Final-state alignment issue in the helicity formalism

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Outline

- The final-state alignment in helicity formalism
  - Introduction
  - Traditional approach

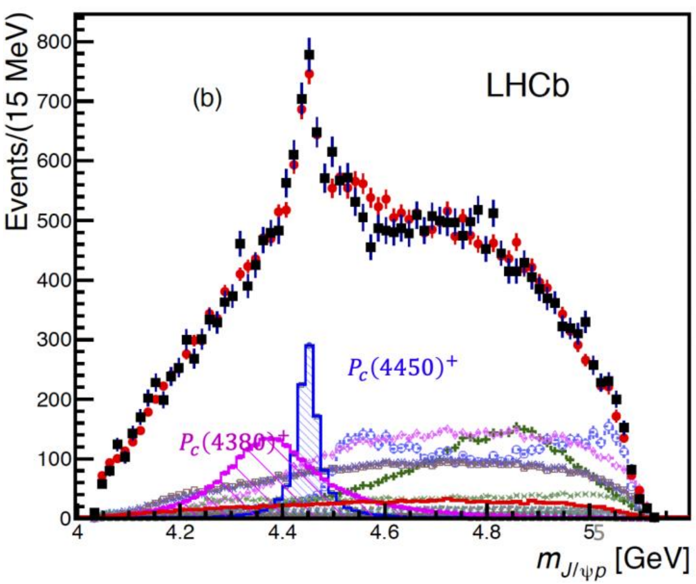
- Potential weak points & proposal of new techniques
  - *Adv.High Energy Phys.* 2020 (2020) 6674595
  - *Chinese Phys. C* **45** (2021) 063103 (today's menu)

Link to the previous talk

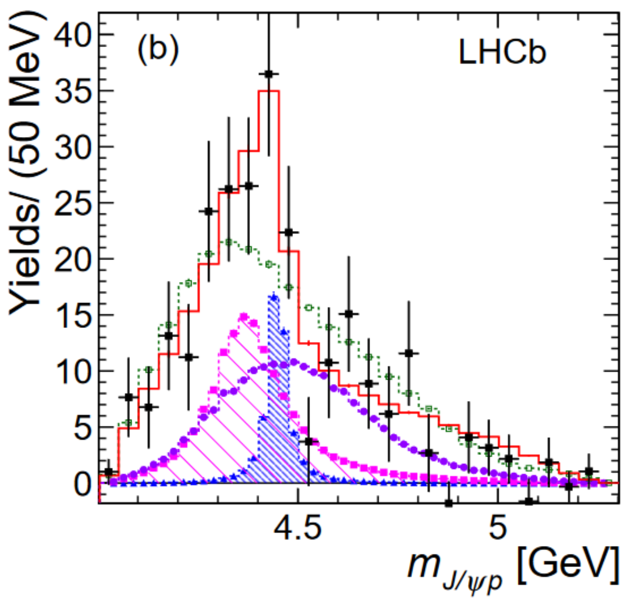
Reminder

- A series analyses @ LHCb indicate the existence of pentaquark-like structures in $J/\psi p$ system
- Two amplitude analysis included

$\Lambda_b^0 \to J/\psi p K^-$ PRL 115(2015)072001



$\Lambda_b^0 \to J/\psi p \pi^-$ PRL 117(2016)082003



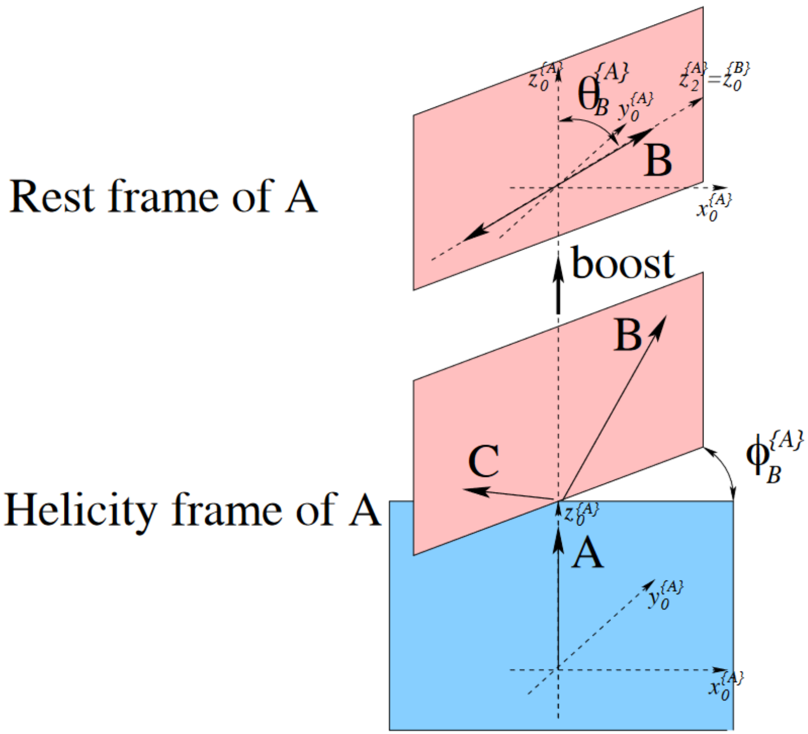
- Helicity-based decay amplitude

$$\lambda = \vec{s} \cdot \vec{p}$$

Helicity formalism: two-body decay

- Widely used for constructing angular sector of decay amplitudes

Two-body decay: $A \rightarrow BC$



Initial state: $|\lambda_A>$ $(x_0^{\{A\}}, y_0^{\{A\}}, z_0^{\{A\}})$

$R(\phi_B^{\{A\}}, \theta_B^{\{A\}}, 0)$

Final state: $|\lambda_B, -\lambda_c>$ $(x_0^{\{B\}}, y_0^{\{B\}}, z_0^{\{B\}})$

$$|\lambda_A> = \sum_{\lambda_{A'}} D^{J_A}_{\lambda_A, \lambda_{A'}}(\phi_B^{\{A\}}, \theta_B^{\{A\}}, 0)\, |\lambda_A'>$$

