Quantum Fluids in the Universe - ISAPP

Friday 09/06/2023

Positivity Bounds for Effective Field Theories With Spontaneously Broken Lorentz Invariance

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Outline

- Positivity Bounds for EFTs with Spontaneous Breaking of Lorentz Invariance

  with Paolo Creminelli, Leonardo Senatore, Matteo Delladio, Oliver Janssen

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- Backreaction Mechanism in Ghost-Free Massive Gravity

  with Miguel Zumalacarreguì, Giovanni Tambalo, Lavinia Heisenberg

Positivity Bounds

The Lorentz-invariant framework

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The Wilson coefficients of an Effective Field Theory are not entirely free. Their signs determine the presence/absence of superluminal propagation about non trivial backgrounds

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How to connect them?

- S-Matrix
- Conserved Currents
- Other “bridge” operators?

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A stronger bound is obtained studying the $\pi\pi \to \pi\pi$ scattering

The analytic S-matrix provides a link between IR and the UV physics

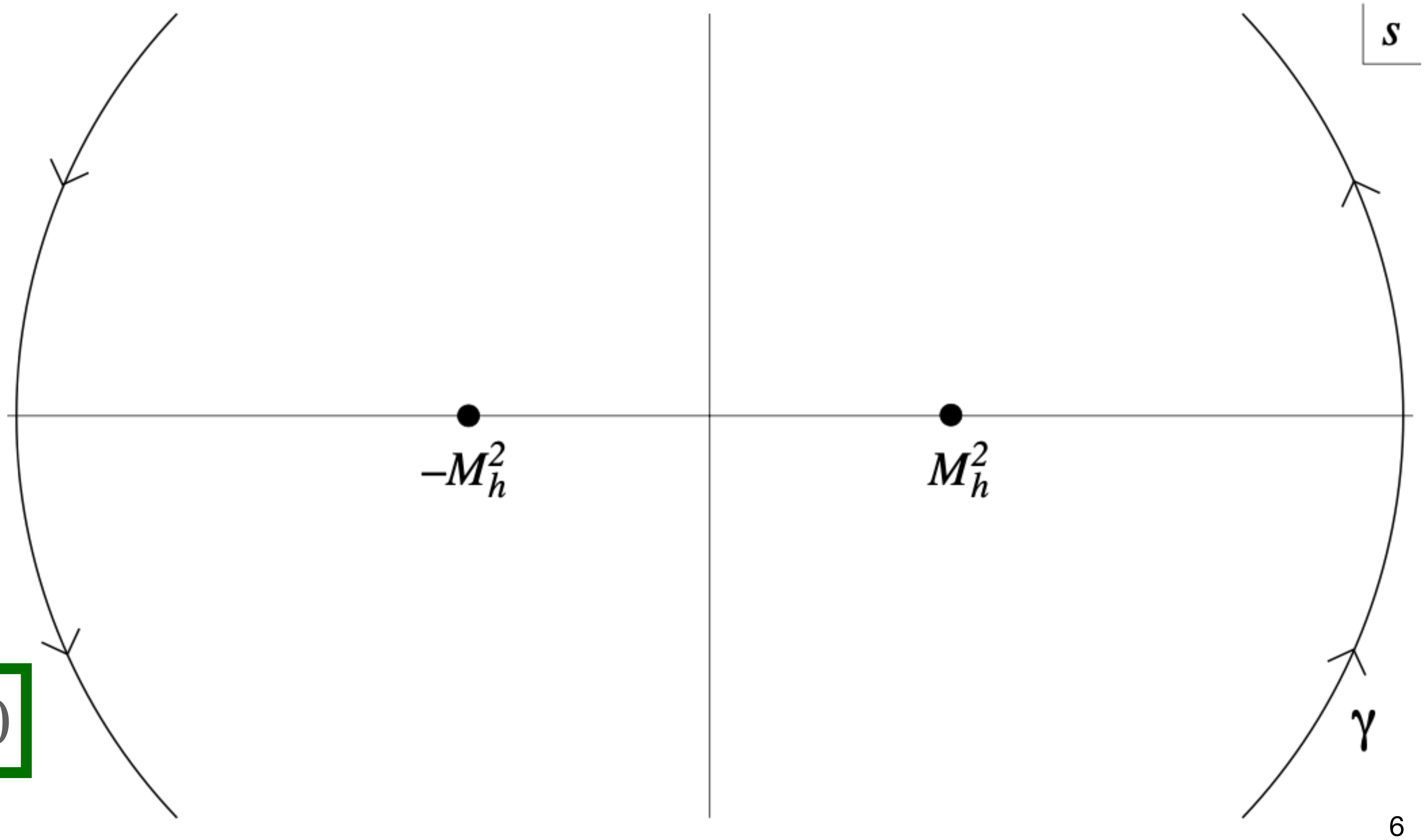
IR amplitude in the forward limit:

$$A_{\pi\pi\to\pi\pi}(s) = \frac{2c_3}{\Lambda^4}s^2$$

$$\frac{2c_3}{\Lambda^4} = \mathrm{Res}[\frac{A_{\pi\pi\to\pi\pi}(s)}{s^3}]_{s=0} = -2\mathrm{Res}[\frac{A_{\pi\pi\to\pi\pi}(s)}{s^3}]_{s=M_h^2} > 0$$

