September 2022

50 years of lepton pair production

Keith Ellis
IPPP, Durham

The beginning…

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Observation of Massive Muon Pairs in Hadron Collisions*

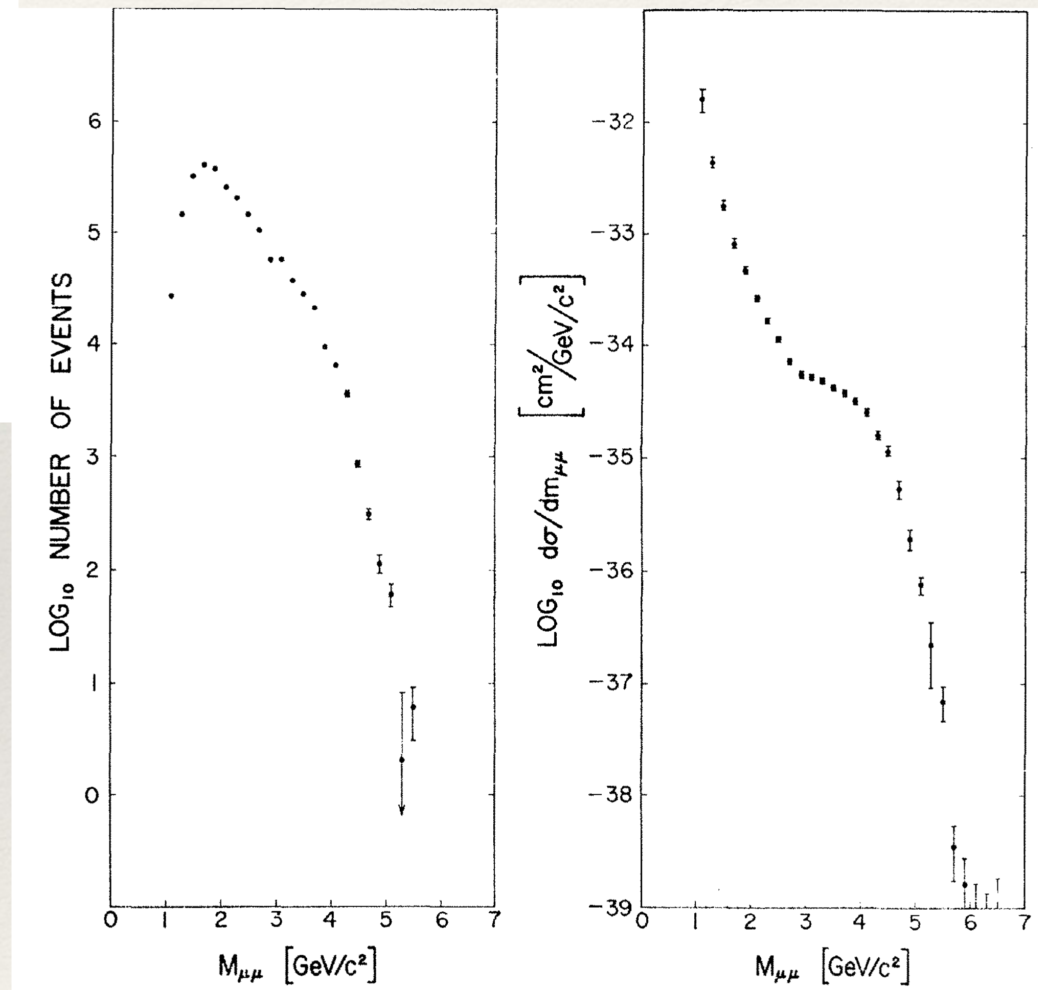
J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, and B. G. Pope
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

E. Zavattini
CERN Laboratory, Geneva, Switzerland
(Received 8 September 1970)

September 1970

Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV/c^2 have been observed in collisions of high-energy protons with uranium nuclei. At an incident energy of 29 GeV, the cross section varies smoothly as $d\sigma/dm_{\mu\mu} \approx 10^{-32}/m_{\mu\mu}^{5}$ cm^2 (GeV/c)$^{-2}$ and exhibits no resonant structure. The total cross section increases by a factor of 5 as the proton energy rises from 22 to 29.5 GeV.

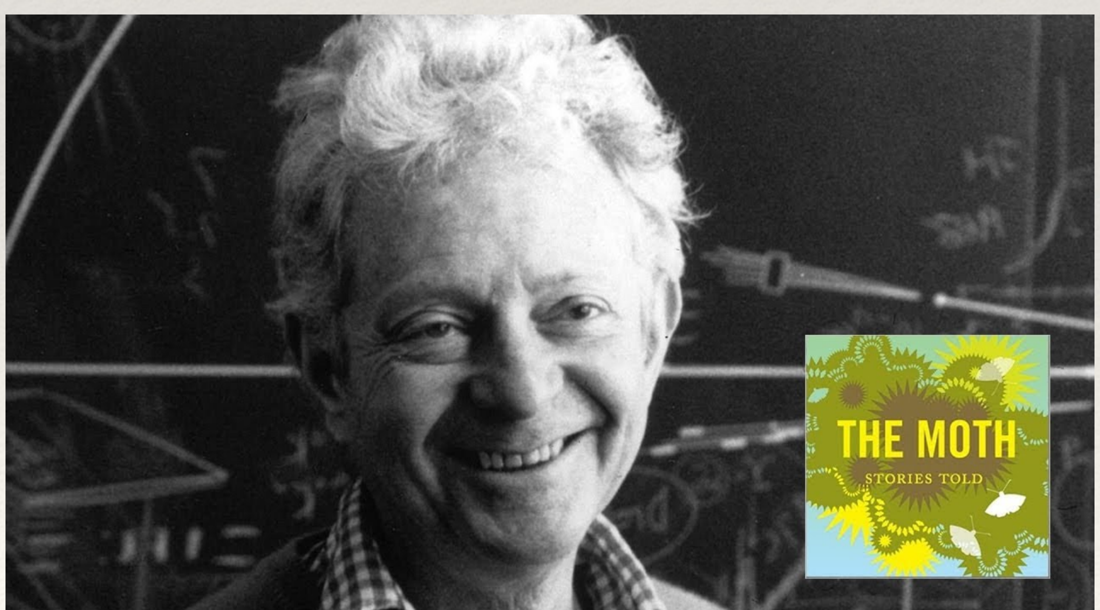


- "As seen both in the mass spectrum and the resultant cross section there is no forcing evidence of any resonant structure."
- Leon credits Yamaguchi and Okun for suggesting lepton pair processes.
- An Italian connection - talked eloquently about the role of Gilberto Bernardini in restoring his sense of wonder about the natural world

Leon on Drell-Yan

> I come now to the Drell-Yan process i.e. dilepton production in hadronic collisions, (sigh!) named by Feynman after an experiment at BNL by Christenson. Here there is consid-

Lederman, Batavia Conference, 9th International Symposium on Lepton and Photon Interactions at High Energy, (1979)



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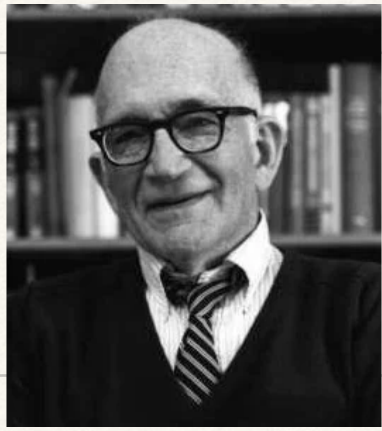


Drell-Yan

- Drell and Yan showed that the parton model could be derived if the impulse approximation was valid.
- To accomplish this, they had to impose a transverse momentum cut-off for the particles that appeared in the quantum field theory.

Assumed anti-parton distributions= parton distributions!

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\mathscr{F}(\tau) = \frac{4\pi\alpha}{3Q^2}\frac{1}{Q^2}\int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F'_{2\bar{a}}(x_2)$$

No color factor!

Unknown! parton charges

- Rapid fall-off of the cross section, despite the fact that the partons were point-like particles (in contrast to DIS).

cf, Altarelli, Brandt & Preparata, PRL (1970)

The first Drell Yan prediction

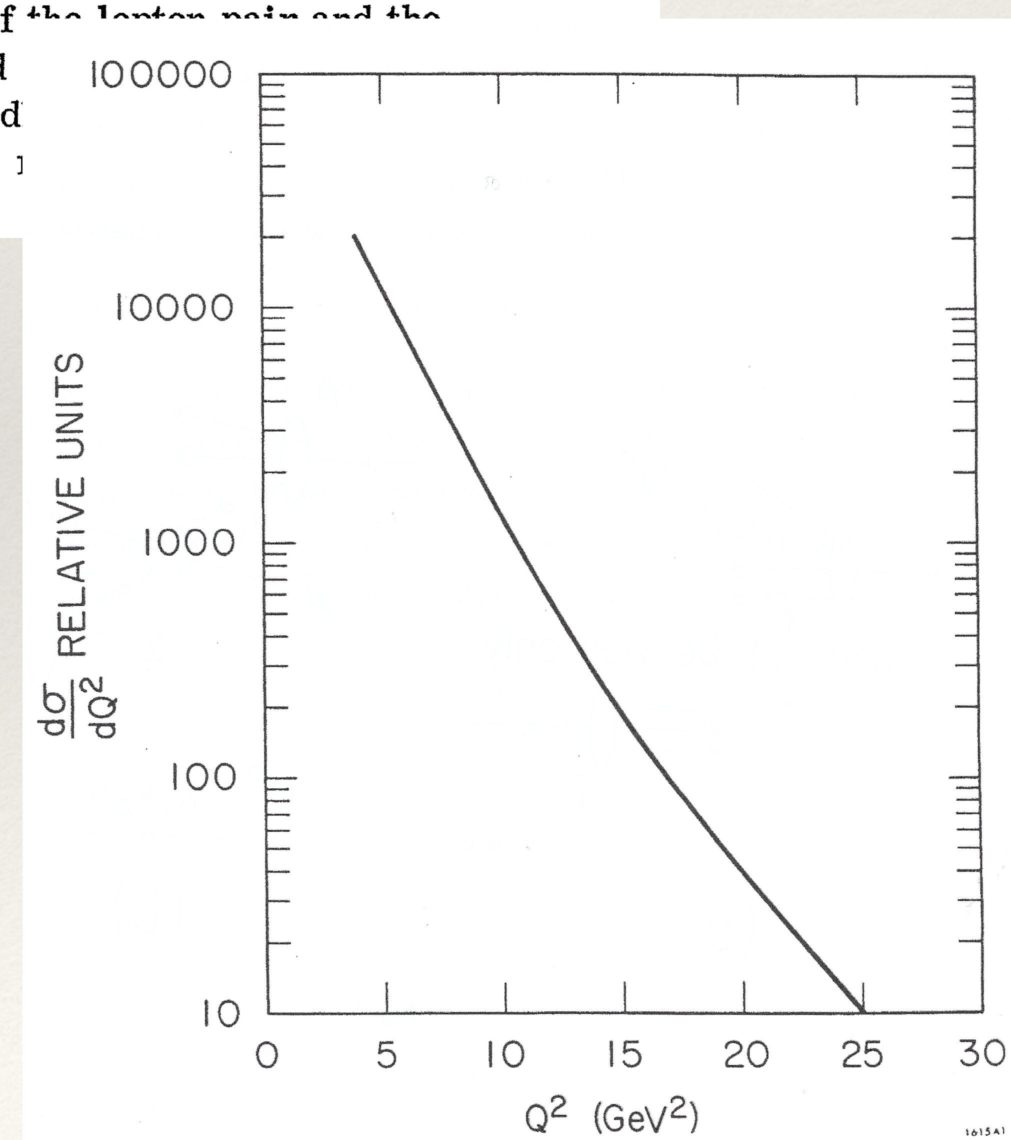
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