Angular momentum conservation:
Non-zero amplitude when $\lambda_B - \lambda_c = \lambda_A'$

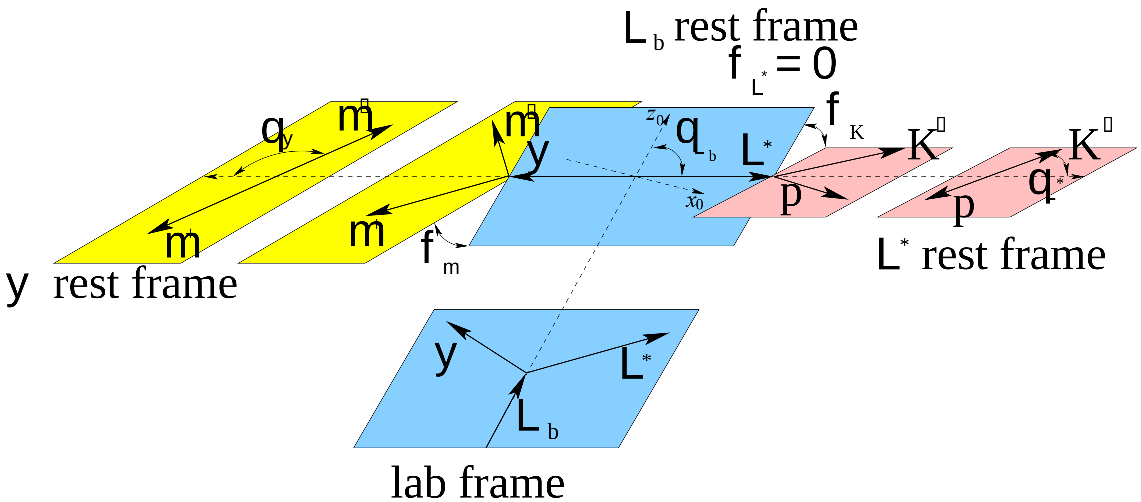
So amplitude proportional to:

$$D^{J_A}_{\lambda_A, \lambda_{B-\lambda_C}}(\phi_B^{\{A\}}, \theta_B^{\{A\}}, 0)$$

A graphic description. A more detailed derivation available in [Annals Phys.281,774(2000)]

Multi-body decays

- Considered as a “chain” of multiple two-body decays
  - Example: $\Lambda_b^0 \to J/\psi(\to \mu^+\mu^-)\Lambda^*(\to pK)$



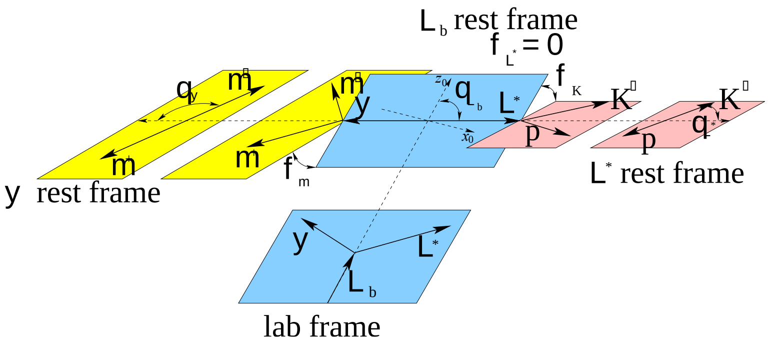
$$\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\,\lambda_p,\,\Delta\lambda_\mu} = \sum_n R_{\Lambda_n^*}(m_{Kp})\,\mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p} \sum_{\lambda_\psi} e^{i\,\lambda_\psi\phi_\mu}\; d^{1}_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi)$$

$$\times \sum_{\lambda_{\Lambda^*}} \mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\,\lambda_\psi}\; e^{i\,\lambda_{\Lambda^*}\phi_K}\; d^{\frac{1}{2}}_{\lambda_{\Lambda_b^0},\,\lambda_{\Lambda^*}-\lambda_\psi}(\theta_{\Lambda_b^0})\; d^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\,\lambda_p}(\theta_{\Lambda^*})$$

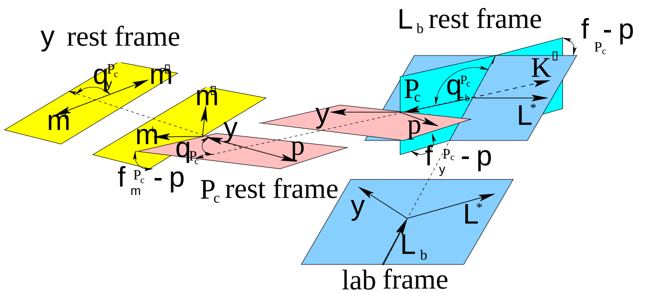
Multiple decay chains

- Still use the pentaquark analysis as example

- Λ^* decay chain (reference chain):
  - $\Lambda_b^0 \to J/\psi(\to \mu^+\mu^-)\Lambda^*(\to pK)$



