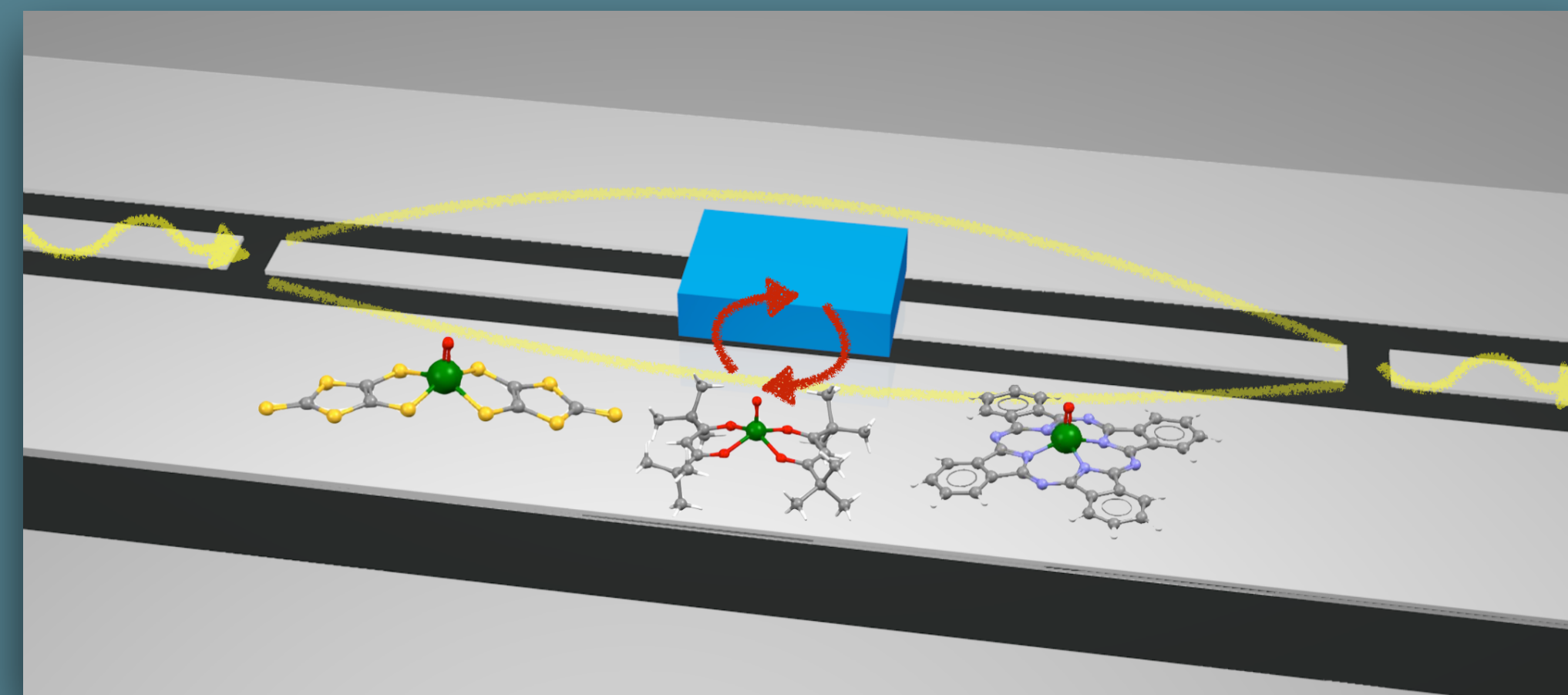
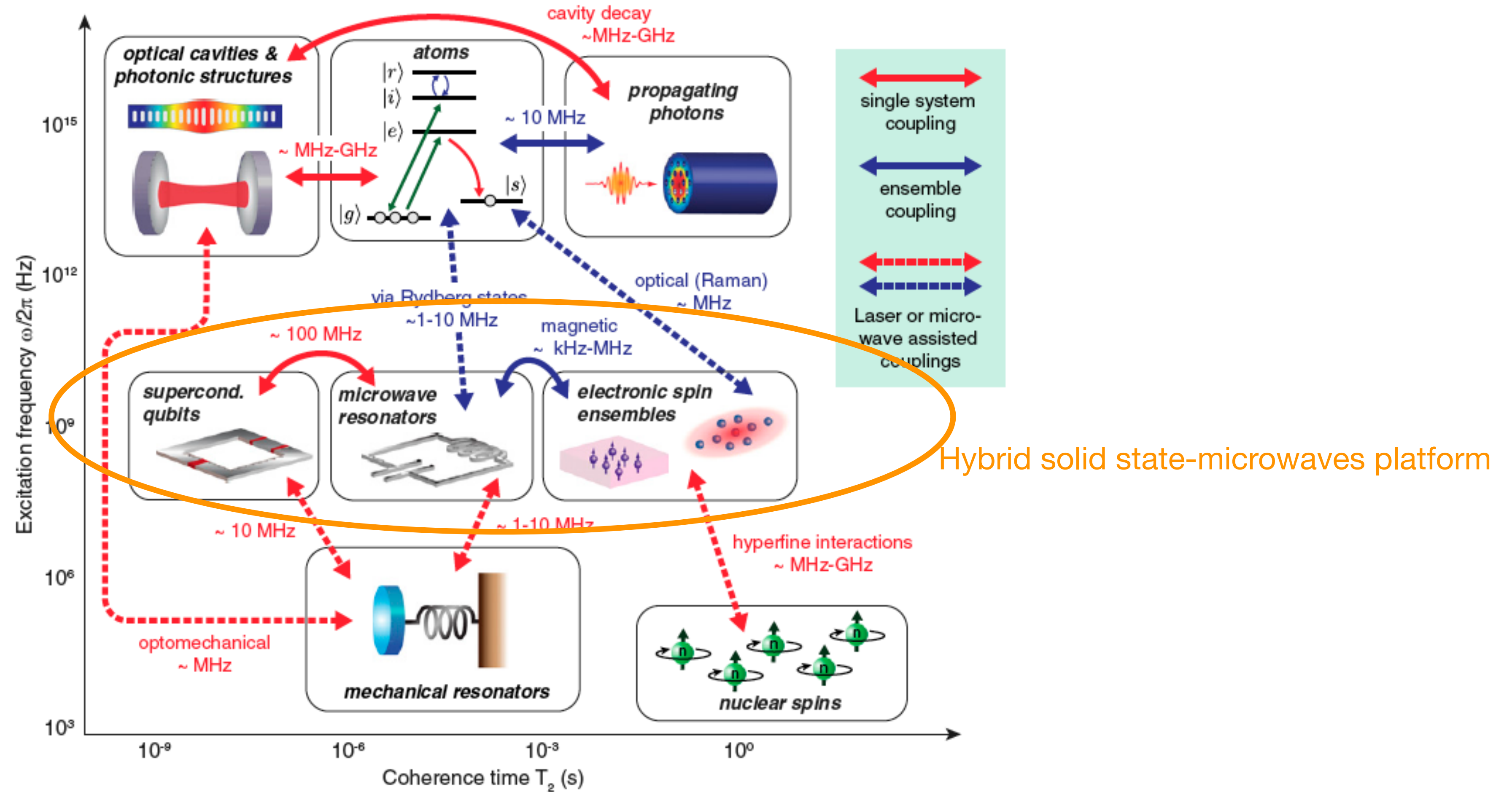


# HYBRID SUPERCONDUCTING CIRCUITS WITH SPINS



<https://www.lowtlab.unimore.it/>

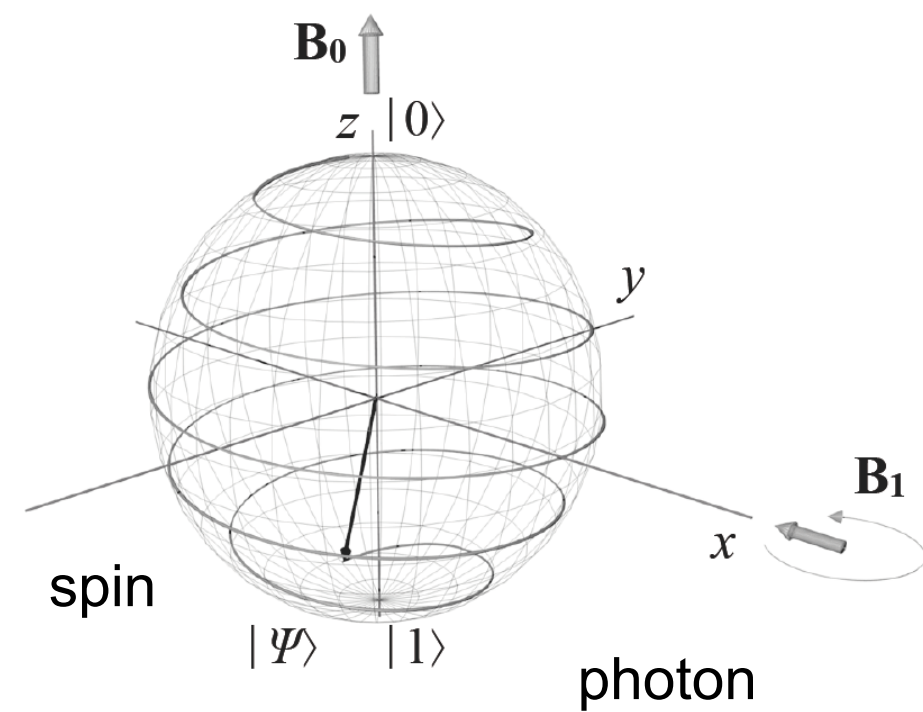
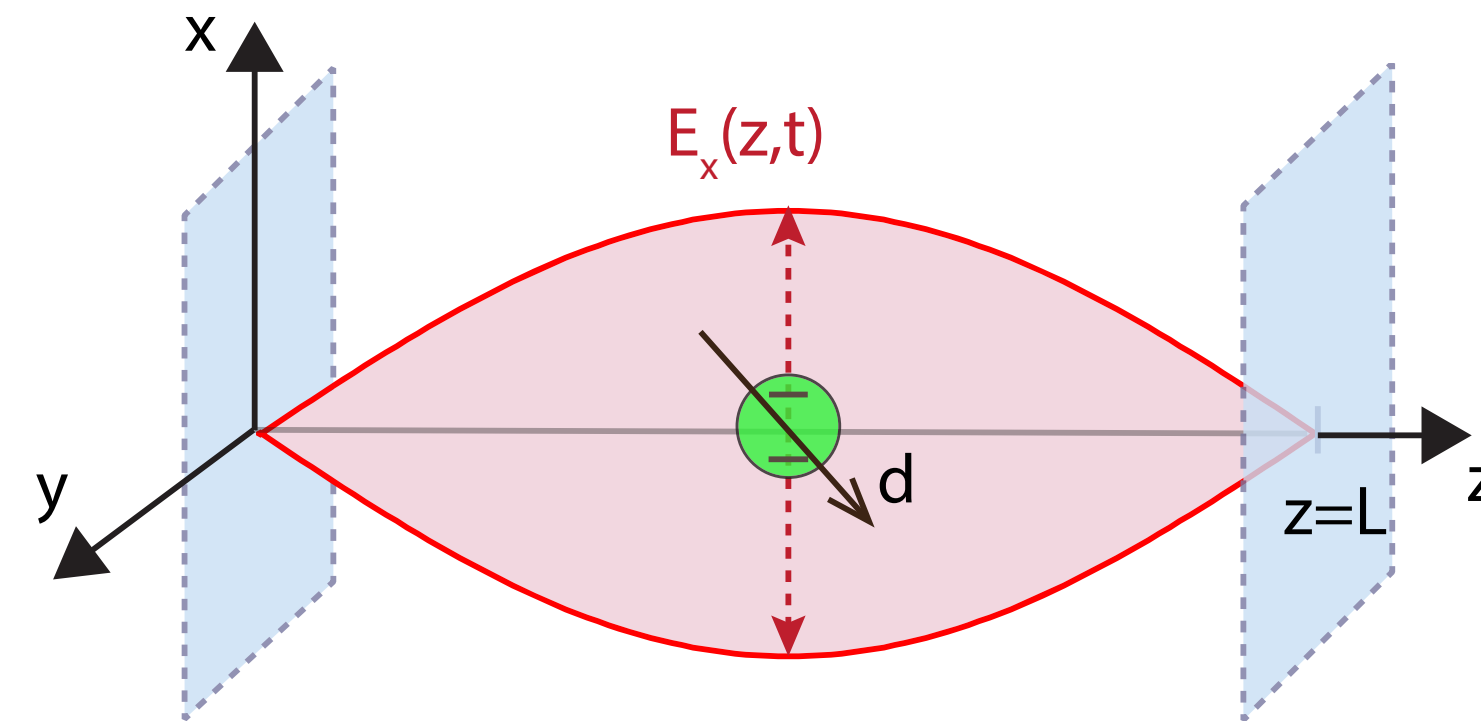
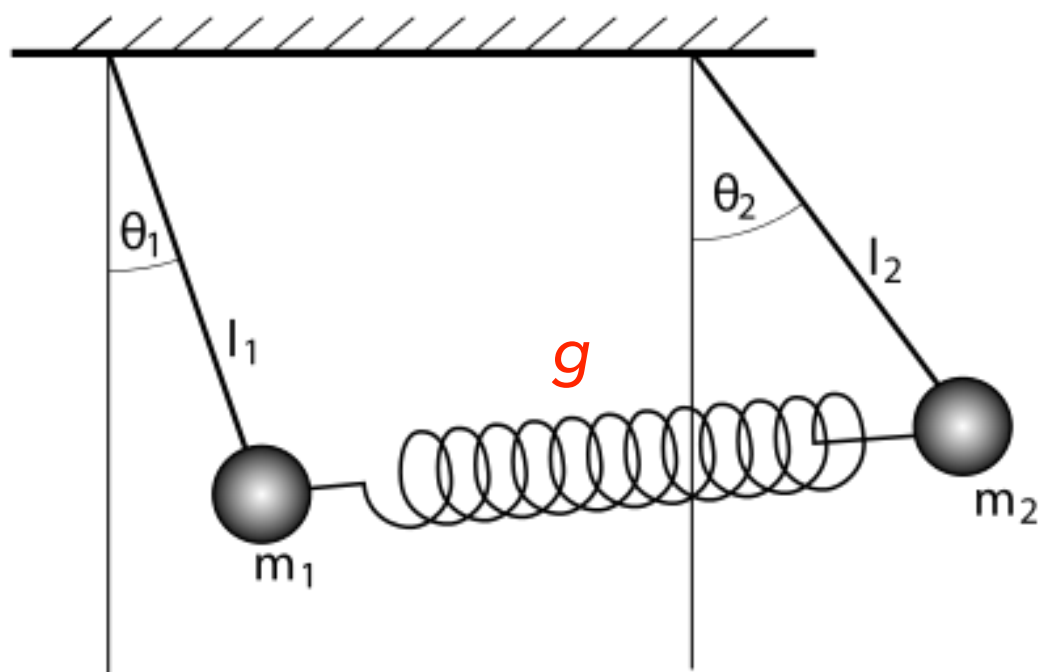
# Platforms for Solid States Sciences & Technologies



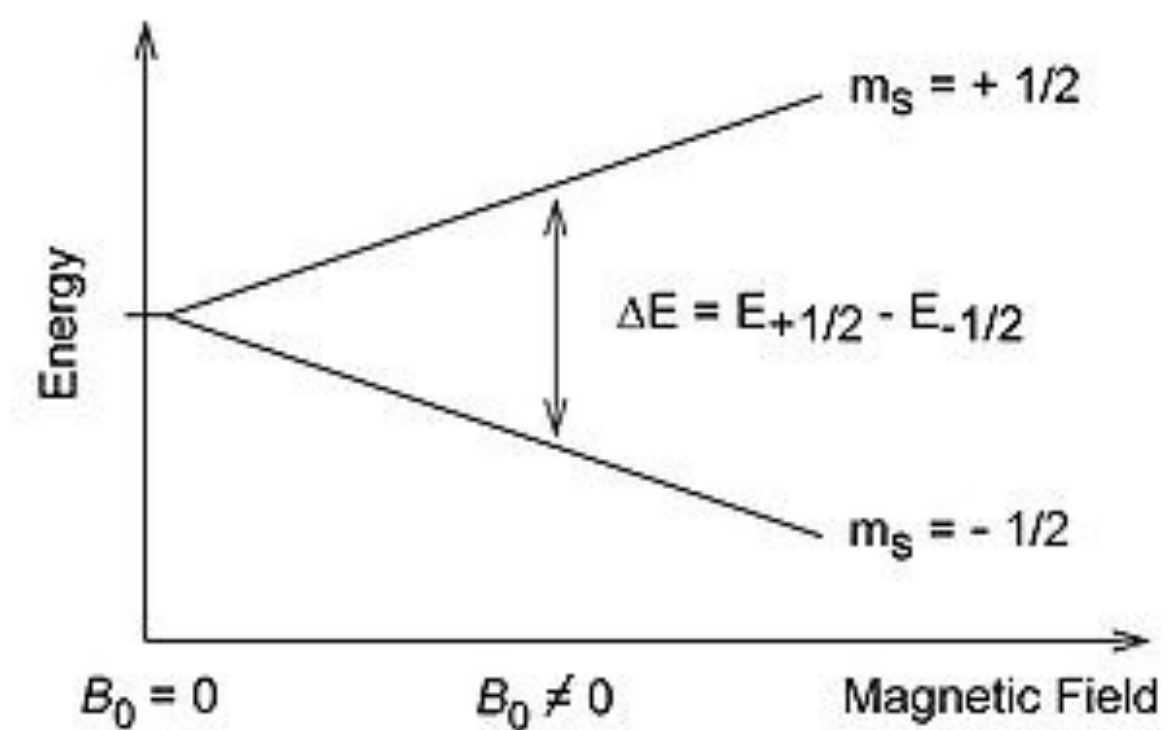


# two coupled oscillators: the case of spin-photon

classical model for two coupled oscillators:



