Gravitational Wave Observations

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INTERNATIONAL SCHOOL ON ASTROPARTICLE PHYSICS *"Quantum Fluids in the Universe"*

Lecture 1: 2023 Jun 06

Discovery and Detectors

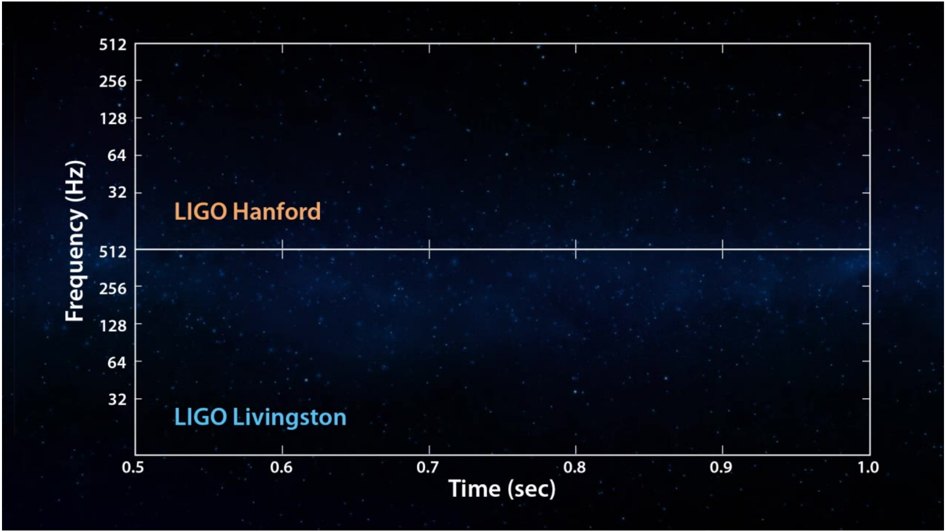
GHENT UNIVERSITY

Gravitational wave observations: course plan

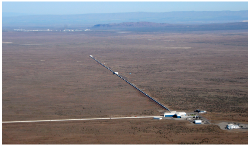
Lecture 1: Discovery and the detectors that made it possible

Lecture 2: Modelling and data analysis

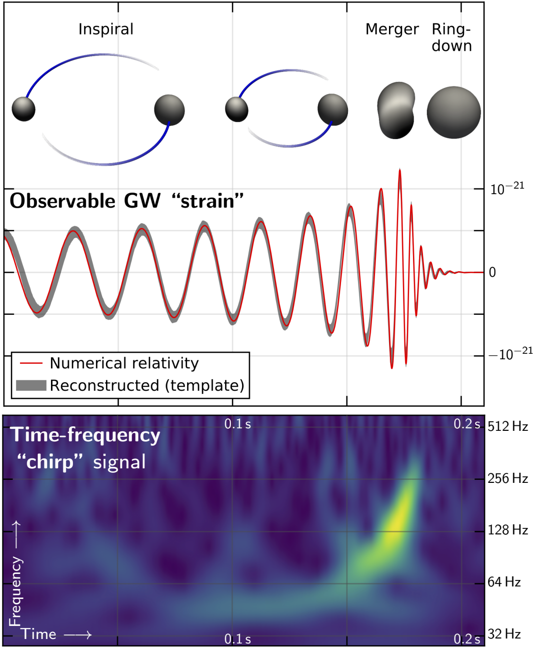
Lecture 3: Observational science and future prospects


Frequency (Hz)
512
256
128
64
32
LIGO Hanford
512
256
128
64
32
LIGO Livingston
0.5
0.6
0.7
0.8
0.9
1.0
Time (sec)

GW150914: BBH



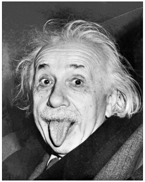
LVC: Abbott+, PRL **116**, 061102 (2016)



What are gravitational waves?

Ripples in the curvature of spacetime!

