CNR IFN Istituto di Fotonica e Nanotecnologie

Critical Quantum Sensing

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National Research Council

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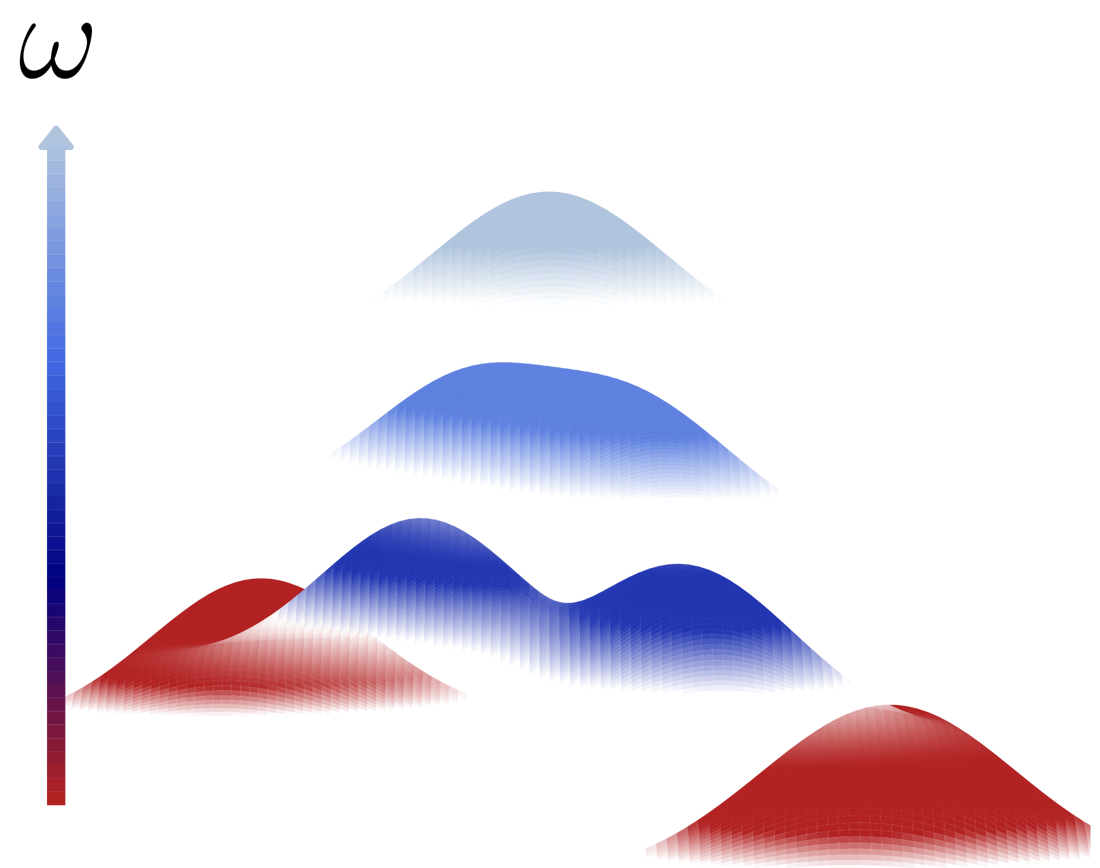
04/10/2022

Outline

Finite-component phase transitions ⟷ **Quantum sensing**



Outline

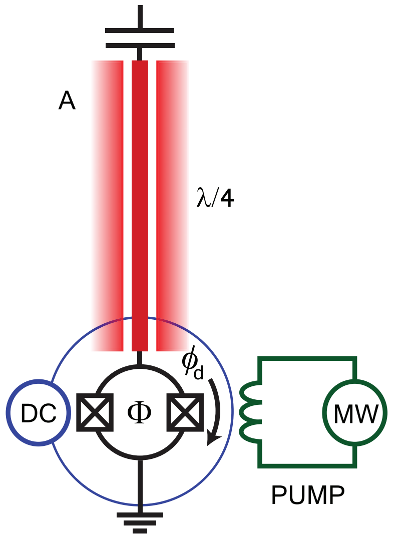
Finite-component phase transitions ⟷ **Quantum sensing**

- **Fundamental limits** — Resources: $\langle \hat{N} \rangle$

- **Practical applications**

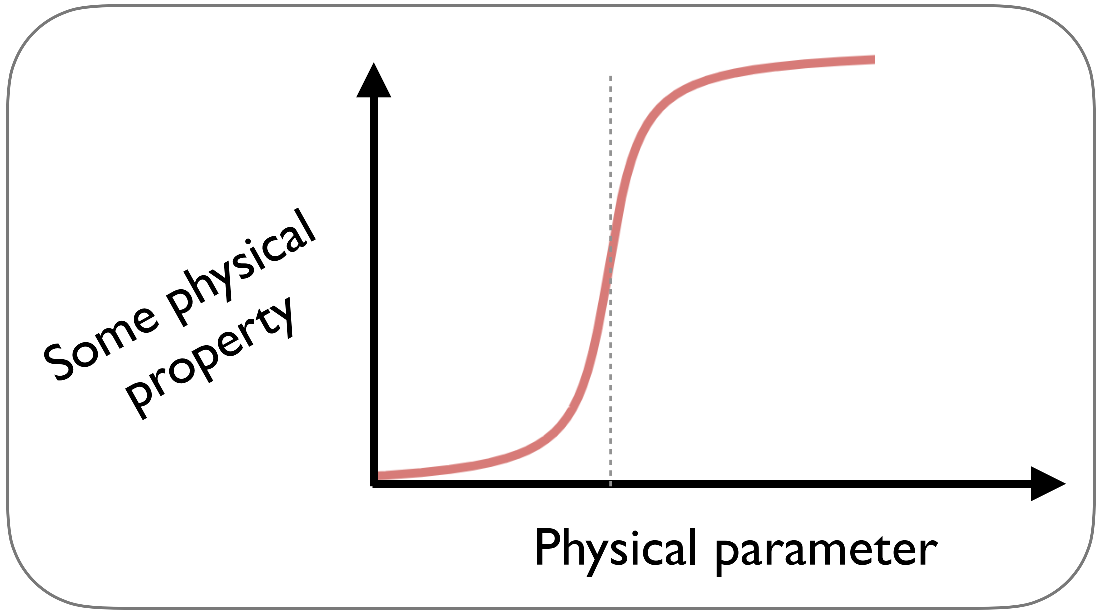
  1- Quantum Magnetometry

  2- Qubit readout

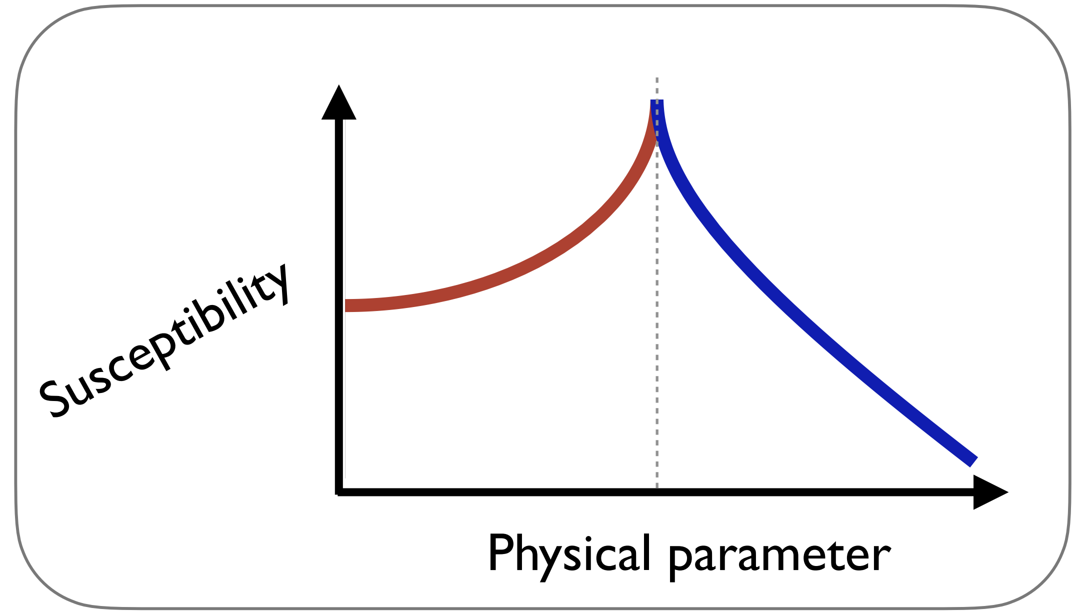


Introduction

Critical phase transition



High sensitivity



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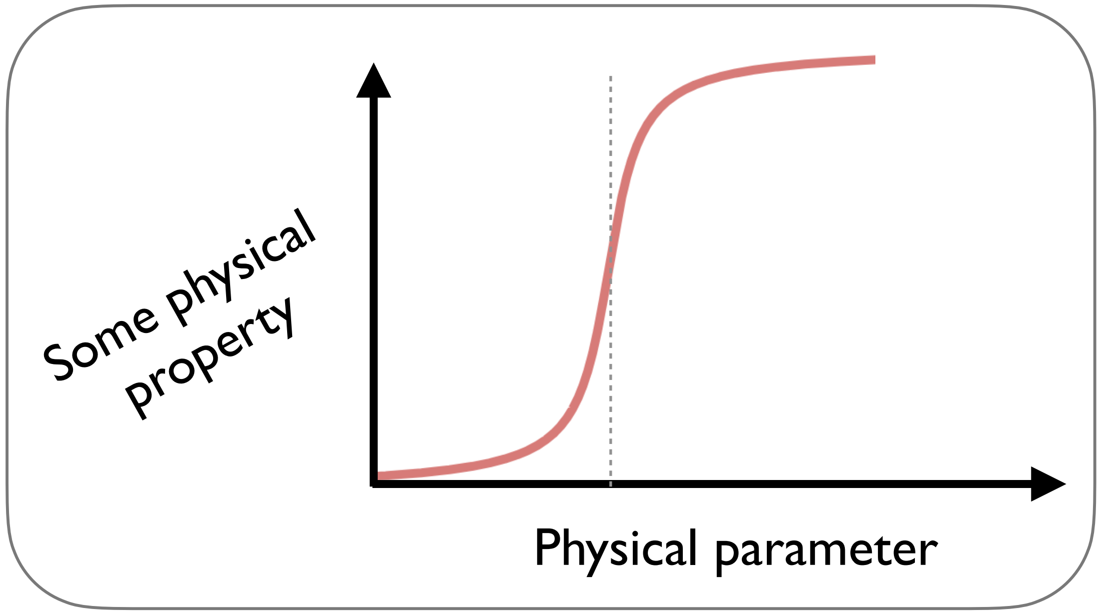

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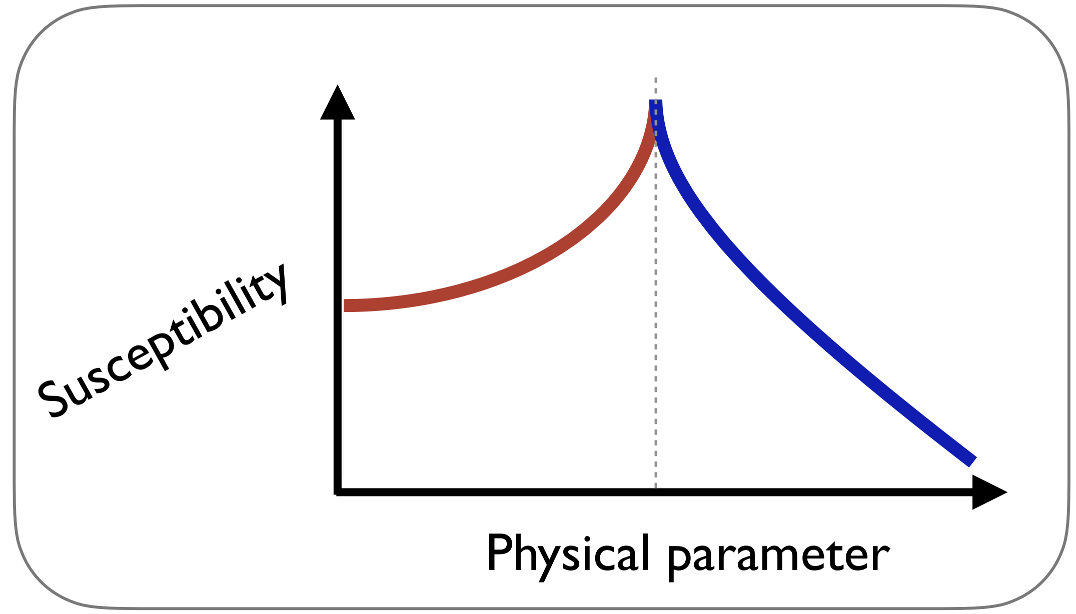
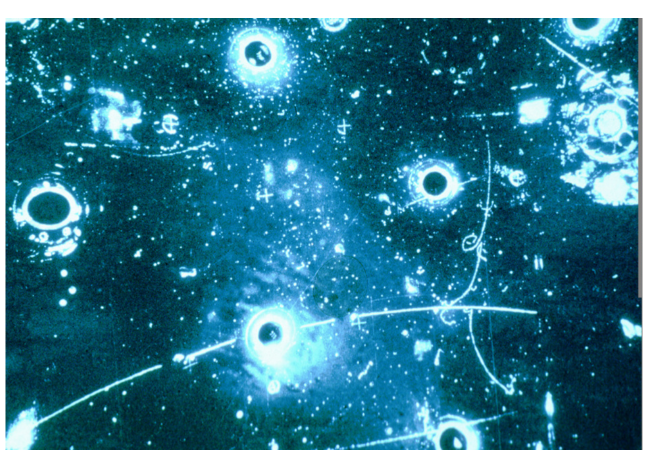


Critical sensors

Bubble chamber

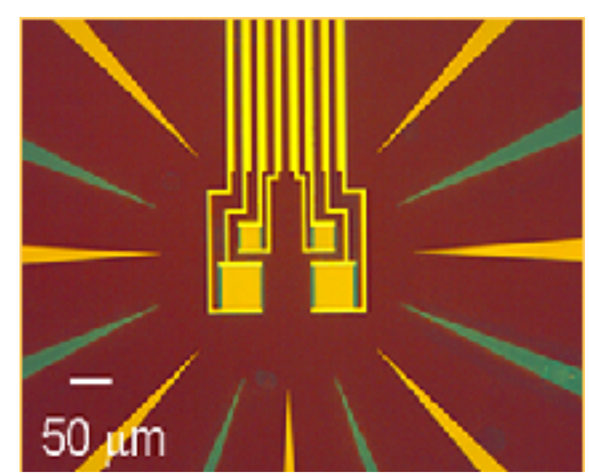
(Liquid-gas)



(CERN image archives)

Transition-edge sensors

(Superconductor-normal)

