QUANTUM COLLAPSE MODELS AND THEIR EXPERIMENTAL TESTS

Fabrizio Napolitano on behalf of the VIP Collaboration

INFN
Istituto Nazionale di Fisica Nucleare

John Templeton Foundation

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MUSEO STORICO DELLA FISICA E CENTRO STUDI E RICERCHE ENRICO FERMI

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Wave-function Collapse Problem

Schrödinger Equation

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

linear and deterministic

Wave function reduction postulate:

$$\frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}} \xrightarrow{\text{measurement}} \begin{array}{l} \xrightarrow{\text{half of total cases}} |a_1\rangle \\ \xrightarrow{\text{half of total cases}} |a_2\rangle \end{array}$$

non-linear and stochastic

Wave-function Collapse Problem

Why the quantum properties of microscopic systems, e.g. the possibility of being in the superposition of different states at once, do not carry over to larger objects?

—— *How and why do we have a boundary between the two dynamics?* ——→

Will isolated quantum system manifest linear and deterministic Schrödinger evolution forever?
→ direct impact on quantum technologies

Superposition principle may progressively break down when atoms glue together to form larger systems (Karolyhazi, Ghirardi, Rimini, Weber, Pearle, Diosi, Penrose, Adler, Bassi, etc.). But what triggers the wave function Collapse?

Schrödinger Equation

$$i\hbar \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$$

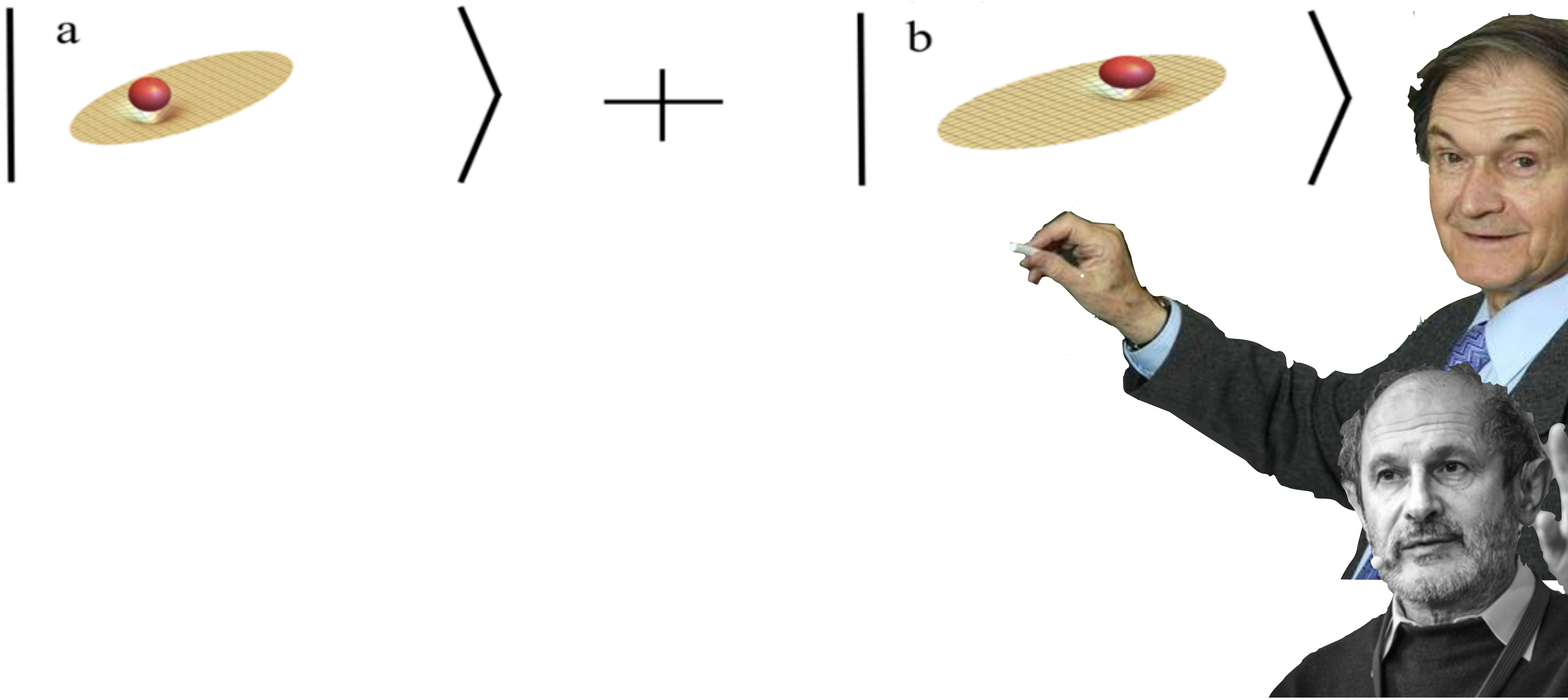
linear and deterministic

Wave function reduction postulate:

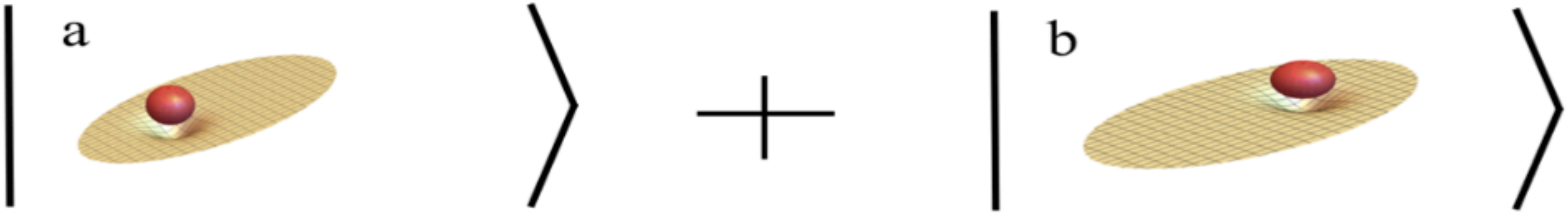
$$\begin{tikzcd} & & {|a_1\rangle} \\ \frac{|a_1\rangle + |a_2\rangle}{\sqrt{2}} \arrow[r, "\text{measurement}"] & {} \arrow[ur, "\text{half of total cases}"] \arrow[dr, "\text{half of total cases}"'] & \\ & & {|a_2\rangle} \end{tikzcd}$$

non-linear and stochastic

Diósi-Penrose (DP) Collapse model



Diósi-Penrose (DP) Collapse model



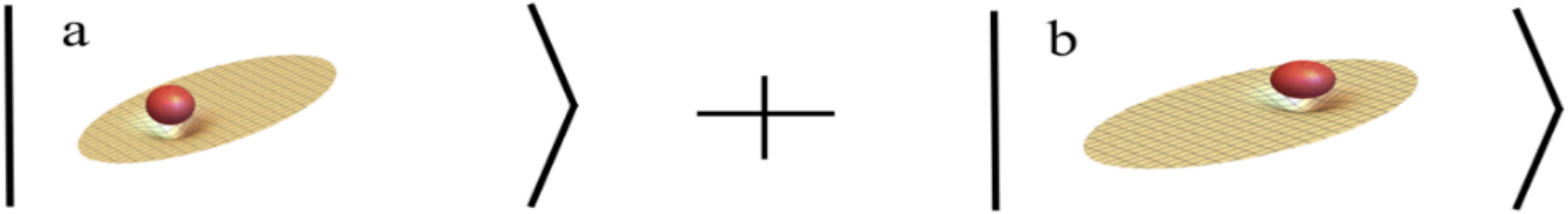
"*as soon as a 'significant' amount of space-time curvature is introduced, the rules of quantum linear superposition must fail*" (R. Penrose)

$$\Delta E_{\mathrm{DP}}(\mathbf{d}) = -8\pi G \int \mathrm{d}\mathbf{r} \int \mathrm{d}\mathbf{r}' \, \frac{\mu(\mathbf{r}) \left[\mu(\mathbf{r}' + \mathbf{d}) - \mu(\mathbf{r}')\right]}{|\mathbf{r} - \mathbf{r}'|}$$

Measures how rare the superposition is in gravitational terms

R. Penrose, Found. Phys. 44, 557–575 (2014), R. Penrose, Gen. Relativ. Gravit. 28, 581-600 (1996), L. Diósi, Phys. Rev. A **40**, 1165–1174 (1989).

Diósi-Penrose (DP) Collapse model



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$$\tau_{\mathrm{DP}} = \frac{\hbar}{\Delta E_{\mathrm{DP}}}$$

- ● Proton: m $\simeq 10^{-27}$ Kg, R $\simeq 10^{-15}$ m, $\tau_{\mathrm{DP}} \simeq 10^{6}$ years
- ● Dust grain: m $\simeq 10^{-12}$ Kg, R $\simeq 10^{-5}$ m, $\tau_{\mathrm{DP}} \simeq 10^{-8}$ s

R. Penrose, Found. Phys. 44, 557-575 (2014), R. Penrose, Gen. Relativ. Gravit. 28, 581-600 (1996), L. Diósi, Phys. Rev. A **40**, 1165–1174 (1989).

Continuous Spontaneous Localization (CSL) model

The CSL model is a stochastic and non-linear modification of the Schrödinger equation

$$d|\psi_t\rangle = [-\frac{i}{\hbar}Hdt+\sqrt{\lambda}\int d^3x(N(x)-\langle N(x)\rangle_t)dW_t(x)-\frac{\lambda}{2}\int d^3x(N(x)-\langle N(x)\rangle_t))^2dt]|\psi_t\rangle$$

Schrödinger

$N(x)$ $\langle N(x)\rangle_t$ *Particle density operator & non linearity*

$W_t(x)$ *Stocasticity*

λ
Collapse strength

$r_c = 1/\sqrt{\alpha},$
Correlation length

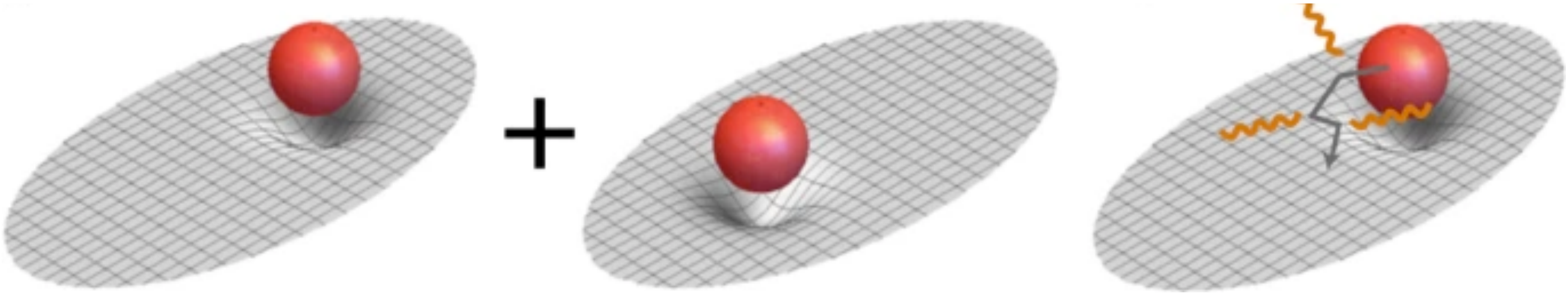
$W_t(x) = W_t(x)(\alpha)$

G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A 42, 78 (1990)

S. L. Adler, JPA 40, (2007) 2935, Adler, S.L.; Bassi, A.;

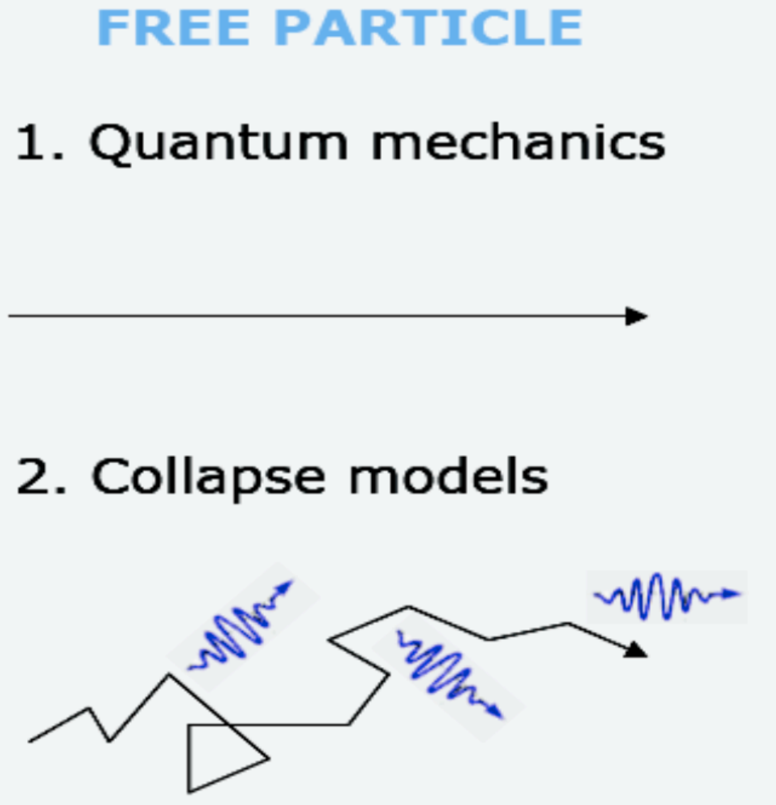
Donadi, S., JPA 46, (2013) 245304.

Testing Collapse Models with Gamma Ray spectroscopy



Collapse happens → the centre of mass is shifted towards the localized wave function position → since the process is random this results in a diffusion process

Deviation from standard QM: emission of radiation from charged particles



Q. Fu, Phys. Rev. A 56, 1806 (1997)
S. L. Adler and F. M. Ramazanoglu, J. Phys. A40, 13395 (2007);
J. Phys. A42, 109801 (2009)
S. L. Adler, A. Bassi and S. Donadi,
J. Phys. A46, 245304 (2013)
S. Donadi, D. A. Deckert and A. Bassi, Annals of Physics 340, 7086 (2014)

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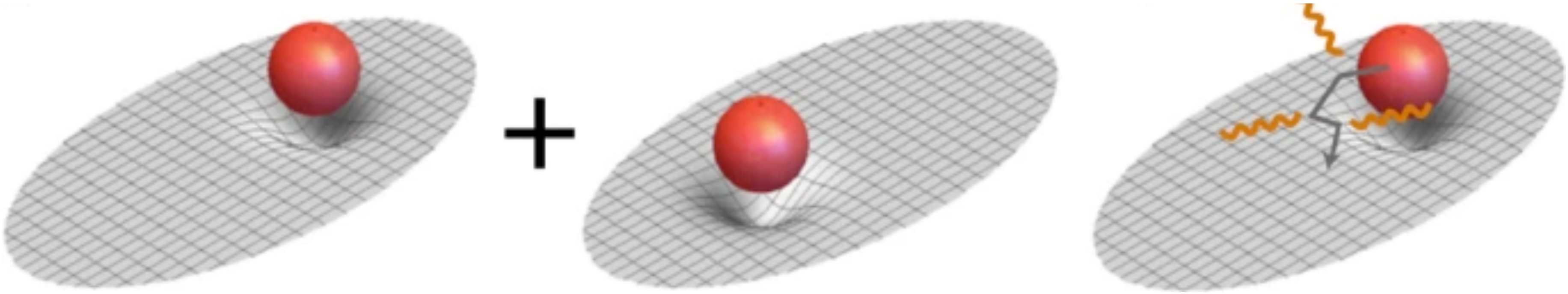
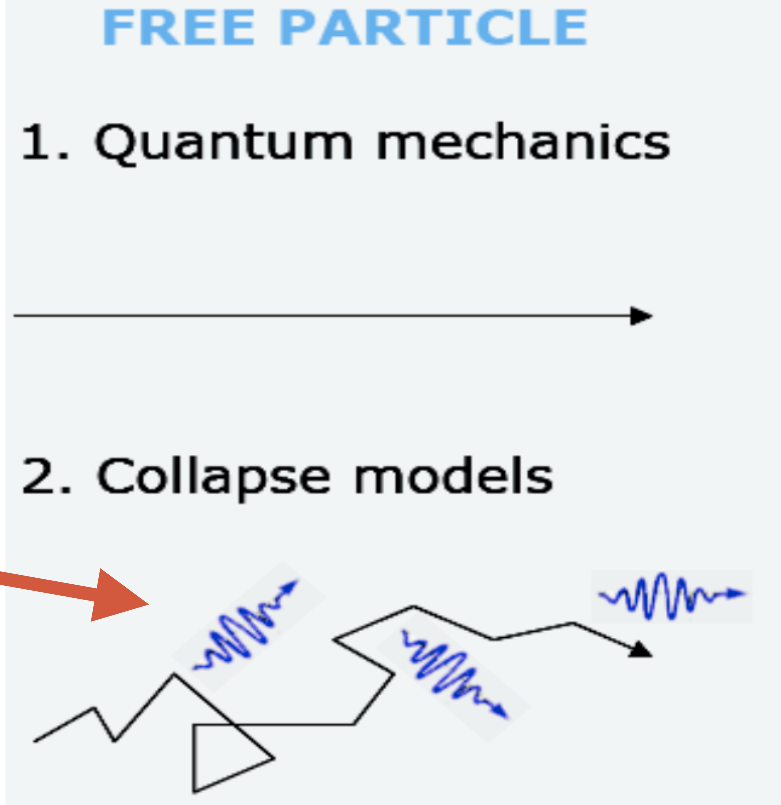