Cauchy theorem relates the Wilson coefficient to a UV strictly positive quantity

Unitarity and Analiticity of the UV completion imply $c_3 > 0$



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→ Troubles

- **No guarantee that the EFT can be extrapolated to a Lorentz-invariant UV theory (i.e. perturbations of a fluid)**
- **No clear connection between UV and IR physics**

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Back to Scattering Amplitudes

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"Fully forward" configuration ($\cos\phi_{q,p} \approx 1$) enhances the contribution of c_6 and c_8

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$\neq$

Bounds on P(X)

Summing up

Conclusions

- EFT’s coefficients are not arbitrary. “Healthy” theories obey Positivity Bounds

- “Cosmological” Positivity Bounds provide useful theoretical priors and robustness tests of already existing “Minkowskian” bounds

- S-matrix not necessarily well defined at arbitrarily high energies due to SSB of Lorentz Invariance

- Lorentz breaking as tool to extract information from h.d.o. in some specific kinematical regimes

Backreaction Mechanism in Ghost-Free Massive Gravity

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The massless propagator is not recovered in the $m \to 0$ limit

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A ghost dof rides on top of the helicity-0 mode π

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Clever structure of the potential s.t. all higher derivatives operators $(\partial^2\pi)^n$ are total derivatives

Ghost-Free Massive Gravity

$$\mathcal{L}_{dRGT} = \frac{M_{Pl}^2}{2}\sqrt{-g}\Big(R[g] - \mathcal{U}[g,f]\Big)$$

Clever Potential term

$$\mathcal{U}[g,f] = -\frac{m^2}{2}\sum_{n=0}^{4}\alpha_n e_n(\mathcal{K})$$

Fundamental building-block

$$\mathcal{K}^a_b \equiv \delta^a_b - \left(\sqrt{g^{-1}f}\right)^a_{\ b}$$

$$e_0(\mathcal{K}) = \epsilon^{abcd}\epsilon_{abcd} = 4!$$

$$e_1(\mathcal{K}) = \epsilon^{abcd}\epsilon_{a'bcd}\mathcal{K}^{a'}_a = 3![\mathcal{K}]$$

$$e_2(\mathcal{K}) = \epsilon^{abcd}\epsilon_{a'b'cd}\mathcal{K}^{a'}_a\mathcal{K}^{b'}_b = 2!([\mathcal{K}]^2 - [\mathcal{K}^2])$$

$$e_3(\mathcal{K}) = \epsilon^{abcd}\epsilon_{a'b'c'd}\mathcal{K}^{a'}_a\mathcal{K}^{b'}_b\mathcal{K}^{c'}_c = ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3])$$

$$e_4(\mathcal{K}) = \epsilon^{abcd}\epsilon_{a'b'c'd'}\mathcal{K}^{a'}_a\mathcal{K}^{b'}_b\mathcal{K}^{c'}_c\mathcal{K}^{d'}_d = \det[\mathcal{K}]$$

$$\det\left[\frac{\delta\mathcal{L}_{dRGT}}{\delta\dot{\phi}^a\delta\dot{\phi}^b}\right] = 0$$

⇓

No ghost instabilities

⇓

dRGT graviton propagates 5 dofs

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Averaged Evolution
$\neq$
Evolution of the Average

Smoothing

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Smoothing the evolution equation introduces extra sources for the long-wavelength modes ϕ_l

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Averaged inhomogeneities appear in brackets as extra sources after smoothing of equations

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Basic ingredients:

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Conformally flat metric

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A Monster

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$$\mathcal{U}_2 = 12 - 6[X] + [X]^2 - [X^2]$$

A *minimal* potential

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- compute fields equations at second order in π
- smooth-out fields equations
- get a dynamical evolution for ψ_l

Tackling Massive Gravity

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Solving the above equations linearly in π_s and ψ_s yields $\pi_s(\rho_s; \psi_l, \dot{f})$:

$$[\partial\pi_s \partial\pi_s]_\Lambda \simeq \frac{\int d^3p}{(2\pi)^3} K(p,k) P_m(p)$$

Thank you