

Martinelli '70  
*Accademia dei Lincei. Roma*  
*26 Settembre 2022*

Weak interactions on the lattice with Guido  
*Luciano Maiani*  
*Sapienza and CERN*

**1976 Polyakov Meeting  
Bohr Institute. Copenhagen**  
Courtesy of Paolo di Vecchia



**Born in Napoli, 1952**, graduated in Physics at Sapienza Univ. in 1975, research associate at Laboratori Nazionali di Frascati dell'INFN (1977-87) and, from 1988, full professore in Theoretical Physics at Sapienza University of Roma.

**At Universita La Sapienza in Roma:**

Director of the Physics Dept., 2007- 2010, President of Ateneo della Scienza e della Tecnologia della Sapienza.

Guido has been Director of *Scuola Internazionale Superiore di Studi Avanzati (SISSA)*, Trieste, 2010-2015, has chaired the *Commissione Scientifica Nazionale Fisica Teorica* of INFN, has been member of *Scientific Policy Committee*, CERN, and President of *Comitato Consultivo dell'ANVUR* (designated by ERC).



Guido Martinelli

Professor of Theoretical Physics La Sapienza Roma Italy  
Verified email at roma1.infn.it - [Homepage](#)

Theoretical physics

FOLLOW

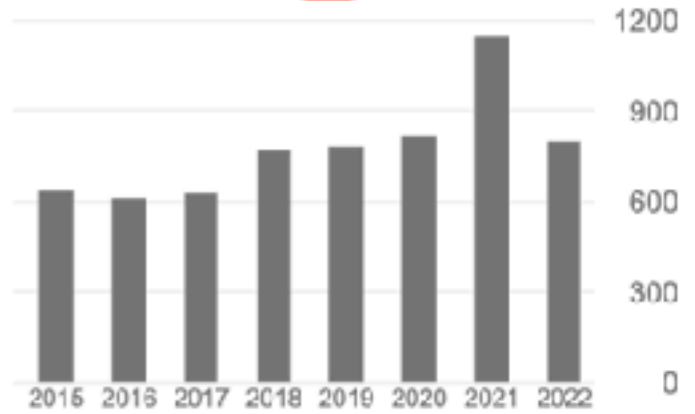
GET MY OWN PROFILE

Cited by [VIEW ALL](#)

	All	Since 2017
Citations	30525	4944
h-index	93	37
i10-Index	251	105

TITLE	CITED BY	YEAR
<a href="#">A general method for non-perturbative renormalization of lattice operators</a> G Martinelli, C Pittori, CT Sachrajda, M Testa, A Vladikas Nuclear Physics B 445 (1), 91-105	1087	1995
<a href="#">Leptonic decay of heavy flavors: A theoretical update</a> G Altarelli, N Cabibbo, G Corbo, L Maiani, G Martinelli Nuclear Physics B 208 (3), 365-380	1084	1982
<a href="#">Large perturbative corrections to the Drell-Yan process in QCD</a> G Altarelli, RK Ellis, G Martinelli Nuclear Physics B 157 (3), 461-497	1060	1979
<a href="#">Chiral symmetry on the lattice with Wilson fermions</a> M Bochicchio, L Maiani, G Martinelli, G Rossi, M Testa Nuclear Physics B 262 (2), 331-355	717	1985
<a href="#">The <math>\Delta S=1</math> effective Hamiltonian including next-to-leading order QCD and QED corrections</a> M Ciuchini, E Franco, G Martinelli, L Reina Nuclear Physics B 415 (2), 403-459	711	1994
<a href="#">Vector boson production at colliders: a theoretical reappraisal</a> G Altarelli, RK Ellis, M Greco, G Martinelli Nuclear Physics B 246 (1), 12-44	638	1984
<a href="#">Model-independent constraints on <math>\Delta F=2</math> operators and the scale of new physics</a> M Bona, M Ciuchini, E Franco, V Lubicz, G Martinelli, F Parodi, M Pierini, ... Journal of High Energy Physics 2008 (03), 049	602	2008
<a href="#">Leptoproduction and Drell-Yan processes beyond the leading approximation in chromodynamics</a> G Altarelli, RK Ellis, G Martinelli Nuclear Physics B 143 (3), 521-545	573	1978
<a href="#">2000 CKM-triangle analysis a critical review with updated experimental inputs and theoretical parameters</a> M Ciuchini, V Lubicz, G D'Agostini, E Franco, G Martinelli, F Parodi, ... Journal of High Energy Physics 2001 (07), 013	558	2001

only 500+



Public access [VIEW ALL](#)