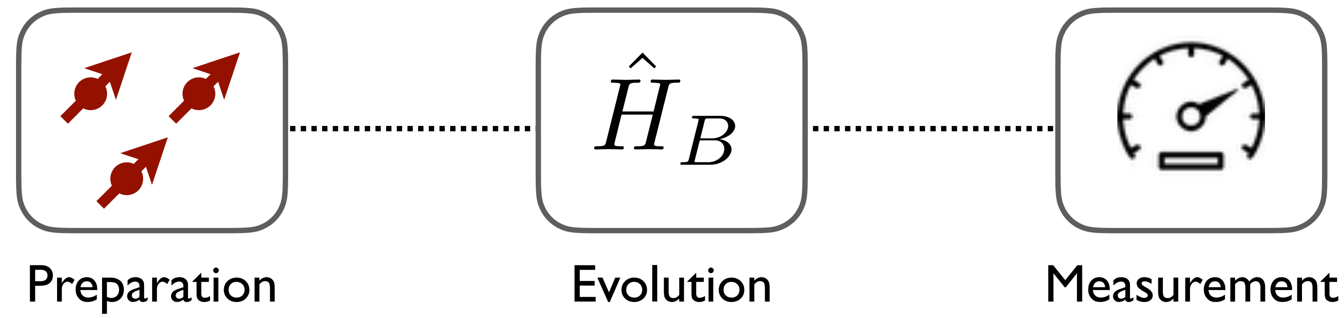


(NIST image archives)

Introduction

Quantum sensing

$$\hat{H}_B = \hat{H}_{sys} + B\,\hat{H}_I$$



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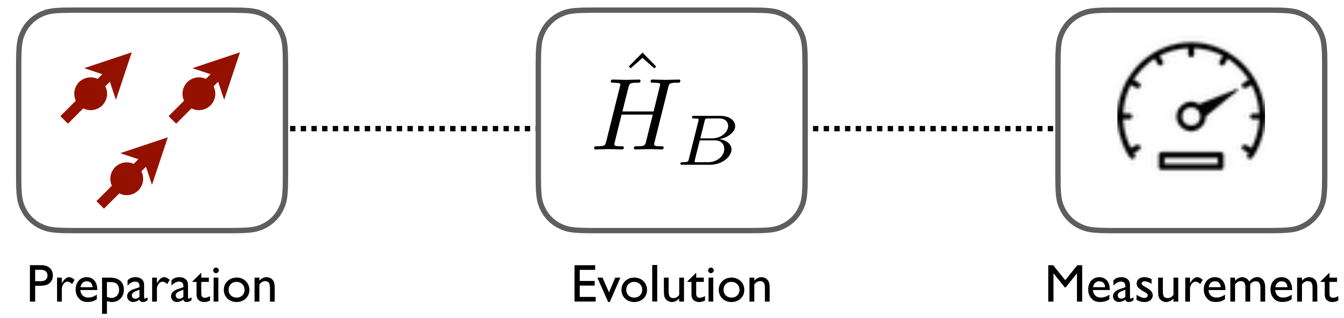

Estimation error:

$$\delta B = \frac{1}{\sqrt{G_B}}$$

↓ Quantum Fisher Information

Classical probes

$$G_B \sim N$$

Quantum probes

$$G_B \sim N^2$$

Heisenberg limit

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Quantum sensing

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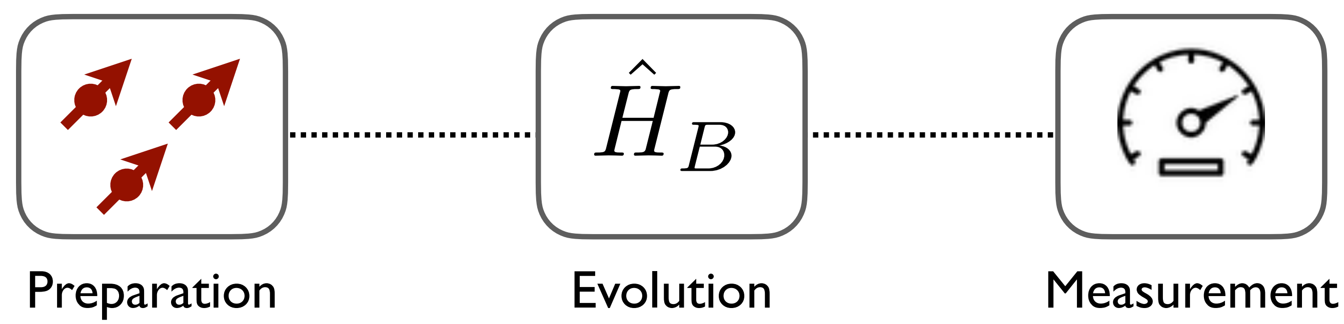

Estimation error: $\delta B = \dfrac{1}{\sqrt{G_B}}$

↓

Quantum Fisher Information

Classical probes

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Heisenberg limit

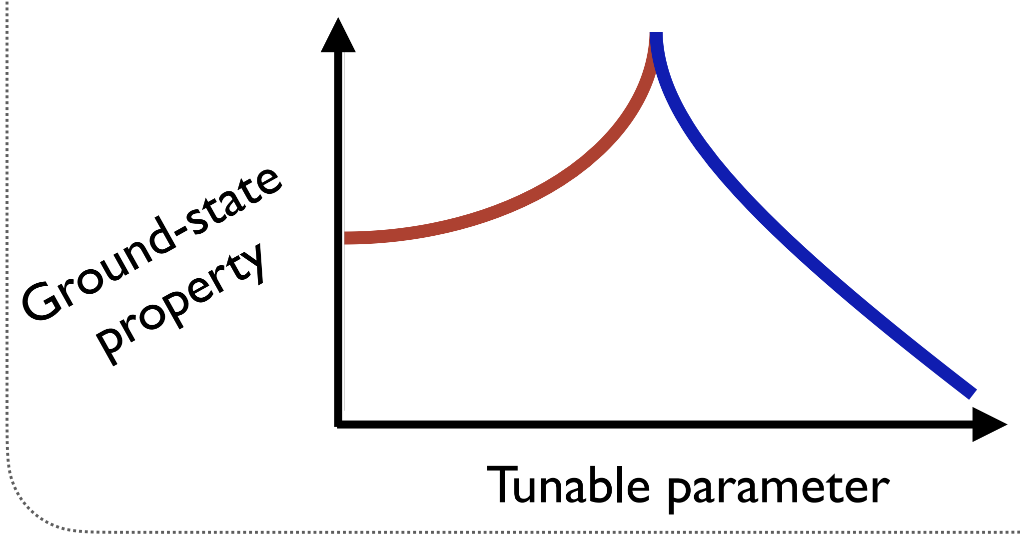
Critical quantum sensing: $\hat{H}_{sys}$ → **Quantum phase transition**

P. Zanardi et al., Phys. Rev. A **78**, 042105 (2008)

K. Macieszczak et al., Phys. Rev. A **93**, 022103 (2016)

Introduction

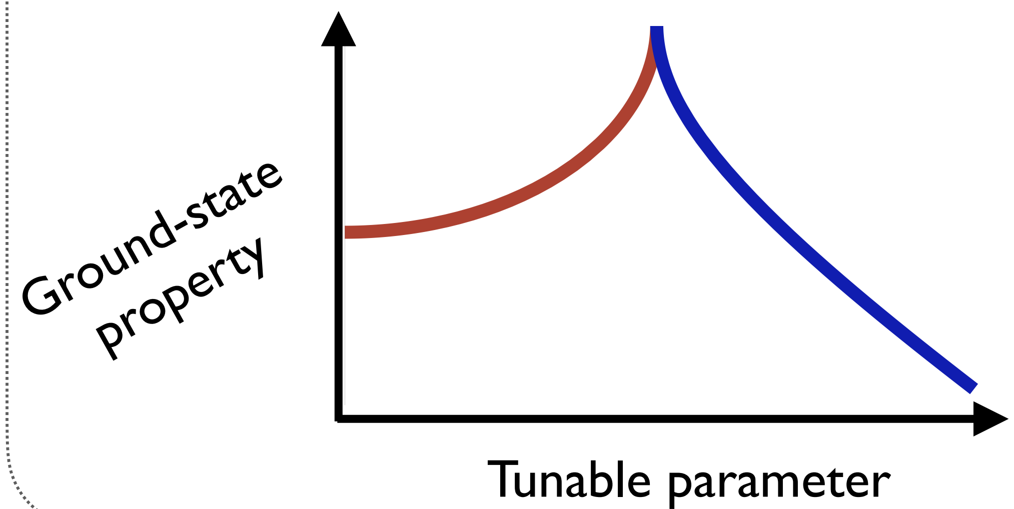
Diverging Susceptibility



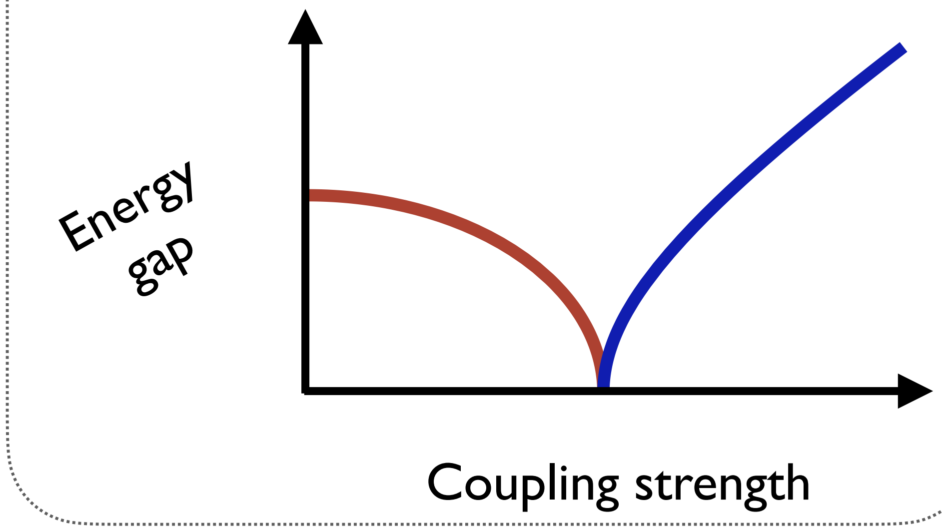
Infinite precision

Introduction

Diverging Susceptibility



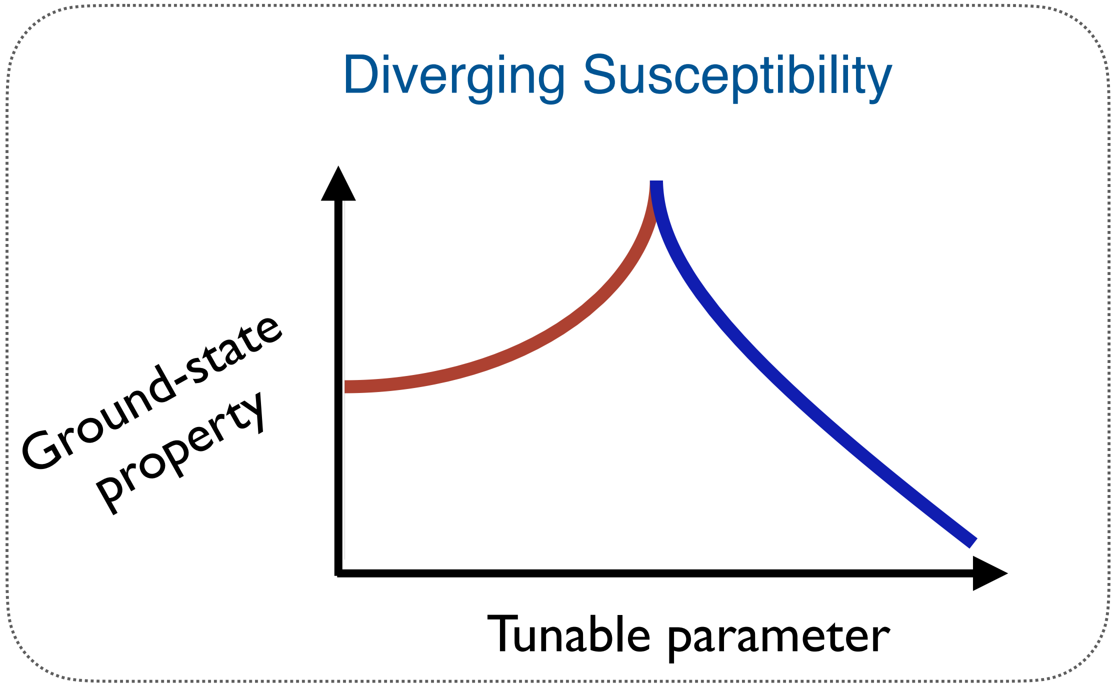
Closing energy gap

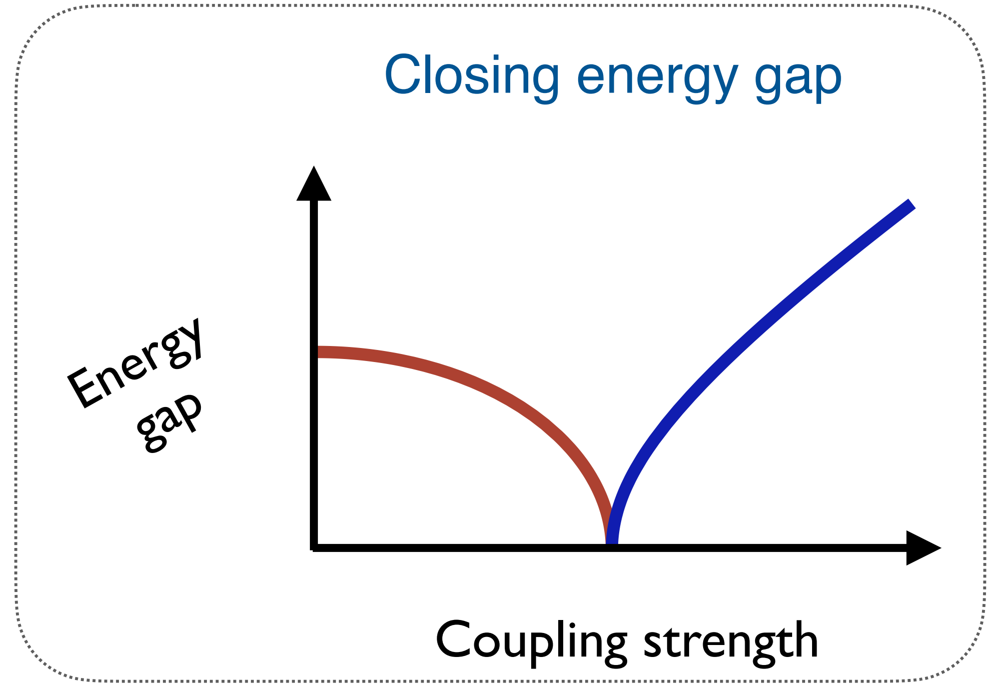


Infinite precision

Infinite time..

Introduction

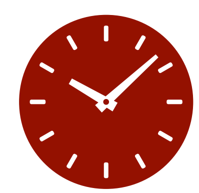




Infinite precision

Infinite time..

