Implementation of mass-resolution effect in amplitude analysis

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Introduction

$$\underbrace{\mathrm{PDF_{obs}}} = [\underbrace{\mathrm{PDF_{phys}}} \otimes \underbrace{G}] \times \underbrace{\epsilon}$$

Efficiency

Data distribution

Detector Resolution

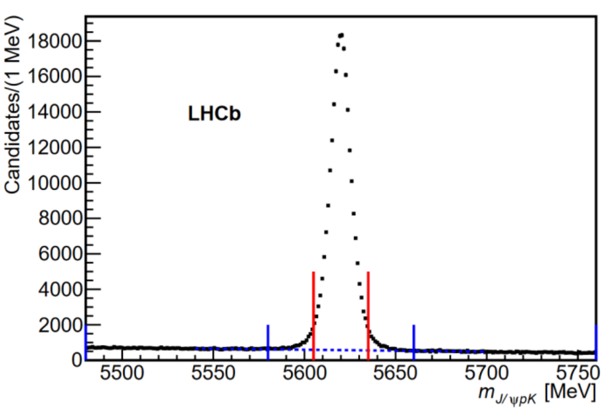
Kinematic properties
of physics progress

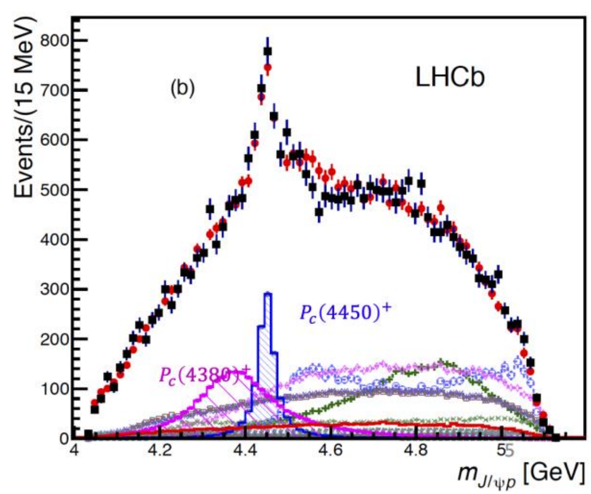
- Typical resolution (σ) @ LHCb ~ **MeV**

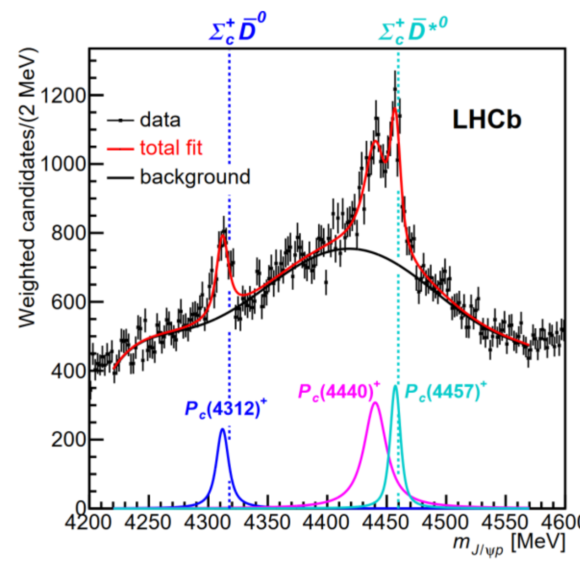
$P_c(4312)^+, P_c(4440)^+,$
$P_c(4457)^+$

$P_c(4380)^+, P_c(4450)^+$

Λ_b^0







$\Gamma \lll \sigma$
$\mathrm{PDF_{phys}} \otimes G \approx G$

$\Gamma \sim 100\mathrm{MeV} \gg \sigma$
$\mathrm{PDF_{phys}} \otimes G \approx \mathrm{PDF_{phys}}$

$\Gamma \sim 10\mathrm{MeV} \sim \sigma$
$\mathrm{PDF_{phys}} \otimes G$

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$$\underline{\mathrm{PDF_{obs}}} = \underline{[\mathrm{PDF_{phys}}} \otimes \underline{G]} \times \underline{\epsilon}$$