Collapse happens → the centre of mass is shifted towards the localized wave function position → since the process is random this results in a diffusion process

Deviation from standard QM: emission of radiation from charged particles

→ Anomalous amount of radiation can prove the collapse models



FREE PARTICLE

1. Quantum mechanics

2. Collapse models

Q. Fu, Phys. Rev. A 56, 1806 (1997)
S. L. Adler and F. M. Ramazanoglu, J. Phys. A40, 13395 (2007);
J. Phys. A42, 109801 (2009)
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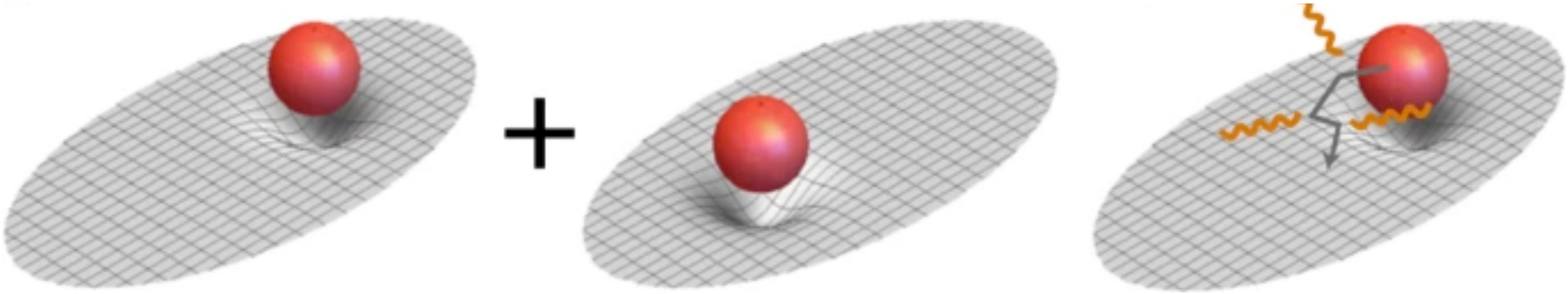


We search for spontaneous radiation emission from a germanium crystal and the surrounding materials in the experimental apparatus.

Theoretical prediction for the expected spontaneous emission rate

DP - s. e. photons rate:

$$\frac{d\Gamma_t}{d\omega} = \frac{2}{3}\frac{Ge^2N^2N_a}{\pi^{3/2}\varepsilon_0 c^3 R_0^3 \omega},$$

*

CSL - s. e. photons rate:

$$\frac{d\Gamma_t}{d\omega} = \frac{\lambda\hbar e^2 N^2 N_a}{4\pi^2 \epsilon_0 c^3 m_0^2 r_C^2 E}$$

*

→ Calculated in collaboration with L. Diosi, A. Bassi & S. Donadi

where:

λ - collapse strength

r_C - correlation length

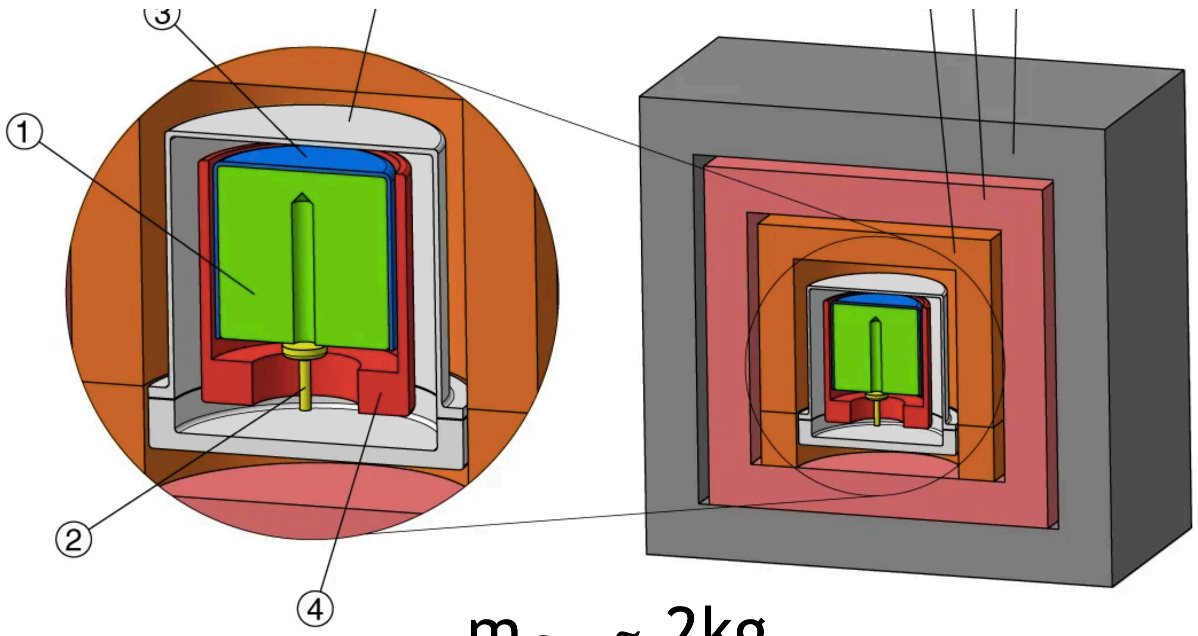
→ see e. g. *S. L. Adler, JPA 40, (2007) 2935, Adler, S.L.; Bassi, A.; Donadi, S., JPA 46, (2013) 245304.*

R_0 - size of the particle mass density → See e.g. Diósi, *L. J. Phys. Conf. Ser. 442, 012001 (2013)., Penrose, R. Found. Phys. 44, 557-575 (2014).*

**: photon rates for energies > 100 keV*

Measurement and MC validation

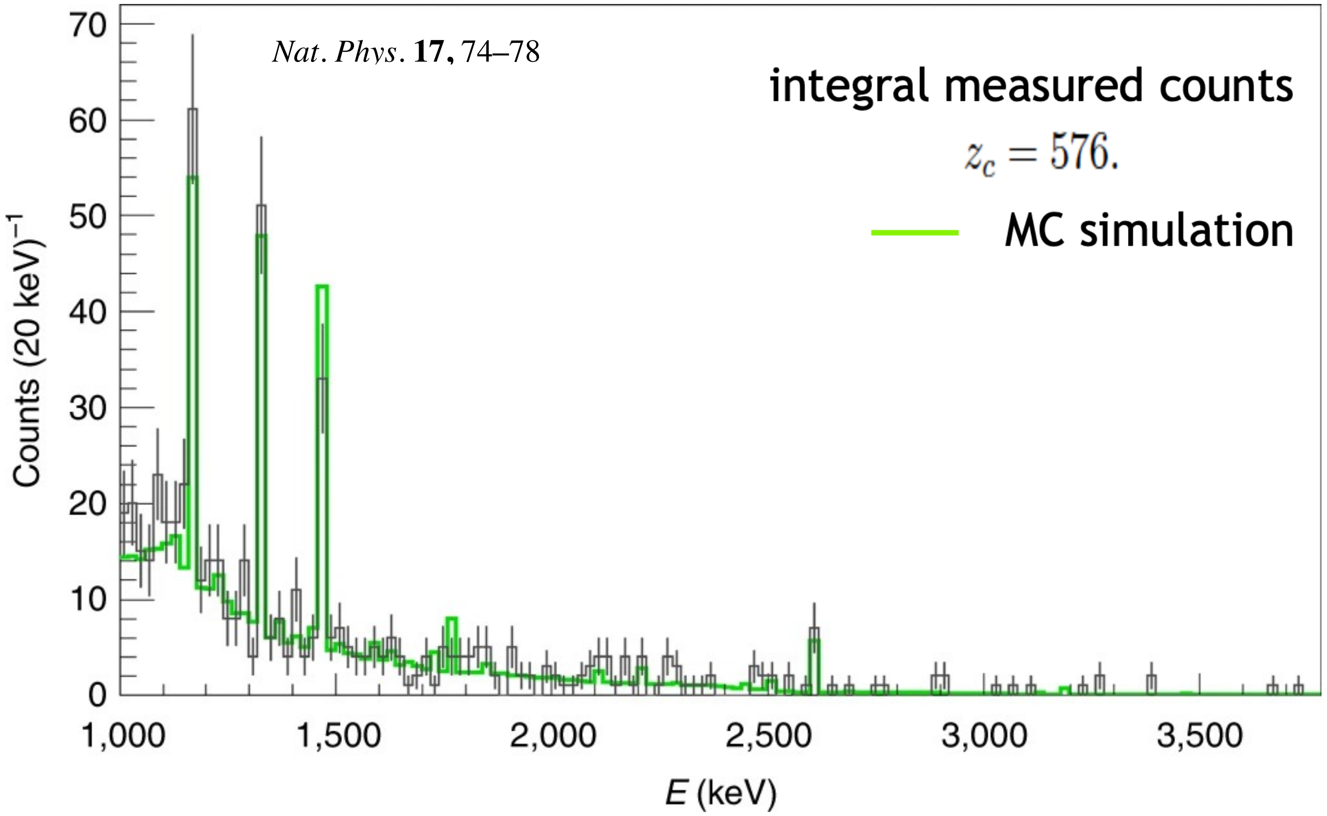
Coaxial p-type high purity germanium (**HPGe**)



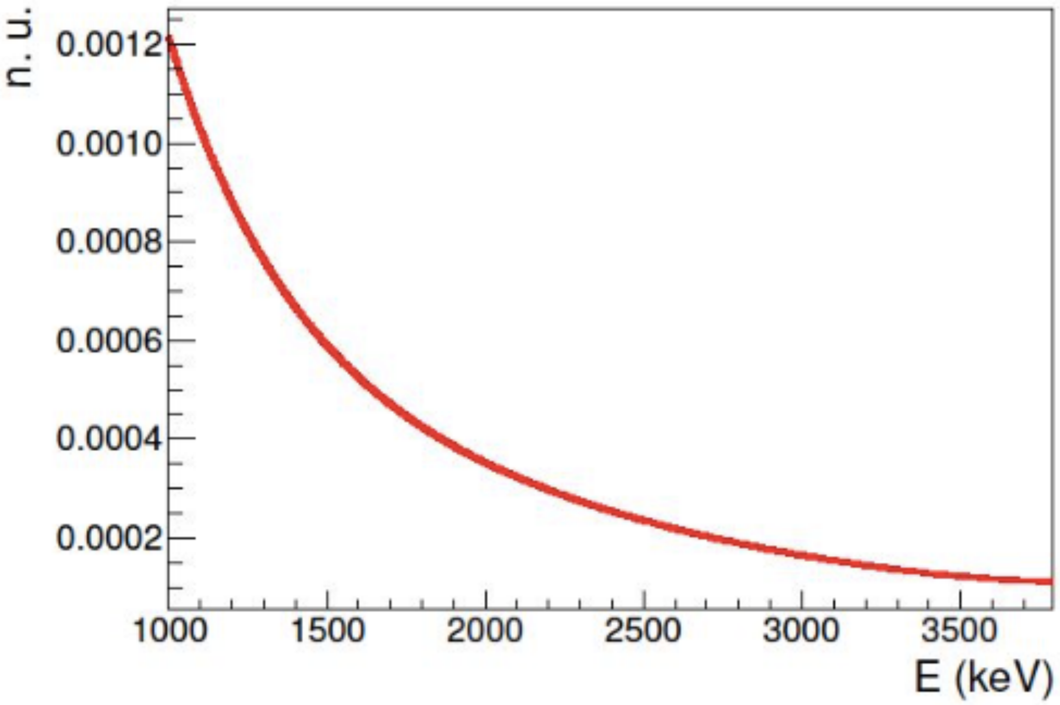
m_{Ge} ~ 2kg

- the activities are measured for each component
- the MC simulation accounts for:
    1. emission probabilities and decay schemes for each radio-nuclide in each material
    2. photons propagation and interactions
    3. detection efficiencies.

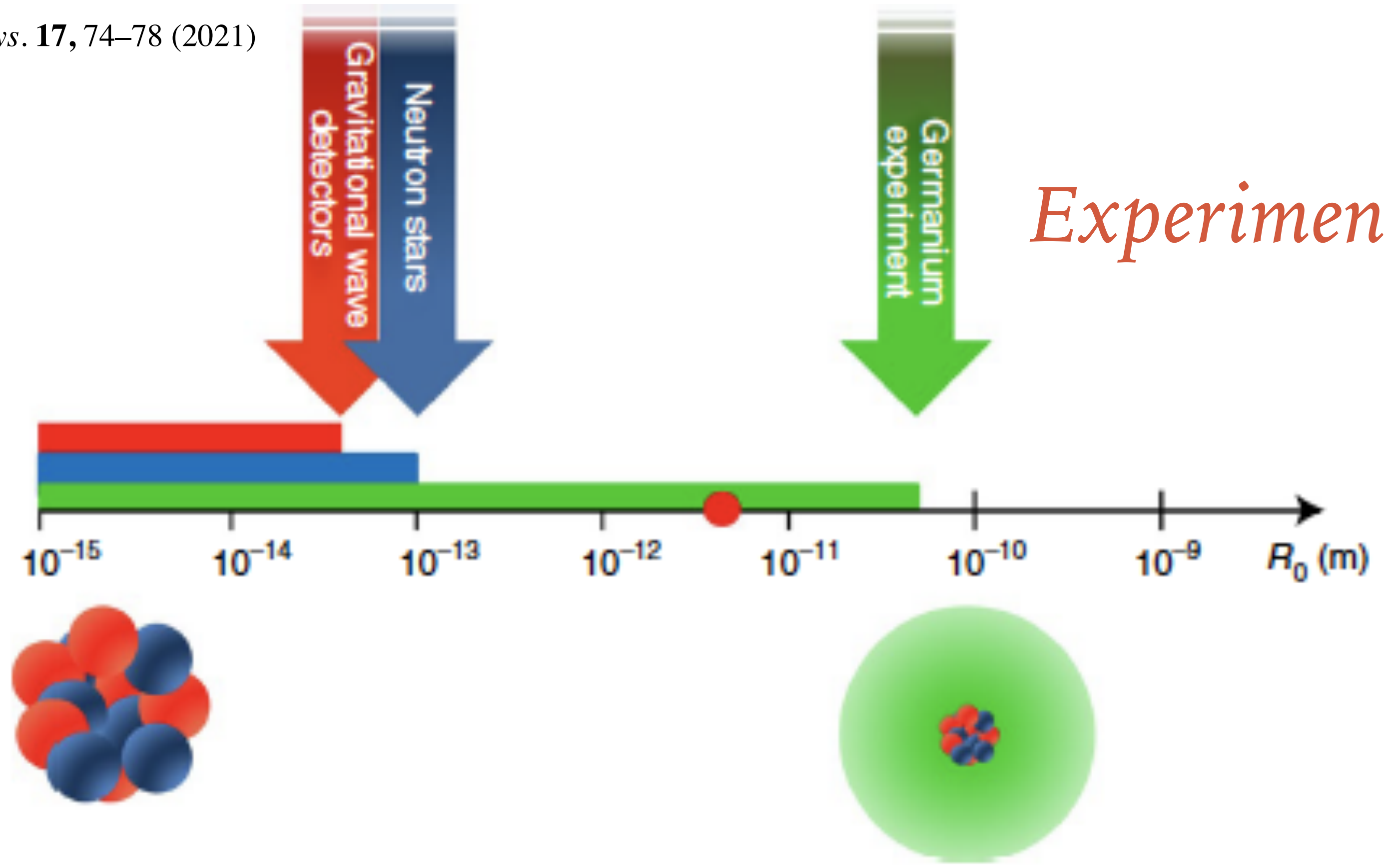
The simulation describes 88% of the integral counts:


Nat. Phys. **17**, 74–78

expected signal contribution



Results



Nat. Phys. **17,** 74–78 (2021)

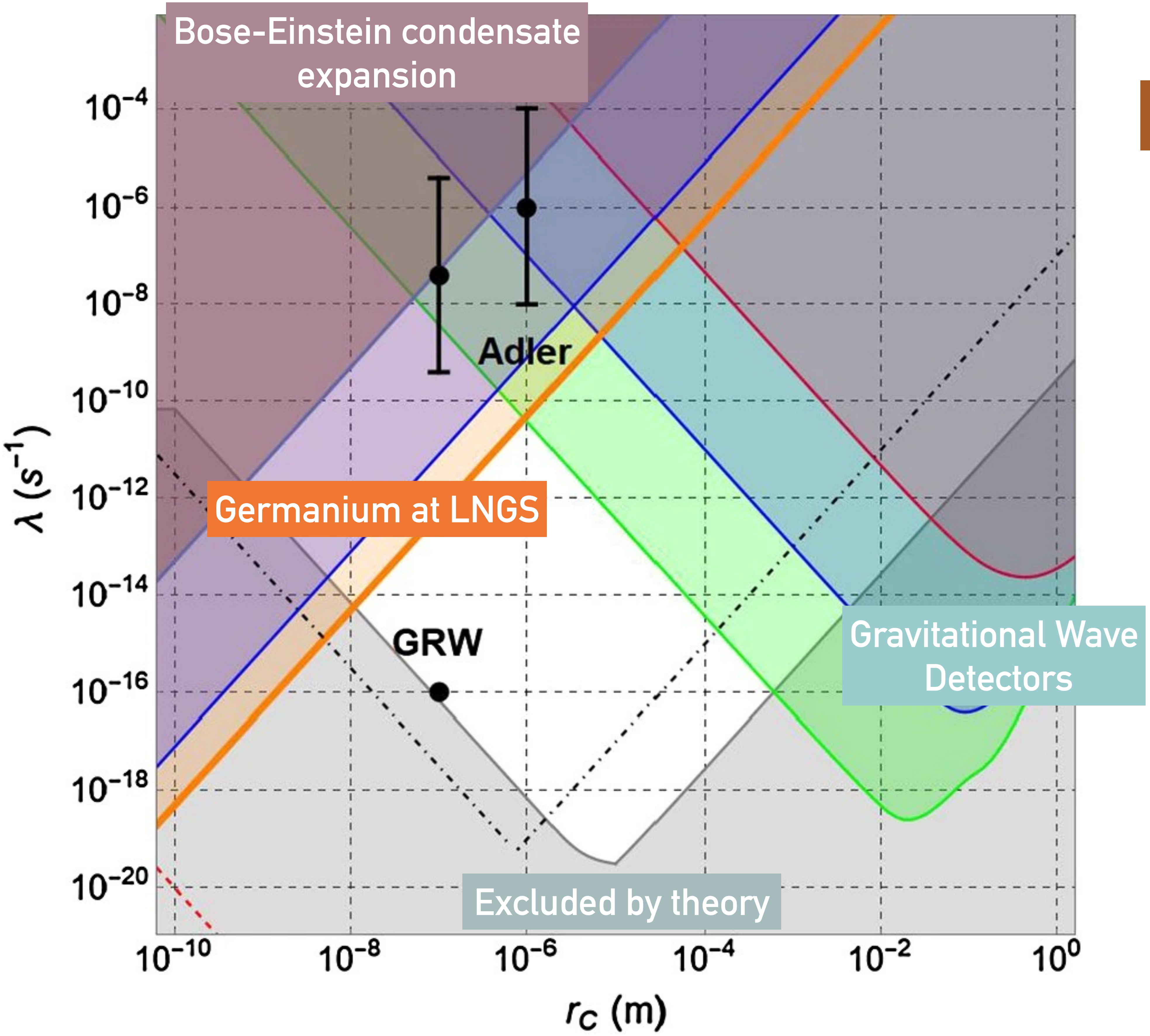
Experimental: $R_0 > 0.54 \times 10^{-10}$ *m*

If R_0 *is the size of the nucleus' wave function as suggested by Penrose, in a germanium crystal* R_0^2 *is the mean square displacement of a nucleus in the lattice which, for Ge at liquid nitrogen temperature amounts to:*

Theoretical: $R_0 = 0.05 \times 10^{-10}$ *m*

DP model ruled out in the present formulation

Underground test of gravity-related wave function collapse". Nature Physics 17, pages 74–78 (2021)



CSL Model

Donadi, S., Piscicchia, K., Del Grande, R. *et al.*. *Eur. Phys. J. C* 81, 773 (2021).

Planning for the future activities - outline

Near future: next 2 years

- Work with theoreticians
  - Generalized models
  - Cancellation effects
  - Study energy dependence on Z

Mid future: next 4/5 years

- Low-energy frontier
  - Upgrading front-end electronics
  - Shape dependence on different targets

Long future: next 7/8 years

- Dedicated setup for collapse measurement, (Dark Matter, Cosmology)

Theory development - X-rays spontaneous radiation

Near future: next 2 years

DP is ruled out in present formulation!

In collaboration with the theoretical groups: Diosi, Penrose, Adler, Bassi … we are developing generalized models e.g.：

- Add dissipation terms to the master equation and stochastic nonlinear Schrödinger equation of the DP theory, to counteract the runaway energy increase
- Non-Markovian correlation function

Generalized models lead to **strong dependence on the emission energy in relation to the atomic structure**

Theory development - X-rays spontaneous radiation

Near future: next 2 years

$$\left.\frac{d\Gamma}{dE}\right|_t^{CSL} = N_{atoms}\cdot\frac{\hbar\, e^2\,\lambda}{4\,\pi^2\,\epsilon_0\,c^3\,m_0^2\,r_C^2\,E}\cdot$$

$$\cdot\left\{\underline{N_p^2+N_e}+2\cdot\underline{\sum_{o\,o'\ \text{pairs}} N_{eo}\,N_{eo'}\,\frac{\sin\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}{\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}+\underline{\sum_o N_{eo}\,\frac{\sin\left(\frac{\rho_o\,E}{\hbar\,c}\right)}{\left(\frac{\rho_o\,E}{\hbar\,c}\right)}}\cdot\Big[(N_{eo}-1)\cos\left(\frac{\rho_o\,E}{\hbar\,c}\right)}-\underline{2\,N_p}\Big]\right\}$$

Naive term, low-energy p,e- contribution without taking into account their coherent emission

Cancellation

Coherent emission

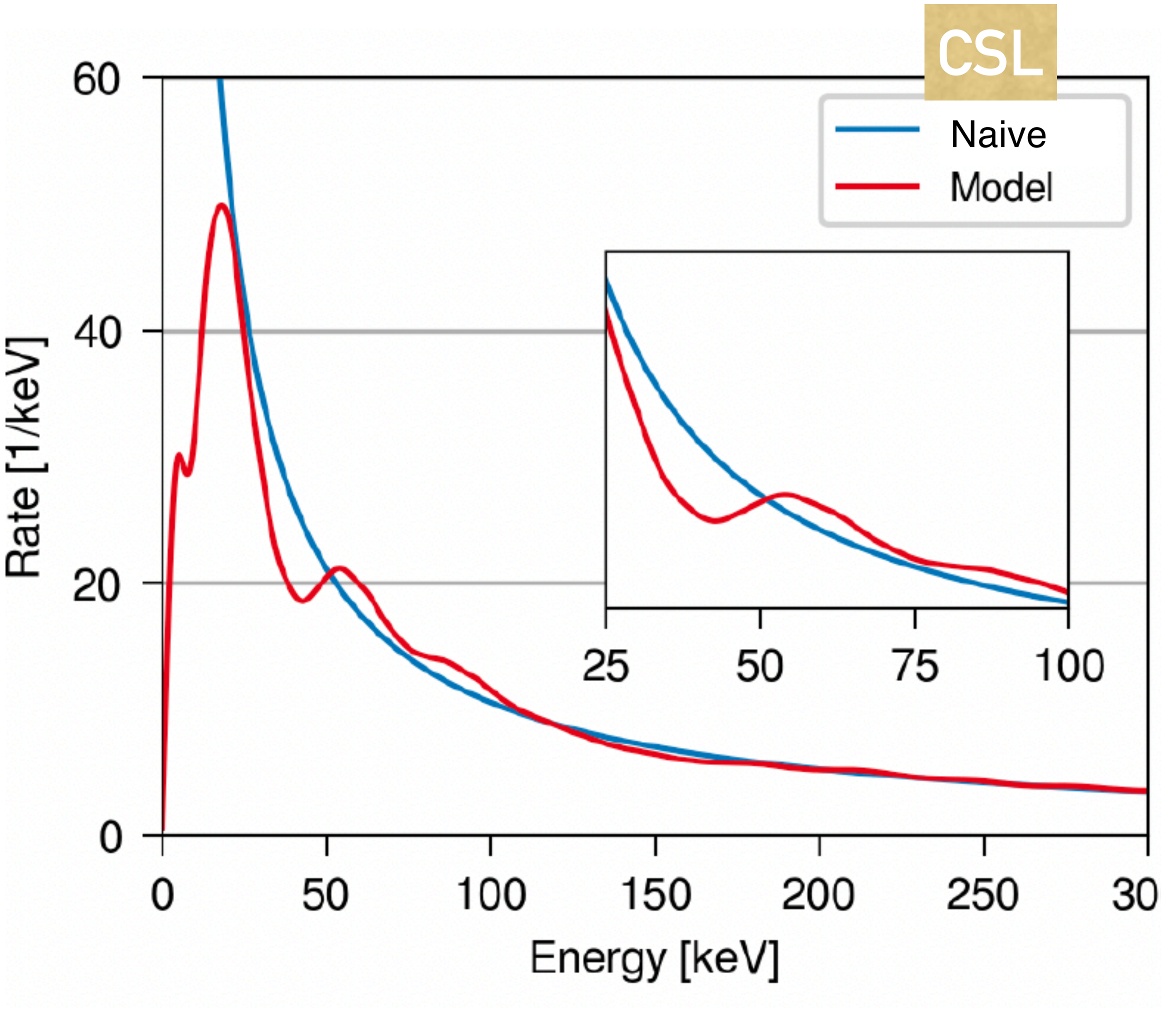
At each energy the atomic structure influences the shape of the expected spontaneous emission spectrum

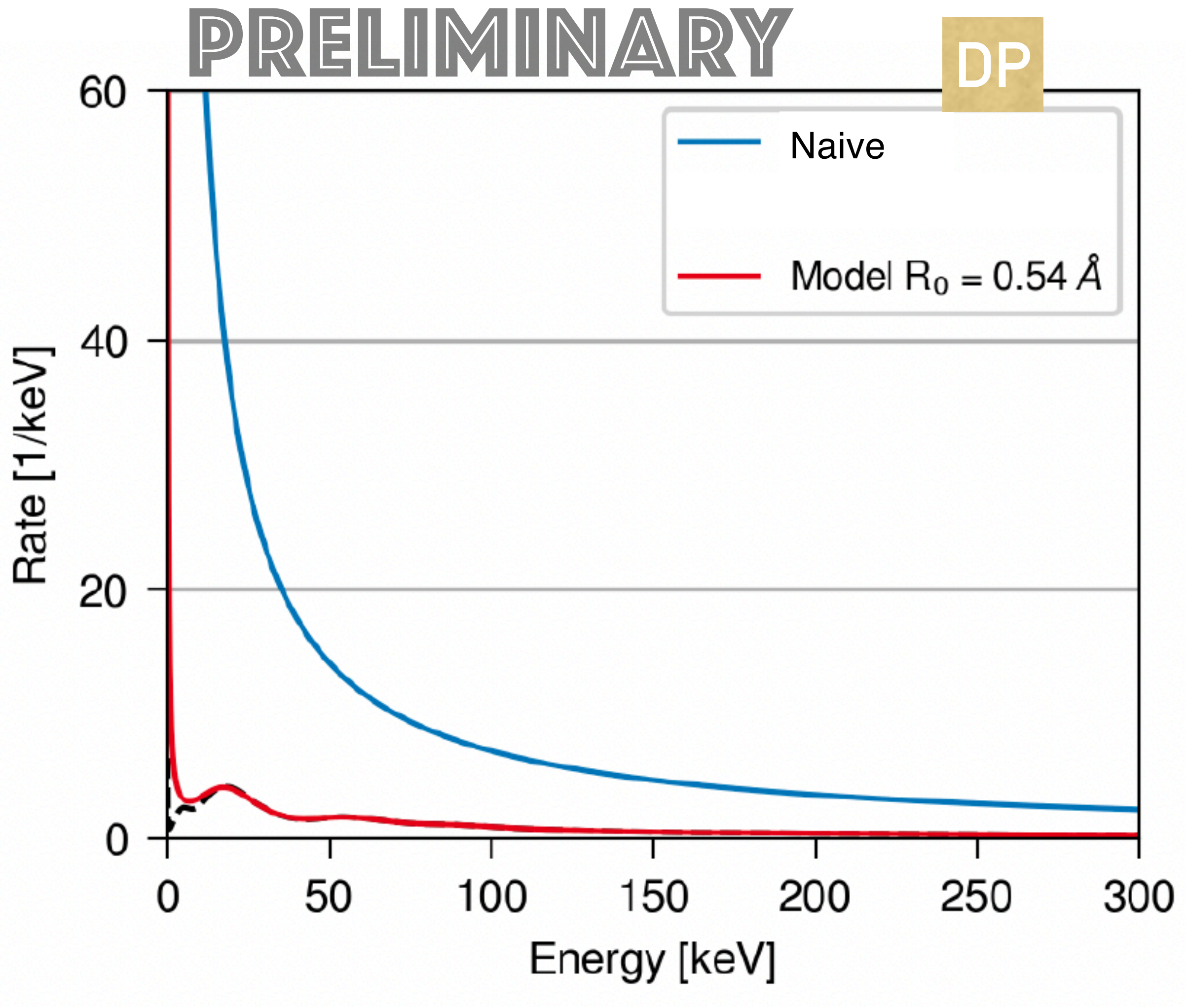
Theory development - X-rays spontaneous radiation

Near future: next 2 years

— $$\frac{1}{E}\left(N_p^2+N_e\right)$$

— $$\frac{1}{E}\left\{N_p^2+N_e+2\cdot\sum_{o\,o'\text{ pairs}} N_{eo}\,N_{eo'}\,\frac{\sin\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}{\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}+\sum_o N_{eo}\,\frac{\sin\left(\frac{\rho_o\,E}{\hbar\,c}\right)}{\left(\frac{\rho_o\,E}{\hbar\,c}\right)}\cdot\left[(N_{eo}-1)\cos\left(\frac{\rho_o\,E}{\hbar\,c}\right)-2\,N_p\right]\right\}$$