Time is a resource
CQM



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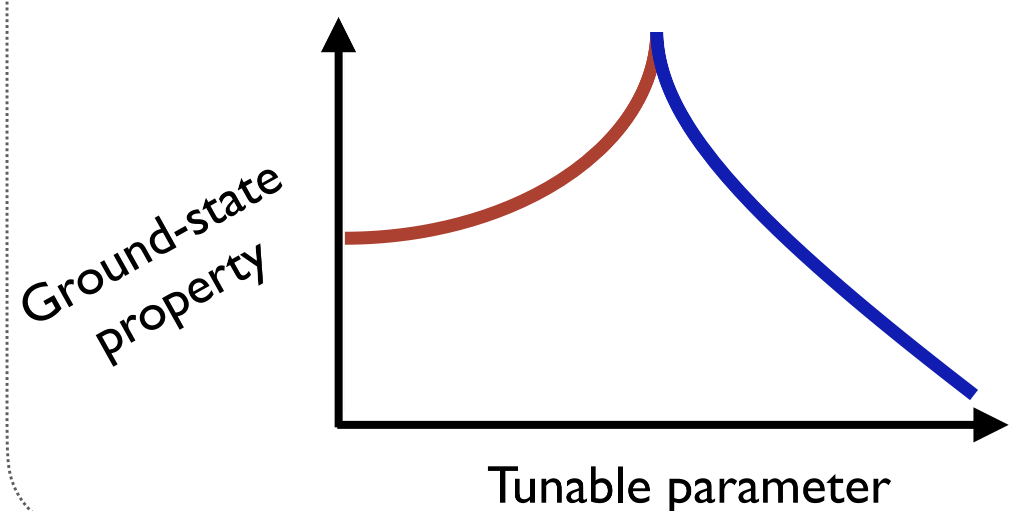

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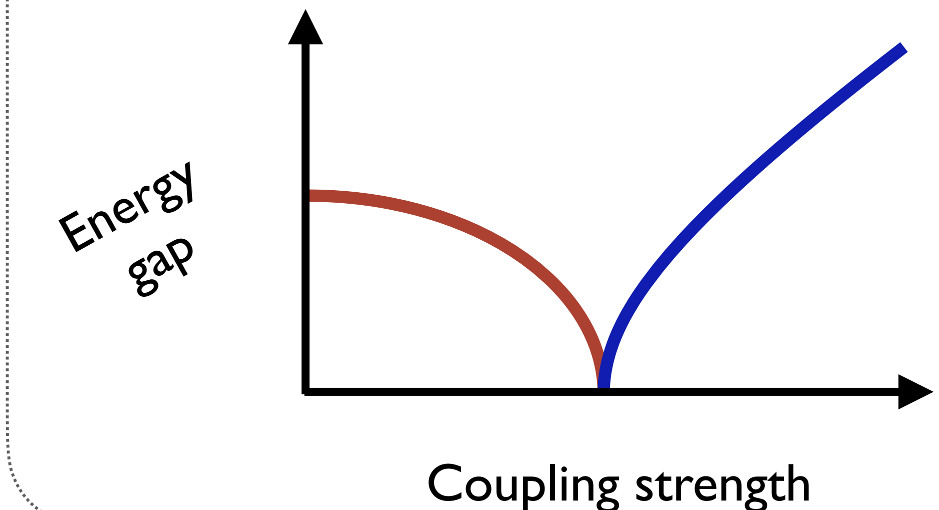

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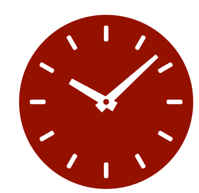
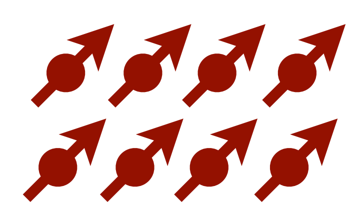


PHYSICAL REVIEW X **8,** 021022 (2018)

At the Limits of Criticality-Based Quantum Metrology: Apparent Super-Heisenberg Scaling Revisited

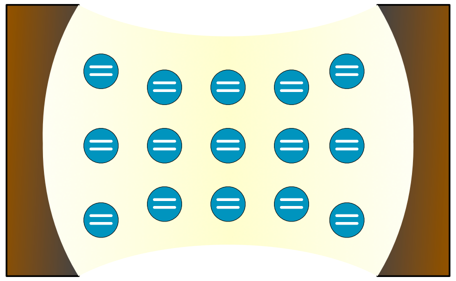
Marek M. Rams,[1,*] Piotr Sierant,[1,†] Omyoti Dutta,[1,2] Paweł Horodecki,[3,‡] and Jakub Zakrzewski[1,4,§]

For spins



$$G_B \sim t^2 N^2$$

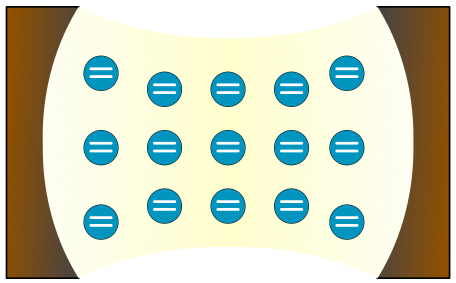
Dicke model



$$H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sum_{i=1}^{N} \sigma_i^z + \frac{g}{\sqrt{N}} (a + a^\dagger) \sum_{i=1}^{N} \sigma_i^x$$

Introduction

Dicke model



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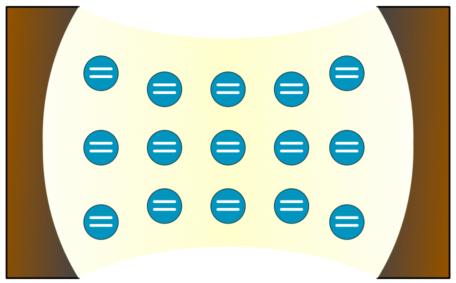
Thermodynamic limit $N \to \infty$

Introduction

Dicke model



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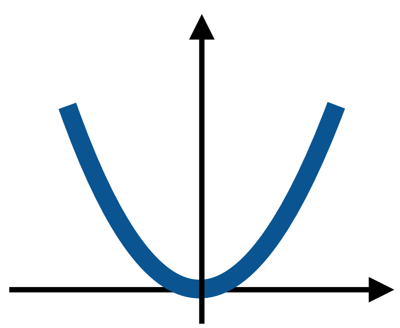
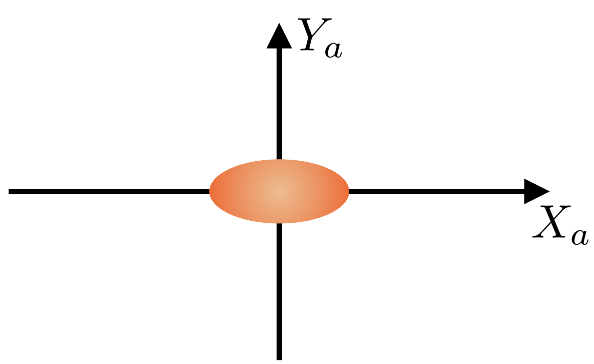
Thermodynamic limit $N \to \infty$

Normal Phase $g < \frac{\sqrt{\omega_c \omega_q}}{2}$

Harmonic potential



(Squeezed) vacuum



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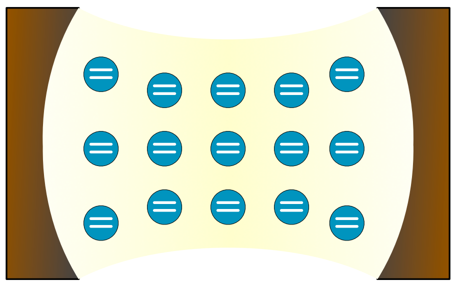

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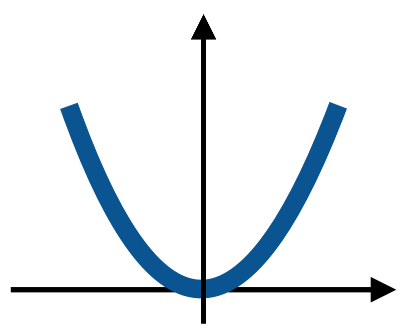

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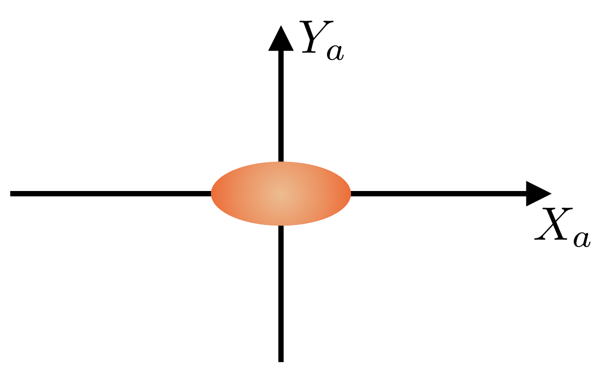
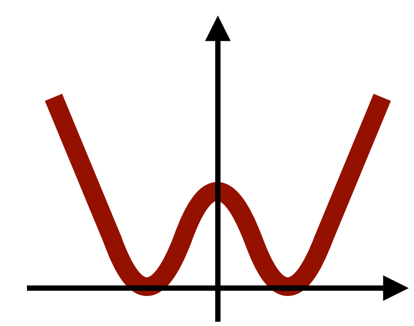
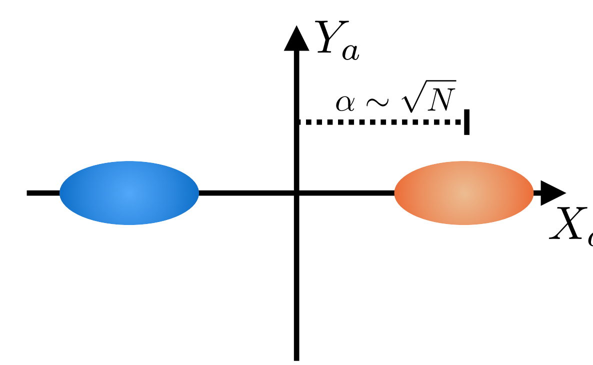


Superradiant Phase $g > \frac{\sqrt{\omega_c \omega_q}}{2}$

Double-well Potential



Superposition



Introduction

Finite-component quantum phase transitions

~~$N \to \infty$~~ $\quad\quad$ $N \sim 1$

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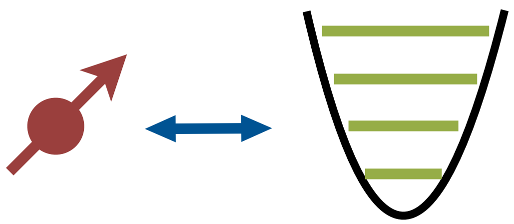
Quantum Rabi model



$$H_{\text{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

Superradiant phase transition in the **scaling limit**

$$\Omega/\omega_0 \to \infty$$

L. Bakemeier, et al., Phys. Rev. A 85, 043821 (2012).

M.-J. Hwang, et al., Phys. Rev. Lett. 115, 180404 (2015).

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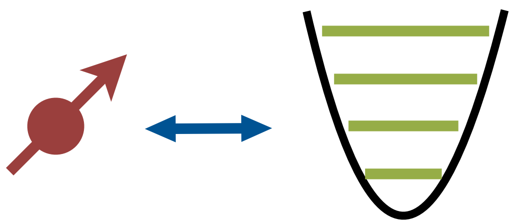

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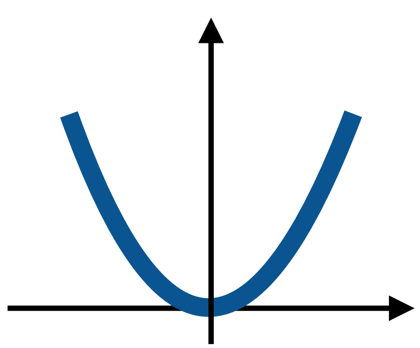

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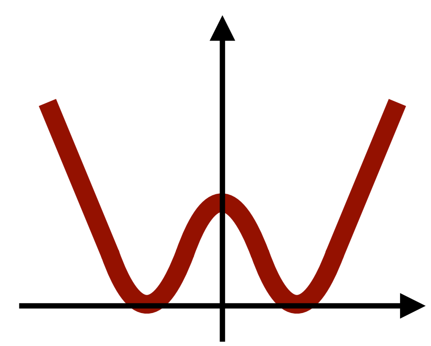

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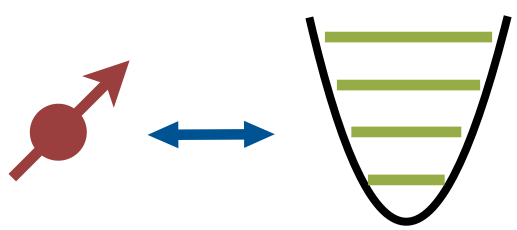

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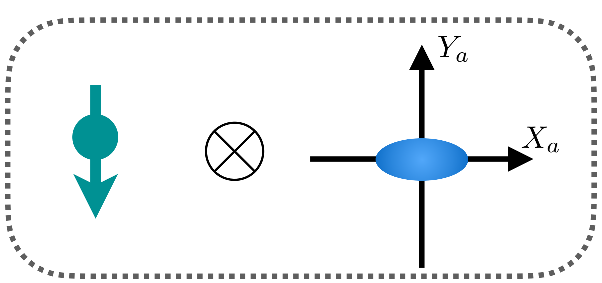
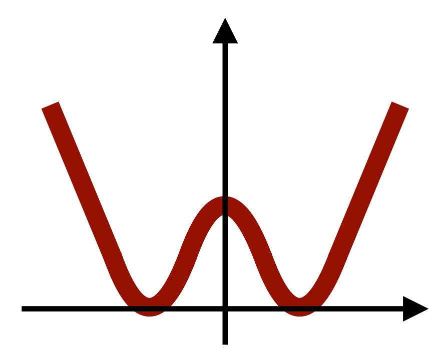


$|\downarrow\rangle \otimes \mathcal{S}_\lambda|0\rangle$

Superradiant phase $\lambda > \lambda_c$



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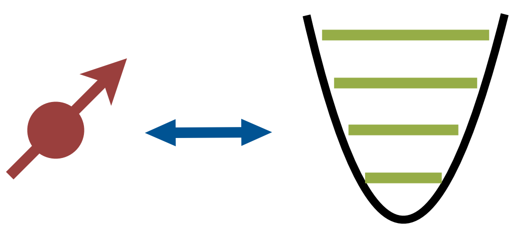

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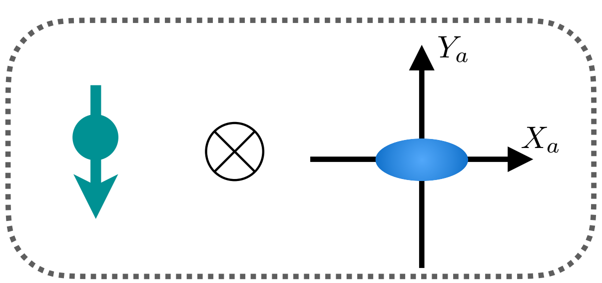

