

# Quantum information and quantum metrology for Fundamental Physics

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## **Entanglement and Relativity**

How do we quantify entanglement?





 $|L\rangle$ 







# ENTANGLEMENT AND ITS USES



### HOW DOES THIS WORK IN A <u>RELATIVISTIC</u> QUANTUM THEORY?

## Quantum field theory in curved spacetime



### **Theoretical predictions**

Particle creation by spacetime dynamics Hawking radiation Davies-Fulling-Unruh effect **Classical spacetime + quantum fields** Combines QT with GR at low energies scales reachable by cutting-edge experiments



We challenge the belief that only analogue systems can test QFTCS predictions

## PARTICLES FROM FIELDS



## **KILLING OBSERVERS**

#### **INSIGHTS**

- particles present ill-defined subsystems!
- particles well-defined only for killing observers
- particle interpretation may change with change of Killing vector field



#### **KILLING OBSERVERS**

different timelike Killing vectors K and K  $\Rightarrow$  different splits of basis in pos/neg

$$\{u_p, u_p^*\} \longrightarrow \{\bar{u}_p, \bar{u}_p^*\}$$

Bogoliubov transformation

$$\bar{a}_p = \int_{q \in \mathcal{P}} [\alpha_{pq}^* a_q - \beta_{pq}^* a_q^{\dagger}],$$

Squeezed states

$$|0\rangle = e^{\sum_{i \neq j} \gamma_{ij} a_i^{\dagger} a_j^{\dagger} - \gamma_{ij}^* a_i a_j} |\bar{0}\rangle$$

## **Bogoliubov transformations**



### **Bogoliubov transformation**

- Realizes a linear transformation of the modes:  $\tilde{a}_m = \sum_n (\alpha_{mn} a_n + \beta_{mn}^* a_n^{\dagger})$
- Alphas: passive terms (beam-splitter like)
- Betas: active terms (two-mode squeezers)



Examples: change of observer, space-time dynamics, moving cavity

# **EXAMPLE: UNRUH EFFECT**

Minkowski spacetime in 1+1 dimensions

(flat spacetime = no gravity!)



$$0_k \rangle^{\mathcal{M}} \sim \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n_k\rangle_I |n_k\rangle_{II},$$

$$\cosh r = (1 - e^{-2\pi\Omega})^{-1/2}, \qquad \Omega = |k|c/a$$

Rob is causally disconnected from region II



Timelike killing observers

thermal state trace

- (a) inertial observer
- (b) uniformly accelerated observers

Similar effect in black holes: Hawking radiation

## some results



## FLAT SPACETIME

# Alice and <u>Rob</u>

I. Fuentes-Schuller & R. Mann PRL (2005)

#### THEORETICAL PHYSICS

### To Escape From Quantum Wierdness, Put the Pedal to the Metal





Entanglement • observer-dependent
• degrades with acceleration , vanishes for ∞ acceleration

# Entanglement sharing

Where did the lost entanglement between Alice and Bob go?

$$|0_k\rangle^{\mathcal{M}} \sim \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r |n_k\rangle_I |n_k\rangle_{II},$$

**Bosonic field** 





multipartite entanglement!

Alsing, Fuentes-S, Mann, Tessier PRA 2006 Adesso, Fuentes-S, Ericsson PRA 2007

#### **Fermionic field**



Mode in region II Alice Rob

**Bipartite entanglement between** Alice and mode in region II

important differences between fermions and bosons.

## CURVED SPACETIME

## SPACETIME AS A CRISTAL

Curve spacetimes generally do not admit timelike killing vector fields...

particular spacetimes with asymptotically flat regions



# BLACK HOLES & COSMOLOGY

# Alice falls into a black hole



## Entanglement cosmology

### Ball, Fuentes-S, Schuller PLA 2006



"History of the universe encoded in entanglement"

### toy model

- expansion rate  $\sigma$ expansion factor  $\epsilon$
- calculate entanglement asymptotic past S=0 asymptotic future  $S=S(\sigma,\epsilon)$
- excitingly, can solve for  $\sigma = \sigma(S) \qquad \quad \epsilon = \epsilon(S)$

### Fermionic entanglement cosmology

### Fuentes, Mann, Martin-Martinez, Moradi PRD 2010

Fermionic fields in 3+1 dimensions: more realistic model



"The Universe entangles less fermionic fields"

## REVIEW

### **Observer-dependent entanglement**

Paul M Alsing<sup>1</sup> and Ivette Fuentes<sup>2</sup>

Published 18 October 2012  $\boldsymbol{\cdot}$  © 2012 IOP Publishing Ltd

Classical and Quantum Gravity, Volume 29, Number 22

Citation Paul M Alsing and Ivette Fuentes 2012 Class. Quantum Grav. 29 224001

Entanglement is observer dependent Spacetime dynamics create entanglement Horizons (black holes) degrade entanglement



# Localized systems for Relativistic Quantum Information



## Systems for relativistic quantum information



Cavities Trapped BECs





Traveling wave-packets

1. Entanglement between localized systems

cavities

detectors

localized wave-packets



•gravity effects on quantum properties

• Propose earth-based and space-based experiments

## **Covariance Matrix Formalism**

Convenient for systems consisting of N bosonic modes — N harmonic oscillators

$$\hat{x}_{(2n-1)} = \frac{1}{\sqrt{2}} (\hat{a}_n + \hat{a}_n^{\dagger}) \quad \text{generalized position}$$
$$\hat{x}_{(2n)} = \frac{-i}{\sqrt{2}} (\hat{a}_n - \hat{a}_n^{\dagger}) \quad \text{generalized momentum}$$

Covariance matrix:

$$\sigma_{ij} = \langle \hat{x}_i \hat{x}_j + \hat{x}_j \hat{x}_i \rangle - 2 \langle \hat{x}_i \rangle \langle \hat{x}_j \rangle$$

Displacement vector:

$$\hat{d} = \begin{pmatrix} \langle \hat{x}_1 \rangle \\ \langle \hat{x}_2 \rangle \\ \vdots \\ \langle \hat{x}_{2n} \rangle \end{pmatrix}$$



For Gaussian states the covariance matrix and displacement vector replace the density matrix

Collect in vector form:  $\hat{X}$ 

$$= \begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_{2n} \end{pmatrix}$$

### Gaussian states





|0>

Coherent states

 $\frac{1}{\cosh(r)}\sum_{n} \tanh r^n |n_a\rangle |n_b\rangle$ 

vacuum



Squeezed states

Thermal states

Gaussian states can be described using the first and second moments. They are defined as states for which higher moments vanish.

### Example: vacuum state

 $|0\rangle = |0_1\rangle |0_2\rangle \dots |0_N\rangle = |0\rangle^{\bigotimes N}$ 



Second moments:

Covariance matrix:

 $\left\langle \hat{x}_i \hat{x}_j \right\rangle = \frac{1}{2} \left\langle 0 \right| \left( \hat{a}_i \hat{a}_j^{\dagger} + \hat{a}_i^{\dagger} \hat{a}_j \right) \left| 0 \right\rangle = \delta_{ij}$ 

$$\sigma = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} = \mathbf{I}$$

## Example: single-mode squeezed coherent state

$$U(r) = e^{\frac{r}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})}$$
  

$$D(\alpha) = e^{(\alpha \hat{a} - \alpha^* \hat{a}^{\dagger})}$$
  

$$|\phi(r, \alpha)\rangle = U(r)D(\alpha) |0\rangle$$
Displacement vector:  $\hat{d} = \begin{pmatrix} \alpha \\ \alpha^* \end{pmatrix}$   

$$S(r, \alpha) = U(r)D(\alpha)$$

Second moments:

 $\langle \hat{x}^2 \rangle = \langle \hat{p}^2 \rangle = \langle \phi | (\hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger}\hat{a}) | \phi \rangle = \cosh(2r) \qquad \langle \hat{x}\hat{p} \rangle = \langle \hat{p}\hat{x} \rangle = \frac{1}{2} \langle \phi | (\hat{a}\hat{a}^{\dagger} + \hat{a}^{\dagger}\hat{a}) | \phi \rangle = -\sinh(2r)$ 

Covariance matrix: 
$$\sigma = \begin{pmatrix} \cosh(2r) & -\sinh(2r) \\ -\sinh(2r) & \cosh(2r) \end{pmatrix}$$