General relativity



Einstein equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

spacetime ↑ matter

coupling

metric $g_{\mu\nu}$

curvature $R_{\mu\nu\lambda\sigma}$ “Riemann tensor”

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & & & \\ & \mathcal{P} & & \\ & & \mathcal{P} & \\ & & & \mathcal{P} \end{pmatrix}$$

ρ: energy density

$\mathcal{P}$: pressure

non-linear

Linearized GR

Small perturbations h about a flat background:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

flat background metric, $\eta_{\mu\nu}$ small deformations, $h_{\mu\nu} \ll 1$

"trace reversed" $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$ (convenient variable)

In Lorenz gauge, where $\partial_\mu \bar{h}^{\mu\alpha} = 0$,

$\Box \bar{h}_{\mu\nu} = 0$ **wave equation!** for relative deformation or *strain* $h \equiv \frac{\delta l}{l}$

GW polarizations

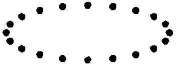
Two physical degrees of freedom — rest are "gauge" d.o.f

Gauge transformation possible to **TT gauge** where d.o.f are transverse and traceless

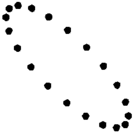
Two polarizations: $+$ and $\times$ ("plus" and "cross")

$$\begin{pmatrix} 0 & & & \\ & h_+ & h_\times & \\ & h_\times & -h_+ & \\ & & & 0 \end{pmatrix}$$

transverse, traceless in **TT gauge**



$+$ polarization



$\times$ polarization

Are gravitational waves physical?

1916–1918: **Einstein**'s calculations (incl. flux, quadrupole formula, ...)

1922: **Eddington** brought up the importance of gauge artefacts

1936: **Einstein** and **Rosen** claimed that GWs could not exist!

Robertson (reviewer) was convinced otherwise → **Infeld** → **Einstein**

1956: **Pirani** rephrased in terms of co-ordinate independent observables

1957: **Feynman** demonstrated GWs could transmit energy (Chapel Hill)

Bondi | **Weber** (resonant bar detectors)

1975: **Hulse-Taylor** binary pulsar ⇒ GWs exist! (Nobel Prize 1993)

2015: Direct detection by **LIGO-Virgo** (Nobel Prize 2017)

Pulsars

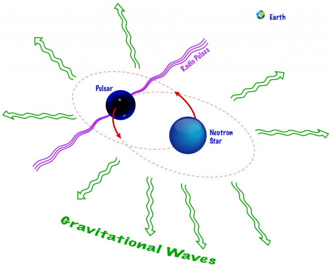
Spinning neutron stars | strong magnetic fields $\mathcal{O}(10^8\text{-}10^{15})$ G

radio emission along magnetic axis | not aligned with spin axis

$\Rightarrow$ pulses of radio emission

most precise clocks!

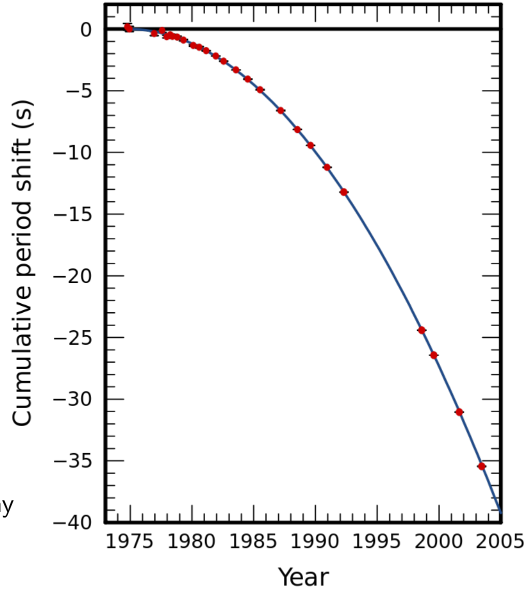
Jocelyn Bell 1967



$$\dot{P} = -\frac{192\pi}{5c^5}\frac{\mu}{M}\left(\frac{2\pi G M}{P}\right)^{5/3}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}$$

PSR B1913+16 orbital decay

Hulse-Taylor (1975)



How can we detect GWs?

rigid rulers	not practicable
resonant mass	bar detectors | lunar GW antenna!
proper length between “freely falling” masses	LIGO | ET | LISA
modulation of time dilation	pulsar timing array
CMB polarization	Planck | BICEP

GW strain

Radiation zone: $R \ll \lambda \ll d_L$

$$h_{ij}^{\mathrm{TT}} \simeq \frac{2G}{c^4 d_L} \ddot{I}_{ij}(t - \frac{r}{c})$$