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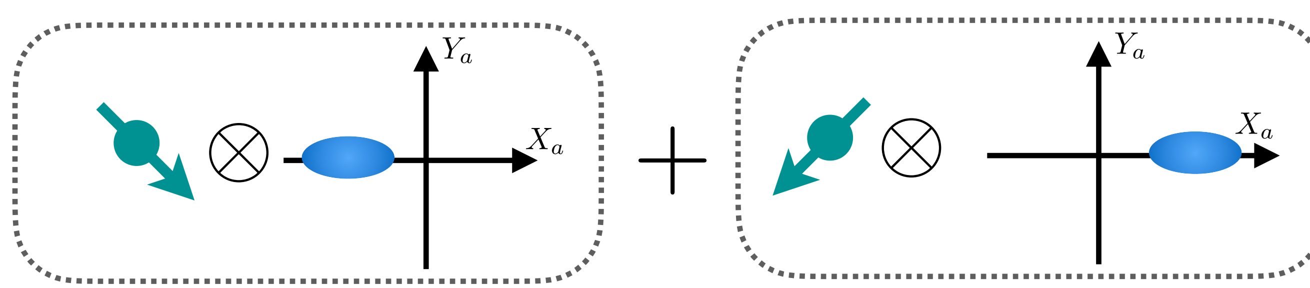

$$|\swarrow\rangle \otimes \mathcal{D}_{-\alpha}\mathcal{S}_\chi|0\rangle + |\searrow\rangle \otimes \mathcal{D}_\alpha\mathcal{S}_\chi|0\rangle$$

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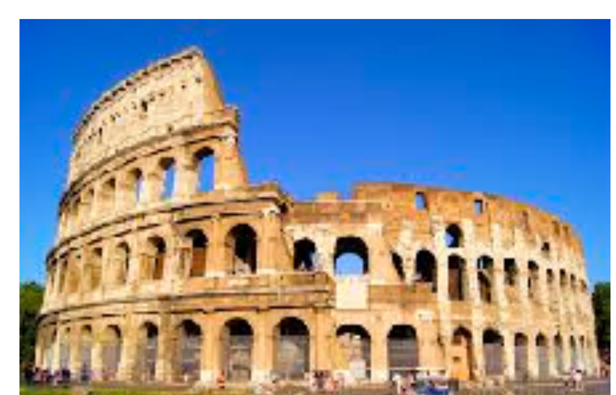
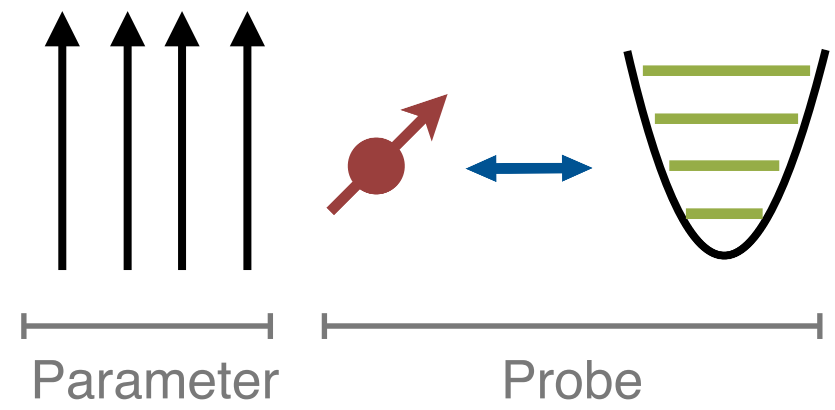


Finite-component critical probe

$$H_{\text{Rabi}} = \omega_0 a^\dagger a + \frac{\Omega}{2}\sigma_z - \lambda(a + a^\dagger)\sigma_x$$

Scaling limit $\Omega/\omega_0 \gg 1$



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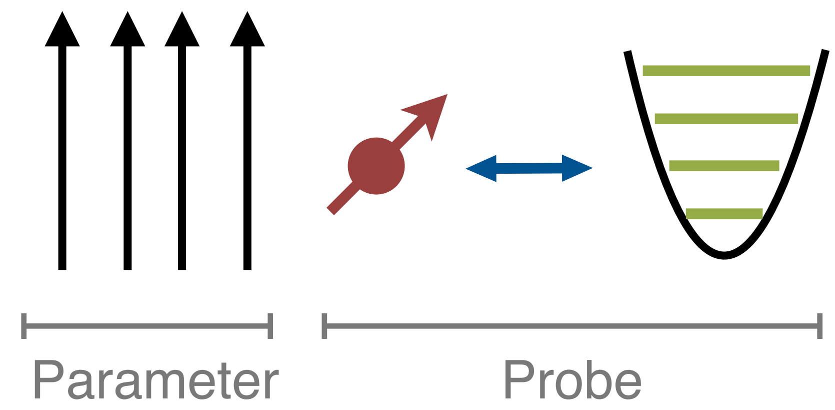

$$g = \lambda/\sqrt{\Omega\omega_0}$$

Quantum phase transition

$$g \to 1$$

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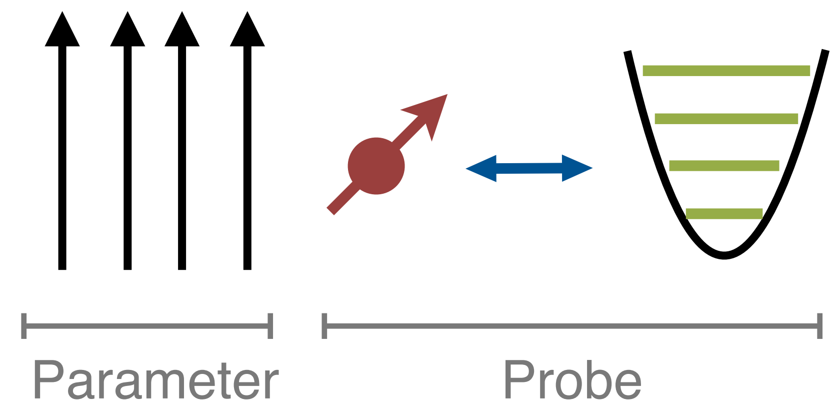


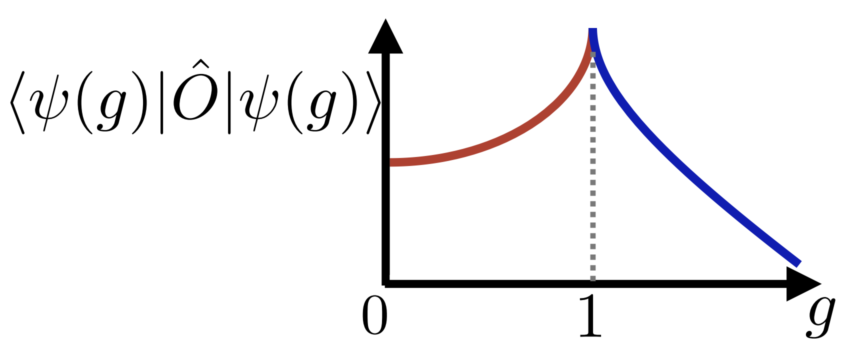
$$g = \lambda/\sqrt{\Omega\omega_0}$$

Quantum phase transition $g \to 1$

Estimation protocol

1) Prepare trivial ground state

$$|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$$



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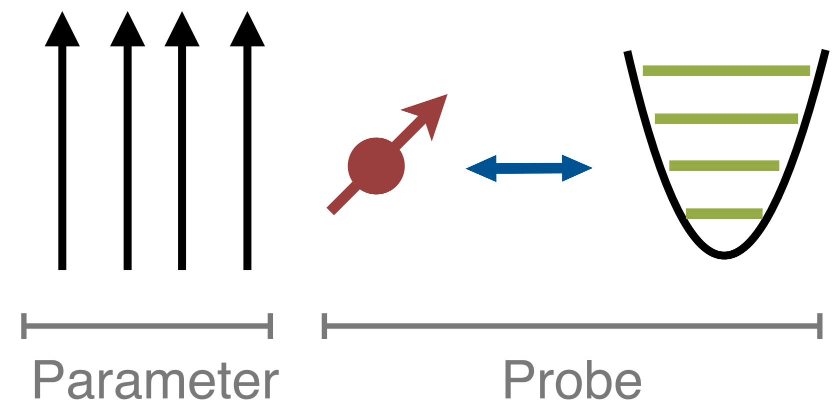

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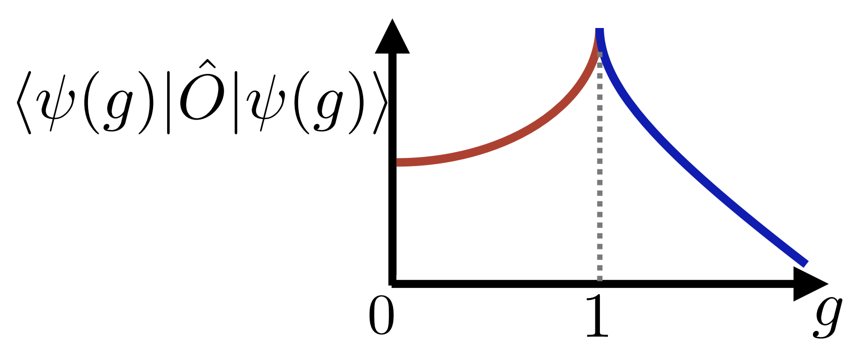
2) Adiabatic sweep

$$g(t) : 0 \longrightarrow 1$$



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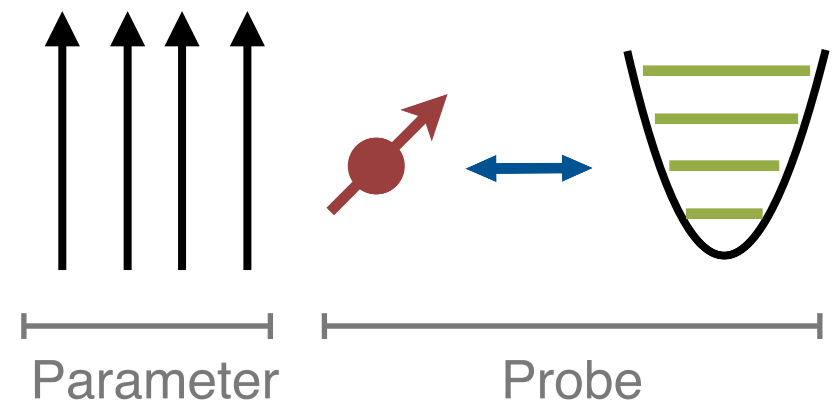


$$g = \lambda/\sqrt{\Omega\omega_0}$$

Quantum phase transition

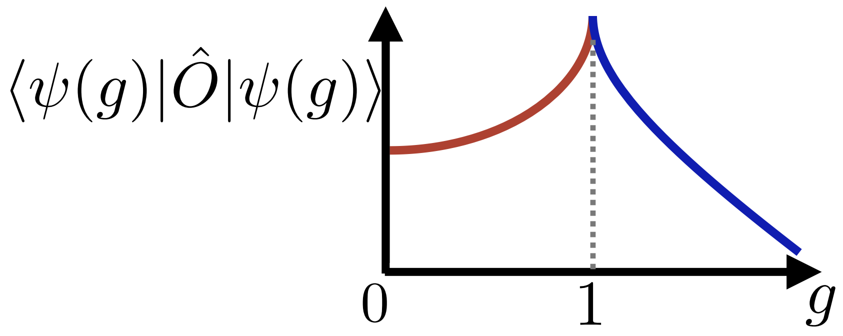
$$g \to 1$$

Estimation protocol

1) Prepare trivial ground state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

2) Adiabatic sweep $g(t) : 0 \longrightarrow 1$

3) Measure ground state $\langle\psi(g)|\hat{O}|\psi(g)\rangle$



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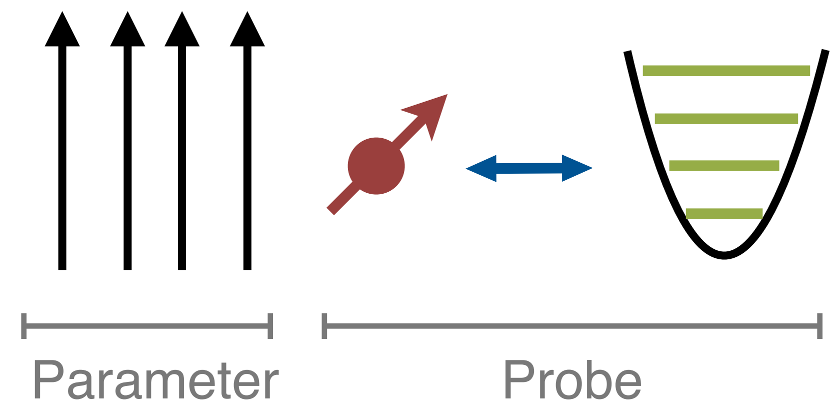

$$g = \lambda/\sqrt{\Omega\omega_0}$$ → Quantum phase transition $g \to 1$

Estimation protocol

1) Prepare trivial ground state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

2) Adiabatic sweep $g(t) : 0 \longrightarrow 1$

3) Measure ground state $\langle\psi(g)|\hat{O}|\psi(g)\rangle$

Driven-dissipative case

1) Prepare trivial initial state $|\psi(g)\rangle = |0\rangle \otimes |\downarrow\rangle$

2) Long-time evolution $\lim_{t\to\infty} |\psi(t)\rangle$

3) Measure steady state $\langle\psi(t)|\hat{O}|\psi(t)\rangle$

Evaluation

Quantum Fisher information

Ground state $|\psi(g)\rangle = \hat{S}(\xi)|0\rangle \otimes |\downarrow\rangle$

Squeezing $\xi = -\frac{1}{4}\log(1 - g^2)$

Q.F.I. $G_A = 4[\langle \partial_A \psi | \partial_A \psi \rangle + (\langle \partial_A \psi | \psi \rangle)^2]$

Protocol duration

Evaluation

Quantum Fisher information

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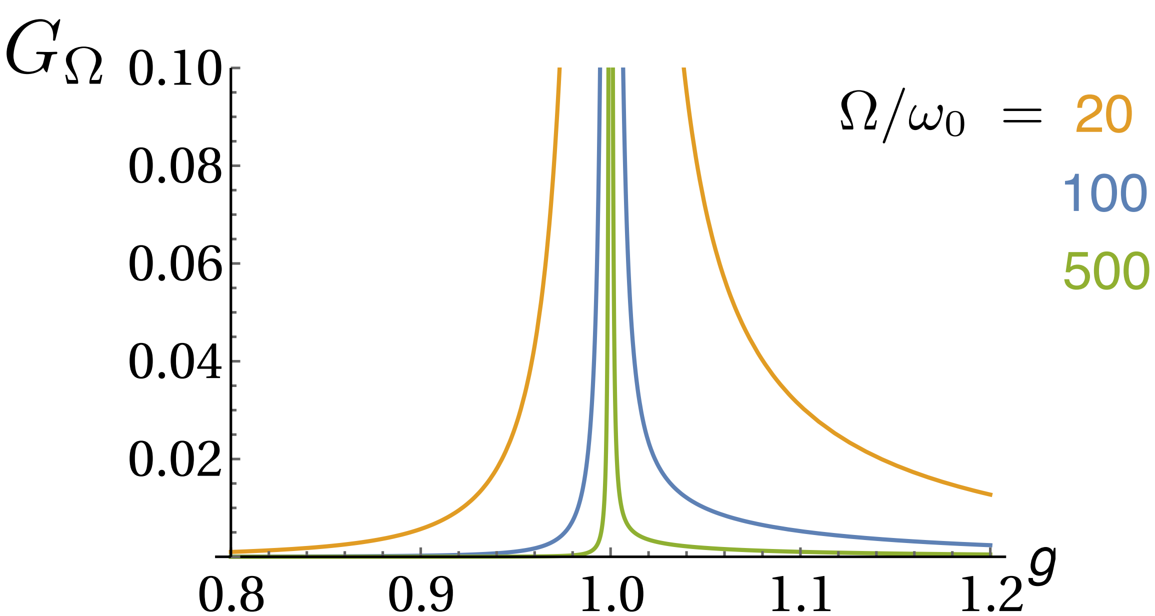

