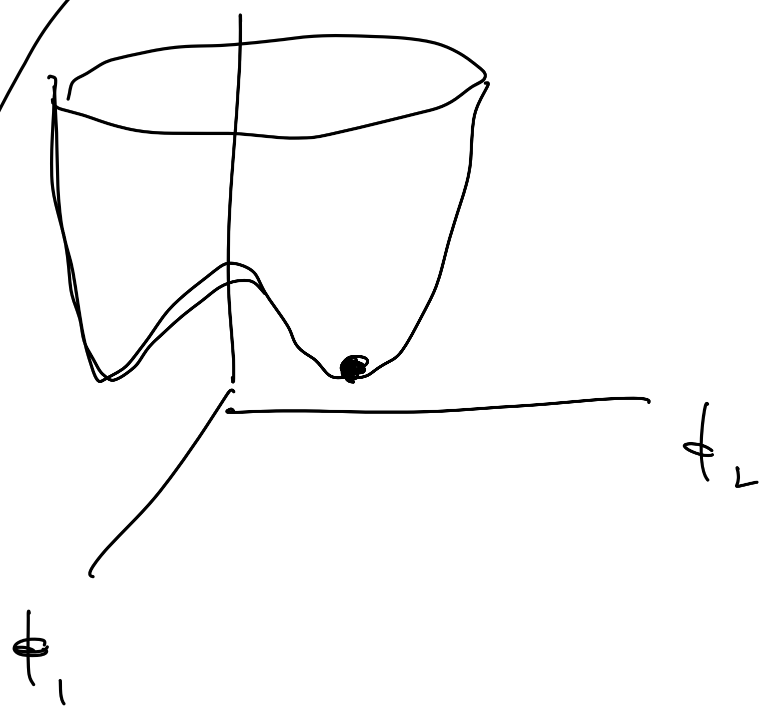
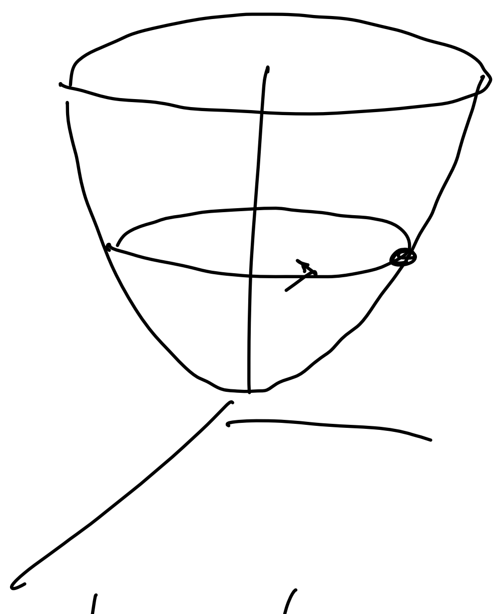
Possible UV completion of superfluid

$U(1): \Phi \to \Phi e^{i\alpha}$

$$\mathcal{L} = |\partial\Phi|^2 - m^2|\Phi|^2 - \lambda|\Phi|^4$$





$\varphi \to \varphi + \alpha$

$$\Phi = \rho(x)\, e^{i\varphi(x)} \qquad \varphi = \mu t + \pi$$

$$\frac{\delta S}{\delta \rho} = 0 \qquad \rho^2 = f\left((\partial\phi)^2\right) + \text{"}\partial X\text{"}$$

