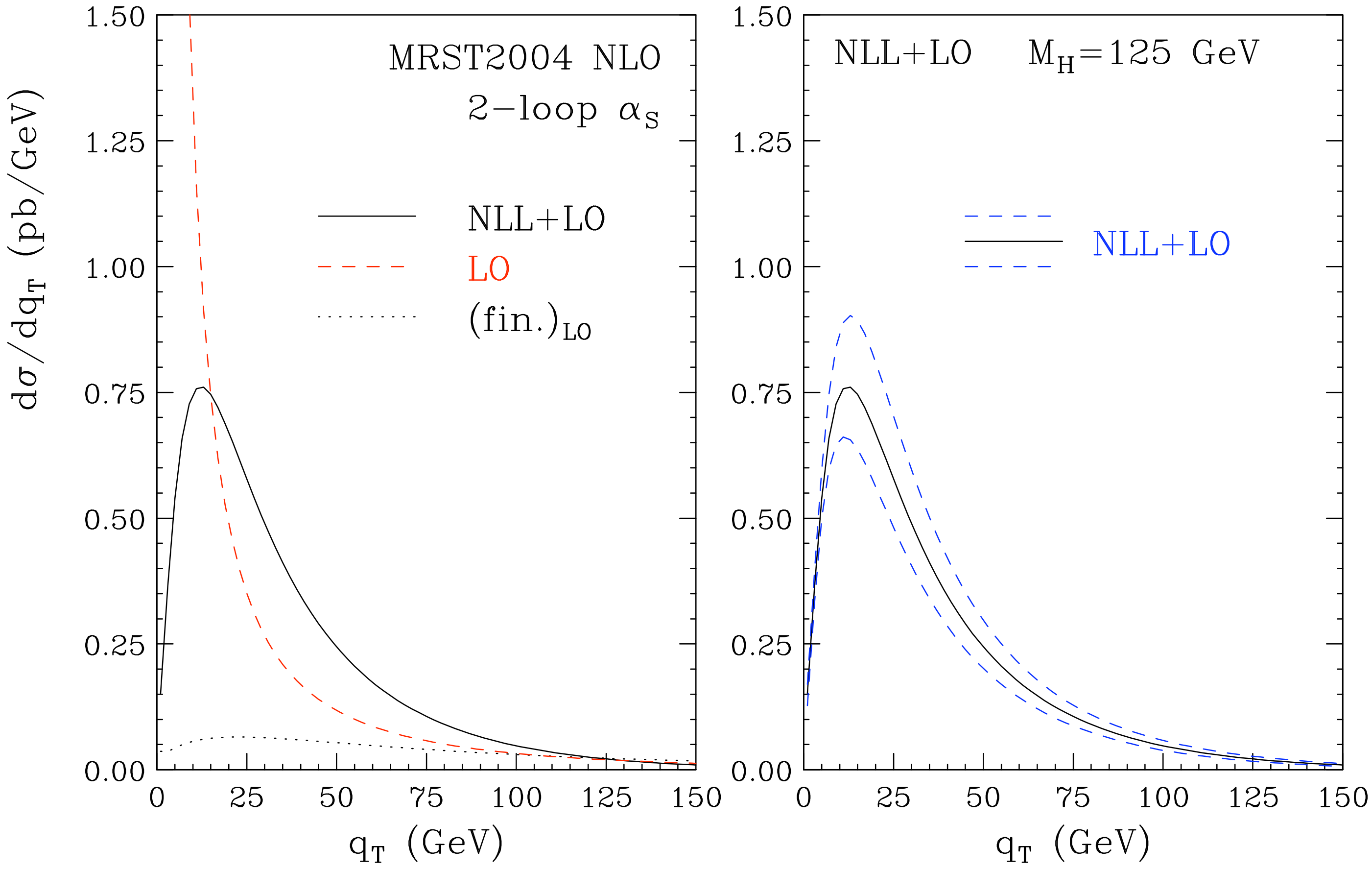
$$\sigma_n(Q^2,x)=\sum_{k=1}^{2n}\sigma_{nk}\left[\frac{\log^{k-1}(1-x)}{1-x}\right]_+ +\sigma_{n0}\delta(1-x)+r_n(x)$$