- P_c^+ decay chain:
  - $\Lambda_b^0 \to P_c^+(\to J/\psi(\to \mu^+\mu^-)p)K$

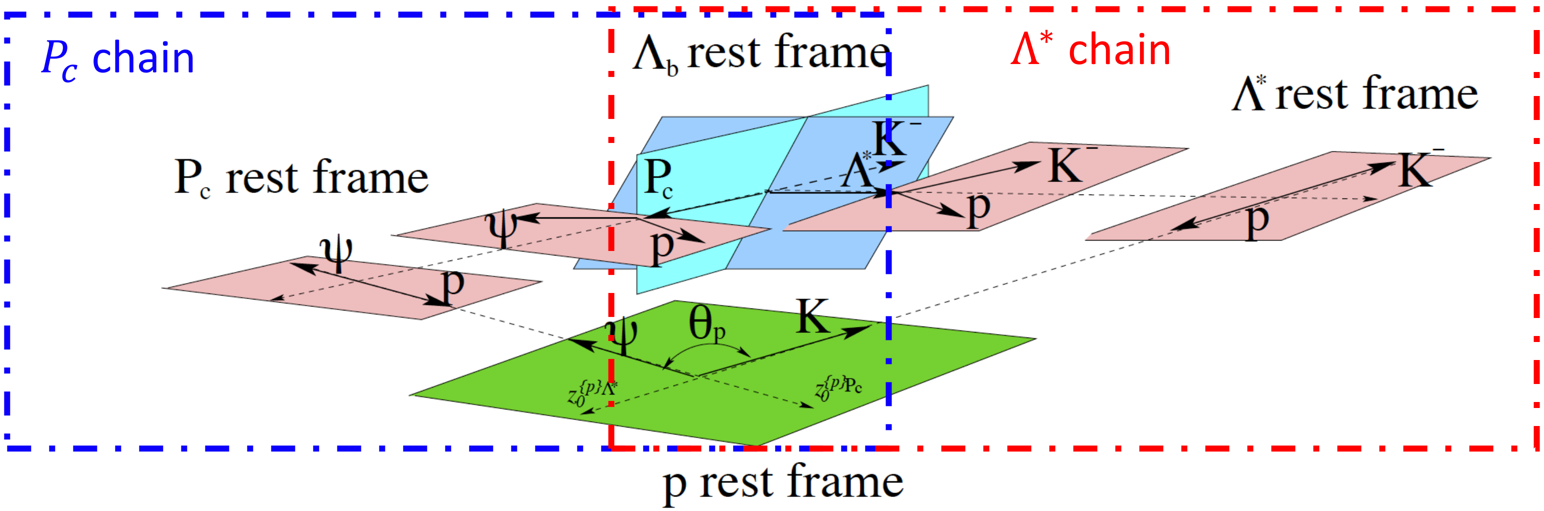


- Start from the same initial state (by-definition)
- Final states should also be defined in the same frame
  - Does not happen automatically
  - Need a final-state alignment

The alignment angle

- Traditional approach for final-state alignment
  - Express final (x, y, z) axis using particle momentum
  - Find out the relation between different chains

$\Lambda_b^0 \to P_c^+(\to J/\psi(\to \mu^+\mu^-)p)K^-$ $\Lambda_b^0 \to J/\psi(\to \mu^+\mu^-)\Lambda^*(\to pK)$



$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}=\pm\frac{1}{2}} \sum_{\lambda_p=\pm\frac{1}{2}} \sum_{\Delta\lambda_\mu=\pm 1} \left| \mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} + e^{i\,\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c}} \boxed{d^{\frac{1}{2}}_{\lambda_p^{P_c},\lambda_p}(\theta_p)}\, \mathcal{M}^{P_c}_{\lambda_{\Lambda_b^0},\lambda_p^{P_c},\Delta\lambda_\mu} \right|^2$$

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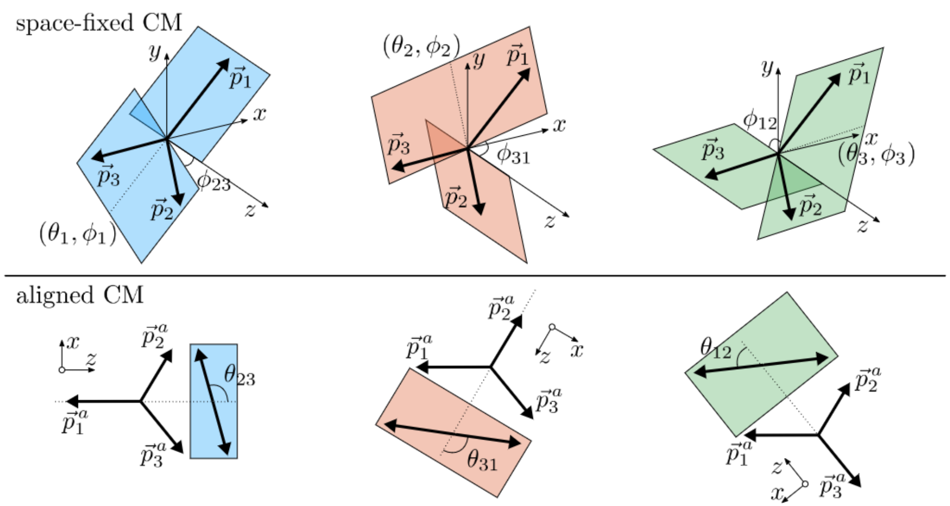
The DPD formula