May1970!

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of
two initial hadrons, respectively. General scaling properties and
inelastic electron scattering are discussed. In particular, a rapid
section as $Q^2/s \to 1$ is predicted as a consequence of the observed
elastic scattering structure function νW_2 near threshold.

- Predictions are
  - approximate scaling $\frac{Q^3 d\sigma}{dQ} = F(\tau),\ \tau = Q^2/s,$
  - angular dependence, $(1 + \cos^2\theta)$
  - A^1 dependence on nucleon number.



Asymptotic freedom expands it scope

- The publication of the DGLAP equation with its physical picture of parton evolution, raised the issue of whether the Drell-Yan model could be extended to QCD.
- Politzer (1977) deserves credit for outlining the factorization idea.
- Unlike in the parton model, the transverse momentum is now unbounded.
- Transverse momentum in Drell-Yan processes (APP) and AEM (1979) followed Politzer's lead regulating collinear/soft singularities by continuing off-shell, (which turned out to be a tricky procedure).

cf, Sachrajda, 2/1978 - Lepton pair production and the Drell-Yan formula in QCD

The Ellis-Martinelli collaboration.

- 10 papers in all (1978-1985)
- 5 papers AEM
- 2 papers EMP
- 1 paper AEMG
- 2 papers EM (Perturbative lattice gauge theory)

We had written a previous paper including (erroneous) corrections to DY as a postscript

LARGE PERTURBATIVE CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD *

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Center for Theoretical Physics,
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G. MARTINELLI
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Laboratori Nazionali di Frascati,
Frascati 00044, Italy

Received 17 April 1979

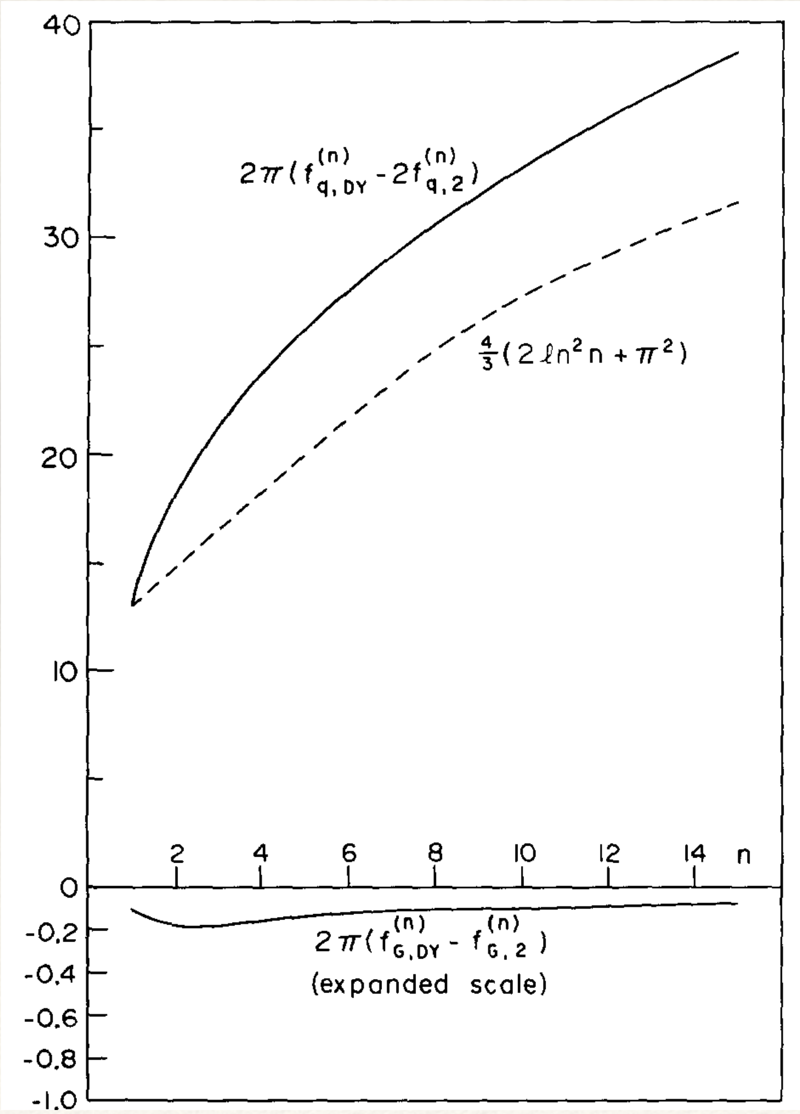
Marciano(1975) - Dimensional Regularization and Mass singularities

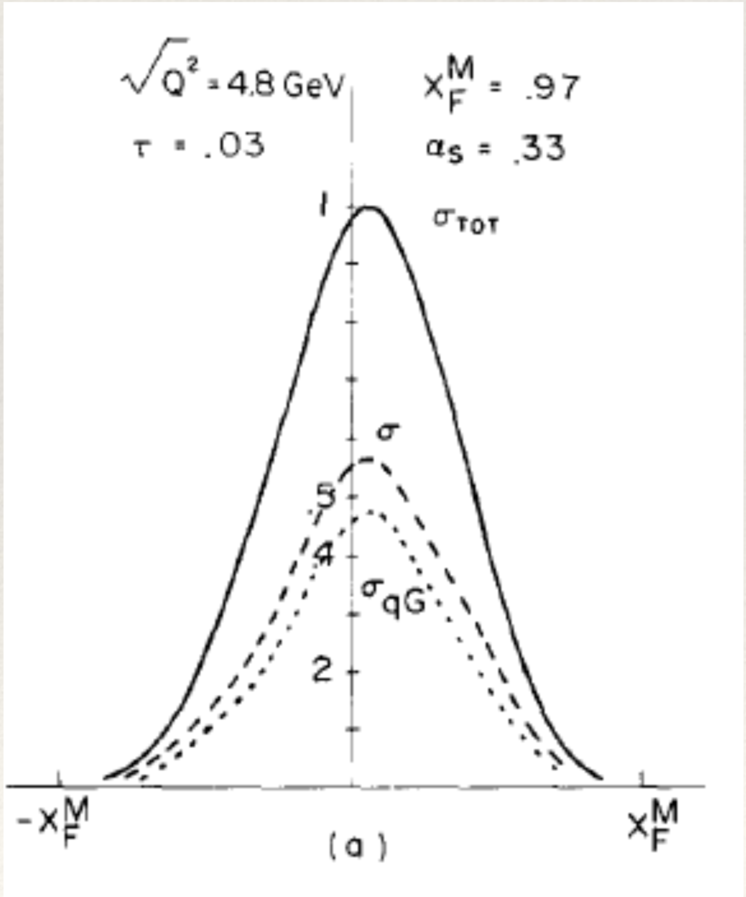
First QCD corrections for hadron-hadron interactions

$$\alpha_s f_q(z) = C_F \frac{\alpha_s}{2\pi}\left[\left(1+\frac{4\pi^2}{3}\right)\delta(1-z) + 2(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + \frac{3}{(1-z)_+} - 6 - 4z\right]$$

$$\alpha_S f_G(z) = \frac{1}{2}\frac{\alpha_s}{2\pi}\left[(z^2+(1-z)^2)\ \ln(1-z) + \frac{9}{2}z^2 - 5z + \frac{3}{2}\right]$$

- Correction relative to DIS
- $\dfrac{\alpha_S}{2\pi} \approx \dfrac{1}{20}$
- Simple origin for the large size of the corrections;
- Phenomenology, x_F distribution;