0 articles	76 articles
not available	available

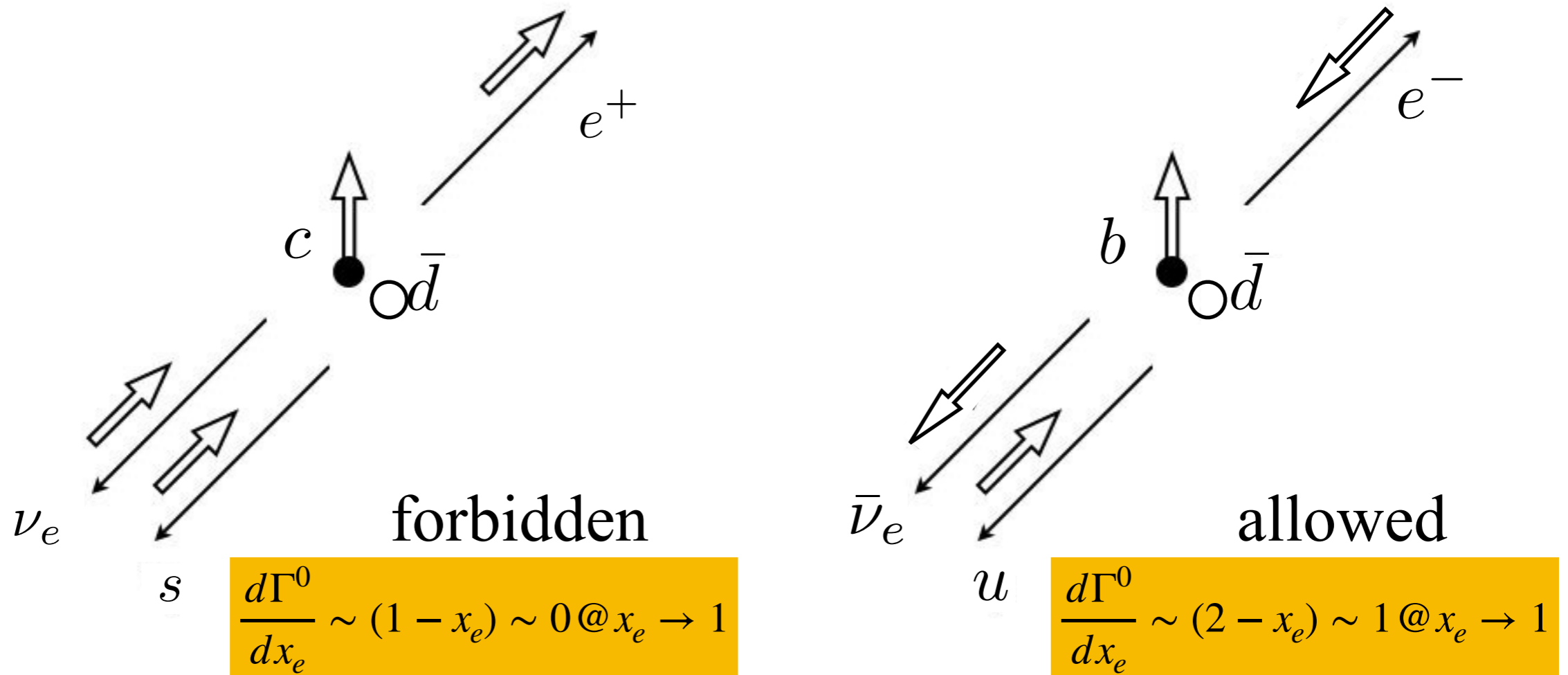
Based on funding mandates

# 1. Charm semileptonic decay (1982)

- The charm quark mass is definitely larger than  $\Lambda_{\text{QCD}}$ , the invariant mass of the hadronic final system is also  $\gg \Lambda_{\text{QCD}}$ , and we can use parton model + perturbative QCD to compute the semileptonic width and the energy spectrum of the emerging charged lepton.
- While in Paris, Cabibbo and myself worked out the semileptonic rate of the charm quark (1978) and later (with G. Corbo') the electron energy spectrum.
- After discovery of the  $b$  quark, with even a larger mass, we thought our formulae for the energy lepton spectrum near the end point could be used to determine the Cabibbo-Kobayashi-Maskawa matrix element  $V_{ub}$ , not determined by the total rate (dominated by the  $b \rightarrow c$  transition).
- Paolo Franzini (then still in Cornell with CLEO) observed however that the lepton end point in  $b$  decay corresponds to small hadron masses and therefore non perturbative QCD corrections come in.

# Semileptonic decays of c vs. b quarks

Charged lepton energy end point configurations in c and b decay



charm decay is dominated by inelastic, large mass, hadron configurations: perturbative QCD corrections are adequate.

beauty decay is dominated by small mass, hadron configurations: resummation of perturbative QCD corrections is required.

# QCD experts get in

- The two Guidos, Altarelli and Martinelli, came in, with the crucial resummation of the perturbative terms and the result provided a valuable (and much used) tool in the estimate of  $V_{ub}$  from inclusive rates:

G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, *Leptonic Decay of Heavy Flavors: A Theoretical Update*, Nucl. Phys. B 208 (1982) 365.

- The other side of the story:

an alternative method is to obtain  $V_{ub}$  from *exclusive rates* provided by the lattice computation of the the form factor of B meson  $\rightarrow$  light flavoured vector meson.

Inclusive average:

$$|V| = (4.25 \pm 0.12_{-0.14}^{+0.15} \pm 0.23) \times 10^{-3}$$

Lattice QCD:  $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$

see:

Ceccucci (CERN), Z. Ligeti (LBNL), Y. Sakai (KEK)

CKM Quark-Mixing Matrix (PdG 2021)

## 2. Weak Interactions on the Lattice (1984)

- After Ken Wilson's papers, early lattice QCD work was focussed on the calculation of the hadron spectrum.
- Exceptional work was done in Italy, starting from conception and realization of the APE supercomputer (N. Cabibbo, G. Parisi and coll.)
- An example of the preparatory work done in Italy for hadron spectroscopy is:

### Hadron spectroscopy in lattice QCD

F Fucito, G Martinelli, C Omero, G Parisi, R Petronzio, F Rapuano  
Nuclear Physics B 210 (3), 407-421 (1982)

- Cabibbo, Martinelli and Petronz started a new turn in 1983: a lattice QCD calculation of mesonic matrix elements of four-quark operators

### Weak interactions on the lattice

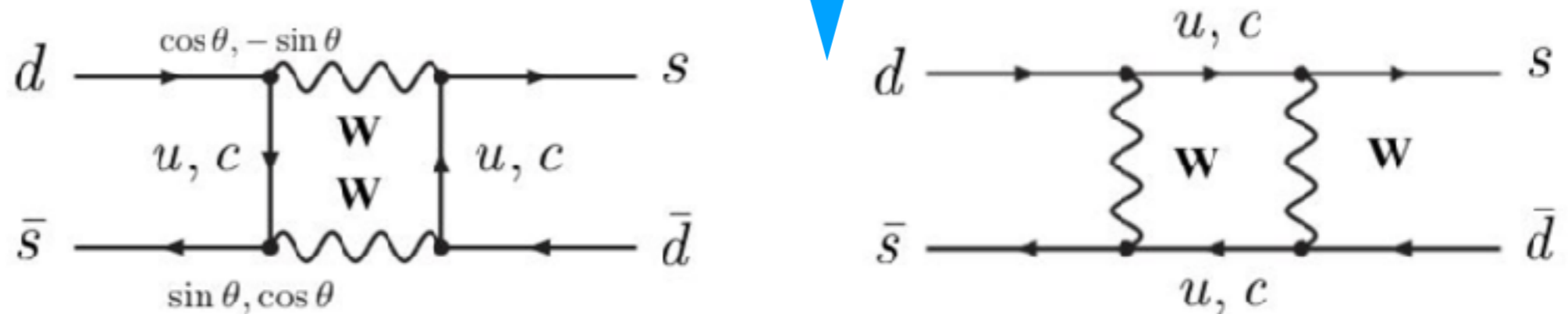
N Cabbibo, G Martinelli, R Petronzio Nuclear Physics B 244 (2), 381-391 (1984)

# A long story, going back to the works of Ioffe & Shabalin, Low...the GIM Mechanism...

- Ioffe and Shabalin
- F. Low (1967-1968)

$$m_{K_1} - m_{K_2} = \frac{1}{2m_K} \left( \frac{G}{\sqrt{2}} \right)^2 \frac{\Lambda^2}{8\pi^2} \langle K^0 | j_\mu^{K^0}(0) j_\mu^{K^0}(0) | \bar{K}^0 \rangle,$$