*g* coupling strength



$\gamma_s(T_1, T_2)$   
spin damping

$\kappa = f_0/Q_L$   
cavity losses

- The Rabi model

$$H = H_{ph} + H_{TLS} + H_{TLS-ph}$$

$$H_{ph} = h\nu_0 a^\dagger a$$

$$H_{TLS} = h\nu_{TLS} \sigma_{TLS}^+ \sigma_{TLS}^-$$

$$H_{int} = -(\gamma\mu\mathbf{S}) \cdot \mathbf{B} \quad \text{dipole interaction}$$

$$H_{int} = -g(\hat{a} + \hat{a}^\dagger)(\sigma_+ + \sigma_-),$$

$$H_{\text{Rabi}} = \omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) - \frac{1}{2} \omega_q \sigma_z - g(\hat{a} + \hat{a}^\dagger)(\sigma_- + \sigma_+).$$

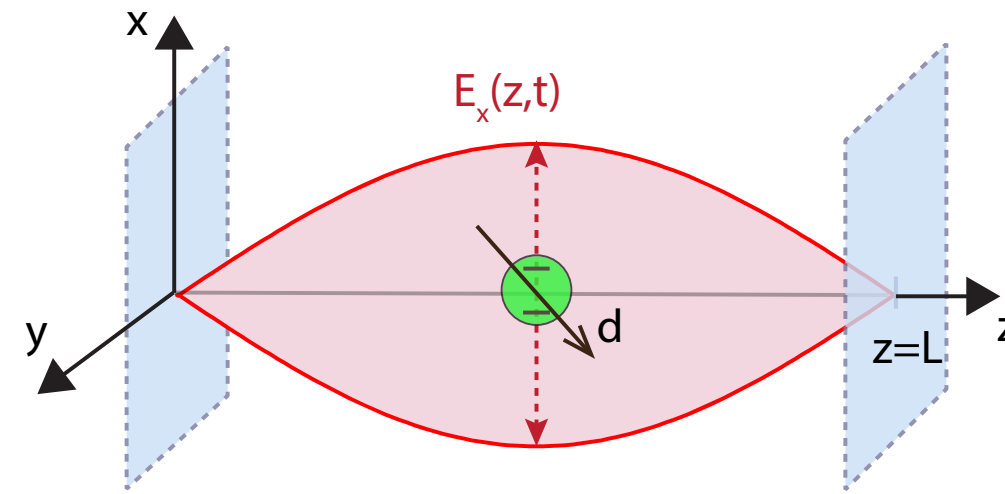
$$H_{int} \Rightarrow \hat{a}^\dagger \sigma_- + \hat{a} \sigma_+ + \hat{a}^\dagger \sigma_+ + \hat{a} \sigma_-$$

Rotating Wave Approximation (RWA):

$$g \ll \omega_q, \omega_c, \text{ and also } |\omega_c - \omega_q| \ll |\omega_c + \omega_q|.$$

Janes Cumming hamiltonian in RWA:

$$H_{\text{JC}} = \omega_c \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) - \frac{1}{2} \omega_q \sigma_z - g(\hat{a}^\dagger \sigma_- + \hat{a} \sigma_+).$$

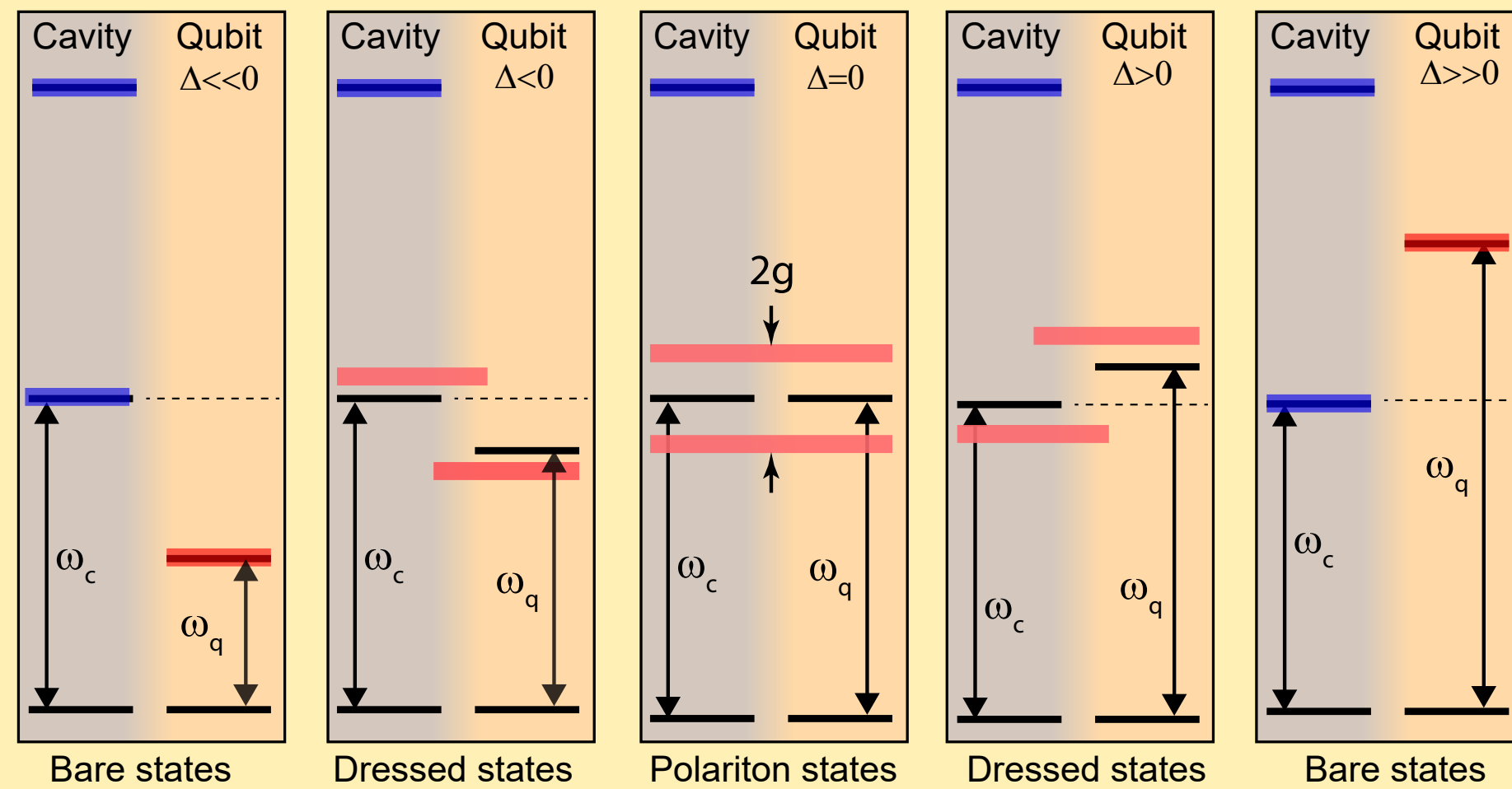


**peculiarities of spin systems:**

- dipolar coupling with single spin is weak
- diamagnetic term can be neglected (?)
- applications specifically developed for magnetic systems

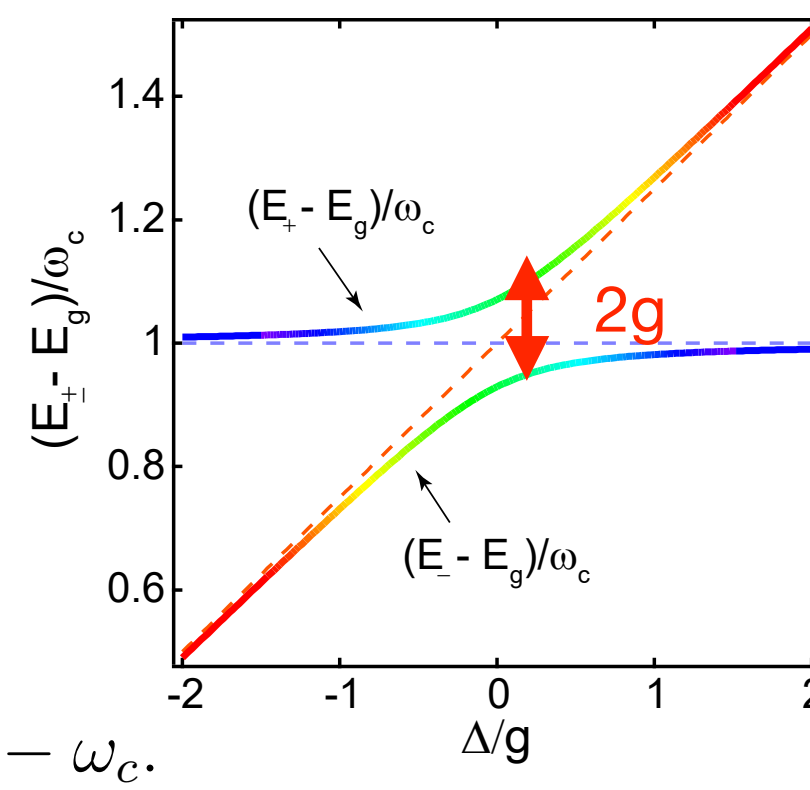
# states & energy levels

- $|g\rangle|0\rangle \rightarrow$  qubit in ground state, no photons in the cavity
- $|g\rangle|n+1\rangle \rightarrow$  qubit in ground state,  $n+1$  photons in the cavity
- $|e\rangle|n\rangle \rightarrow$  qubit in excited state,  $n$  photons in the cavity



## polariton-like branches

$$E_{\pm} - E_g = \omega_c \mp \frac{1}{2} \sqrt{4g^2 + \Delta^2} + \Delta/2.$$

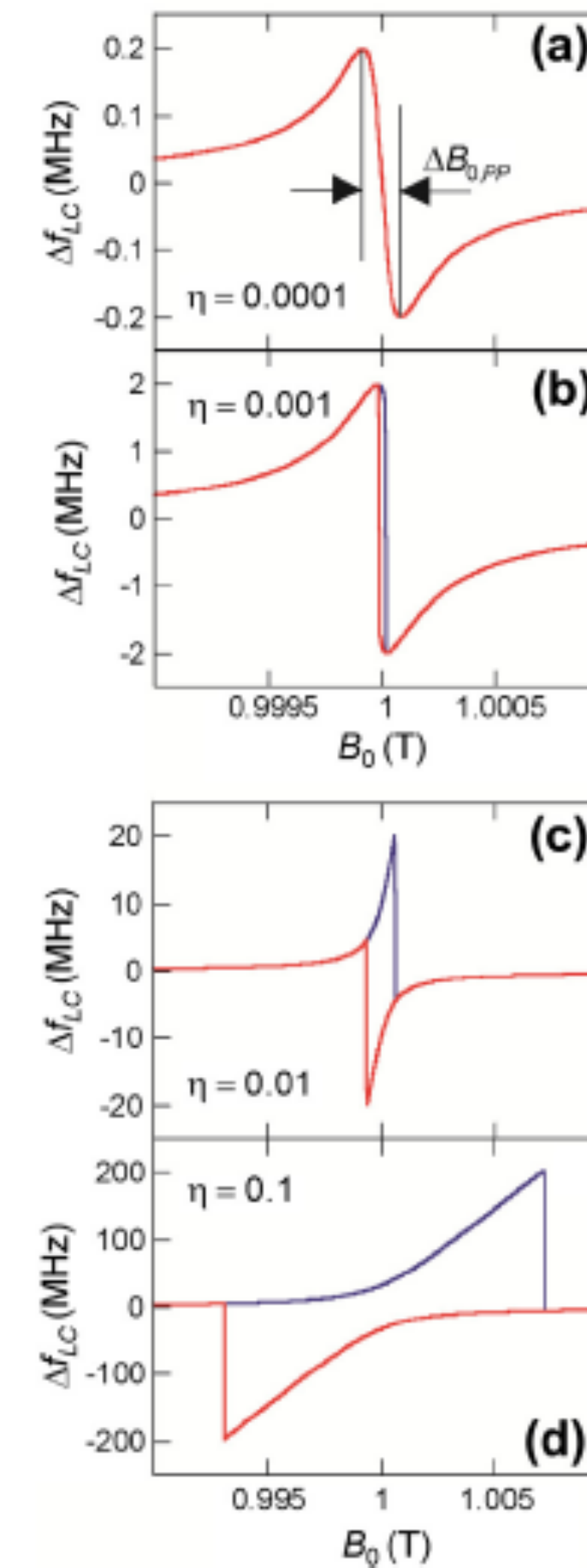


$$\Delta = \omega_q - \omega_c.$$

# phenomenology: transmission spectra

*weak coupling:  $g < \gamma; k$*

coupling strength  $g$

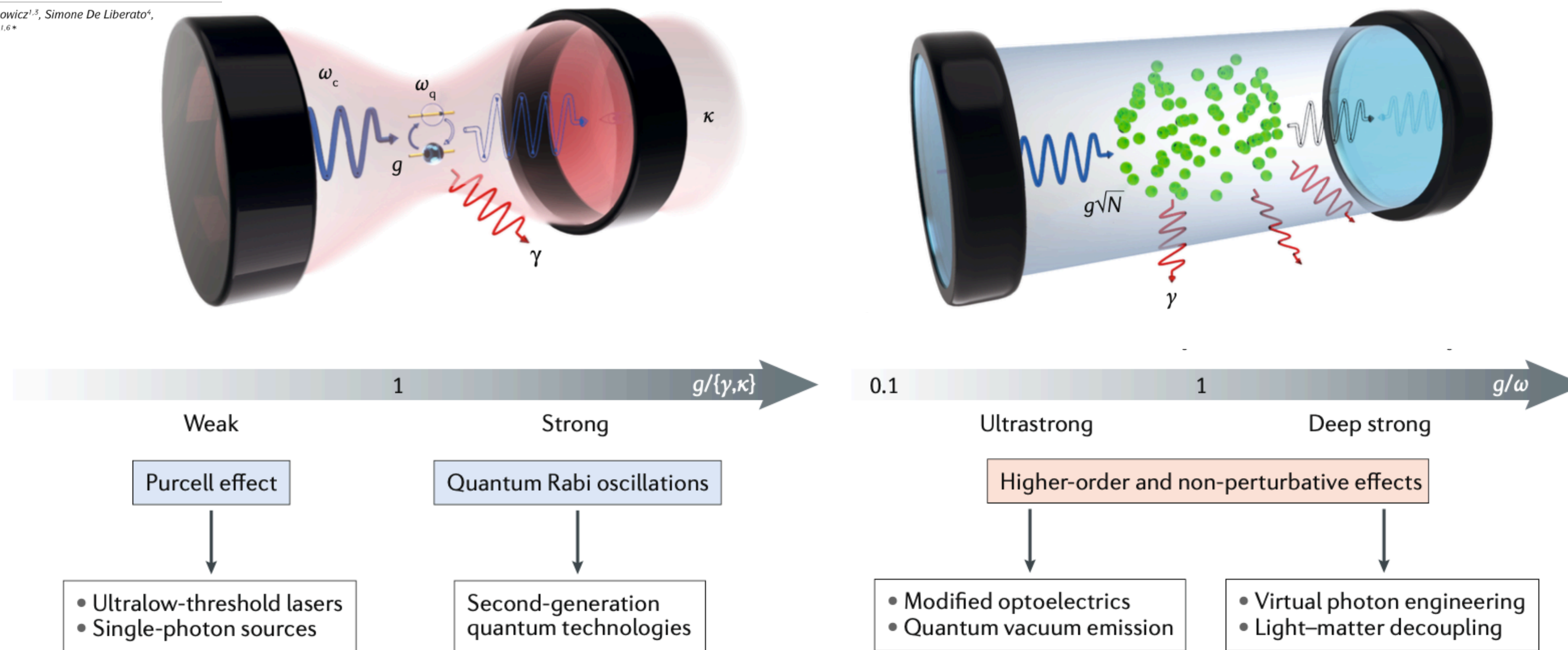


*strong coupling:  $g > \gamma; k$*



# Ultrastrong coupling between light and matter

Anton Frisk Kockum<sup>1,2\*</sup>, Adam Miranowicz<sup>1,3</sup>, Simone De Liberato<sup>4</sup>, Salvatore Savasta<sup>1,5</sup> and Franco Nori<sup>1,6\*</sup>



Ultrastrong coupling (USC) and deep strong coupling regimes:

- processes that do not conserve the number of interactions and ground state that contains virtual excitations
- potential applications in quantum technology, nonlinear optics, modified chemical reactions and enhancement of various quantum phenomena

Forn-Diaz et al. Rev. Mod. Phys. 91, 025005 (2019)

Kockum et al. Nat. Rev. Phys. 1, 20 (2019)