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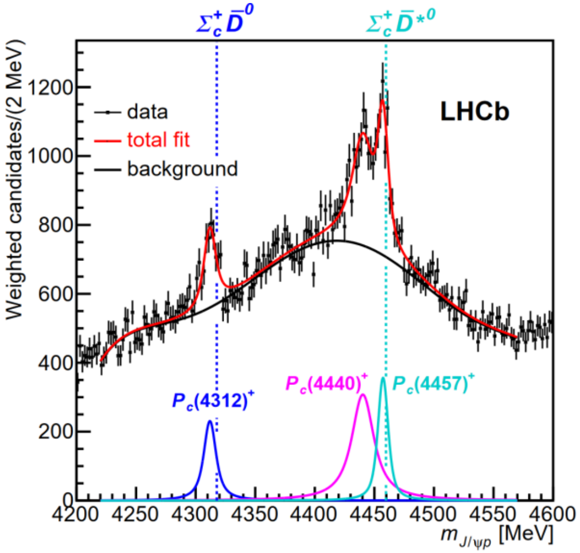
Today's menu:

Challenges of implementing mass resolution effect in AmAn on huge dataset

Several tricks as potential solutions

Use $\Lambda_b^0 \to J/\psi p K^-$ Run1+2 AmAn as example (CERN-THESIS-2021-314, LHCb unofficial)

$P_c(4312)^+, P_c(4440)^+, P_c(4457)^+$



$\Gamma \sim 10\mathrm{MeV} \sim \sigma$

$\mathrm{PDF_{phys}} \otimes \mathrm{Res}$

Amplitude fit on huge dataset

arXiv:0905.0724

- Unbinned maximum likelihood fit (sFit)

$$
\begin{aligned}
-\ln L(\vec{\omega}) &= -\alpha \sum_i W_i \ln \mathcal{P}_{sig}(m_{Kp,i}, \Omega_i|\vec{\omega}) \\
&= -\alpha \sum_i W_i \ln \frac{\epsilon(m_{Kp}, \Omega) \times [(|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \times \Phi(m_{Kp,i})) \otimes G(m_{Kp,i}, \Omega_i)]}{I(\vec{\omega})} \\
&\approx -\alpha \sum_i W_i \ln \frac{\epsilon(m_{Kp}, \Omega) \times \Phi(m_{Kp,i}) \times [|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \otimes G(m_{Kp,i}, \Omega_i)]}{I(\vec{\omega})} \\
&= -\alpha \sum_i W_i \ln \epsilon(m_{Kp,i}, \Omega_i)\Phi(m_{Kp}) - \alpha \sum_i W_i \ln \frac{|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \otimes G(m_{Kp,i}, \Omega_i)}{I(\vec{\omega})},
\end{aligned}
$$

W_i: single-event weight for BKG subtraction; M: matrix element; G: resolution function

ϵ: PHSP-dependent efficiency; Φ: PHSP density; I: normalization factor

$$
\begin{aligned}
I(\vec{\omega}) &= \int\int \epsilon(m_{Kp}, \Omega) \times [(|\mathcal{M}(m_{Kp}, \Omega|\vec{\omega})|^2 \times \Phi(m_{Kp})) \otimes G(m_{Kp}, \Omega)] dm_{Kp} d\Omega \\
&\approx \int\int \epsilon(m_{Kp}, \Omega) \times \Phi(m_{Kp}) \times [|\mathcal{M}(m_{Kp}, \Omega|\vec{\omega})|^2 \otimes G(m_{Kp}, \Omega)] dm_{Kp} d\Omega. \\
&= C \sum_j (|\mathcal{M}(m_{Kp,j}, \Omega_j|\vec{\omega})|^2 \times \Phi(m_{Kp})) \otimes G(m_{Kp,j}, \Omega_j),
\end{aligned}
$$

Challenges: speed of the fit (1)