- GIM (1970)



$$\Lambda^2 \sim m_c^2; j_\mu \cdot j_\mu = (\bar{s}\gamma_\mu(1 - \gamma_5)d)(\bar{s}\gamma^\mu(1 - \gamma_5)d);$$

- Gaillard & Lee (1974): Vacuum saturation

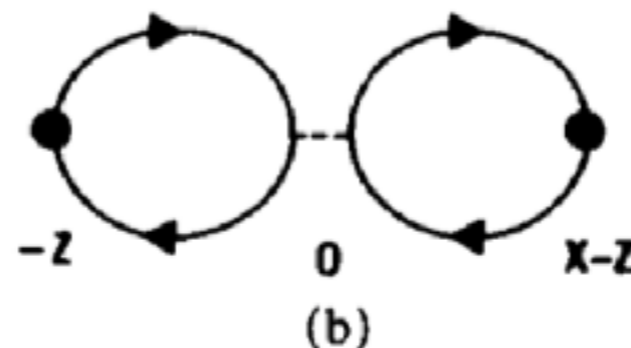
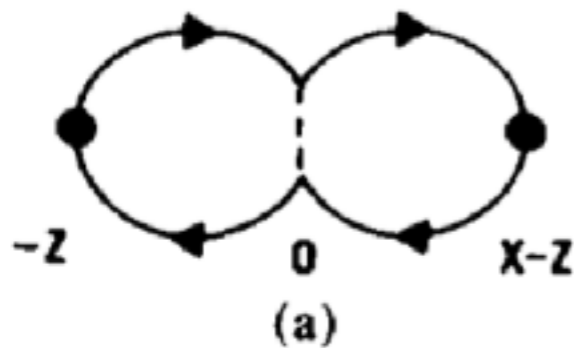
$$\langle K^0 | (\bar{s}\gamma_\mu(1 - \gamma_5)d)(\bar{s}\gamma^\mu(1 - \gamma_5)d) | \bar{K}^0 \rangle \propto f_K^2 m_K^2$$



# What you have to put on the lattice...and CMP result

- The operator in the origin of coordinates
- propagators from origin to anywhere

$$\pm \sum_{y,z} \langle \text{Tr} \{ S(-z) S^+(-z) \gamma_{L,R}^\mu S(y) S^+(y) \gamma_L^\mu \} \rangle_U \delta(y_0 + z_0 - t)$$



## CMP:

As a second test we have evaluated the  $K_0 - \bar{K}_0$  matrix element

$$\frac{\langle K_0 | (\bar{s} \gamma_\mu (1 - \gamma_5) d) (\bar{s} \gamma_\mu (1 - \gamma_5) d) | \bar{K}_0 \rangle}{m^2} = (4a^2)(10 \pm 1) 10^{-2} \quad [\sim 7.7 \times 10^{-2}].$$

Shifman, Vainstein,  
Zhakarov, vacuum  
saturation in QCD  
(1976)

Cabibbo, Martinelli,  
Petronzio (1983)

The fact that our results are close to those obtained by a well-accepted approximation scheme is a clear indication of the validity of the method.

# An exciting program for non-leptonic weak interactions

Cabibbo, Martinelli, Petronzio (1983): **We are confident that our method can lead to an accurate determination of matrix elements which are essential for the theoretical predictions of important physical quantities, such as  $\Delta T = \frac{3}{2}$  rates for K meson decays and K –  $\bar{K}$  mixing, including CP violation. A new method is, however, required for a full evaluation of the  $O_{5,6}$  matrix elements.**

New groups got into the game

C. W. Bernard, T. Draper, A. Soni, H. D. Politzer and M. B. Wise,  
Chiral Perturbation Theory and the  $\Delta I = 1/2$  Rule, UCLA/84/TEP/14, Calt 68-1211, (1984)  
R. C. Brower, G. Maturana, M. Belen Gavela and R. Gupta,  
Calculation of Weak Transitions in Lattice, Phys. Rev. Lett. **53** (1984), 1318

- Focus on chiral symmetry in lattice QCD: **H. Nielsen, M. Nonomiya (1981)**
- It is broken on the lattice by the Wilson term....
- and so must it be...otherwise the singlet axial current in QCD could not be anomalous ( $\pi^0 \rightarrow \gamma\gamma$  !!!)
- but in the continuum limit *chiral symmetry for non anomalous currents has to be recovered*. This was shown by the Roma group:

M. Bochicchio, L. Maiani, G. Martinelli, G. C. Rossi and M. Testa,  
Chiral Symmetry on the Lattice with Wilson Fermions  
Nucl. Phys. B **262** (1985), 331

**Now: a fully fledged industry**

# Pseudoscalar meson couplings and non-leptonics

- During the 1980s, the decay constants of the pseudoscalar mesons,  $\pi$ ,  $K$ ,  $D$ ,  $D_s$  have been successfully computed in Lattice QCD by the groups listed before, in particular by the Roma-Paris (Orsay)-Southampton group.
- A large effort by us and by the UCLA group (C. Bernard, A. Soni and coll.) went to lattice calculations of  $K \rightarrow \pi$  non-leptonic decay amplitudes with  $\Delta I = 3/2, 1/2$ , following
  - the QCD renormalization group results (Lee and Gaillard, Altarelli and Maiani 1974)
  - the successive the penguin diagram analysis (Shifman, Vainstein and Zakharov)
  - the chiral symmetry prescriptions in Bochicchio et al and: L. Maiani, G. Martinelli, G. C. Rossi and M. Testa, Nucl. Phys. B **289** (1987), 505
    - use the LG and AM hamiltonian at lattice level;
    - work with  $a^{-1} > 2 \text{ GeV}$  i.e. dynamical charm quarks to justify the use of LG-AM hamiltonian (renormalised at a momentum scale  $\mu \gg m_c$ ;
    - let SVZ penguins develop non perturbatively by QCD interactions on the lattice:

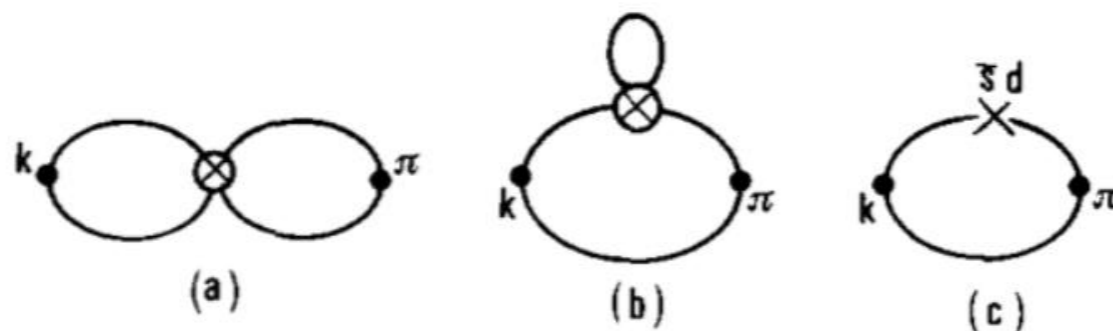


Fig. 2. The “eight” (a), “eye” (b), and counterterm (c) diagrams for  $K-\pi$  three-point correlations.