# Magnetic Coupling strength

*how can we increase the magnetic coupling?*

Single spin coupling:  $g_s \propto B_{ac}$

The spin-photon coupling rate  $g/h$  is assumed to be equal to half the Rabi frequency of an electron under the resonator magnetic vacuum field, which has amplitude  $B_{vac}$  and direction perpendicular to  $B_0$ :

$$\frac{g}{h} = \frac{\gamma_e B_{vac}}{4} \quad B_{vac} \approx \frac{\mu_0 I_{vac}}{2w}, \quad I_{vac} = \pi \sqrt{\frac{h}{Z_0}} \nu_0 \Rightarrow I_{vac}[A] \approx 1.14 \times 10^{-17} \nu_0[\text{Hz}]$$

AIP Advances 4, 087122 (2014); <https://doi.org/10.1063/1.4893242>

PRL 102, 083602 (2009)

PHYSICAL REVIEW LETTERS

week ending  
27 FEBRUARY 2009

**Cavity QED Based on Collective Magnetic Dipole Coupling:  
Spin Ensembles as Hybrid Two-Level Systems**

Atac Imamoglu

$g = g_s \sqrt{N}$  collective coupling

$$g_s \sim \text{Hz}$$

$$\text{if } N \sim 10^{12} \\ g = g_s \sqrt{N} \sim \text{MHz}$$

$$\text{if } N \sim 10^{18} \\ g = g_s \sqrt{N} \sim \text{GHz}$$

Coupling depends on the effective overlapping of the spin density and the e.m. excitation, that is on the filling factor:

$$g_i^2 = \chi_{\text{eff}} \omega^2 \xi, \quad \xi = \frac{\int \int \int_{V_{\text{YIG}}} \mu_0 \vec{H}^* \vec{H} dV_{\text{YIG}}}{\int \int \int_V \mu_0 \vec{H}^* \vec{H} dV}$$

# OUTLINE

- GENERAL PROBLEM hybrid circuits (scalability)**

- IMPLEMENTATION**

  - strategies to enhance the coupling:

  - resonators: YBCO circuits

  - spin systems: MolSpin & YIG

- TOWARDS USC**

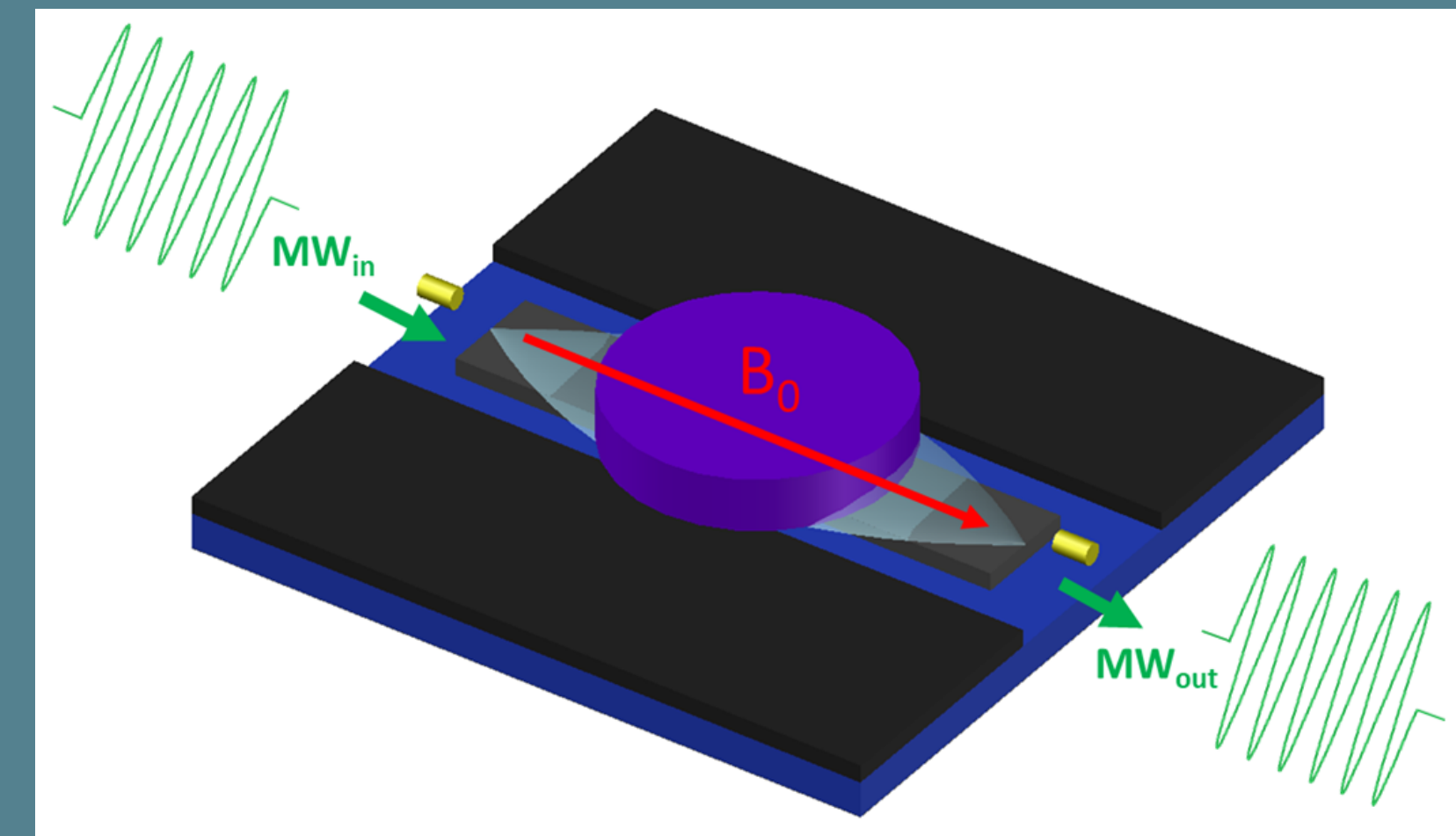
  - experiments & model

    - CW spectra: examples of S21 dispersion data analysis

- PESPECTIVES**

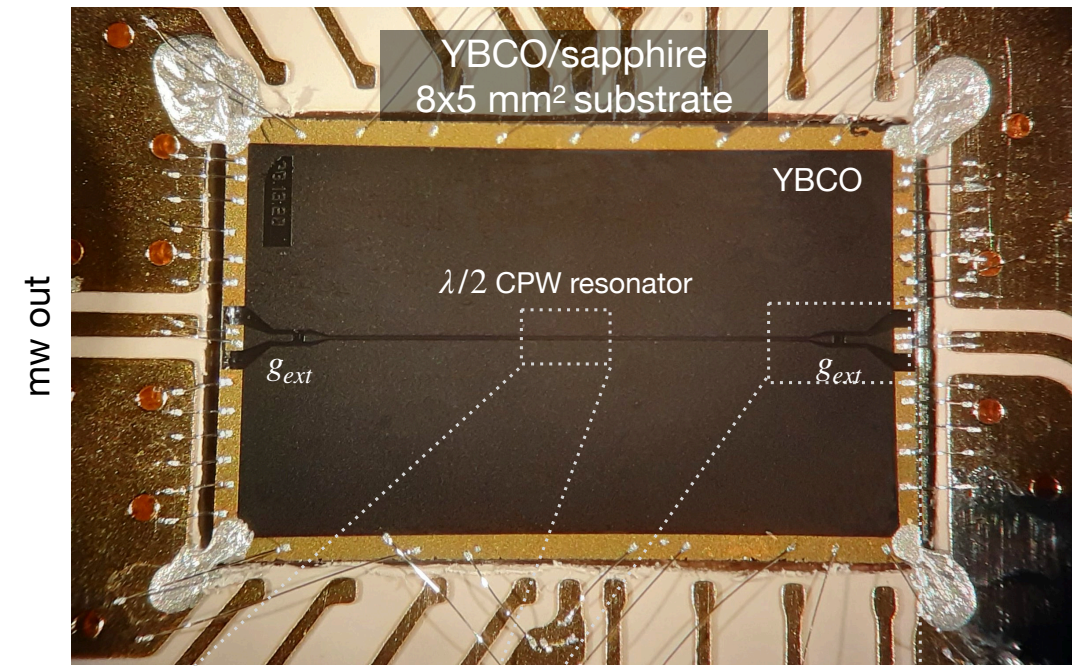
  - new phenomena

  - Q-sensing with scalable architectures





# Planar superconducting resonators



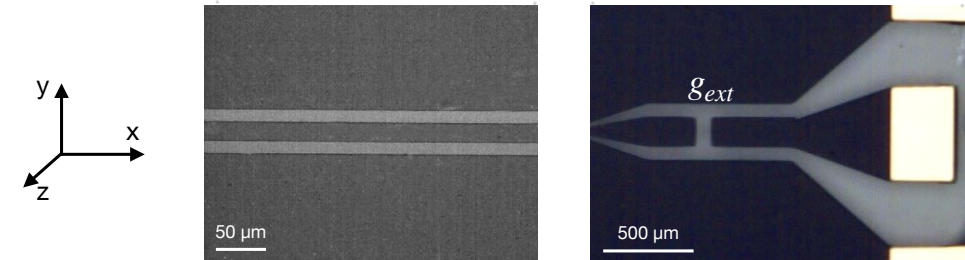
YBCO thickness:  
330 nm

YBCO coplanar back-plated waveguide resonator

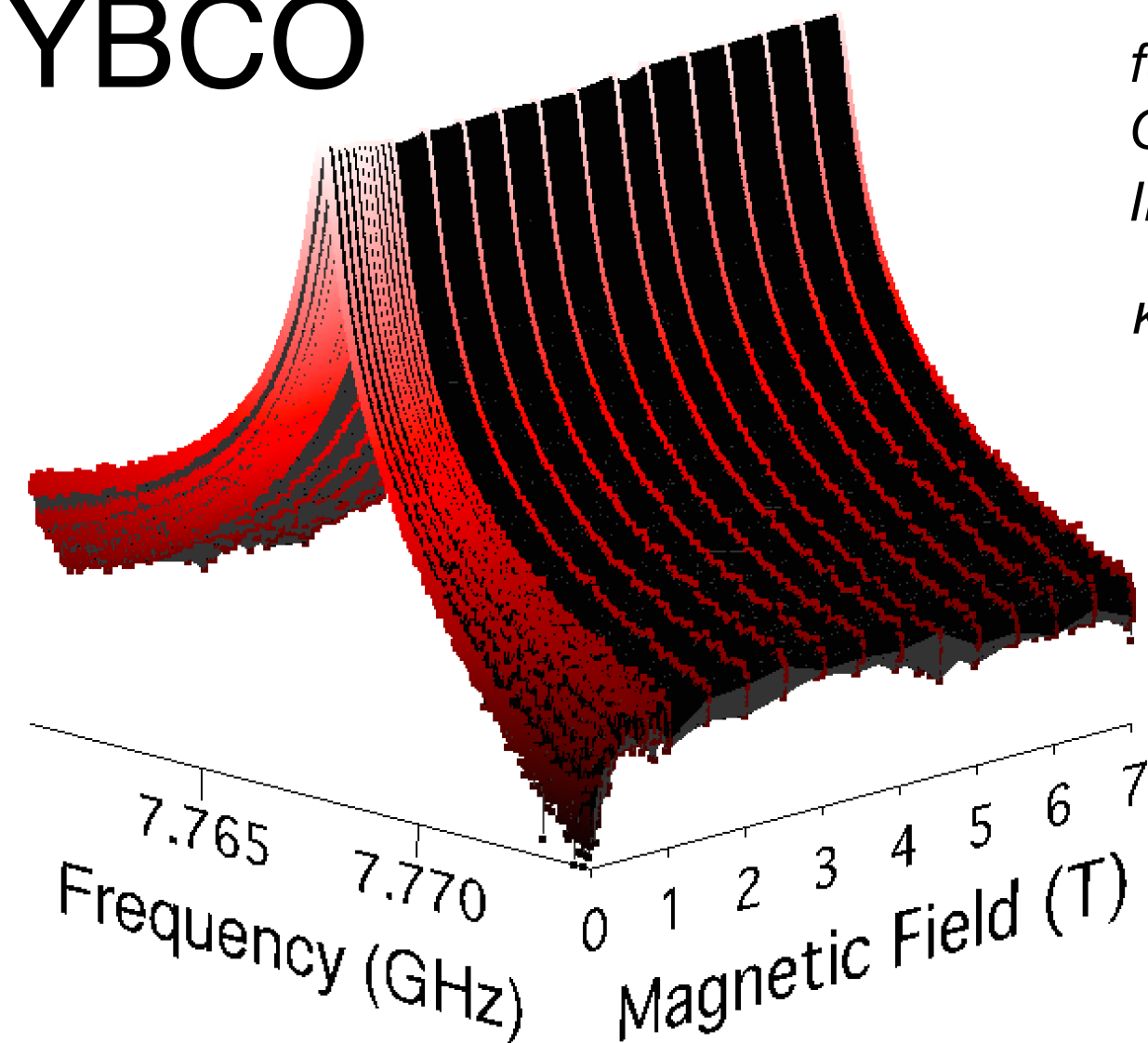
YBCO (330 nm) (Ceraco GmbH):  $T_c=87$  K

sapphire (430  $\mu\text{m}$ ):  $\epsilon_r \approx 11$ ;  $\tan(\delta) \sim 10^{-6}$  at low T

$w=200$   $\mu\text{m}$ ,  $s=73$   $\mu\text{m}$ :  $Z_0 \approx 50$  Ohm



## YBCO



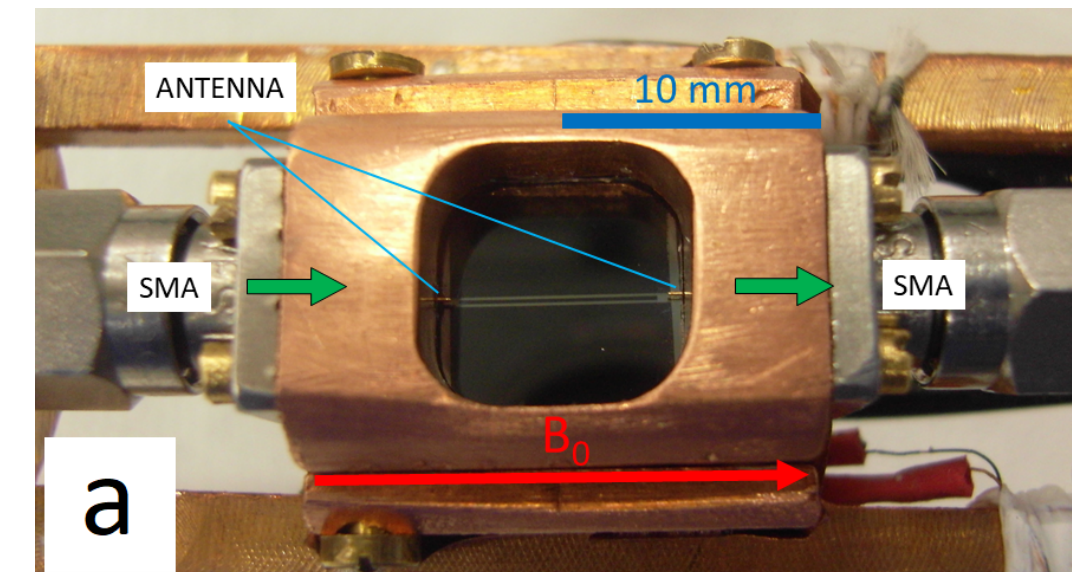
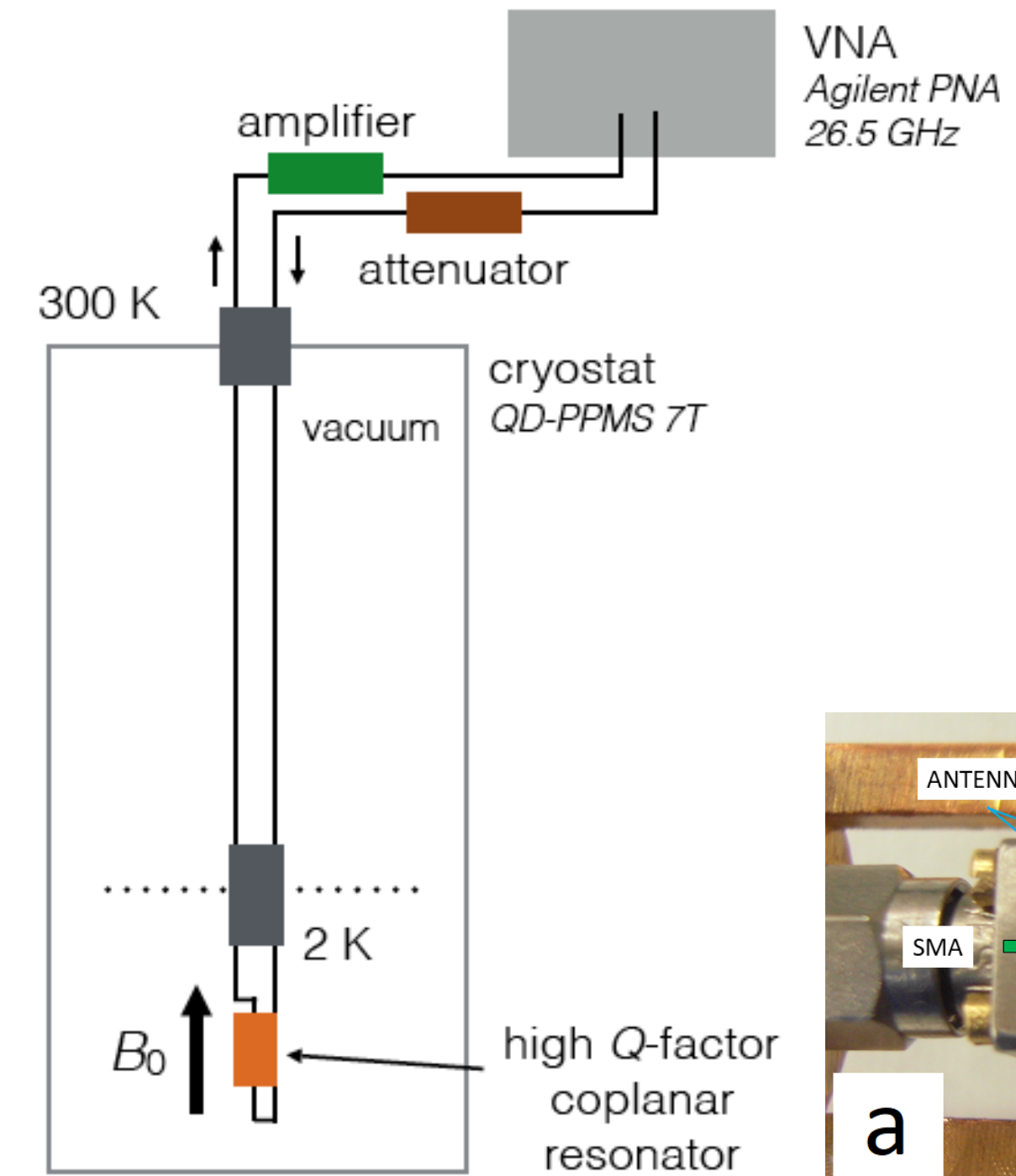
$f_0=7.7553$  GHz