Separable state:  $S_a(r_a, \alpha_a) \otimes S_b(r_b, \alpha_b) |00\rangle$ 

## Example: two-mode squeezed state

$$|\psi_{ab}\rangle = \frac{1}{\cosh(r)} \sum_{n} tanh^{n}(r) |n_{a}\rangle |n_{b}\rangle$$

Frist moments:

$$\langle \hat{x}_a \rangle = \left\langle \psi_{ab} \right| \frac{1}{\sqrt{2}} (\hat{a}_a + \hat{a}_a^{\dagger}) \left| \psi_{ab} \right\rangle = 0 = \left\langle \hat{x}_b \right\rangle$$

Displacement vector: 
$$\hat{d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Covariance matrix:

Second moments:

$$\langle \hat{x}_a \hat{x}_a \rangle = \frac{1}{2} \langle \psi_{ab} | (\hat{a}_a \hat{a}_a^{\dagger} + \hat{a}_a^{\dagger} \hat{a}_a) | \psi_{ab} \rangle = \cosh(2r)$$

$$\sigma = \begin{pmatrix} \cosh(2r) & 0 & \sinh(2r) & 0 \\ 0 & \cosh(2r) & 0 & -\sinh(2r) \\ \sinh(2r) & 0 & \cosh(2r) & 0 \\ 0 & -\sinh(2r) & 0 & \cosh(2r) \end{pmatrix}$$

# State Evolution

$$\sigma_f = S^T \sigma_i S$$
 S is the symplectic matrix  
It plays an important role in QFT



There are computable measures of entanglement if the states are Gaussian

Example:

Negativity  $N(v_i)$   $v_i$  is the smallest eigenvalue of  $i\Omega\sigma^{PT}$ 

Partial transpose

## QFT in the symplectic formalism



Articles

# Entanglement generation in relativistic quantum fields

Nicolai Friis 🛛 🕿 & Ivette Fuentes

Pages 22-27 | Received 06 Apr 2012, Accepted 10 Jul 2012, Published online: 20 Aug 2012

66 Download citation 2 https://doi.org/10.1080/09500340.2012.712725

$$S = \begin{pmatrix} \mathcal{M}_{11} \ \mathcal{M}_{12} \ \mathcal{M}_{13} \ \dots \\ \mathcal{M}_{21} \ \mathcal{M}_{22} \ \mathcal{M}_{23} \ \dots \\ \mathcal{M}_{31} \ \mathcal{M}_{32} \ \mathcal{M}_{33} \ \dots \\ \vdots \qquad \vdots \qquad \vdots \qquad \ddots \end{pmatrix}$$

$$\mathcal{M}_{mn} = \begin{pmatrix} \Re(\alpha_{mn} - \beta_{mn}) & \Im(\alpha_{mn} + \beta_{mn}) \\ -\Im(\alpha_{mn} - \beta_{mn}) & \Re(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

## Example: inertial cavity

Minkowski coordinates (t, x)

 $\Box \phi(t,x)=0~$  field equation

solutions: plane waves+ boundary

$$u_k(x,t) = \frac{1}{\sqrt{k\pi}} \sin\left[\frac{k\pi}{L}(x-x_A)\right] e^{-i\omega_k t},$$
$$\omega_k = \frac{1}{L} \sqrt{(k\pi)^2 + m^2}, \quad \text{discrete spectrum}$$

creation and annihilation operators

$$x_{1} \cdot x_{2} \cdot \xi_{1} \cdot \xi_{2}$$

$$t_{a}$$

$$t_{a}$$

$$t_{a}$$

$$t_{a}$$

$$t_{a}$$

$$t_{a}$$

 $\hat{\boldsymbol{\phi}}(\boldsymbol{x},\boldsymbol{t}) = \sum_{n} (u_n(t,x)\hat{a}_n + u_n^*(t,x)\hat{a}_n^{\dagger})$ 

 $\hat{a}_n |0\rangle = 0$  vacuum state  $|m_n\rangle = \hat{a}_n^{\dagger m} |0\rangle$  particle states

## Non-uniform motion



Voyage to Alpha Centauri: Entanglement degradation of cavity modes due to motion

David Edward Bruschi, Ivette Fuentes, and Jorma Louko Phys. Rev. D **85**, 061701(R) – Published 15 March 2012

### **Bogoliubov transformations**

$$\tilde{a}_m = \sum_n \left( \alpha_{mn}^* a_n - \beta_{mn}^* a_n^\dagger \right)$$

$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n) \text{ and } \beta_{mn} = -(\tilde{\phi}_n, \phi_m^*)$$

### Entangling Moving Cavities in Noninertial Frames

T. G. Downes, I. Fuentes, and T. C. Ralph Phys. Rev. Lett. **106**, 210502 – Published 25 May 2011

Bogoliubov transformations  

$$\begin{pmatrix} \hat{x}'_{1} \\ \vdots \\ \hat{x}'_{2n} \end{pmatrix} = \begin{pmatrix} \Re(\alpha_{11} - \beta_{11}) & \cdots & \Im(\alpha_{1n} + \beta_{1n}) \\ \vdots & \ddots & \vdots \\ -\Im(\alpha_{n1} + \beta_{n1}) & \cdots & \Re(\alpha_{nn} + \beta_{nn}) \end{pmatrix} \begin{pmatrix} \hat{x}_{1} \\ \vdots \\ \hat{x}_{2n} \end{pmatrix}$$

$$\alpha_{nm} = (u'_{n}, u_{m}) \qquad \beta_{nm} = (u'_{n}, u^{*}_{m})$$

$$\phi, \psi) = i \int_{\Sigma} d\Sigma^{\mu} [\psi^{*} \partial_{\mu} \phi - \phi \partial_{\mu} \psi^{*}]$$

Can't be computed but...

$$S = \begin{pmatrix} \mathcal{M}_{11} \ \mathcal{M}_{12} \ \mathcal{M}_{13} \ \cdots \\ \mathcal{M}_{21} \ \mathcal{M}_{22} \ \mathcal{M}_{23} \ \cdots \\ \mathcal{M}_{31} \ \mathcal{M}_{32} \ \mathcal{M}_{33} \ \cdots \\ \vdots \qquad \vdots \qquad \vdots \qquad \ddots \end{pmatrix}$$
$$\mathcal{M}_{mn} = \begin{pmatrix} \Re(\alpha_{mn} - \beta_{mn}) \ \Im(\alpha_{mn} + \beta_{mn}) \\ -\Im(\alpha_{mn} - \beta_{mn}) \ \Re(\alpha_{mn} + \beta_{mn}) \end{pmatrix}$$

# Approximation...





$$\alpha = \alpha^{(0)} + \alpha^{(1)} + \alpha^{(2)} + O(h^3),$$
  
$$\beta = \beta^{(1)} + \beta^{(2)} + O(h^3),$$

 $h \ll 1$  computable transformations

During periods of inertial or uniformly accelerated motion

$T_t = -$	e <sup>id</sup>	ω <sub>1</sub> t . : · Ο .	 e <sup>it</sup>	$\begin{matrix} 0\\ \vdots\\ \omega_n t\end{matrix}$	0 : 0	···· ··	0 : 0	
		0 : 0	··· ··	0 : 0	$e^{-i\omega_1 t}$ : 0	··· 、 ···	$\begin{matrix} 0\\ \vdots\\ e^{-i\omega_n t}\end{matrix}$	Ţ

Т

$$T_{\tau} = \frac{\begin{pmatrix} e^{i\omega_{1}\tau} & \dots & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & 0 & \dots & e^{i\omega_{n}\tau} & 0 & \dots & 0 \\ 0 & \dots & e^{i\omega_{n}\tau} & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & 0 & \dots & 0 & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & e^{-i\omega_{1}\tau} & \dots & 0 \\ \vdots & \ddots & \vdots & 0 & \dots & e^{-i\omega_{n}\tau} \end{pmatrix}$$

Consider the vacuum as the initial state in region 1

Region I $\sigma_0 = I$ Region II $\sigma_{II} = S^T I S$ Region III $\sigma_{II} = S^{-1T} T_{\tau}^T S^T S T_{\tau} S^{-1}$ 