- Complicated amplitude formula

$$
\begin{aligned}
-\ln L(\vec{\omega}) &= -\alpha \sum_i W_i \ln \mathcal{P}_{sig}(m_{Kp,i}, \Omega_i|\vec{\omega}) \\
&= -\alpha \sum_i W_i \ln \frac{\epsilon(m_{Kp}, \Omega) \times [(|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \times \Phi(m_{Kp,i})) \otimes G(m_{Kp,i}, \Omega_i)]}{I(\vec{\omega})} \\
&\approx -\alpha \sum_i W_i \ln \frac{\epsilon(m_{Kp}, \Omega) \times \Phi(m_{Kp,i}) \times [|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \otimes G(m_{Kp,i}, \Omega_i)]}{I(\vec{\omega})} \\
&= -\alpha \sum_i W_i \ln \epsilon(m_{Kp,i}, \Omega_i)\Phi(m_{Kp}) - \alpha \sum_i W_i \ln \frac{|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \otimes G(m_{Kp,i}, \Omega_i)}{I(\vec{\omega})},
\end{aligned}
$$

$$
|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}=\pm\frac{1}{2}, \lambda_p=\pm\frac{1}{2} \Delta\lambda_\mu=\pm 1} |\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu} + e^{i\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} d^{\frac{1}{2}}_{\lambda_p^{P_c}, \lambda_p}(\theta_p) \mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c}, \Delta\lambda_\mu}
+ e^{i\Delta\lambda_\mu \alpha_\mu^{Z_{cs}}} \sum_{\lambda_p^{Z_{cs}}} d^{\frac{1}{2}}_{\lambda_p^{Z_{cs}}, \lambda_p}(\theta_p^{Z_{cs}}) \mathcal{M}^{Z_{cs}}_{\lambda_{\Lambda_b^0}, \lambda_p^{Z_{cs}}, \Delta\lambda_\mu^{Z_{cs}}}|^2.
$$

A complicated function to describe decay properties @ 6D PHSP

Challenges: speed of the fit (2)

- Huge number of data & MC events

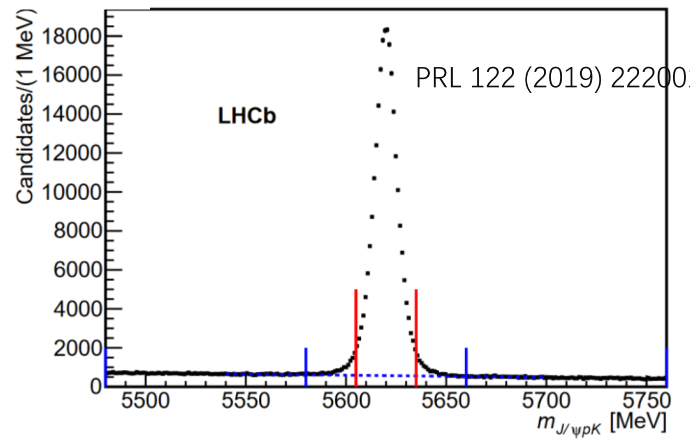
$$-\ln L(\vec{\omega}) = -\alpha \sum_i W_i \ln \epsilon(m_{Kp,i}, \Omega_i)\Phi(m_{Kp}) - \alpha \sum_i W_i \ln \frac{|\mathcal{M}(m_{Kp,i}, \Omega_i|\vec{\omega})|^2 \otimes G(m_{Kp,i}, \Omega_i)}{I(\vec{\omega})},$$

$$I(\vec{\omega}) = C \sum_j (|\mathcal{M}(m_{Kp,j}, \Omega_j|\vec{\omega})|^2 \times \Phi(m_{Kp})) \otimes G(m_{Kp,j}, \Omega_j),$$