$Q_L \approx 30000$

$IL = -16.5$  dB

$\kappa = f_0/Q_L = 0.5$  MHz

# CW spectroscopy

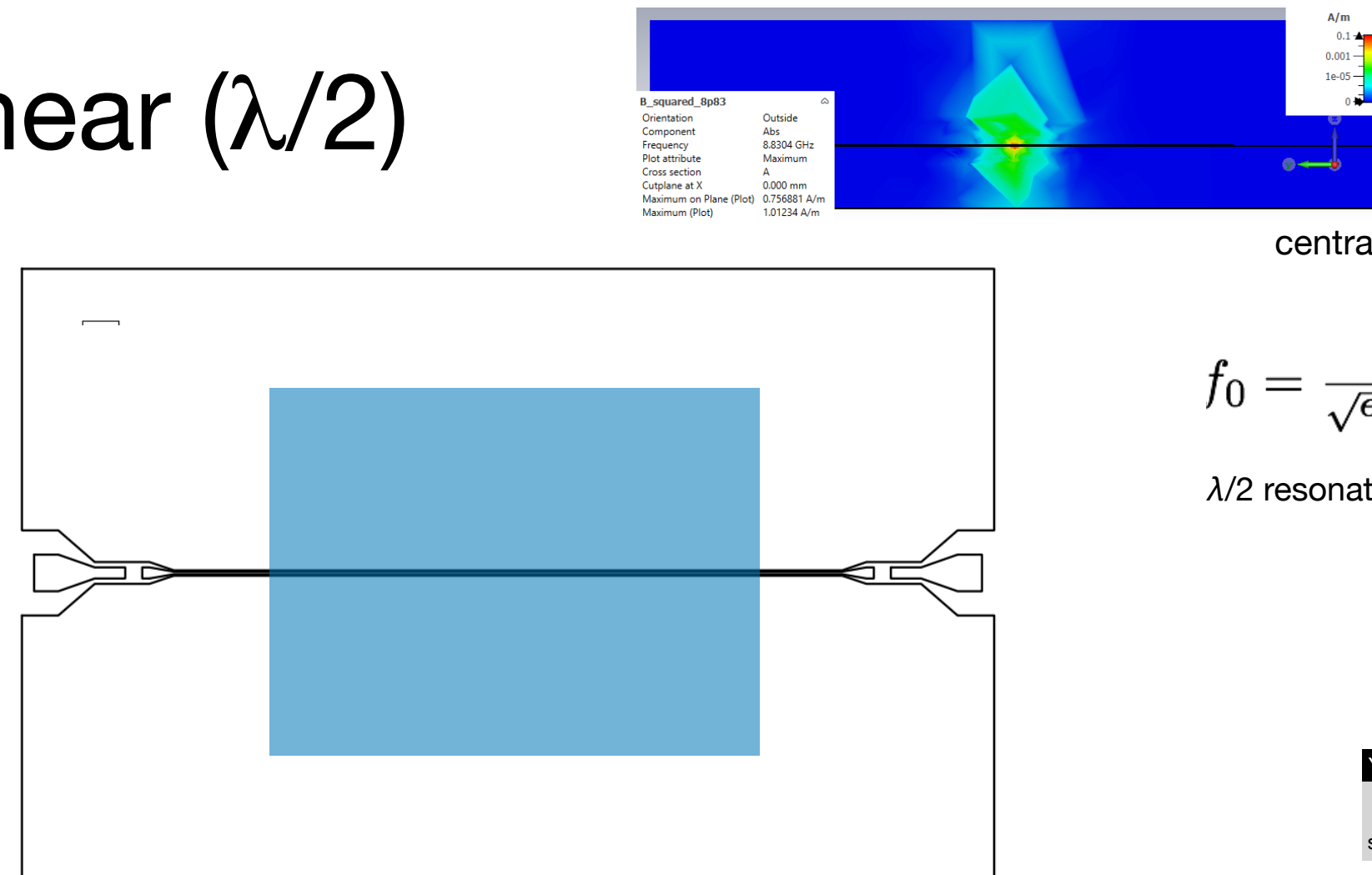


C. Bonizzoni, A. Ghirri, M. Affronte *Advances in Physics X* 3:1, 1435305, DOI: 10.1080/23746149.2018.1435305 (2018)

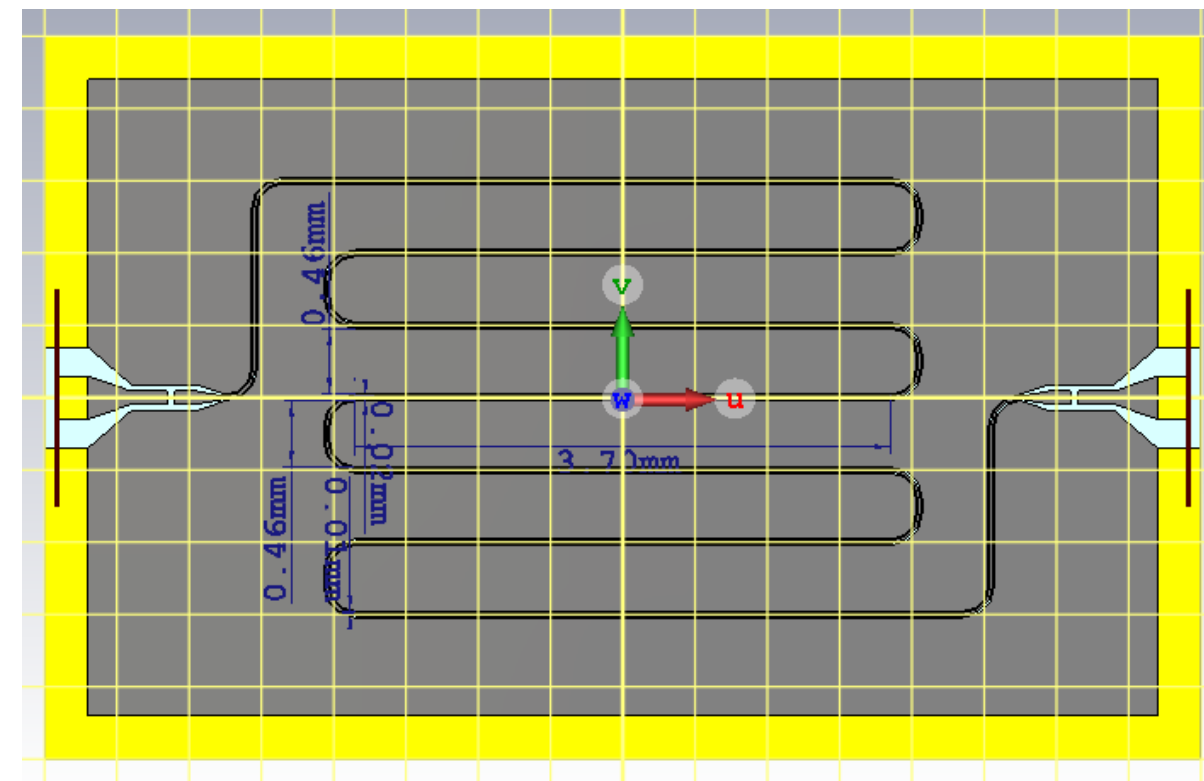
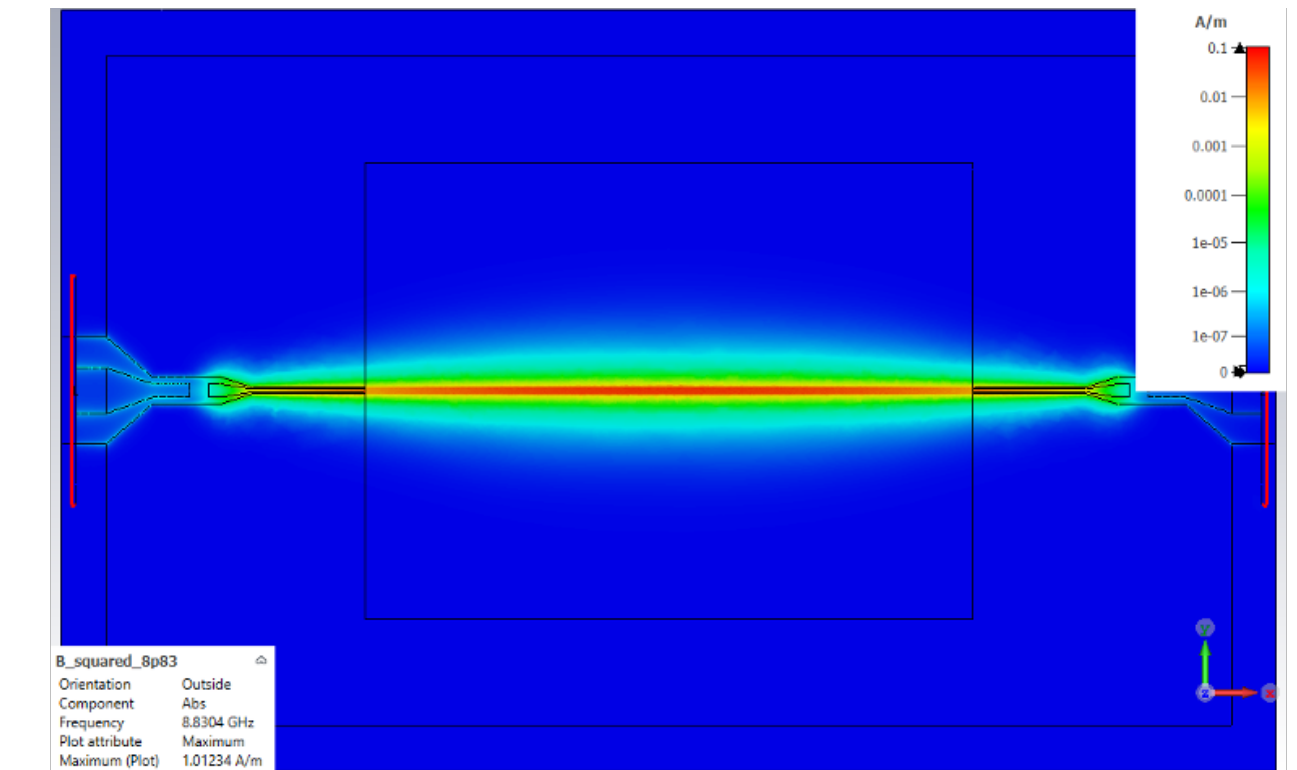
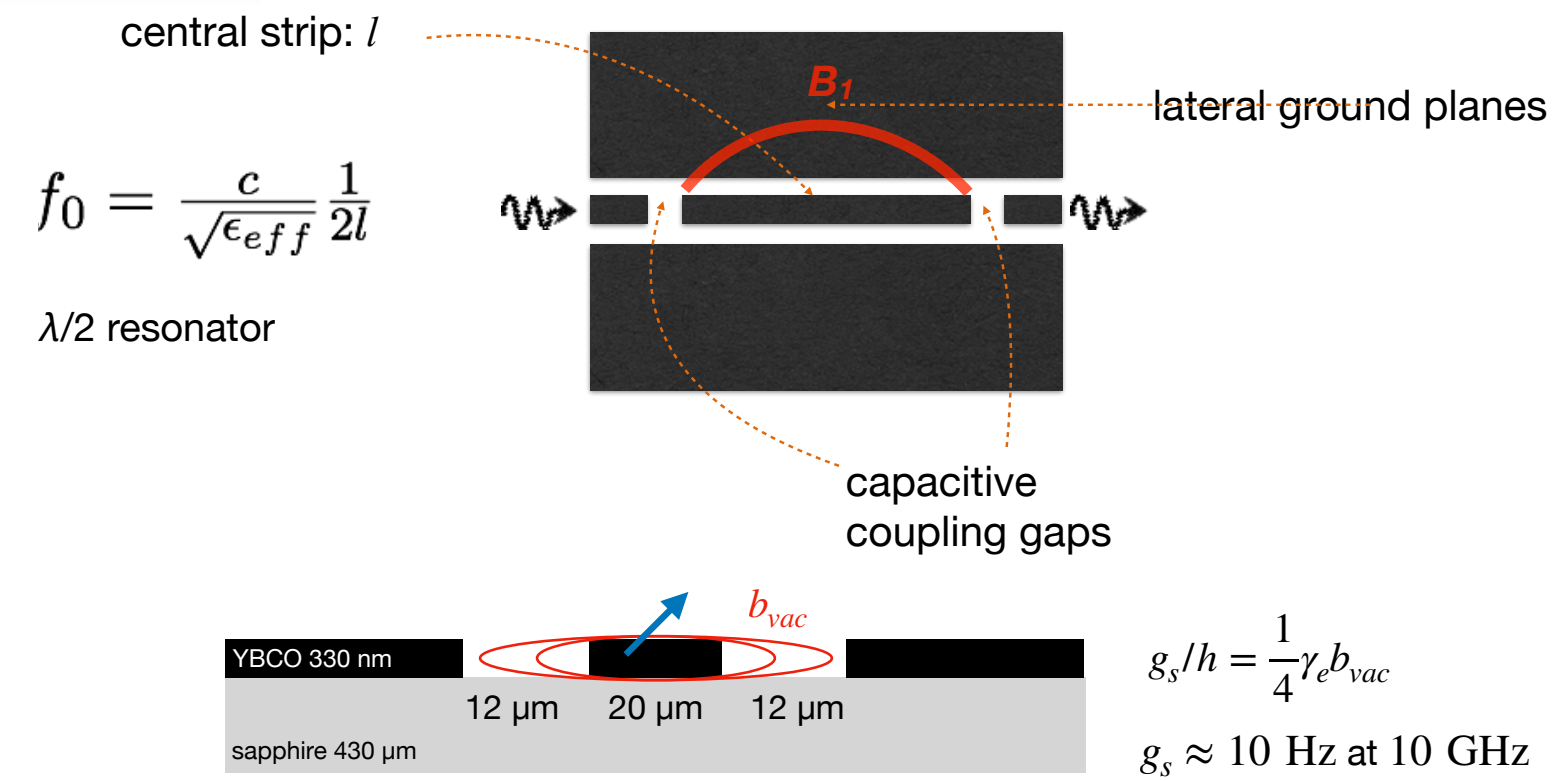


# Optimizing the effective volume & $B_{ac}$ strenght

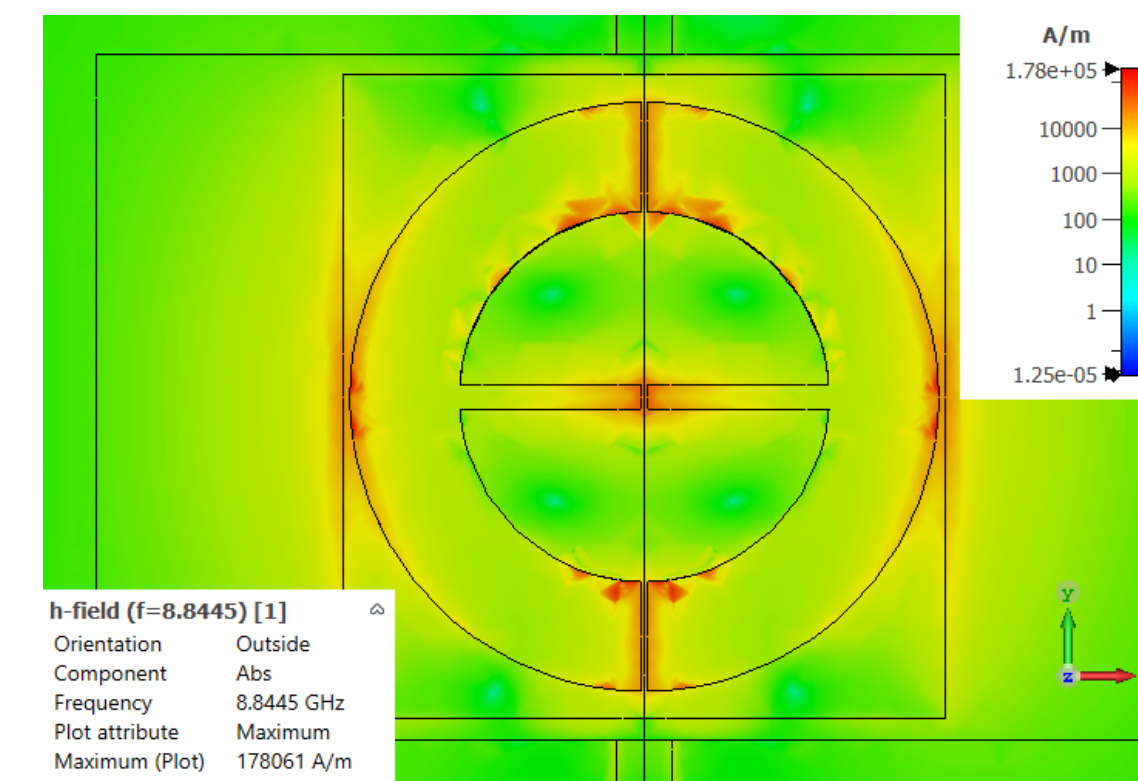
Linear ( $\lambda/2$ )



