CosmicAntiNuclei

Investigation of wave function properties

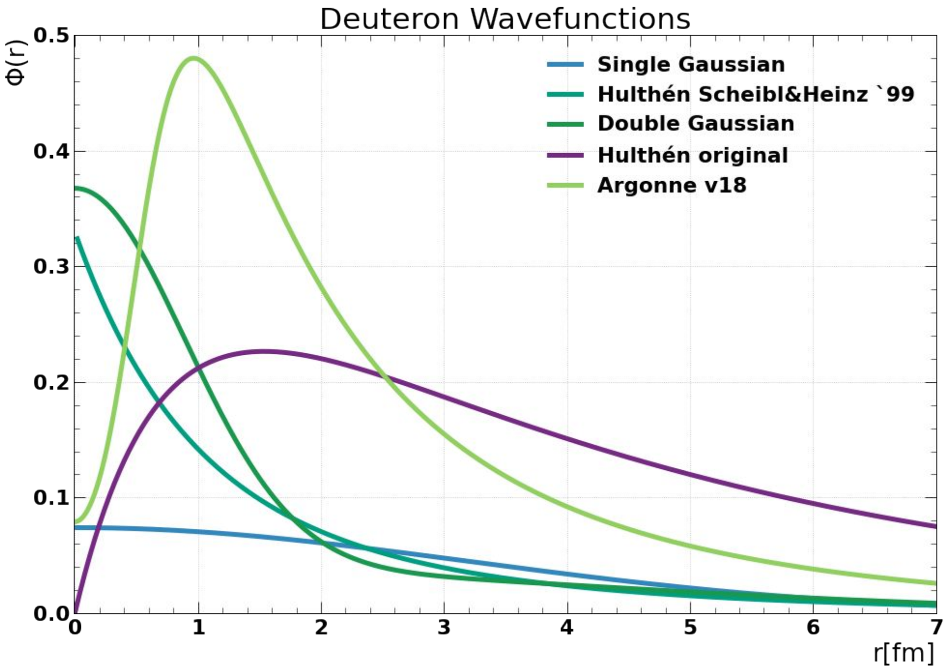
Maxi Horst

The deuteron wave function

Overview

There are several possible wave functions for the deuteron

- Simplistic:
  Single Gaussian
- Experimental data ('50s):
  Double Gaussian
- From *pion field theory* ('50s):
  Hulthén (important note later!)
- From modern χ_{EFT}:
  Argonne ν_{18}



The deuteron wave function

Calculation of properties

Since the deuteron is a mixture of S- and D-State one can express the wavefunction

$$\phi_{\mathrm{d}}(\vec{r}) = \frac{1}{\sqrt{4\pi}\,r}\left[u(r) + \frac{1}{\sqrt{8}}w(r)\,S_{12}(\hat{r})\right]\chi_{1m}$$

u(r) = S-Wave, w(r) = D-Wave
S_{12} = Spin tensor, χ_{1m}= Spinor

The most important observables for the wavefunction are

- Radius r_d

$$r_d = \frac{1}{2}\left\{\int_0^\infty r^2\left[u^2(r) + w^2(r)\right]dr\right\}^{1/2}$$

- Quadrupole moment Q_d

$$Q_d = \frac{1}{20}\int_0^\infty r^2 w(r)\left[\sqrt{8}u(r) - w(r)\right]dr$$

- Magnetic moment μ_d

$$\mu_d = \mu_s - \frac{3}{2}(\mu_s - \frac{1}{2})P_D$$

μ_s: $\mu_p + \mu_n$

The deuteron wave function

Problems with Hulthén

- When calculating the deuteron size we realized that the Hulthén give r_d~52 fm

$$N \cdot (e^{-\alpha r} - e^{-\beta r})$$

The deuteron wave function

Problems with Hulthén

- When calculating the deuteron size we realized that the Hulthén give r_d~52 fm

$$\phi_d(\vec{r}) = \frac{1}{\sqrt{4\pi}r}\left[u(r) + \frac{1}{\sqrt{8}} w(r) S_{12}(\hat{r})\right] \chi_{1m}$$

$u(r)$ = S-Wave, $w(r)$ = D-Wave
S_{12} = Spin tensor, χ_{1m}= Spinor

- The wave function we have been using is **not** **ϕ**(r) but *u(r)*
(= harmonic oscillator wave function to the Hulthén potential)

- The wave function used in Scheibl & Heinz and Kachelrieß et al. is the actual deuteron wave function

$$\varphi_d(r) = \sqrt{\frac{ab(a+b)}{2\pi(a-b)^2)}} \frac{e^{-ar} - e^{-br}}{r}$$

The deuteron wave function

Properties of wavefunctions

- For the Gaussian and Double Gaussian we can only calculate the size since we only have the S-Wave (magnetic moment no D-Wave contribution $\mu_d = \mu_p + \mu_n$)