We are interested in the region $x\to 1$

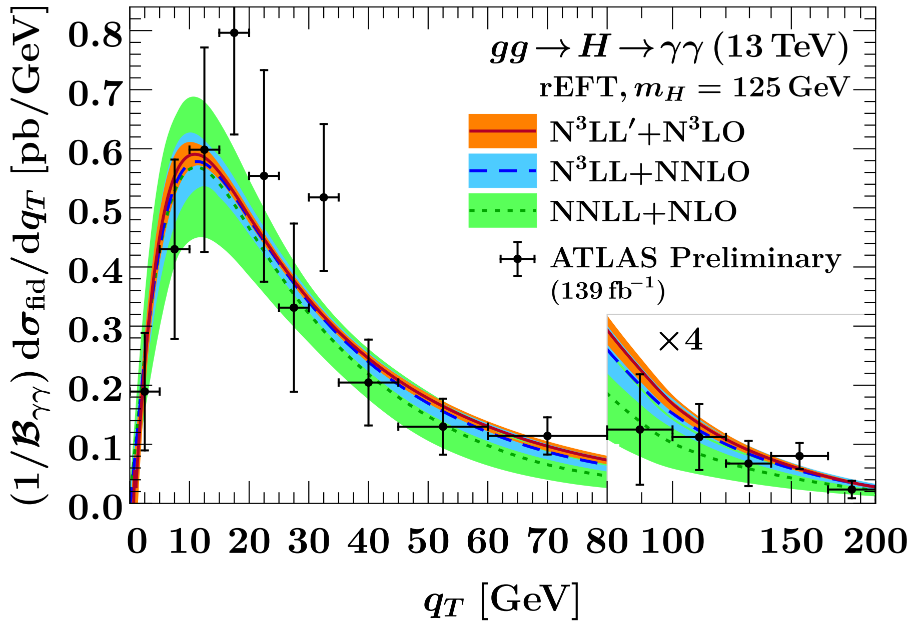
$$\int_{1-x_s}^{1}dx\,\sigma_n(Q^2,x)=\sum_{k=1}^{2n}\frac{\sigma_{nk}}{k}\log^k(1-x_s)+\sigma_{n0}+R_n(x_s)$$

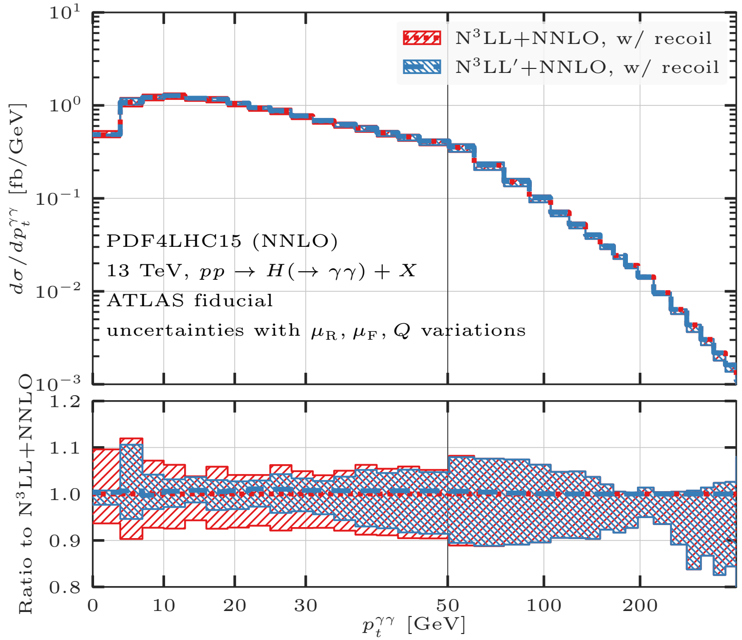
"Convergence" spoiled if $\alpha_s\log^2(1-x_s)\sim 1$; all-order resummation needed

An example: The q_T spectrum of Higgs production at the LHC; $x = 1 - \frac{q_T^2}{Q^2}$ [Bozzi, Catani, de Florian, Grazzini 2006 following original work by Collins, Soper and Sterman 1984]



An example: The q_T spectrum of Higgs production at the LHC; $x = 1 - \frac{q_T^2}{Q^2}$





left: [Billis et al 2021]

right: [Re, Rottoli, Torrielli 2021]

2. Resummation

To be definite, consider the production of a weakly-interacting, massive final state of squared mass Q^2 (Drell-Yan pair, Higgs boson) at squared energy s.

In this case $x = \frac{Q^2}{s}$ and $x \to 1$ in the threshold (soft emission) limit.

Resummation performed in terms of Mellin moments (factorization of phase space):

$$\tilde{f}(N) = \int_0^1 dx\, x^{N-1} f(x); \qquad f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN\, x^{-N} \tilde{f}(N)$$

where $c > \mathrm{Re}\, \bar{N}_i$; $\bar{N}_i$ singular points of $\tilde{f}(N)$.