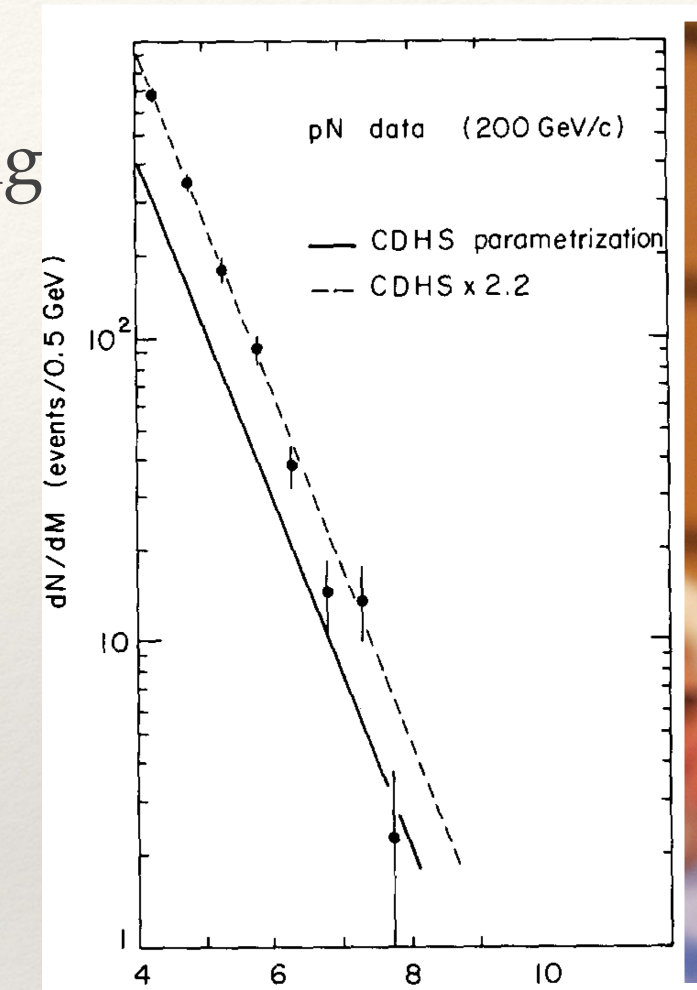


Drell-Yan data and K-factor

- Data lay above the DY prediction, leading to the introduction of a "K-factor"

$$\left(\frac{d\sigma}{dQ^2}\right)_{EXP} = K \left(\frac{d\sigma}{dQ^2}\right)_{NAIVE\ D.Y.}$$

- From ~4 experiments $K \geq 2$
- Telegdi question

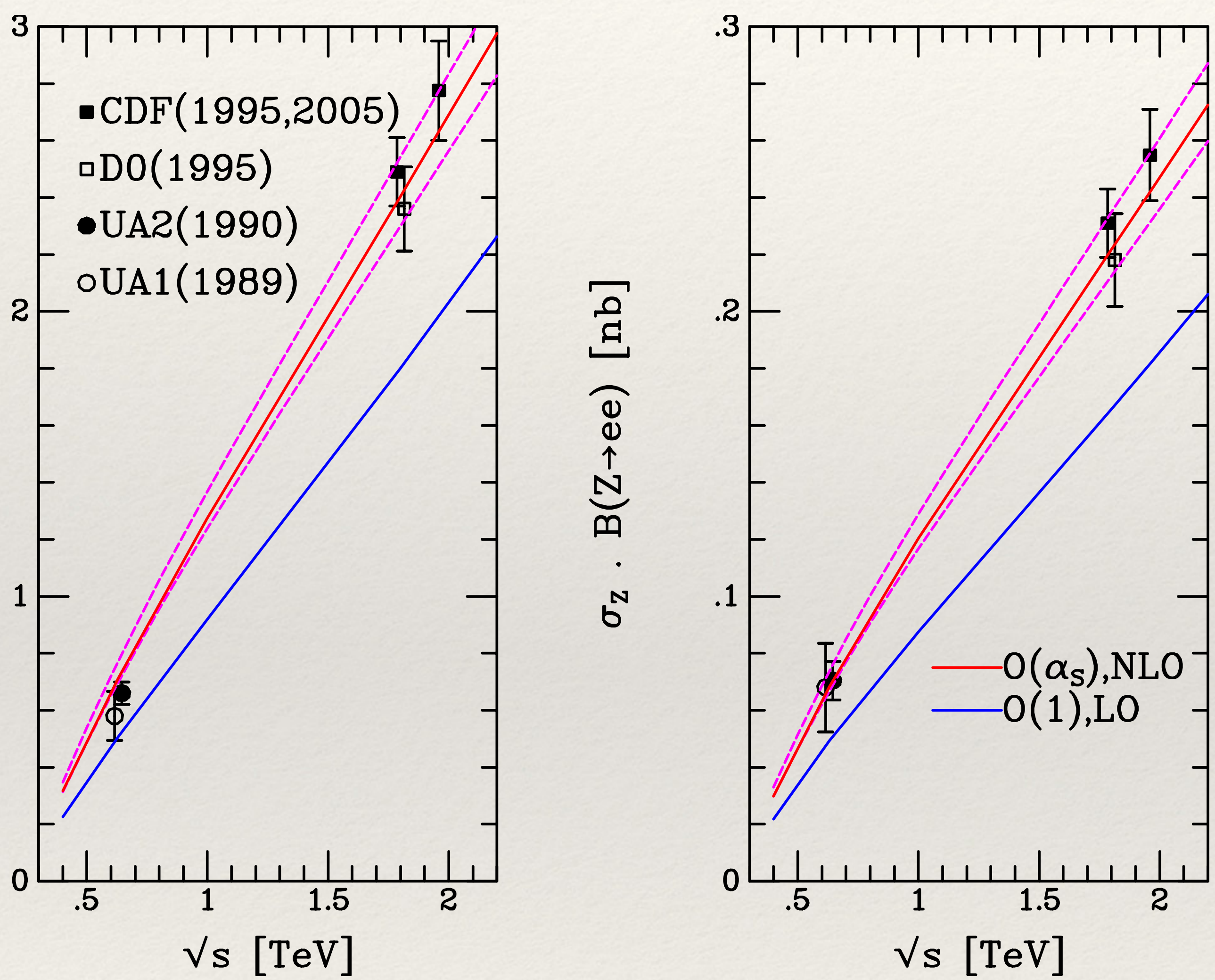


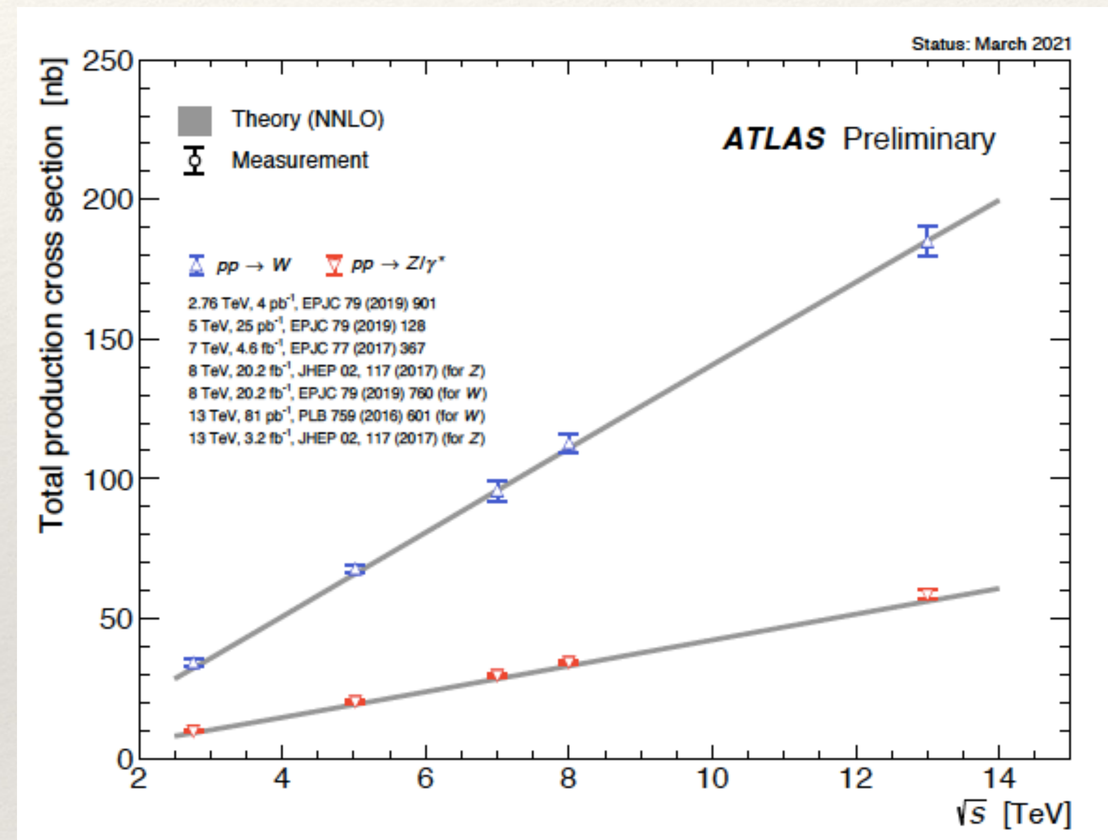
$K = (d^2\sigma/dx_1dx_2)_{exp}/(d^2\sigma/dx_1dx_2)_{DY\ model}$.

Reaction	pN	$\bar{p}$N	π^-N	π^+N	π^-H_2	$(\pi^- - \pi^+)$N
K	2.2 ± 0.4	2.4 ± 0.5	2.2 ± 0.3	2.4 ± 0.4	2.4 ± 0.4	2.2 ± 0.4
Events	960	44	5607	2073	138	–