## Basic building block



$$B = T_t S T_\tau S^{-1}$$
$$\sigma_B = B^T B$$

General trajectory: grafting 
$$B_F = B_1 B_2 \dots B_n$$

### We can approximate any trajectory

(

## "Integration" method for quantum fields



$$\hat{A}_{mn} = i(\omega_m - \omega_n)\hat{\alpha}_{mn} \int_{\tau_0}^{\tau_f} e^{-i(\omega_m - \omega_n)(\tau - \tau_0)} h(\tau) d\tau ,$$
$$\hat{B}_{mn} = i(\omega_m + \omega_n)\hat{\beta}_{mn} \int_{\tau_0}^{\tau_f} e^{-i(\omega_m + \omega_n)(\tau - \tau_0)} h(\tau) d\tau .$$

Regular Article - Theoretical Physics | Open Access | <u>Published: 31 August 2020</u> Evolution of confined quantum scalar fields in curved spacetime. Part I

Spacetimes without boundaries or with static boundaries in a synchronous gauge

#### Luis C. Barbado 🖂, Ana L. Báez-Camargo & Ivette Fuentes

The European Physical Journal C80, Article number: 796 (2020)Cite this article1100 Accesses1 Citations2 AltmetricMetrics

Regular Article - Theoretical Physics | Open Access | Published: 29 October 2021 Evolution of confined quantum scalar fields in curved spacetime. Part II

Spacetimes with moving boundaries in any synchronous gauge

#### Luis C. Barbado ⊠, Ana L. Báez-Camargo & Ivette Fuentes

 The European Physical Journal C
 81, Article number: 953 (2021)
 Cite this article

 415
 Accesses
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### Entanglement

Computable measures for Gaussian states

Separable initial states of two modes  $S_a(s) \otimes S_b(s) |00\rangle$ 

Negativity  $N(v_i) \quad v_i$  is the smallest eigenvalue of  $i\Omega\sigma^{PT}$ 

$$\mathcal{N} = \left(\Re \left(e^{i\omega_a t} \beta_{ab}\right)^2 + \left(\Im \left(e^{i\omega_a t} \beta_{ab}\right) \cosh(s) - \Im \left(e^{i\omega_a t} \alpha_{ab}\right) \sinh(s)\right)^{1/2}\right)$$

$$\alpha = \alpha^{(0)} + \alpha^{(1)} + \alpha^{(2)} + O(h^3),$$
  
$$\beta = \beta^{(1)} + \beta^{(2)} + O(h^3),$$

Friis and Fuentes JMO (invited) 2012

### entanglement generated



Friis, Bruschi, Louko & Fuentes PRD 2012 Friis and Fuentes invited at JMO 2012 Bruschi, Louko, Faccio & Fuentes 2012

general trajectories continuous motion including circular acceleration

initial separable squeezed state

entanglement: negativity

 $\mathcal{N} = \left(\Re\left(e^{i\omega_a t}\beta_{ab}\right)^2 + \left(\Im\left(e^{i\omega_a t}\beta_{ab}\right)\cosh(s) - \Im\left(e^{i\omega_a t}\alpha_{ab}\right)\sinh(s)\right)^{1/2}\right)$ 

### motion and gravity create entanglement



Friis, Bruschi, Louko, Fuentes PRD (R) 2012

#### results

non-uniform motion creates entanglement

gravity creates entanglement

### quantum gates



Friis, Huber, Fuentes, Bruschi PRD 2012 Bruschi, Lee, Dragan, Fuentes, Louko PRL 2012 Bruschi, Louko, Faccio & Fuentes NJP 2013 Bruschi, Sabin, Kok, Johansson, Delsing & Fuentes SR 2016

> the relativistic motion of quantum systems can be used to produce quantum gates

two-mode squeezer beam splitter

multi-qubit gates: Dicke states Multi-mode squeezer Cluster states

### Cavity moving at relativistic speeds using superconducting circuits





### Generating Multimode Entangled Microwaves with a Superconducting Parametric Cavity

C. W. Sandbo Chang, M. Simoen, José Aumentado, Carlos Sabín, P. Forn-Díaz, A. M. Vadiraj, Fernando Quijandría, G. Johansson, I. Fuentes, and C. M. Wilson Phys. Rev. Applied **10**, 044019 – Published 8 October 2018

### Cavities moving in curved spacetime



#### PAPER • OPEN ACCESS Dynamical Casimir effect in curved spacetime

Maximilian P E Lock<sup>1,2</sup> D and Ivette Fuentes<sup>4,2,3</sup> Published 7 July 2017 • © 2017 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft <u>New Journal of Physics</u>, <u>Volume 19</u>, <u>July 2017</u>

Citation Maximilian P E Lock and Ivette Fuentes 2017 New J. Phys. 19 073005

# We find corrections due to curvature effects

### Teleporation with an accelerated partner

N. Friis, A. R. Lee, K. Truong, C. Sabín, E. Solano, G. Johansson, and I. Fuentes Phys. Rev. Lett. **110**, 113602 – Published 12 March 2013



The fidelity of teleportation is affected by motion It is possible to correct by local rotations and trip planning

### Spacetime effects on satellite-based quantum communications

David Edward Bruschi, Timothy C. Ralph, Ivette Fuentes, Thomas Jennewein, and Mohsen Razavi Phys. Rev. D **90**, 045041 – Published 28 August 2014



#### Observable effects in satellite-based quantum cryptography

$$\Phi = \int_0^{+\infty} d\omega \left[ \phi_{\omega}^{(u)} a_{\omega} + \phi_{\omega}^{(v)} b_{\omega} + \text{h.c.} \right]$$

$$a_{\omega_0}(t) = \int_0^{+\infty} d\omega \, e^{-i\omega t} F_{\omega_0}(\omega) \, a_\omega.$$
 Traveling wave-packet

$$S = \begin{pmatrix} \Theta \mathbb{1}_2 & \sqrt{1 - \Theta^2} \mathbb{1}_2 \\ -\sqrt{1 - \Theta^2} \mathbb{1}_2 & \Theta \mathbb{1}_2 \end{pmatrix}$$

Spacetime acts as a beam-splitter

# Spacetime effects on satellite-based quantum cryptography

Bruschi, Ralph, Fuentes, Jennewein, Razavi PRD 2014



### Testing the effects of gravity and motion on quantum entanglement in space-based experiments

David Edward Bruschi<sup>1</sup>, Carlos Sabín<sup>2</sup>, Angela White<sup>3,6</sup>, Valentina Baccetti<sup>4</sup>, Daniel K L Oi<sup>5</sup> and Ivette Fuentes<sup>2</sup>

Published 21 May 2014 • © 2014 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft

New Journal of Physics, Volume 16, May 2014



Effects of gravity and motion on entanglement





Figure 2. Negativity N vs. difference in gravitational field strength between initial and final orbits  $\delta\phi$ , after the first change in velocity  $\Delta v_l$ . The acceleration of the satellite is  $a = 10^{-3} \text{ m/s}^2$  (solid, blue),  $a = 2 \cdot 10^{-3} \text{ m/s}^2$  (red,dashed),  $a = 3 \cdot 10^{-3} \text{ m/s}^2$ (black, dotted) while  $L = 100 \ \mu\text{m}$ ,  $c_s = 1 \text{ mm/s}$ , giving rise to  $h^2 \simeq 0.05$  and  $\Omega_1 = 2\pi \times 50$  Hz. The initial squeezing is r = 1/2.