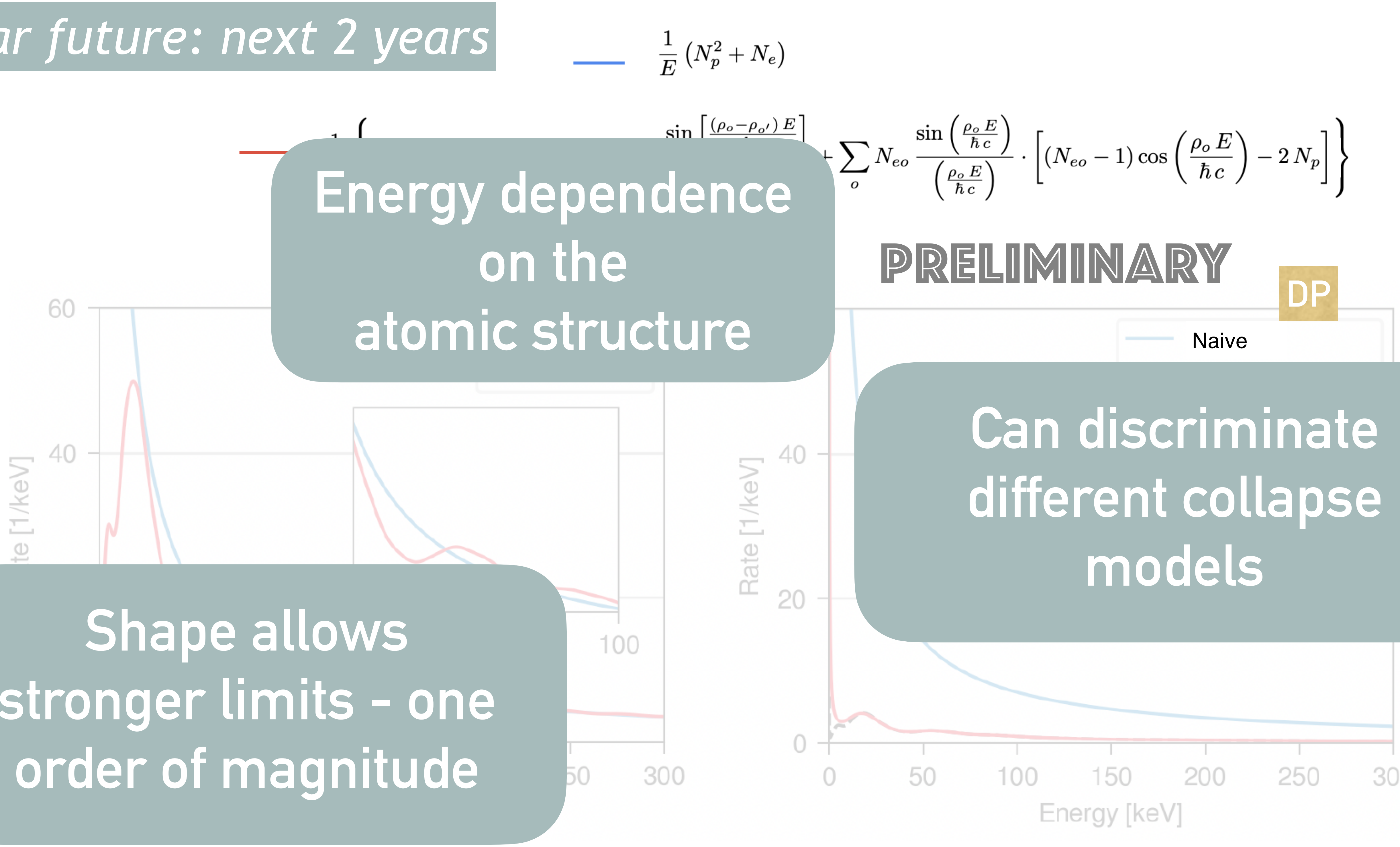




At each energy the atomic structure influences the shape of the expected spontaneous emission spectrum

Theory development - X-rays spontaneous radiation

Near future: next 2 years



Energy dependence on the atomic structure

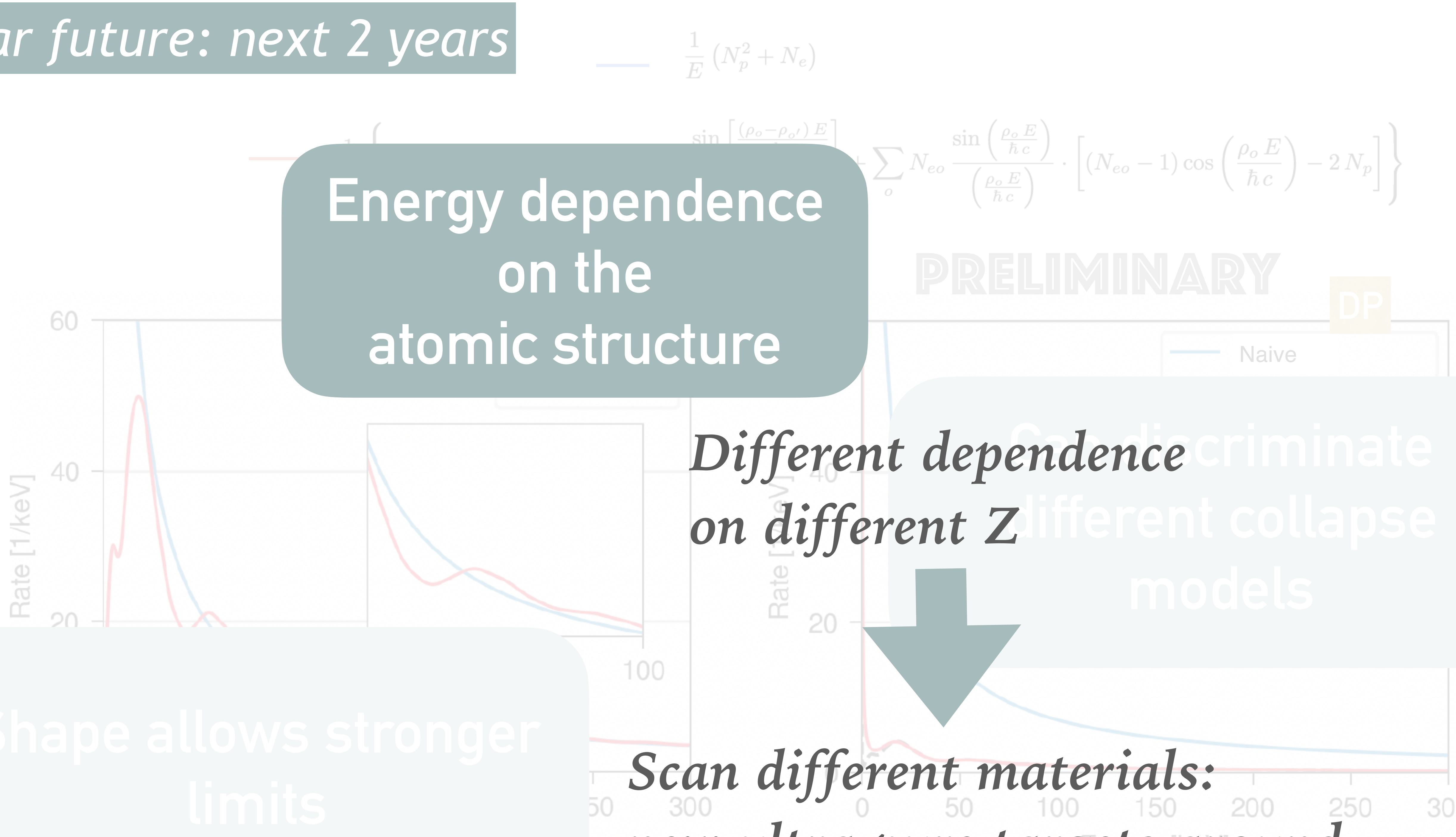
Can discriminate different collapse models

Shape allows stronger limits - one order of magnitude

At each energy the atomic structure influences the shape of the expected spontaneous emission spectrum

Theory development - X-rays spontaneous radiation

Near future: next 2 years



Energy dependence on the atomic structure

Different dependence on different Z

Scan different materials: new ultra pure targets around detector

Aluminum, Silver, Gold, etc

Can discriminate different collapse models

Shape allows stronger limits

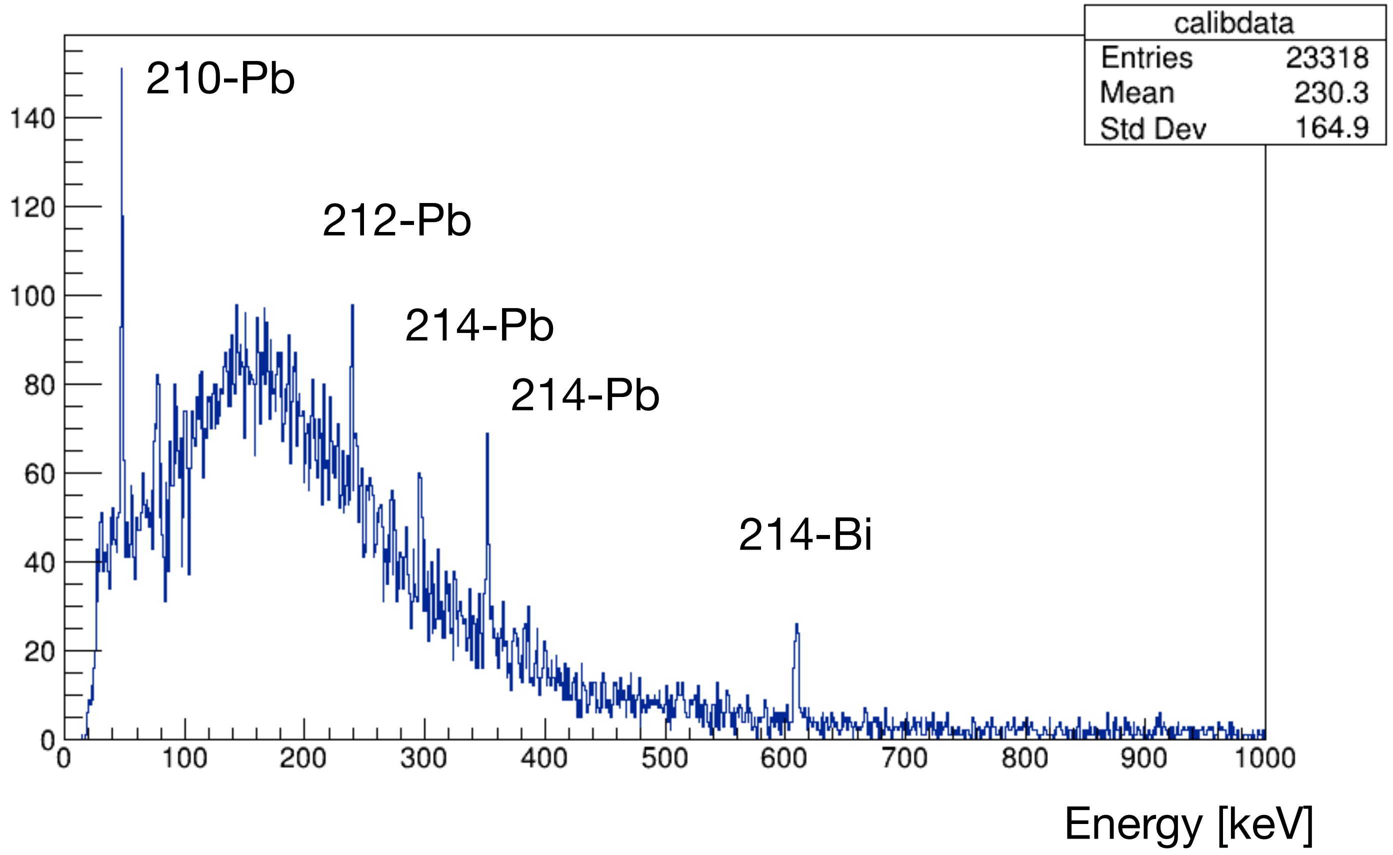
At each energy the atomic structure influences the shape of the expected spontaneous emission spectrum

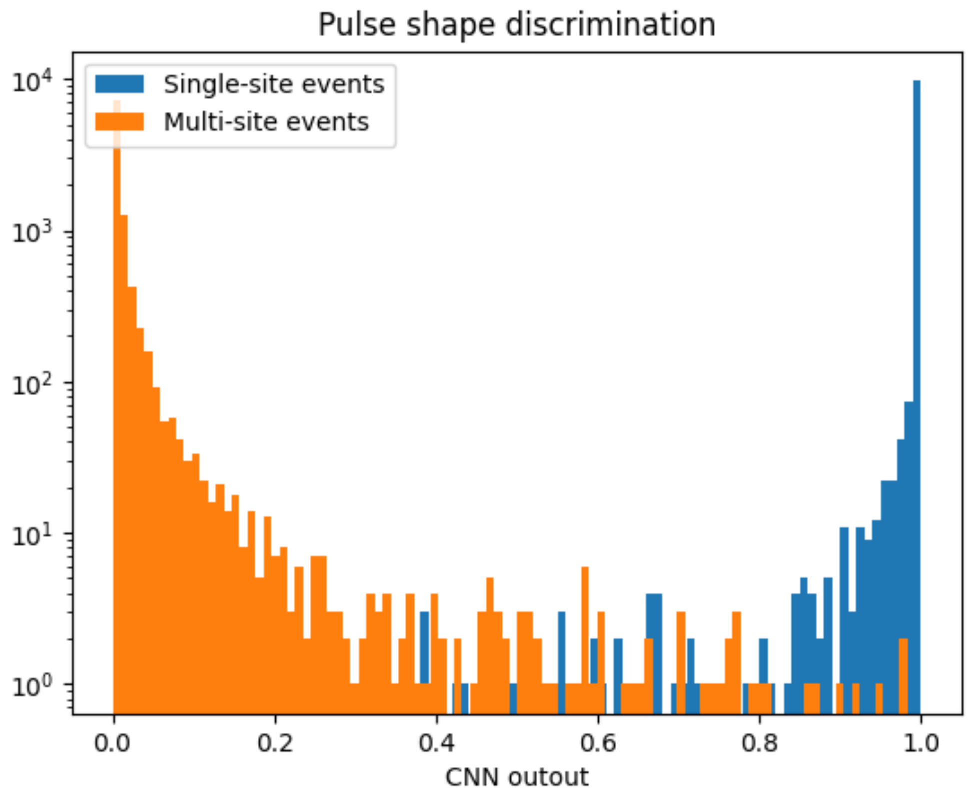
Spontaneous Radiation - Dedicated BEGe-like detector

Mid future: next 4/5 years

Stronger limits possible at low energy with a Broad Energy (BE)-like Germanium

- Possible to exploit shape dependence to enhance limit setting on different models
- Using a ~1 kg BEGe for preliminary studies, dedicated to spontaneous radiation
- Using ML techniques for pulse shape discrimination
  - Further enhance physics capabilities





Spontaneous Radiation - BEGe detector

Mid future: next 4/5 years

Stronger limits possible at low energy with a Broad Energy (BE) Germanium

- Possible to exploit shape dependence to enhance limit setting on different models
- Using a ~1 kg BEGe for preliminary studies, dedicated to spontaneous radiation
- Using ML techniques for pulse shape discrimination
    - Further enhance physics capabilities

- Plan to use different ultra-pure materials with different Z, TBD
- Plan to reach lower energies using different front-end electronics
- Test setup equipped with low noise DAQ
- High insulation low noise amplifier
    - Gain towards lower energies

BEGe schieldin
improved BEGe

Dedicated setup

Long future: next 7/8 years

Need low energy capability & ultra low noise

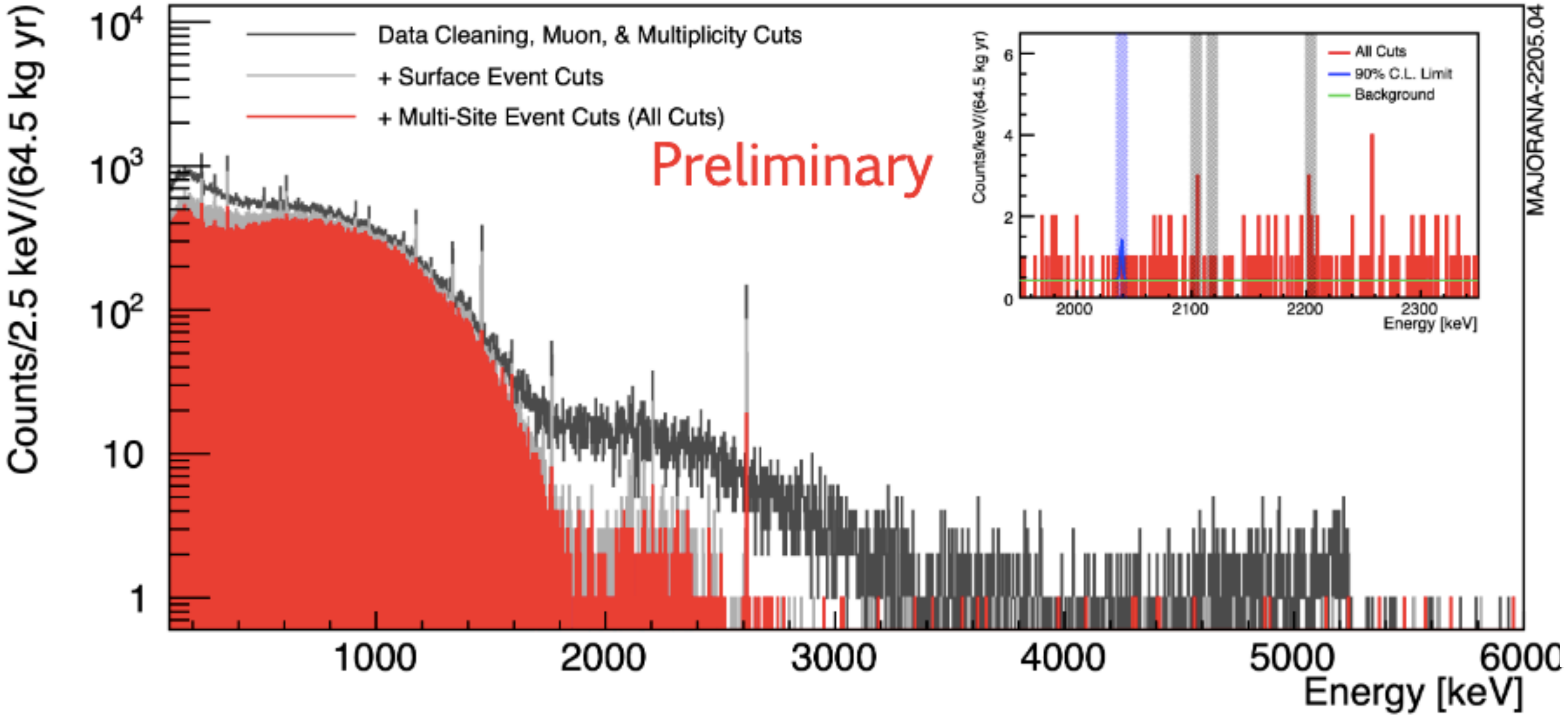
- Low energy needed to maximize physics capabilities of collapse studies
- High resolution needed to distinguish shape dependence
- Need to run with different targets, optimized for spontaneous emission parameters