“eye” (b) develops the penguin  
(c) is the counterterm to be added  
to obtain the physical penguin

# non-leptonics (cont'd)

- We obtained very encouraging results for  $\Delta I = 3/2$ ,  $K^+ \rightarrow \pi^+\pi^0$  amplitude

$$\frac{\langle K | H_W | \pi^+\pi^0 \rangle}{m_K} = (7 \pm 2) \cdot 10^{-8}$$

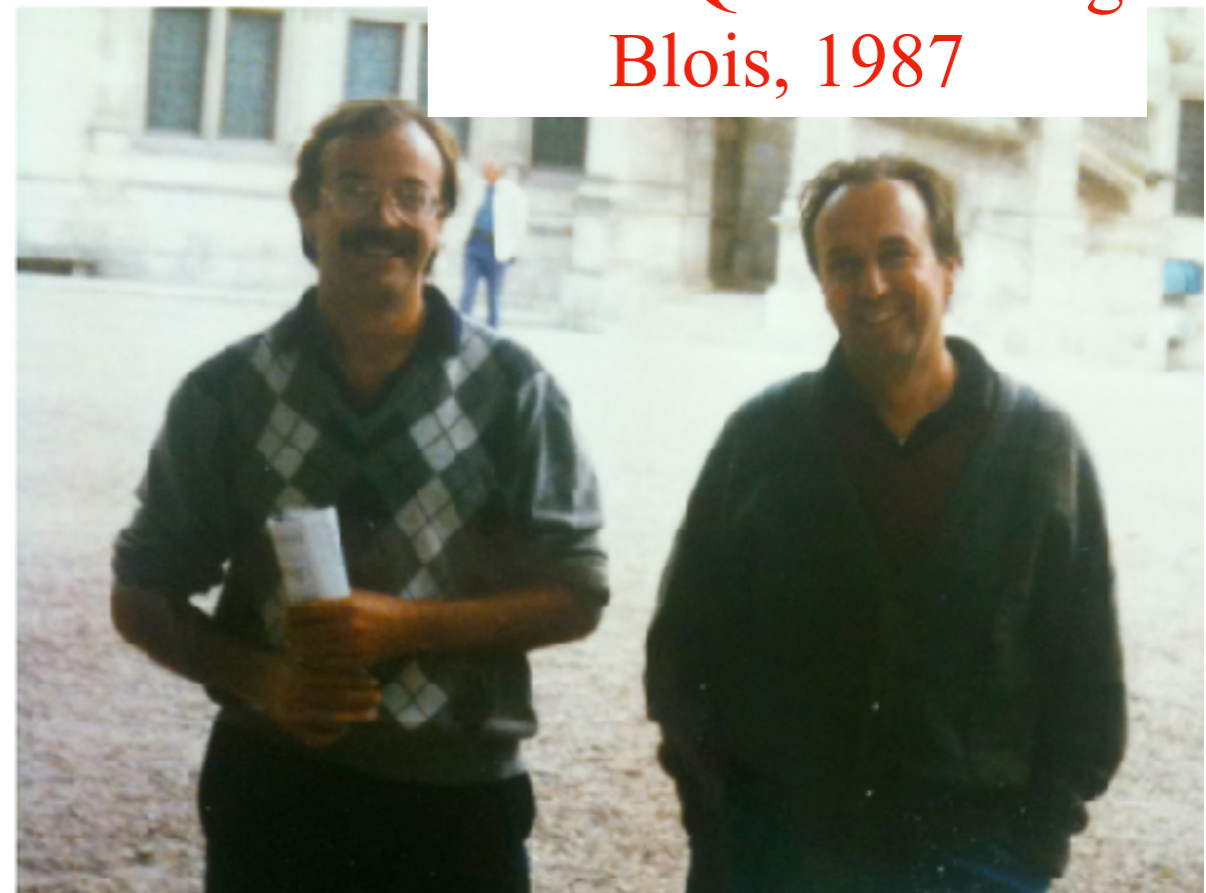
M. B. Gavela, L. Maiani, S. Petrarca, F. Rapuano, G. Martinelli, O. Pene, C. T. Sachrajda, Nucl. Phys. B **306** (1988) 677.

- to be compared with the experimental value of [expt:  $3.7 \cdot 10^{-8}$ ]
- Preliminary results have been presented at the Blois Conference on Lattice QCD

M. B. Gavela, L. Maiani, S. Petrarca, G. Martinelli and O. Pene, *First Results for the Amplitude in K Decays, With Quenched Lattice QCD and Wilson Fermions*, Phys. Lett. B **211** (1988), 139

Lattice QCD Meeting  
Blois, 1987

- However, high statistics is required for the non-perturbative subtractions needed to isolate the  $\Delta I = 1/2$ ,  $K \rightarrow \pi$  amplitude
- this made the calculation not possible with the then available computer power and we could not continue the program;
- our UCLA competitors, as well, could not make progresses on  $\Delta I = 1/2$  amplitudes for a long time.



# 3. The pseudoscalar coupling of B mesons on the lattice (1991)

- In the calculations of the 1980s, the value of the inverse lattice spacing ( $a^{-1}$ ) was generally taken between 1-3 GeV. Thus, on these lattices, it was not possible to simulate hadrons containing the b-quark ( $m_b > a^{-1}$ ).
- An interesting technique for the study of heavy-quark physics on the lattice was proposed by E. Eichten and B. Hill (*Phys. Lett. B* **234** (1990), 511):
- expand the heavy-quark propagator in inverse powers of the quark mass and take the leading term, corresponding to propagation only along the time direction

$$\text{Action} = b^\dagger (i\partial_0 + gA_0)b$$

- coupling the static b to a fully dynamical light quark, it became possible to consider the local heavy-light axial current  $A_0 = b^\dagger(x)\gamma_0\gamma_5q(x)$  in Lattice QCD.
- To compute the pseudoscalar coupling  $f_B$  from the correlation function  $G(x) = \langle A_0(x)A_0(0) \rangle$ , one has to isolate the one-particle intermediate state (i.e. the ground state B meson) and:

$$\langle 0 | A_0 | B \rangle = f_B p_0$$

- The first calculation of  $f_B$  in the static limit was done by the Roma-Southampton group (Allton, Sachrajda, Lubicz, Maiani and Martinelli) on a lattice with  $a^{-1} = 2$  GeV with the surprising result

$$310 \pm 25 \pm 50 \text{ MeV}$$

with  $f_B > f_D$ , contrary to expectation.