Critical scaling Q.F.I.

$$G_A \simeq \frac{1}{32\,A^2(1-g)^2}$$

Protocol duration

Evaluation

Quantum Fisher information

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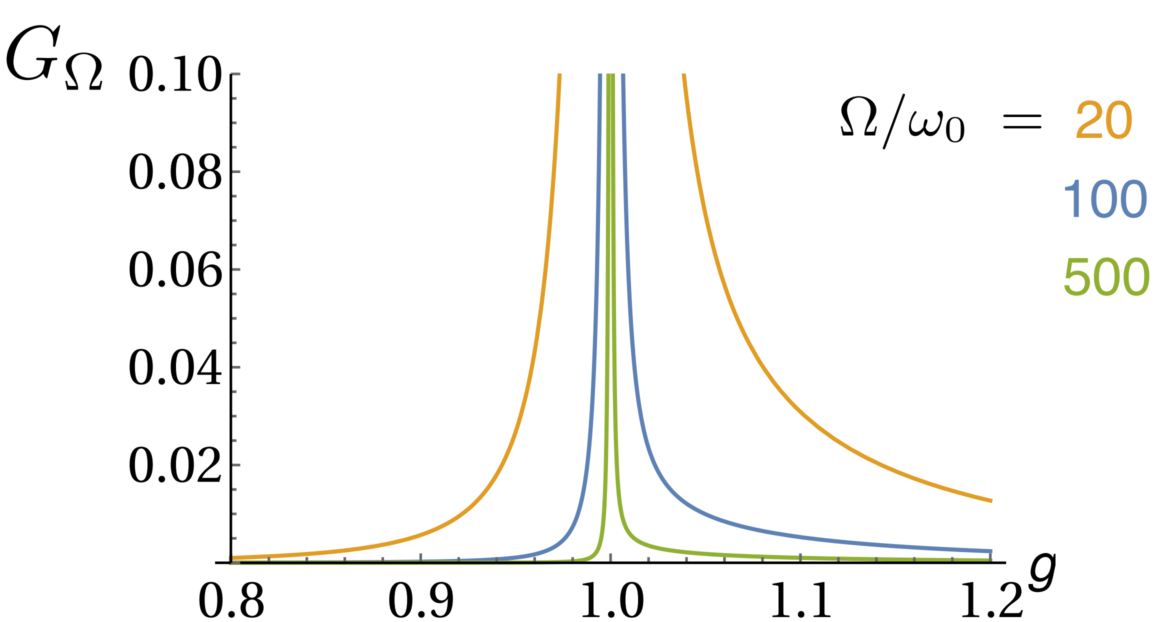
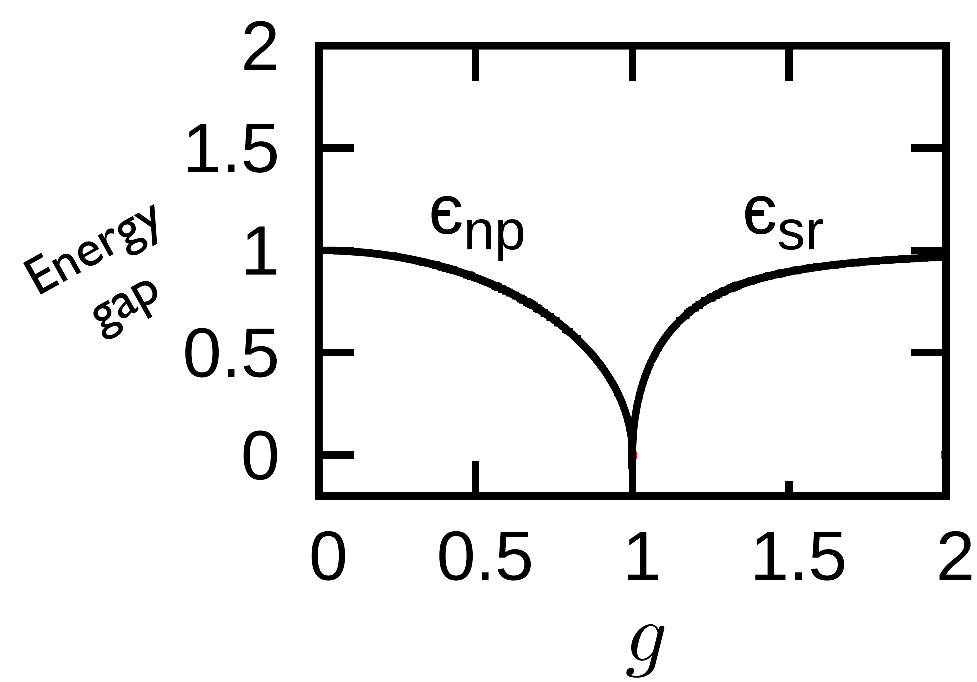


Critical scaling Q.F.I. $G_A \simeq \frac{1}{32\,A^2(1-g)^2}$

Protocol duration

$\epsilon_{\rm np} = \omega_0\sqrt{1 - g^2}$



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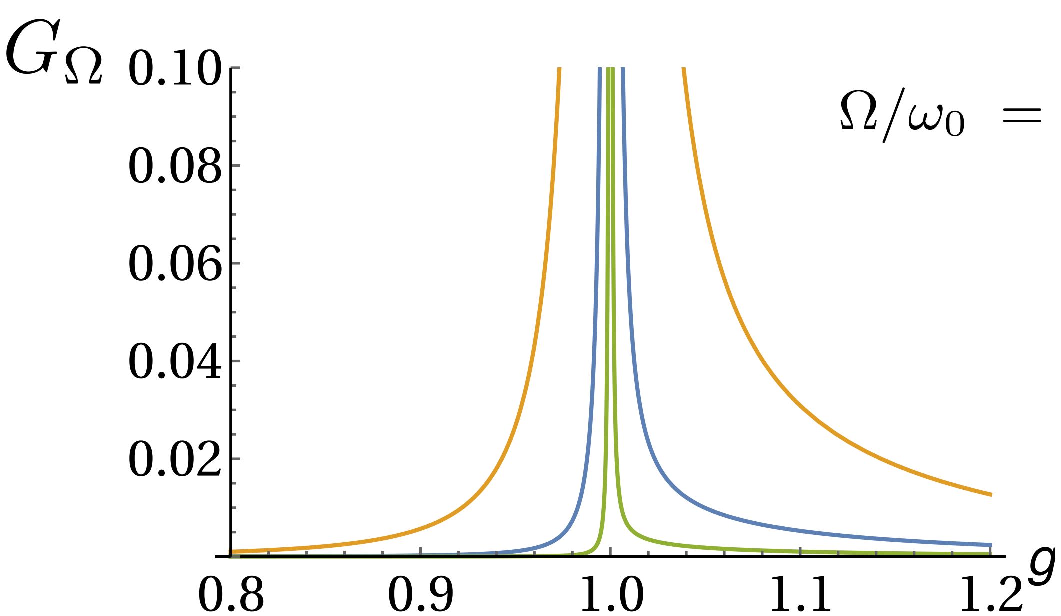

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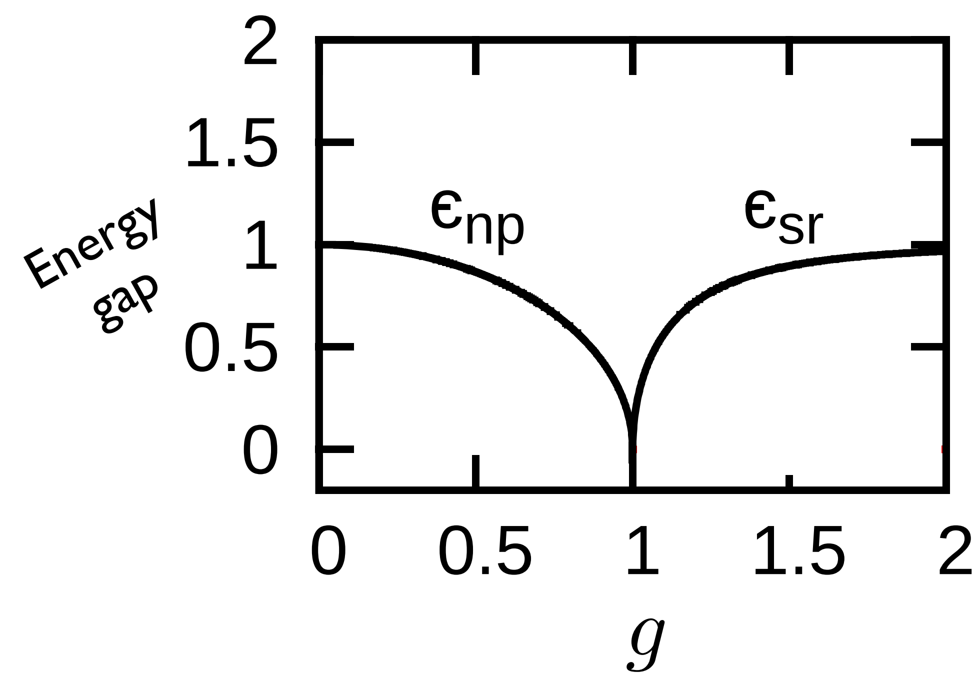

(from time-dependent perturbation theory)

Adiabatic evolution $v(g) \ll \frac{2g}{1+g^2}\,\omega_0\,(1-g^2)^{3/2}$

Critical scaling Evolution time $T = \int_0^g \frac{ds}{v(s)} \sim \frac{1}{\omega_0\sqrt{1-g}}$

Results

Analysis of the scaling of the estimation precision G_{ω_0}

Probe number $\langle \hat{N} \rangle$ Time

Hamiltonian: $G_{\omega_0} \sim \langle \hat{N} \rangle^2 T^2$ Saturate Heisenberg limit

Driven-dissipative: $G_{\omega_0} \sim \langle \hat{N} \rangle T$ Optimal in noisy Q. Metrology

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Optimal scaling in spite of the critical slowing down!

- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

Take-home message

Finite-component phase transitions ⟷ **Quantum metrology**

- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

Take-home message

Finite-component phase transitions ⟷ **Quantum metrology**

$N \gg 1$ → $\Omega/\omega_0 \gg 1$

Termodynamic limit → Scaling limit

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$N \gg 1$

Termodynamic limit

→

$\Omega/\omega_0 \gg 1$

Scaling limit

Fundamental interest

Ultimate Precision limits

$\langle \hat{N} \rangle$

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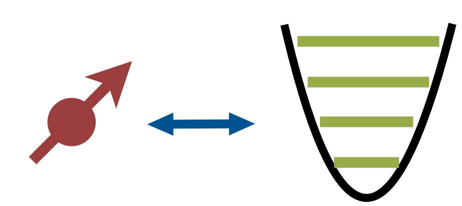
Termodynamic limit → Scaling limit

Fundamental interest

Ultimate Precision limits  $\langle \hat{N} \rangle$

Practical consequences

Few-body critical probe 

- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

Recent works

Dynamical protocols

- Y Chu, S Zhang, B Yu, J Cai, PRL **126**, 010502 (2021).

- L. Garbe, O. Abah, S. Felicetti, R. Puebla,
Quantum Science and Technology 7 (3), 035010 (2022).

- S. Wald, S..V. Moreira, and F. L. Semião
Phys. Rev. E 101, 052107 (2020)

Exponential speed-up

- K. Gietka, L. Ruks, and T. Busch,
Quantum 6, 700 (2022).

- L. Garbe, O. Abah, S. Felicetti, R. Puebla,
preprint at arXiv:2112.11264 (2021).

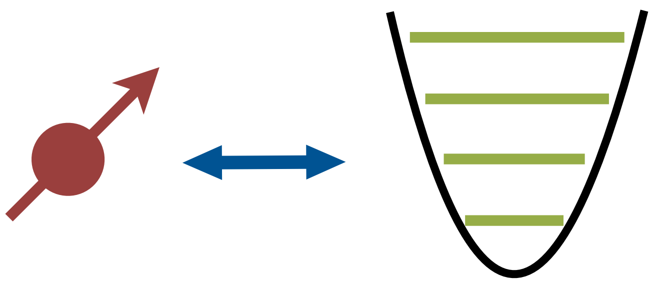
Continuous measurements

- T, Ilias, D. Yang, Susana F. Huelga, and M. B. Plenio
PRX Quantum 3, 010354 (2022)

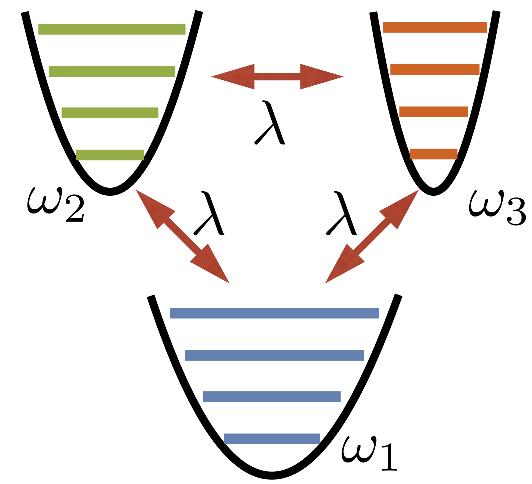
Finite-component phase transitions

A universal feature of ultrastrong coupling

Quantum Rabi model



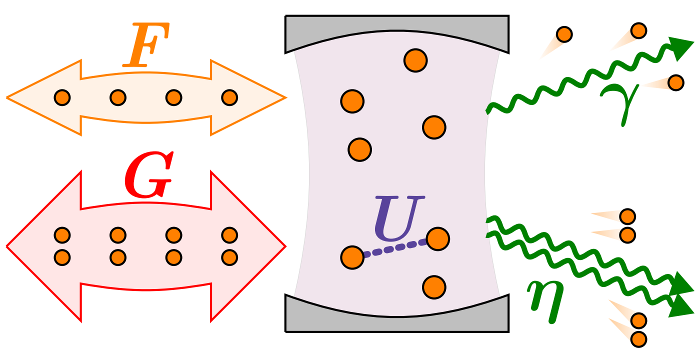
Nonlinear quantum resonators



- S. Felicetti and A. Le Boité, Phys. Rev. Lett. **124**, 040404 (2020).