$\uparrow$ retarded time

changing quadrupole moment

Order of magnitude estimates —

$$h \sim \frac{G}{c^4}\frac{\ddot{I}}{d_L} \qquad I \sim M R^2 \qquad \ddot{I} \sim \omega^2 I$$

$M \sim 1\mathrm{kg}, \quad R \sim 1\mathrm{m}, \quad \omega \sim 1 s^{-1} \qquad d_L \gg c/\omega \quad \Rightarrow \quad h \ll \frac{G}{c^5} M R^2 \omega^3$

$G = 6.64 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

$c = 3 \times 10^8 \mathrm{ms^{-1}}$

$$h \sim \frac{G}{c^4}\frac{MR^2\omega^2}{d_L} = \frac{10^{-44}}{d_L/\mathrm{m}} \quad \text{or} \quad h \ll 10^{-53}$$

spacetime is **stiff**

GW energy

$$L_{\mathrm{GW}} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{I}_{ij}\dddot{I}^{ij}\rangle$$

gravitational **Larmor formula**

Order of magnitude estimates —

$$L_{\mathrm{GW}} \sim \frac{G}{c^5}\dddot{I}^2 \qquad I \sim M R^2 \qquad \dddot{I} \sim \omega^3 I$$

$M \sim 1\mathrm{kg}, \quad R \sim 1\mathrm{m}, \quad \omega \sim 1s^{-1}$ $\qquad d_L \gg c/\omega$

$G = 6.64 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

$c = 3 \times 10^8 \mathrm{ms^{-1}}$

$$L_{\mathrm{GW}} \sim \frac{(1W)^2}{c^5/G} \sim 10^{-53}\,\mathrm{W}$$

GW energy

$$L_{\mathrm{GW}} = -\frac{dE}{dt} = \frac{1}{5}\frac{G}{c^5}\langle \dddot{\bar{I}}_{ij}\dddot{\bar{I}}^{ij}\rangle$$

gravitational **Larmor formula**

Order of magnitude estimates —

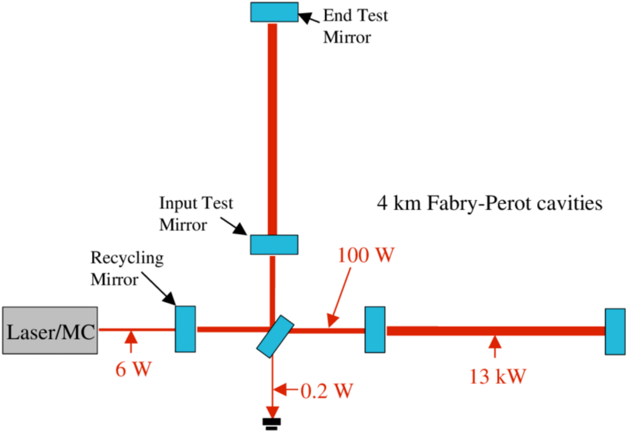
$$L_{\mathrm{GW}} \sim \frac{G}{c^5}\dddot{I}^2 \qquad I \sim M\,R^2 \qquad \dddot{I} \sim \omega^3 I \sim I\,\frac{v^3}{R^3}$$

virial $\Rightarrow \quad \frac{GM}{R} \approx v^2$

$$L_{\mathrm{GW}} \sim \frac{G}{c^5}\left(\frac{Mv^3}{R}\right)^2 = \frac{c^5}{G}\left(\frac{GM}{Rc^2}\right)^2\left(\frac{v}{c}\right)^6 \approx \frac{c^5}{G}\left(\frac{v}{c}\right)^{10}$$

“Planck luminosity” $\frac{c^5}{G} = 3.63 \times 10^{52}\ \mathrm{W}$

Interferometric GW detectors



Can we measure a strain of 10^{-21}?

Over 4 km detectors: *distances of* 10^{-18} m*!*

nucleus of an atom: 10^{-15} m

molecules (mirror surface): 10^{-10} m

Can we measure a strain of 10^{-21}?