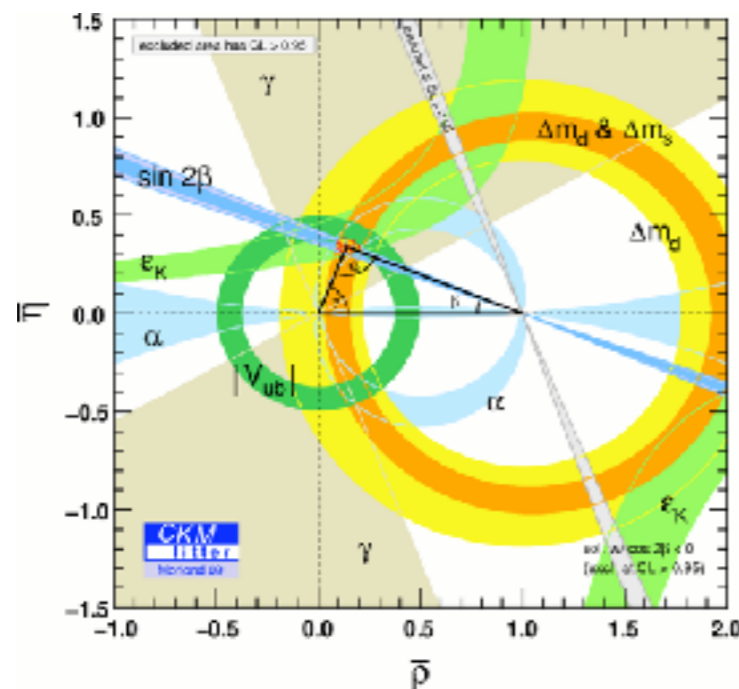
# $f_B$ , $M_{top}$ and $\sin 2\beta$

- We realized soon a totally unexpected effect of the large value of  $f_B$ .
- Combined with a high value of the top quark mass, above the lower limit then available (100 GeV), a large  $f_B$  had the effect to push the CP-violating asymmetry in  $B \rightarrow J/\Psi + K_S$  decays to a much larger level than previously estimated.
- We found that  
 $M_{top} > 140$  GeV and  $200$  GeV  $< f_B < 300$  MeV would lead to values  
 $\sin(2\beta) = 0.6 - 0.8$

M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Nucl. Phys. B 369 (1992) 139.

*The lower bound to the asymmetry is, in this case, about a factor 5 larger than previously found, which corresponds to about a factor 25 reduction in the luminosity required for a given statistical error in the asymmetry measurement.*

*...We wait eagerly for new data from Fermilab to tell us if the t-quark is below 130 GeV. or not..*



All went well

Present world average, including some other measurements, is  
 $\sin 2\beta = 0.699 \pm 0.017$ . CKM Quark-Mixing Matrix (PdG 2021)

Top quark was observed in 1994,  
 $M_{top} = 173$  GeV

# ...at present

$f_P$	Allton et al. <sup>a</sup> (1991)	Lattice <sup>d</sup> (2021)	from $P^+ \rightarrow \tau^+ \nu_\tau$ <sup>e</sup> (2008)
$f_D$	$180 \pm 25 \pm 30$ MeV	$212.0 \pm 0.7$ MeV	$205.8 \pm 8.5 \pm 2.5$ MeV
$f_B$	$310 \pm 25 \pm 50$ <sup>b</sup> $260 \pm 25 \pm 50$ <sup>c</sup> MeV	$190.0 \pm 1.3$ MeV	$216 \pm 22$ MeV
$f_P \sqrt{M_P} = A(1 - \frac{C}{M_P})$	$C = 1.2$ GeV	$C = 0.76$ GeV	$C = 0.93$ GeV

Table 1:

## Notes:

a: C. S. Allton, *et al.*, *A Lattice Computation of the B-Meson Decay Constant*, Nucl. Phys. B **399** (1991) 598.

b: lattice spacing  $a^{-1} = 2$  GeV.

c: lattice spacing  $a^{-1} = 1.8$  GeV, determined using the values of  $f_\pi$  and  $f_K$ .

d: dynamical quark flavours on the lattice:  $u, d, s, c$ . Flavour Lattice Averaging Group (FLAG) 2021, arXiv:2111.09849.

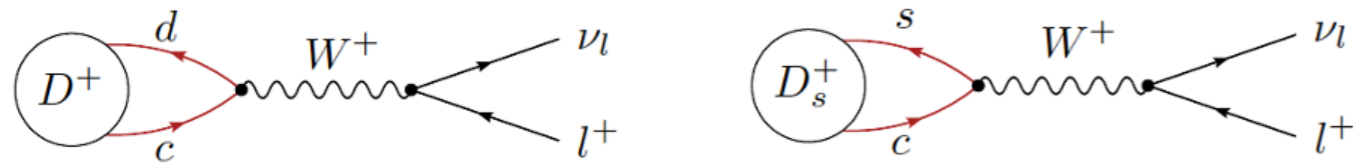
e: J. Rosner, S. Stone, *Decay Constants of Charged Pseudoscalar Mesons*, arXiv:0802.1043.

- In QCD, one expects the asymptotic scaling law:  $f_P \sqrt{M_P} = \text{const.}$  for large quark mass.
- Setting:  $f_P \sqrt{M_P} = A(1 - \frac{C}{M_P})$ , one would expect  $C = \mathcal{O}(\Lambda_{QCD}) \sim 0.25$  GeV
- A sizeable deviation was seen by Allton et al. and is confirmed in more recent estimates of  $f_B$  vs  $f_D$  (see Table)....???...

