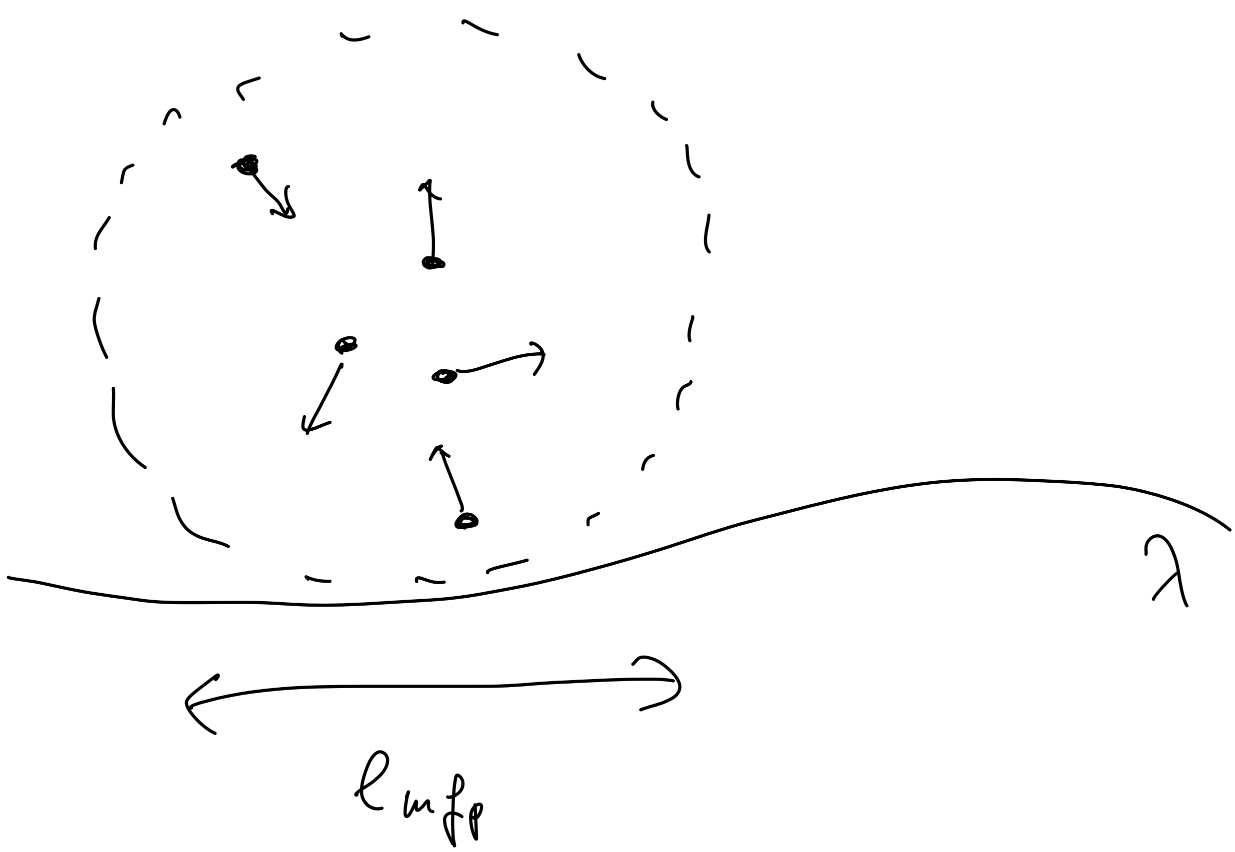
EFT's for fluids, superfluids, ...