	Radius r_d [fm]	Quadrupole moment Q_d [fm^2]	Magnetic moment μ_d [μ_N]
Gaussian	1.9595	–	(0.88)
Double Gaussian	1.9311	–	(0.88)
χEFT	1.954	0.2681	0.847177
Hulthén	1.9809	0.292166	0.84706
Experiment	1.97535(85)	0.2859(3)	0.85744

The deuteron wave function

Future plans

- Our predictions using the “Hulthén” wave function (without 1/r) are not usable
- Bhawani will calculate the Wigner function for the real Hulthén and also take another look at χ_{EFT} since the way we did it had some flaws (Fitting before doing the integration)
- That way we will have predictions for 4 wave functions
- We can at least compare the same wave functions as there are in the B_2 predictions and see if they are equal to the MC predictions

To-do list

- Me:
  - Start implementing the new source model into EPOS
  - Use new fit to protons to produce new spectra (100M statistics has finally finished!)
  - Add deuterons to the antideuteron spectra
- Sushanta:
  - Run more Pythia events (if not done yet)
  - Implement HM trigger
  - Make a plot with the multiplicity distribution of Pythia (before and after HM trigger)

Backup

Functions used for calculations

$$\phi_d(r) = \frac{e^{-\frac{r^2}{2d^2}}}{(\pi d^2)^{3/4}},$$

d=3.2 fm

$$\phi_d(r) = \pi^{-3/4}\left[\frac{\Delta^{1/2}}{d_1^{3/2}} e^{-r^2/(2d_1^2)} + e^{i\alpha}\frac{(1-\Delta)^{1/2}}{d_2^{3/2}} e^{-r^2/(2d_2^2)}\right]$$

d1=3.979 fm, d2=0.89, Δ=0.581

$$u(r) = N \cdot (e^{-\alpha r} - e^{-\beta r})$$

$$w(r) = N \cdot \eta(1 - e^{-\tau r})^5 \cdot e^{-\alpha r} \cdot \left(1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2}\right)$$

α=0.2316fm^{-1}, β=1.61 fm^{-1}, τ=1, η=0.026

$$u(r) = r^{3/2} \sum_{i=1}^{N} A_i \exp(-a_i r^3)$$

$$w(r) = r \sum_{i=1}^{N} B_i \exp(-b_i r^3).$$

i	A_i	a_i	B_i	b_i
1	-2.31737065809	0.35274596179	-0.16442140495	4.27551981801
2	0.02659110800	0.00732058085	0.02868983216	0.02949360502
3	-0.28877337807	5.38000986025	0.00074392415	0.00062004296
4	0.99922786191	0.36067988379	0.05707754763	0.09089766773
5	0.11094070754	0.06921749879	0.01167238661	0.00953758791
6	0.01077521733	0.00225041184	0.00370587438	0.00281824630
7	0.00274077964	0.00052965934	0.08509685731	0.30274669392
8	0.65409540449	0.26723780768		
9	0.99185931871	0.41270669565		
10	0.05693893648	0.02247865903		

Argonne v_{18} Fit
Parameters

$$u(r) = r^{3/2} \sum_{i=1}^{N} A_i \exp(-a_i r^3)$$
$$w(r) = r \sum_{i=1}^{N} B_i \exp(-b_i r^3).$$

N	u(r)	w(r)
3	2.24E-05	4.46E-06
4	3.26E-06	3.58E-06
5	6.45E-07	3.58E-06
6	4.27E-07	3.58E-06
7	4.24E-07	4.30E-07
8	4.25E-07	4.31E-07
9	4.10E-07	4.32E-07
10	4.03E-07	4.32E-07
11	4.04E-07	4.31E-07

χ^2/ndf for N parameters

i	A_i	a_i	B_i	b_i
1	-2.31737065809	0.35274596179	-0.16442140495	4.27551981801
2	0.02659110800	0.00732058085	0.02868983216	0.02949360502
3	-0.28877337807	5.38000986025	0.00074392415	0.00062004296
4	0.99922786191	0.36067988379	0.05707754763	0.09089766773
5	0.11094070754	0.06921749879	0.01167238661	0.00953758791
6	0.01077521733	0.00225041184	0.00370587438	0.00281824630
7	0.00274077964	0.00052965934	0.08509685731	0.30274669392
8	0.65409540449	0.26723780768		
9	0.99185931871	0.41270669565		
10	0.05693893648	0.02247865903		

Paper references for the experimental values

Deuteron size:

https://journals-aps-org.eaccess.ub.tum.de/prl/pdf/10.1103/PhysRevLett.80.468

Quadrupole moment:

https://www.sciencedirect.com/science/article/abs/pii/037594748390516X

Magnetic moment:

https://physics.nist.gov/cgi-bin/cuu/Value?mudsmun (CODATA)