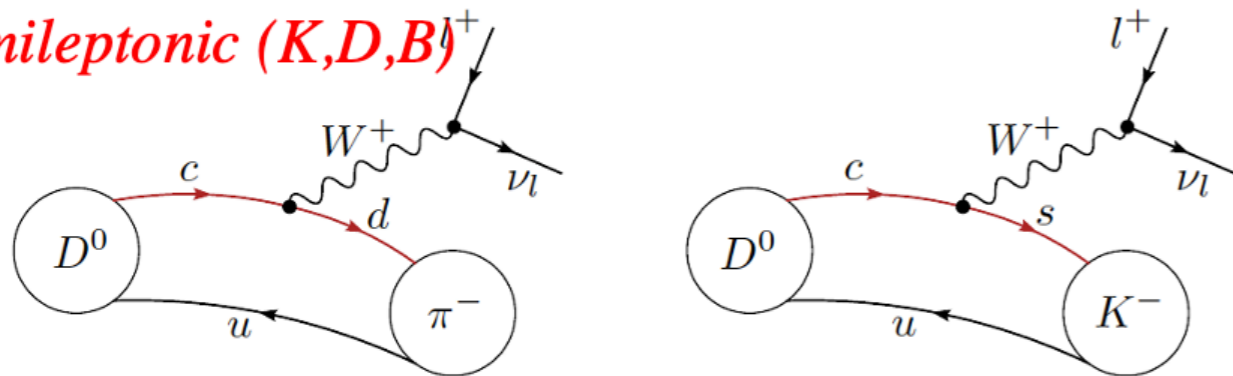
# Guido' summary figure: Weak Interactions on the Lattice

**Guido Martinelli.** *QCD and Supercomputers*, in: *Bruno Touscheck 100 YEARS*, Roma, Frascati 2-4 December 2021; Luisa Bonolis, Luciano Maiani, Giulia Pancheri Editors, Springer (2022)

## Leptonic ( $\pi, K, D, B$ )



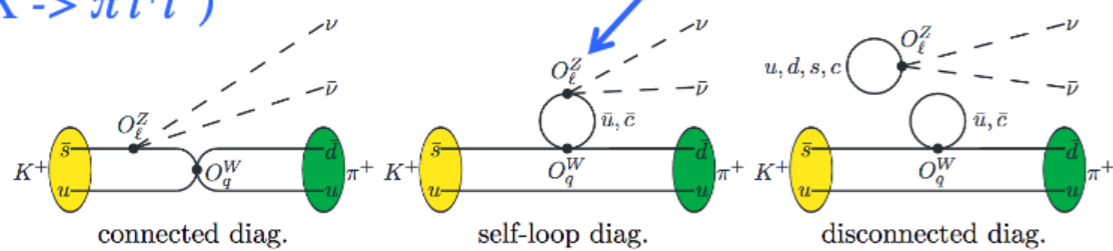
## Semileptonic ( $K, D, B$ )



## (some) Radiative and Rare

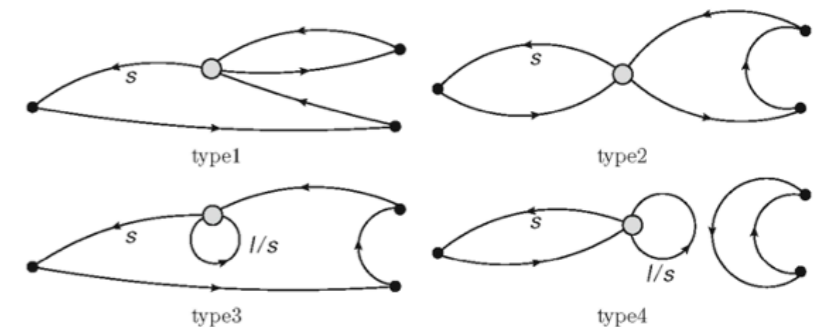
(also  $K \rightarrow \pi l^+ l^-$ )

long distance effects

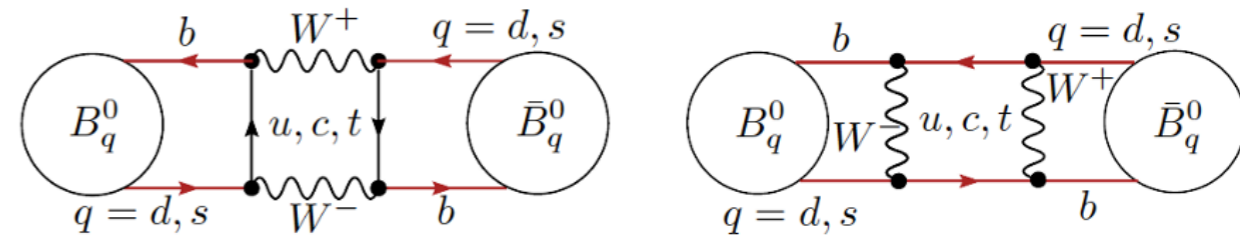


**Non-leptonic**  
but only below the  
inelastic threshold  
(may be also  
3 body decays)

$B \rightarrow \pi\pi, K\pi$ , etc. No!



**Neutral meson mixing (local)**



+ some long distance contributions to K and D neutral meson mixing + short distance contributions to  $B \rightarrow K^{(*)} l^+ l^-$  decays, not shown in the figure.

*Synthetic overview of the amplitudes most frequently computed for Weak Interaction Phenomenology*

**Left panel:** Leptonic, Semi-leptonic and Radiative decays (also for baryons and electromagnetic transitions not shown in the figure).

**Right Panel:** Non-leptonic decays of Kaons, Neutral meson mixing of  $B_q$  mesons (D meson and Kaon mixing not shown in the figure).

Lattice QCD computed also some long distance contributions to K and D neutral meson mixing and short distance contributions to  $B \rightarrow K^{(*)} l^+ l^-$  decays, not shown in the figure.



# Thirty years later.... Unanticipated hadrons: what are they?

## The virtues of heavy quarks

( $m_Q \gg \Lambda_{QCD}$ ):

- the inclusive decays are calculable deep inelastic processes;
- $c\bar{c}$  or  $b\bar{b}$  bound states involve short distance forces: a calculable spectrum of charmonia/bottomonia;
- inside hadrons,  $c\bar{c}$  or  $b\bar{b}$  pairs are not easily created or destroyed: a hadron decay into  $J/\Psi$  or  $\Upsilon + \dots$  indicates a valence  $c\bar{c}$  or  $b\bar{b}$  pair  $\rightarrow$  heavy-quark counting is possible.