("phases of matter")

⊛ why? Universality

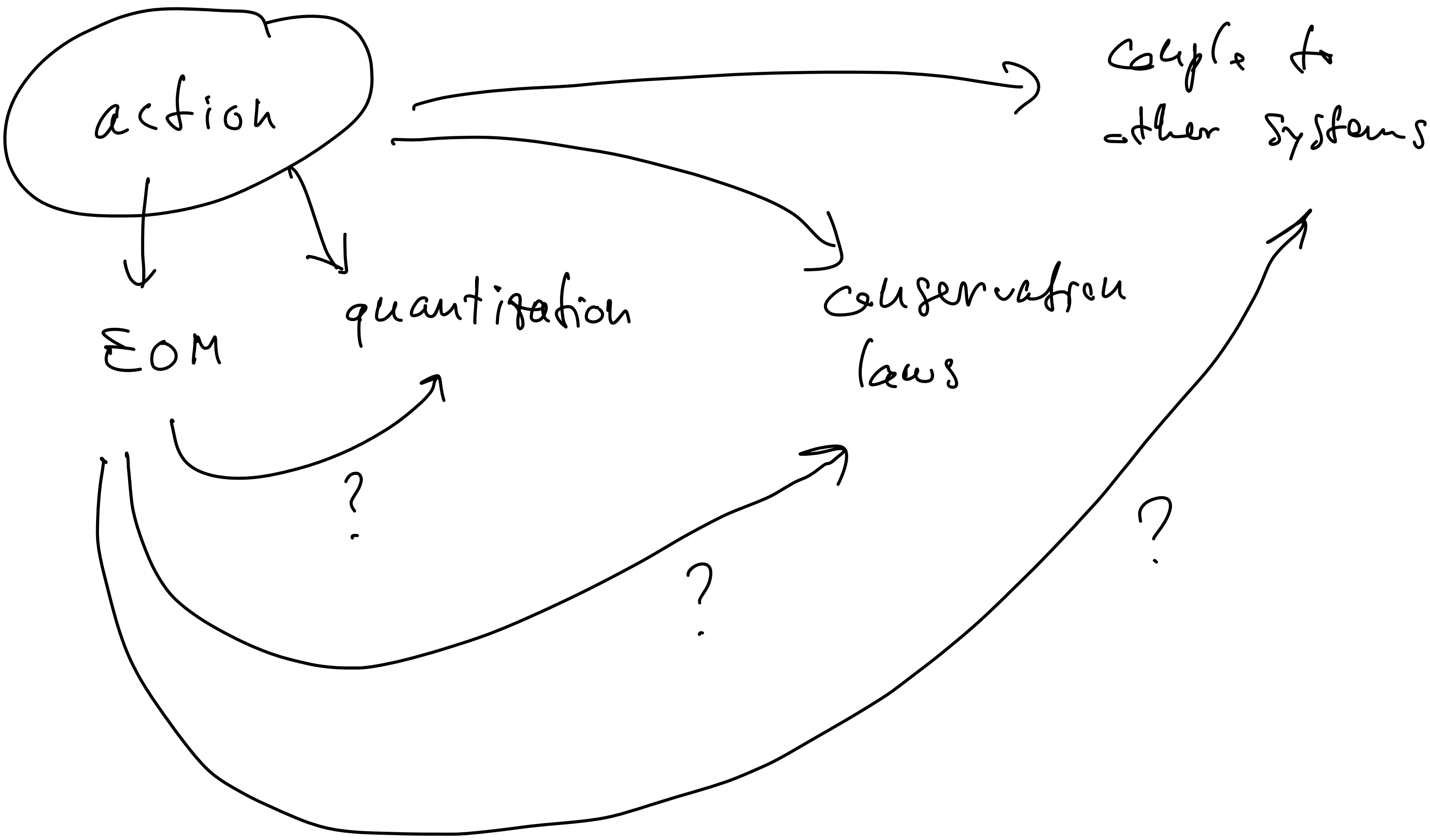
Hydrodynamics = oldest effective theory



$$\begin{cases} \dot{\rho} + \vec{\nabla}\cdot(\rho \vec{v}) = 0 \\ \dot{\vec{u}} + (\vec{u}\cdot\vec{\nabla})\,\vec{u} = -\dfrac{\vec{\nabla} p}{\rho} \end{cases} \qquad p = p(\rho)$$

eq. of state

EOM



(*) What

Goldstone dynamics of fluids, superfluids, solids, ...

<u>Goldstone</u>? Some S.S.B. always there in "<u>condensed matter</u>"

<u>bosons</u>

- <u>No</u>: Fermi liquids
  Electronic properties of solids
  Finite T effects (dissipation)

"finite density matter"

↓

<u>Superfluid</u> = cleanest, simplest "CM" system

Traditionally: zero viscosity, BEC, etc.

