Black Holes as Probes for Ultralight DM

Bruno Bucciotti

9th June

Supervisor: Enrico Trincherini

$10^{-21}\,\mathrm{eV} \lesssim \mu \lesssim 10\,\mathrm{eV}$

Light complex scalar
minimally coupled to gravity

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Easy to get light scalars

$$\phi \to \phi + c$$

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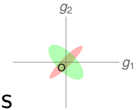

Assume no interactions

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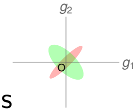
minimally coupled to gravity

Easy to get light scalars
$\phi \to \phi + c$

Interplay with black holes

Assume no interactions



Goals

- Analytically understand the zoo of solutions
- Check numerical results
- Extend previous work
- Emphasize horizon effects

ϕ (classical EoM)

What is the field profile?
Dependence on μ?
Relation $|\phi|_{r\to\infty}$ to $|\phi|_{r\to r_{BH}}$?

$$\Box\phi - \mu^2\phi = 0 \qquad \text{(classical EoM)}$$

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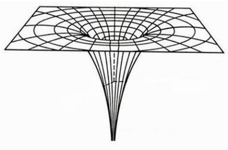

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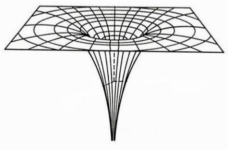

- BH domination at first, no self gravity
- Nonrotating black hole

$\rightarrow$ Schwarzschild metric

What is the field profile?
Dependence on μ?
Relation $|\phi|_{r\rightarrow\infty}$ to $|\phi|_{r\rightarrow r_{BH}}$?

Hairy solutions

$$\phi \to e^{-i\omega t} Y_{l,m}\phi(r), \qquad \omega = \mu, \quad V_l = r_{BH}\mu^2 r^3 - l(l+1)r(r - r_{BH})$$

$$r(r - r_{BH})\frac{\mathrm{d}}{\mathrm{d}r}\left(r(r - r_{BH})\frac{\mathrm{d}\phi}{\mathrm{d}r}\right) + V_l(r)\phi = 0$$

$l = 0$ in [Hui '19]

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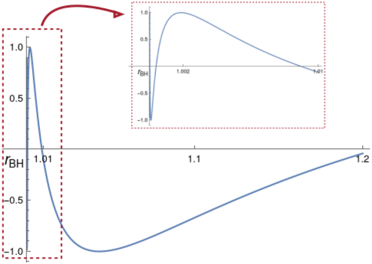
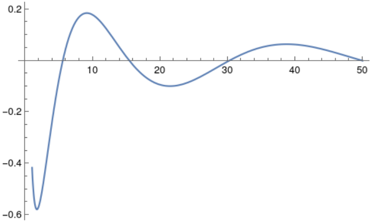


Near horizon limit
(numerically hard)



Far field

Local WKB

$l = 0$ in [Hui '19]

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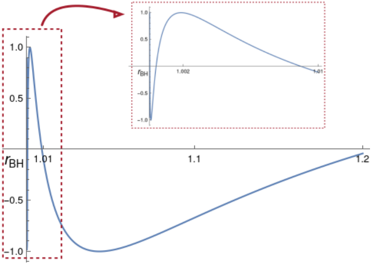

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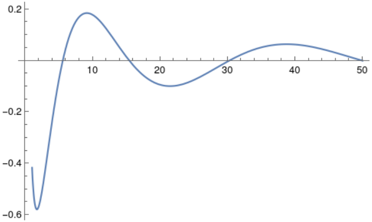


Far field

Local WKB

Can they be 'glued'?

$l = 0$ in [Hui '19]

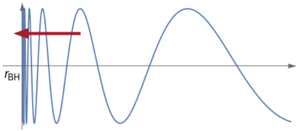
Global WKB $\qquad r_{SgrA^*} \simeq 10^7 \mathrm{km} \simeq 10^{17} \mathrm{eV}^{-1}$

$$(r - r_{BH})^{-i\mu r_{BH}} \longleftrightarrow \frac{1}{r^{3/4}} e^{-2i\mu\sqrt{r_{BH} r}}$$

$$|\phi_l| \propto V_l^{-1/4}(r)$$

l	0	1	2
$\mu r_{BH} \gtrsim$	0.3	0.7	1.2

Impose causal infalling b.c. at horizon
(Nontrivial implications:
Love numbers, no hair thms.)



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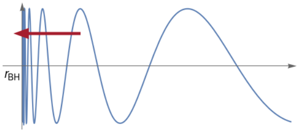
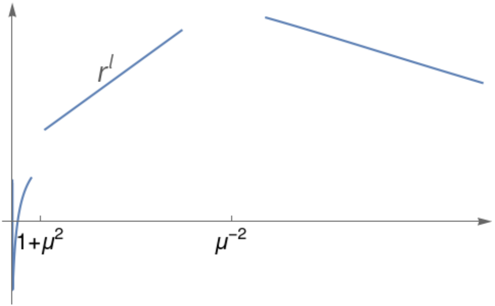


Small μr_{BH}?