# Experimental test of photonic entanglement in accelerated reference frames

<u>Matthias Fink</u> <sup>⊡</sup>, <u>Ana Rodriguez-Aramendia</u>, <u>Johannes Handsteiner</u>, <u>Abdul Ziarkash</u>, <u>Fabian</u> <u>Steinlechner</u>, <u>Thomas Scheidl</u>, <u>Ivette Fuentes</u>, <u>Jacques Pienaar</u>, <u>Timothy C. Ralph</u> & <u>Rupert Ursin</u> <sup>⊡</sup>

Nature Communications 8, Article number: 15304 (2017) Cite this article



entanglement under uniform acceleration is conserved

Future experiments: non-uniform acceleration and Satellite-based experiments

# Quantum estimation of the Schwarzschild spacetime parameters of the Earth

David Edward Bruschi, Animesh Datta, Rupert Ursin, Timothy C. Ralph, and Ivette Fuentes Phys. Rev. D **90**, 124001 – Published 1 December 2014



$$oldsymbol{g} = ext{diag}\left(-\left(1-rac{2M}{r}
ight), rac{1}{1-rac{2M}{r}}, r^2, r^2 \sin^2 heta
ight)$$

Estimating the Schwarzschild radius of the Earth Using traveling wave-packets

$$\frac{\Delta r_S}{r_S} \ge \frac{8\,\sigma\,r_A\,(r_A+L)}{\sqrt{N(\Omega_1^2+\Omega_2^2)}\,r_S\,L\,\sinh r} \sim 5.8\times 10^{-7}$$

# Quantum-metrology estimation of spacetime parameters of the Earth outperforming classical precision

Jan Kohlrus, David Edward Bruschi, and Ivette Fuentes Phys. Rev. A **99**, 032350 – Published 29 March 2019



Orbit	Light's trajectory	$\Delta r_S/r_S$	$\Delta R_E/R_E$	$\Delta h/h$
LEO	$Lab1 \leftrightarrows Lab2$	$8.50 \times 10^{-10}$	$1.12 \times 10^{-9}$	$3.56 \times 10^{-9}$
	$Lab1 \rightarrow Lab1$	$8.87 \times 10^{-10}$	$1.17 \times 10^{-9}$	$3.71 \times 10^{-9}$
	$Lab2 \rightarrow Lab2$	$8.15 \times 10^{-10}$	$1.07 \times 10^{-9}$	$3.41 \times 10^{-9}$
	$Sat \rightarrow Lab1$	$1.77 \times 10^{-9}$	$2.33 \times 10^{-9}$	$7.43 \times 10^{-9}$
	$Sat \rightarrow Lab2$	$1.63 \times 10^{-9}$	$2.14 \times 10^{-9}$	$6.83 \times 10^{-9}$
VLEO	$Lab1 \rightarrow Lab1$	$6.06 \times 10^{-10}$	$6.30 \times 10^{-10}$	$1.57 \times 10^{-8}$
	$\text{Sat} \to \text{Lab 1}$	$1.21 \times 10^{-9}$	$1.26 \times 10^{-9}$	$3.15 \times 10^{-8}$

TABLE II: Precision bounds obtained through the quantum metrology scheme described above, for the different possible configurations of the reflecting and downlink schemes. Results for uplinks are very similar to those in downlinks.







Available theoretical and experimental PhD studentships

Home UK students at University of Southampton (start this or next year) International students, Emmy Studentship at Keble College, Oxford (start next year)

Available theoretical and experimental postdoctoral positions

Informal inquires: i.fuentes-guridi@soton.ac.uk

look at my website for more information about my group https://ivettefuentes.weebly.com

## Relativistic Quantum Metrology and Analogue experiments



### Many fundamental questions unanswered



What is the nature of dark matter?

Is dark energy driving the accelerated expansion of the Universe?

What physics dominated the Universe at early times?

Does the equivalence principle holds for quantum systems?

### Underpinning our difficulty to find answers



Quantum physics and General Relativity are incompatible

### Technology and instruments must be developed first







# The interplay of theory and experiment



Quantum technologies can help us put the pieces of the puzzle together

### Bose-Einstein Condensate (BEC)







Cold atoms in a thermal distribution

Atoms in a BEC form a lump

### **BEC in spacetime**

A covariant formalism is available Phonons are a relativistic quantum field



### Analogue spacetimes and simulations

#### **Analogue Systems**

moving fluid superfluid helium Bose–Einstein condensate gravity waves in water electromagnetic waves in a dielectric medium



#### Published: 12 October 2014

### Observation of self-amplifying Hawking radiation in an analogue black-hole laser

#### Jeff Steinhauer

Nature Physics 10, 864-869 (2014) | Cite this article

13k Accesses | 227 Citations | 388 Altmetric | Metrics

#### Published: 12 June 2017

#### Rotational superradiant scattering in a vortex flow

<u>Theo Torres, Sam Patrick, Antonin Coutant, Maurício Richartz, Edmund W. Tedford & Silke Weinfurtner</u>

Nature Physics 13, 833-836 (2017) | Cite this article

11k Accesses | 141 Citations | 240 Altmetric | Metrics

### Quantum analogue of a Kerr black hole and the Penrose effect in a Bose-Einstein condensate

D. D. Solnyshkov, C. Leblanc, S. V. Koniakhin, O. Bleu, and G. Malpuech Phys. Rev. B **99**, 214511 – Published 24 June 2019

### $Covariant formulation \qquad \mathcal{L} = -\sqrt{-g} \left\{ g^{\mu\nu} \partial_{\mu} \hat{\Phi}^{\dagger} \partial_{\nu} \hat{\Phi} + \left( \frac{m^2 c^2}{\hbar^2} + V \right) \hat{\Phi}^{\dagger} \hat{\Phi} + U \left( \hat{\Phi}^{\dagger} \hat{\Phi}, \lambda_i \right) \right\},$

BEC: barotropic, irrotational and inviscid fluid, in a covariant formalism.

- i) simulating a spacetime metric
- ii) simulating the effects of spacetime dynamics on a phononic field.



1. Superfluid regime (barotropic and inviscid)

Bogoliubov approximation so  $\left\langle \hat{\Phi} \right\rangle \approx \phi$  and  $\left\langle \hat{\psi}^{\dagger} \hat{\psi} \right\rangle \ll |\phi|^2$  $\hat{\Phi} = \phi \left( 1 + \hat{\psi} \right) \qquad \phi = \sqrt{\rho} e^{i\theta}$  2. In the long wavelength regime

Velocity flows

$$|k| \ll rac{\sqrt{2}}{\xi} \left(1 + rac{\hbar^2}{2m^2\xi^2 u_0^2}
ight) \min\left[1, rac{m u_0 \xi}{\sqrt{2}\hbar}
ight],$$

Scattering length

phonons should have wavelengths far longer than the healing length

 $u_{\mu}=rac{\hbar}{m}\partial_{\mu} heta$ 

Healing length 
$$\xi = \frac{1}{\sqrt{\lambda\rho}}$$
.  $\lambda = 8\pi a$ .  
 $U(\phi^{\dagger}\phi, \lambda) = \frac{1}{2}\lambda |\phi^{\dagger}\phi|^{2} + \cdots$  Strength of the interaction

The fluctuations are massless phonons

Plugging the Bogoliubov approximation in the equations of motion for the Lagrangian yields

$$\frac{1}{\sqrt{-G}}\partial_{\mu}\sqrt{-G}G^{\mu\nu}\partial_{\nu}\hat{\psi} = 0 \qquad \qquad v^{\mu} = \frac{c}{|u|}u^{\mu} \qquad \text{Normalized}$$

$$G_{\mu
u} = rac{
ho c}{c_s} \left[ g_{\mu
u} + \left( 1 - rac{c_s^2}{c^2} 
ight) rac{v_\mu v_
u}{c^2} 
ight]. \qquad \qquad c_s^2 = rac{c^2 c_0^2}{c_0^2 + \left| u 
ight|^2}$$

Normalized velocity flows

Effective speed of sound

$$c_0^2 = rac{\hbar^2}{2m^2}
ho \partial_
ho^2 U\left(
ho,\lambda
ight) = rac{\hbar^2}{2m^2}\lambda
ho$$

The equations of motion of the mean field  $\phi$  can be split into real and imaginary components resulting in a continuity equation

$$\nabla_{\mu}\left(\rho u^{\mu}\right)=0$$

and an equation directly relating bulk field properties with potentials:

$$u|^{2} = c^{2} + \frac{\hbar^{2}}{m^{2}} \left\{ V + \partial_{\rho} U\left(\rho, \lambda\right) - \frac{\nabla_{\mu} \nabla^{\mu} \sqrt{\rho}}{\sqrt{\rho}} \right\}$$

$$G_{\mu
u} \propto \begin{pmatrix} -\left(c_s^2 - v^2\right)/c^2 & -v_i/c \\ -v_j/c & \delta_{ij} \end{pmatrix}$$

Acoustic metric commonly used. We can obtain it in our covariant formulation in the following way.