QFT: Superfluid = system w/

homogeneous, isotropic state

$|\psi\rangle$ that:

$$\boxed{\begin{array}{l} 1) \text{ breaks a } U(1) \text{ charge } \hat{Q}. \\ \hline 2) \text{ has a finite density for } \hat{Q}. \end{array}}$$

$U(1)$ charge: $\qquad (\partial_\mu J^\mu = 0)$

$$\hat{Q} = \int d^3x \; \hat{J}^0$$

2) $\Longleftrightarrow$ $\langle\psi| \hat{J}^0(x) |\psi\rangle \equiv n \neq 0$

<u>breaks boosts.</u>

$\Rightarrow \hat{J}^0 |\psi\rangle \neq 0$

<u>If</u> $\hat{Q}|\psi\rangle = q|\psi\rangle \qquad q \neq 0$

$(q \to \infty \text{ as } Vol \to \infty)$

$$\langle\psi|\overbrace{[\hat{Q},\hat{O}(x)]}^{\delta O(x)}|\psi\rangle = \delta\langle O(x)\rangle$$

order parameter

$$\| $$

$$\langle\psi| q\hat{O} - \hat{O} q|\psi\rangle = 0$$

$$\Rightarrow \left(\text{SSB} \Longleftrightarrow |\psi\rangle \text{ is } \underline{\text{not}} \text{ an eigenstate of } \hat{Q} \right)$$

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1) SSB $\longrightarrow$ Goldstone $\varphi(x)$ $\quad$ $(U(1))$

$$U(1): \quad \varphi(x) \longrightarrow \underline{\varphi(x)+\alpha} \qquad \alpha = \text{const}$$

$$\left[\text{Ex:} \quad \Phi(x) \longrightarrow \underline{e^{i\alpha}}\, \Phi(x) \right.$$

$$\langle \Phi(x) \rangle \neq 0$$

$$\Phi(x) = \rho(x)\, e^{i\varphi(x)}$$

$\Rightarrow \quad \phi \to \phi + \alpha \quad]$

$U(1)$: $\partial_\mu \phi \to \partial_\mu \phi$ $\qquad (\partial \phi)^2 = \partial_\mu \phi \partial^\mu \phi$

Lorentz: $(\partial\phi)^2, \quad (\Box \phi)^2, \quad \ldots$

$$\mathcal{L} = \underset{\substack{\uparrow \\ \text{generic function}}}{P}\Big(\underbrace{(\partial\phi)^2}_{\equiv X} \Big) + \cancel{\text{higher } \partial\text{'s}} \to \text{low energy, long distances}$$

$\eta_{\mu\nu} = (+,-,-,-)$

2)

$$J^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 2P'(X)\,\partial^\mu\phi$$

$$\underbrace{\langle \hat{J}^0(x) \rangle}_{\text{const}} \neq 0 \quad \Rightarrow \quad \underbrace{\langle \partial^0 \hat{\phi} \rangle}_{\text{const}} \neq 0$$

phonon field

$$\hat{\phi}(x) = \mu \cdot t + \hat{\pi}(x)$$

chemical pot. (later)

$$X = (\partial\phi)^2 = (\mu\,\delta^0_\alpha + \partial_\alpha \pi)(\mu\,\eta^{\alpha 0} + \partial^\alpha \pi)$$

$$= \mu^2 + 2\mu\dot{\pi} + (\partial\pi)^2$$

$$= \mu^2 + 2\mu\dot{\pi} + \dot{\pi}^2 - (\vec{\nabla}\pi)^2$$

$$P(X) = P(\mu^2) + P'(\mu^2)\left(2\mu\dot{\pi} + \dot{\pi}^2 - (\nabla\pi)^2\right)$$

$$+ \frac{1}{2!} P''(\mu^2)\left(2\mu\dot{\pi} + \dot{\pi}^2 - (\nabla\pi)^2\right)^2$$

$$+ \dots$$

$T^{\mu\nu}$?