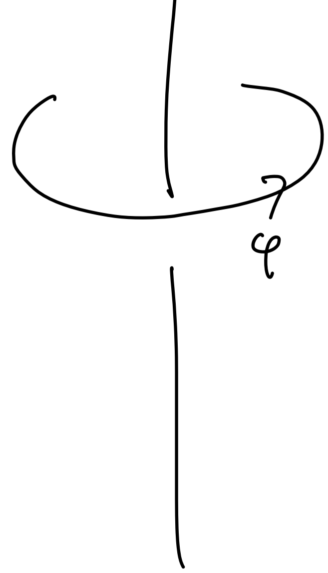
$$(\partial\rho)^2 + \rho^2 \overbrace{(\partial\phi)^2}^{X} - m^2\rho^2 - \lambda\rho^4$$

$$P(X) = \frac{X(m^2 - X)}{4\lambda} + \text{loop corrections}$$

Vortex Lines, $\phi \longleftrightarrow A_{\mu\nu}$ duality

Vortex solution

ρ^z



$$\phi = \mu t + \varphi$$

HW: check that it solves the EOM

$$\left(\partial_\mu \left(2P'(X)\, \partial^\mu \phi \right) = 0 \right)$$

$$\varphi \to \varphi + 2\pi$$

$$\phi \to \phi + 2\pi$$

Not single valued!

$$\left(\Phi \propto e^{i\phi} \to \Phi \right)$$

~ magnetic monopole in E&M

In E&M: $\underline{\underline{A_\mu}} \longleftrightarrow \underline{\underline{\tilde{A}_\mu}}$ magnetic dual gauge field

$$F_{\mu\nu} \longleftrightarrow \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

$$\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu$$

(Polchinski, "Dualities in field theory and string theory")

For us:

$$\phi \longleftrightarrow A_{\mu\nu} \quad \text{2-form}$$

(D-dim: $\phi \longleftrightarrow A_{\mu_1 \cdots \mu_{D-2}}$ (D-2)-form)

$$S_\phi = \int P\left((\partial\phi)^2\right) d^4x$$

$$A_{\mu\nu} = - A_{\nu\mu}$$

Master action

$$S = S[\underset{\text{1-form}}{V_\mu}, \underset{\text{2-form}}{A_{\mu\nu}}]$$

$$\equiv \int d^4x \left(P(V^2) - \epsilon^{\mu\nu\rho\sigma} V_\mu \partial_\nu A_{\rho\sigma} \right)$$

(1) $\dfrac{\delta S}{\delta A_{\rho\sigma}} = 0$

$$\epsilon^{\mu\nu\rho\sigma} \partial_\nu V_\mu = 0$$

$$\Downarrow$$

$\exists\, \phi(x)$ s.t. $V_\mu = \partial_\mu \phi$

$$\Downarrow$$

$$S \to S_\phi = \int d^4x \, P(\partial\phi)^2$$

$$\cancel{\underbrace{- \epsilon^{\mu\nu\rho\sigma} \partial_\mu \phi \, \partial_\nu A_{\rho\sigma}}_{\partial_\mu \left(\epsilon^{\mu\nu\rho\sigma} \phi \, \partial_\nu A_{\rho\sigma} \right)}}$$

(2) $\dfrac{\delta S}{\delta V_\mu} = 0$

$$\underline{\underline{2P'(V^2) V^\mu}} - \underbrace{\epsilon^{\mu\nu\rho\sigma} \partial_\nu A_{\rho\sigma}}_{\equiv f^\mu} = 0$$

$$\Downarrow$$

$$V^\mu \parallel \underline{\underline{f^\mu}}$$

$$4P'^2(V^2) V^2 = f^2$$

$$\Downarrow$$

$$V^2 = V^2(f^2)$$

$$2P'(V^2)V^\mu = f^\mu$$

$$4P'^2(V^2)V^2 = f^2 \implies V^2 = V^2(f^2)$$

$$S = \int P(V^2) - V^\mu f_\mu$$

$$= \int F(f^\mu f_\mu) \qquad f^\mu = \epsilon^{\mu\nu\rho\sigma}\partial_\nu A_{\rho\sigma}$$

$$\equiv S_A[A_{\mu\nu}]$$

$P((\partial\phi)^2)$ & $F(f^2)$

describe the same physics.