Large-x region mapped onto large-N:

$$\int_0^1 dx\, x^{N-1} \left[\frac{\log^{k-1}(1-x)}{1-x} \right]_+ \sim \log^k N + \text{subleading logs}$$

Resummed result in N space:

$$\tilde{\sigma}^{\text{res}}(N) = g_0(\alpha_s) \exp \left[\frac{1}{\alpha_s} g_1(\alpha_s \log N) + g_2(\alpha_s \log N) + \alpha_s g_3(\alpha_s \log N) + \ldots \right]$$

[Catani, Trentadue 1989; Sterman 1987]

3. A difficulty: the Landau pole

The functions $g_1(\alpha_s \log N), g_2(\alpha_s \log N), \ldots$ have a logarithmic branch cut on the positive real N axis. A consequence of resummation.

Functions of

$$\alpha_s\left(\frac{Q^2}{N}\right) = \frac{\alpha_s(Q^2)}{1 - b_0\alpha_s(Q^2)\log N}\left[1 + \mathrm{NLL}\right]$$

(a proof by renormalization group

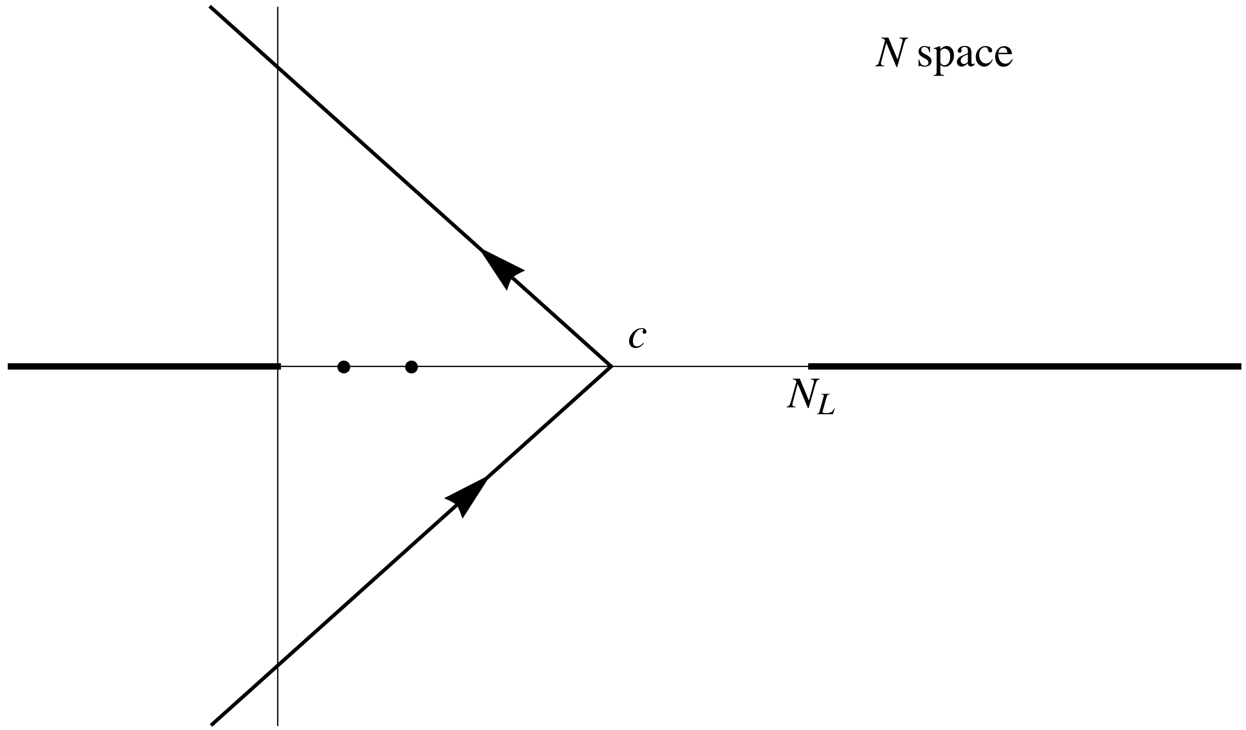
[Contopanagos, Laenen, Sterman 1997; Forte, R 2002])

Branch cut for

$$\mathrm{Re}\, N > N_L = \exp\frac{1}{b_0\alpha_s(Q^2)}$$

The inverse Mellin transform does not exist.

