Dual Shapiro steps in a Josephson junctions array

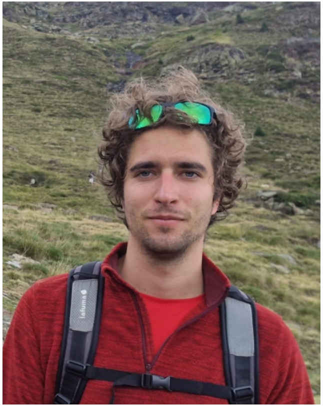
Theoretical elements and experimental evidence

Nicolò Crescini, Institut Néel - CNRS

cQED@Tn – October 2022

Introducing the Grenoble team

This talk is presented on behalf of the Bloch Oscillations working group.



Samuel Cailleaux

N.Roch

O.Buisson

K.Murch

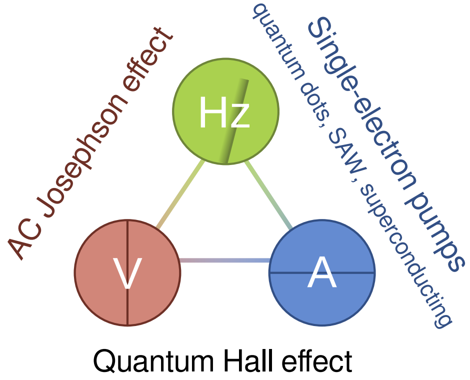
D.Basko

W.Guichard

https://arxiv.org/abs/2207.09381

The fundamental units of quantum electrodynamics

The quantum metrology electrical triangle.



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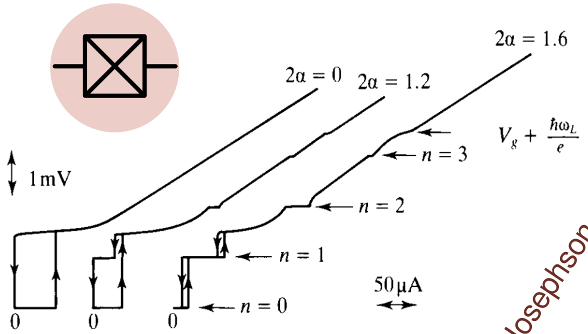
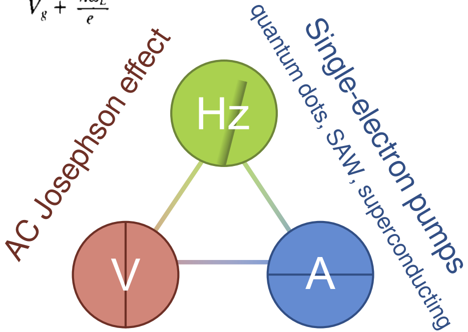


$$\Delta V = n\Phi_0 \cdot f$$

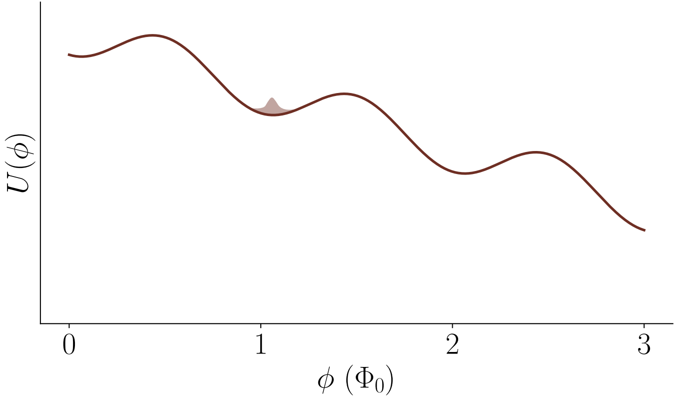
[1]M. Tinkham - Introduction to Superconductivity



Quantised transport

Shapiro steps — voltage plateaux

The phase dynamics of a current biased Josephson junction is analogue to the one of a particle in a washboard potential, tilted by a current..



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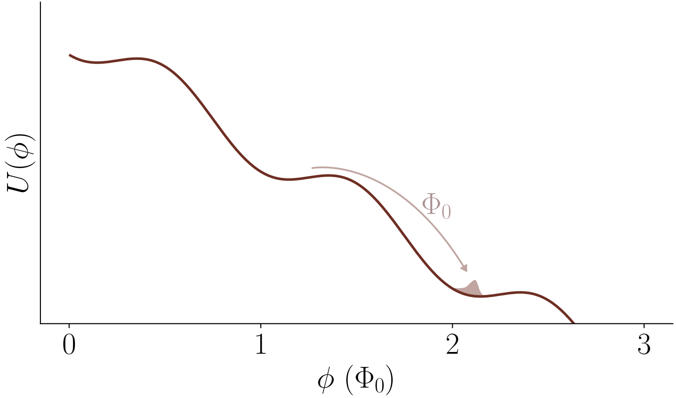

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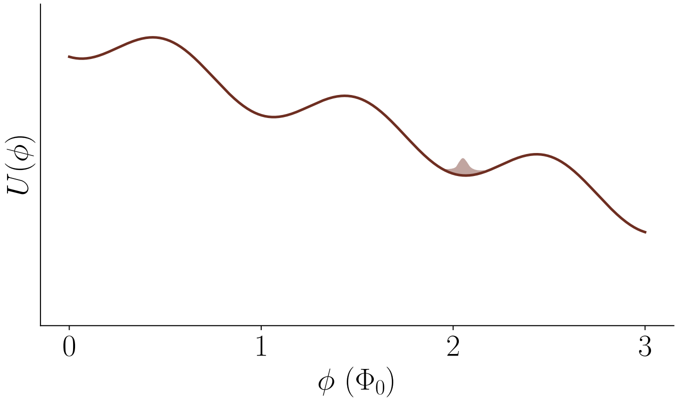