NA3, Badier et al,

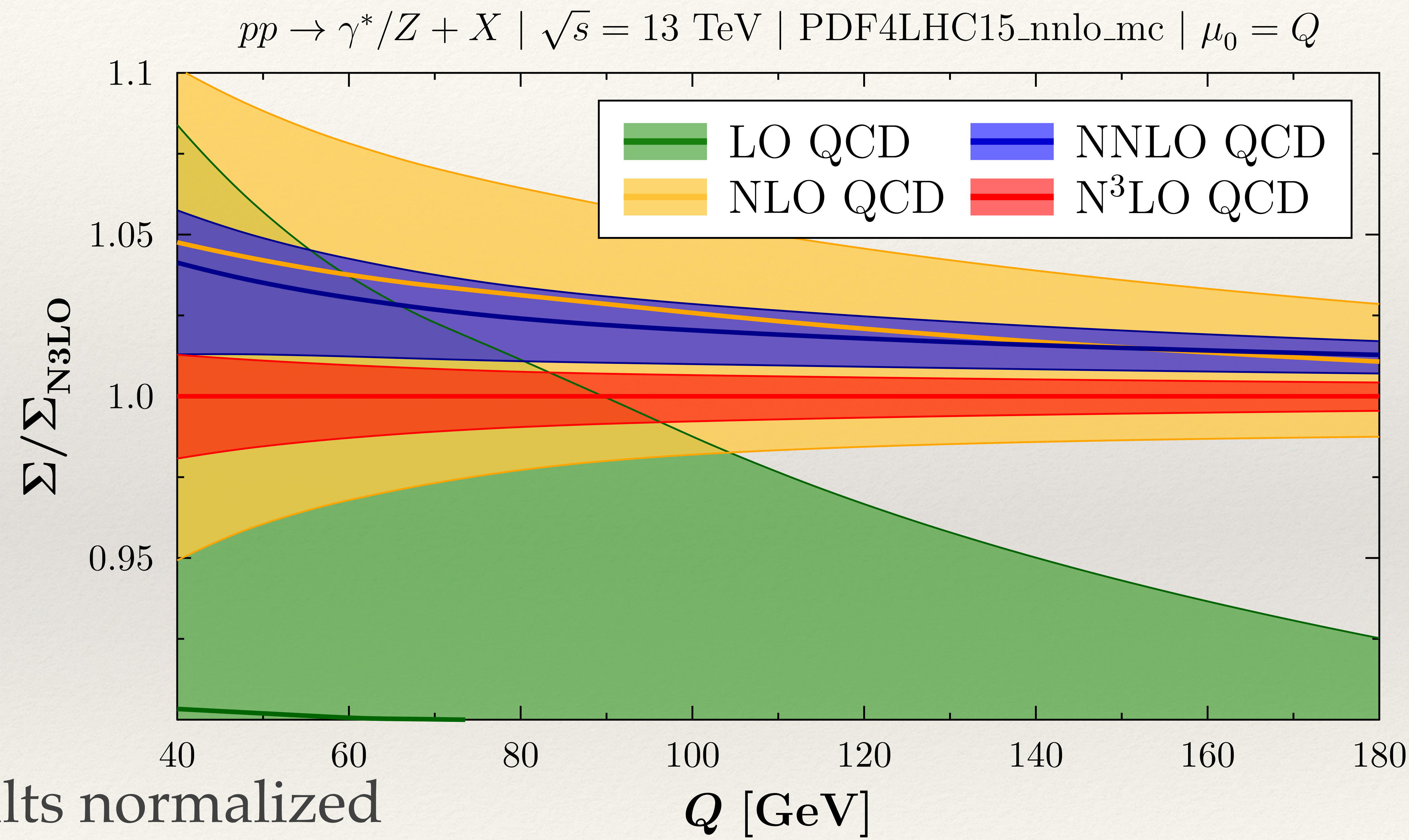
Experimental Situation for massive boson prediction





- Plots show the necessity of NLO corrections, and current ATLAS results compared with NNLO calculations.

N³LO results for Z/γ* production



- Results normalized to N³LO

Baglio et al, 2209.06138

If this were a proper history….

- First NNLO calculation of Drell-Yan process Hamberg, Van Neerven, Matsuura
- Issue of whether initial state interactions compromise factorization raised Brodsky, Bodwin and Lepage
    - Low order demonstration of factorization for Drell-Yan process,Lindsay, Ross, Sachrajda (1983)
  - Situation was summarized in 2004 by Collins, Sterman,Soper "recent work has, we believe, established its validity at all orders. Nevertheless, …. there is plenty of room for improvement in our understanding."

Factorization in Drell-Yan

- Simple classical argument (Basu et al, 1984)
- Consider a particle with charge e travelling in the positive z direction with constant velocity β.
- At large γ, constant fields at arbitrary time, not in sync with the arrival of the particle.
- However such large fields are pure gauge, and do not have physical consequences.
- Argument suggests corrections to DY picture at order $1/Q^4$

$$A^\mu(t,\vec{x}) = \int dt' d\vec{x}' \frac{J^\mu(t',\vec{x}')}{|\vec{x}-\vec{x}'|} \delta(t' + |\vec{x}-\vec{x}'| - t)$$

$$J^t(t,\vec{x}) = e\,\delta(\vec{x} - \vec{r}(t)) \qquad \vec{r}(t) = \beta t \hat{z}$$

$$J^z(t,\vec{x}) = e\,\beta\,\delta(\vec{x} - \vec{r}(t))$$

$$A^t(t,\vec{x}) = \frac{e\gamma}{\sqrt{[x^2 + y^2 + \gamma^2(\beta t - z)^2]}}$$

$$A^x(t,\vec{x}) = 0$$

$$A^y(t,\vec{x}) = 0$$

$$A^z(t,\vec{x}) = \frac{e\gamma\beta}{\sqrt{[x^2 + y^2 + \gamma^2(\beta t - z)^2]}}$$

,

NNLO results

- In a recent paper (2202.07738) we tried to document all the processes calculated at NNLO.
- About 50% are available in MCFM.
- We use both q_T slicing and jettiness slicing.

Most apart from heavy quark and jet production are generalizations of Drell-Yan

Process	MCFM	Process	MCFM
$H + 0$ jet [8–14]	✓ [15]	$W^\pm + 0$ jet [16–18]	✓ [15]
$Z/\gamma^* + 0$ jet [11, 17–19]	✓ [15]	ZH [20]	✓ [21]
$W^\pm\gamma$ [18, 22, 23]	✓ [24]	$Z\gamma$ [18, 25]	✓ [25]
$\gamma\gamma$ [18, 26–28]	✓ [29]	single top [30]	✓ [31]
$W^\pm H$ [32, 33]	✓ [21]	WZ [34, 35]	✓
ZZ [1, 18, 36–40]	✓	W^+W^- [18, 41–44]	✓
$W^\pm + 1$ jet [45, 46]	[3]	$Z + 1$ jet [47, 48]	[4]
$\gamma + 1$ jet [49]	[5]	$H + 1$ jet [50–55]	[6]
$t\bar{t}$ [56–61]		$Z + b$ [62]	
$W^\pm H$+jet [63]		ZH+jet [64]	
Higgs WBF [65, 66]		$H \to b\bar{b}$ [67–69]	
top decay [31, 70, 71]		dijets [72–74]	
$\gamma\gamma$+jet [75]		$W^\pm c$ [76]	
$b\bar{b}$ [77]		$\gamma\gamma\gamma$ [78]	
HH [79]		HHH [80]	