- Similar as traditional helicity formalism, but do the rotations in an alternative way



$$
\begin{aligned}
(M^{\Lambda}_{\lambda,\xi})_{\text{LHCb}} = & \sum_{\nu,\mu} D^{J*}_{\Lambda,\nu}(\phi_1, \theta_1, \phi_{23}) \\
& \times \Bigg(\sum_{s}^{\Lambda^* \to pK} \sum_{\tau} \sqrt{2} n_s \, d^{1/2}_{\nu,\tau-\mu}(0) \, H^{0\to(23),1}_{\tau,\mu} \; X_s(\sigma_1) \; d^{s}_{\tau,\lambda}(\theta_{23}) \, H^{(23)\to 2,3}_{\lambda,0} \\
& \quad + \sum_{s}^{P_c \to J/\psi p} \sum_{\tau,\mu',\lambda'} \sqrt{2} n_s \, d^{1/2}_{\nu,\tau}(\hat{\theta}_{3(1)}) \, H^{0\to(12),3}_{\tau,0} \; X_s(\sigma_3) \; d^{s}_{\tau,\mu'-\lambda'}(\theta_{12}) \, H^{(12)\to 1,2}_{\mu',\lambda'} \; d^{1/2}_{\lambda'\lambda}(\zeta^{2}_{3(1)}) \; d^{1}_{\mu'\mu}(\zeta^{1}_{3(1)}) \Bigg) \\
& \times \sqrt{3} \, e^{i\mu(\phi_{23}+\phi''_{+})} \, d^{1}_{\mu\xi}(\theta_{+}) \, H^{1\to\mu^+,\mu^-}_{\lambda_+,\lambda_-} ,
\end{aligned}
\tag{C5}
$$

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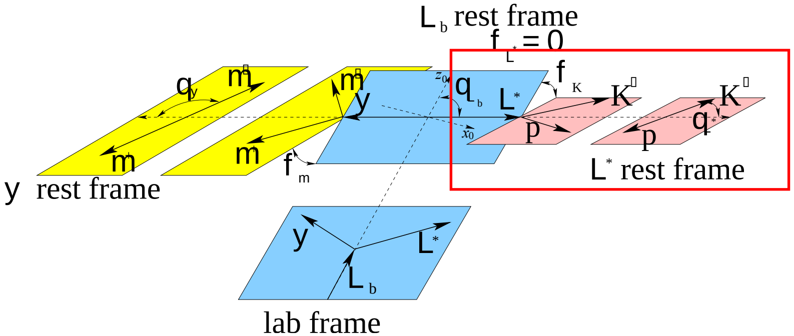
The DPD formula

- Similar as traditional helicity formalism, but do the rotations in an alternative way
- In principle should lead to the same result as the traditional helicity formalism
- Numerical comparisons made between DPD and 2015 formalism for $\Lambda_b^0 \to J/\psi p K^-$ amplitude analysis. Interesting behaviors observed.

The particle ordering issue

- Several changes have to be made for a consistent result of DPD & traditional formalism
  - Particle ordering issue:



$$\mathcal{M}^{\Lambda^*}_{\lambda_{\Lambda_b^0},\lambda_p,\Delta\lambda_\mu} = \sum_n R_{\Lambda_n^*}(m_{Kp})\, \mathcal{H}^{\Lambda_n^*\to Kp}_{\lambda_p} \sum_{\lambda_\psi} e^{i\lambda_\psi\phi_\mu}\, d^1_{\lambda_\psi,\Delta\lambda_\mu}(\theta_\psi)$$
$$\times \sum_{\lambda_{\Lambda^*}} \mathcal{H}^{\Lambda_b^0\to\Lambda_n^*\psi}_{\lambda_{\Lambda^*},\lambda_\psi}\, e^{i\lambda_{\Lambda^*}\phi_K}\, d^{\frac{1}{2}}_{\lambda_{\Lambda_b^0},\lambda_{\Lambda^*}-\lambda_\psi}(\theta_{\Lambda_b^0})\, d^{J_{\Lambda_n^*}}_{\lambda_{\Lambda^*},\lambda_p}(\theta_{\Lambda^*})$$

In the $\Lambda^* \to pK$ decay node, define the decay angles using the helicity angles of proton, rather than that of kaon (change the "particle ordering")

$$\phi_p = \phi_K + \pi\ (if\ \phi_K < 0),\ \phi_p = \phi_K - \pi\ (if\ \phi_K > 0)$$
$$\theta_p = \pi - \theta_K$$

The particle ordering issue

- Effect of changing the ordering

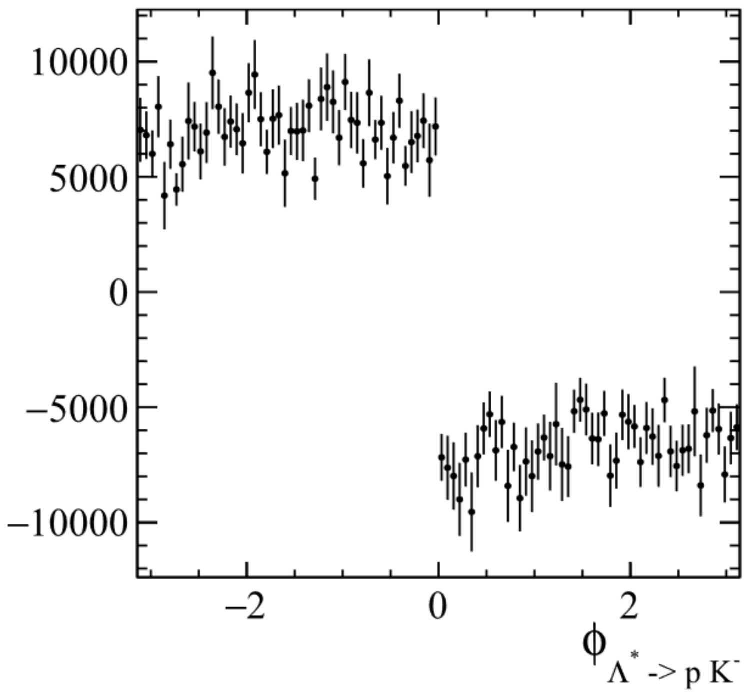
$$\phi_p = \phi_K + \pi\ (if\ \phi_K < 0), \phi_p = \phi_K - \pi\ (if\ \phi_K > 0)$$