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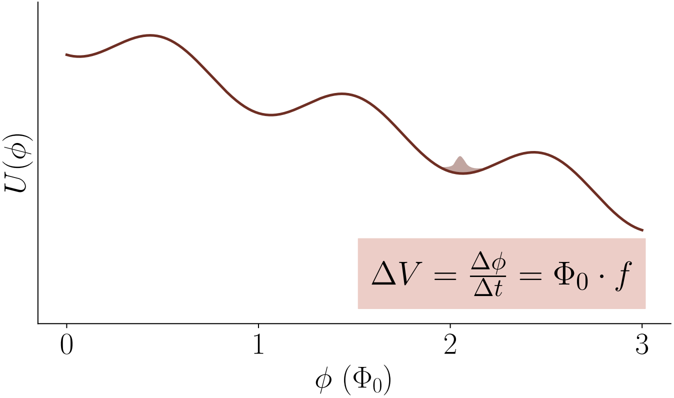
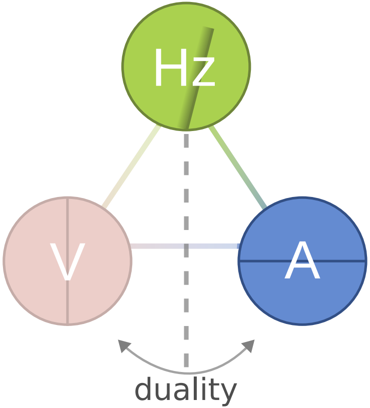


Quantised transport

The quantum metrology electrical triangle.

- $E_J/E_C \gg 1$
- Classical (localised) ϕ
- Tilted washboard $U(\phi)$
- Fluxons transport
- Shapiro steps



- $E_J/E_C \sim 1$
- Classical (localised) Q
- First Bloch band $U(Q)$
- Cooper pairs transport
- Dual Shapiro steps

$$[\phi, Q] = 2ie$$

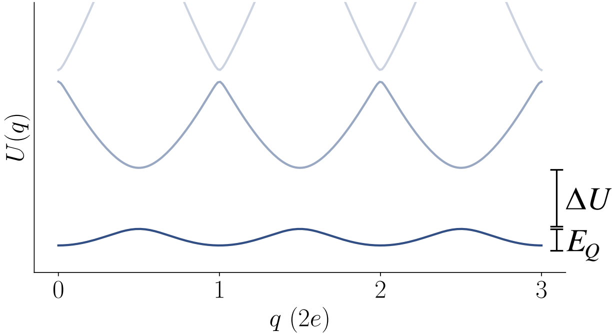
For $E_J \simeq E_C$, the Hamiltonian is

$$H = 4E_C\,(Q/2e)^2 - E_J\cos(\phi),$$

and can be recast to Bloch bands. If charge fluctuations are small enough, the dynamics can be restricted to the first band, obtaining

$$H = \sum_s U^{(s)}(q) \sim E_Q\cos(\pi Q/e).$$

Bloch bands



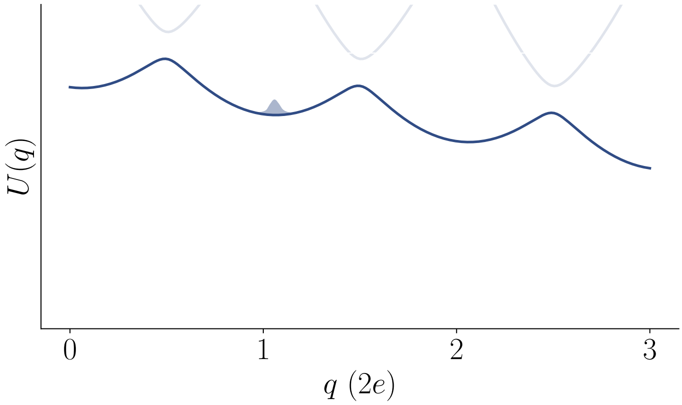
It is possible to realise a potential periodic in charge with a small Josephson junction.

K.K. Likharev, A.B. Zorin - J. Low Temp. Phys. 59, 347–382 (1985)

Quantised transport

Dual Shapiro steps — current plateaux

The $\cos(Q)$ potential is dual to the washboard's $\cos(\phi)$, and the tilting is provided by a voltage bias instead of a current one.



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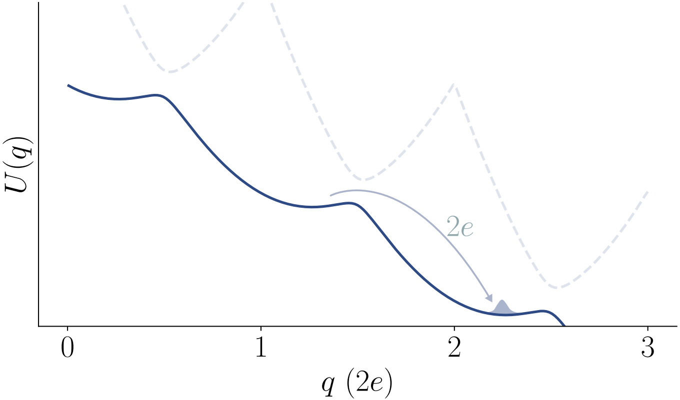