Take place in driven-dissipative systems

Pumped Kerr resonators



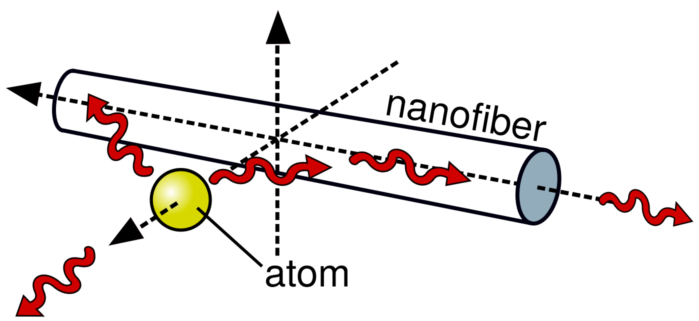
- N. Bartolo, F. Minganti, W. Casteels, and C. Ciuti, Phys. Rev. A **94**, 033841 (2016).

- R. Rota, F. Minganti, C. Ciuti, and V. Savona, Phys. Rev. A **112**, 110405 (2019).

Finite-component phase transitions

Implementations in quantum technologies

Atomic Systems



- A. Dareau et al., PRL **121**, 253603 (2018).
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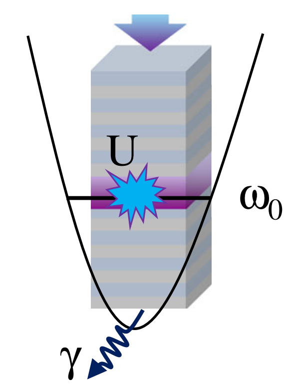
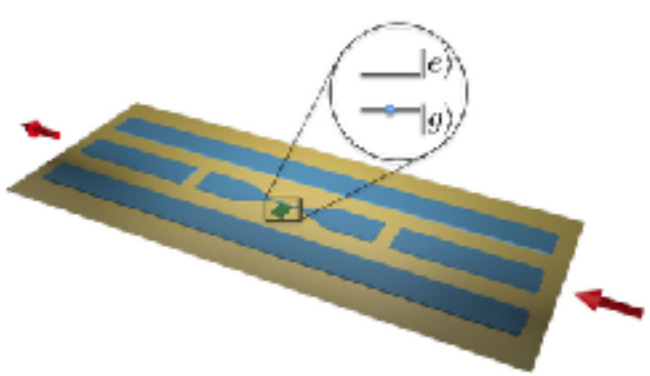
Polaritonics



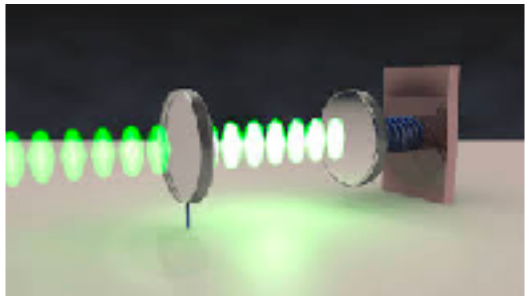
- S. R. K. Rodriguez et al., PRL 118, 247402 (2017).
- T. Fink et al., Nat. Phys 14, 365 (2018).

Circuit QED



- D. Marcovic et al., PRL **121**, 040505 (2018).

Opto/electro-mechanics



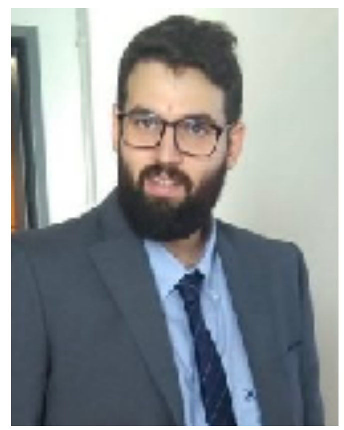
- G. Peterson et al., PRL **123**, 247701(2019).

Aalto University

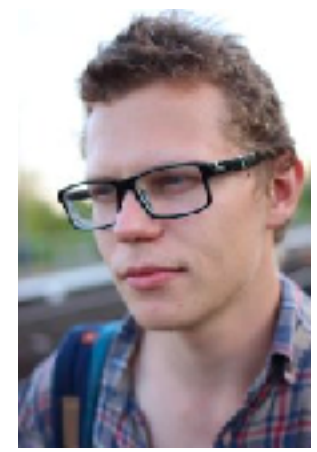
RIKEN

CNR IFN Istituto di Fotonica e Nanotecnologie

Roberto
di Candia



Fabrizio
Minganti



Kirill
Petrovnin

G. Sorin
Paraoanu

- R. Di Candia *, F. Minganti *, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, arXiv:2107.04503 (2021).

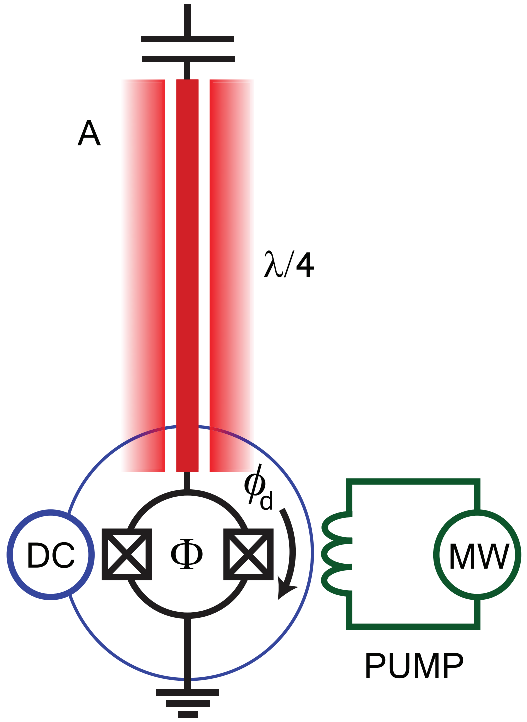
Parametric quantum sensor

Master equation

$$\dot{\rho} = -i[H, \rho] + \kappa N (a^\dagger \rho a - 1/2 \{aa^\dagger, \rho\})$$

Hamiltonian

$$\hat{H}_{\mathrm{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2} \hat{a}^2$$



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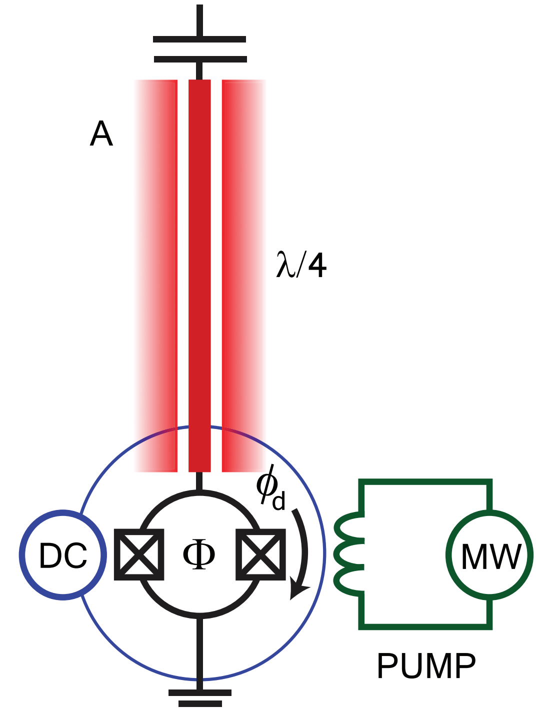
Weak (but finite!) nonlinearity

$$\chi \ll 1$$

→ **Critical PT steady state**



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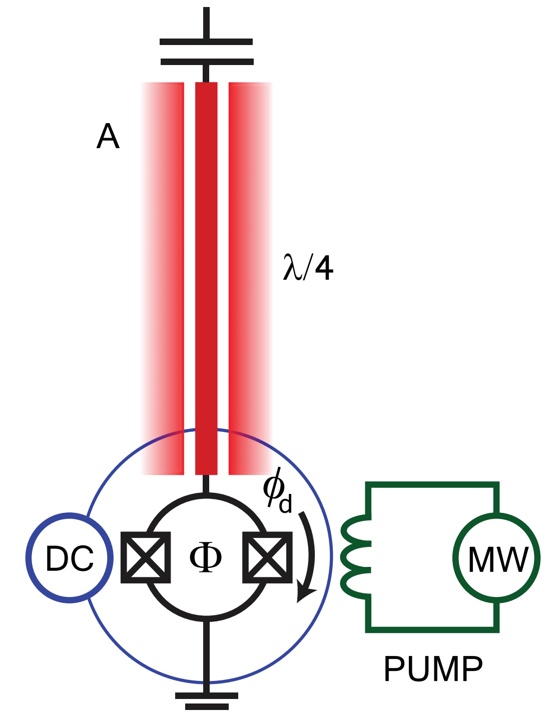

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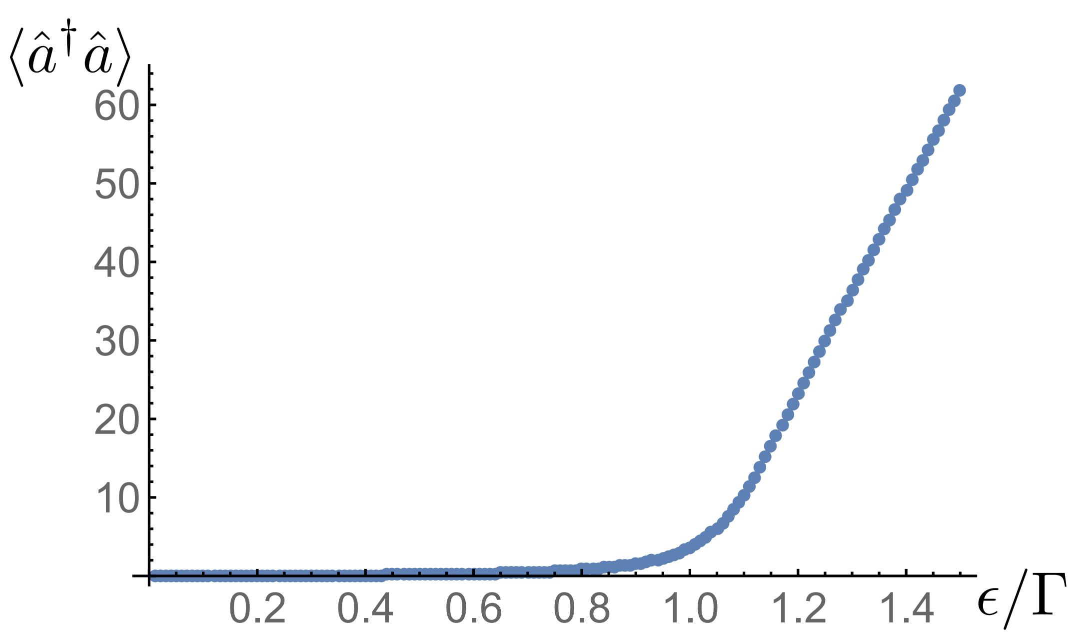

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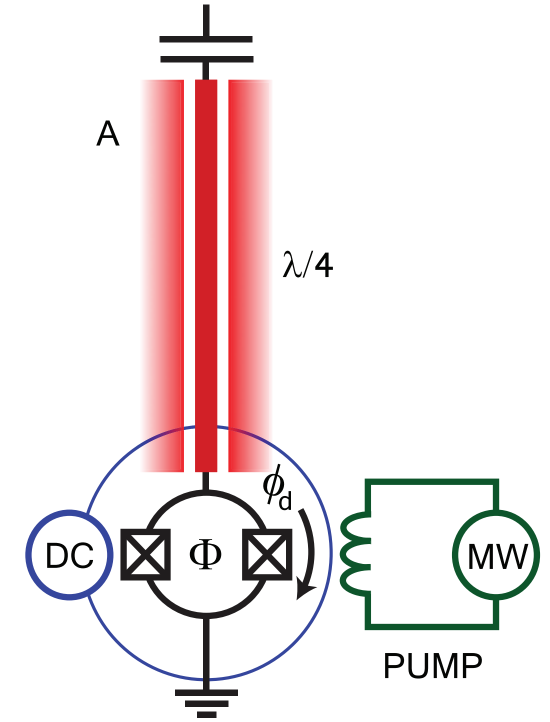

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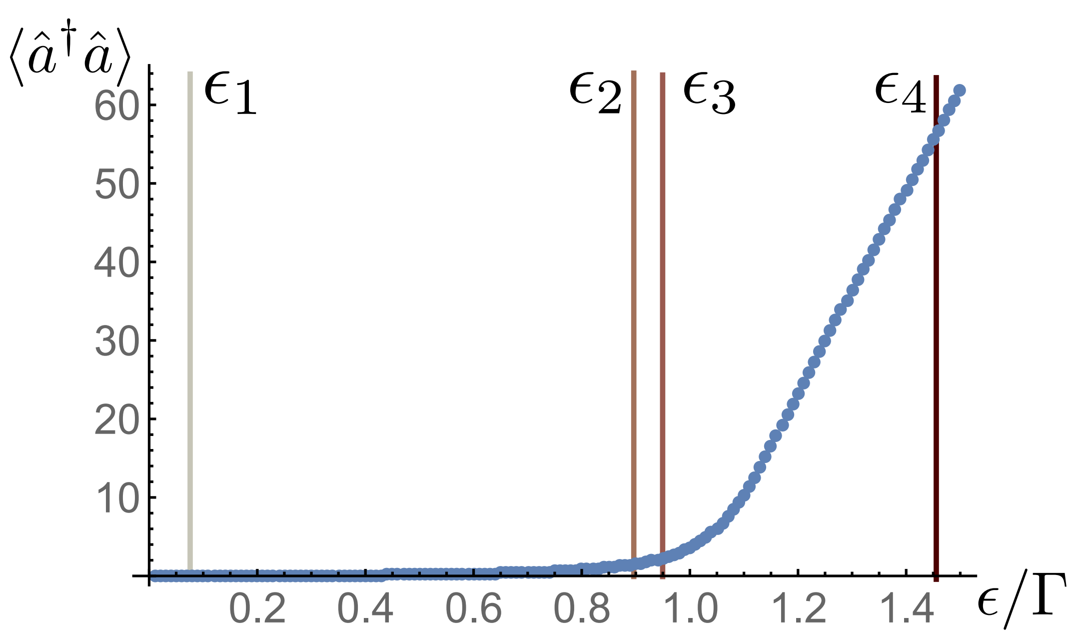

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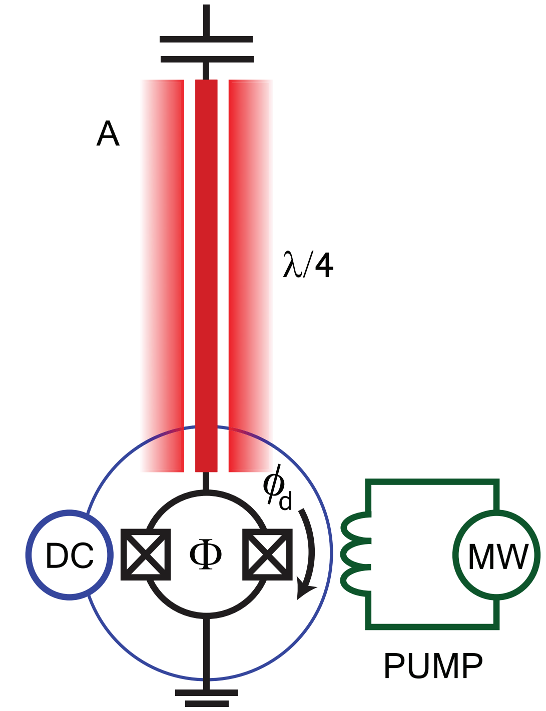