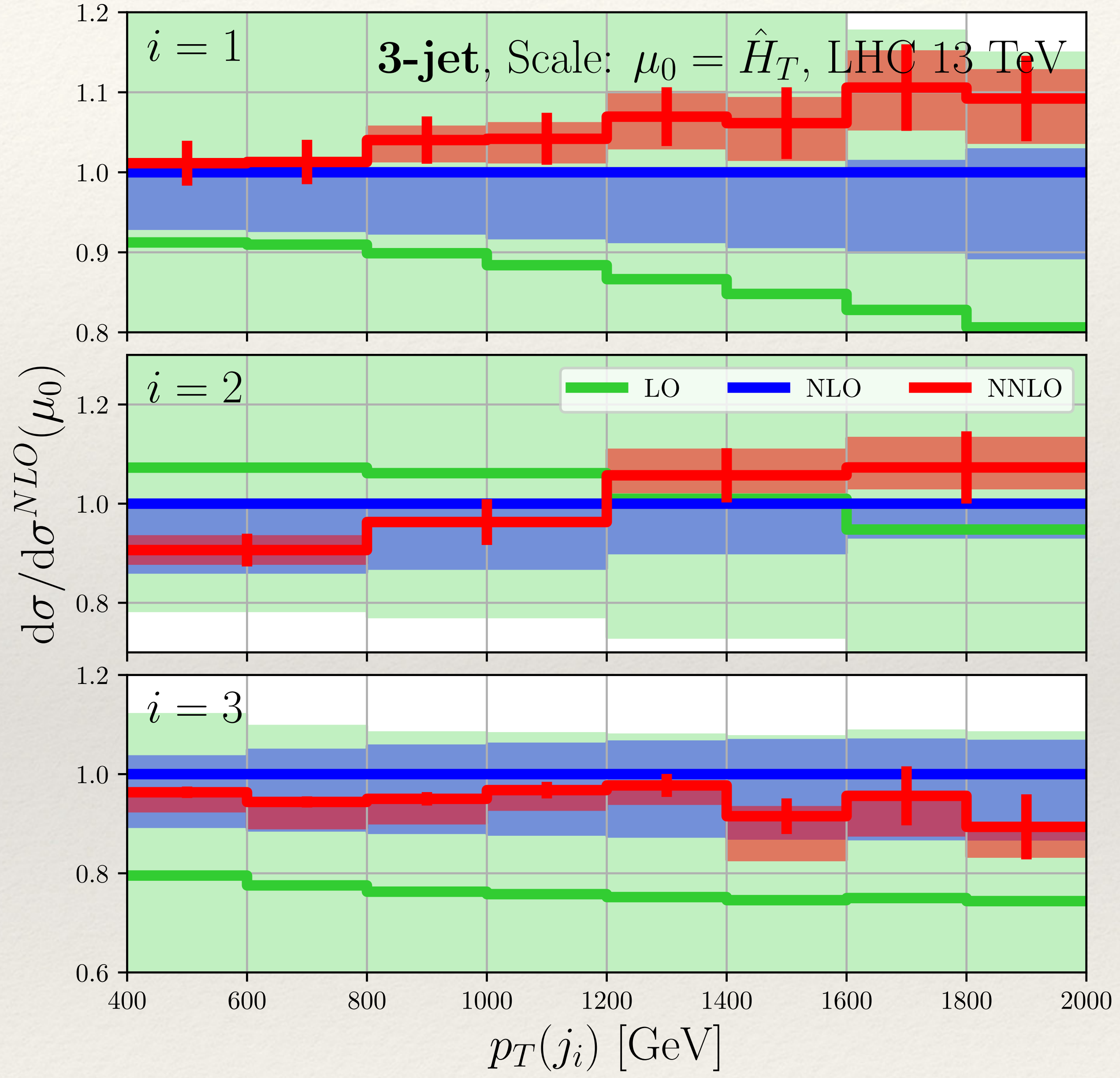
Examples of NNLO results from MCFM

Process		target		MCFM		
	$\sigma_{NLO^\star}$	σ_{NNLO}	δ_{NNLO}	σ_{NNLO}	δ_{NNLO}	
$pp \to H$	29.78(0)	39.93(3)	10.15(3)	39.91(5)	10.13(5)	nb
$pp \to Z$	56.41(0)	55.99(3)	−0.42(3)	56.03(3)	−0.38(3)	nb
$pp \to W^-$	79.09(0)	78.33(8)	−0.76(8)	78.41(6)	−0.68(6)	nb
$pp \to W^+$	106.2(0)	105.8(1)	−0.4(1)	105.8(1)	−0.4(1)	nb
$pp \to \gamma\gamma$	25.61(0)	40.28(30)	14.67(30)	40.19(20)	14.58(20)	pb
$pp \to e^-e^+\gamma$	2194(0)	2316(5)	122(5)	2315(5)	121(5)	pb
$pp \to e^-\bar{\nu}_e\gamma$	1902(0)	2256(15)	354(15)	2251(2)	349(2)	pb
$pp \to e^+\nu_e\gamma$	2242(0)	2671(35)	429(35)	2675(2)	433(2)	pb
$pp \to e^-\mu^-e^+\mu^+$	17.29(0)	20.30(1)	3.01(1)	20.30(2)	3.01(2)	fb
$pp \to e^-\mu^+\nu_\mu\bar{\nu}_e$	243.7(1)	264.6(2)	20.9(3)	264.9(9)	21.2(8)	fb
$pp \to e^-\mu^-e^+\bar{\nu}_\mu$	23.94(1)	26.17(2)	2.23(3)	26.18(3)	2.24(2)	fb
$pp \to e^-e^+\mu^+\nu_\mu$	34.62(1)	37.74(4)	3.12(5)	37.78(4)	3.16(3)	fb
$pp \to ZH$	780.0(4)	846.7(5)	66.7(6)	847.3(7)	67.3(6)	fb
$pp \to W^\pm H$	1446.5(7)	1476.1(7)	29.6(10)	1476.7(8)	30.2(4)	fb

Table 4. NLO results, computed using MCFM with NNLO PDFs (denoted $\sigma_{NLO^\star}$), total NNLO cross sections from `vh@nnlo` ($W^\pm H$ and ZH only) and MATRIX (remaining processes, using the extrapolated result from Table 6 of Ref. [24]) and the target NNLO coefficients (δ_{NNLO}, with $\delta_{NNLO} = \sigma_{NNLO} - \sigma_{NLO^\star}$). The result of the MCFM calculation (0-jettiness, fit result b_0 from Eq. (3.9)) is shown in the final column.

State of the art: 2→3 at NNLO

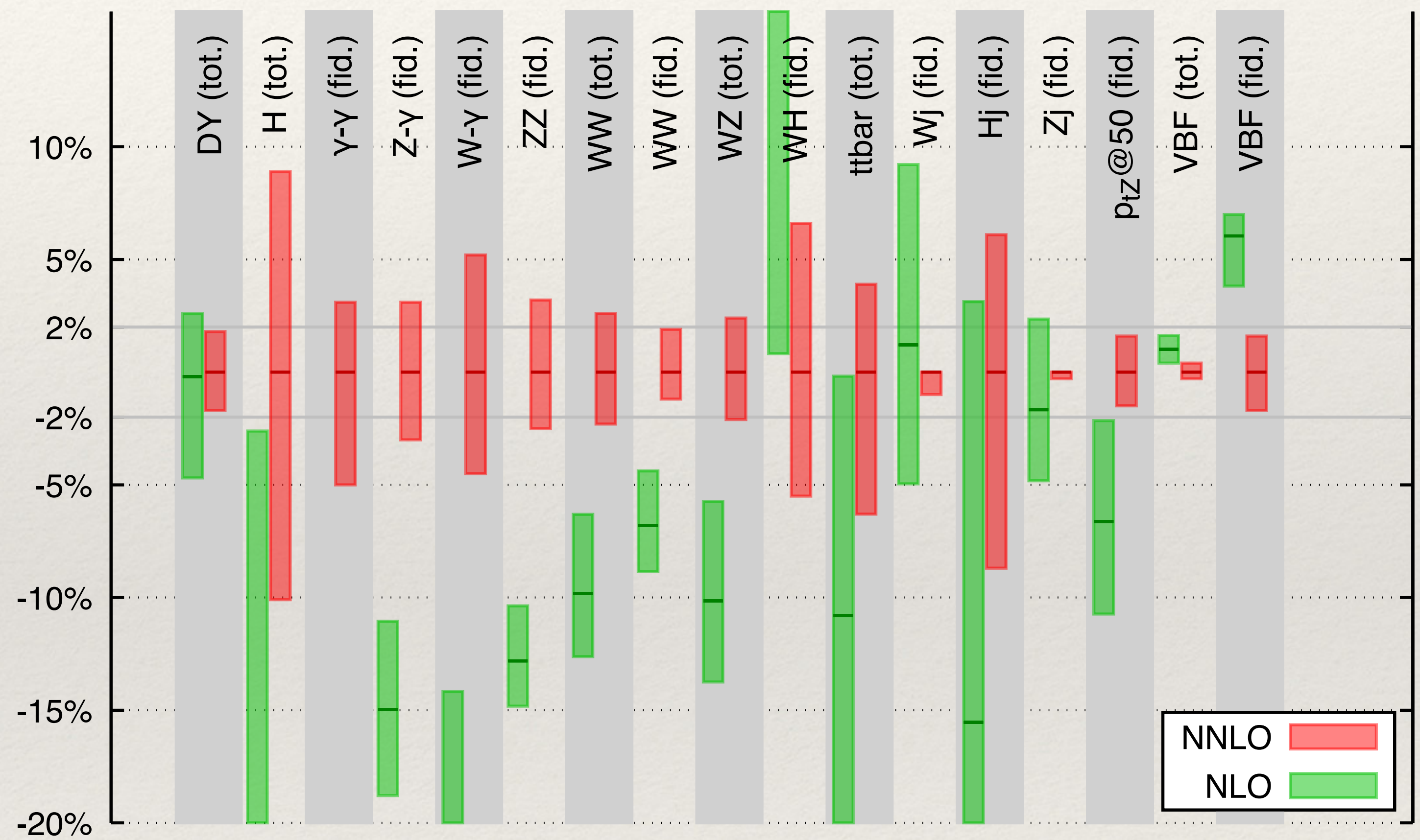
- Goes beyond the Drell-Yan paradigm;
- Exact except for leading colour for double virtual amplitude;
- Calculation of the p_T for the first, second and third jet;
- Evidence for kinematic structure in the corrections.



A first next-to-next-to-leading order study of three-jet production at the LHC, 2106.05331,

Error estimates (2016)

- NNLO results allow us to assess how large the theoretical errors are at NLO and NNLO.
- Many processes have errors at the 2% level.
- However, the NNLO central value lies within the NLO error band in only 4 out of the 17 cases shown.



Salam, Lund conference 2016

Differential distributions

Transverse momentum distribution in DY

- DDT wrote down a very beautiful formula(8/78)
- $$\frac{d\sigma}{dq^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{9sq^2q_T^2} \times \frac{\partial}{\partial \ln q_T^2} \sum_{F=q,\bar{q}} e_F^2 \, D_a^F(x_1, \ln\frac{q_T^2}{\mu^2}) \, D_b^F(x_2, \ln\frac{q_T^2}{\mu^2}) \, T^2(q_T^2, q^2)$$
- Parisi & Petronzio (2/79), based on arguments from electrodynamics, correct the form factor T. Similar conclusion by Curci et al, (3/79).
- The formulations are in b-space, (Fourier conjugate to q_T) and there is the additional result, that the shrinkage of the intercept at q_T is calculable.

$$\frac{\left.\frac{d\sigma}{dp_T^2}\right|_{p_T=0}}{\int dp_T^2 \frac{d\sigma}{dp_T^2}} = \left(\frac{\Lambda}{Q}\right)^\eta, \quad \eta \approx 0.6 \text{ (Balancing semi-hard gluons).}$$

All orders result for q_T distribution

$$\frac{d\sigma}{dQ^2 dy dq_T^2} = \frac{4\pi\alpha^2}{9Q^2 s} \int d^2b \;\exp(iq_T . b) \sum_j e_j^2$$

$$\times \sum_a \int_{x_a}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_a; 1/b) \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_b; 1/b)$$

$$\times \exp\left\{ -\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\ln \frac{Q^2}{\bar{\mu}} A(\alpha_S(\bar{\mu}) + B(\alpha_S(\bar{\mu})\right] \right\}$$

$$+ \frac{4\pi^2\alpha^2}{9Q^2 s} Y(q_T; Q, x_a, x_b)$$

$$A(\alpha_S(\mu) = \sum_{n=0}^{\infty} A^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad A^{(1)} = C_F, A^{(2)} = 2C_F\left\{ C_A(\frac{67}{18} - \frac{\pi^2}{6}) - \frac{10T_F n_f}{9} \right\}$$

$$B(\alpha_s(\mu)) = = \sum_{n=0}^{\infty} B^{(n)} \left(\frac{\alpha_S}{2\pi}\right)^n, \quad B^{(1)} = -3C_F,$$