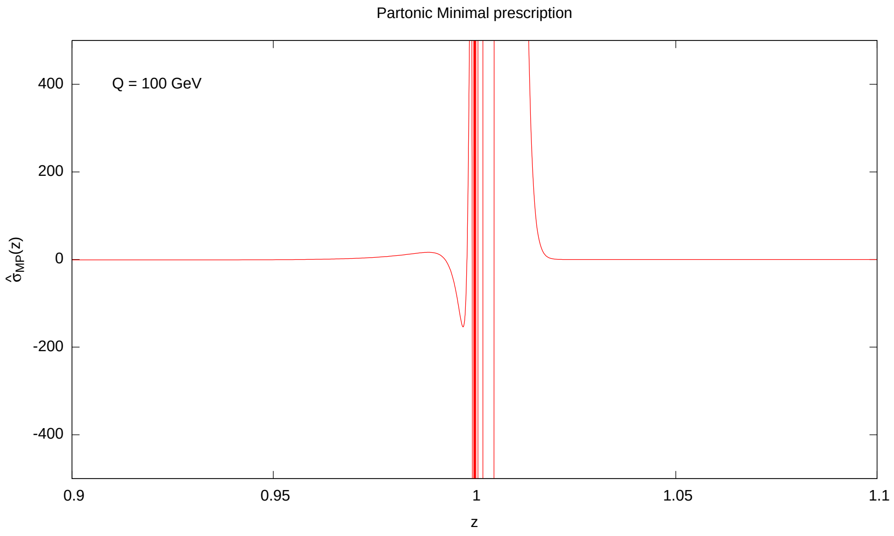
Way out [Catani, Mangano, Nason, Trentadue]: just don't care!

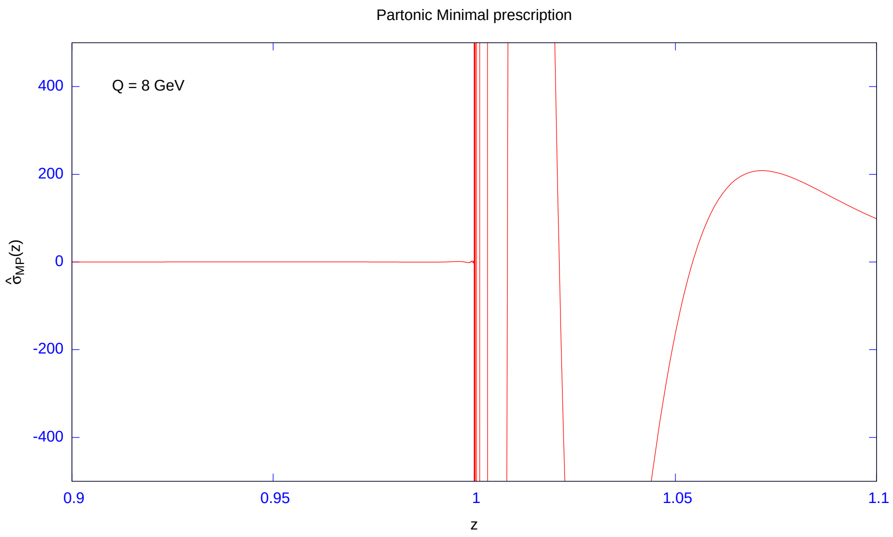


Take the inverse Mellin transform as usual, with $c \ll N_L$.

Usually referred to as the Minimal Prescription.

Drawback: "inverse Mellin" different from zero (and oscillating) for $x > 1$.





[Bonvini, Forte, R 2010]

- **Not a serious problem:** The power expansion of the MP formula is free of ambiguities of the order of powers of $\frac{\Lambda_{\rm QCD}}{Q}$ The ambiguity associated with the asymptotic expansion of the MP resummation is exponentially suppressed, $e^{-\frac{Q}{\Lambda_{\rm QCD}}}$.
- **Alternative prescriptions available**, e.g. Borel sum [Forte, Ubiali, Rojo, R 2006; Abbate, Forte, R 2007; Bonvini, Forte, R 2009]; small differences.

4. When is resummation relevant?

At collider energies, $x = \frac{Q^2}{s}$ typically very small ($\sim 10^{-4}$ for Higgs production at the LHC). However

$$\sigma_{\rm hadr}(Q^2, x) = \int_x^1 \frac{dz}{z}\,\mathcal{L}(z)\sigma(Q^2, \frac{x}{z})$$

What really matters is the range of z that dominates the convolution integral:

$$\frac{x}{z} = \frac{Q^2}{sz}$$

can be significantly larger than x, depending on the shape of parton densities.

Can we make this quantitative?