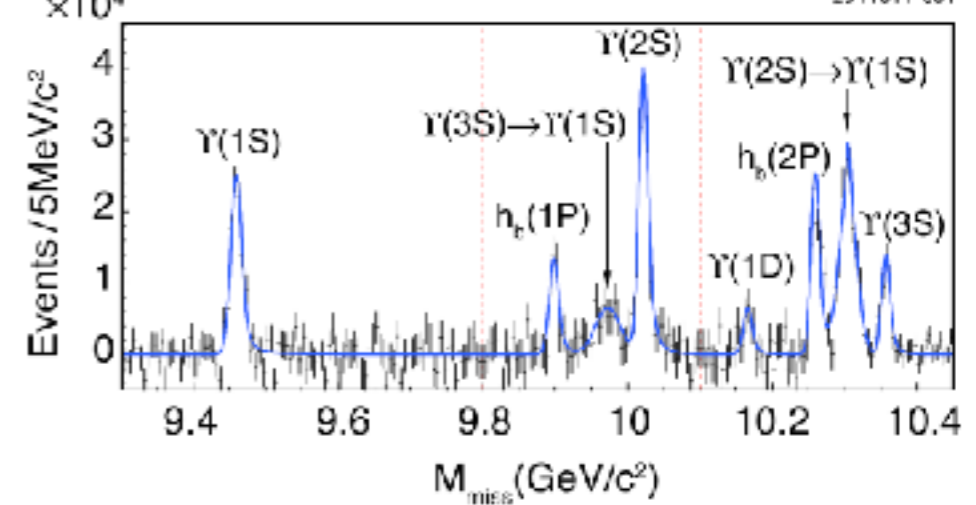
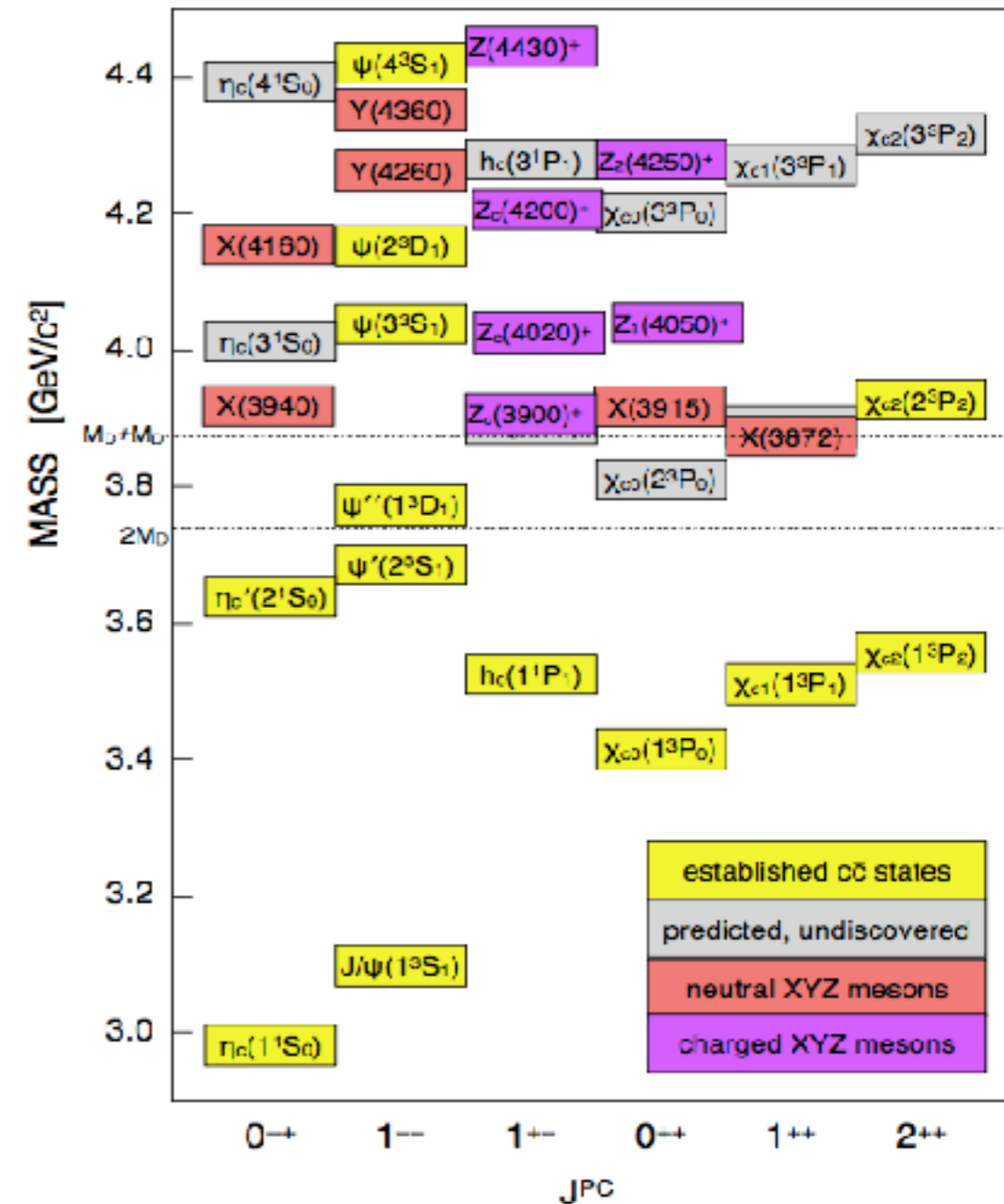


Figure 1: From Belle [31], the mass recoiling

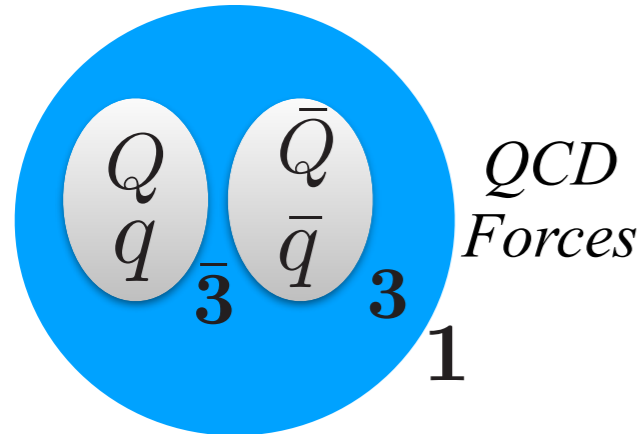


# No consensus, yet



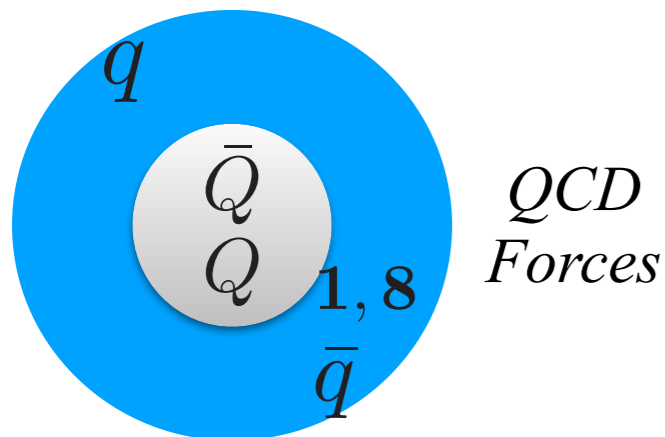
Hadron Molecule

F-K. Guo, C. Hanhart, U-G Meißner, Q. Wang, Q. Zhao, and B-S Zou, arXiv 1705.00141 (2017)



Compact Diquark-Antidiquark

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, Phys. Rev. D 71 (2005) 014028; D 89 (2014) 114010.