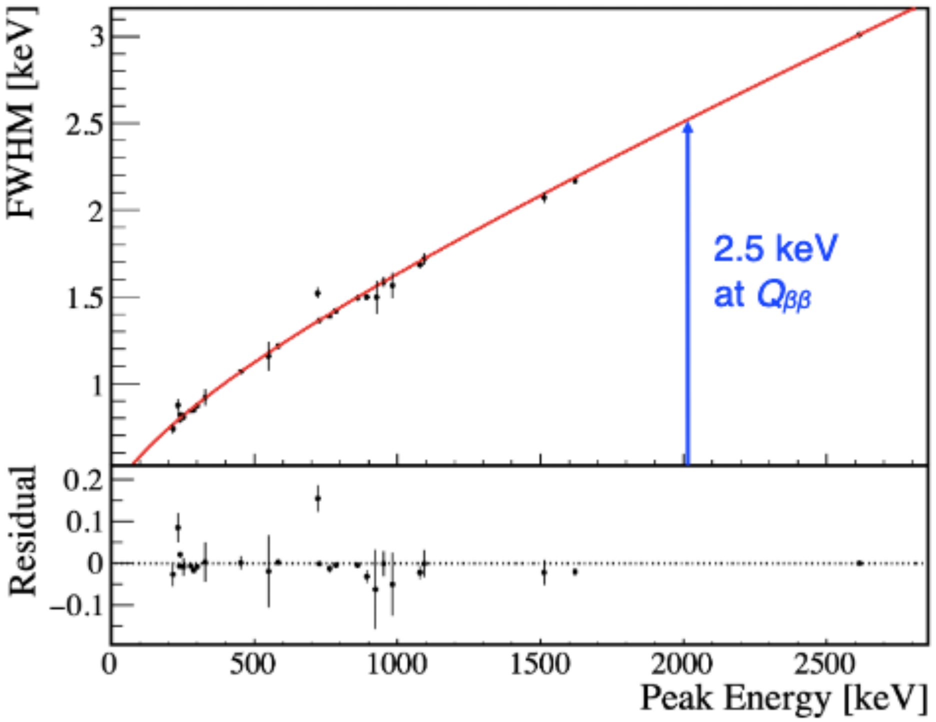
Dedicated setup

Long future: next 7/8 years

- Array of p-type point contact detectors
- > 10 kg total crystal mass
- Low background:
  - Compact shield + active muon veto

Example: MAJORANA demonstrator





Dedicated setup

Long future: next 7/8 years

AJORANA demonstrator

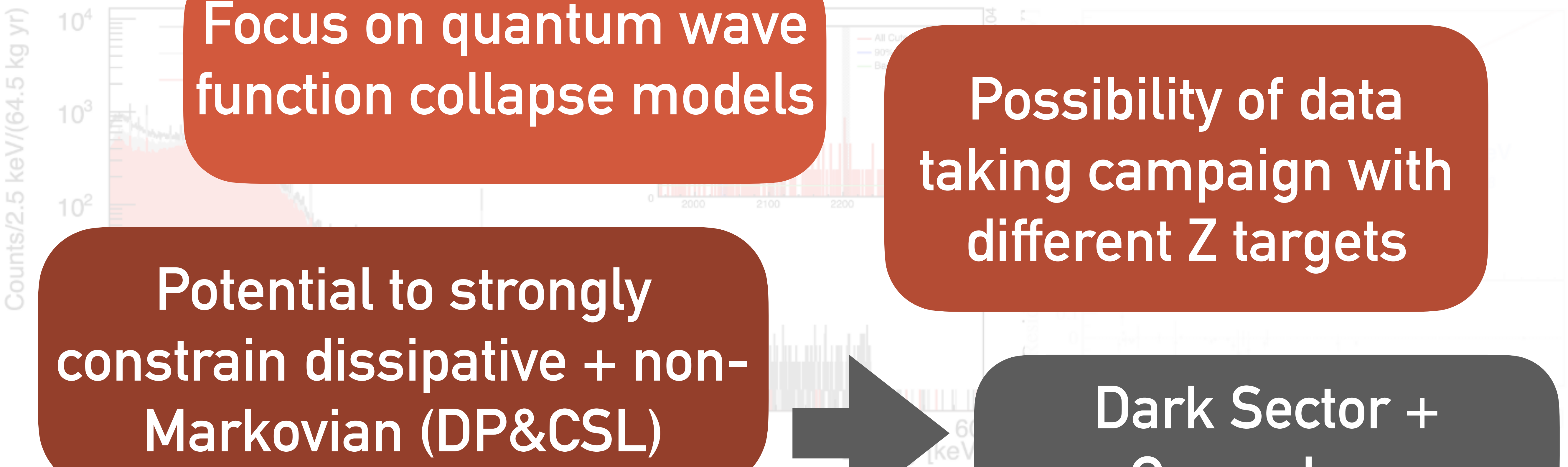
detectors

ystal mass

muon veto

Need a specialized MAJORANA-like Experiment

With a lower energy threshold



Focus on quantum wave function collapse models

Possibility of data taking campaign with different Z targets

Potential to strongly constrain dissipative + non-Markovian (DP&CSL)

→ Dark Sector + Cosmology

Conclusions

- Wave function collapse still an open question
- Many collapse models e.g. Diòsi-Penrose and CSL
  - Each predict emission of spontaneous radiation
- Models are about to be excluded in their simplest formulation
- Near future: Developing new models with theoreticians: non-Markovian and dissipative terms
- Mid future: low energy frontier
  - scan different Z for atomic dependence
  - Upgraded setup with more efficient front-end electronics
- Long future: new, dedicated setup, a la MAJORANA

Thank you for your attention!

Questions?

Diósi-Penrose (DP) Collapse model

$$d|\psi_t\rangle = \left[\underline{-\frac{i}{\hbar}\hat{H}dt} + \underline{\sqrt{\frac{G}{\hbar}}\int d\mathbf{x}(\hat{\mu}(\mathbf{x}) - \langle\hat{\mu}(\mathbf{x})\rangle)dW_t(\mathbf{x})} - \underline{\frac{G}{2\hbar}\int d\mathbf{x}d\mathbf{y}\,\frac{(\hat{\mu}(\mathbf{x})-\langle\hat{\mu}(\mathbf{x})\rangle)(\hat{\mu}(\mathbf{y})-\langle\hat{\mu}(\mathbf{y})\rangle)}{|\mathbf{x}-\mathbf{y}|}}\right]|\psi_t\rangle$$

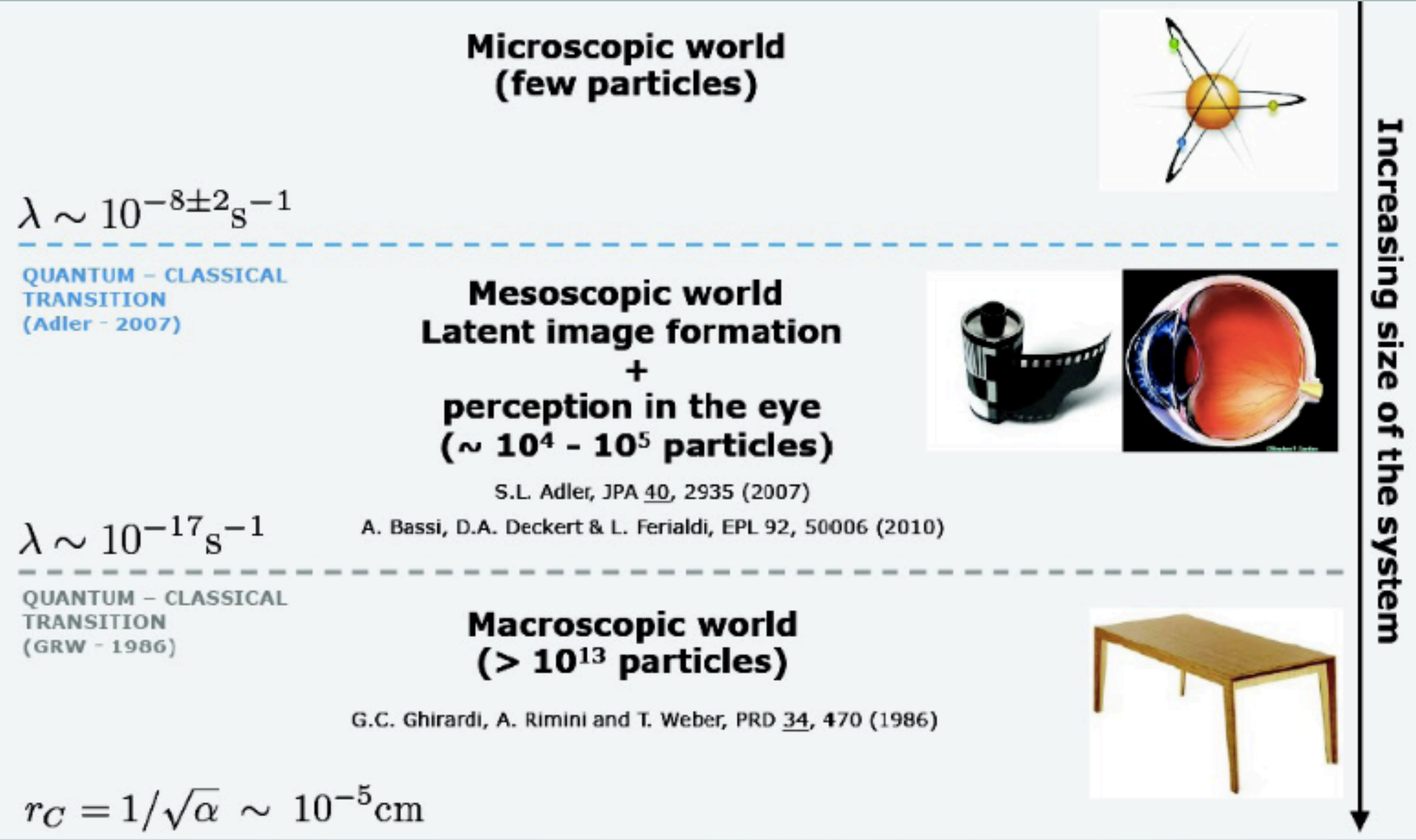
Schrödinger

Specific dynamics for the collapse

Collapse in position, no superluminal signals and amplification mechanism

$$\tau^{-1} = \frac{G}{2\hbar}\int d\mathbf{x}d\mathbf{y}\,\frac{(\hat{\mu}_a(\mathbf{x})-\hat{\mu}_b(\mathbf{x}))(\hat{\mu}_a(\mathbf{y})-\hat{\mu}_b(\mathbf{y}))}{|\mathbf{x}-\mathbf{y}|}$$

R. Penrose, Found. Phys. 44, 557-575 (2014), R. Penrose, Gen. Relativ. Gravit. 28, 581-600 (1996), L. Diósi, Phys. Rev. A **40**, 1165-1174 (1989).



Microscopic world
(few particles)

$\lambda \sim 10^{-8\pm2}\mathrm{s}^{-1}$

QUANTUM – CLASSICAL
TRANSITION
(Adler - 2007)

Mesoscopic world
Latent image formation
+
perception in the eye
(~ 10^4 - 10^5 particles)

S.L. Adler, JPA 40, 2935 (2007)

A. Bassi, D.A. Deckert & L. Ferialdi, EPL 92, 50006 (2010)

$\lambda \sim 10^{-17}\mathrm{s}^{-1}$

QUANTUM – CLASSICAL
TRANSITION
(GRW - 1986)

Macroscopic world
(> 10^{13} particles)

G.C. Ghirardi, A. Rimini and T. Weber, PRD 34, 470 (1986)

$r_C = 1/\sqrt{\alpha} \sim 10^{-5}\mathrm{cm}$

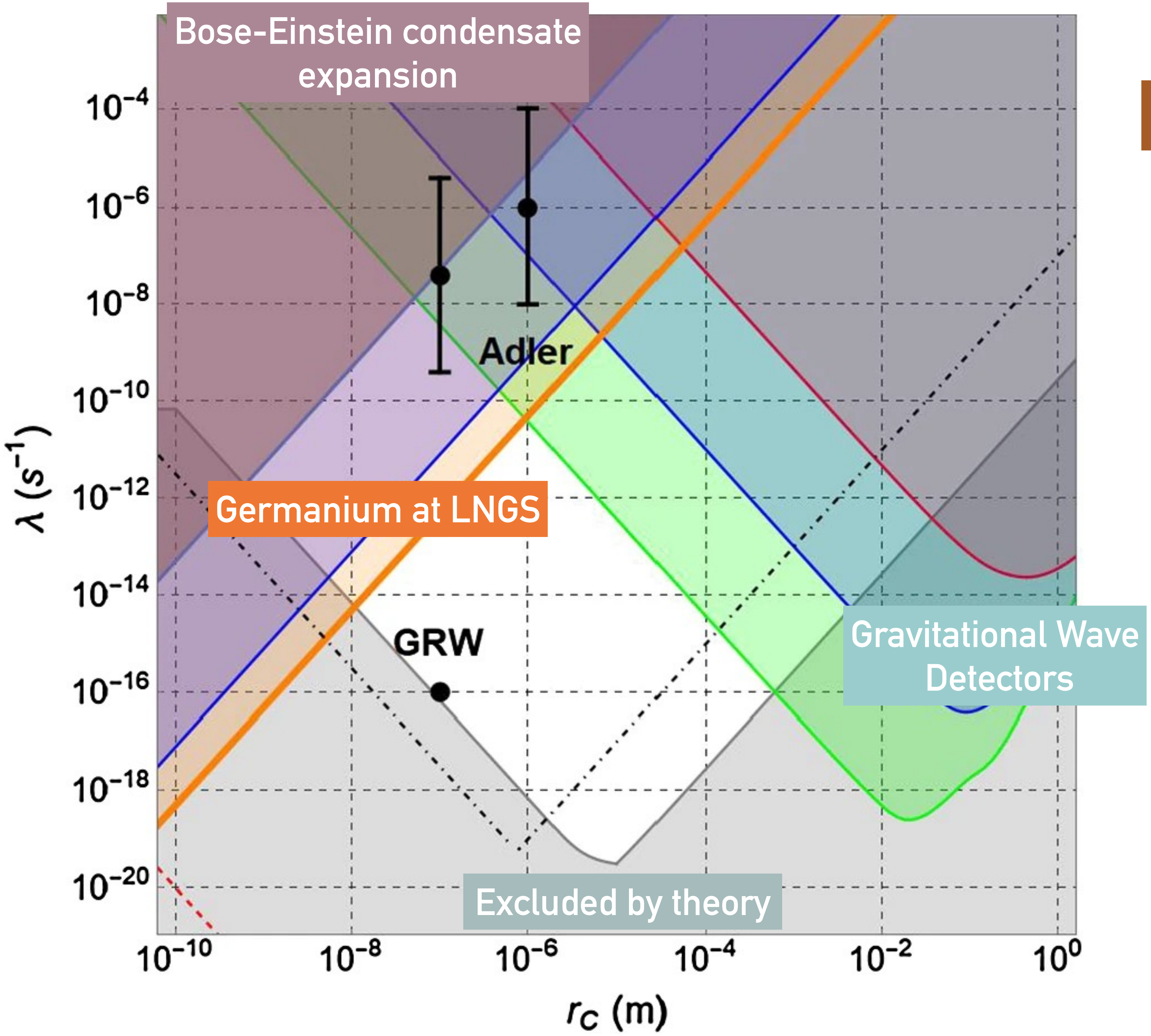
Increasing size of the system

G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A 42, 78 (1990)

S. L. Adler, JPA 40, (2007) 2935, Adler, S.L.; Bassi, A.;

Donadi, S., JPA 46, (2013) 245304.