$l = 1$

$$\mathcal{O}(\mu r_{BH})^{\frac{3}{2}+2l} \quad \sim (\mu r_{BH})^{\frac{3}{2}+2l} r^l \quad \sim \frac{1}{r^{3/4}} \cos(2\mu\sqrt{r_{BH} r})$$

$\mathcal{O}(1)$ variations in the density profile

- ✓ Solution for all distances
- ✓ Analytic control (causality)
- ✓ μ dependence
- ✓ $|\phi_\infty / \phi_{hor.}|$

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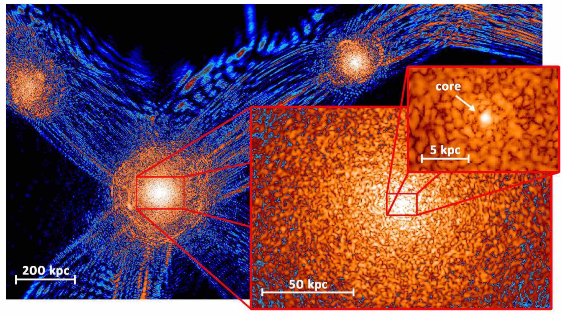
- ✓ μ dependence
- ✓ $|\phi_\infty/\phi_{hor.}|$

$\omega < \mu$ allowed, but needs $Im[\omega] < 0 \rightarrow e^{-Im[\omega]t}$ decay!



Soliton (bound state). [Schive et al. 2014]

Solitons

$M_s \simeq 10^9 M_\odot \propto M_{halo}^{1/3}(?)$

easier → BH domination: $M_{BH} \gg M_s$

harder → Soliton domination (self-gravity): $M_{BH} \ll M_s$

Solitons

$M_s \simeq 10^9 M_\odot \propto M_{halo}^{1/3}(?)$

easier ↙ ↘ harder

BH domination
$M_{BH} \gg M_s$

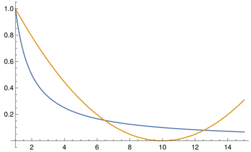
Soliton domination
(self-gravity) $M_{BH} \ll M_s$

Aim to

- Soliton domination
- Solution at all distances
- Causal boundary conditions
- GR gravity close to the horizon
- No non-relativistic approximation

$\alpha\phi_c + (1-\alpha)\phi_{n.c}$

- Large μr_{BH}: b.c. important at large distances
- Small μr_{BH}: b.c. important only $\sim r_{BH}$ away, but maybe this is already outside the sphere of influence of the BH!

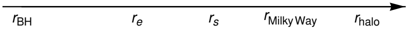


$$l = 0^1, \qquad \nabla^2 \Phi_N = -4\pi G \rho$$

$$V \to V - 2\mu^2 r^4 \Phi_N + (\omega^2 - \mu^2) r^4$$

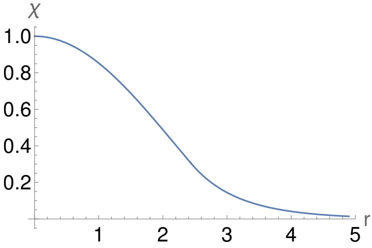
[1] Instabilities at $l > 0$ (?) [Dmitriev '21]

Sketch: given Φ_N, compute ϕ in WKB $\to$ compute ρ, Φ_N

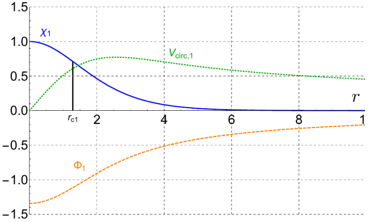


$$r_{BH} \simeq 10^7 \text{km}, \; r_e \simeq 10^{13} \text{km} \simeq 1 \text{pc}$$
$$r_s \simeq 10^{15} \text{km}, \; r_{M.W.} \simeq 10^{17} \text{km}$$

$$r_s = \frac{1}{\mu^2 G M_s}, \; r_{n.l.} = r_s \frac{M_{BH}}{M_s}, \quad \Delta M_s / M_s \simeq 10^{-8} \text{ in } 10^{10} \text{y} \propto \left(\frac{r_{BH}}{r_s}\right)^2 \frac{1}{r_s}$$



Analytic solution



Bar, Blas, Blum, Sibiryakov

Stability is crucial! It selects small μr_{BH}

$$GM_s \ll r_s \to GM_s \ll \frac{1}{\mu^2 GM_s} \to \mu GM_s \ll 1$$

$$\longrightarrow \mu r_{BH} \ll \frac{M_{BH}}{M_s} \ll 1$$

The hierarchy of scales makes boundary conditions unimportant at large distances

$$r_s \gg GM_s \to r_{n.l.} \gg GM_s \frac{M_{BH}}{M_s} = r_{BH}$$

GR corrections far away are small or the soliton is unstable.

For $M_{BH} > M_s$ the known result $GM_{BH} r_s \mu^2 \simeq \mathcal{O}(1)$ gives

$$\mu r_{BH} \ll \sqrt{\frac{M_{BH}}{M_s}}$$

Conclusions

Hairy black hole:

✓ evade no-hair theorems

✓ $\rho(r)$ for spinning DM

⚠ boundary conditions significant when $\mu r_{BH} \gtrsim \mathcal{O}(1)$

Soliton:

✓ $\rho(r)$, with self-gravity

✓ b.c unimportant in soliton domination

✓ b.c unimportant in BH domination, small μr_{BH}

⚠ b.c. significant when $M_{BH} > M_s$ and $\mu r_{BH} \gtrsim \mathcal{O}(1)$

For $SgrA^*$: $M_s \lesssim 10^7 M_\odot, \quad \mu \gtrsim 10^{-17}\mathrm{eV}$

Future directions

Include self interactions. Consider more scalars