Michelson interforemoter:

$$h = \frac{\Delta l}{l} \approx \frac{\lambda_{\text{laser}}}{L_{\text{ifo}}} \approx \frac{10^{-6}\,\text{m}}{10^{3}\,\text{m}} \approx 10^{-9}$$

Fabry-Pérot cavity (wave-optics): light bounces back and forth

$$\text{L} \to L_{\text{eff}} = \quad \sim 140 \times 4\,\text{km} \approx 600\,\text{km}$$

$$\text{Note:}\ \lambda_{\text{GW}} = \frac{c}{f_{\text{GW}}} \approx \frac{3 \times 10^{8}\,\text{ms}^{-1}}{300\,\text{s}^{-1}} \approx 1000\,\text{km}$$

$$h = \frac{\Delta l}{l} \approx \frac{\lambda_{\text{laser}}}{L_{\text{eff}}} \approx \frac{10^{-6}\,\text{m}}{10^{6}\,\text{m}} \approx 10^{-12}$$

Can we measure a strain of 10^{-21}?

We can measure *a fraction* of a wavelength not just dark and bright spots

Photodetector (counts photons): **shot noise** $\Leftrightarrow$ Poisson statistics

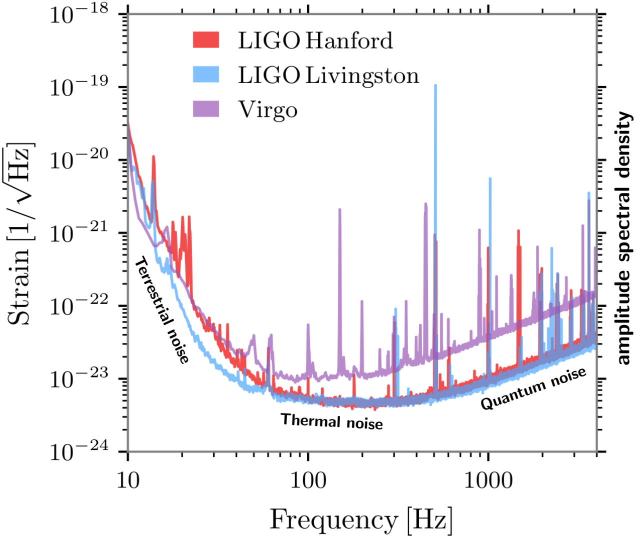
$$\frac{\delta l}{\lambda_{\text{laser}}} \approx \frac{\Delta N_{\text{photon}}}{N_{\text{photon}}} \approx \frac{N_{\text{photon}}^{1/2}}{N_{\text{photon}}} \approx N_{\text{photon}}^{-1/2}$$

$$N_{\text{photon}} = \frac{P_{\text{laser}}\tau}{hc/\lambda_{\text{laser}}} \qquad \tau \lesssim \frac{1}{f_{\text{GW}}}, \qquad 1\,\text{W laser}$$

$$\lesssim \frac{P_{\text{laser}}\lambda_{\text{laser}}}{hc\, f_{\text{GW}}} = \frac{10^{-6}}{6\times 10^{-34}\times 3\times 10^{8}\times 300} \approx 10^{16}\ \text{photons}$$

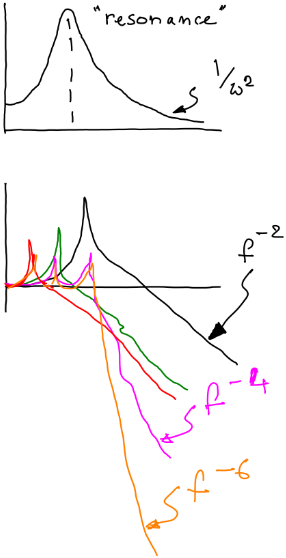
$$h = \frac{\Delta l}{l} \approx \frac{\lambda_{\text{laser}}}{L_{\text{eff}}} \times N_{\text{photon}}^{-1/2} \approx \frac{10^{-6}\ \text{m}}{10^{6}\ \text{m}} \times 10^{-8} \approx 10^{-20}$$