CoPlanar Waveguide ( $\lambda/2$ )



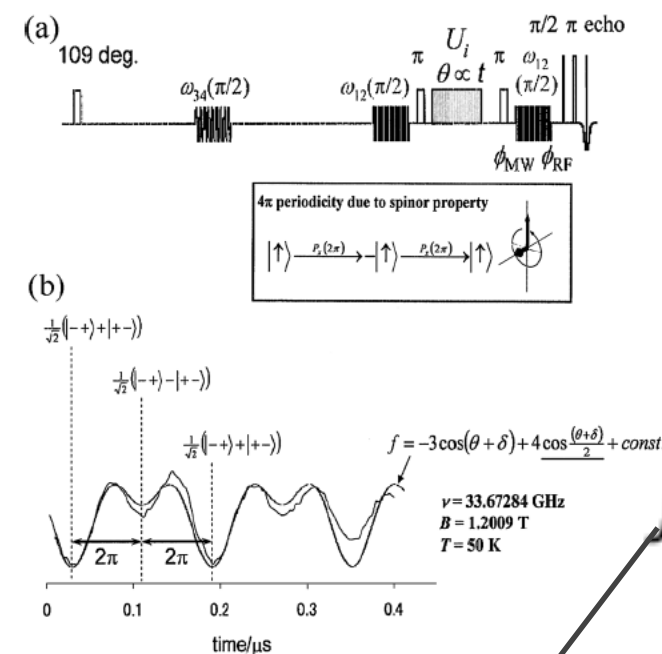
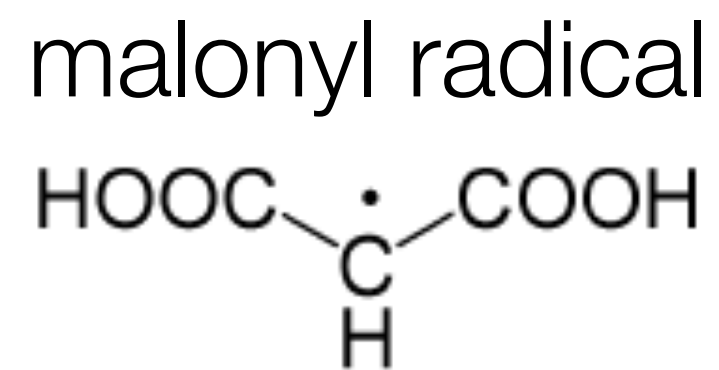
Meander [multi-frequency]



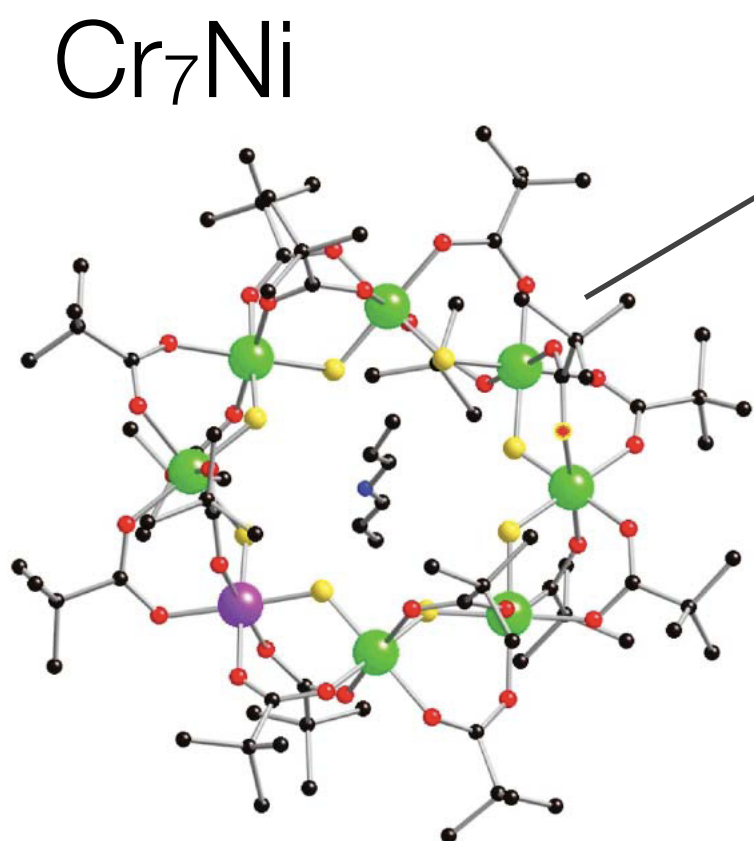
Inverse Anapole  
[radiation' field focused on small volume]

# Paramagnetic centers Molecular spins electron spin coherence $> \mu\text{s}$

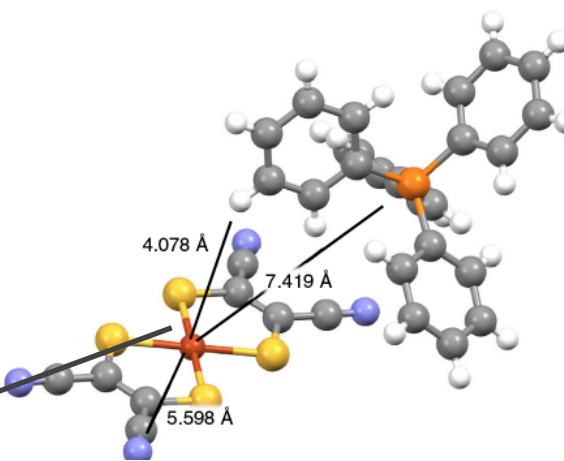
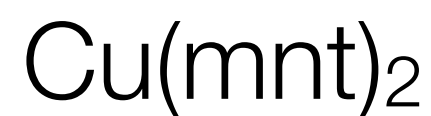
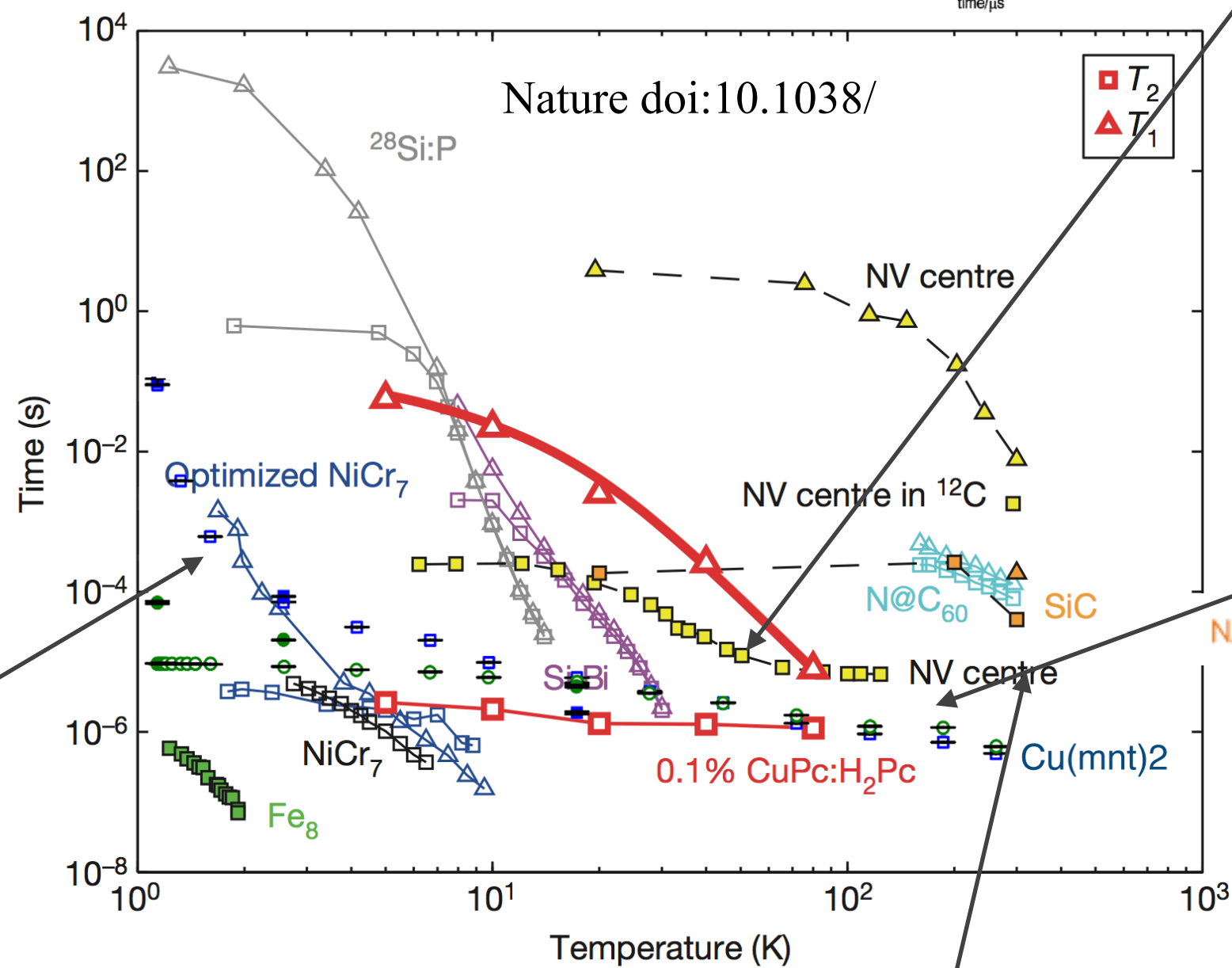
line width  $\sim 10\text{MHz}$



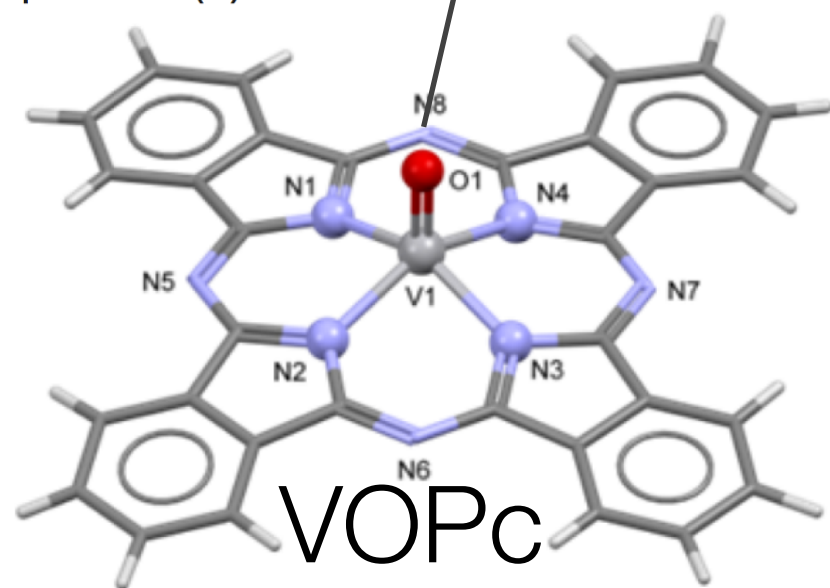
*J. Mater. Chem.*, 2009, 19, 3739–3754



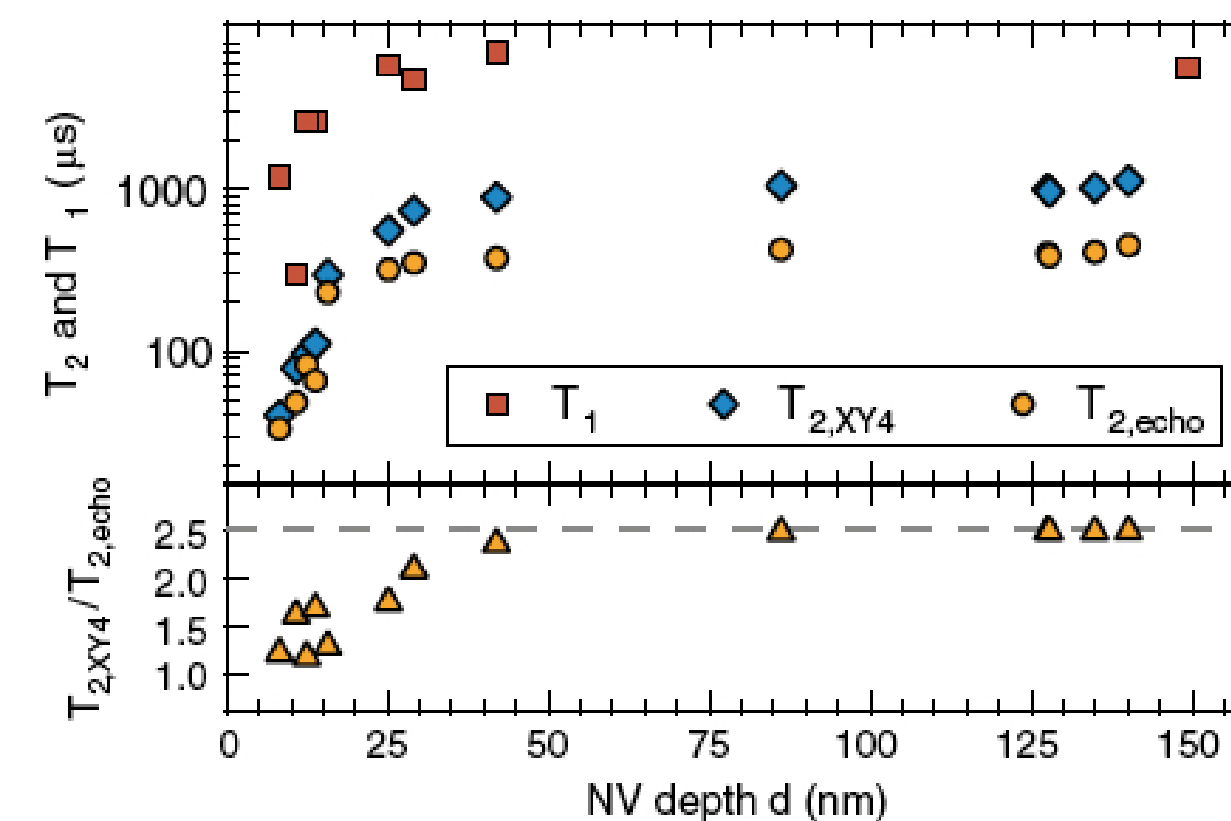
Wedge et al. PRL 108, 107204 (2012)



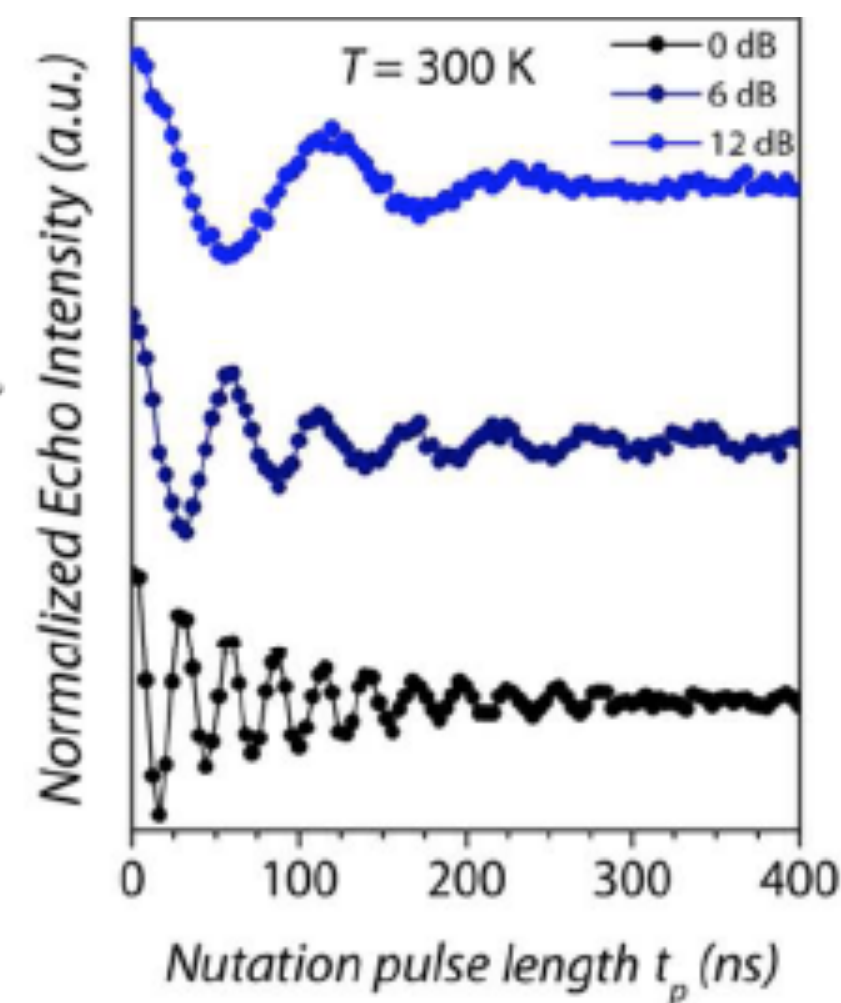
*NATURE COMMUNICATIONS* |5:5304| DOI: 10.1038/ncomms6304