$$S = \int d^4x\, P(\eta^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi)$$

$$\rightarrow \int d^4x \sqrt{g}\, P(g^{\mu\nu}\partial_\mu f\, \partial_\nu f)$$

$$\frac{\delta}{\delta g^{\mu\nu}} \longrightarrow \underset{\substack{\uparrow \\ g=\eta}}{T_{\mu\nu} =} 2P'(X)\, \partial_\mu f\, \partial_\nu f - \eta_{\mu\nu} P(X)$$

cf. $\quad T_{\mu\nu} \underset{\substack{\uparrow \\ \text{for a perfect fluid}}}{=} (\rho + p)\, u_\mu u_\nu - \eta_{\mu\nu}\, p \qquad u_\mu u^\mu = +1$

$$\begin{cases} p = P(X) \\ \rho = 2P'(X)X - P \\ u_\mu = \partial_\mu f / \sqrt{X} \end{cases}$$

$p = f(\rho)$ eq. of state (experemintal / microphysics input)

$$\hookrightarrow \quad P(X) = f\left(2P'(X) - P\right)$$

ODE for $P(X)$ $\Rightarrow$ $P(X)$ completely determined by eq. of state

Zero-T thermodynamics:

$$\begin{cases} (\rho + p) = \mu \cdot n \\ d\rho = \mu \cdot dn \end{cases} \qquad n = \#\text{ density}$$

$$(dp = n \cdot d\mu)$$

$$J^\mu = 2P'(X)\,\partial^\mu \phi$$

$$n = J^\mu \cdot u_\mu = 2P'(X)\frac{X}{\sqrt{X}} = 2P'\sqrt{X}$$

$$\rho = 2P'X - P$$

$$\to p = P$$

$$\Rightarrow \quad \mu = \sqrt{X} \quad \text{chemical potential}$$

$$P(X) \longleftrightarrow p(\mu^2)$$

$$c_s^2 = \frac{dp}{d\rho} = \frac{\frac{dP(X)}{dX}}{\frac{d(2P'(X)X - P)}{dX}} = \frac{P'}{2P''X + \cancel{2}P' - \cancel{P'}}$$

$$= \frac{P'}{2P''X + P'}$$

$$P(X) \supset + P'(\mu^2)\left(\dot{\pi}^2 - (\nabla\pi)^2\right)$$
$$+ \frac{1}{2!} P''(\mu^2)\left(2\mu\dot{\pi}\right)^2$$

$$= \left(P' + 2\mu^2 P''\right)\dot{\pi}^2 - P'(\nabla\pi)^2$$

$$= \underbrace{P'}_{\frac{1}{2}\left(\frac{\rho+p}{\mu^2}\right)} \left[\underbrace{\frac{P' + 2\mu^2 P''}{P'}}_{1/c_s^2} \dot{\pi}^2 - (\nabla\pi)^2 \right]$$

$$-\frac{1}{c_s^2}\ddot{\pi} + \nabla^2\pi = 0$$

$$\ddot{\pi} - c_s^2 \nabla^2 \pi = 0$$

$\Rightarrow$ sound waves propagating at c_s.

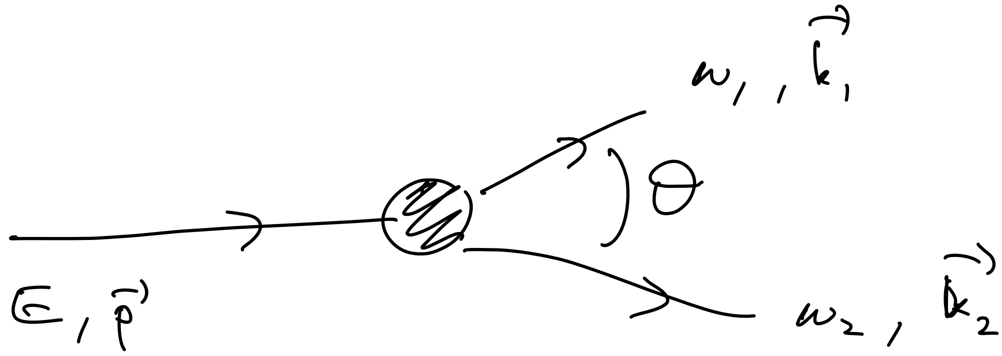
$\cancel{U(1)}$ + finite density

$$\Rightarrow \quad S = \int \underset{\substack{\uparrow \\ \text{eq. of state} \quad p = p(\mu^2)}}{P(X)}\, d^4x$$