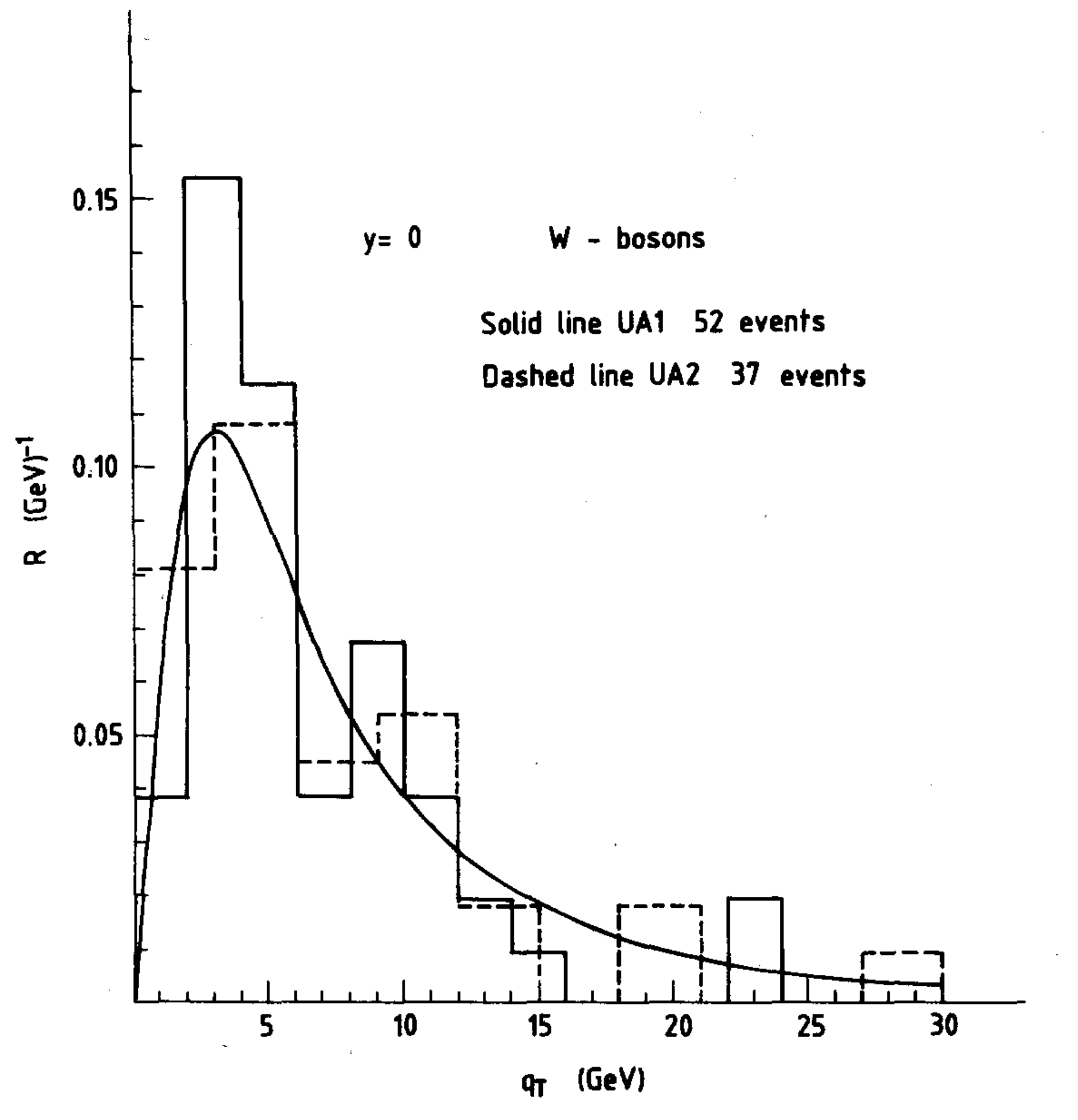
$$B^{(2)} = C_F\left[C_F(\pi^2 - \frac{3}{4} - 12\zeta_3) + C_A(\frac{11\pi^2}{9} - \frac{193}{12} + 6\zeta_3) + T_R n_f(\frac{17}{3} - \frac{4\pi^2}{9})\right]$$

Calculated using EMP results

Collins, Sterman and Soper (1984)

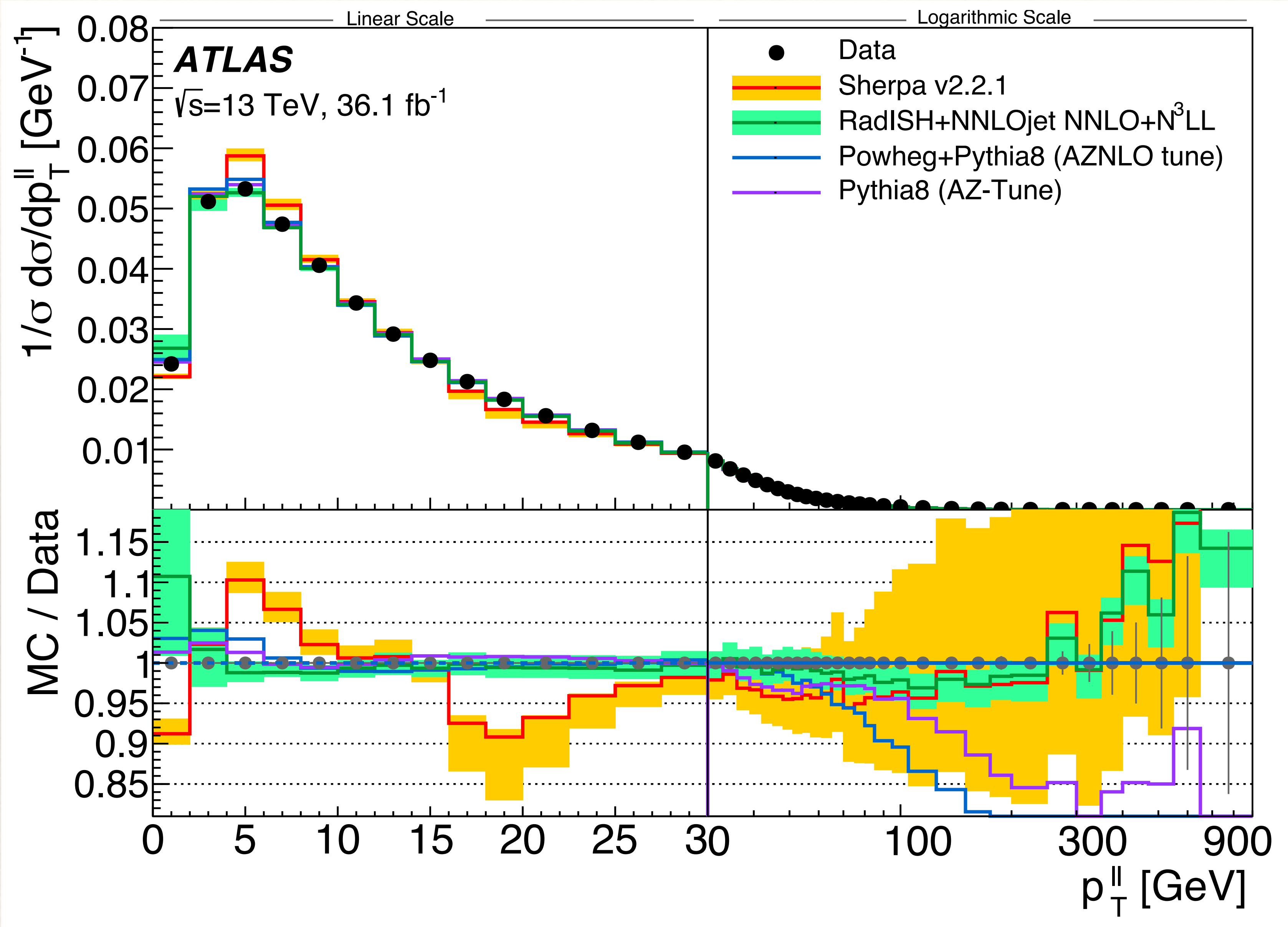
W Discovery!

- At the same time as CSS, we in AEM+Mario Greco produced q_T plots using all the available theoretical information.
- A similar plot using our prediction, with 68 UA1 events, (and without the UA2 data!) was presented by Carlo Rubbia in his Nobel lecture.



Z-p_T (2019)

- LHC results at $\sqrt{s} = 13$ TeV rather more impressive.



1912.02844

To “b” or not to “b”

$$\frac{d\sigma}{dQ^2dydq_T^2} = \frac{4\pi\alpha^2}{9Q^2s}\int d^2b\, e^{iq_T.b} \sum_j e_j^2 \sum_a \int_{x_a}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_a;1/b)\frac{d\xi_B}{\xi_B} f_{b/B}(\xi_b;1/b)$$

$$\times \exp\left\{ -\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\frac{Q^2}{\bar{\mu}}A(\alpha_S(\bar{\mu}) + B(\alpha_S(\bar{\mu})\right]\right\}$$

- b-space, (Fourier conjugate to q_T)
- Advantages
  - Elegant inclusion of transverse momentum conservation.
  - Perturbative predictions for intercept $d\sigma/dq_T^2\Big|_{q_T=0}$
- Disadvantages
  - b-integral extends to infinity; integrate over Landau pole in the coupling.
  - Handled by $b \to b_* = \dfrac{b}{\sqrt{1+(b/b_{lim})^2}}$ so $b_* < b_{lim}$ this substitution changes prediction even at large q_T
  - Difficulties with matching onto fixed order perturbation theory.

Small q_T in SCET language

$$\frac{d^2\sigma}{dq_T^2 dy} = \sigma_0 |\, C_V(M_Z^2, \mu^2)|^2 \sum_{i=q,g}\sum_{j=\bar{q},g} \int_{\xi_1}^1 \frac{dz_1}{z_1} \int_{\xi_2}^1 \frac{dz_2}{z_2} \left[C_{q\bar{q}}(z_1, z_1, q_T^2, M_Z^2, \mu)\ \phi_{i/N_1}(\xi_1/z_1, \mu)\ \phi_{j/N_2}(\xi_2/z_2, \mu) \right]$$

$$C_{q\bar{q}}(z_1, z_1, q_T^2, M_Z^2, \mu) = \frac{1}{4\pi} \int d^2x_\perp e^{-iq_\perp \cdot x_\perp} \left(\frac{x_T^2 M_Z^2}{b_0^2} \right)^{F_{q\bar{q}}} e^{2h_F(L_\perp, a_s)}\ \bar{I}(z_1, L_\perp, a_s)\ \bar{I}(z_2, L_\perp, a_s)$$

$$= \frac{1}{2} \int_0^\infty dx_T x_T\ J_0(x_T q_T)\ \exp(g_F(M_Z^2, \mu, L_\perp, a_s)) \qquad L_\perp = \ln \frac{x_T^2 \mu^2}{b_0^2}$$

$$g_F = -\eta L_\perp - a_s \left[(\Gamma_0 + \eta\beta_0) \frac{L_\perp^2}{2} + O(L_\perp) \right] \qquad \eta = C_F \frac{\alpha_s}{\pi} \ln \frac{M_Z^2}{\mu^2}$$

$$\frac{1}{2} \int_0^\infty dx_T\ x_T\ J_0(x_T q_T) e^{-\eta L_\perp} = \frac{1}{q_T^2} \left(\frac{q_T^2}{\mu^2} \right)^\eta \frac{\Gamma(1-\eta)}{\Gamma(\eta)} \left(\frac{b_0}{2} \right)^{2\eta}$$