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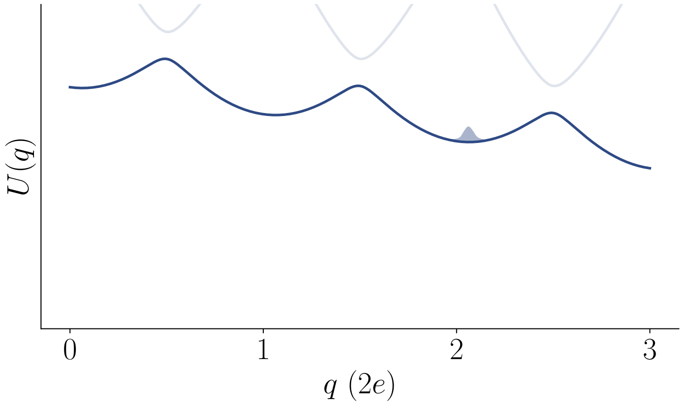

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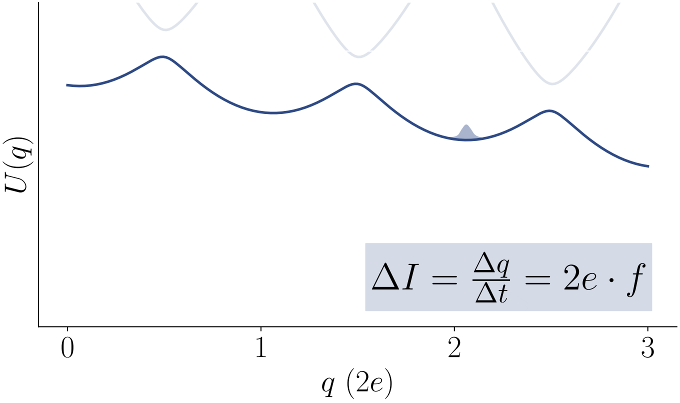


From theory to a scheme

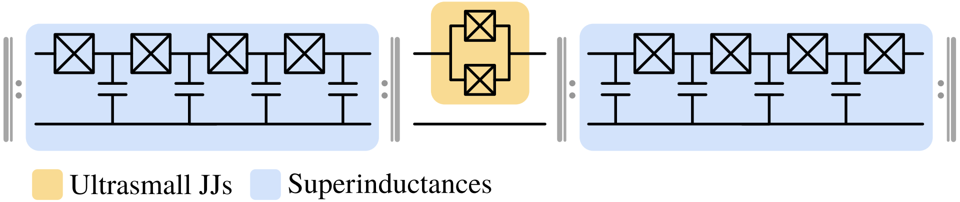
We first devise a scheme that satisfies the theoretical constraints.

Theory

- Bloch bands dynamics
- Localised charge wavefunction
- Negligible thermal fluctuations

Experiment

- Ultrasmall junction, $E_J/E_C \simeq 1$
- Environmental impedance $> R_q$
- Low temperatures, $T \simeq 20\,\mathrm{mK}$

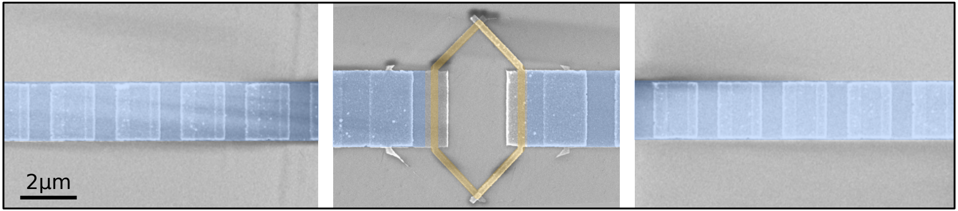


Guichard and Hekking, Phys. Rev. B 81 (2010) + Arndt, Roy, and Hassler Phys. Rev. B 98 (2018)

From a scheme to a device: the Bloch array

We realise the scheme with a superconducting circuit entirely based on Josephson junctions.

The devices are patterned in a single lithographic step, and to reduce ground capacitances we employ a low-ϵ fused-silica wafer.



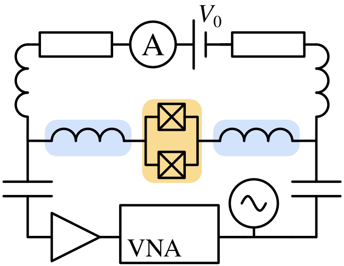
Additional parameters for number crunchers

Ultrasmall junction: $E_J/E_C = 1.6$, $E_Q \simeq 2.5\,\mathrm{GHz}$, $\Delta U \simeq 7.9\,\mathrm{GHz}$, $I_c \simeq 10\,\mathrm{nA}$

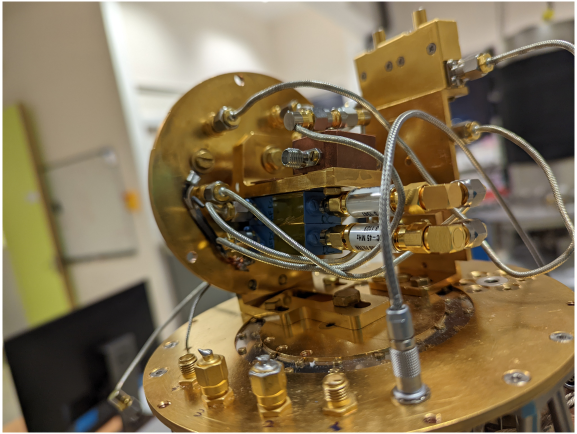
Superinductances: $Z_\mathrm{a} = 8.0\,\mathrm{k\Omega}$, $L_\mathrm{a} = 3.3\,\mu\mathrm{H}$, $E_J/E_C = 250$, $N_\mathrm{a} = 1750$

Down to a measurement setup

Our experimental apparatus allows for simultaneous DC and microwave measurements.