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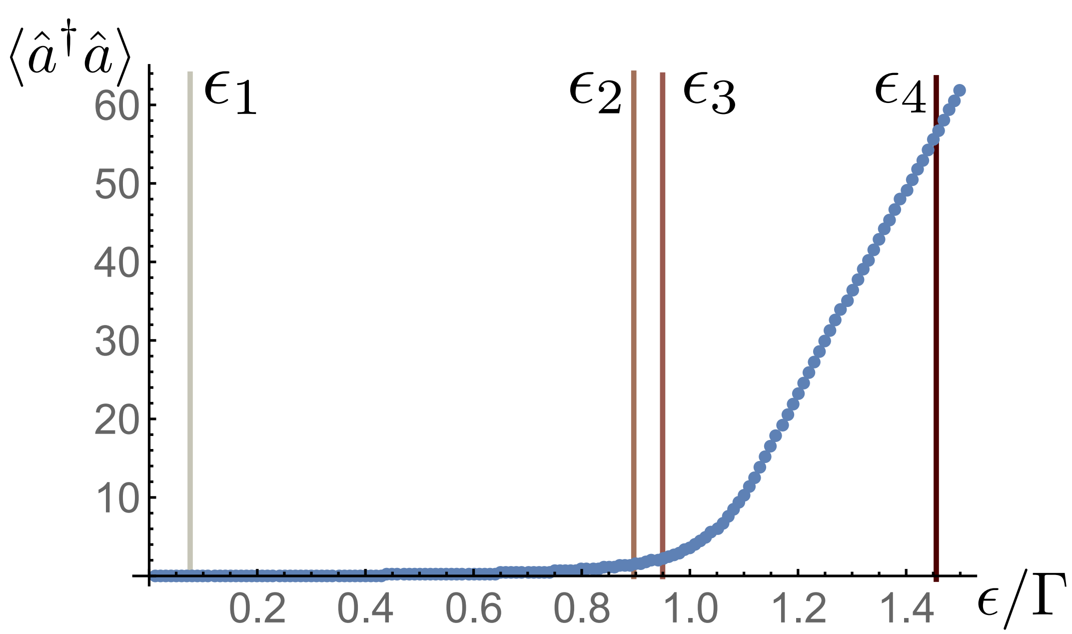

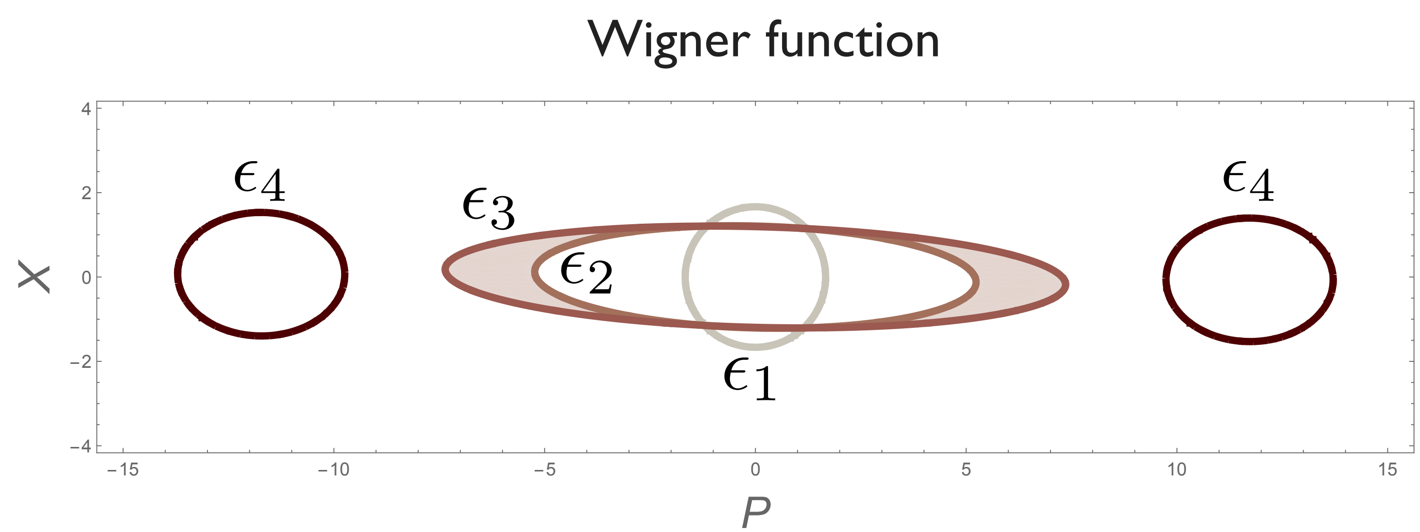

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Magnetometry

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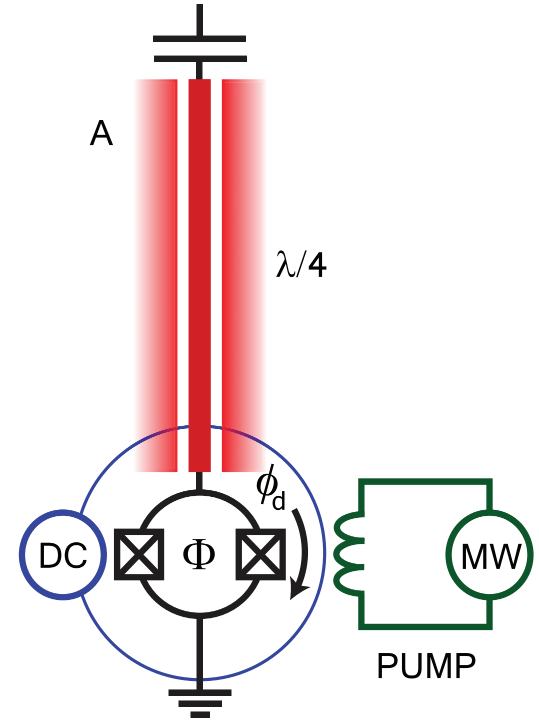
$$\omega = \omega(B)$$

Magnetic field estimation



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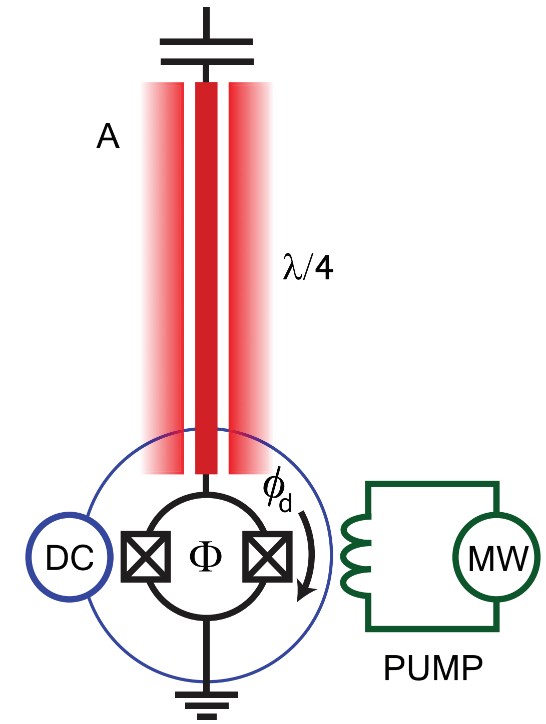

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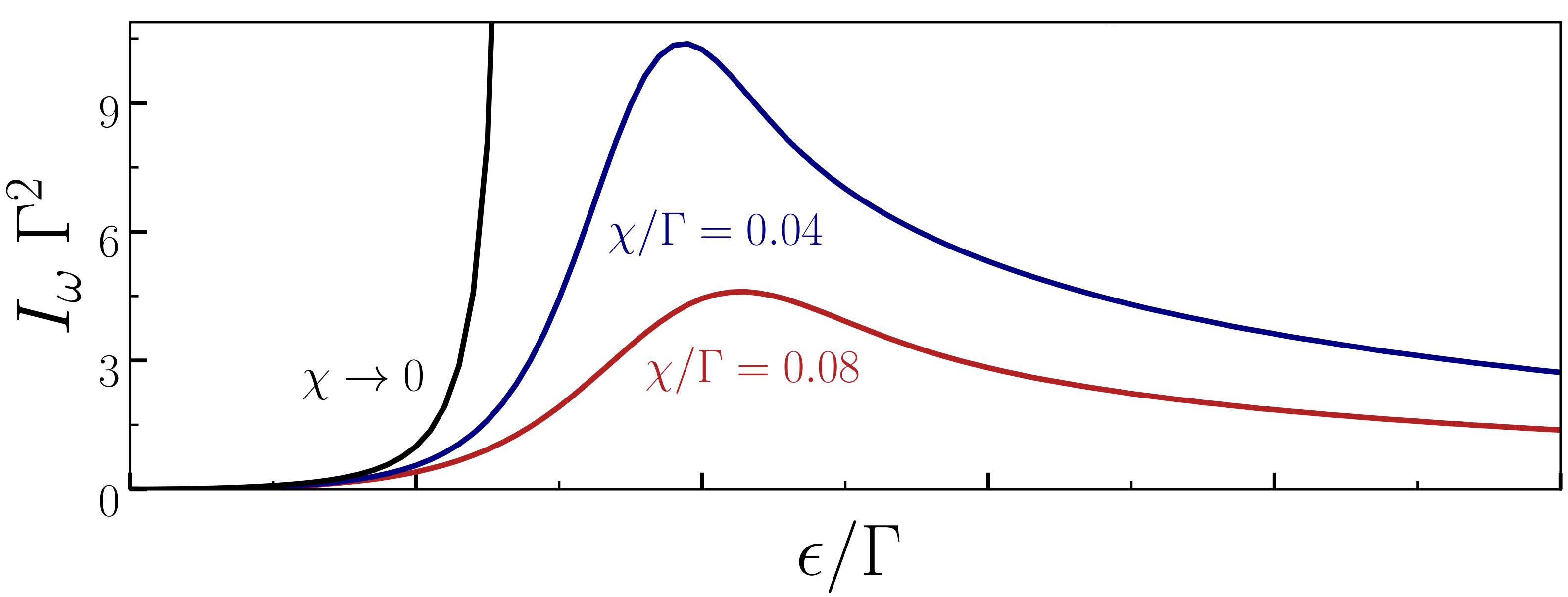

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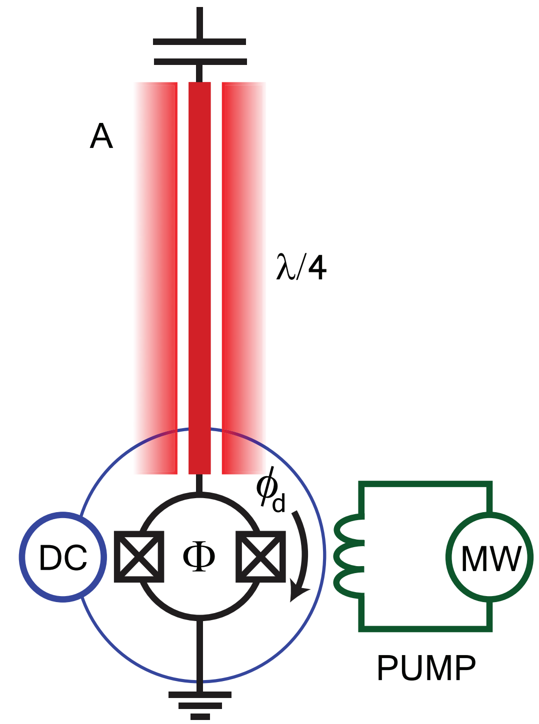


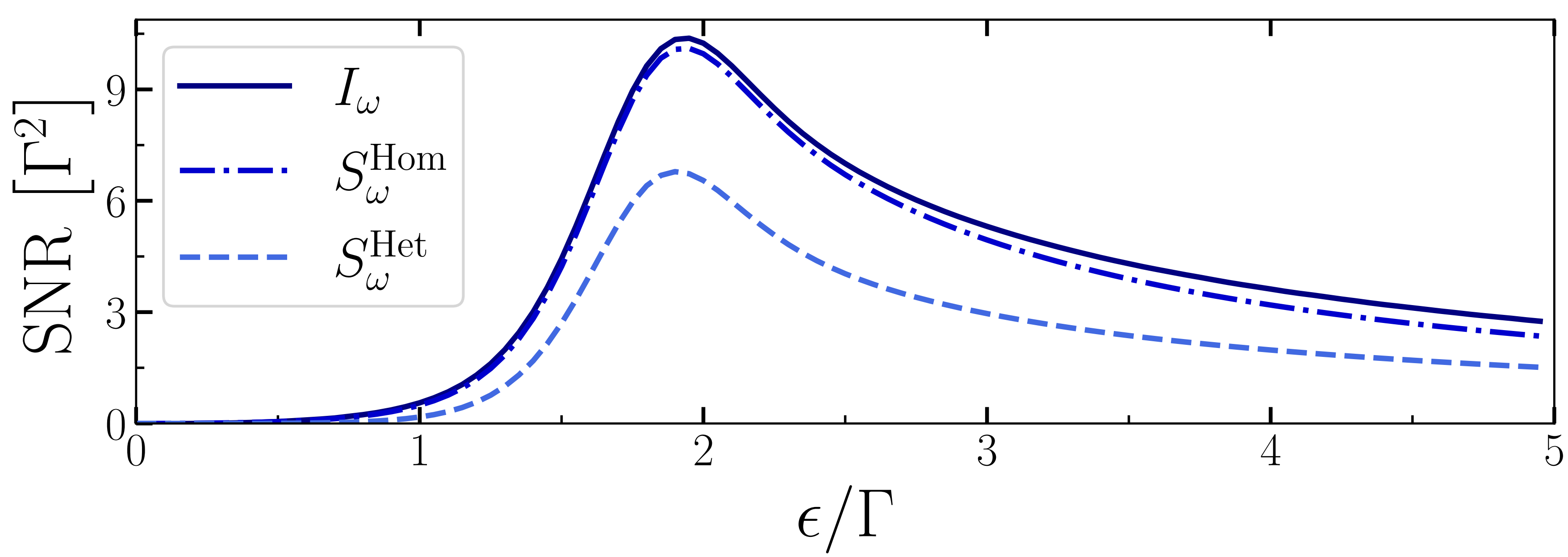
- P. Krantz et al, New J. Phys. **15** 105002 (2013).