### Flat spacetime

We define a "flat" acoustic metric for the phonons in the absence of any simulated fields as the acoustic metric in coordinates with the following conditions:

- 1. The background spacetime is flat, i.e.  $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ ,
- 2. No flows, i.e.  $v_i = 0$ ,
- 3. Static density, i.e.  $\rho_0 = \rho_0(\boldsymbol{x}) \Leftrightarrow \partial_t \rho_0 = 0$ , and
- 4. Unperturbed interaction strength  $\lambda$ , so  $\partial_t \lambda = \partial_x \lambda = 0$ .

The acoustic metric in n + 1 dimensions then has the form

$$G^0_{\mu\nu} = \frac{\rho_0 c}{c_{s0}} \begin{pmatrix} -c_{s0}^2/c^2 \\ & \mathbb{I}_n \end{pmatrix} \qquad \qquad c_{s0} = \frac{\hbar}{m} \sqrt{4\pi\rho a}.$$

### Simplest simulation

Gravitational wave in 1+1 dimension

$$g_{\mu
u} = \eta_{\mu
u} + h_{\mu
u}$$

A gravitational wave moving in the z-direction

$$\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t) & h_\times(t) \bullet 0 \\ 0 & h_\times(t) & -h_+(t) \bullet 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \text{time-dependent of the set o$$

time-dependent perturbations in two different polarisations.

In one dimension

 $ds^{2} = -c^{2} dt^{2} + (1 + h_{+}(t)) dx^{2}$ 

BEC in flat spacetime in the absence of spatial flows

 $v^t = c$  and  $v^x = 0$ , then the line element is conformal to:

 $ds^2 = -c_s^2 dt^2 + dx^2$ 

$$\mathfrak{g}_{ab} = rac{
ho \, c_s}{c} \left( egin{array}{cccc} -c_s^2 & 0 & 0 & 0 \ 0 & 1+h_+(t) & h_ imes(t) & 0 \ 0 & h_ imes(t) & 1-h_+(t) & 0 \ 0 & 0 & 0 & 1 \end{array} 
ight)$$

Look FB resonance and add tupac's references

Now, if we consider that the speed of sound can depend on t:

 $c_s(t) = c_{s0} f(t)$ , the corresponding line element is conformal to:

Therefore, if the speed of sound varies in time such that

$$f(t) = (1 - \frac{A_+ \sin \Omega t}{2}),$$

we find, up to the first order in  $A_+$ :

$$ds^{2} = -c_{s0}^{2} dt^{2} + (1 + A_{+} \sin \Omega t) dx^{2}.$$

So the experimental task is to modulate the speed of sound as:

background scattering length

$$c_s(t) = c_{s0}(1 - \frac{A_+ \sin \Omega t}{2}).$$
 Vary scattering length using magnetic fields  $a = a_{bg}(1 - \frac{\omega}{B - B_0})$  Feshbach resonances

### Recent experiments

#### Open Access

#### Thermalization by a synthetic horizon

Lotte Mertens, Ali G. Moghaddam, Dmitry Chernyavsky, Corentin Morice, Jeroen van den Brink, and Jasper van Wezel

Phys. Rev. Research 4, 043084 – Published 8 November 2022



FIG. 1. Schematic overview of the quench set-up. In the top part of the figure a lightlike geodesic is drawn in a spacetime diagram for flat spacetime (left) and curved spacetime with a horizon at x = 0 (right). In the lower part the corresponding tight binding models are shown with constant hopping (left) and position dependent hopping (right).

28 November 2022 / Evrim Yazgin

#### Black hole simulation began glowing, providing hope for unifying gravity and quantum mechanics

The experiment may prove the existence of Hawking radiation.

#### Article Published: 09 November 2022

### Quantum field simulator for dynamics in curved spacetime

Celia Viermann , Marius Sparn, Nikolas Liebster, Maurus Hans, Elinor Kath, Álvaro Parra-López, Mireia Tolosa-Simeón, Natalia Sánchez-Kuntz, Tobias Haas, Helmut Strobel, Stefan Floerchinger & Markus K. Oberthaler

Nature 611, 260–264 (2022) Cite this article



FIG. 2. Configurable density distribution for hyperbolic and spherical geometry. a) Density distribution of the condensate for hyperbolic geometry and expansion of a phononic wave packet depicted in th

sate (blue under-, rec two-by-two region of p els. **b**) Density distrit the two-dimensional co propagation of the way ing confirming the suc positive curvature. E the initial perturbatio respectively.

RESEARCH ARTICLE PHYSICS

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#### Experimental observation of curved lightcones in a quantum field simulator

Mohammadamin Tajik 💿 🖾 , Marek Gluza 💿 , Nicolas Sebe 💿 , 💷 , and Jörg Schmiedmayer Authors Info & Affiliations Edited by Angel Rubio, Max-Planck-Institut fur Struktur und Dynamik der Materie, Hamburg, Germany; received January 23, 2023; accepted March 24, 2023

May 15, 2023 120 (21) e2301287120 https://doi.org/10.1073/pnas.2301287120

-self-consistency checks -useful to study the behaviour of matter under the influence of fields with non-linear behaviour since their non-linearities can make computer simulations challenging.

#### Physicists Create a Holographic Wormhole Using a Quantum Computer

The unprecedented experiment explores the possibility that space-time somehow emerges from quantum information, even as the work's interpretation remains disputed.

A good analogy can be an invaluable tool in studying a complex or inaccessible system but...

Analogue experiments can't falsify or verify a physical theory (as much as a computer simulation can't do this either)

Attention!

New phenomena can be misinterpreted Analogues cannot truly discover new effects





QUANTUM GRAVITY

🥊 71 🕴 🔳

### **BEC in spacetime**

A covariant formalism is available Phonons are a relativistic quantum field



### Atom interferometer: quantum spatial interferometery



 $\delta a = 1 \left/ \left( \sqrt{N} k T^2 \right) \right.$ 

Single particle detector, local

Interferometry in the <u>spatial domain</u>: limited by time of flight

Compatible with Newtonian physics

### Gravimeters are going **Big**







# But...I want one in my phone



Change of paradigm!

#### PHYSICAL REVIEW A

covering atomic, molecular, and optical physics and quantum information

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#### Trapped-atom interferometer with ultracold Sr atoms

Xian Zhang, Ruben Pablo del Aguila, Tommaso Mazzoni, Nicola Poli, and Guglielmo M. Tino Phys. Rev. A **94**, 043608 – Published 4 October 2016



#### MINIATURIZED ATOM-CHIP GRAVIMETER

Matterwave interferometers based on cold atoms are commonly used as gravimeters. They reach accuracies of up to  $10^{-9}$ g and are nowadays even commercially available.

We have demonstrated a compact quantum gravimeter, which employs an atom chip for the rapid and efficient creation of Bose-Einstein condensates (BEC). At the same time, the atom chip serves for complete state preparation of the atomic cloud and as a retroreflector for the laser beam to create an optical lattice. With the lattice, we split, redirect, and recombine the BEC to form a Mach-Zehnder interferometer and measure the local gravitational acceleration.

To extend the interferometer time and increase the device's sensitivity, we employ the optical lattice for an innovative launch mechanism. In this way, we acquire an intrinsic sensitivity of  $\Delta g/g = 10^{-7}$ , while keeping all atom-optical operations in a volume of less than a one-centimeter cube.



Image by S. Abend and E. Rasel/Leibniz Univ. of Hannover (Physics 9, 131, 2016)

#### **RELATED PUBLICATIONS**

S. Abend et. al. Atom-Chip Fountain Gravimeter Phys. Rev. Lett. 117, 203003 (2016)

### **Quantum frequency interferometry**

Howl & Fuentes arXiv:1902.09883





Uses interactions: collective excitations, entanglement between atoms Implementation of frequency modes: phonons in a BEC (massless quantum field) Interferometry in the <u>frequency (time) domain</u>, non-local

We use parametric amplification produced by the non-linearity introduced by atomic collisions

Compatible with General Relativity: underpinned by QFT in curved spacetime
#### **BECs and quantum frequency interferometry**



- Detector can be miniaturized
- High sensitivity
- High resilience to noise

Howl & Fuentes <u>arXiv:1902.09883</u>





#### Quantum sensors underpinned by QFTCS

- Continuous source gravitational wave detector
- Quantum relativistic clocks
- Dark energy
- Proper acceleration
- Local gravitational fields (UK patent No.1908538.0)
- Gravitational gradient (UK patent No. 2000112.9)
- Curvature
- Spacetime parameters
- Dark Matter!