Repeat PDF calculation for all data events

> 0.3 Million $\Lambda_b^0 \to J/\psi p K^-$ candidates in data

> 2.5 Million simulated $\Lambda_b^0 \to J/\psi p K^-$ events



- To calculate L, the amplitude is to be computed for $\sim$**3 Million** times

Implementation of $\otimes$ G

- An ideal solution:
  - Find a analytical expression of $|M|^2 \otimes G$, but usually not practical
  - **Interference term**
  - **Complicated line-shape functions** (for example K-Matrix with coupled-channel effects considered)
- A naïve solution:
  - Do a standard numerical convolution

$$
\begin{aligned}
|\mathcal{M}|^2 \otimes G &= \int_{-\infty}^{\infty} |\mathcal{M}(m_{J/\psi p} = m_a + x)|^2 G(x) dx \\
&\approx \int_{-\alpha}^{\alpha} |\mathcal{M}(m_{J/\psi p} = m_a + x)|^2 G(x) dx \\
&\approx \sum_{i=-n}^{i=n} |\mathcal{M}(m_{J/\psi p} = m_a + i\Delta x)|^2 G(i\Delta x) \Delta x.
\end{aligned}
$$

Fit Speed $\times \frac{1}{2n}$

Not good for complicated analysis

Need further tricks to speed up the process

No convolution for MC events

- MC events just used for normalization factor

$$I(\vec{\omega}) = C \sum_j (|\mathcal{M}(m_{Kp,j}, \Omega_j|\vec{\omega})|^2 \times \Phi(m_{Kp})) \otimes G(m_{Kp,j}, \Omega_j),$$

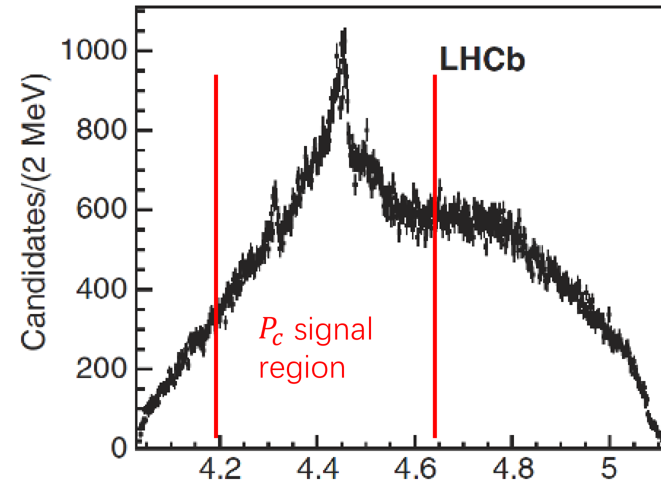
- One can safely (?) remove the convolution operator

$$\begin{aligned} I(\vec{\omega}) &\approx \int\int [\epsilon(m_{Kp}, \Omega) \times |\mathcal{M}(m_{Kp}, \Omega|\vec{\omega})|^2] \otimes G(m_{Kp}, \Omega) dm_{Kp} d\Omega \\ &= \int\int [\epsilon(\Phi + \Phi') \times |\mathcal{M}(\Phi + \Phi'|\vec{\omega})|^2] \times G(\Phi') d\Phi d\Phi' \\ &= \int\int [\epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2] \times G(\Phi') d\Phi d\Phi' \\ &= \int \epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2 d\Phi \times \int G(\Phi') d\Phi' \\ &= \int \epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2 d\Phi, \end{aligned}$$

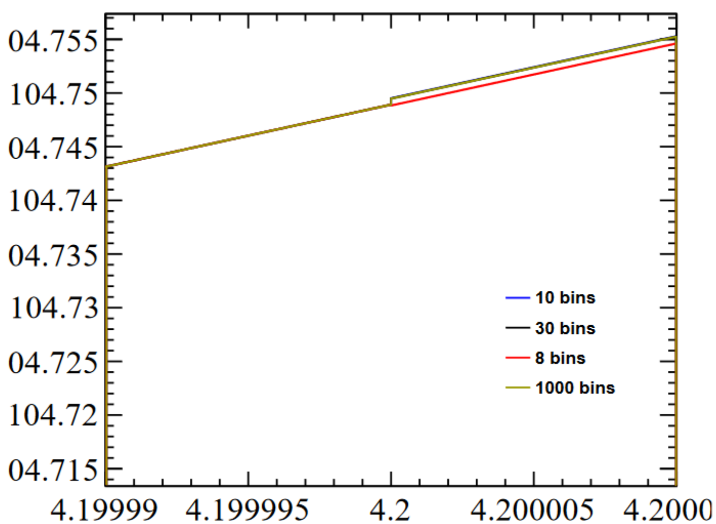
- Effect on the fit speed
  - $\frac{1}{2n} \rightarrow \frac{N_{data}+N_{MC}}{2n\times N_{data}+N_{MC}} \gg \frac{1}{2n}$ considering that $N_{MC} \gg N_{data}$