Mellin transform turns convolutions into ordinary products:

$$\tilde{\sigma}_{\rm hadr}(Q^2, N) = \tilde{\mathcal{L}}(N)\tilde{\sigma}(Q^2, N)$$

$$\begin{aligned}\sigma_{\rm hadr}(Q^2, x) &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\tilde{\mathcal{L}}(N)\tilde{\sigma}(Q^2, N) \\ &= \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, \exp E(N, x)\end{aligned}$$

The exponent

$$E(N, x) = N\log\frac{1}{x} + \log\tilde{\mathcal{L}}(N) + \log\tilde{\sigma}(Q^2, N)$$

has always a mimimum at $N = N_0(x)$ on the real N axis,

$$E'(N_0(x), x) = 0; \qquad E''(N_0(x), x) > 0$$

Saddle-point approximation:

$$\sigma_{\rm hadr}(Q^2,x) \approx \frac{1}{\sqrt{2\pi E''(N_0(x),x)}} x^{-N_0(x)} \tilde{\mathcal{L}}(N_0(x)) \tilde{\sigma}(Q^2, N_0(x))$$

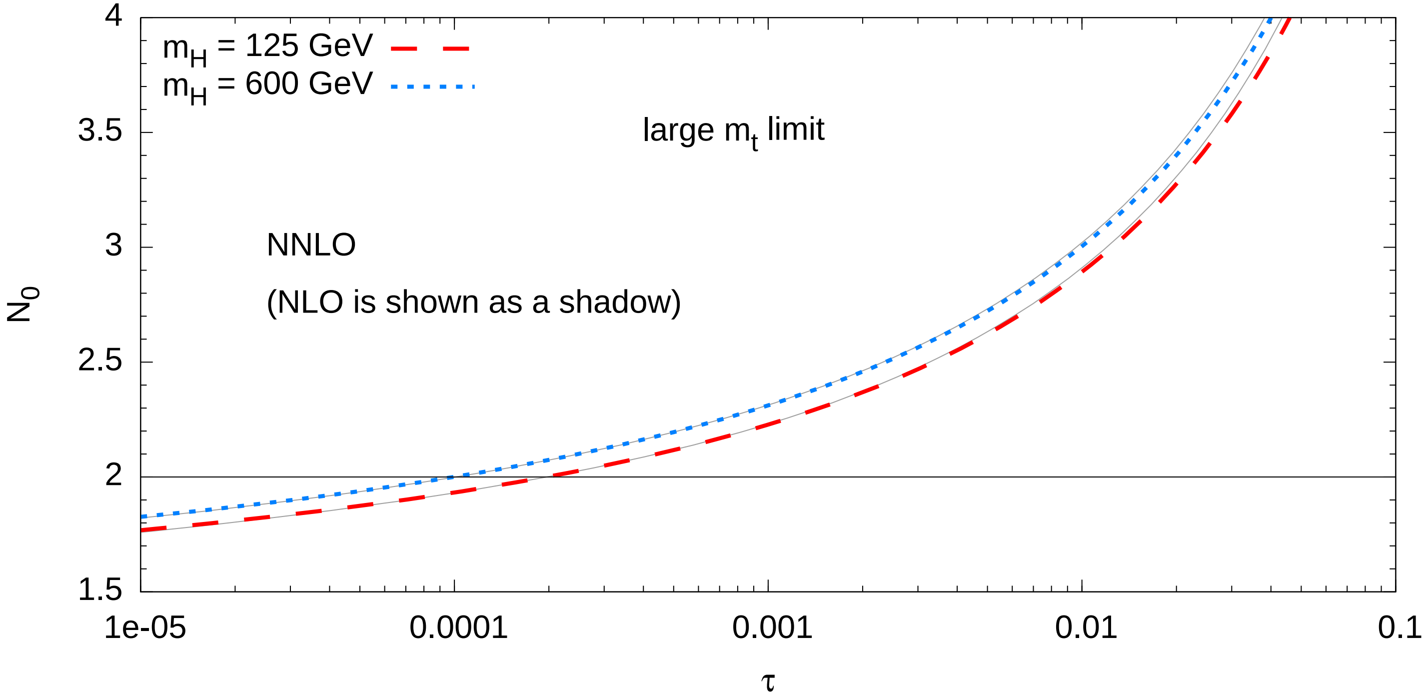
Many nice features:

- **very accurate**
- **both N_0 and E'' essentially independent of $\tilde{\sigma}$, mostly determined by parton luminosity**
- **cross section in physical space expressed as an ordinary (as opposed to convolution) product.**[a]

Is $N_0(x)$ large for interesting values of x?