CSL Model

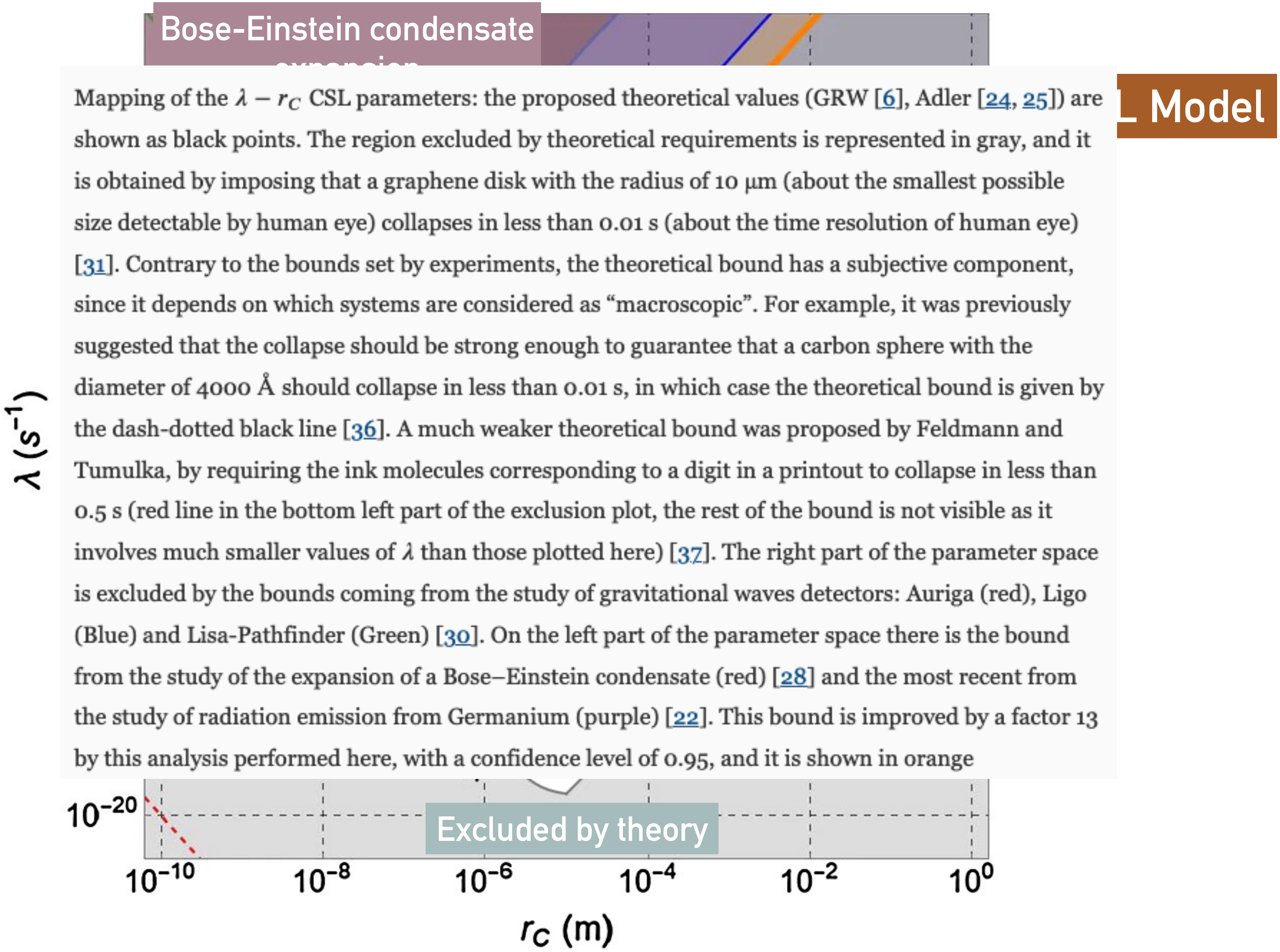

Bose-Einstein condensate expansion

Germanium at LNGS

Gravitational Wave Detectors

Excluded by theory

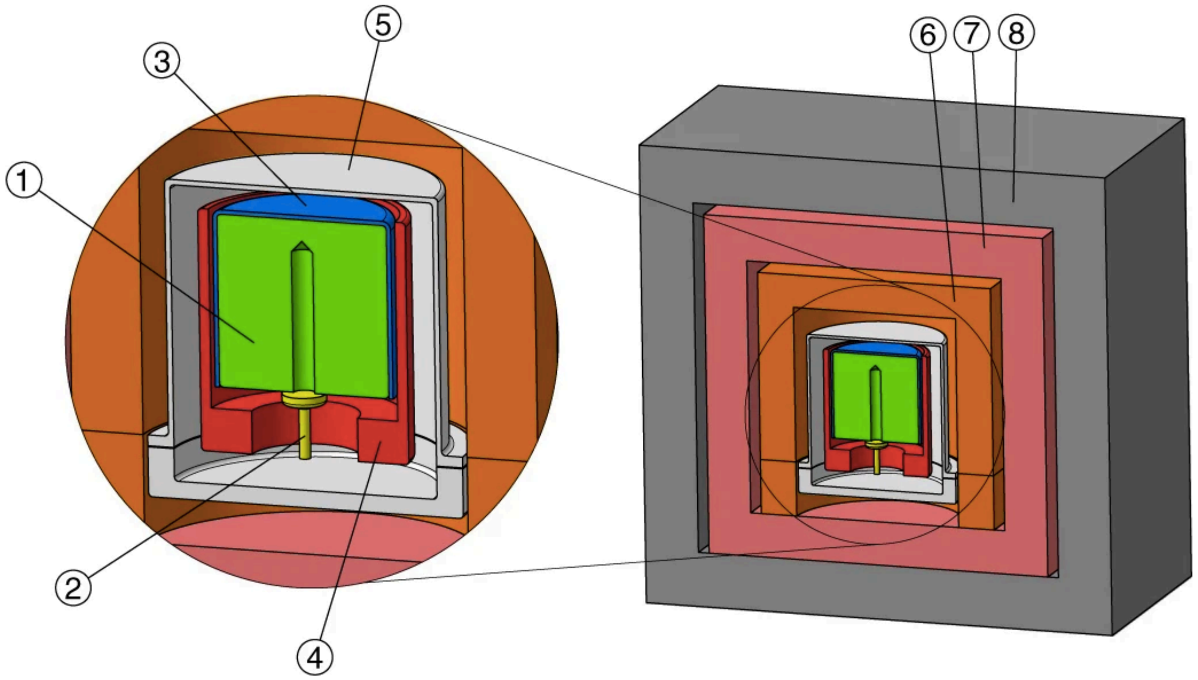
Donadi, S., Piscicchia, K., Del Grande, R. *et al.*. *Eur. Phys. J. C* **81, 773 (2021).**



Mapping of the $\lambda - r_C$ CSL parameters: the proposed theoretical values (GRW [6], Adler [24, 25]) are shown as black points. The region excluded by theoretical requirements is represented in gray, and it is obtained by imposing that a graphene disk with the radius of 10 μm (about the smallest possible size detectable by human eye) collapses in less than 0.01 s (about the time resolution of human eye) [31]. Contrary to the bounds set by experiments, the theoretical bound has a subjective component, since it depends on which systems are considered as "macroscopic". For example, it was previously suggested that the collapse should be strong enough to guarantee that a carbon sphere with the diameter of 4000 Å should collapse in less than 0.01 s, in which case the theoretical bound is given by the dash-dotted black line [36]. A much weaker theoretical bound was proposed by Feldmann and Tumulka, by requiring the ink molecules corresponding to a digit in a printout to collapse in less than 0.5 s (red line in the bottom left part of the exclusion plot, the rest of the bound is not visible as it involves much smaller values of λ than those plotted here) [37]. The right part of the parameter space is excluded by the bounds coming from the study of gravitational waves detectors: Auriga (red), Ligo (Blue) and Lisa-Pathfinder (Green) [30]. On the left part of the parameter space there is the bound from the study of the expansion of a Bose–Einstein condensate (red) [28] and the most recent from the study of radiation emission from Germanium (purple) [22]. This bound is improved by a factor 13 by this analysis performed here, with a confidence level of 0.95, and it is shown in orange

Donadi, S., Piscicchia, K., Del Grande, R. *et al.*. *Eur. Phys. J. C* **81, 773 (2021).**

The experiment at LNGS

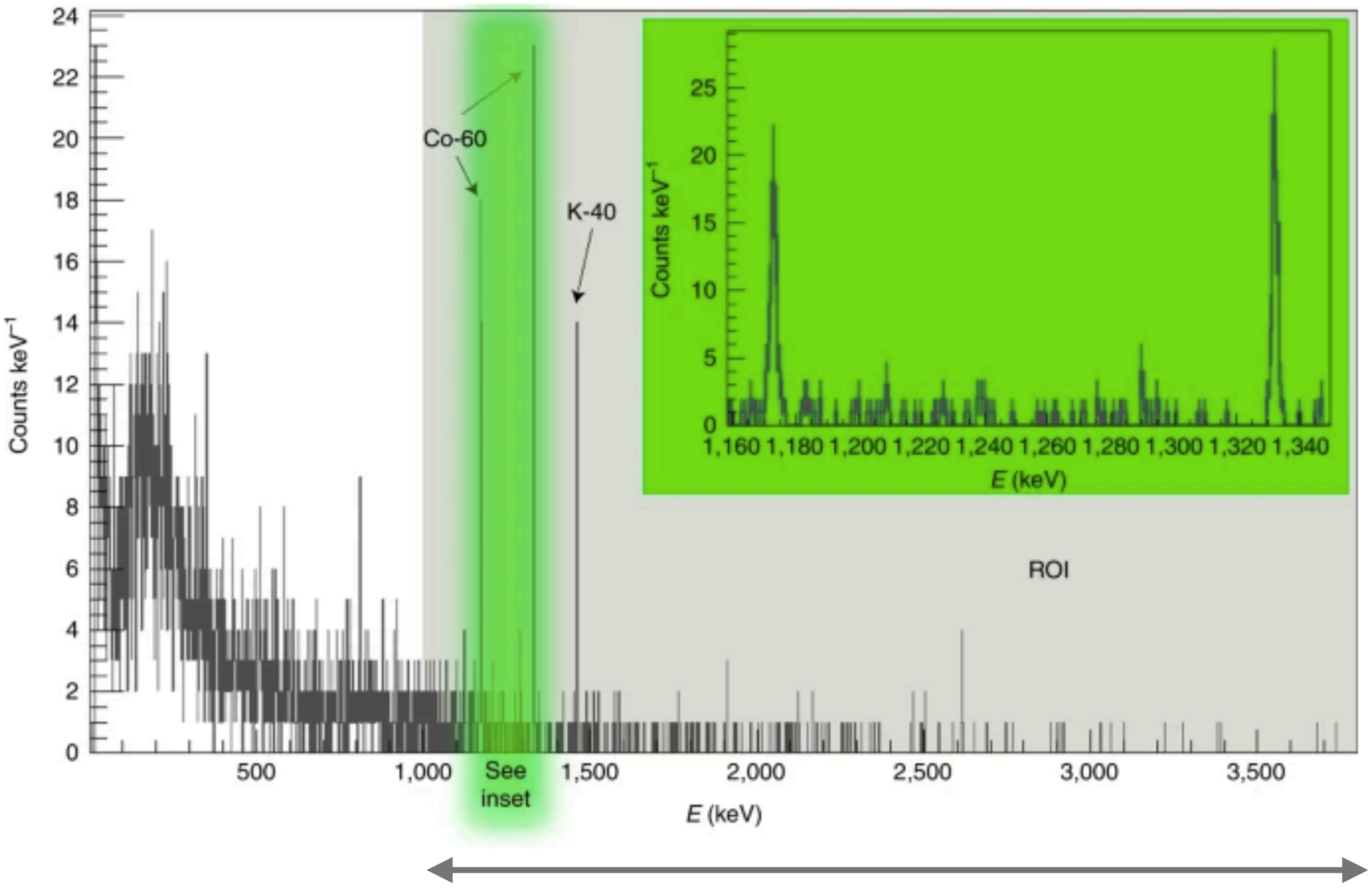


Coaxial p-type high purity germanium detector (**HPGe**):

- Exposure 124 kg · day, m_{Ge} ~ 2kg
- passive shielding: inner - electrolytic copper, outer - lead
- on the bottom and on the sides 5 cm thick borated polyethylene plates give a partial reduction of the neutron flux
- an airtight steel housing encloses the shield and the cryostat, flushed with boil-off nitrogen to minimize the presence of radon.

Measurement and MC validation

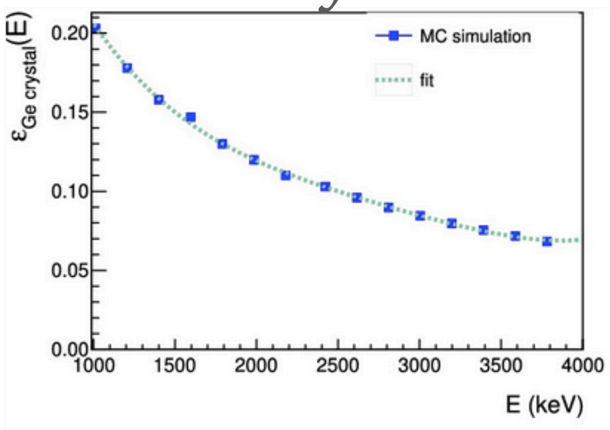
Nat. Phys. **17,** 74–78 (2021)