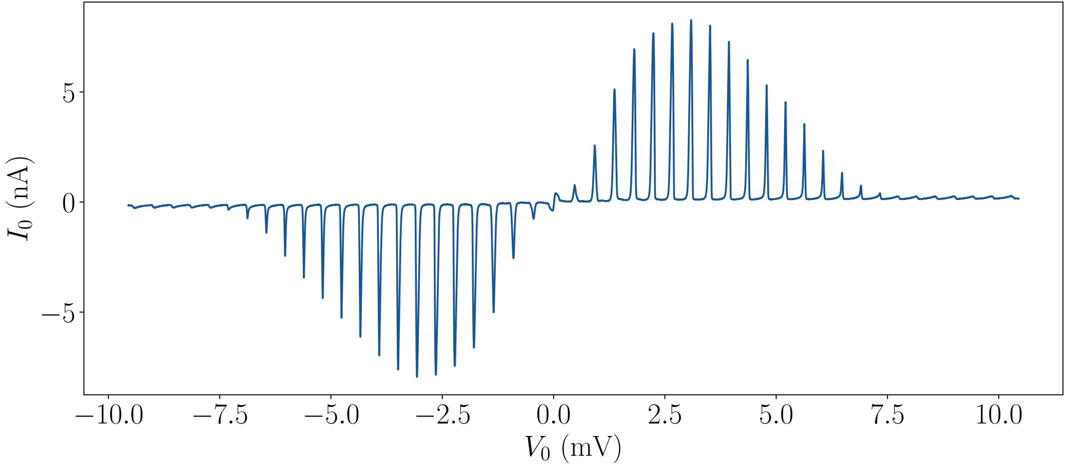
- Low noise RF and DC lines
- Off-chip resistors and bias-tees
- 23 mK working temperature
- Qcodes DAQ



Time to measure!

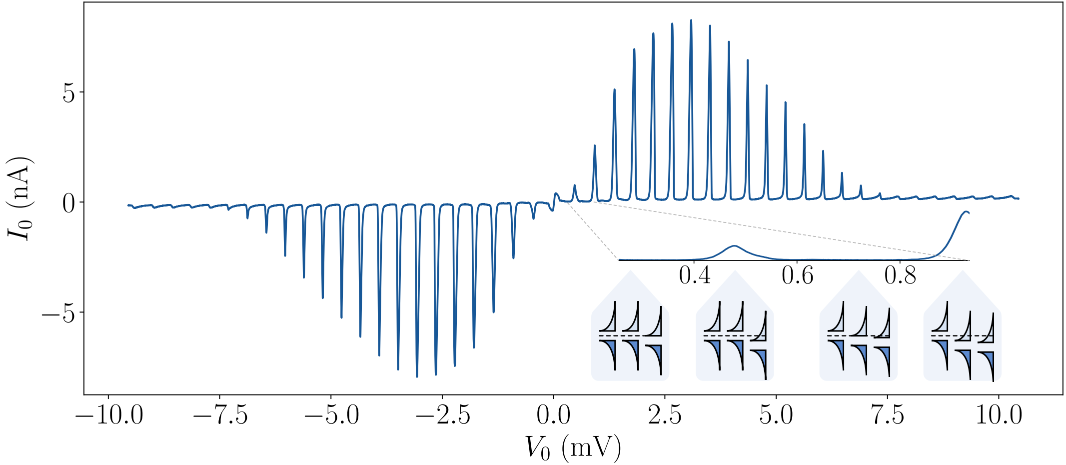
IV characteristics

Probing the current flowing through the device with a low applied voltage.



IV characteristics

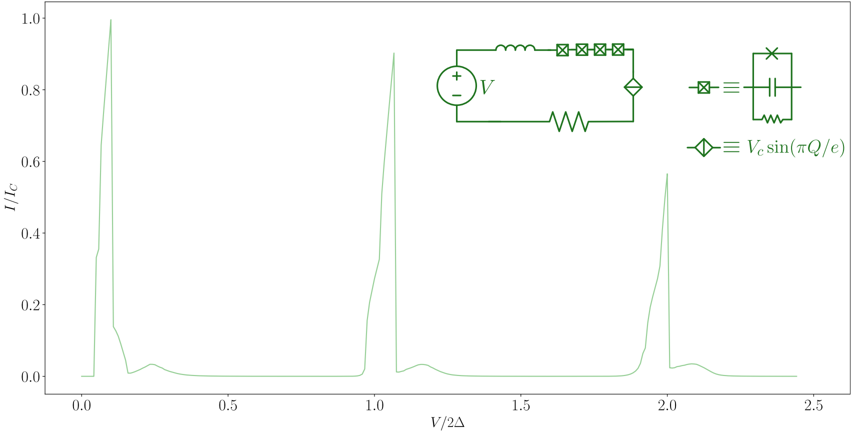
Probing the current flowing through the device with a low applied voltage.



A toy model of the Bloch array

RCSJ simulations confirm the consistency of what we experimentally observe.