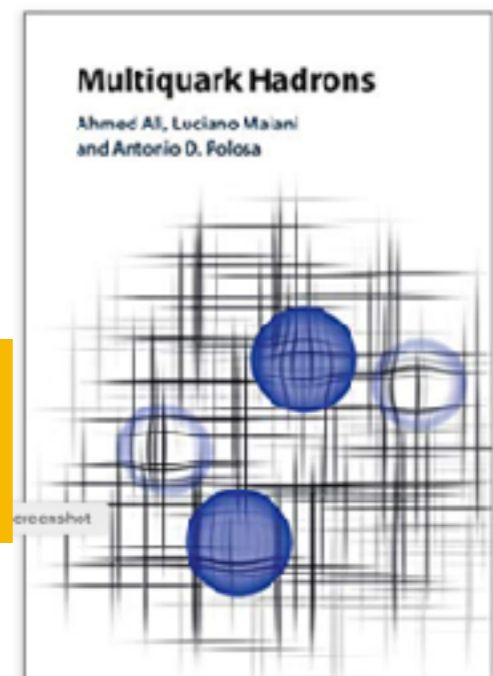


HadroCharmonium (1)  
Quarkonium Adjoint Meson (8)

S. Dubynskiy, S. and M. B. Voloshin, Phys. Lett. B 666, (2008) 344.

E. Braaten, C. Langmack and D. H. Smith, Phys. Rev. D 90 (2014) 01404

For a review, see:  
A. Ali, L. Maiani and A.D. Polosa, *Multiquark Hadrons*, Cambridge University Press (2019)



# The new sensation: doubly heavy tetraquarks

- Doubly heavy tetraquarks have been anticipated long ago

Esposito, Papinutto, Pilloni, Polosa, Tantalò, Phys. Rev. D88, 054029 (2013)

- $\mathcal{T}_c^+ = cc\bar{u}\bar{d}$  seen by LHCb (2021)

- The possibility has been raised that  $I = 0, J^P = 1^+, \mathcal{T}_{bb}^- = bb\bar{u}\bar{d}$  be stable under strong and e.m. decays

M. Karliner and J. L. Rosner, PRL **119** (2017) 202001. E. J. Eichten and C. Quigg, PRL **119** (2017) 202002.; S. Q. Luo et al. Eur. Phys. J. C **77** (2017) 709.

- extended calculations of  $\mathcal{T}_{cc}$  and  $\mathcal{T}_{bb}$  mass have been presented, in the Born-Oppenheimer approximation, analytical

L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. **D100** (2019) 074002,

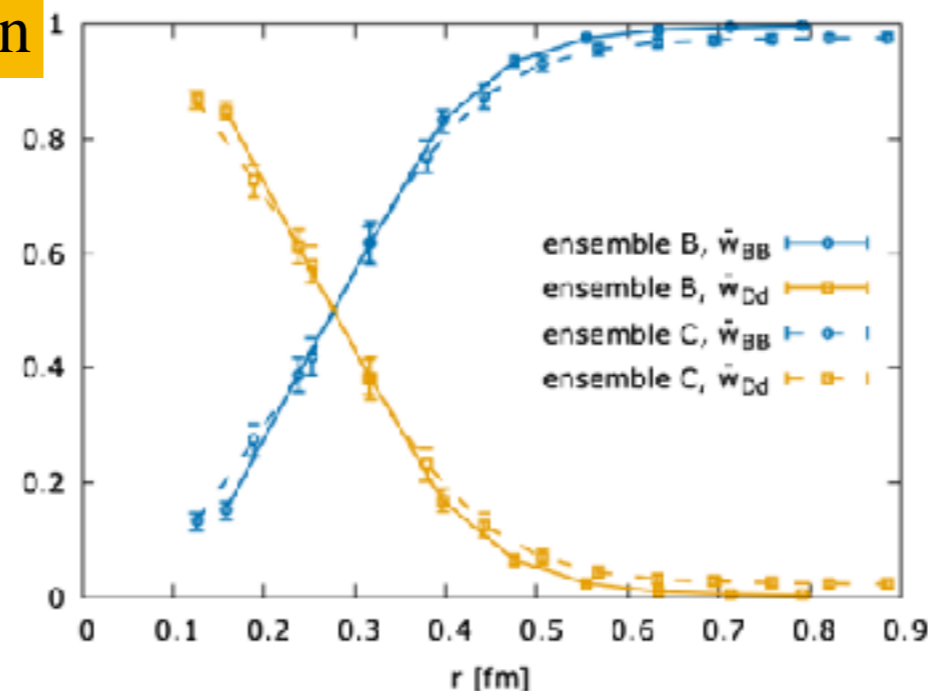
- A recent lattice QCD calculation shows a deeply bound 6 b-quarks dibaryon:  $[bb][bb][bb]$

N. Mathur et al. ArXiv:2205.02862

## Radiography of the double beauty tetraquark on lattice

$Dd = [bb][\bar{q}\bar{q}]; BB = \text{Meson} - \text{Meson}$

- Projection of the lattice  $[bb][\bar{q}\bar{q}]$  result over  $Dd$  (yellow) or  $BB$  (blue) gives a picture of the space arrangement of light quarks at a given  $bb$  distance,  $R$
- Mainly  $[bb]_3[\bar{q}\bar{q}]_3$  at the peak of the  $bb$  wave function  $\sim 0.2$  fm.



←  $B_1 B_1 = \text{Meson-Meson}$

P. Bicudo et al. Phys. Rev. **D 103** (2021) 114506

←  $Dd = [bb][\bar{q}\bar{q}]$

Post classical, unanticipated hadrons in QCD Lattice:  
the challenge of the coming years?

**Congratulations to Guido  
and  
Thank You !**