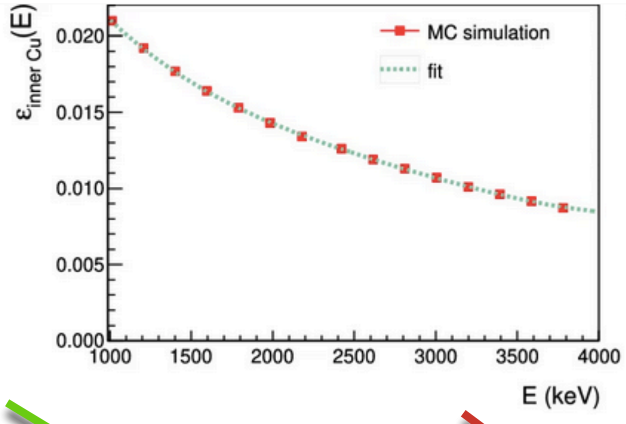
ROI

Measurement and MC validation

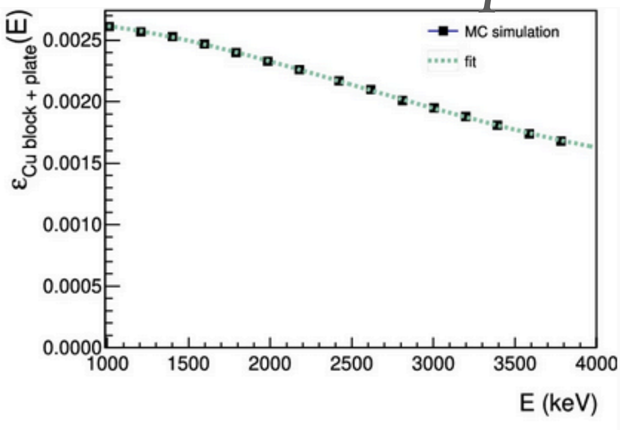
Ge Crystals



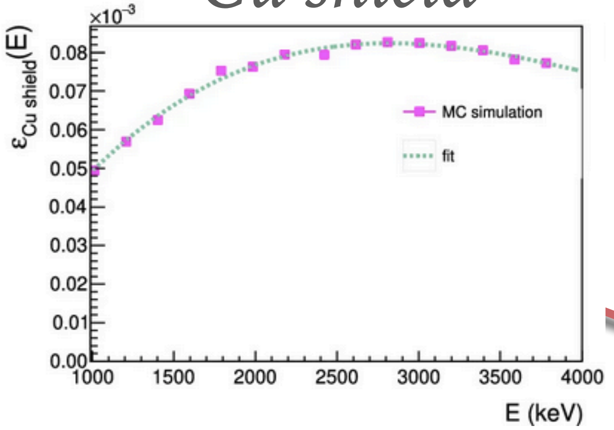
Inner Cu



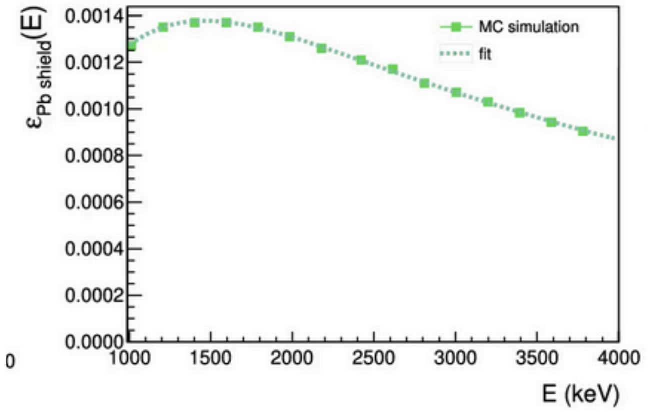
Cu block and plate

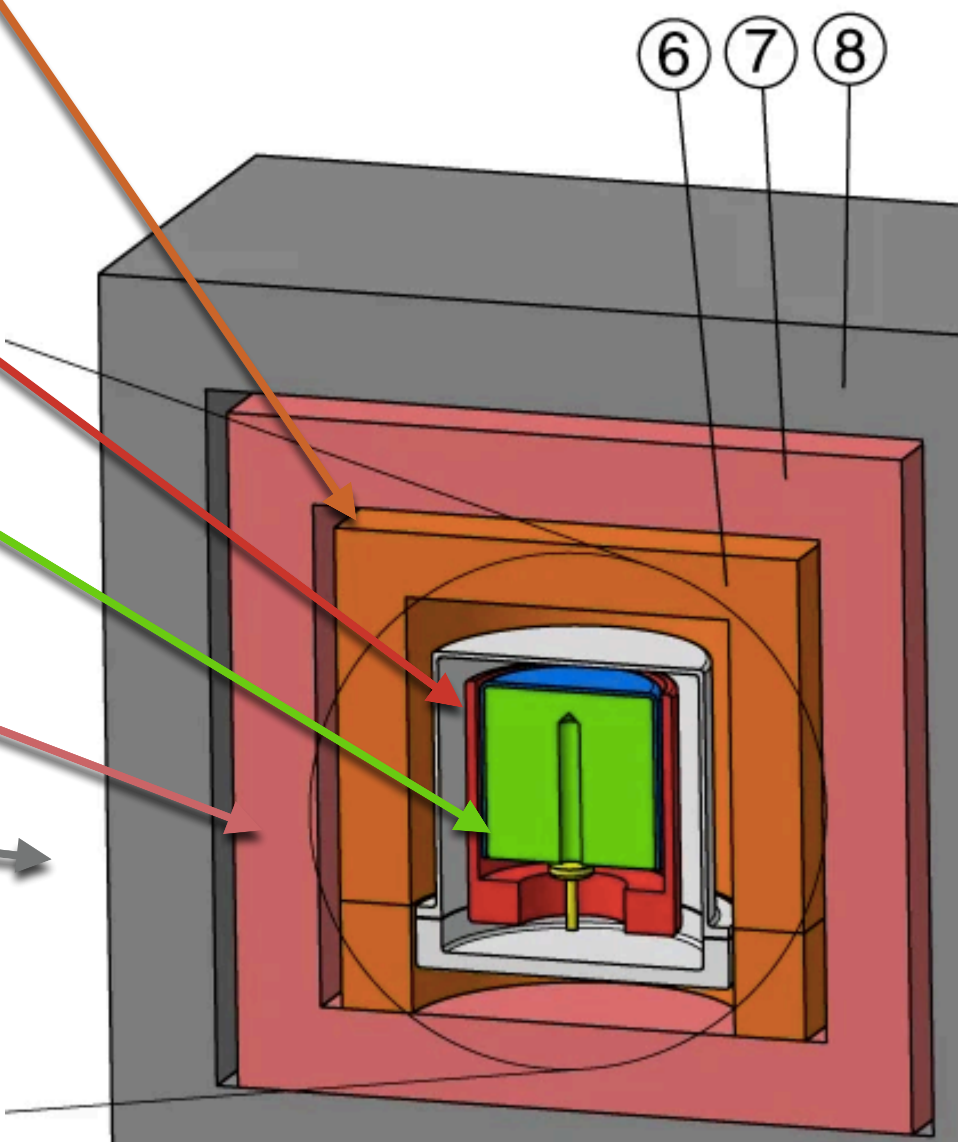


Cu shield

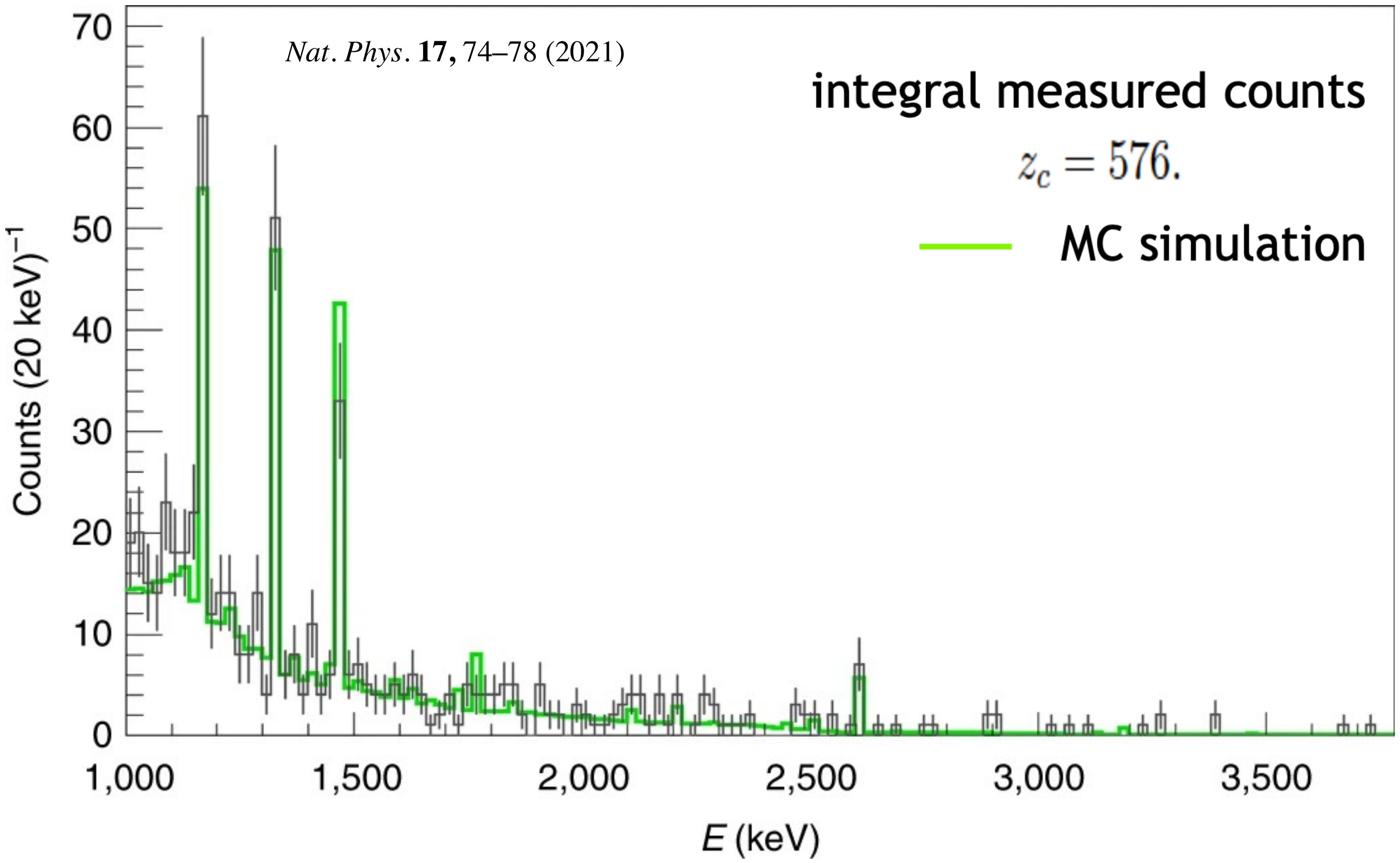


Pb shield





Measurement and MC validation



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integral measured counts

$z_c = 576.$

MC simulation

- the activities are measured for each component
- the MC simulation accounts for:
  1. emission probabilities and decay schemes for each radio-nuclide in each material
  2. photons propagation and interactions
  3. detection efficiencies.

The simulation describes 88% of the integral counts:

$$z_{b,ij} = \frac{m_i\, A_{ij}\, T\, N_{rec,ij}}{N_{ij}}, \qquad z_b = \sum_{i,j} z_{b,ij} = 506.$$

expected signal contribution

The expected signal of spontaneous radiation is obtained weighting the theoretical rate for the detection efficiencies:

- 10^{8} photons generated for each energy for each material
- efficiency functions are obtained by polinomial fits ➡ $\boxed{\epsilon_i(E) = \sum_{j=0}^{c_i} \xi_{ij} E^j}$
- the expected signal contribution is:

$$z_s(R_0) = \sum_i \int_{\Delta E} \frac{d\Gamma_t}{dE}\bigg|_i T \epsilon_i(E)\, dE = \frac{a}{R_0^3}$$

with a = $1.8\ 10^{-29}$ m^3

Results

Lower bound on R_0

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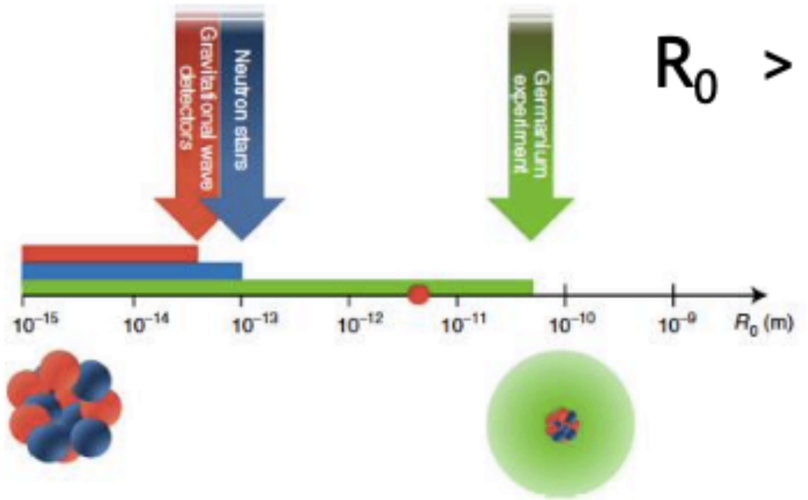


$R_0 > 0.54 \cdot 10^{-10}$ m

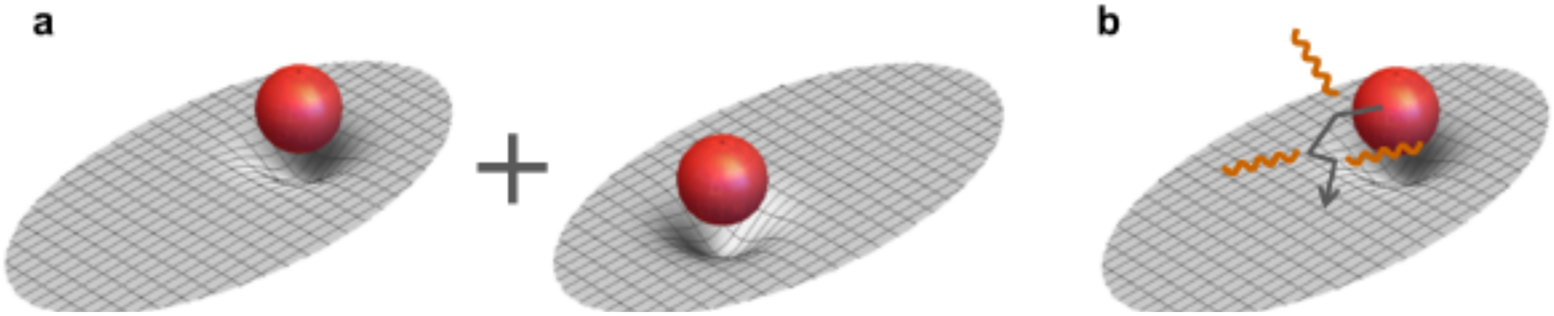
If R_0 is the size of the nucleus's wave function as suggested by Penrose, we have to compare the limit with the properties of nuclei in matter.

In a crystal $R^2_0 = \langle u^2 \rangle$ is the mean square displacement of a nucleus in the lattice, which, for the germanium crystal, cooled to liquid nitrogen temperature amounts to $R_0 = 0.05 \cdot 10^{-10}$ m

"Underground test of gravity-related wave function collapse". Nature Physics 1-5, (2020).

Diósi-Penrose (DP) Collapse model

Penrose: When a system is in a spatial quantum superposition, a corresponding superposition of two different space-times is generated. The superposition is unstable and decays in time. The more massive the system in the superposition, the larger the difference in the two space-times and the faster the wave-function collapse.



$$\Delta E_{\mathrm{DP}}(\mathbf{d}) = -8\pi G \int \mathrm{d}\mathbf{r} \int \mathrm{d}\mathbf{r}' \, \frac{\mu(\mathbf{r})\left[\mu(\mathbf{r}'+\mathbf{d}) - \mu(\mathbf{r}')\right]}{|\mathbf{r}-\mathbf{r}'|}$$

$$\tau_{\mathrm{DP}} = \frac{\hbar}{\Delta E_{\mathrm{DP}}}$$

● **Proton: m ≃ 10^{-27} Kg, R ≃ 10^{-15} m, τ_{DP}≃ 10^{6} years**

● **Dust grain: m ≃ 10^{-12} Kg, R ≃ 10^{-5} m, τ_{DP}≃ 10^{-8} s**

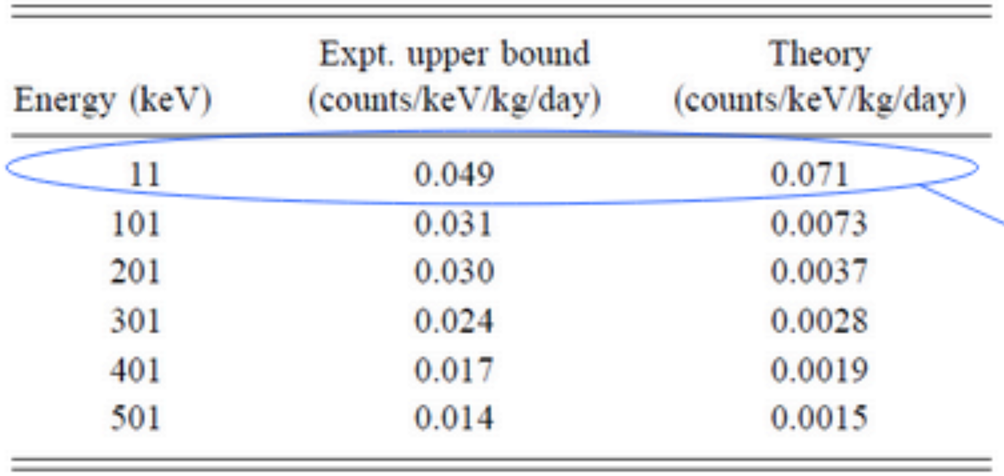
R₀ short-length cutoff: size of particle mass density

L. Diósi and B. Lukács, Ann. Phys. 44, 488 (1987), L. Diósi, Physics letters A 120 (1987) 377, L. Diósi, Phys. Rev. A 40, 1165–1174 (1989), R. Penrose, Gen. Relativ. Gravit. 28, 581–600 (1996), R. Penrose, Found. Phys. 44, 557–575 (2014).

*R. Penrose, Found. Phys. **44**, 557–575 (2014), R. Penrose, Gen. Relativ. Gravit. **28**, 581–600 (1996),* L. Diósi, Phys. Rev. A **40**, 1165–1174 (1989).