#### Demonstrate particle creation by spacetime dynamics!

### **Bose Einstein Condensate in a box**



mean field (ground state)  $\widehat{\Phi} = \phi(1 + \widehat{\psi})$ 

phonons density fluctuations due to interactions

quasi-uniform density

 $\rho = \phi^{\dagger} \phi$ 

Gaunt et. al. PRL 110 200406 (2013)

#### BEC in flat spacetime



$$\mathfrak{g}_{ab} = \left(\frac{n_0^2 c_s^{-1}}{\rho_0 + p_0}\right) \begin{pmatrix} -c_s^2 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Minkowski space but with speed of sound

$$\tau = (c/c_s)t \implies ds^2 = -cdt^2 + dx^2$$

phonons in a cavity-type 1-dimensional trap

$$\omega_n = \frac{n \, \pi \, c_s}{L} \quad \text{spectrum}$$

$$\Box \phi(t, x) = 0$$

$$\phi_n = \frac{1}{\sqrt{n\,\pi}} \sin\frac{n\pi(x-x_L)}{L} \, e^{-i\,\omega_n\,t}$$

Solutions to the K-G equation



$$\mathfrak{g}_{ab} = \left(\frac{n_0^2 c_s^{-1}}{\rho_0 + p_0}\right) \begin{pmatrix} -c_s^2 & 0 & 0 & 0\\ 0 & 1 + h_+(t) & h_\times(t) & 0\\ 0 & h_\times(t) & 1 - h_+(t) & 0\\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In a one-dimensional trap

$$ds^{2} = -c_{s}^{2} dt^{2} + (1 + h_{+}(t)) dx^{2}.$$

$$h_{+}(t) = \epsilon \sin \Omega t$$
 Continuous sources  
 $\omega_n = \frac{n \pi c_s}{L}$  Resonance!

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{+}(t) & h_{\times}(t) & 0 \\ 0 & h_{\times}(t) & -h_{+}(t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

### Field transformations



#### **Bogoliubov transformations**

$$\tilde{a}_m = \sum_n \left( \alpha_{mn}^* a_n - \beta_{mn}^* a_n^\dagger \right)$$

$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n) \text{ and } \beta_{mn} = -(\tilde{\phi}_n, \phi_m^*)$$

$$egin{aligned} eta_{jk}(t) &= & -rac{\epsilon}{2}\sqrt{rac{n}{m}}\,\omega_m\,t\,[-x_L+(-1)^{m+n}(L+x_L)]\delta_{jm}\,\delta_{kn}+\mathcal{O}(\epsilon^2)\ lpha_{jk}(t) &= & 0+\mathcal{O}(\epsilon^2), \end{aligned}$$

Bruschi, Fuentes & Louko PRD (R) 2011

 $\hat{U}_{\epsilon}$ 

#### Field transformations



The dynamics of spacetime produces a Bogoliubov transformation on the field modes

$$\tilde{a}_m = \sum_n \left( \alpha_{mn}^* a_n - \beta_{mn}^* a_n^\dagger \right)$$

$$\alpha_{mn} = (\tilde{\phi}_m, \phi_n) \text{ and } \beta_{mn} = -(\tilde{\phi}_n, \phi_m^*)$$

Bogoliubov transformation for monocromatic gravitational wave

$$egin{aligned} eta_{jk}(t) &= -rac{\epsilon}{2}\sqrt{rac{n}{m}}\,\omega_m\,t\,[-x_L+(-1)^{m+n}(L+x_L)]\delta_{jm}\,\delta_{kn}+\mathcal{O}(\epsilon^2)\ lpha_{jk}(t) &= 0+\mathcal{O}(\epsilon^2), \end{aligned}$$



#### **Application: gravitational wave detector**



#### Phonon creation by gravitational waves

Carlos Sabín<sup>1</sup>, David Edward Bruschi<sup>2</sup>, Mehdi Ahmadi<sup>1</sup> and Ivette Fuentes<sup>1</sup> Published 7 August 2014 • © 2014 IOP Publishing Ltd and Deutsche Physikalische Gesellschaft <u>New Journal of Physics, Volume 16, August 2014</u>

Requires high phonon numbers  $\sim \sqrt{N_0}$ and long phonon lifetimes  $\sim 10$ s

#### Three mode application



**Circuit representation** 



Improves the sensitivity by several orders of magnitude. Squeezing can be much smaller than assumed previously and the system can suffer from short phononic lifetimes.



#### Interferometric transformations



Two-mode squeezing operation

$$\boldsymbol{S}_{s} = \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \cosh r \boldsymbol{1} & \sinh r(\cos \vartheta_{sq} \boldsymbol{\sigma}_{\boldsymbol{z}} + \sin \vartheta_{sq} \boldsymbol{\sigma}_{\boldsymbol{x}}) \\ \boldsymbol{0} & \sinh r(\cos \vartheta_{sq} \boldsymbol{\sigma}_{\boldsymbol{z}} + \sin \vartheta_{sq} \boldsymbol{\sigma}_{\boldsymbol{x}}) & \cosh r \boldsymbol{1} \end{pmatrix},$$

The tritter transformation is

$$\boldsymbol{S}_{tr} = \begin{pmatrix} \cos\theta \mathbf{1} & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & \cos^2(\frac{\theta}{2})\mathbf{1} & -\sin^2(\frac{\theta}{2})\mathbf{1} \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & -\sin^2(\frac{\theta}{2})\mathbf{1} & \cos^2(\frac{\theta}{2})\mathbf{1} \end{pmatrix}$$

#### Howl & Fuentes <u>arXiv:1902.09883</u>

#### Application: gravitational wave detector

Three mode application



#### Interferometric transformations



Two-mode squeezing operation

$$\boldsymbol{S}_{s} = \begin{pmatrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \cosh r \boldsymbol{1} & \sinh r(\cos \vartheta_{sq} \boldsymbol{\sigma_{z}} + \sin \vartheta_{sq} \boldsymbol{\sigma_{x}}) \\ \boldsymbol{0} & \sinh r(\cos \vartheta_{sq} \boldsymbol{\sigma_{z}} + \sin \vartheta_{sq} \boldsymbol{\sigma_{x}}) & \cosh r \boldsymbol{1} \end{pmatrix},$$

The tritter transformation is

$$\boldsymbol{S}_{tr} = \begin{pmatrix} \cos\theta \mathbf{1} & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) & \frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\mathbf{1} + i\cos\vartheta\boldsymbol{\sigma_y}) \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & \cos^2(\frac{\theta}{2})\mathbf{1} & -\sin^2(\frac{\theta}{2})\mathbf{1} \\ -\frac{1}{\sqrt{2}}\sin\theta(\sin\vartheta\boldsymbol{\sigma_z} - i\cos\vartheta\boldsymbol{\sigma_y}) & -\sin^2(\frac{\theta}{2})\mathbf{1} & \cos^2(\frac{\theta}{2})\mathbf{1} \end{pmatrix}$$





# **Quantum metrology**



- Enables ultrasensitive devices for measuring fields, frequencies, time
- Quantum clocks and sensors are being sent to space... relativity cannot be ignored

Used to measure gravitational parameters...

gravitational field strengths accelerations

#### **Quantum Metrology**



 $\langle \psi_{\epsilon} | \psi_{\epsilon+d\epsilon} \rangle \ll 1$ 

Exploit quantum properties of the probe state to estimate with high precision parameters in the theory (Hamiltonian)

### **Quantum Metrology**



#### General framework for RQM

Ahmadi, Bruschi, Sabin, Adesso, Fuentes, Nature Sci. Rep. 2014 Ahmadi, Bruschi, Fuentes PRD 2014

Fisher information in QFT: Analytical formulas in terms of general Bogoliubov coefficients