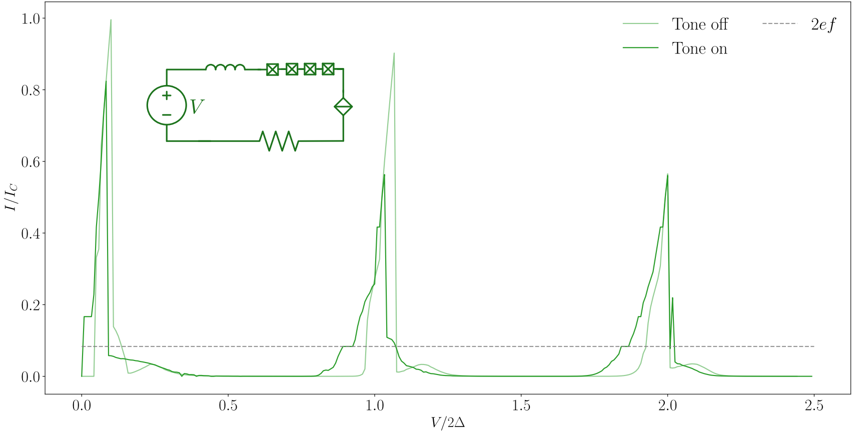
$$V = L\ddot{Q} + R\dot{Q} + V_c \sin\left(\frac{\pi}{e} Q\right) + \sum_i \dot{\phi}_i$$



A toy model of the Bloch array

We push the model a bit further to know what to expect.

$$V + V_{ac} = L\ddot{Q} + R\dot{Q} + V_c \sin\left(\frac{\pi}{e}Q\right) + \sum_i \dot{\phi}_i$$



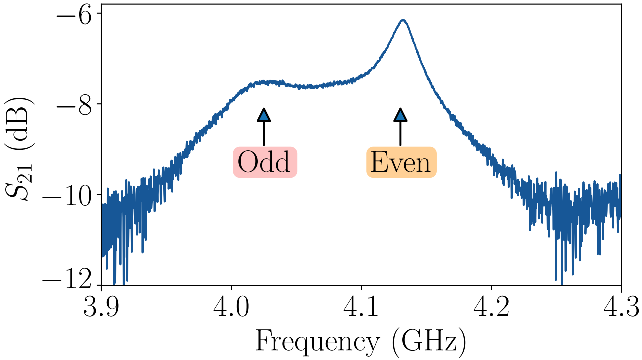
Microwave spectroscopy

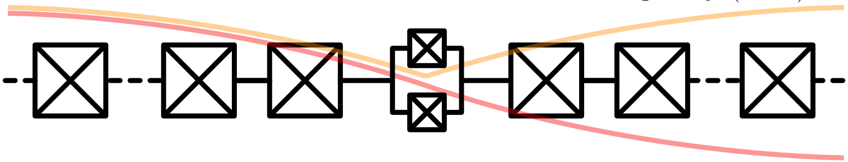
Mode doublets of the Bloch array.

Symmetry matters

The odd modes of the array are coupled to the center, i.e. to the ultrasmall junction.

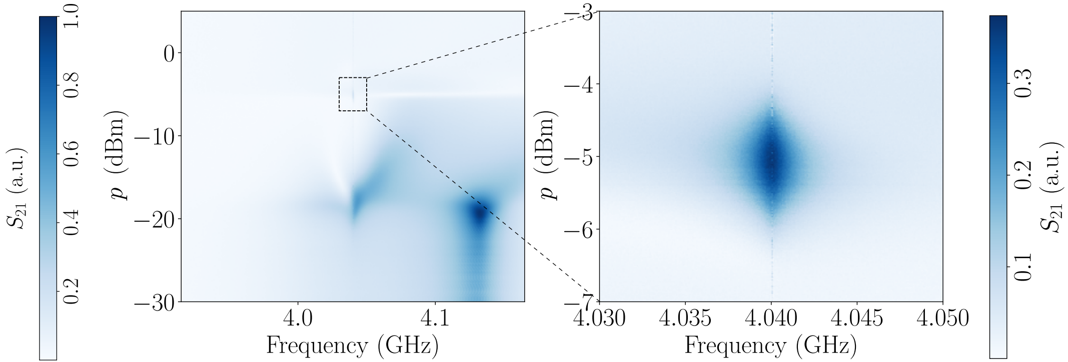
Léger, Roch *et al.* Nat.Comm. 2019
arxiv.org/abs/1910.08340





Microwave properties with tone irradiation

We observe the emergence of a mode synchronous with the tone.



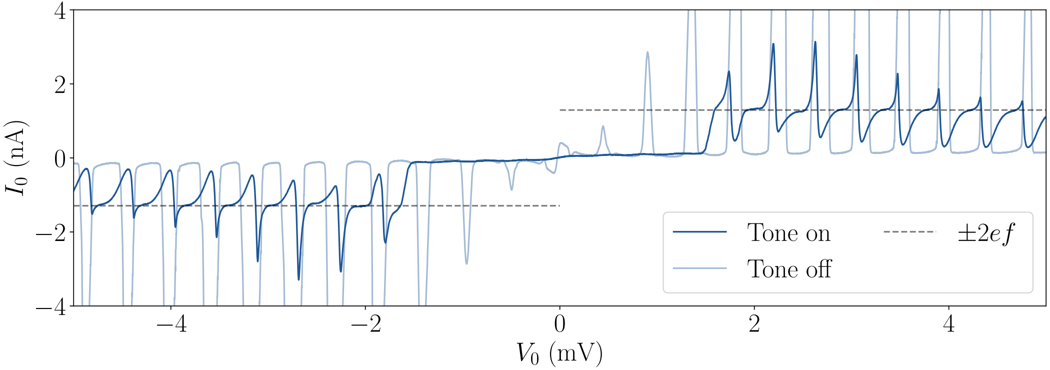
Why is synchronicity relevant?