First limit from Ge detector measurement

Q. Fu, Phys. Rev. A 56, 1806 (1997) → upper limit on λ comparing with the radiation measured with isolated slab of Ge (raw data not background subtracted)
H. S. Miley, et al., Phys. Rev. Lett. 65, 3092 (1990)



Energy (keV)	Expt. upper bound (counts/keV/kg/day)	Theory (counts/keV/kg/day)
11	0.049	0.071
101	0.031	0.0073
201	0.030	0.0037
301	0.024	0.0028
401	0.017	0.0019
501	0.014	0.0015

TABLE I. Experimental upper bounds and theoretical predictions of the spontaneous radiation by free electrons in Ge for a range of photon energy values.

Comparison with the lower energy bin, due to the *non-relativistic constraint*

$$\frac{d\Gamma(E)}{dE} = c\,\frac{e^2\lambda}{4\pi^2 r_C^2 m^2 E} = (4)\cdot(8.29\ 10^{24})\cdot(8.64\ 10^4)\,\frac{e^2\lambda}{4\pi^2 r_C^2 m^2 E} \le \left.\frac{d\Gamma(E)}{dE}\right|_{ex}$$

4 valence electrons are considered
BE ~ 10 eV « energy of emitted γ ~ 11 keV
***quasi-free* electrons**

(Atoms / Kg)
in Ge

1 day

S. L. Adler, F. M. Ramazanoglu, J. Phys. A40, (2007) 13395
J. Mullin, P. Pearle, Phys. Rev. A90 (2014), 052119

$\lambda < 2 \times 10^{-16}\ s^{-1}$ non-mass proportional
$\lambda < 8 \times 10^{-10}\ s^{-1}$ mass proportional

S. L. Adler, A. Bassi and S. Donadi, J. Phys. A46, 245304 (2013)
S. Donadi, D. A. Deckert and A. Bassi, Annals of Physics 340, 70-86 (2014)
KP et al., Entropy 2017, 19(7), 319

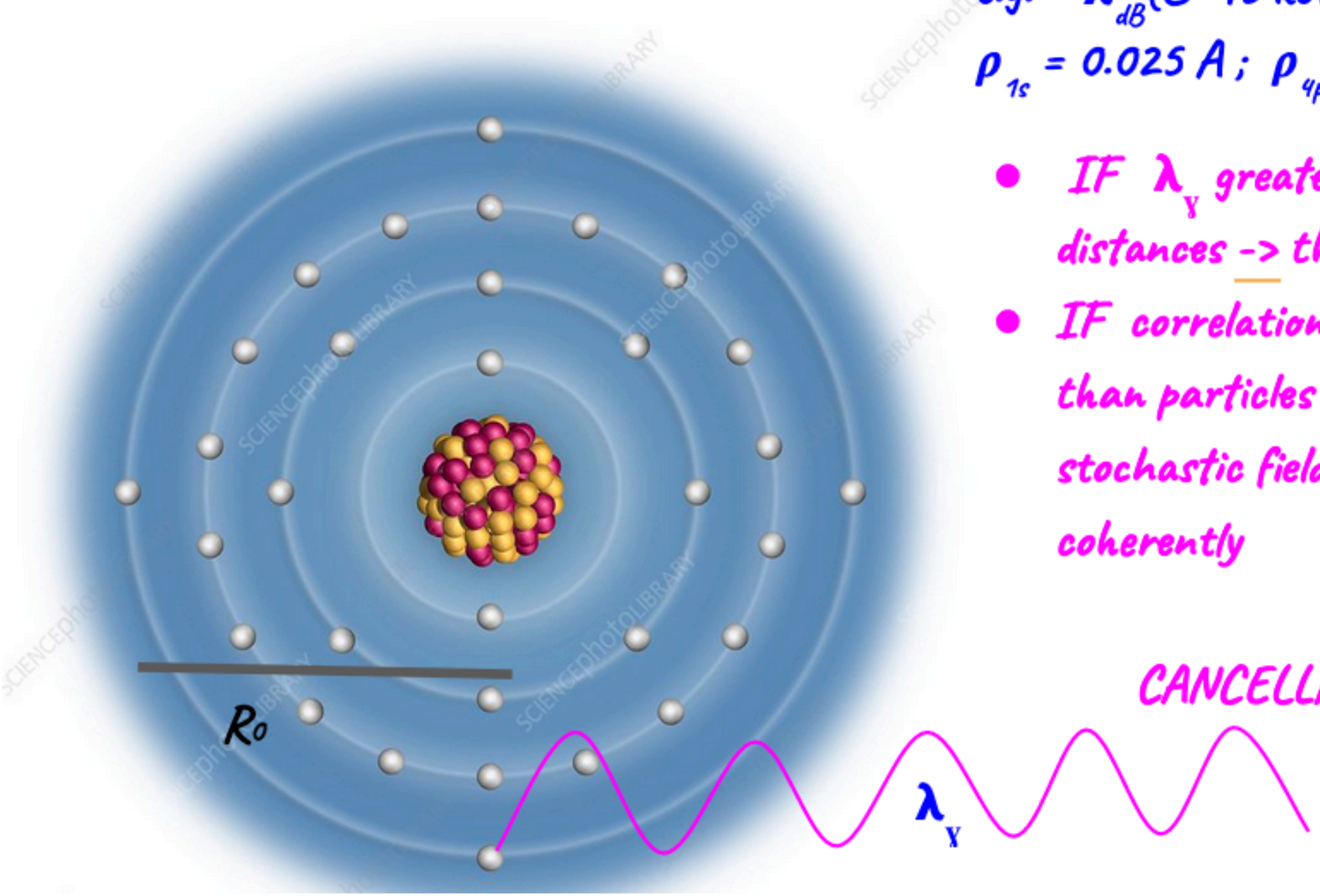
X-rays spontaneous radiation

the CSL

In the low-energy regime, the photon w.l. is comparable to the atomic orbits dimensions

e.g. $\lambda_{dB}(E=15\ keV) = 0.8\ A$

$\rho_{1s} = 0.025\ A$; $\rho_{4p} = 1.5\ A$

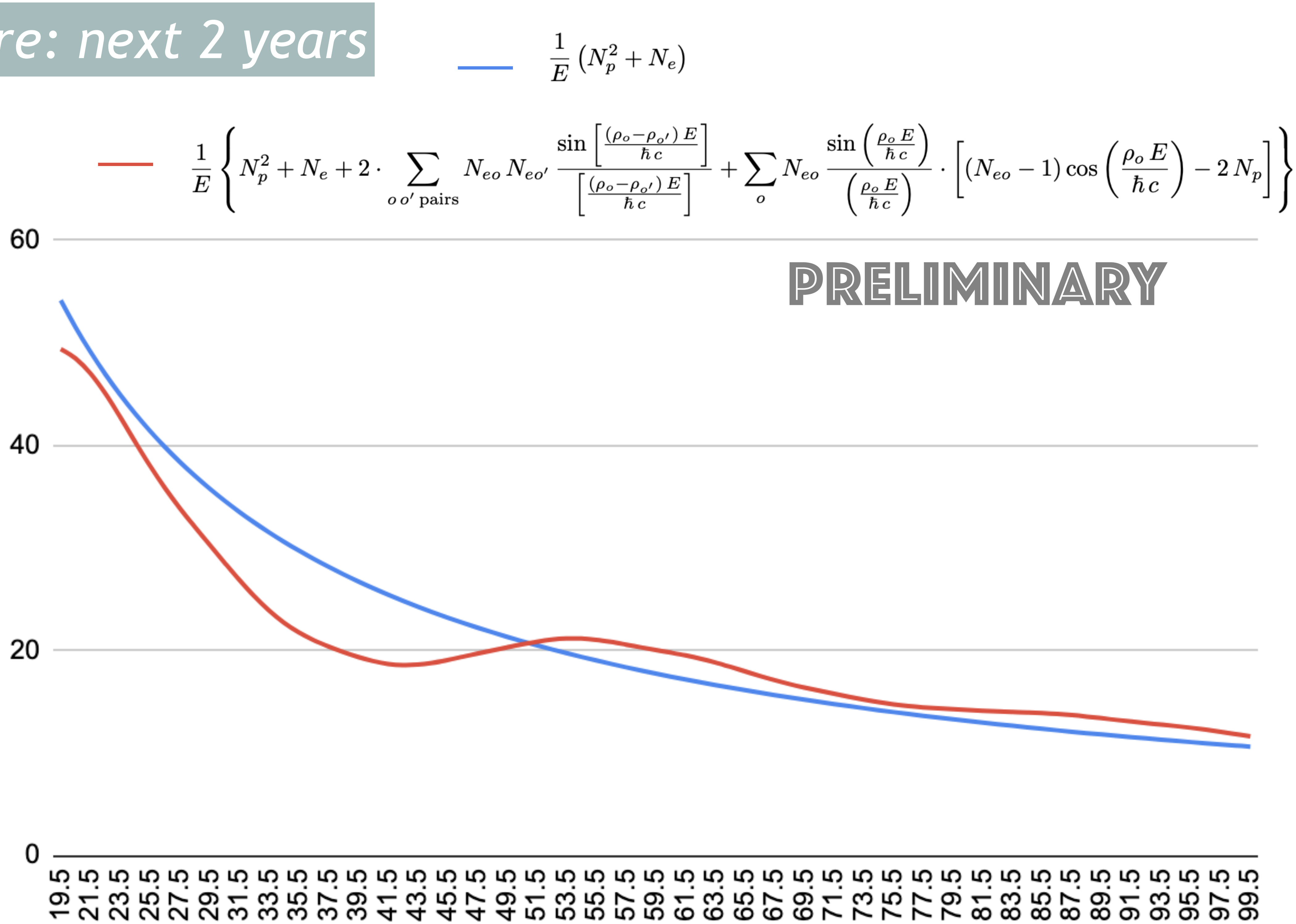


- IF λ_{γ} greater than particles distances -> they emit coherently
- IF correlation length greater than particles distances -> the stochastic field vibrates them coherently

CANCELLATION

Theory development - X-rays spontaneous radiation

Near future: next 2 years



$$\frac{1}{E}\left(N_p^2+N_e\right)$$

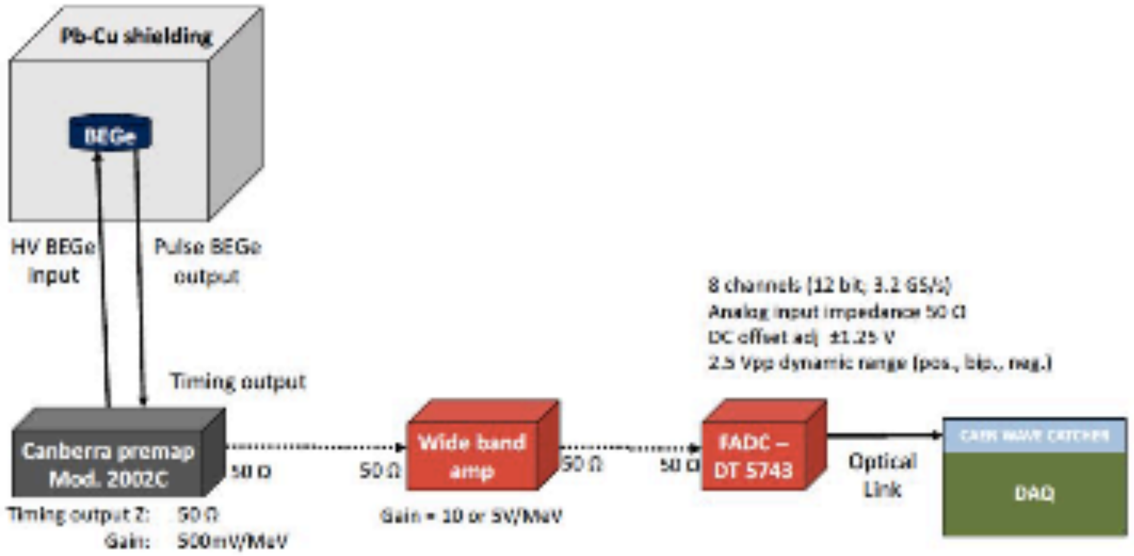
$$\frac{1}{E}\left\{N_p^2+N_e+2\cdot\sum_{o\,o'\text{ pairs}} N_{eo}\,N_{eo'}\,\frac{\sin\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}{\left[\frac{(\rho_o-\rho_{o'})\,E}{\hbar\,c}\right]}+\sum_o N_{eo}\,\frac{\sin\left(\frac{\rho_o\,E}{\hbar\,c}\right)}{\left(\frac{\rho_o\,E}{\hbar\,c}\right)}\cdot\left[(N_{eo}-1)\cos\left(\frac{\rho_o\,E}{\hbar\,c}\right)-2\,N_p\right]\right\}$$

At each energy the atomic structure influences the shape of the expected spontaneous emission spectrum

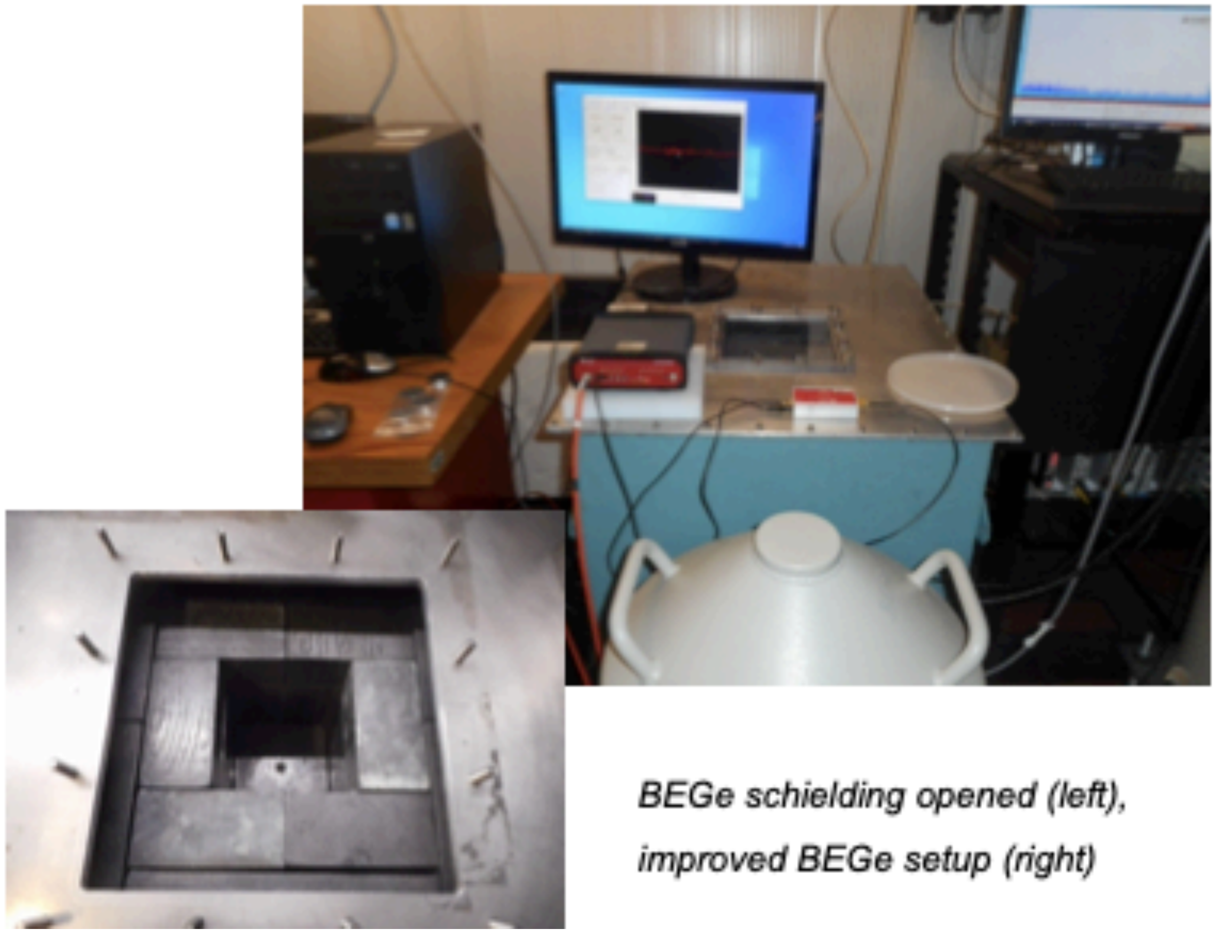
Spontaneous Radiation - BEGe detector

Mid future: next 4/5 years

Stronger limits possible at low energy with a Broad Energy (BE) Germanium



Block diagram of improved BEGe experimental apparatus



BEGe schielding opened (left), improved BEGe setup (right)

- Test setup equipped with low noise DAQ
- High insulation low noise amplifier
  - Gain towards lower energies