$$X = \partial_\mu \phi\, \partial^\mu \phi \qquad \phi = \mu t + \pi(x)$$

$$S \to \underset{\substack{\uparrow \\ \text{const}}}{S_0} + \underset{\substack{\uparrow \\ \mathcal{O}(\pi^2)}}{S_2} + \underset{\substack{\uparrow \\ \mathcal{O}(\pi^3)}}{S_3} + \underset{\substack{\uparrow \\ \mathcal{O}(\pi^4)}}{S_4} + \dots$$

Phonon decay



$$\begin{cases} \vec{p} = \vec{k}_1 + \vec{k}_2 \\ \cancel{c_s} p = \cancel{c_s} k_1 + \cancel{c_s} k_2 \end{cases}$$

$$p^2 = k_1^2 + k_2^2 + 2\vec{k}_1 \cdot \vec{k}_2$$

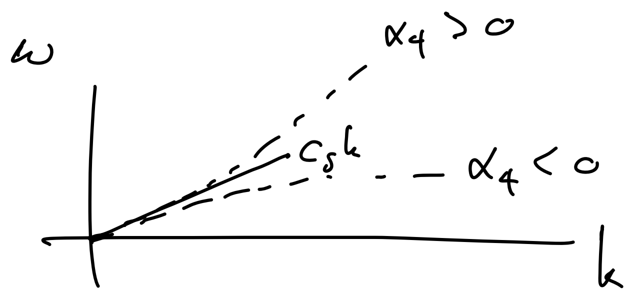
$$p^2 = k_1^2 + k_2^2 + 2 k_1 k_2$$

only possible for $\underline{\underline{\theta = 0}}$

<u>collinear kinematics</u>

$\rightarrow$ $\omega^2 = c_s^2 k^2 + \alpha_4 k^4 + \alpha_6 k^6 + \dots$

(including higher derivative corrections)



For $\underline{\alpha_4 > 0}$, decay is kinematically allowed in a range of θ

$$S_2 = \underbrace{\left(\frac{\rho + p}{\mu^2 c_s^2}\right)}_{\equiv Z} \int d^4x \; \frac{1}{2}\left(\dot{\pi}^2 - c_s^2 (\vec{\nabla}\pi)^2\right)$$

$(\phi = \mu t + \pi)$

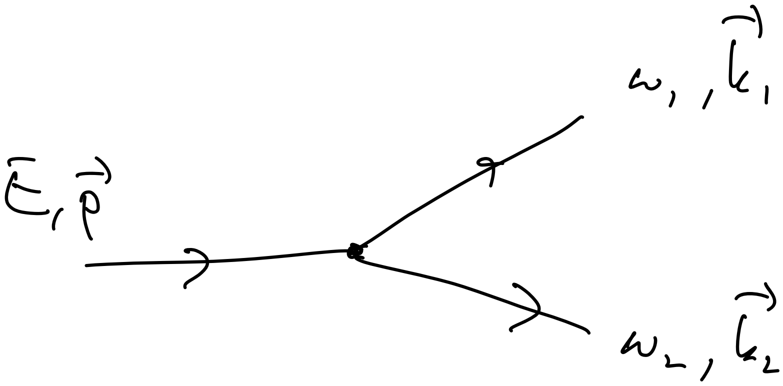
$$S_3 = \int d^4x \Big[2P'' \mu \, \dot{\pi}\left(\dot{\pi}^2 - (\nabla\pi)^2\right) + \frac{4}{3} P''' \mu^3 \dot{\pi}^3 \Big]$$

$\rho + p \sim P'$

$$\equiv \underbrace{\left(\frac{\rho + p}{\mu^2 c_s^2}\right)}_{Z} \int d^4x \left[\frac{\alpha \, \dot{\pi}^3}{3!} + \frac{\beta \, \dot{\pi} (\nabla\pi)^2 c_s^2}{2!} \right]$$