Single-mode Two-mode channels for small parameters

$$H = \epsilon^{-2} \Re \bigg[ 4 \cosh r (f_{\alpha}^{n} + f_{\beta}^{n} + f_{\alpha}^{m} + f_{\beta}^{m}) + 4 \cosh^{2} r (2|\beta_{nm}(t)|^{2} - f_{\alpha}^{n} + f_{\beta}^{n} - f_{\alpha}^{m} + f_{\beta}^{m}) - 4 \sinh^{2} r (-f_{\alpha}^{n} + f_{\beta}^{n} - f_{\alpha}^{m} + f_{\beta}^{m} + 2\beta_{nm}(t)^{2} - 2\alpha_{nm}(t)^{2}) + 4 \sinh r \Re [\mathcal{G}_{nm}^{\alpha\beta} + \mathcal{G}_{nm}^{\alpha\beta}] - 4 \cosh^{4} r |\beta_{nm}(t)|^{2} - \frac{1}{2} \sinh^{2} 2r (2|\alpha_{nm}(t)|^{2} - 3|\beta_{nm}(t)|^{2} - \beta_{nm}(t)^{2} \bigg].$$

$$f_{\alpha}^{i} = \frac{1}{2} \sum_{n \neq k, k'} |\alpha_{ni}|^{2}$$
$$f_{\beta}^{i} = \frac{1}{2} \sum_{n \neq k, k'} |\beta_{ni}|^{2}$$
$$\mathcal{G}_{ij}^{\alpha\beta} = \sum_{n \neq k, k'} \alpha_{ni} \beta_{nj}^{*}$$

#### **Detector sensitivity**



$$\Delta \epsilon \ge \frac{1}{\sqrt{MF_Q}}$$
$$\Delta \epsilon \ge \frac{m}{\sqrt{2\pi\hbar}} \frac{\alpha^3}{\theta N_0^2 \sqrt{N_p \tau t}} \sqrt{\frac{L^7}{a^3}} \frac{\sqrt{nl} \left(l-n\right)^2}{\left(l^2+n^2\right)}$$

m atomic mass  $\theta$  tritter angle  $N_0$  number of atoms in the ground state  $N_p$  number of phonons in the two-mode squeezed state  $\alpha = \sqrt{A}/L$ , A area, L length a scattering length  $M = \tau/t$  number of measurements  $\tau$  integration time t interaction time (lifetime of the phonons)  $\Omega = \omega_n + \omega_l$  the frequency of the gw  $\omega_j = j\pi c_s/L$  phonon frequency n, l mode numbers

#### **Detector sensitivity**



tational wave sources [18, 22]. In this figure, the abbreviations are: BH, collapse to black hole; NS/NS, neutron star coalescence; NS evol, secular evolution of a nonaxisymmetric neutron star.

### Constraints



Quantum decoherence of phonons in Bose–Einstein
condensates

Richard Howl<sup>1</sup> (D), Carlos Sabín<sup>2</sup>, Lucia Hackermüller<sup>3</sup> and Ivette Fuentes<sup>1,4</sup> Published 29 November 2017 • © 2017 IOP Publishing Ltd

Journal of Physics B: Atomic, Molecular and Optical Physics, Volume 51, Number 1

Citation Richard Howl et al 2018 J. Phys. B: At. Mol. Opt. Phys. 51 015303

	$(\gamma^{La}_{k_BT\ll\mu})^{-1} \ (1.56)$	$(\gamma^{La}_{k_BT\gg\mu})^{-1} \ (1.56)$	$(\gamma^{Be,0})^{-1}$ (1.55)	$t_{1/2}$ (3.3)
$t \approx$	$\frac{640}{3\pi}\frac{\hbar^{7}\beta^{4}Ln_{0}^{3}a_{s}^{2}}{lm^{3}}$	$\frac{8}{\pi}\frac{\beta L}{a_s l}$	${640\over 3\pi^6}{mL^5n_0\over\hbar l^5}$	$\frac{1}{50}\frac{m}{\hbar n_0^2 a_s^4}$
$r \propto$	$\frac{\hbar^8\beta^4L^2n_0^{9/2}a_s^{7/2}}{l^2m^4}$	$\frac{\hbar^2 L^2 \beta n_0^{3/2} a_s^{1/2}}{l^2 m}$	$\frac{L^6 n_0^{5/2} a_s^{3/2}}{l^6}$	$\frac{L}{ln_{0}^{1/2}a_{s}^{5/2}}$
$eta \omega_l$	$2\sqrt{\pi}\frac{\hbar l\sqrt{n_0 a_s}}{Lm}$		weak interactions	$ a_s  n_0^{1/3} \ll 1$
phonon regime	$\sqrt{\frac{\pi l^2}{L^2 n_0 a_s}} \ll 1$		ultracold regime	$\frac{1}{4\pi} \frac{m}{\hbar^2 \beta  n_0  a_s} \ll 1$

Table 8.2: Approximate values for the limitations to the phonon and condensate life times and resulting proportionalities of the maximum two-mode squeeze factor from SECTION 8.1 in the case where the respective damping or particle losses become dominant.

The two bottom rows show a measure  $\beta \omega_l$  for the thermal occupation of the initial state, which should be minimal, and restrictions from the diluteness condition and the assumptions of the phononic and the ultracold regime.

To see where one could optimize and which boundaries will be encountered, all quantities above are given in terms of the parameters characterizing an individual experiment.

P. Juschitz, Two-mode Phonon Squeezing in Bose-Einstein Condensates for Gravitational Wave Detection <u>arXiv:2101.05051</u>

# Search for yet unknow sources



Know sources: 10 kHz

Exotic sources: primordial black holes boson stars Early Universe Cosmology Phase transitions Preheating after inflation, Cosmic strings Dark matter Ultralight 10<sup>-8</sup>- 10<sup>14</sup> Hz Decay: Penrose CCC

#### Ultra-Light Dark Matter (bosonic)



search for *coherent effects of the entire field*, not sing hard particle scatterings

#### Select an area to comment on

Generic Candidates: Light pseudo-Nambu-Goldstones (axions and "axion like particles" — ALPs); Massive hidden vector bosons (aka "dark photons"); Light scalars (moduli/dilatons...)

Slide by John March-Russel



# Different principle

Interferometer arm length L

Resonance

 $\frac{\Delta L}{L}$ 

 $\Omega = \omega_n + \omega_m$ quantum excitations wave

Quantum Weber Bar

# Temperature

### Weber bar

#### BEC

**T**∼ 4 K





Initial quantum states Squeezing Parametric amplification



# How can it work if its so small?



Speed of sound:  $C_s = 10$  mm/s  $L = 10^{-1} \cdot 10^{-3}$  mm Speed of light:  $C = 2.99 \times 10^{11}$  mm/s L = 2.99Km-2990Km

#### Broadband: 3D

#### Spherical BEC





#### We are studying different geometries



This three-dimensional projection of the Milky Way galaxy onto a transparent globe shows the probable locations of the three confirmed black-hole merger events observed by the two LIGO detectors—GW150914 (dark green), GW151226 (blue), GW170104 (magenta)—and a fourth confirmed detection (GW170814, light green, lower-left) that was observed by Virgo and the LIGO detectors. Also shown (in orange) is the lower significance event, LVT151012. Image credit: LIGO/Virgo/Caltech/MIT/Leo Singer (Milky Way image: Axel Mellinger).

# **Commercial applications**

$$oldsymbol{g} = ext{diag}\left(-f(r), rac{1}{f(r)}, r^2, r^2 \sin^2 artheta
ight)$$
 $f(r) = 1 - r_S/r$ 

Phononic gravimeter:

Same sensitivity but much smaller system.