[a] Useful for comparisons with SCET results [Bonvini, Forte, Ghezzi, R 2012; Bonvini, Forte, Rottoli, R 2015]

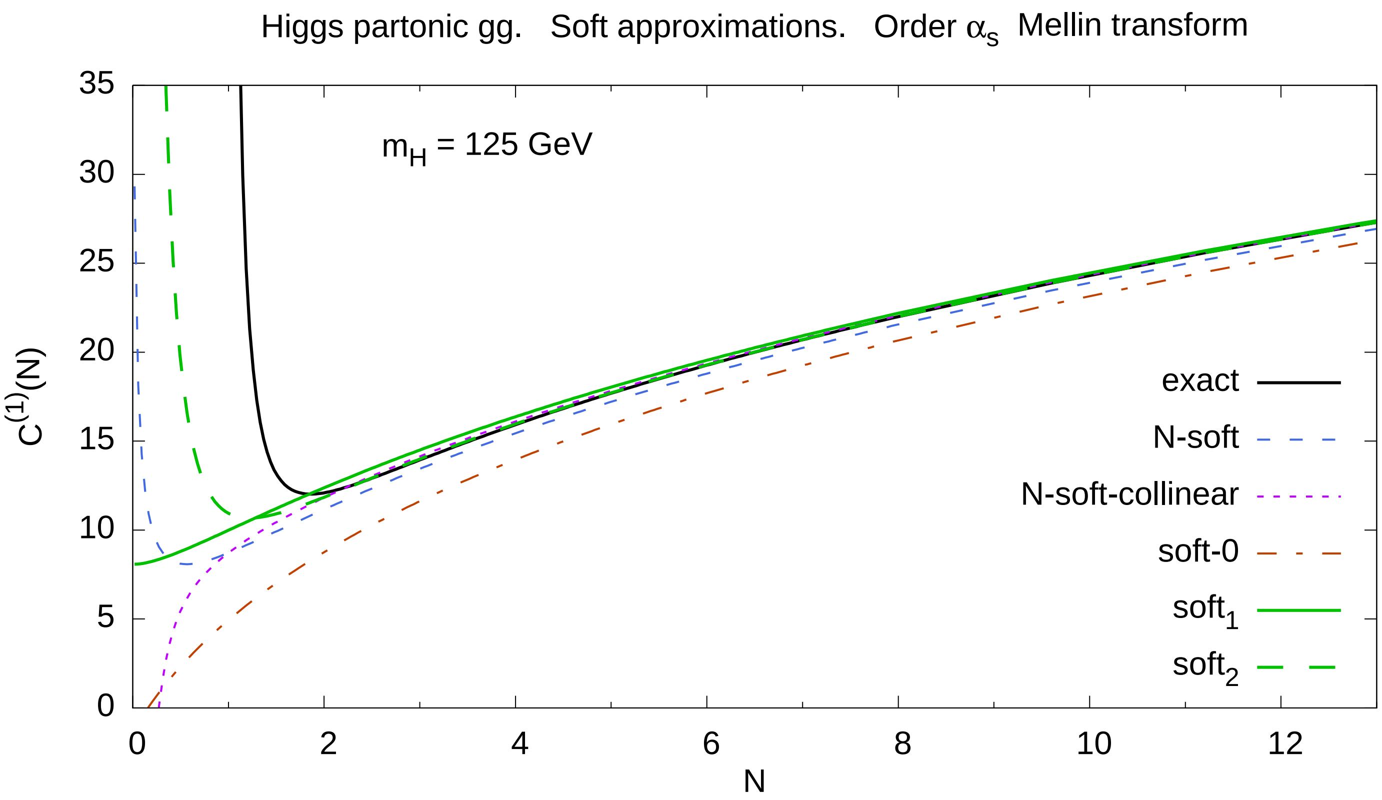
Not really:



[Bonvini, Forte, R 2012]