HW: compute α, β in terms of

$$c_s^2 \, , \quad \frac{dc_s^2}{d\log X}$$



$$\vec{p} \parallel \vec{k}_1 \parallel \vec{k}_2$$
$$\omega_1 = c_s k_1$$
$$\omega_2 = c_s (p - k_1)$$
$$E = c_s p$$

$$iM = \frac{1}{Z^{3/2}} \cdot Z \Big[i\alpha (-iE)(i\omega_1)(i\omega_2)$$

$$+ i c_s^2 \beta \Big((-iE)(-i\vec{k}_1)\cdot(-i\vec{k}_2)$$

$$+ (i\omega_1)(i\vec{p})\cdot(-i\vec{k}_2)$$

$$+ (i\omega_2)(i\vec{p})\cdot(-i\vec{k}_1) \Big]$$

$$\rightarrow \frac{1}{Z^{1/2}} \Big[-\alpha c_s^3 p k_1 (p - k_1)$$

$$- 3\beta c_s^3 p k_1 (p - k_1) \Big]$$

$$= - \frac{\mu c_s^4}{\sqrt{\rho + p}} (\alpha + 3\beta)\ p\, k_1 (p - k_1)$$

$$d\Gamma = \frac{1}{2E} |\mathcal{M}|^2 \, d\Pi_f$$

final state phase space

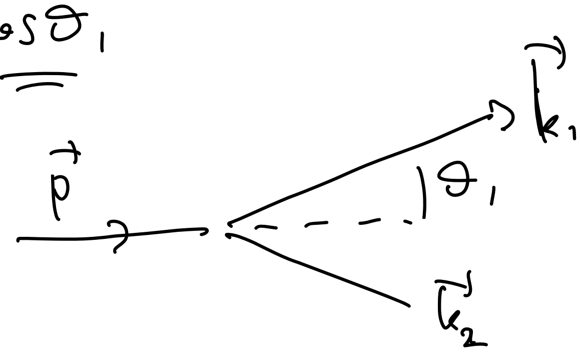
$c_s p$ ....

$$d\Pi_f = \frac{d^3 k_1}{(2\pi)^3 2\omega_1} \frac{d^3 k_2}{(2\pi)^3 2\omega_2} (2\pi)^4 \delta(E - \omega_1 - \omega_2) \cdot \delta^3(\vec{p} - \vec{k}_1 - \vec{k}_2)$$

$$\longrightarrow \frac{d^3 k_1}{(2\pi)^3 2\omega_1} \frac{1}{2\omega_2} \frac{1}{c_s} (2\pi) \delta(p - k_1 - |\vec{p} - \vec{k}_1|)$$

$$(\vec{k}_2 = \vec{p} - \vec{k}_1)$$

$$|\vec{p} - \vec{k}_1| = \sqrt{p^2 + k_1^2 - 2 p k_1 \underline{\underline{\cos\theta_1}}}$$



$$d^3 k_1 = k_1^2 \, d\varphi_1 \, d\cos\theta_1$$

$$\delta(f(x)) = \frac{\delta(x - x_0)}{|f'(x_0)|} \qquad (f(x_0) = 0)$$

$$d\Pi_f \to \frac{d\varphi_1\, d\cos\theta_1\, dk_1 k_1^2}{(2\pi)^2\, 2\omega_1\, 2\omega_2\, c_s} \quad \frac{\delta(\cos\theta_1)}{pk_1 / \sqrt{p^2 + k_1^2 - 2pk_1}}$$

$$\to \frac{dk_1\, k_1^2}{(2\pi)(2\omega_1)(2\omega_2)\, c_s} \cdot \frac{p - k_1}{pk_1}$$

$$d\Gamma = \frac{1}{16\pi} \frac{\mu^2 c_s^4}{(\rho + P)} (\alpha + 3\beta)^2\, \underline{\underline{(p - k_1)^2}}\, \underline{\underline{k_1^2}}\, dk_1$$

$$\Gamma = \int_0^{p/2 \leftarrow \text{Bose}} \cdots = \boxed{\frac{\mu^2 c_s^4}{16\pi(\rho + P)} (\alpha + 3\beta)^2 \frac{p^5}{60}}$$

related to c_s^2, $\frac{d c_s^2}{d\log X}$