Bloch oscillations are the phenomenon dual to the AC Josephson effect which need to synchronise to the external tone in order to give steps.

IV characteristics with tone irradiation

The corresponding IV curve features flat steps with current $2ef$.

We measure the IV curve with the tone parameters of the Bloch mode.



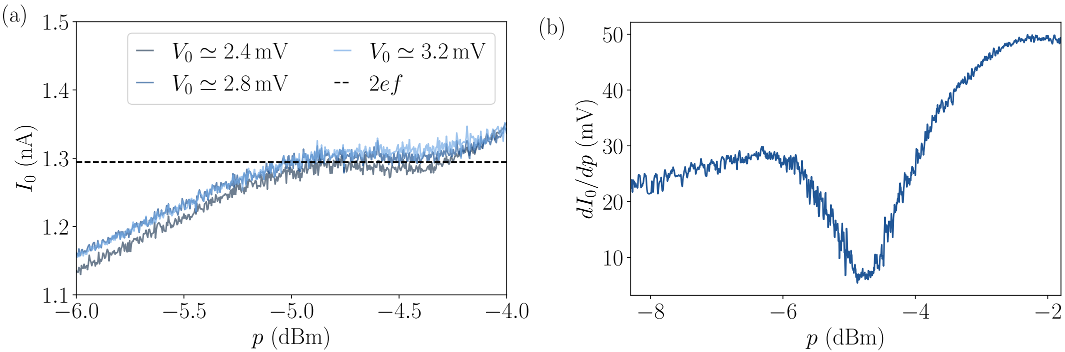
IV characteristic under microwave irradiation

With $f = 4.04$ GHz and $p = -5.0$ dBm, we get plateaux of current $2ef$ before every peak.

Power dependence

There is a clear singularity of the system corresponding to the mode power.

The power dependence of the step is studied by measuring the step current as a function of the pump power, and by modulating the pump amplitude.



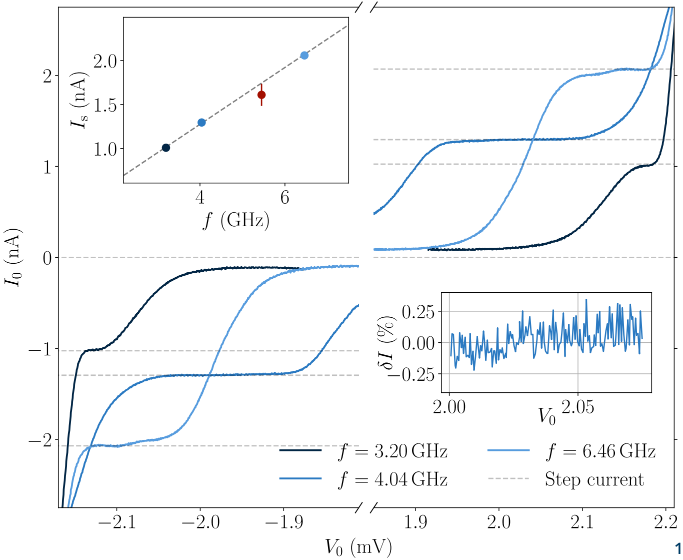
Both the measurements indicate that at the power where we observe the Bloch mode there is a singularity of the system.

Frequency dependence of the current plateaux

Using different array modes we verify the expected scaling of current vs. frequency.

Metrology

The measurement of the step current $2e \cdot f$ at different f relates frequency to current through the Cooper pair charge $2e$. This can be used to get a quantum metrological definition of the Ampére.



Conclusions and outlook

Take home messages of what we understood.

Bloch array with microwaves OFF

- Observation and modelisation of 2Δ-spaced peaks
- Characterisation of the microwave dynamics
- Simultaneous DC and microwave study of the sample

Bloch array with microwaves ON

- Detection of a microwave mode synchronous with the tone
- Corresponding emergence of flat $2ef$ current plateaux in the IV curve
- Same phenomenology observed at four different frequencies

What's next?

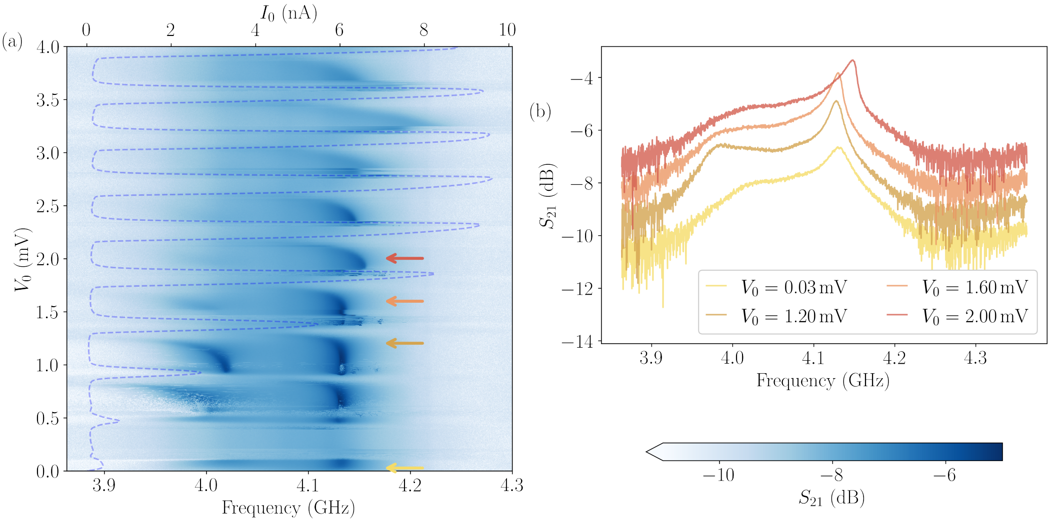
After this first evidence, the phenomenon needs to be studied. A systematic analysis of the various parameters will allow to master the Bloch array and improve it. Plus: there are many interesting behaviours of the device which are currently under study.