#### Phononic gradiometer:

Improves the state of the art by at least two orders of magnitude





#### 1. W02020249974 - QUANTUM GRAVIMETERS AND GRADIOMETERS PCT Biblio. Data Description Claims Drawings ISR/W0SA/A17(2)[a] National Phase Notices Documents Submit observation PermaLink Report Type: International Search Report in XML 🔻 Report Language: English - Original Document 🔻 Disclaimer The image version (PDF) available on PATENTSCOPE is the official version. This online html version is provided to assist users. Despite the great care taken in its compilation to ensure a precise and accurate representation of the data appearing on the printed document/images, errors and/or omissions cannot be excluded due to the data transmittal, conversion and inherent limitations of the (optional) machine translation processes used. Hyperlinks followed by this symbol 🛷, are to external resources that are not controlled by WIPO. WIPO disclaims all liability regarding the above points. Part 1: 1 2 3 4 5 6 Part 2: A B C D E PATENT COOPERATION TREATY PCT **INTERNATIONAL SEARCH REPORT** [PCT Article 18 and Rules 43 and 44] International application No Applicant's or agent's file reference PCT/GB2020/051434 DJC96140P.WO (Earliest) Priority Date (day/month/year International filing date (day/month/year) 13 June 2019 12 June 2020

Setup	Running time	Length	$\Delta r_S/r_S$
[14, 15, 17] Atom. Int.	$100\mathrm{s}-8\mathrm{h}$	$0.2-2.5\mathrm{m}$	$10^{-9}$
[16] BEC-chip	$100\mathrm{s}$	$10^{-2} { m m}$	$10^{-10}$
Phononic MZI	6 s	$10^{-4}{ m m}$	$10^{-8}$



# Quantum technologies could be useful for Fundamental Physics

Main aim: Develop new instruments window to scales that have not been explored before New Physics!



Quantum technologies can help us put the pieces of the puzzle together



# Relativistic effects on quantum clocks

# Quantum clocks (quantum 1.0)



Two-level atom





Optical clocks routinely achieving 10<sup>-17</sup> - 10<sup>-18</sup> systematic uncertainty

# Time

#### **Quantum mechanics**

Time is absolute (Galilean trans.).
Space and time are different.
Time is a parameter.
Space is an operator.
Particles can be in a superposition of positions at once.





#### Relativity

Space and time are not different. Time is observer dependent Time flows at different rates in different points in space (Lorentz trans.)

#### Proper time



# Einstein's light clock

# Classical light

# Quantum: photons





Lindkvist, Sabin, Fuentes, Dragan, Svensson, Delsing, Johansson PRA (2014)

#### Relativistic quantum clock model

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#### Motion and gravity effects in the precision of quantum clocks

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Clock: one mode of the field in a coherent or squeezed state

How does motion, curvature and gravity affect the clock ticks and the clock's precision?

### Relativistic quantum clock model

Localized relativistic quantum field

#### Relativistic quantum clock model

Clock: one mode of the field in a coherent state.

How does motion, curvature and gravity affect the clock ticks and the clock's precision?

#### Precision: Quantum Fisher information



FIG. 2: a) Ratio of the transformed and original QFI for an initial coherent state as a function of h, for  $\theta_a = \pi$  and initial photon numbers N > 1. The solid(dashed) curve is for  $\theta_0 = 0(\pi/2)$ ). b) Ratio of the transformed and original QFI for an initially squeezed vacuum as a function of h, for  $\theta_a = \pi$  and initial photon numbers N = 1 (blue), N = 5(red) and N = 10 (green). The solid(dashed) curves are for  $\theta_0 = 0(\pi/2)$ ).

#### **Implementing the twin-paradox**

Lindkvist, Sabin, Fuentes, Dragan, Svensson, Delsing, Johansson PRA (2014)







#### What new did we learn:

Quantum particle creation makes clock tick slower
### Precision of quantised light clocks in a Schwarzschild spacetime

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 $x_{\rm t} = \bar{r}_0 + l/2(1-\chi)$ 

FIG. 1. Vertical light clock in the gravitational field of the Earth. The mirrors are held together by a  $L_{\nu} = 20cm$  rigid rod.

FIG. 4. The horizontal and vertical cavities intersect at  $x = \bar{r}_0$ . This figure shows where the mirrors are located.

#### General Relativity and Quantum Cosmology

#### [Submitted on 16 Apr 2022]

#### Gravitational time dilation in extended quantum systems: the case of light clocks in Schwarzschild spacetime

Tupac Bravo, Dennis Rätzel, Ivette Fuentes



FIG. 1. Coordinate system used to quantize the field in horizontal (red) and vertical (green) cavities.





And for quantum clocks with squeezed vacuum input states

$$\Delta_{\mathbf{h},k}(\tau_0) = \frac{1}{2\sqrt{\mathcal{M}}} \frac{1}{\sqrt{N_p(N_p+1)}} \frac{L}{\pi c k}$$
$$\Delta_{\mathbf{v},k}(\tau_0) = \Delta_{\mathbf{h},k}(\tau_0) \left(1 + \frac{r_{\mathrm{S}}L}{4\bar{r}_0^2}\chi\right)$$



## Clock made of atoms in a trap

## Notion of time?





Use collective modes of vibration

## Testing gravitational induced collapse





## Large mass experiments

# Spatial superpositions in a double-well potential

Schrödinger Cat state





## Test Penrose's gravitational induced collapse

Decoherence time must be longer than the collapse time

$$\tau \sim \frac{\hbar}{E_G}$$
  $E_G = \frac{13Gm^2N^2}{14R}.$ 

Non-uniform spherical just touching

## Gravitational self-energy

For a uniform sphere

$$E_G = \begin{cases} \frac{6GM^2}{5R} \left(\frac{5}{3}\lambda^2 - \frac{5}{4}\lambda^3 + \frac{1}{6}\lambda^5\right) & \text{if } 0 \le \lambda \le 1, \\ \frac{6GM^2}{5R} \left(1 - \frac{5}{12\lambda}\right) & \text{if } \lambda \ge 1, \end{cases}$$



where  $\lambda := b/(2R)$ 



Gravitational collapse depends strongly in the mass geometry: signature

a) and b) can collapse at shorter times depending on the ellipticity

Howl, Penrose & Fuentes, NJP 2019

# Decoherence and noise

Howl, Penrose & Fuentes, NJP 2019

- Three-body recombination
- Two-body losses
- Thermal cloud interactions
- Foreign atom interactions
- Decoherence from the trapping potential

Advantage: atom-atom interactions can be controlled in a BEC

It would be convenient to supressed them after the superposition has been created.

 $\gamma = 1/(8\pi)$  A <sup>133</sup>Cs BEC with 4x10<sup>9</sup> atoms and R=1  $\mu m$  would collapse in approximately 2 seconds  $\gamma = 8\pi$  2 s would occur when  $N \approx 10^9$  and R = 0.1 mm or  $N \approx 10^8$  and  $R = 1 \,\mu$ m.





# Confidential: shaking of the building

Work in progress with Roger, Richard and Devang



$$\Delta E_G \Delta t = \frac{h}{2}$$

So far, the community has worked on making the gravitational self-energy larger by increasing the mass to get times that are short enough to see collapse. For this  $N > 10^9$ 

For much smaller masses collapse times are very long. We propose to measure a very small energy uncertainty due to gravity: shaking of the building

A very small energy difference between wells can excite and atom to the next energy level. Experiments can detect single atoms in a higher state!

## Other BEC states

• Work in progress with Penrose & Westbrook



In a BEC atoms are not bound together and can tunnel between left and right wells.

We are studying gravitational self-energy for a variety of BEC quantum states

## Gravitational self-energy for spheroidals



Figure 4. Both plots are of the gravitational self-energy of the difference between displaced uniform spherical and spheroidal mass distributions,  $E_G$ , against b/(2R), where R is the radius of the sphere and b is the distance between the centres of the states. All mass distributions have the same total mass and volume. The solid line is for the spherical case, and the various dashed and dotted lines are for the a), b), c) and d) spheroidal configurations illustrated in Figure 3. The left plot is for  $\epsilon = 0.5$  (ellipticity e = 0.87), and the right plot is for  $\epsilon = 0.01$  (ellipticity e = 0.99995).

## BEC geometries and density

BEC densities are not uniform quadratic and Gaussian densities



Result: non-uniform densities produce shorter collapse times



Figure 7. On the left is the gravitational self-energy of the difference between displaced spherical BECs (in the TF approximation) and displaced uniform spheres,  $E_G/(GM^2R^{-1})$ , against b/(2R) where R is the radius of the spheres, M is their mass and b is the distance between the centres of the sphere states. On the right is  $E_G/(GM^2R^{-1})$  against b of spherical <sup>133</sup>Cs BECs in the TF and Gaussian approximations with 10<sup>6</sup> atoms, the same trapping frequency  $\omega_0 = 100$  Hz, and with the standard scattering length in the former regime, but with zero scattering in the latter so that we have an ideal BEC in that case.



# Using Quantum technologies we hope to understand better physics at the interplay of quantum and gravity.