M. Atzori et. al. *JACS* 138, 2154-2157 (2016)



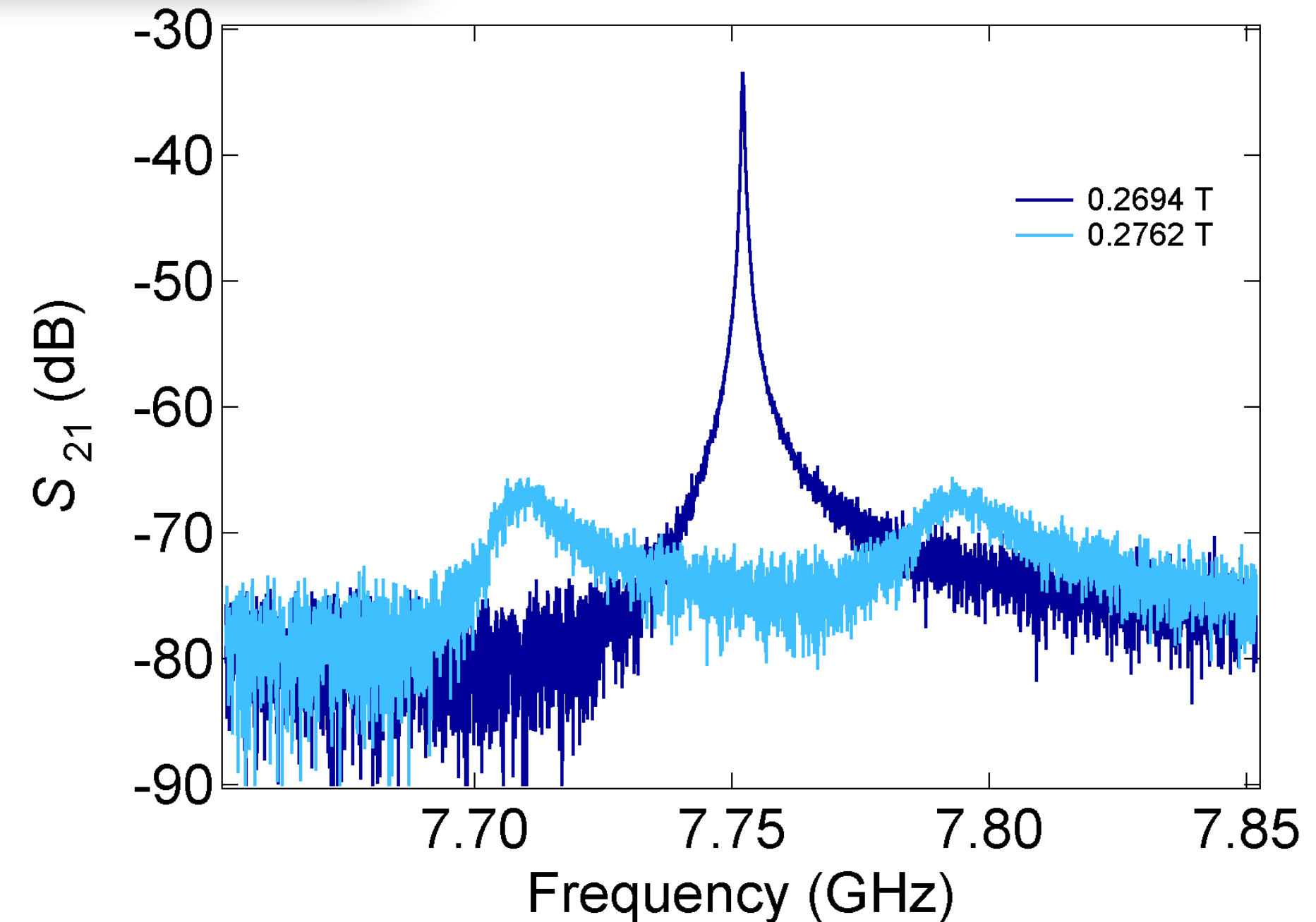
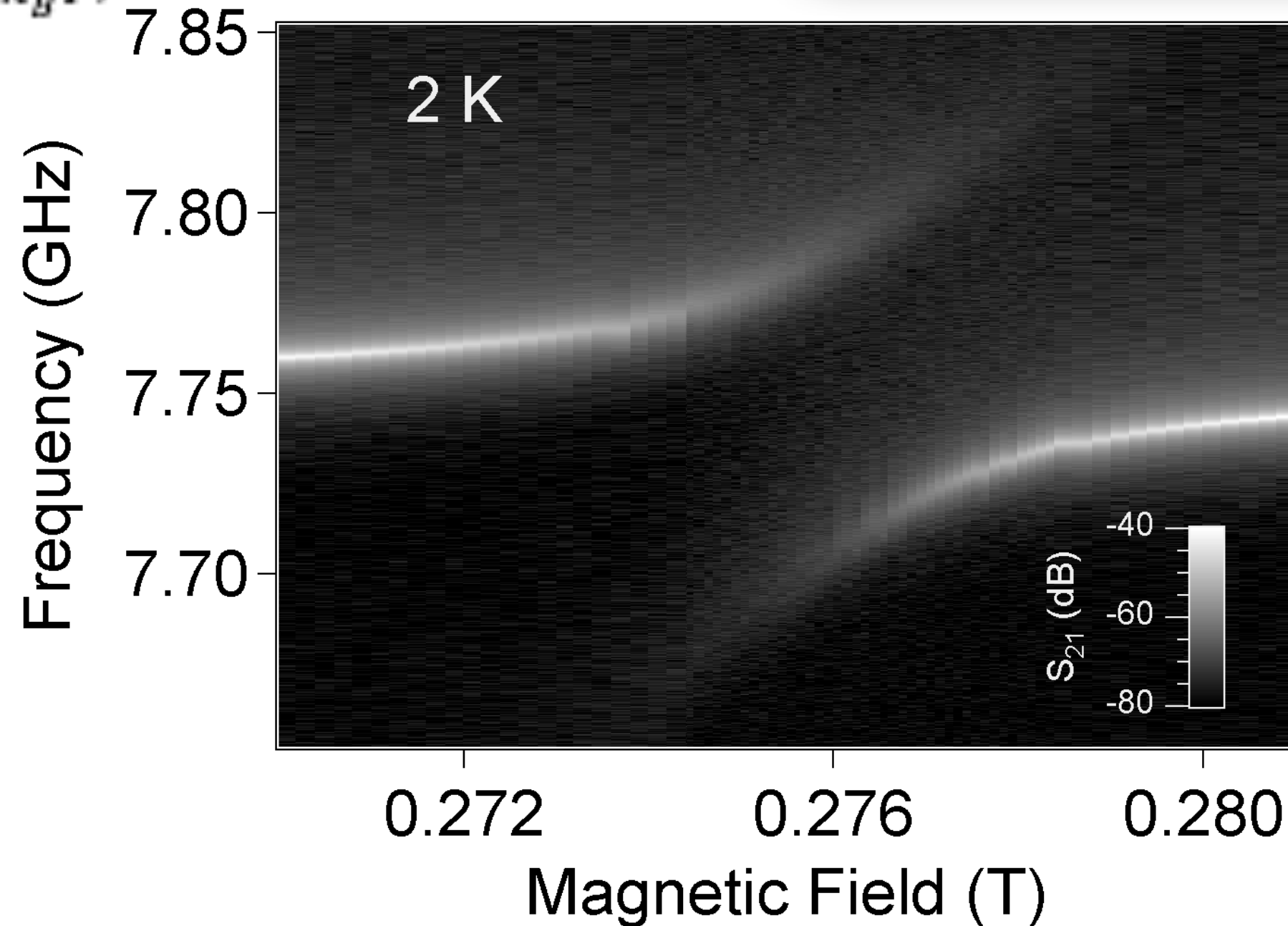
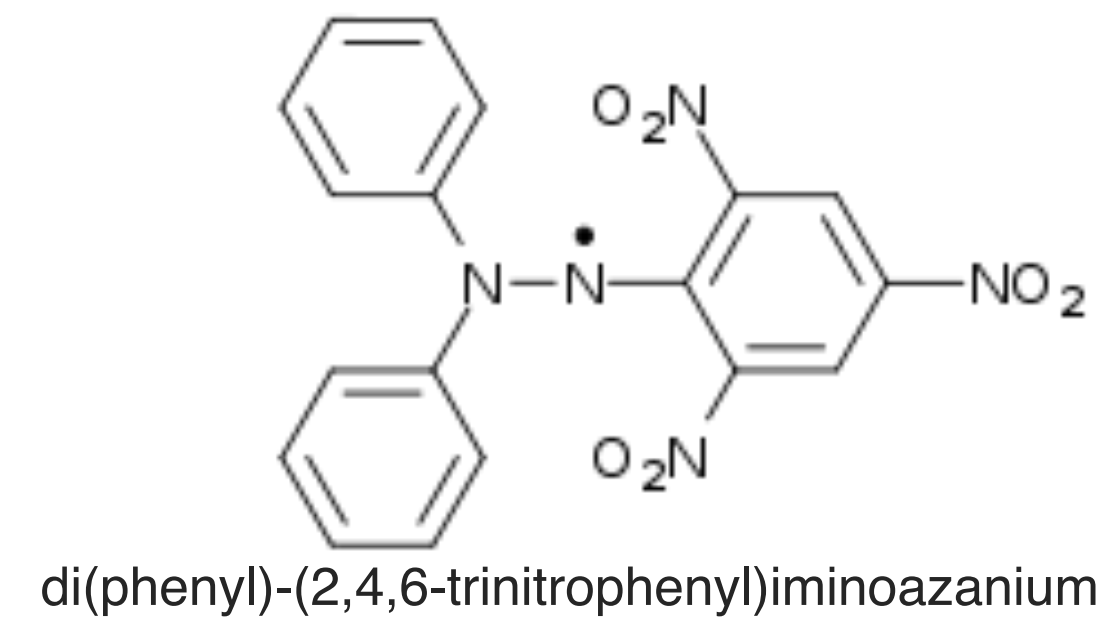
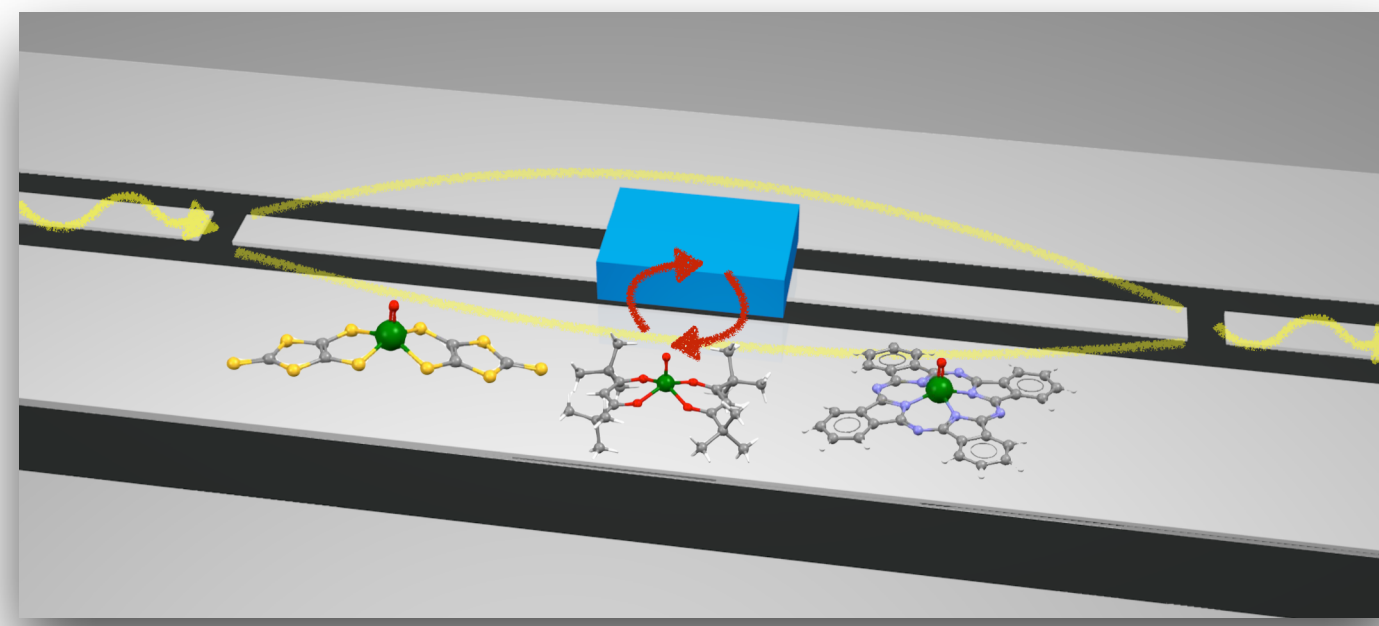
*Molecular Spins in the Context of Quantum Technologies*  
 A. Ghirri, A. Candini, M. Affronte  
*Magnetochemistry* 3(1), 12, (2017)  
 doi: [10.3390/magnetochemistry3010012](https://doi.org/10.3390/magnetochemistry3010012)





# Paramagnetic centres: Radical DPPH

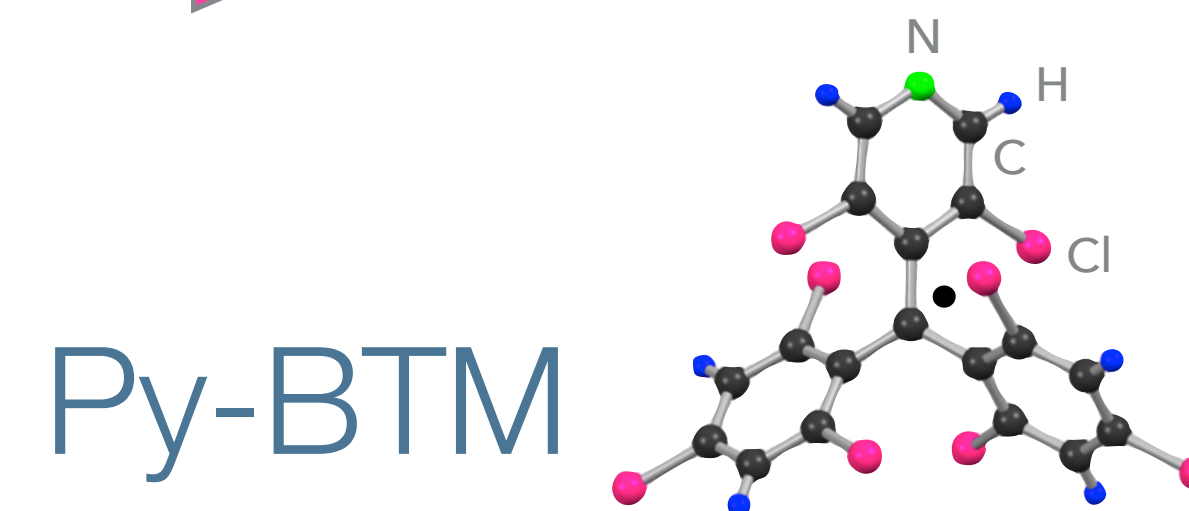
$$N_p = N \tanh\left(\frac{\hbar\omega}{2k_B T}\right)$$



spin ensemble:  
 $\gamma_s = 3$  MHz @ 77 K,  $\gamma_s = 14$  MHz @ 2 K

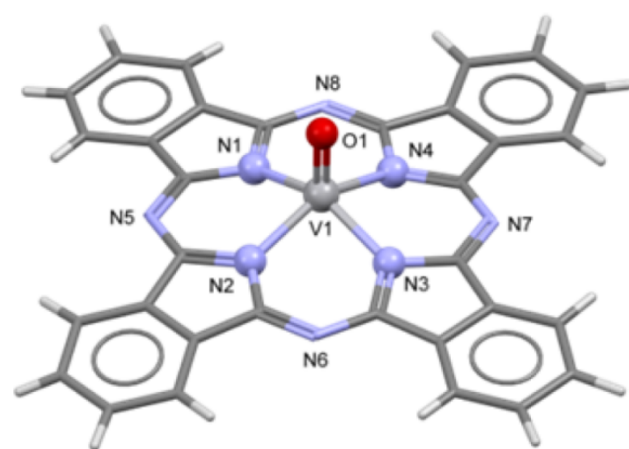
resonator:  $Q_L = 20000$ ;  $\kappa \approx 0.5$  MHz