Preprint online: https://arxiv.org/abs/2207.09381

Thank you for your time!

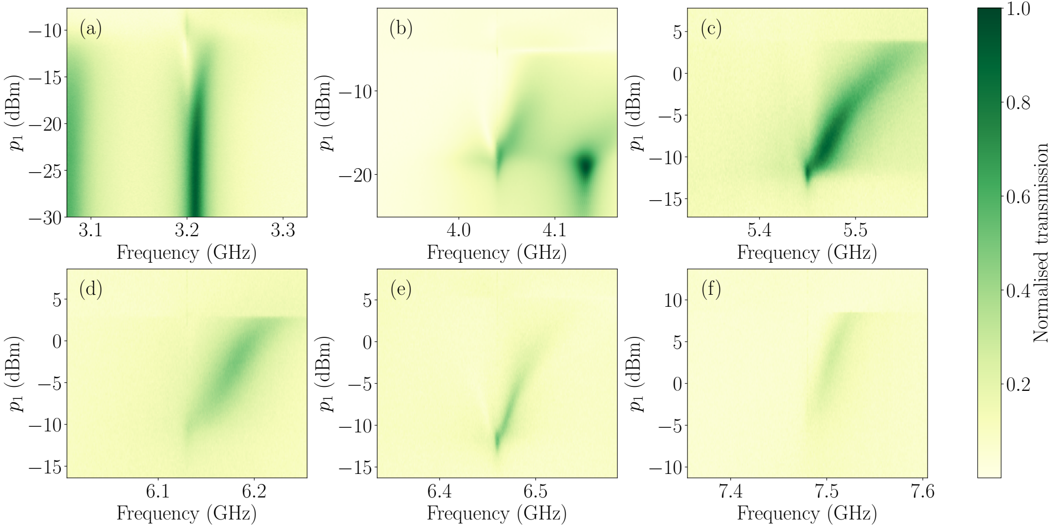
Backup slide 1: microwave spectroscopy with DC bias

We measure the GHz-frequency properties of the sample with a DC bias.



Backup slide 2: modes used for the steps

The central junction is a SQUID.



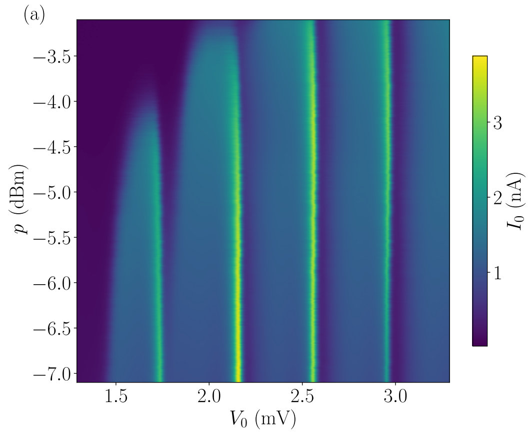
Backup slide 3b: power dependence

Full steps IV as a function of the tone's power.

The effect of higher power is to gradually reduce the peaks height, and therefore the step, starting from the one closer to zero voltage.

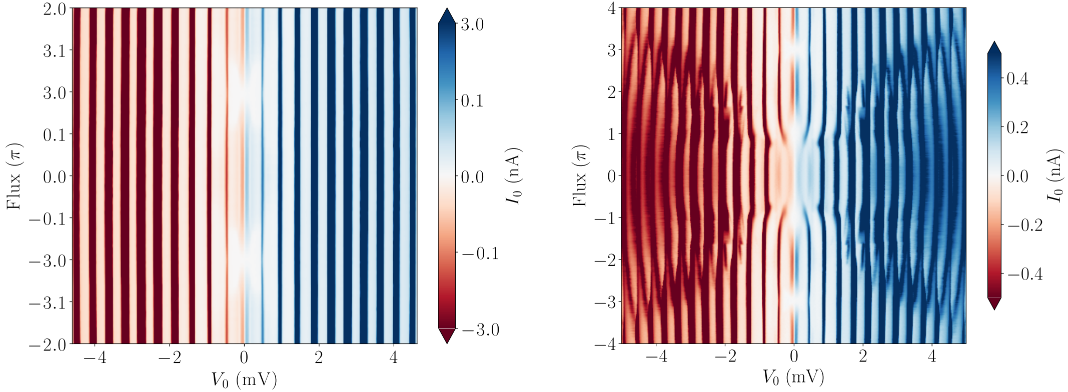
On the to-do list

This observation suggests that it would be better to observe the same effects with lower power, not to compromise the array's properties.