  - A discontinuous change at $\phi_K = 0$
  - Additional 2π phase angle when $\phi_K < 0$ w.r.t. $\phi_K > 0$
  - Namely $e^{i\lambda_{\Lambda^*}\times 2\pi} = -1$ term in Λ^* chain amplitude
    - Does not influence the P_c chain amplitude
  - Leading to an **opposite** performance of **interference** term when $\phi_K < 0$ w.r.t. $\phi_K > 0$
  - Get **cancelled** when combine $\phi_K < 0$ and $\phi_K > 0$ datasets
- The traditional approach is blind for the 2π factor

Interference of P_c and Λ^* chains

- Check the interference term in each ϕ_K regions using 2015 formula

$$\left|M_{\Lambda^*+P_c}\right|^2 - |M_{\Lambda^*}|^2 - \left|M_{P_c}\right|^2$$

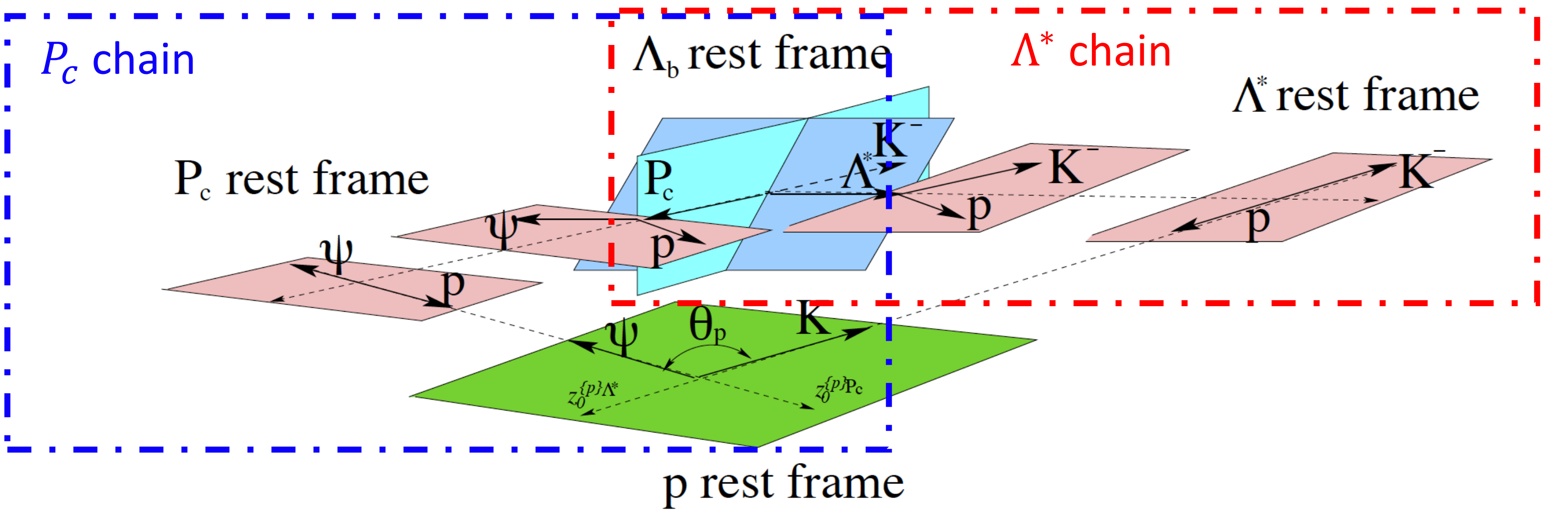


Discontinuous at $\phi = 0$

Interference get totally cancelled when combining samples with $\phi_K > 0$ and $\phi_K < 0$

Proton spin axis alignment

- The initial spin axis is the same for P_c and Λ^* chain, and the final spin axis for proton should also be the same
  - The overall rotations related to proton should be the same in the P_c and Λ^* chain



Basic idea for the check

- Check the rotations related to proton involved in the amplitude formalism, for both P_c and Λ^* chains
- Use the SU2 representation for rotation operators:
  - To describe spin rotation of spin-half particle
  - To make the 2π difference visible

Rotation along z-axis: $$R_z(\alpha) = \begin{pmatrix} e^{-i\alpha/2} & 0 \\ 0 & e^{i\alpha/2} \end{pmatrix},$$

Rotation along y-axis: $$R_y(\beta) = \begin{pmatrix} \cos(\beta/2) & -\sin(\beta/2) \\ \sin(\beta/2) & \cos(\beta/2) \end{pmatrix}.$$

Rotation along any axis: $$R(\alpha, \vec{b}) = R_z(\phi)R_y(\theta)R_z(\alpha)R_y(-\theta)R_z(-\phi)$$

Rotations of two-body decay nodes

Proton-alignment rotation

Rotations related to proton

- Based on 2015 paper's angle definitions:

$$R_{\Lambda^*} = R(\theta_{\Lambda^*}, \vec{p}_p^{\Lambda^*} \times \vec{p}_K^{\Lambda^*}) R(\phi_K, \vec{p}_{\Lambda^*}^{\Lambda_b^0}) R(\theta_{\Lambda_b^0}, \vec{p}_{\Lambda^*}^{\Lambda_b^0} \times \vec{p}_{\Lambda_b^0}^{lab}),$$

$$R_{P_c}^{\pm} = R(\pm\pi, \vec{p}_K^{\Lambda^*}) R(\theta_p, \vec{p}_p^{P_c} \times \vec{p}_\psi^{P_c}) R(\theta_{P_c}, \vec{p}_p^{P_c} \times \vec{p}_\psi^{P_c}) R(\phi_\psi^{P_c}, \vec{p}_{P_c}^{\Lambda_b^0}) R(\theta_{\Lambda_b^0}^{P_c}, \vec{p}_{P_c}^{\Lambda_b^0} \times \vec{p}_{\Lambda_b^0}^{lab}) R(\phi_{P_c}, \vec{p}_{\Lambda_b^0}^{lab}),$$

Λ^* and P_c chain results in opposite y-axis in the proton rest frame, rotate along z-axis by $\pm\pi$ to recover it.