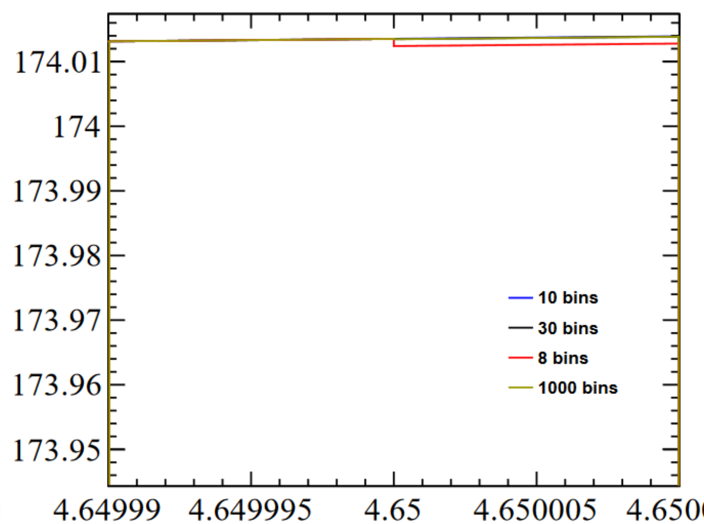
Only convolution for near-peak events

- Convolution not necessary for events far away from P_c peaks
  - Mass shape varies slowly
  - $\boldsymbol{PDF_{phys} \otimes Res \approx PDF_{phys}}$ **outside of** $\boldsymbol{P_c}$ **signal region**
- Need to check continuity of PDF



CERN-THESIS-2021-314 LHCb unofficial $m_{J/\psi p}$ [GeV]





PDF constructor in fit framework

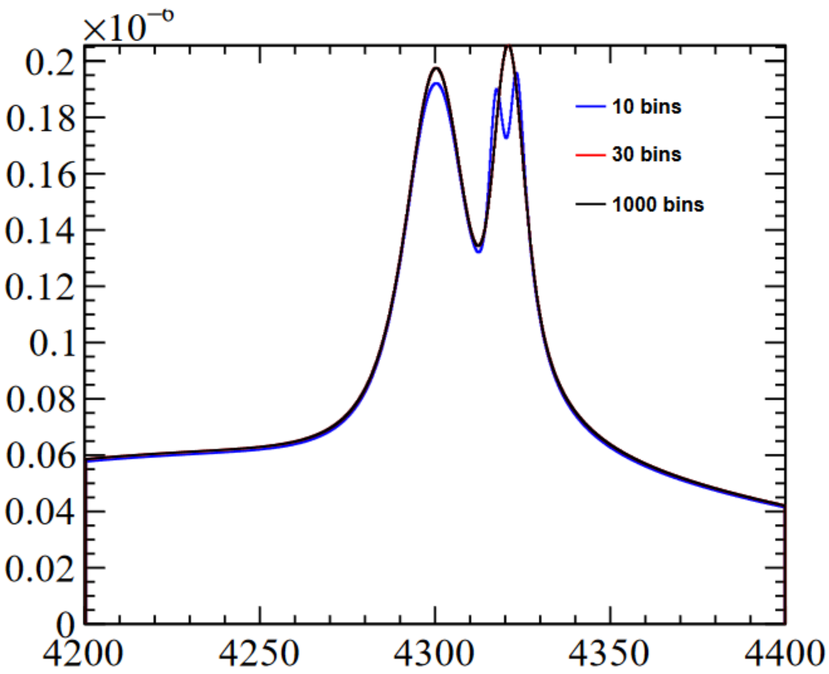
- Rewrite the PDF constructor, only recalculate the **mass-dependent part of amplitude** when doing convolution
  - No need to recalculate $\boldsymbol{\Lambda^*}$ **chain amplitude** & $\boldsymbol{P_c}$ **angular terms**