Signal-to-noise ratio

Fix measurement

$$SNR_\omega(\hat{O}) = \frac{[\partial_\omega \langle \hat{O}\rangle_\omega]^2}{\Delta O_\omega^2}$$

$$\Delta O_\omega^2 = \langle \hat{O}^2\rangle_\omega - \langle \hat{O}\rangle_\omega^2$$



Qubit readout

Master equation

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Hamiltonian

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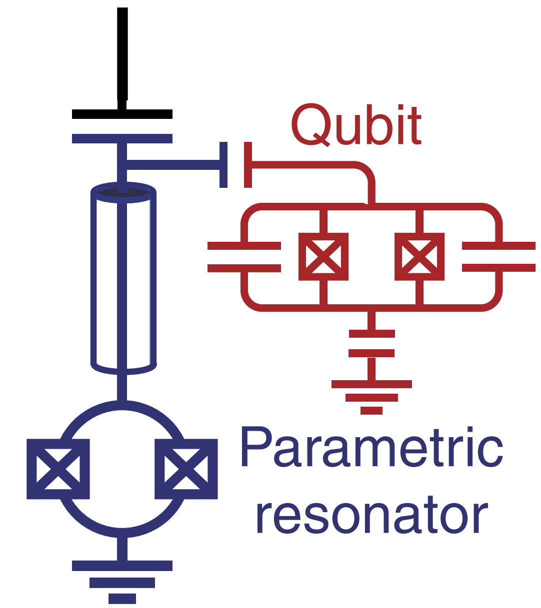

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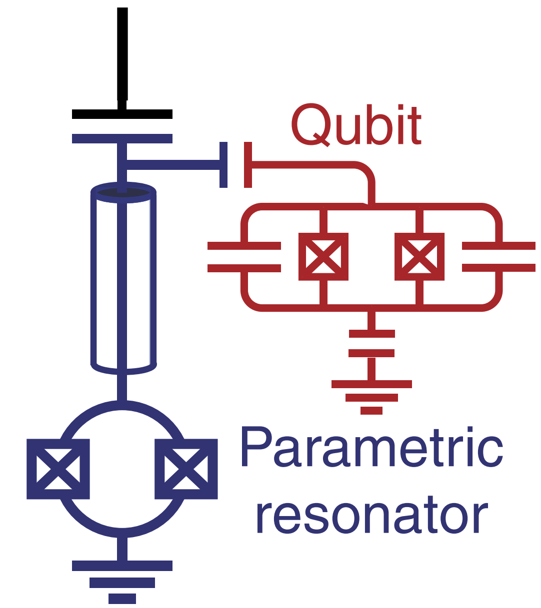
Dispersive coupling $H_{qc} = \delta\omega \; \hat{\sigma}_z \hat{a}^\dagger \hat{a}$



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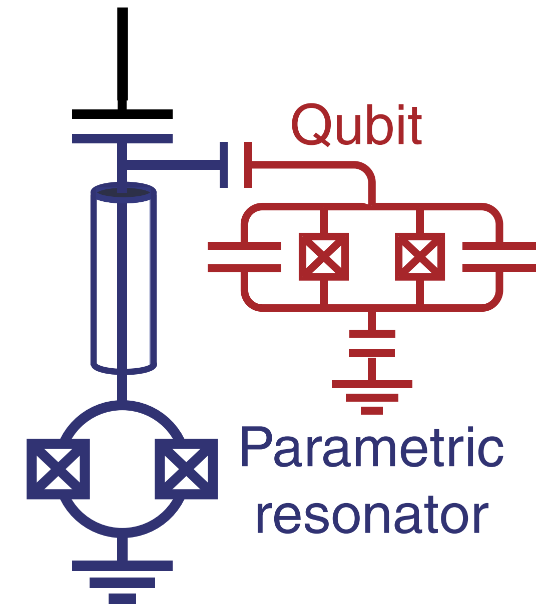

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State discrimination:

$$|0\rangle \longrightarrow \omega' = \omega - \delta\omega$$

$$|1\rangle \longrightarrow \omega' = \omega + \delta\omega$$

Qubit readout

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Hamiltonian

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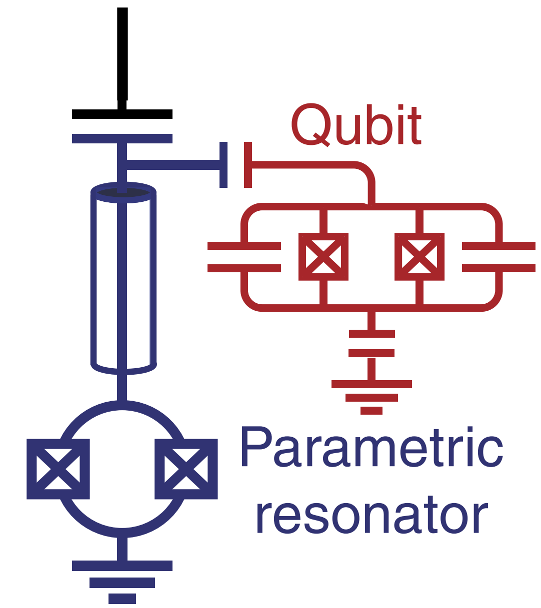

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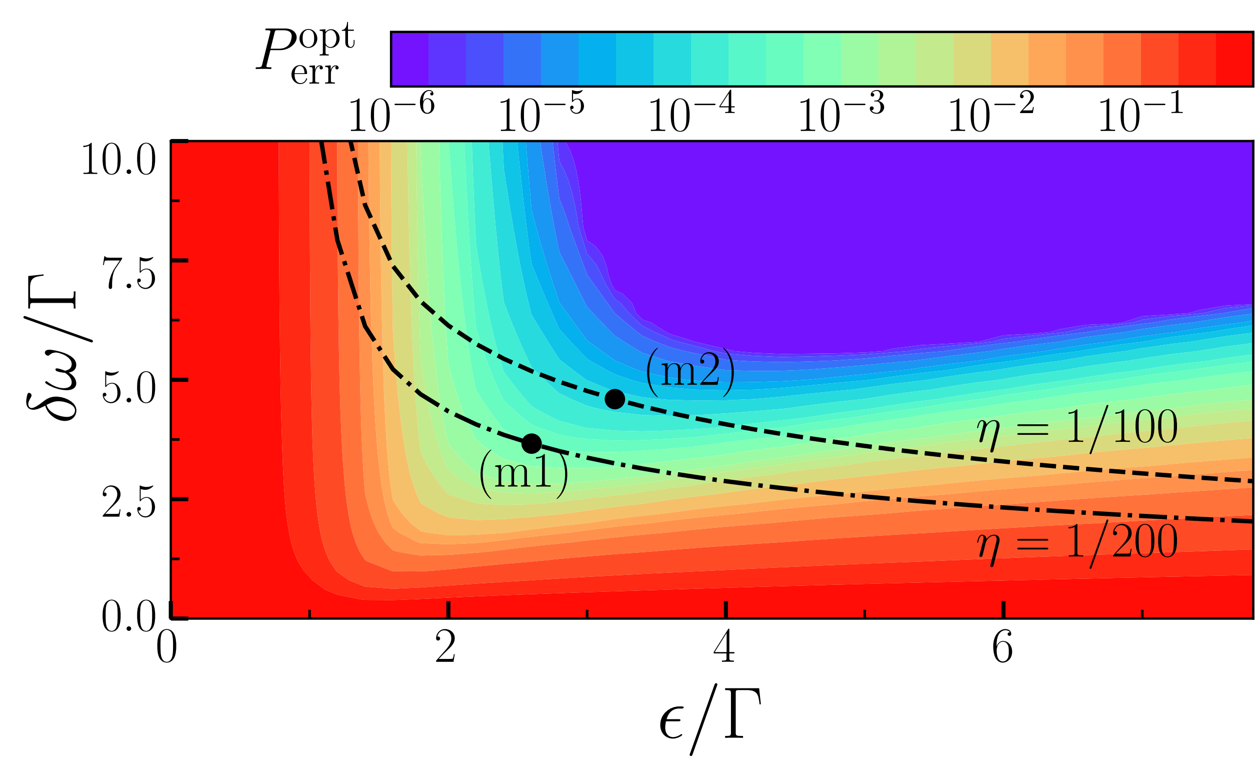
Readout error



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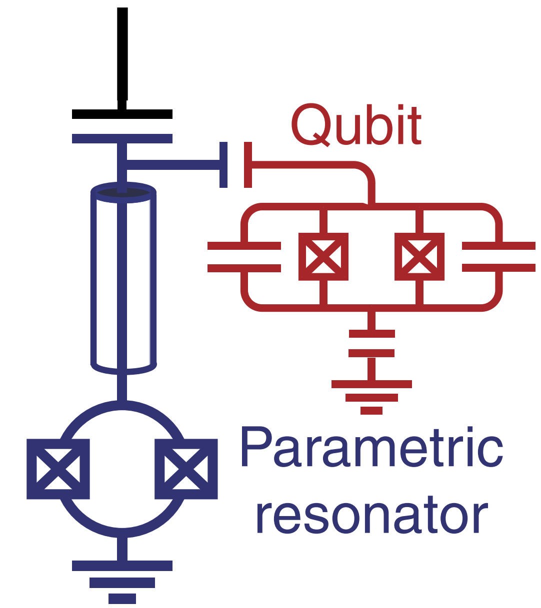
$$\dot{\rho} = -i[H,\rho] + \kappa N(a^\dagger \rho a - 1/2\{aa^\dagger, \rho\})$$

Hamiltonian

$$\hat{H}_{\text{Kerr}}/\hbar = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2}(\hat{a}^{\dagger 2} + \hat{a}^2) + \chi \hat{a}^{\dagger 2}\hat{a}^2$$

Dispersive coupling $H_{qc} = \delta\omega\ \hat{\sigma}_z \hat{a}^\dagger \hat{a}$



- P. Krantz et al, Nat. Comm. **7** 11417 (2016).

State discrimination:

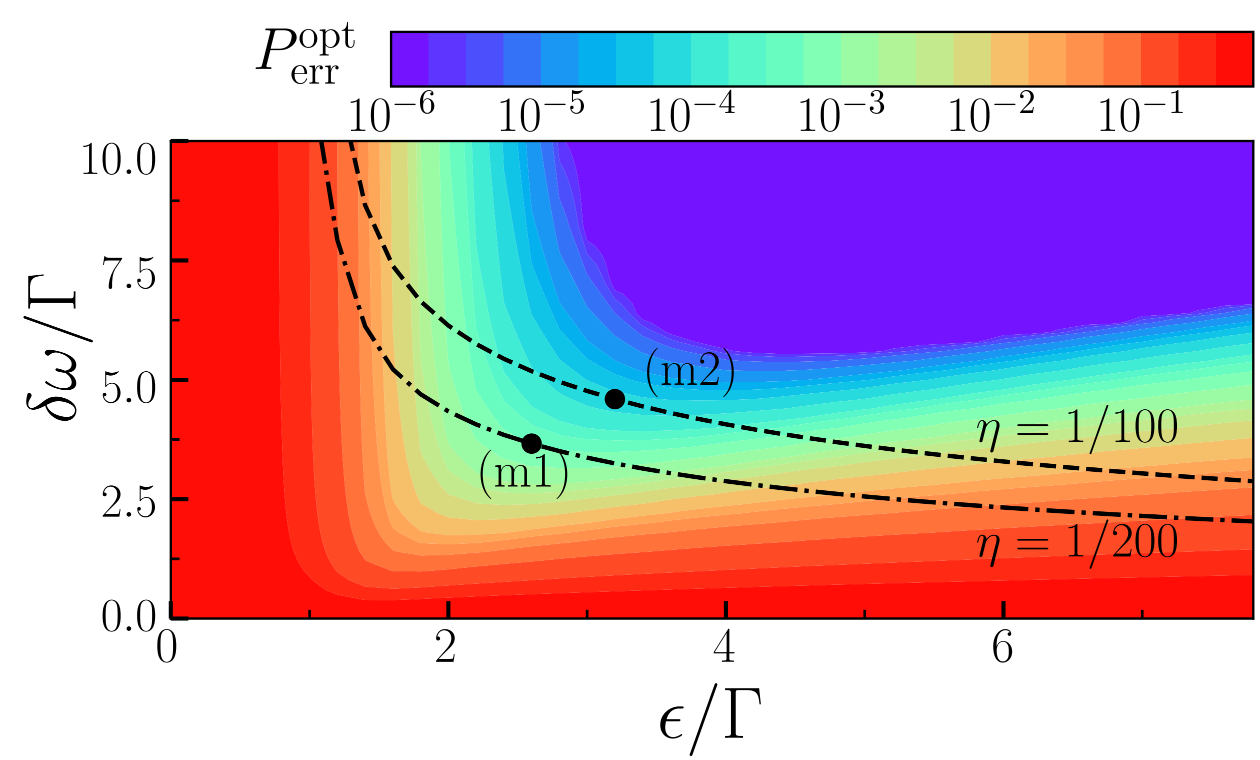
$$|0\rangle \longrightarrow \omega' = \omega - \delta\omega$$

$$|1\rangle \longrightarrow \omega' = \omega + \delta\omega$$

Qubit degradation is proportional to:

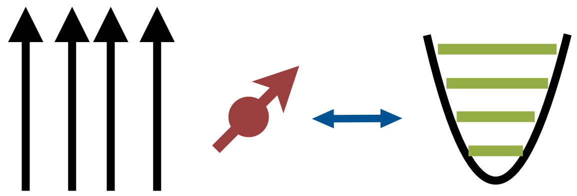
$$\eta = N\delta\omega^2/(4g^2)$$

Readout error



Conclusions

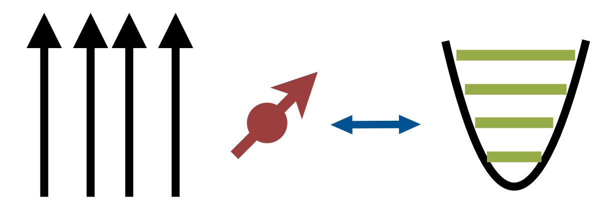
- **Finite-component PT for optimal quantum sensing**



- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

Conclusions

- **Finite-component PT for optimal quantum sensing**

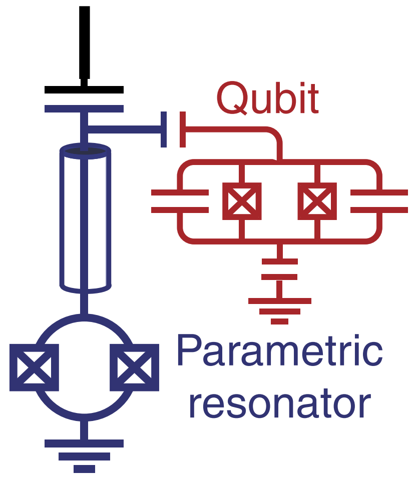


- L. Garbe, M. Bina, A. Keller, M. G. A. Paris, and S. Felicetti, Phys. Rev. Lett. **124**, 120504 (2020).

- **Critical parametric quantum sensor**

1- Quantum Magnetometry

2- Qubit readout



- R. Di Candia*, F. Minganti*, K.V. Petrovnin, G. S. Paraoanu, and S. Felicetti, arXiv:2107.04503 (2021).

Thanks!