- Use proton, instead of kaon to define Λ^* decay angle:

$$R_{P_c}^{\pm} = R(\pm\pi, \vec{p}_p^{P_c} \times \vec{p}_\psi^{P_c}) R(\theta_p, \vec{p}_p^{P_c} \times \vec{p}_\psi^{P_c}) R(\theta_{P_c}, \vec{p}_p^{P_c} \times \vec{p}_\psi^{P_c}) R(\phi_\psi^{P_c}, \vec{p}_{P_c}^{\Lambda_b^0}) R(\theta_{\Lambda_b^0}^{P_c}, \vec{p}_{P_c}^{\Lambda_b^0} \times \vec{p}_{\Lambda_b^0}^{lab}) R(\phi_{P_c}, \vec{p}_{\Lambda_b^0}^{lab}),$$

$$R_{\Lambda^*} = R(\theta_{\Lambda^*}, \vec{p}_K^{\Lambda^*} \times \vec{p}_p^{\Lambda^*}) R(\phi_p, \vec{p}_{\Lambda^*}^{\Lambda_b^0}) R(\theta_{\Lambda_b^0}, \vec{p}_{\Lambda^*}^{\Lambda_b^0} \times \vec{p}_{\Lambda_b^0}^{lab}),$$

Λ^* and P_c chain results in opposite z-axis in the proton rest frame, rotate along y-axis by $\pm\pi$ to recover it.

Boosts in different chains

- Λ^* and P_c chains boost from Λ_b^0 rest frame to proton rest frame in different ways. The proton rest frame can be different in these two chains.
- For boost operators:

Boost along z-axis:

$$B_z(\gamma) = \begin{pmatrix} e^{-\gamma/2} & 0 \\ 0 & e^{\gamma/2} \end{pmatrix}$$

Boost along any axis:

$$B(\gamma, \vec{a}) = R_z(\phi) R_y(\theta) B_z(\gamma) R_y(-\theta) R_z(-\phi)$$

Boosts of Λ^* chain

$$B_{\Lambda^*} = B(-y_p^{\Lambda^*}, \vec{p}_p^{\Lambda^*}) B(-y_{\Lambda^*}^{\Lambda_b^0}, \vec{p}_{\Lambda^*}^{\Lambda_b^0}),$$

Boosts of P_c chain

$$B_{P_c} = B(-y_p^{P_c}, \vec{p}_p^{P_c}) B(-y_{P_c}^{\Lambda_b^0}, \vec{p}_{P_c}^{\Lambda_b^0}).$$

Not used for the boost operator in spin space, just to derive the rotation operator to associate the proton rest frames from different chains

Compare two chains

- For a proper alignment, we expect

A rotation to transfer between two different proton rest frames

$$\boxed{B_{\Lambda^*} B_{P_c}^{-1}} R_{\Lambda^*} = R_{P_c}^{\pm}.$$

- Use a small sample of $\Lambda_b^0 \to J/\psi p K^-$ data, calculate the exact form of the matrices in the left and right side of these equations, and check the distribution of

$$D^{\pm} = \sum_{i,j} |L_{i,j} - R_{i,j}^{\pm}|^2,$$

$D^{\pm} = 0$: Left = Right
$D^{\pm} = 8$: Left = -Right

Conclusion: Half of the candidates have an additional “-1” sign compared to the rest.