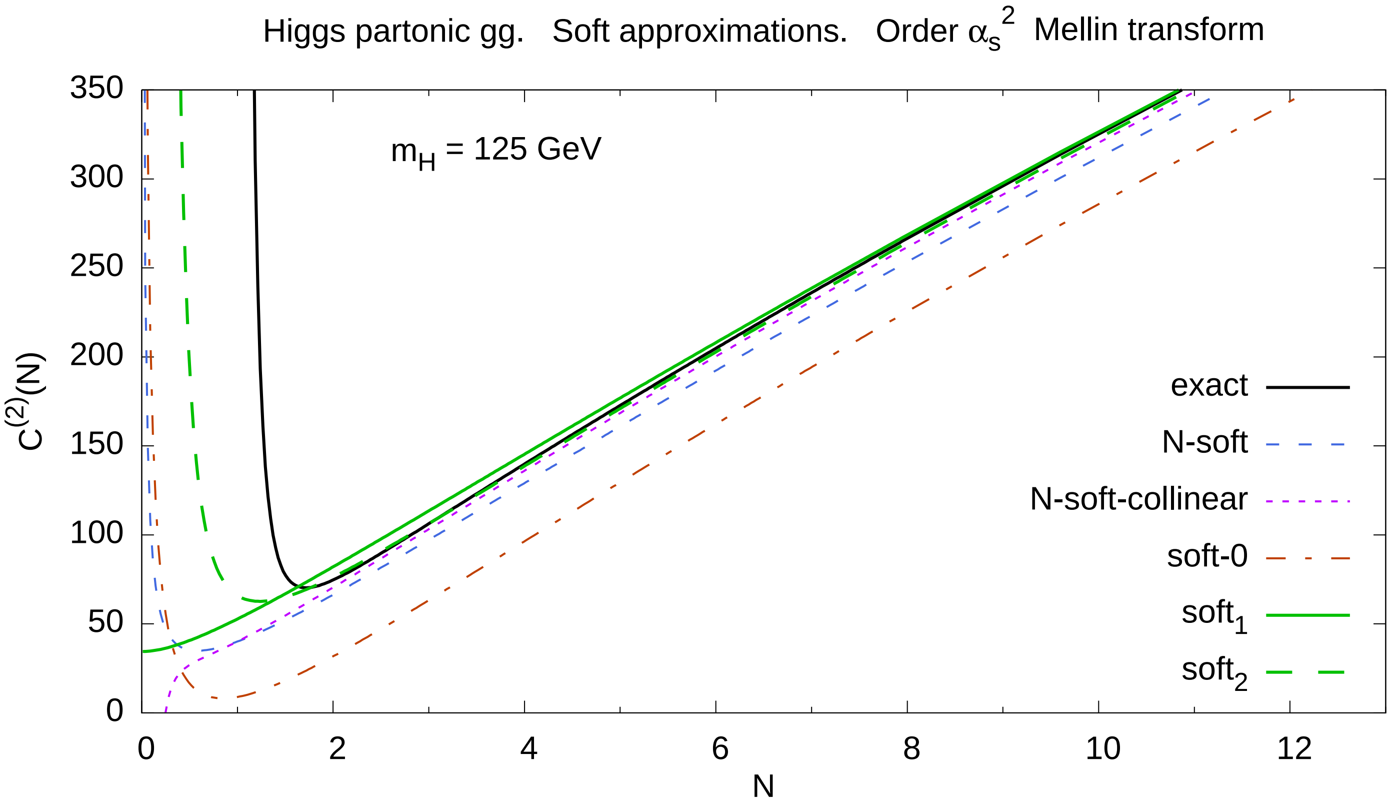
But ...