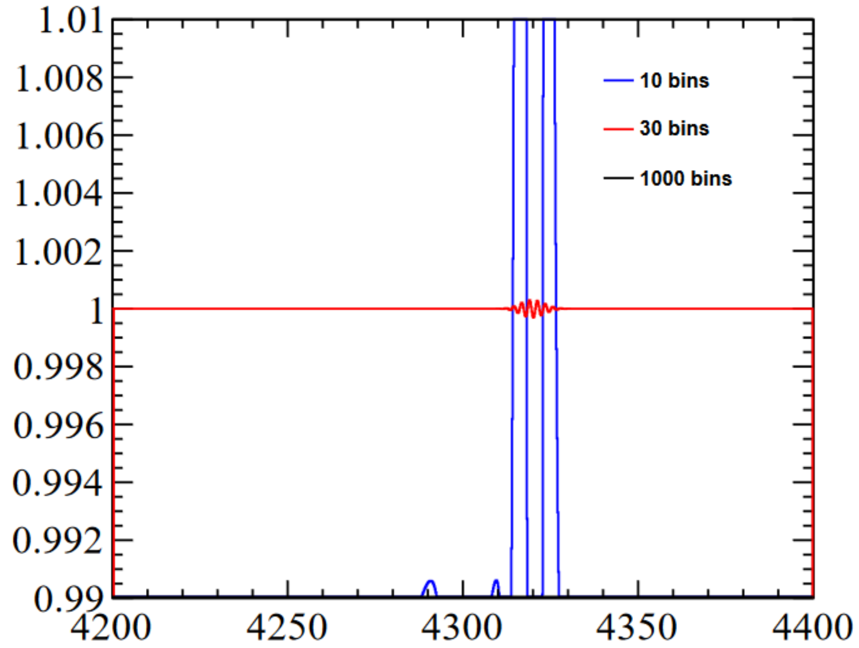
$$
\begin{aligned}
|\mathcal{M}|^2 &= \sum_q |M_{\Lambda^*,Z_{cs},q} + \sum_j f_{P_{c,j},q} R_{P_{c,j}}|^2 \\
&= \sum_q (|M_{\Lambda^*,Z_{cs},q}|^2 + M^*_{\Lambda^*,Z_{cs},q} \times \sum_j f_{P_{c,j},q} R_{P_{c,j}} + \\
&\quad M_{\Lambda^*,Z_{cs},q} \times \sum_j f^*_{P_{c,j},q} R^*_{P_{c,j}} + \sum_{j,k} f_{P_{c,j},q} f^*_{P_{c,k},q} R_{P_{c,j}} R^*_{P_{c,k}}),
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{M}|^2 \otimes G = \sum_q (&|M_{\Lambda^*,Z_{cs},q}|^2 \\
&+ M^*_{\Lambda^*,Z_{cs},q} \times \sum_j f_{P_{c,j},q} (R_{P_{c,j}} \otimes G) \\
&+ M_{\Lambda^*,Z_{cs},q} \times \sum_j f^*_{P_{c,j},q} (R^*_{P_{c,j}} \otimes G) \\
&+ \sum_{j,k} f_{P_{c,j},q} f^*_{P_{c,k},q} (R_{P_{c,j}} R^*_{P_{c,k}} \otimes G))
\end{aligned}
$$