Detector noise



next week

Terrestrial noise: vibration isolation



Seismic noise | anthropogenic noise

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = f$$

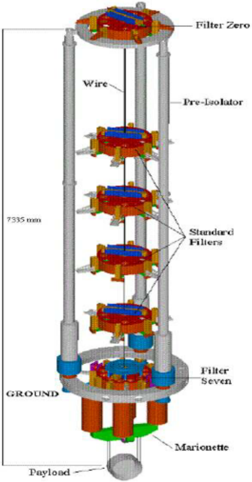
$$x = \frac{f}{\omega_0^2 - \omega^2 + i\gamma\omega}$$

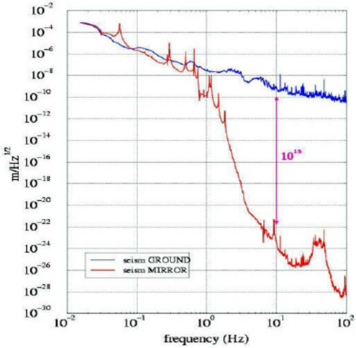
$$x \propto \omega^{-2} \qquad (\omega \gg \omega_0)$$

multi-stage pendulum | inverted pendulum

low natural frequency

Multi-stage suspension system





Virgo super-attenuator

Thermal noise

Mirror and its coating get heated up $\Rightarrow$ Brownian motion!

$$S_n(\omega) \approx \frac{4k_B T}{M\omega^2}\mathrm{Re}\left[\frac{i\omega}{\omega_0^2 - \omega^2 + i\omega_0^2\phi(\omega)}\right]$$

$$\Rightarrow \sqrt{S_n(f)} \sim f^{-5/2}$$

Thermal motion of atoms is the limiting factor! CRYOGENICS

Quantum noise

Photon shot noise | radiation pressure noise

$\Rightarrow$ standard quantum limit

Quantum squeezing to surpass quantum limit?

Frequency-dependent squeezing!

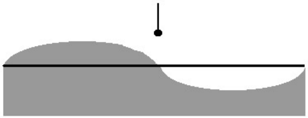
Gravity gradient noise / Newtonian noise

Newtonian gravity fluctuates, e.g. due to surface waves!

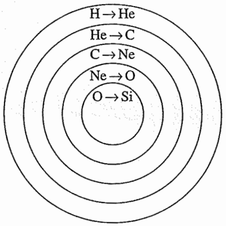
important for future detectors

Cannot be filtered

active subtraction?



Death of main sequence stars



(gravito-thermal instability) negative specific heat ⇒

red giant 'core-halo' structure (hot core, cool outer layers)

progenitor up to 8 $M_\odot$: planetary nebula + **white dwarf** *(e^- degeneracy)*

maximum mass of white dwarf $\approx 1.3\, M_\odot$ (Chandrashekhar limit)

above that ⇒ **neutron star** *(n degeneracy)*

progenitor 8 – 25 $M_\odot$: **core collapse supernova** (SNe Ib, Ic, II)

most of heavier elements up to ^{56}Fe produced in SNe explosions

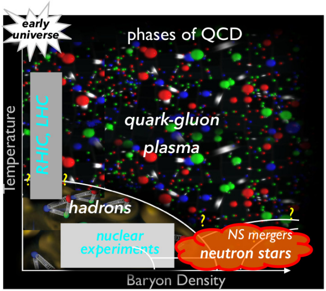
first stars: very low **metallicity**

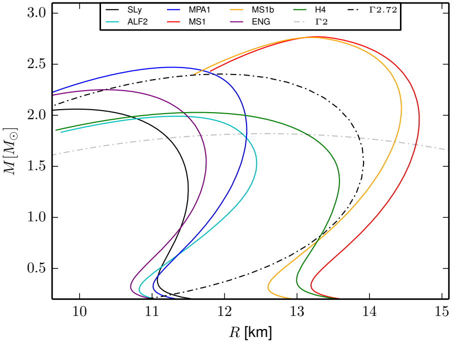
Neutron stars

$M_{NS} \gtrsim 1.3\, M_\odot$ $R_{NS} \lesssim 12\,\text{km}$ $\omega \gtrsim \text{kHz}$ solar mass in a city!

giant nucleus; cold ball of strong interactions

equation-of-state $\rho = \mathcal{P}(\rho)$ $\Leftrightarrow$ $M(R)$ maximum mass?





Pulsars

Spinning neutron stars — strong magnetic fields $\mathcal{O}(10^8\text{-}10^{15})$ G

radio emission along magnetic axis — not aligned with spin axis

$\Rightarrow$ pulses of radio emission

most precise clocks!