Backup slide 4a: flux dependence

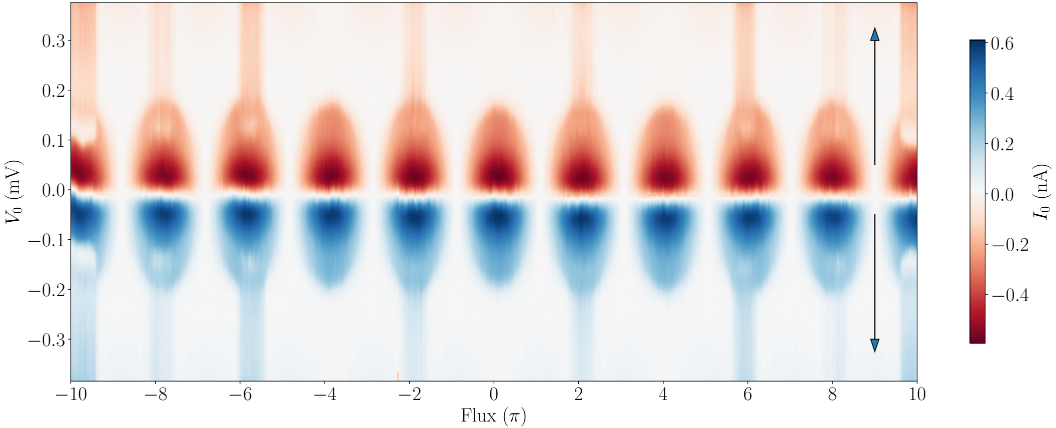
The central junction is a SQUID, so we expect some flux dependence.



Hofstadter's butterfly?

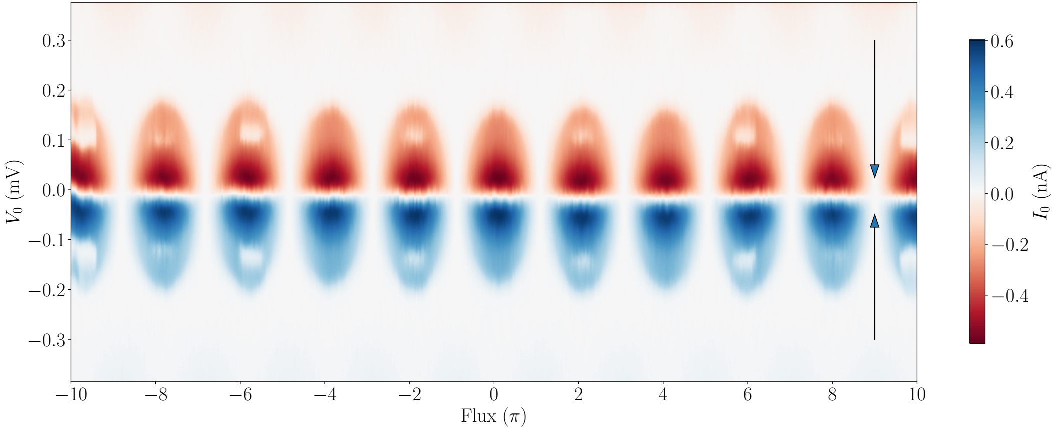
Backup slide 4b: flux dependence

Low voltage currents have some 2π and some 4π dependence.



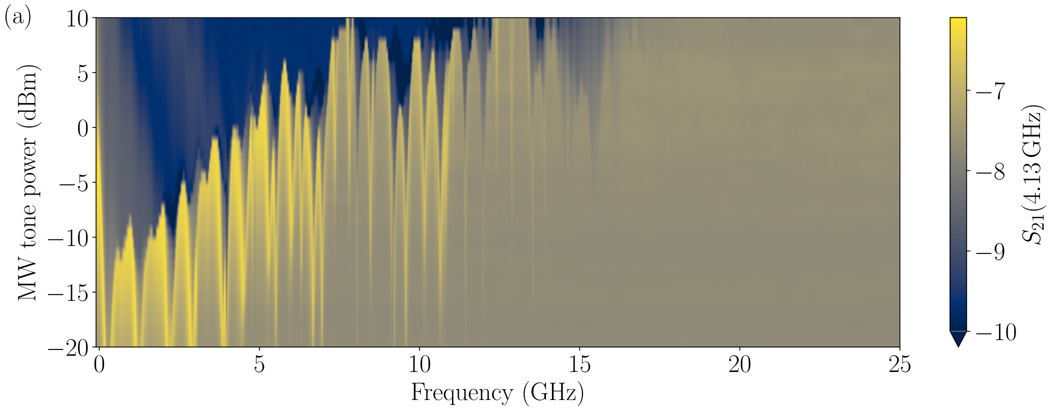
Backup slide 4b: flux dependence

Low voltage currents have some 2π and some 4π dependence.



Backup slide 5: two-tones spectroscopy

We use two-tones spectroscopy to characterise the FSR and plasma frequency of the array.



Backup slide 6: RCSJ simulations

Details on the performed simulations.

The IV curve is the numerical simulation of a Bloch array comprising:

- $N\times$ RCSJ junctions in series
- non-linear capacitor V_c
- an inductance L
- a damping resistor R

Total current though each junction

where ϕ_i is the superconducting phase drop on the i^{th} junction.

$$\begin{aligned} I_{\text{tot}} &= I_C + I_R + I_J \\ I_C &= C\ddot{\phi}_i, \\ I_R &= s(\dot{\phi}_i)\frac{1}{R_N}\dot{\phi}_i, \\ I_J &= \left(1 - s(\dot{\phi}_i)\right) I_c \sin\left(\frac{\phi_i}{\phi_0}\right). \end{aligned} \tag{1}$$

And we solve the equation of motion

$$V = L\ddot{Q} + R\dot{Q} + V_c \sin\left(\frac{\pi}{e}Q\right) + \sum_i \dot{\phi}_i$$

With the following parameters

- $N = 4$
- $dt = 0.02\,RC$
- $L = 5\,\mu\text{H}$
- $R = 1\,\text{k}\Omega$
- $V_c \simeq 50\,\mu\text{V}$