Thermodynam dictionary :

$(\partial\phi)^2 = \mu^2$ $\quad$ P = pressure

$2P'X - P = \rho$

$$2P'\sqrt{(\partial\phi)^2} = n \quad \text{\# density}$$
$$= J^0$$

$$f^2 = n^2 = (J^0)^2$$

$$F = -\rho$$

$$P(\mu^2) \longleftrightarrow \rho(n^2)$$

$$dP = n\,d\mu \longleftrightarrow d\rho = \mu\,dn$$

Symmetries: $U(1)$ $\phi \to \phi + \alpha$

$$\partial_\mu\phi \to \partial_\mu\phi$$
$$\wr\wr$$
$$V_\mu$$
$$f^\mu(V^\mu) \to f^\mu = \epsilon^{\mu\nu\rho\sigma}\partial_\nu A_{\rho\sigma}$$

gauge invariance: $A_{\mu\nu} \to A_{\mu\nu} + \partial_\mu\xi_\nu - \partial_\nu\xi_\mu$

arbitrary $\xi_\mu = \xi_\mu(x)$

Background + fluctuations:

$$\phi(x) = \mu t + \pi(x)$$

$$A_{\mu\nu} = \begin{cases} A_{00} = 0 \\ A_{0i} = \delta A_{0i}(x) \equiv n \cdot a_i(x) \\ A_{ij} = -\frac{1}{3} n \epsilon_{ijk} x^k + \delta A_{ij}(x) \end{cases}$$

$$\equiv n \epsilon_{ijk} \left(-\frac{1}{3} x^k + b^k(x) \right)$$

gauge transf:

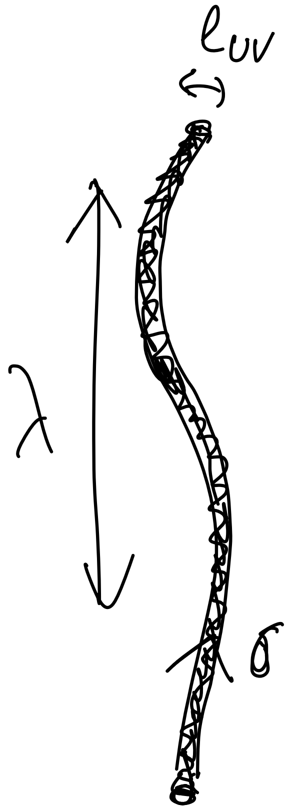
$$\vec{a} \to \vec{a} + \dot{\vec{\xi}} - \vec{\nabla}\xi_0$$

$$\vec{b} \to \vec{b} + (\vec{\nabla} \times \vec{\xi})$$

Invariants? $\dot{\vec{b}} - \vec{\nabla} \times \vec{a} \propto \vec{u}(x)$

velocity field

$$\vec{\nabla}\cdot\vec{b} \propto \delta n \propto \delta\rho \propto \delta p$$



Fluctuations of vortex line

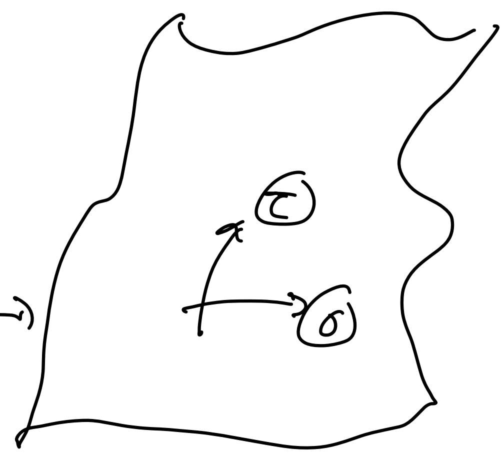
$$\psi(x) \neq \mu t + \varphi$$

"Effective" string theory description?

$$\lambda \gg \ell_{UV}$$

String action?

$$\underbrace{X^{\mu}(\tau, \sigma)}_{\text{world sheet coordinates}}$$



$$S \overset{?}{=} S_{N.G.} = \int T \, d\sigma d\tau \sqrt{|\det \partial_\alpha X^\mu \partial_\beta X_\mu|}$$