Performance

- Numerical convolution of 30 bins result in **a precision of < 0.1% level**





- Extra time consumption: **10% level**

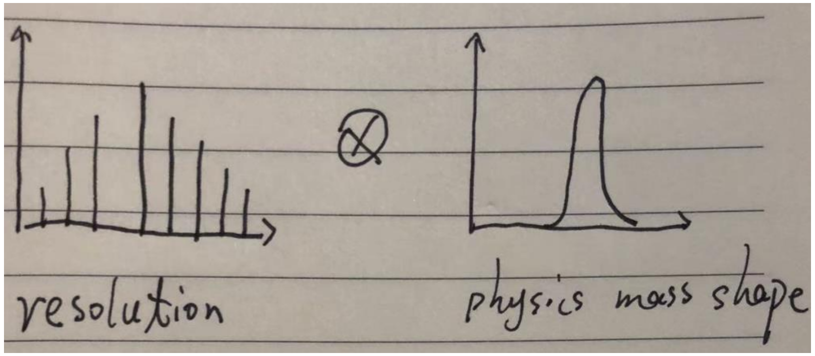
Other tips

Set a lower limit on natural widths

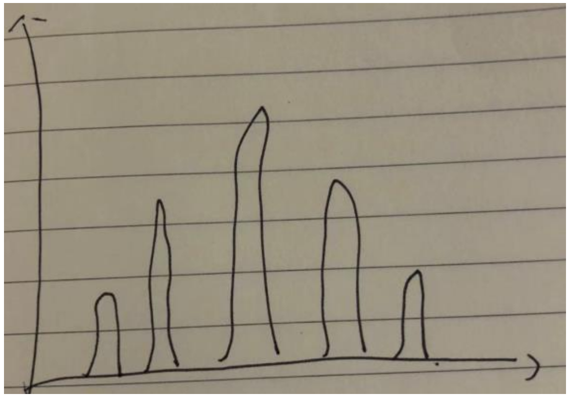
- Numerical convolution only valid when $\Gamma >>$ bin width

$$
\begin{aligned}
|\mathcal{M}|^2 \otimes G &= \int_{-\infty}^{\infty} |\mathcal{M}(m_{J/\psi p} = m_a + x)|^2 G(x)dx \\
&\approx \int_{-\alpha}^{\alpha} |\mathcal{M}(m_{J/\psi p} = m_a + x)|^2 G(x)dx \\
&\approx \sum_{i=-n}^{i=n} |\mathcal{M}(m_{J/\psi p} = m_a + i\Delta x)|^2 G(i\Delta x)\Delta x.
\end{aligned}
$$

"Sampling" the PDF in each Gaussian bins



Γ < bin width



Set a lower limit on natural widths

- A super narrow natural width may also break the statement:
  - One can safely remove the convolution operator for MC events

$$
\begin{aligned}
I(\vec{\omega}) &\approx \int\int [\epsilon(m_{Kp}, \Omega) \times |\mathcal{M}(m_{Kp}, \Omega|\vec{\omega})|^2] \otimes G(m_{Kp}, \Omega) dm_{Kp} d\Omega \\
&= \int\int [\epsilon(\Phi + \Phi') \times |\mathcal{M}(\Phi + \Phi'|\vec{\omega})|^2] \times G(\Phi') d\Phi d\Phi' \\
&= \int\int [\epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2] \times G(\Phi') d\Phi d\Phi' \\
&= \int \epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2 d\Phi \times \int G(\Phi') d\Phi' \\
&= \int \epsilon(\Phi) \times |\mathcal{M}(\Phi|\vec{\omega})|^2 d\Phi,
\end{aligned}
$$

Correct in math

But in real life it suffers from stat. fluctuation of MC sample

- If super narrow peak with reasonable fit fraction
  - Huge (crazy) weights for few events under the narrow peak
  - Hurt the effective stat. power of MC

$$neff = \frac{(\sum Weights)^2}{(\sum Weight^2)}$$

Conclusion

- When you find it necessary to implement detector resolution effect in an amplitude analysis, maybe you find it not friendly on the fit speed, which is already very slow

- Several tricks might be helpful:
  - No convolution for MC events
  - No convolution for data events far away from peaking region
  - Try to reorganize the PDF constructor to avoid repeating calculate the same things

- Would be good to set a reasonable lower limit on the natural width of states of interest

Thank you for your attention !

Any questions or comments ?