RGE's for SCET quantities

- $$\frac{d}{d\ln\mu} F_{qq}(L_\perp, \mu) = 2\Gamma^F_{\text{cusp}}$$
- $$\frac{d}{d\ln\mu} h^F(L_\perp, \mu) = 2\Gamma^F_{\text{cusp}}(\mu)\, L_\perp - 2\gamma^q(\mu)$$

$$\frac{d}{d\ln\mu} C_V(-M^2\,\mu) = \left[\Gamma^F_{\text{cusp}}(\mu)\ \ln\frac{-M^2}{\mu^2} + 2\gamma^q(\mu)\right] C_V(-M^2\,\mu)$$

Sect-based resummation: New information on the constants

- The more recent information on the constants in this formula will be used later on.

$$\beta(\alpha_s) = -2\alpha_s \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = -0.12 - 0.015 - 0.0018 - 0.0012 - 0.000095$$

$$\Gamma^i_{\text{cusp}}(\alpha_s) = \sum_{n=0}^{\infty} \Gamma^i_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = 0.133 + 0.023 + 0.0037 + 0.00058 + 0.00065$$

$$\gamma(\alpha_s) = \sum_{n=0}^{\infty} \gamma_n \left(\frac{\alpha_s}{4\pi}\right)^{n+1} = -0.1 + 0.00035 - 0.0019 + 0.00000029$$

- Numerical values are in the MSbar scheme, with $n_f = 5$ and $\alpha_S = \pi/10$

Behaviour for $q_T = 0$

$$\frac{1}{2}\int_0^\infty dx_T\ x_T\ J_0(x_T q_T) e^{-\eta L_\perp} = \frac{1}{q_T^2}\left(\frac{q_T^2}{\mu^2}\right)^\eta \frac{\Gamma(1-\eta)}{\Gamma(\eta)}\left(\frac{b_0}{2}\right)^{2\eta}$$

- For $0 < \eta < 1,\ \ x_T \sim 1/q_T$
- As η approaches 1, higher order terms in g_F are needed
- $g_F = -\eta L_\perp - a_s\Big[(\Gamma_0 + \eta\beta_0)\frac{L_\perp^2}{2} + O(L_\perp)\Big]$
- $\langle x_T \rangle$ can not become arbitrarily large, even for $q_T = 0$.
- The value of μ that keeps $L_\perp$ small stays near $q_* = M_Z \exp\left(-\frac{\pi}{2C_F\alpha_S(q_*)}\right)$

Becher, Neubert,Wilhelm

Radish (resummation in momentum space)

Bizon et al, 1805.05916

- Resummation performed in momentum space
- q_T generated by the recoil against all emissions
- Two different mechanisms
  - Sudakov (exponential) suppression, when $k_{Ti} \sim q_T$
  - Azimuthal cancellations when $k_{Ti} \gg q_T$ (latter dominant when $q_T \to 0$).
  - Cumulative cross section given by,

$$\Sigma(q_T) = \sigma_0 \int_0^\infty \langle dk_1 \rangle\, R'(k_{T1})\, e^{-R(k_{T1})}\, \epsilon^{R'(k_{T1})}$$

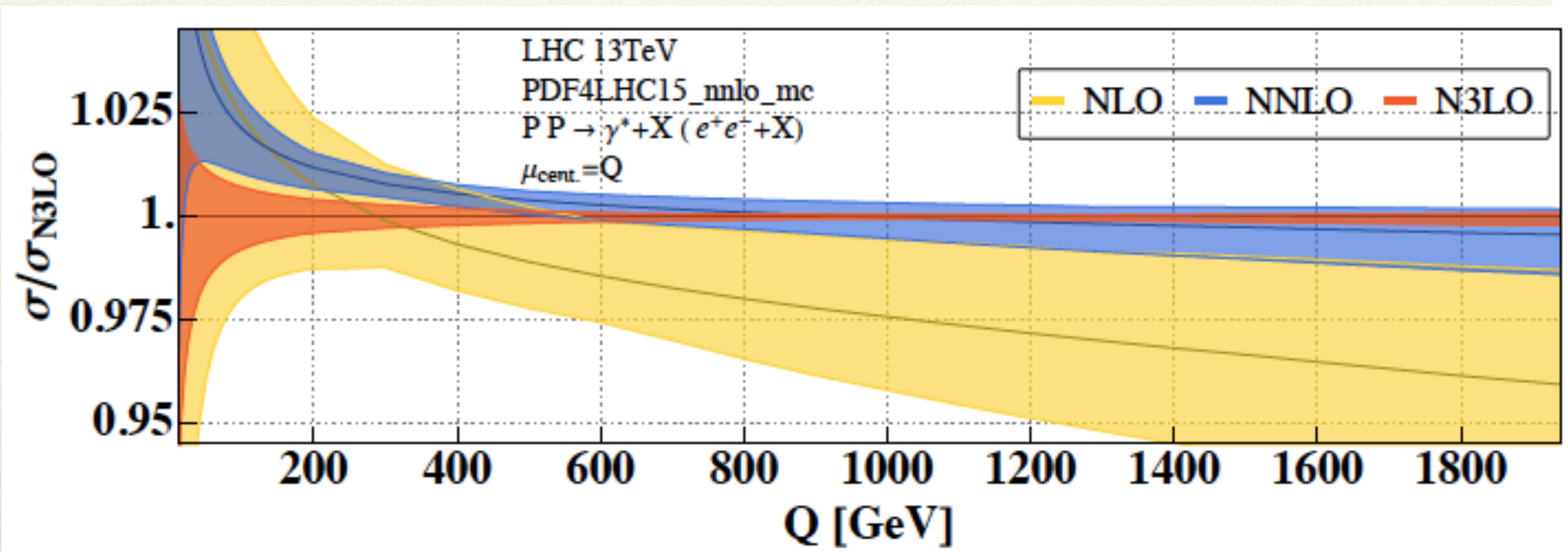
$$\times \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{j=2}^{n+1} \int_{\epsilon k_{T1}}^{k_{T,1}} \langle dk_j \rangle\, R'(k_{T1})\, \Theta(q_T - |\vec{q}_{n+1}|) \qquad \vec{q}_{n+1} = \sum_{j=1}^{n+1} \vec{k}_{T,j}$$

N^3LO

All N^3LO results are of the Drell-Yan type

- Higgs Boson Gluon-Fusion Production in QCD at Three Loops, 1503.06056.
- Inclusive Production Cross Sections at N^3LO, 2209.06138
- Drell{Yan lepton-pair production: q_T resummation at N^3LL accuracy and fiducial cross sections at N3LO, 2103.04974.
- Rapidity Distribution in Drell-Yan Production to Third Order in QCD, 2107.09085
- Fiducial Drell-Yan production at the LHC improved by transverse-momentum resummation at N^4LL+N^3LO,2207.07056
- Third-Order Fiducial Predictions for Drell-Yan Production at the LHC, 2203.01565.
- Transverse Mass Distribution and Charge Asymmetry in W Boson Production to Third Order in QCD, 2205.11426
- Fully Differential Higgs Boson Production to Third Order in QCD, 2102.07607