Nambu Goto $\qquad$ tension

μ = spacetime index

α, β = w.s. indices

$$g^{ind}_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu = \partial_\alpha X^\mu \partial_\beta X^\nu \cdot \eta_{\mu\nu}$$

Subtleties:

1) $T = T((\partial\phi)^2) \neq$ const

neglect, for simplicity

2) $\eta_{\mu\nu}$ in $g^{ind}_{\alpha\beta} \rightarrow f((\partial\phi)^2)\eta_{\mu\nu} + g((\partial\phi)^2)\partial_\mu\phi\,\partial_\nu\phi$

neglect for simplicity

3) there is a <u>more relevant</u> term,

invisible in the f description

$$S \supset S_{KR} = \lambda \int_{w.s.} A^{(2)}$$

$\uparrow$

Kalb-Ramond

$$= \lambda \int d\tau\, d\sigma\, \overbrace{A_{\mu\nu}}^{\sim\, \epsilon_{ijk} x^k} \partial_\tau X^\mu \partial_\sigma X^\nu$$

invariant under all the symmetries,

under gauge transf only up to total ∂'s $\Longrightarrow$ $\lambda = \text{const}$

$$= \int d^4x\, A_{\mu\nu}(x)\ \lambda \int d\tau\, d\sigma\, \delta^4(x^\mu - X^\mu(\tau,\sigma))\, \partial_\tau X^\mu \partial_\sigma X^\nu$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{J^{\mu\nu}(x)}$$

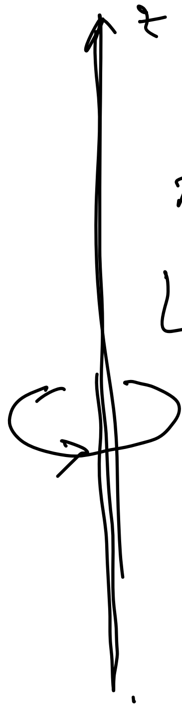
$J^{\mu\nu}(x)$ ~ source for $A_{\mu\nu}$

For a charged particle:

$$S \supset q\int A_\mu\, dX^\mu = q\int_{\substack{\text{world}\\\text{line}}} d\tau\, A_\mu\, \partial_\tau X^\mu$$

$$= \int d^4x\, A_\mu(x) \underbrace{\int d\tau\, \delta^4(x^\mu - X^\mu(\tau))\, q\, \partial_\tau X^\mu}_{J^\mu(x)}$$

Back to straight string



$$A^{(2)} = n\left(\frac{1}{2}r^2\, d\varphi\wedge dz - \frac{\Gamma}{2\pi}\log\frac{r}{r_0}\, dt\wedge dz\right)$$

$$f = \mu t \qquad + \qquad \varphi$$

$$\Gamma = \oint \vec{v}\cdot d\vec{\ell}$$

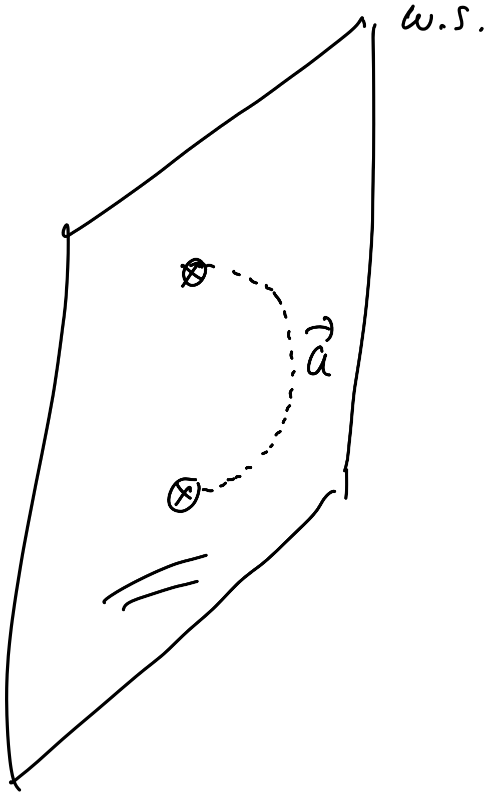
$dt \wedge dz$

λ (up to n, 2π)