Results

- Based on old formula:
  - No requirement on ϕ_K

Interference cancelled between them

Left = right   Left = -right

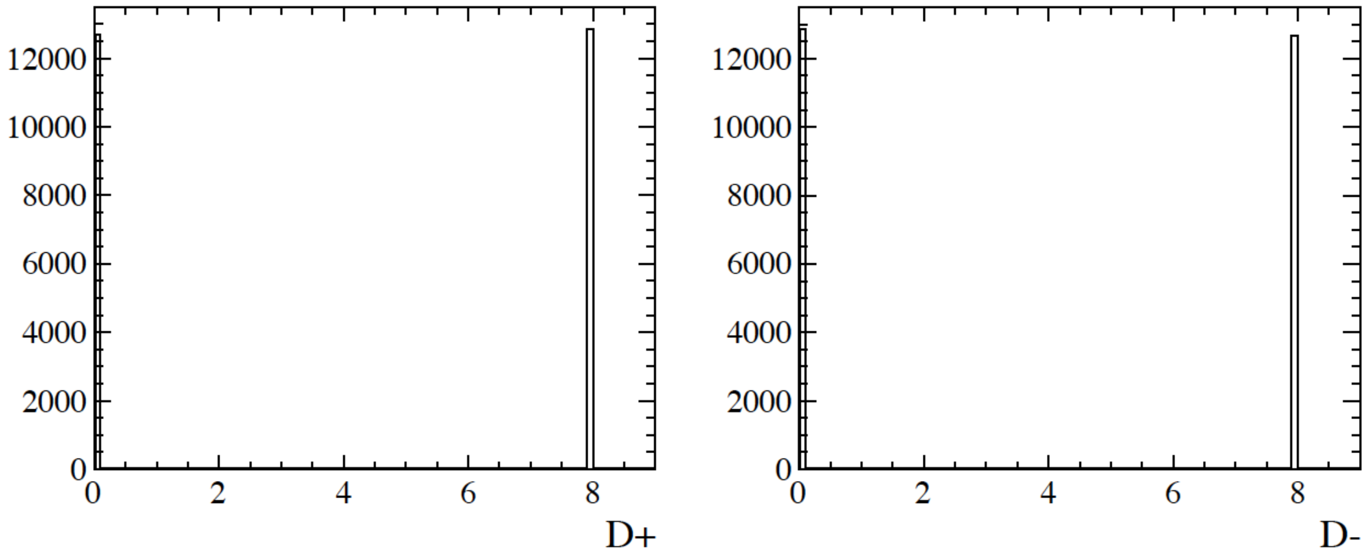


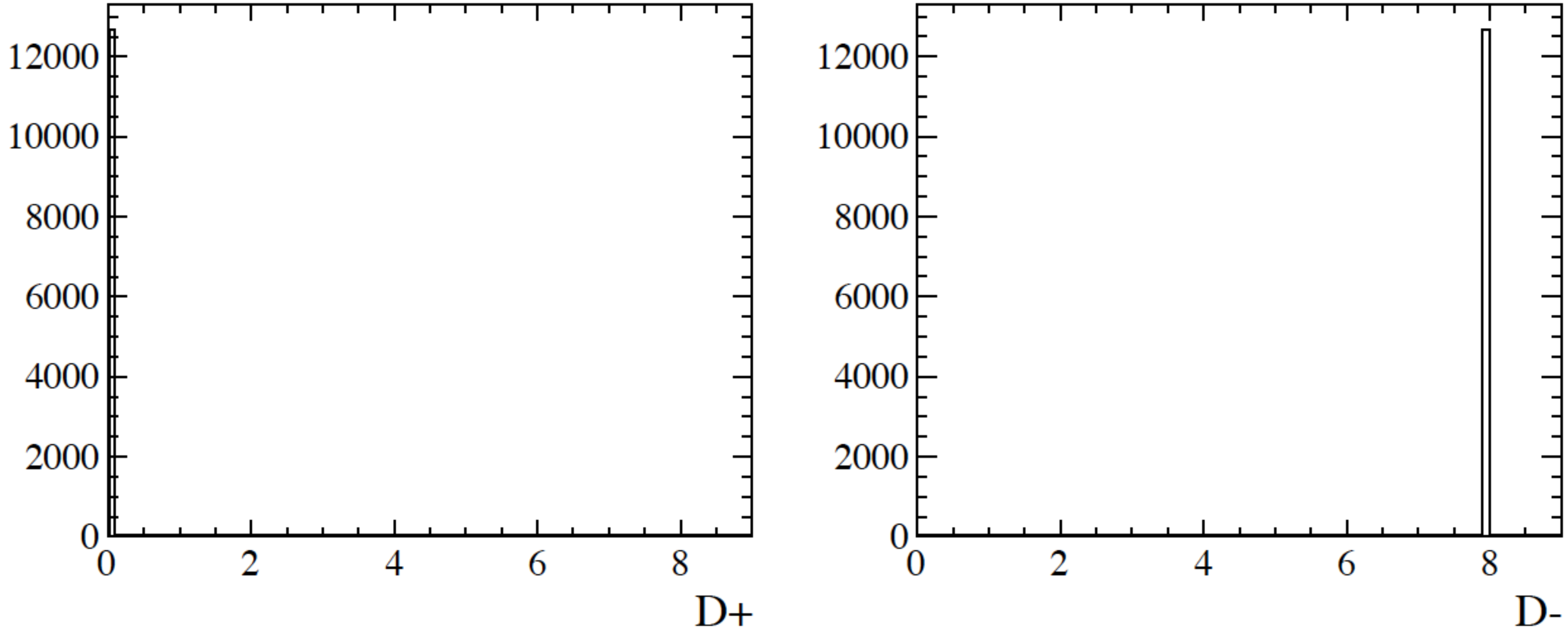
Figure 2: Distribution of $D^{\pm}$, based on SU2 representation, using 2015 paper's ordering.

Conclusion: The “-1” sign is related to the sign of ϕ_K.

Results

- Based on old formula:
    - Require $\phi_K < 0$

No more than a global phase shared by all candidates



Conclusion: the proton can be well aligned with the new particle ordering

Results

- Use proton, instead of kaon to define Λ^* decay angle:

No more than a global phase shared by all candidates

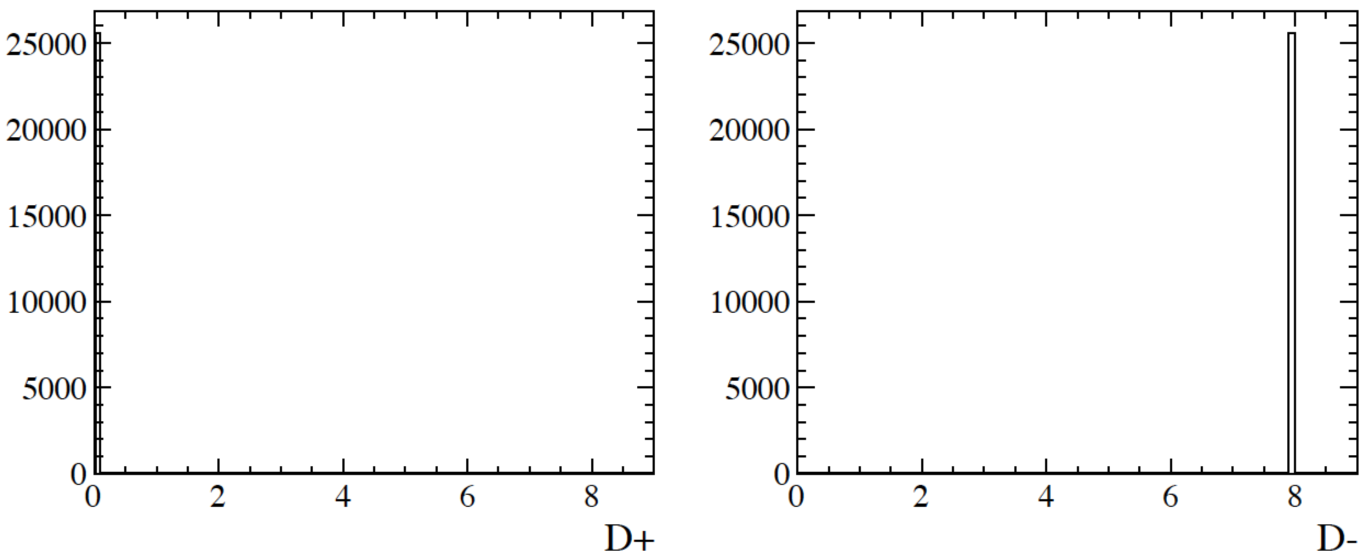
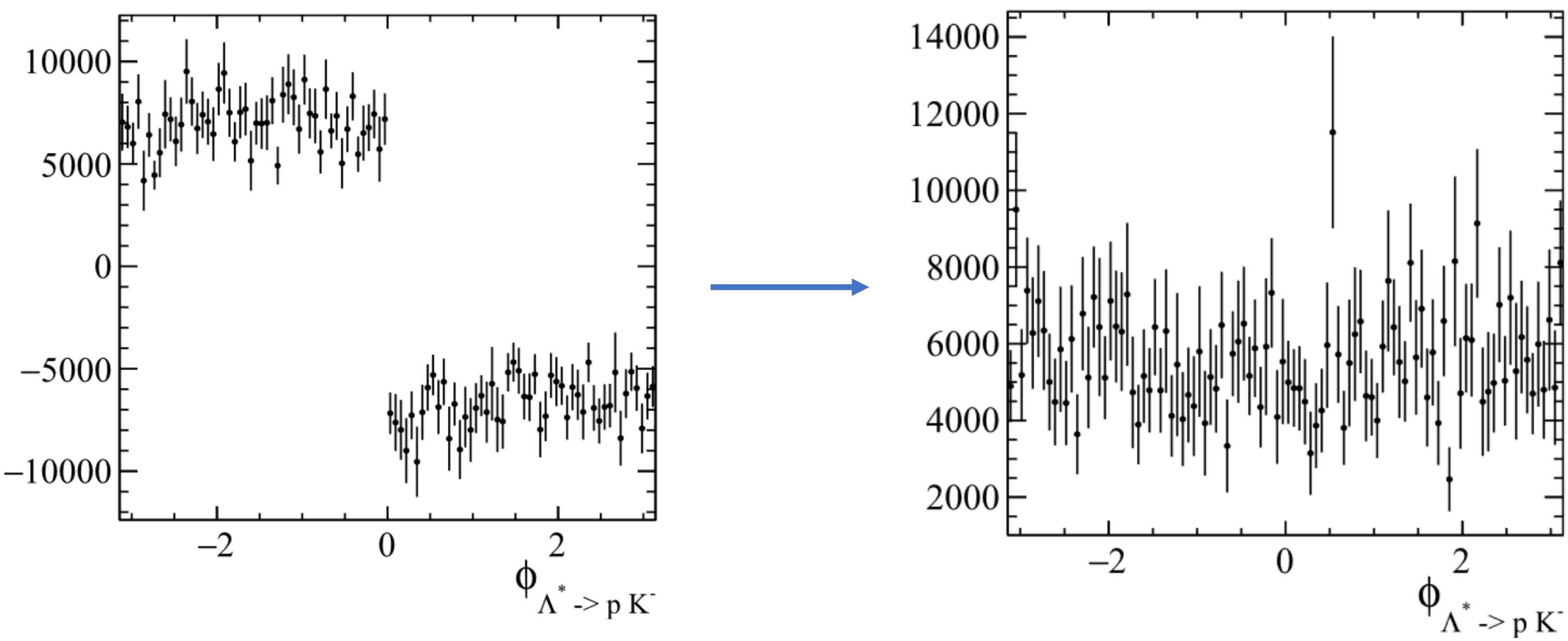


Figure 5: Distribution of $D^{\pm}$, based on SU2 representation, using particle ordering in Ref. [1]..

Conclusion: the proton can be well aligned with the new particle ordering

Results

- Use proton, instead of kaon to define Λ^* decay angle:
  - Discontinuous of the interference also disappear



Conclusion

- Final-state alignment is necessary for the construction of multi-body decay amplitudes with multiple decay chains

- For the validation of the alignment, one could directly write down rotation operators associated with each final-state particle, and check if the total effect is the same in all chains
  - Important to select a good representative to visualize all quantum effects

Thank you for your attention !
Questions or comments ?

Take the particle-2 convention

- For each $A \to BC$ decay node, add a $(-1)^{J_C - \lambda_C}$ term in the amplitude
  - $\Lambda_b^0 \to \Lambda^*(\to pK)J/\psi, \Lambda_b^0 \to P_c(\to J/\psi p)K$
  - Add $(-1)^{J_\psi - \lambda_\psi}$ for $\Lambda_b^0 \to \Lambda^* J/\psi$ decay node
  - Add $(-1)^{J_p - \lambda_p}$ for $P_c \to J/\psi p$ decay node
- Decay angle of $A \to BC$ is defined using B, and the corresponding rotation aligns C to the $-z$ direction
- Add a π rotation along y-axis, to align C to z-axis
  - For writing the amplitude of following C decays in a consistent way; Or for doing the final-state alignment in a consistent way if C is a final-state particle
  - $d^{J_C}_{-\lambda_C,\lambda_C}(\pi) = (-1)^{J_C - \lambda_C}$

Redefinition of ϕ_μ

- The J/ψ is the "particle-2" in $\Lambda_b^0 \to \Lambda^* J/\psi$ node
- In 2015 paper, J/ψ was aligned to z-axis by a π rotation along x-axis
- Now we use particle-2 convention, corresponding to a π rotation along y-axis
- The resulting x, y axis after the π rotation is opposite. Need a redefinition of ϕ_μ to be consistent with particle-2 convention