Drell-Yan – Inclusive N³LO cross-section

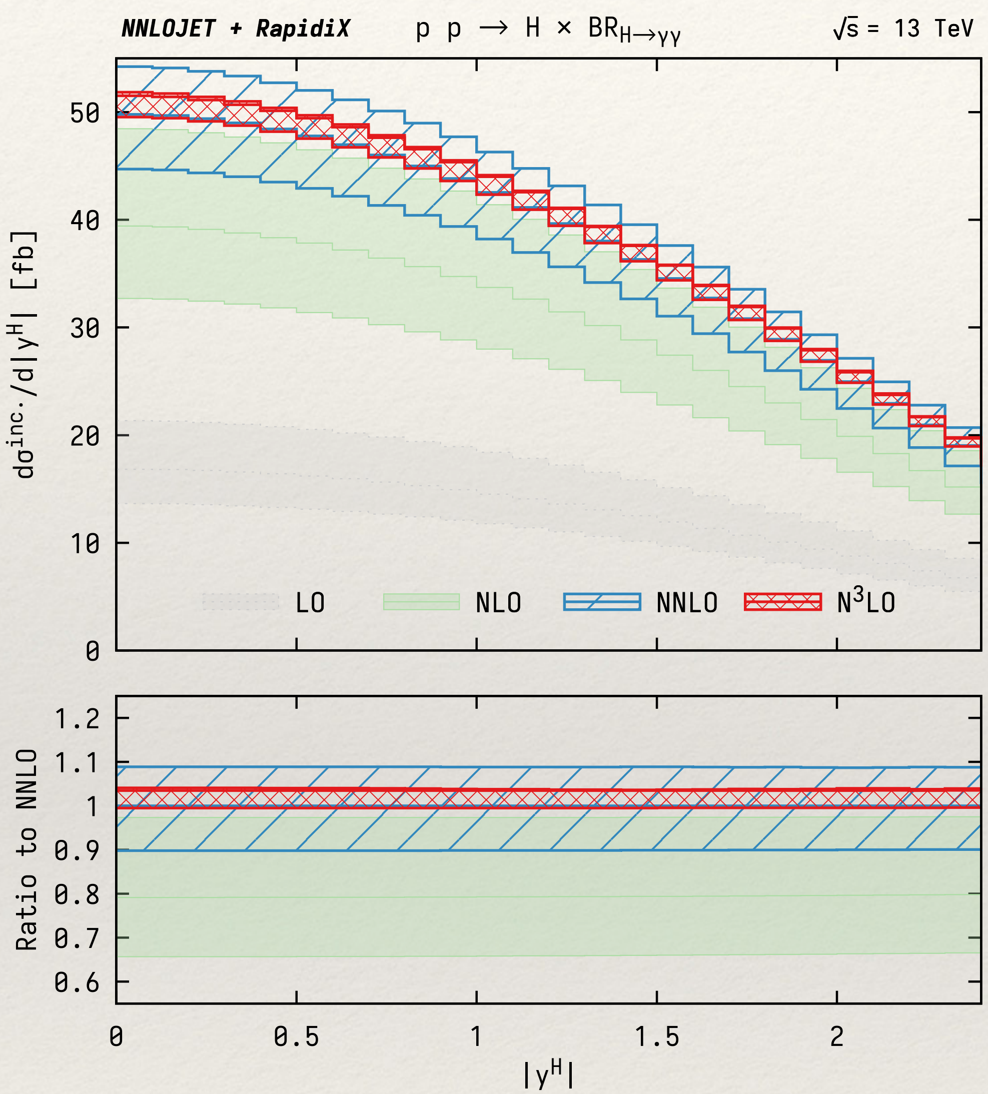


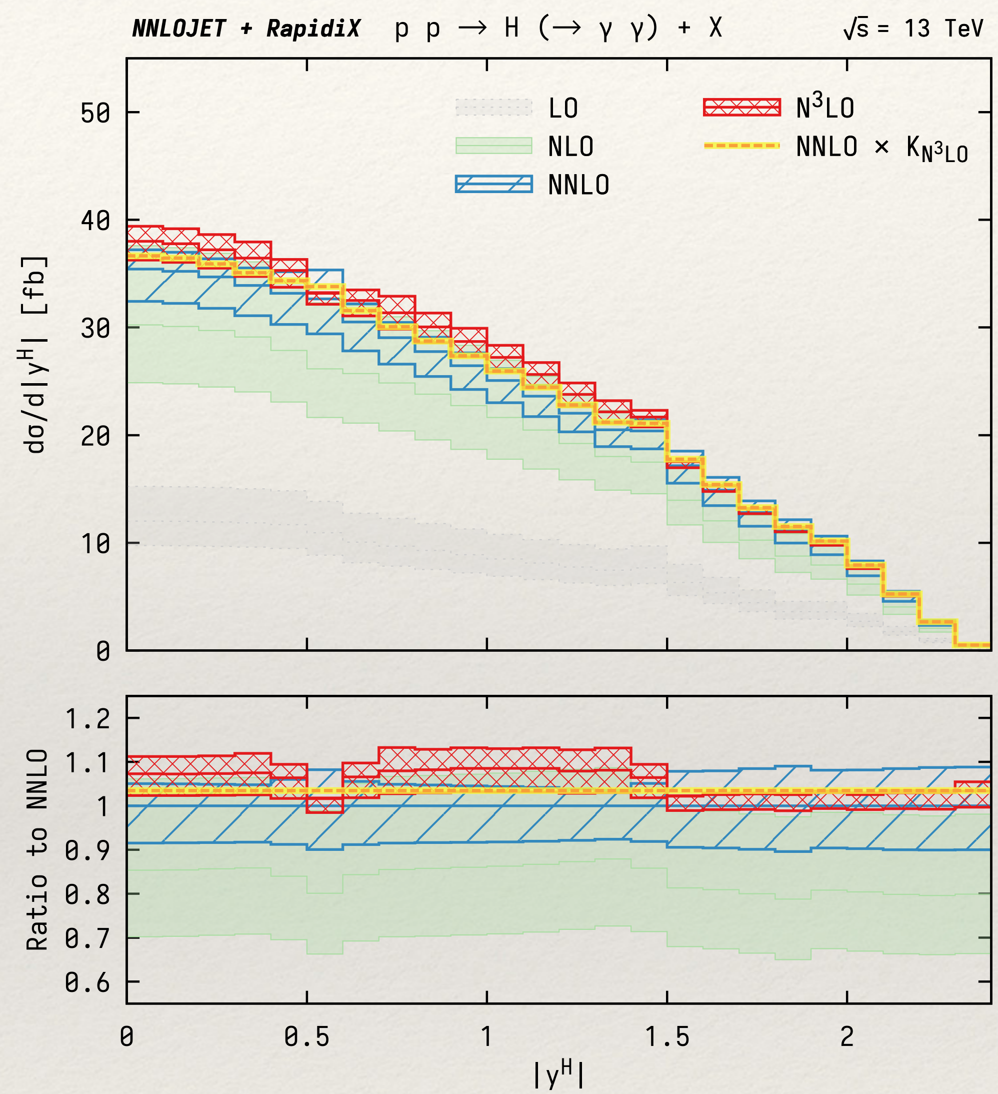
Q/GeV	$K_{\text{QCD}}^{\text{N}^3\text{LO}}$	$\delta(\text{scale})$	$\delta(\text{PDF}+\alpha_S)$	$\delta(\text{PDF-TH})$
30	0.952	$^{+1.5\%}_{-2.5\%}$	$\pm 4.1\%$	$\pm 2.7\%$
50	0.966	$^{+1.1\%}_{-1.6\%}$	$\pm 3.2\%$	$\pm 2.5\%$
70	0.973	$^{+0.89\%}_{-1.1\%}$	$\pm 2.7\%$	$\pm 2.4\%$
90	0.978	$^{+0.75\%}_{-0.89\%}$	$\pm 2.5\%$	$\pm 2.4\%$
110	0.981	$^{+0.65\%}_{-0.73\%}$	$\pm 2.3\%$	$\pm 2.3\%$
130	0.983	$^{+0.57\%}_{-0.63\%}$	$\pm 2.2\%$	$\pm 2.2\%$
150	0.985	$^{+0.50\%}_{-0.54\%}$	$\pm 2.2\%$	$\pm 2.2\%$

- Estimated errors at percent level
- N³LO result lies outside NNLO error band
- Not yet a consistent N³LO calculation, because full N³LO pdf's are not yet available, (but see 2207.04739)

Duhr et al, 2001.07717, 2007.13313, 2111.10379, Chen et al, 2203.01565

N^3LO (Higgs to $\gamma\gamma$)



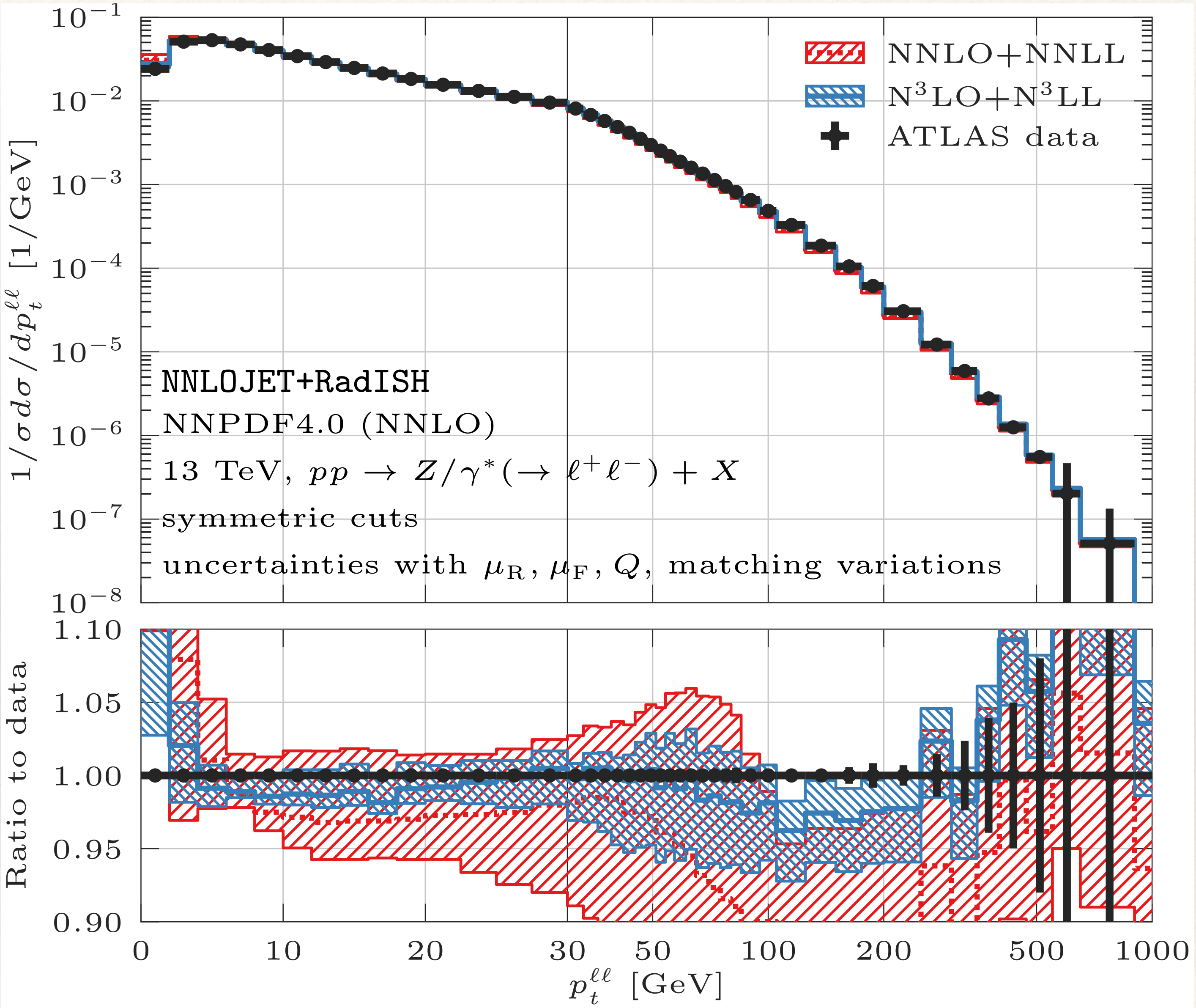


- Inclusive cross section has a simple K-factor and a well-behaved perturbation theory (left plot)
- Cross section with selection cuts, in central region cross section is larger than expected from inclusive K factor (right plot)

Fully Differential Higgs Boson Production to Third Order in QCD, Chen et al, 2102.07607

Fiducial Cross sections at 3rd order

- Modern thrust is to go beyond inclusive results to fiducial cross sections.
- Predictions reach 1%
- Understanding of q_T distribution fundamental for W mass measurement.



Third order fiducial predictions for Drell-Yan at the LHC 2203.01565

Conclusion

- With exception of heavy quark and jet production, the most important high energy processes studied at colliders are really the production of massive bosons, (W,Z/γ*, H, WW, WZ, ZZ, Wγ etc.).
- Our understanding of the processes builds on simple results for lepton pair production, including AEM. After our early calculation, a whole subfield has been spawned, see e.g HP2
- I am delighted to be included in this celebration of Guido's career;
- I admire his tremendous energy, his wide-ranging achievements, and joyful approach to life;
- Happy birthday!