Expand about a straight line

$$X^\mu(\tau,\sigma) = (0,0,z)$$

$$S_{KR} \supset \int dt\, dz \left[n\lambda a_i \partial_z X^i\right]$$



N.G. tension

IR cutoff

$$\frac{dE}{dz} = T - \frac{n^2 \lambda^2}{(\rho + p)} \frac{1}{4\pi} \log\left(L / \ell_{UV} \right)$$

UV cutoff

renormalization

RG eq for T:

$$\frac{d\,T(k)}{d\log k} = - \frac{n^2 \lambda^2}{(\rho + p)} \frac{1}{4\pi}$$

typical scale of the process:

- size of box
- wavelength of perturbations
- distance from other strings

Ex: **Kelvin waves**

gauge choice (σ, τ):

$$\tau = t = X^0$$
$$\sigma = z = X^3$$

$$X^a(t,z) = 0 + \delta X^a(t,z)$$

$a = 1,2$

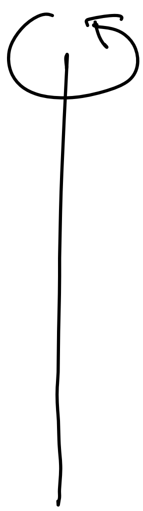
$$S_{KR} + S_{NG} \longrightarrow \int dt\, dz \left[-\frac{1}{3} n \lambda \epsilon_{ijk} X^k \dot{X}^i \partial_z X^j - T(k) \underbrace{|\partial_z \vec{X}|}_{\text{length element of string}} \right]$$

$$A_{\mu\nu} \partial_\tau X^\mu \partial_\sigma X^\nu$$

(neglecting $\dot{X}^i \dot{X}^j$ terms in NG.)

$$\longrightarrow \int dt\,dz \Big[-\frac{1}{2}\, n \lambda\, \underline{\underline{\epsilon_{ab}}}\, \delta X^{a}\, \delta \dot{X}^{b}$$

$$- T(k) \sqrt{\underset{\substack{\uparrow \\ (\partial_z X^3)^2}}{1} + (\partial_z \underline{\underline{\delta X^{a}}})^2} \Big]$$



$$SO(2) = U(1)$$

$$\phi \equiv \frac{1}{\sqrt{2}} \left(\delta X^{1} + i\, \delta X^{2} \right)$$

$$S \rightarrow \int dt\,dz \Big[\underbrace{n \lambda\, \phi^{*}\, i \partial_t \phi}_{\substack{\uparrow \\ S_{KR}}} - \underbrace{T(k)\sqrt{1 + 2|\partial_z \phi|^2}}_{\substack{\uparrow \\ S_{NR}}} \Big]$$

$$\underset{\substack{\uparrow \\ \text{small } \phi \text{ expansion}}}{\simeq} \int dt\,dz \Big[n \lambda\, \phi^{*}\, i \partial_t \phi - \underset{\text{const}}{\cancel{T(k)}}$$

$- T(k)\, |\partial_z \phi|^2 + \text{interactions}\left(|\partial_z \phi|^{2n} \right) \Big]$

Linearized EOM:

$$n\lambda\, i\partial_t \phi + T(k)\, \partial_z^2 \phi = 0$$

$$\phi \sim e^{-i\omega t} e^{ikz} \qquad \phi = \phi(t,z)$$

$$n\lambda\, \omega - T(k)\, k^2 = 0$$

$$\rightarrow \quad \omega = \frac{1}{n\lambda} T(k)\, k^2 \sim \boxed{k^2 \log k}$$

Kelvin 1880

$\rightarrow$ polarization : circular