Jocelyn Bell 1967

Black holes

Schwarzschild solution (2016) Schwarzschild radius $= \frac{2GM}{c^2}$

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2GM}{rc^2}\right)} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

Astrophysical BH rotating $\Rightarrow$ **Kerr solution**: mass m; "spin" $\vec{a}$

No restriction on mass | astrophysically expect few $M_\odot$ – 60 $M_\odot$

lower mass gap, PISN mass gap | primordial BHs?

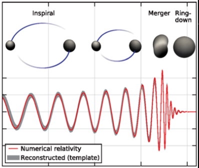
fixed shape (in a given field): no horizon deformability

no-hair theorem $\Rightarrow$ quasinormal mode $\omega, \tau =$ function(m, a)

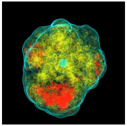
area theorem Hawking temperature $T_H \sim \frac{1}{M}$

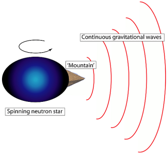
cosmic censorship $\Rightarrow$ maximum a given m

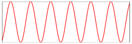
Gravitational-wave sources

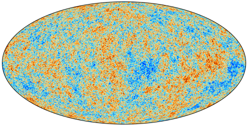


	Modelled	Unmodelled
Transient	Compact binary coalescences NS-NS, NS-BH, BBH	Bursts Supernova explosions
Persistent	Continuous waves Spinning deformed NS	Stochastic background Astrophysical + Cosmological



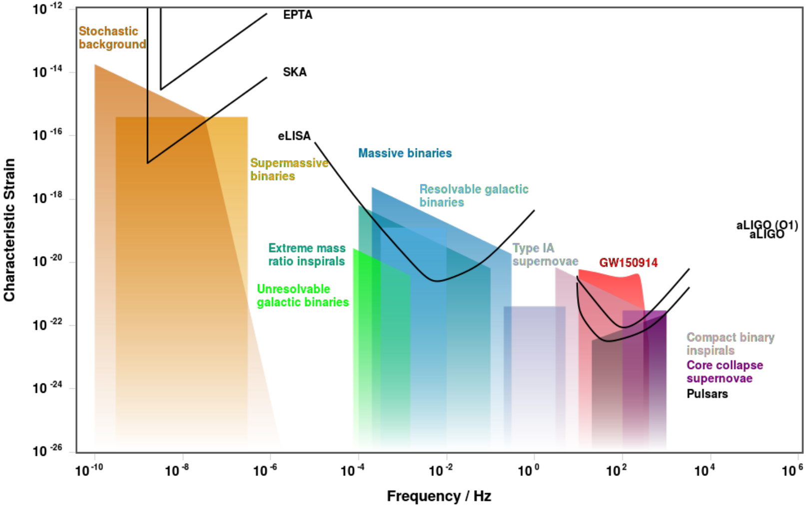






Other gravitational-wave detectors

Moore, Cole, & Berry, http://rhcole.com/apps/GWplotter/



A new window to the observable universe!

electromagnetic waves	gravitational waves
detect **intensity**	detect **amplitude**
$N_{obs} \sim \text{sensitivity}^{3/2}$	$N_{obs} \sim \text{sensitivity}^{3}$
incoherent superposition	**coherent** $\Rightarrow$ sensitivity to **phase**
strongly interacting	**weakly interacting**
affected by gas and dust	affected minimally by medium
deep imaging on small area	all sky sensitivity
high angular resolution	poor angular resolution
wavelength $\ll$ / $<$ size of source	wavelength $\sim$ / $>$ size of source
$\Rightarrow$ **image**	analogous to **sound**

Geometrized units

$c = 1,\ G = 1$ $\qquad$ $[M] = [L] = [t]$

Measure in **seconds**!

distances in seconds ✓

masses in seconds?

Schwarzschild radius of the Sun $= 3\,\mathrm{km}$

$$\Rightarrow \quad 2M_\odot \approx 10^{-5}\,\mathrm{s} = 0.01\,\mathrm{ms}$$

$$\Rightarrow \quad 60M_\odot \approx 0.3\,\mathrm{ms}$$