$$\tilde{\sigma}(m_H^2, N) = 1 + \alpha_s(m_H^2) C^{(1)}(N) + O(\alpha_s^2)$$



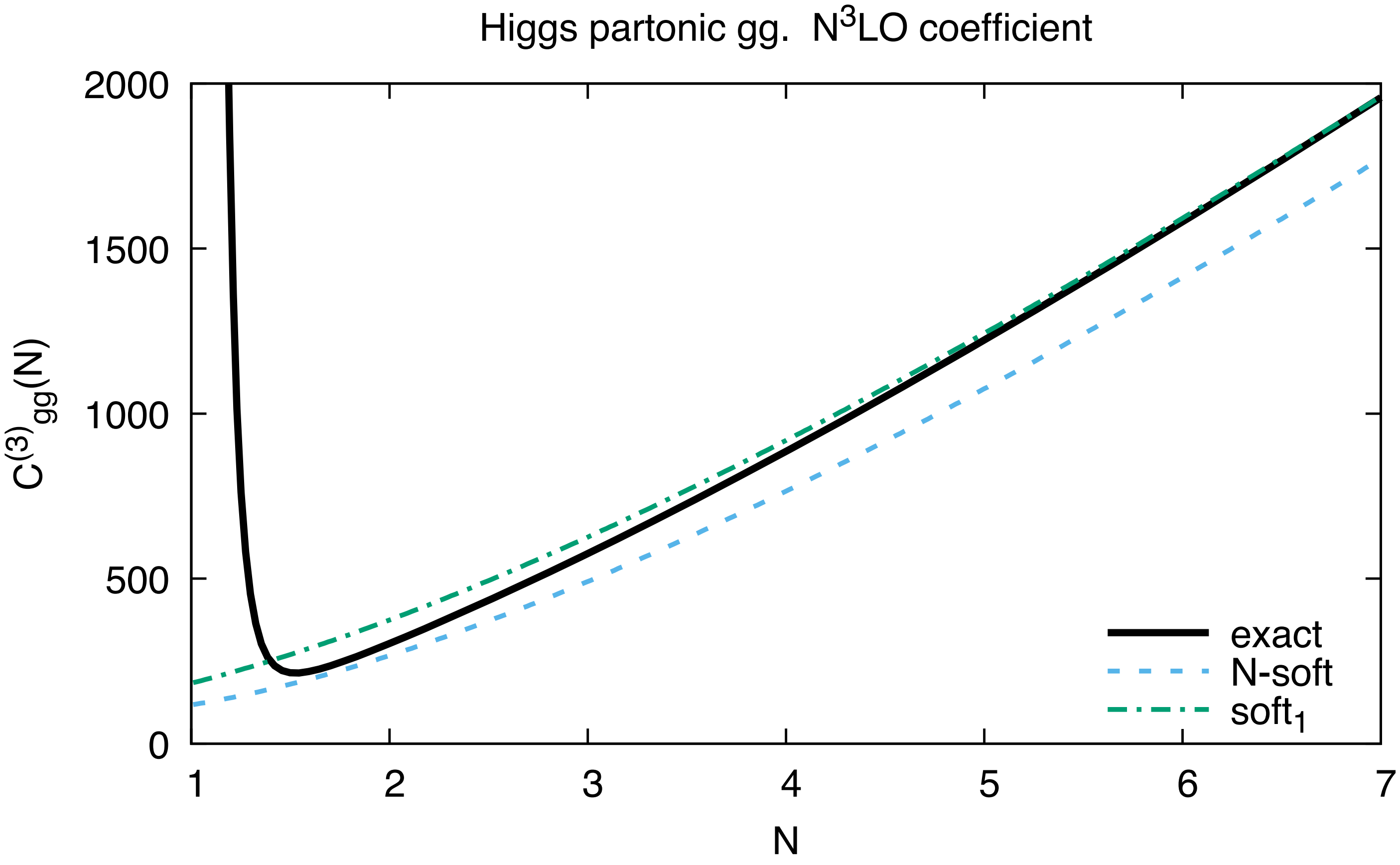
[Ball, Bonvini, Forte, Marzani, R 2012]

$$\tilde{\sigma}(m_H^2, N) = 1 + \alpha_s(m_H^2)C^{(1)}(N) + \alpha_s^2(m_H^2)C^{(2)}(N) + O(\alpha_s^3)$$



[Ball, Bonvini, Forte, Marzani, R 2012]

$$\tilde{\sigma}(m_H^2, N) = 1 + \alpha_s(m_H^2)C^{(1)}(N) + \alpha_s^2(m_H^2)C^{(2)}(N) + \alpha_s^3(m_H^2)C^{(2)}(N) + O(\alpha_s^4)$$



[Bonvini, R 2022]

Known perturbative coefficients dominated by log contributions down to small values of N.

Questions:

- **Is it true also for other processes?**
- **Is it true also at higher orders?**
- **Why?**

5. Improved log approximations

Include cleverly-chosen subleading terms in order to improve the accuracy. The inverse Mellin of powers of $\log N$,

$$\mathcal{D}_k^{\log}(x) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N} \log^k N = \left[\frac{\log^{k-1}\log x}{\log x}\right]_+$$

differs from what is found in explicit perturbative calculations

$$\mathcal{D}_k(x) = \left[\frac{\log^{k-1}(1-x)}{1-x}\right]_+$$

by non-logarithmically enhanced terms:

$$\log x = -(1-x) + O((1-x)^2))$$

Even better:

$$\log(1-x) \to \log \frac{1-x}{\sqrt{x}}$$

(also a subleading correction) for kinematical reasons (upper bound in the k_T integration).

In general

$$\mathcal{D}_k^{\log}(x) \to \hat{\mathcal{D}}_k(x) = \left[\frac{\log^{k-1}(1-x)}{1-x}\right]_+ - \frac{\log^{k-1}\sqrt{x}}{1-x}$$

Mellin transform computable in terms of polygamma functions; no unphysical branch cut on the negative real N axis.

6. Differential resummation of soft logarithms

- **Transverse momentum distributions**
  - **resummation of powers of $\log N$ at fixed q_T to NLL** [De Florian, Kulesza, Vogelsang, 2006]
  - **generalization to all-order accuracy** [Forte, Rota, R 2021]
- **Rapidity distributions**
- **Fully differential: work in progress**

Many thanks to

- **the organizers: Luca, Marco and Vittorio**
- **Marco Bonvini, Stefano Forte, Paolo Nason, Maria Ubiali**

Best wishes Guido!