$g_c \gg \kappa, \gamma_s$



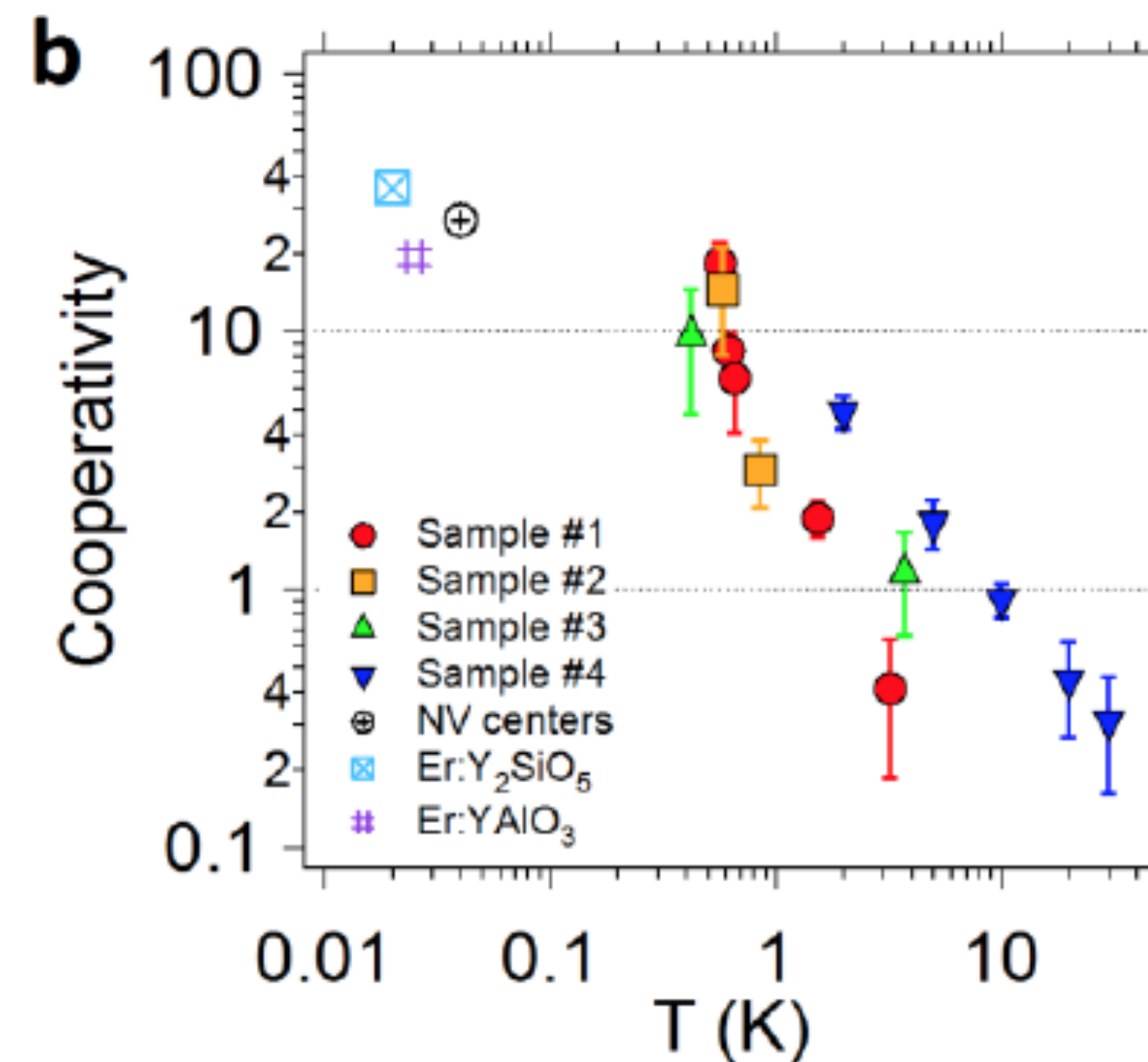
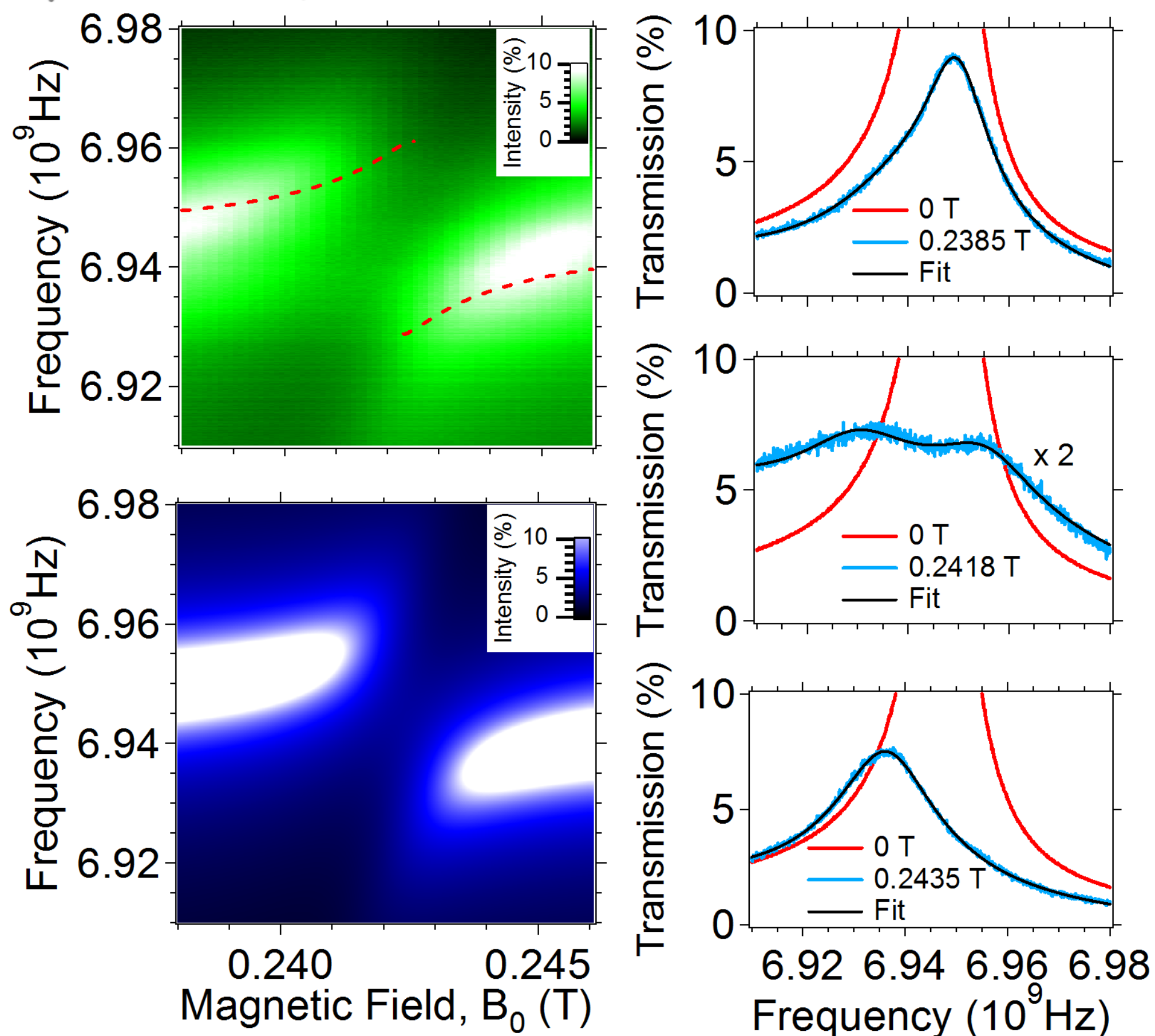
$$C = \frac{g^2}{\gamma\kappa} = 4300$$

# VOPc



$S = \frac{1}{2}$  system ( $\text{VO}^{+2}$ )  
**10% PELLETS VOPc:TiOPc**

M. Atzori et. al. *JACS* 138, 2154-2157 (2016)



$$\Omega = 25 \pm 4 \text{ MHz}$$

$$\gamma = 32 \pm 5 \text{ MHz}$$

$$\kappa \approx 1.0 \pm 0.2 \text{ MHz}$$

$$C = \frac{\Omega^2}{\gamma\kappa} \approx 18 \gg 1$$

**HIGH COOPERATIVITY**

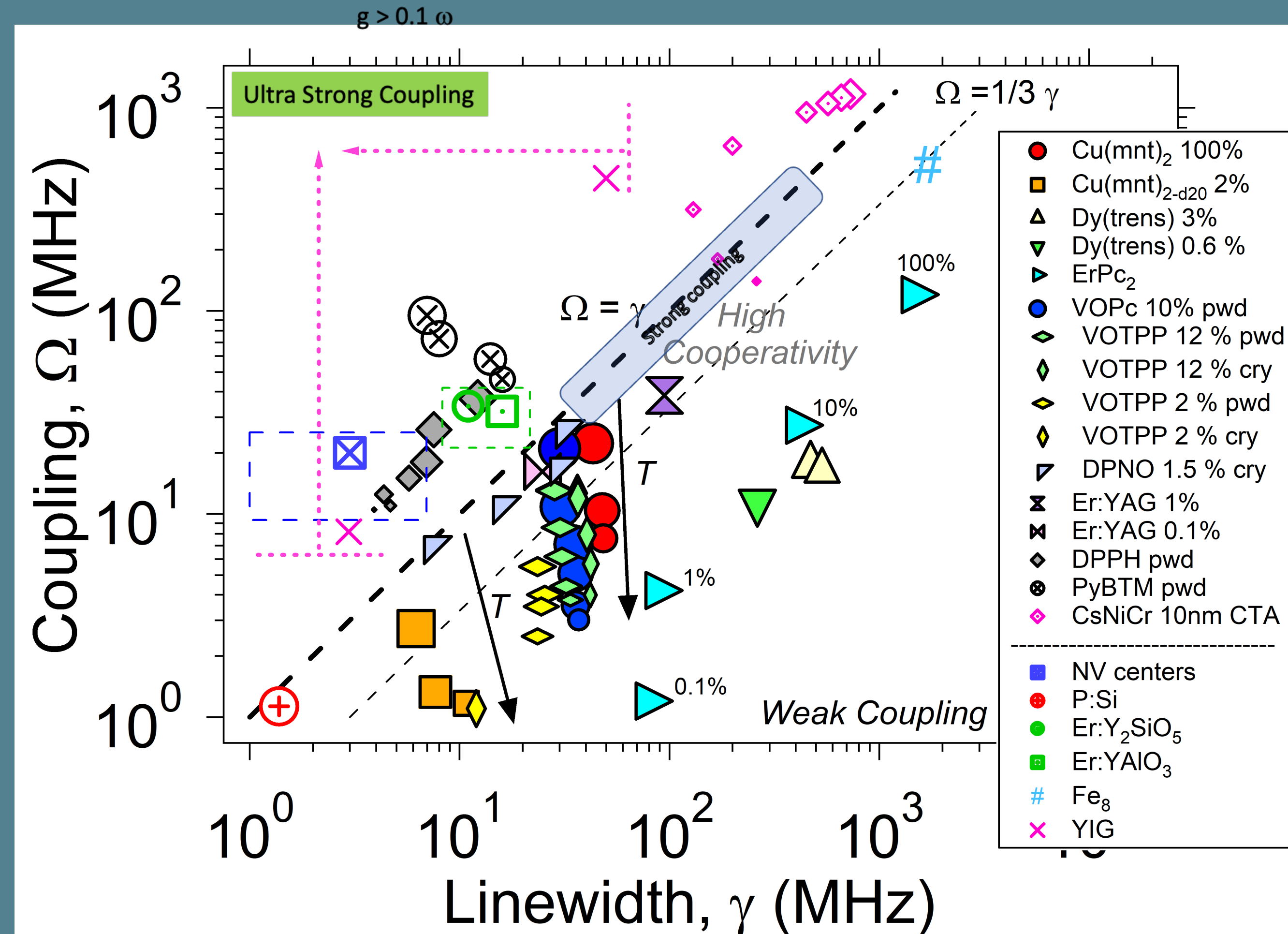
$\Omega = g_s \sqrt{N}$  collective coupling

$$\Omega_s = 0.05 \pm 0.01 \text{ Hz} \quad N_{\text{eff}} \approx 2 \cdot 10^{17}$$

C. Bonizzoni et al. *Scientific Reports* 7, (2017) 13096

# MOLECULAR SPINS EMBEDDED IN SUPERCONDUCTING CIRCUITS

long coherence time  
strong el-ph coupling





# spin waves in ordered magnets

For a mono domain sample and uniform mode we can use **Landau-Liftshitz-Gilbert Eq.** as a macroscopic description of **precession** of uniform magnetization vector:

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H}_{eff} - \frac{\lambda}{M_s} \vec{M} \times (\vec{M} \times \vec{H}_{eff})$$

$$\vec{H}_{eff} = \vec{H}_{magnst} + \vec{H}_{exch} + \vec{H}_{anis} + \vec{H}_{ext}$$

Shape effects are taken into account by the demagnetization factor:

$$B_x^i = B_x^0 - N_x M_x; \quad B_y^i = B_y^0 - N_y M_y; \quad B_z^i = B_z^0 - N_z M_z .$$

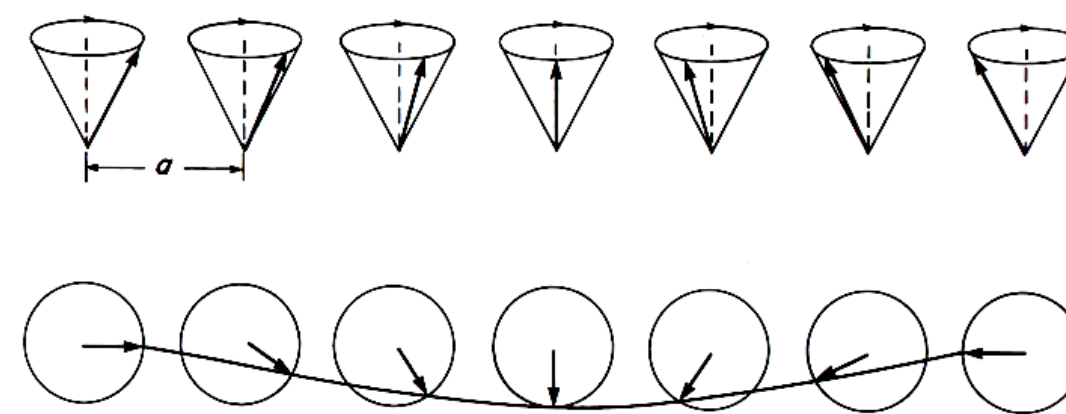
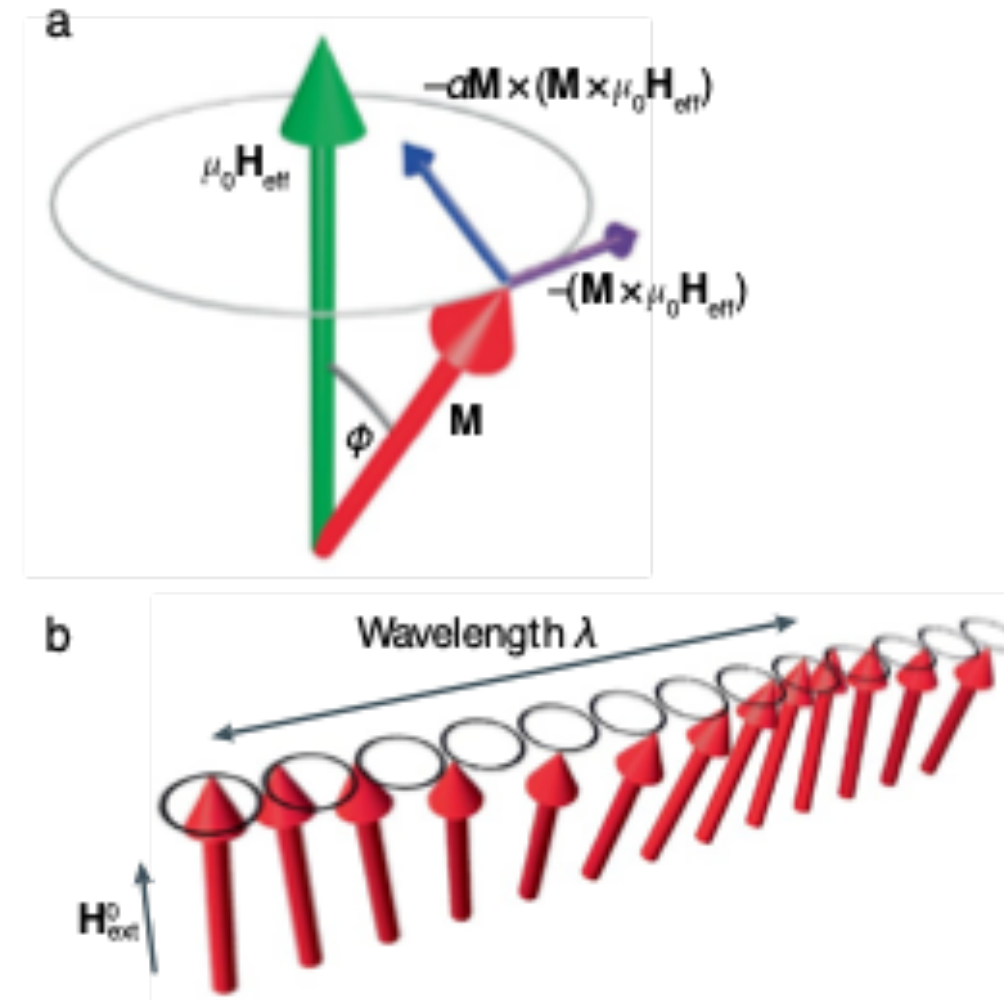
(CGS)  $\omega_0^2 = \gamma^2 [B_0 + (N_y - N_z)M][B_0 + (N_x - N_z)M] ;$

(SI)  $\omega_0^2 = \gamma^2 [B_0 + (N_y - N_z)\mu_0 M][B_0 + (N_x - N_z)\mu_0 M] .$

Equivalently, a **microscopic description of spin waves** can be obtained by considering both N.N. exchange and dipole-dipole interactions:

$$\hat{H} = -\frac{1}{2} \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \mu \mathbf{H}_e \cdot \sum_i \mathbf{S}_i - \frac{1}{2} \sum_{ij, i \neq j} \frac{\mu^2}{|\mathbf{R}_{ij}|^3} \left[ 3(\mathbf{S}_i \cdot \hat{\mathbf{R}}_{ij})(\mathbf{S}_j \cdot \hat{\mathbf{R}}_{ij}) - \mathbf{S}_i \cdot \mathbf{S}_j \right],$$

Eur. Phys. J. B **71**, 59–68 (2009)  
DOI: [10.1140/epjb/e2009-00279-y](https://doi.org/10.1140/epjb/e2009-00279-y)

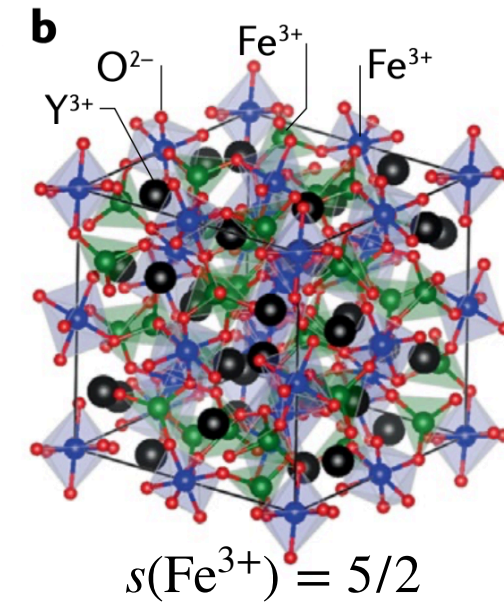
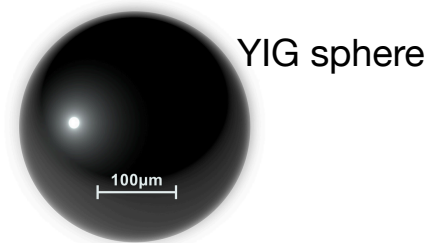
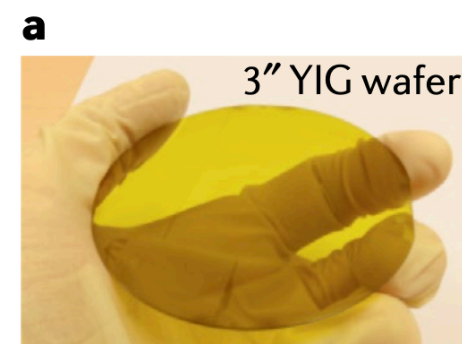




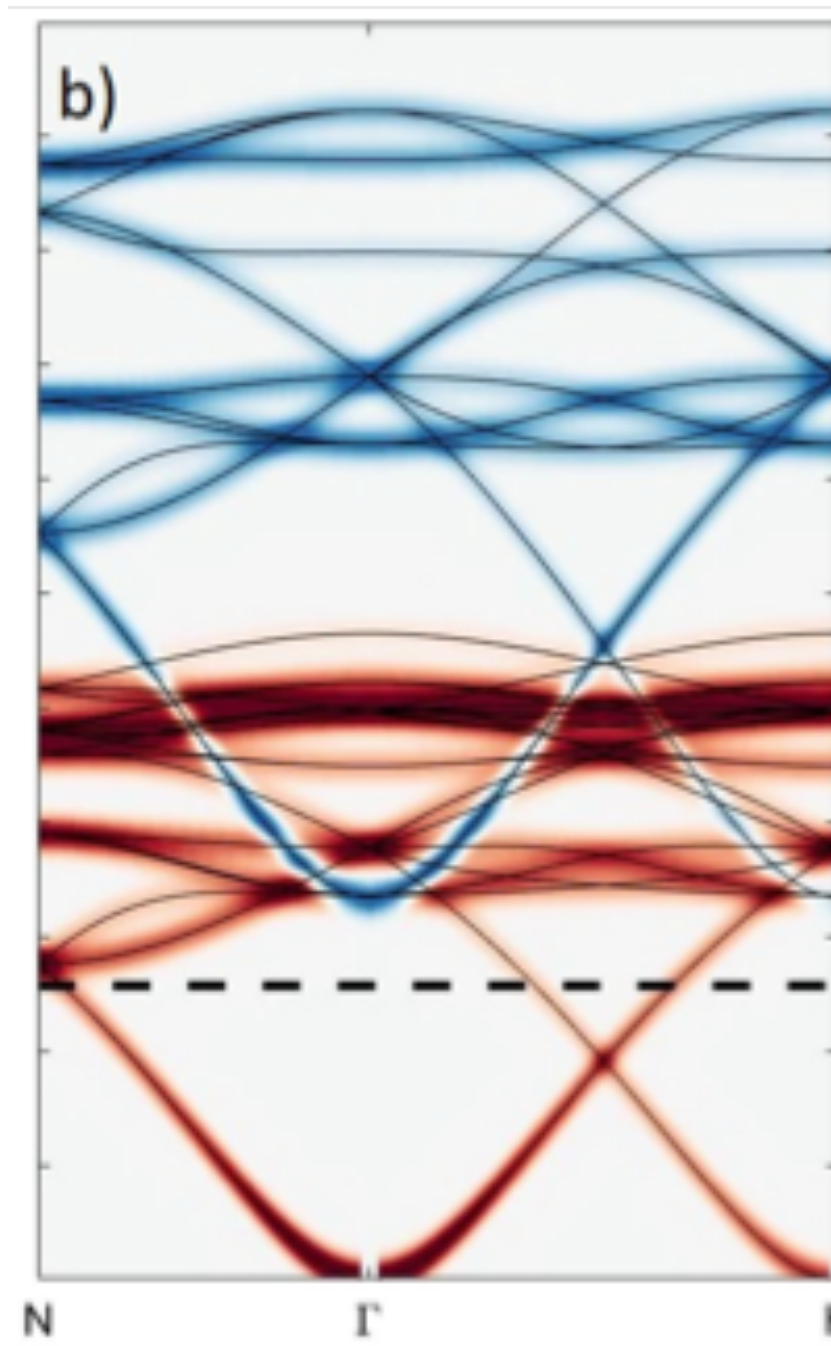
# YIG samples

## $\text{Y}_3\text{Fe}_5\text{O}_{12}$ (YIG)

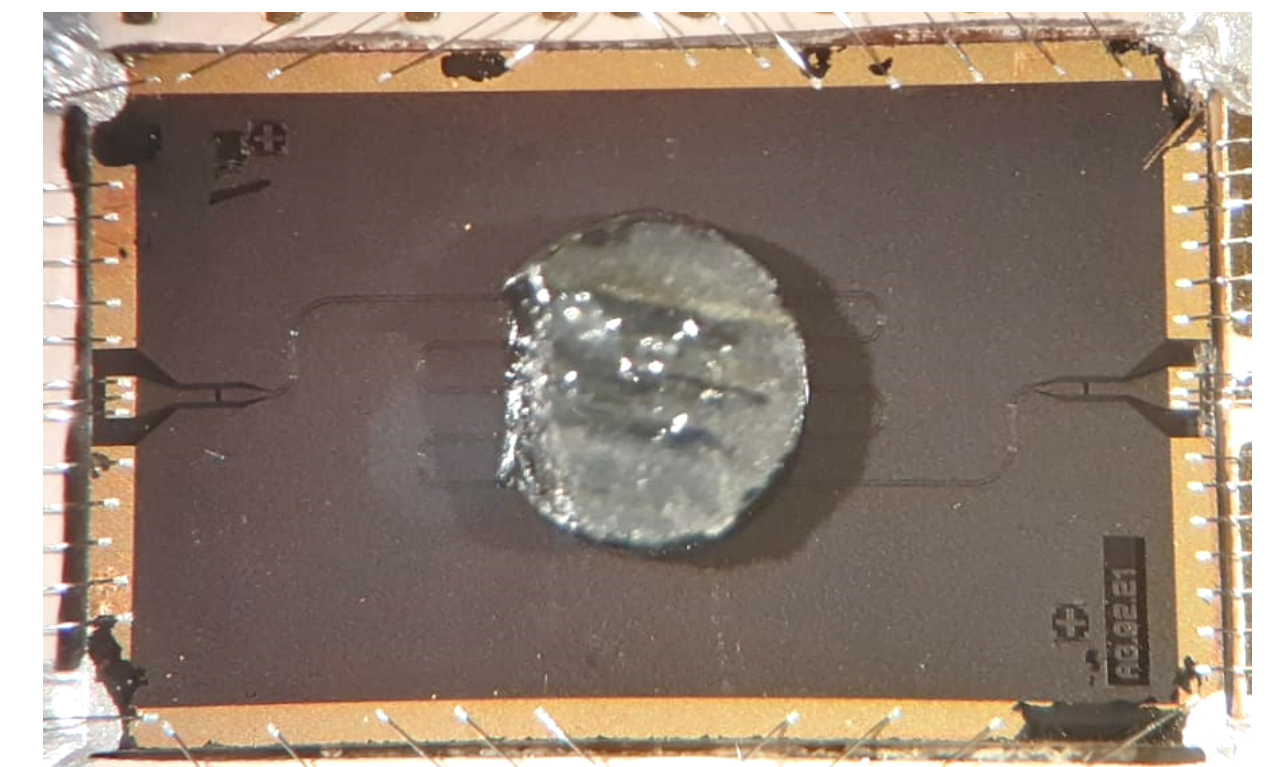
- Ferrimagnetic
- $\rho_s = 4.22 \times 10^{27} \text{ m}^{-3}$
- Low Gilbert damping:  
 $\alpha \sim 10^{-4} - 10^{-5}$



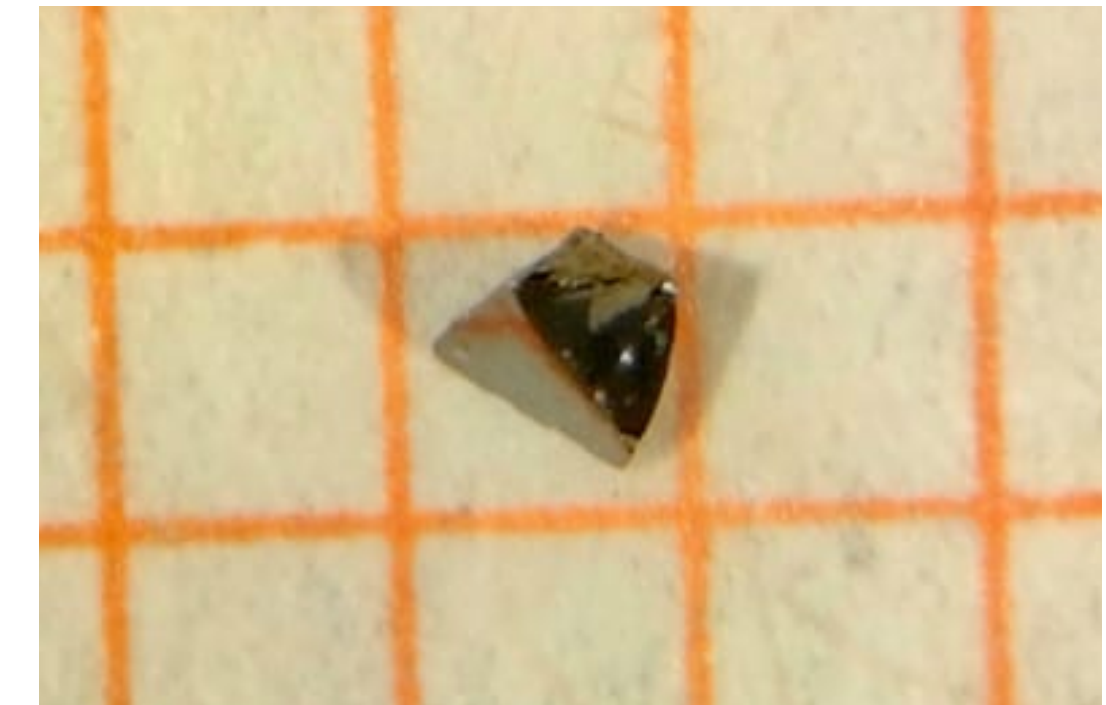
spin wave dispersion in bulk



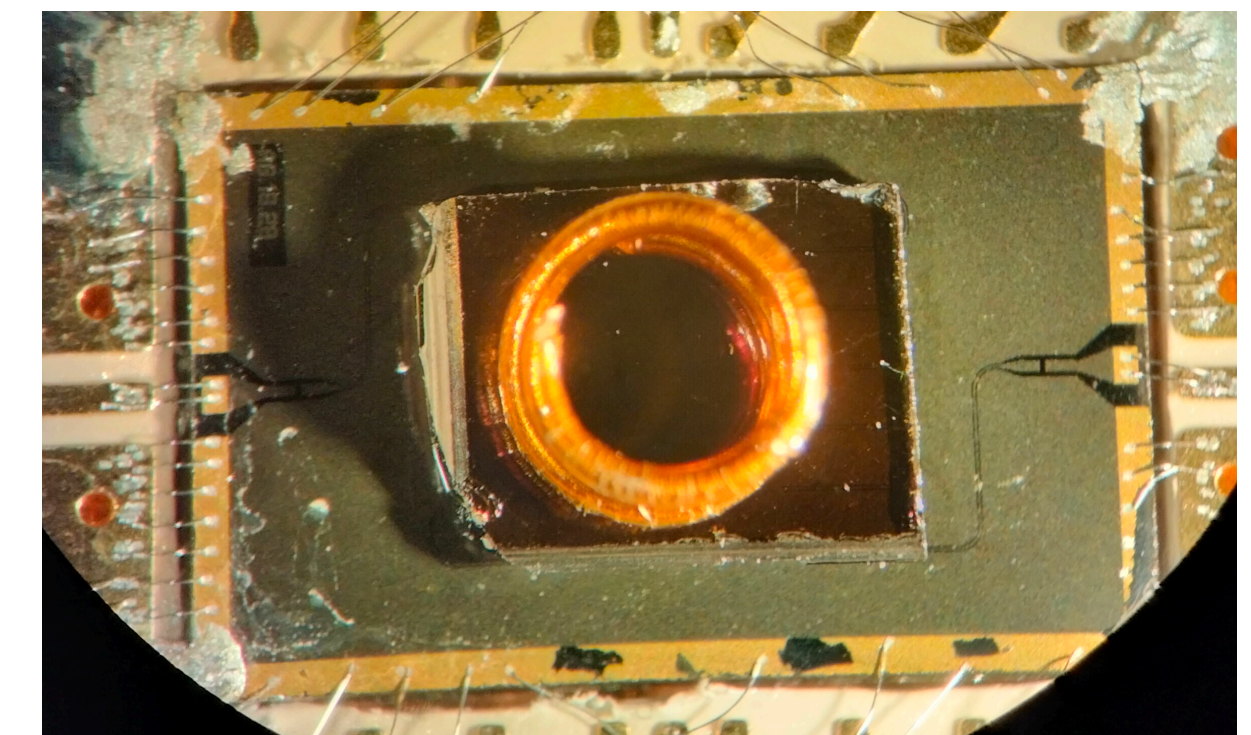
*npj Quantum Materials* (2017)2:63 ; doi:10.1038/s41535-017-0067-y



Sample #1: Bulk YIG Crystal,  
from Istanbul kindly provided by R. Bulat



from Japan, kindly provided by G. Ruoso INFN

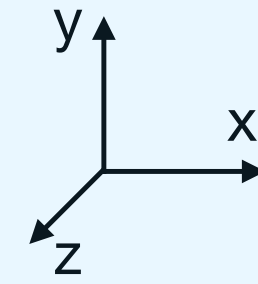
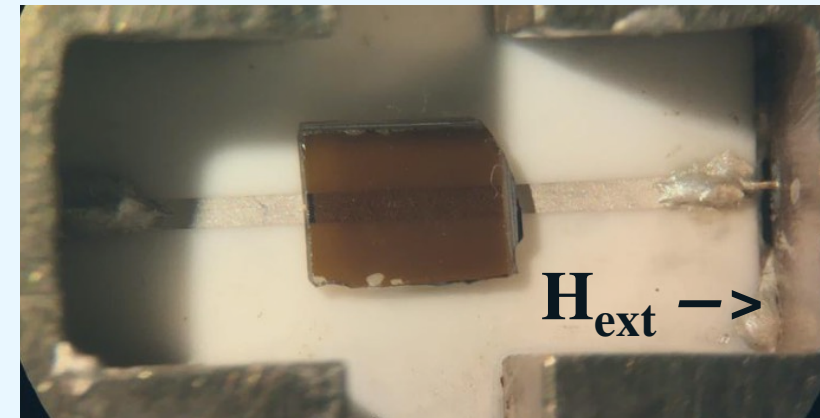


Sample #2: **YIG/GGG film** with dimension  $\approx 4 \times 3 \text{ mm}^2$  ;  $w = (5 \text{ \& } 20 \text{ }\mu\text{m})$   
commercial from [Matesi](#)

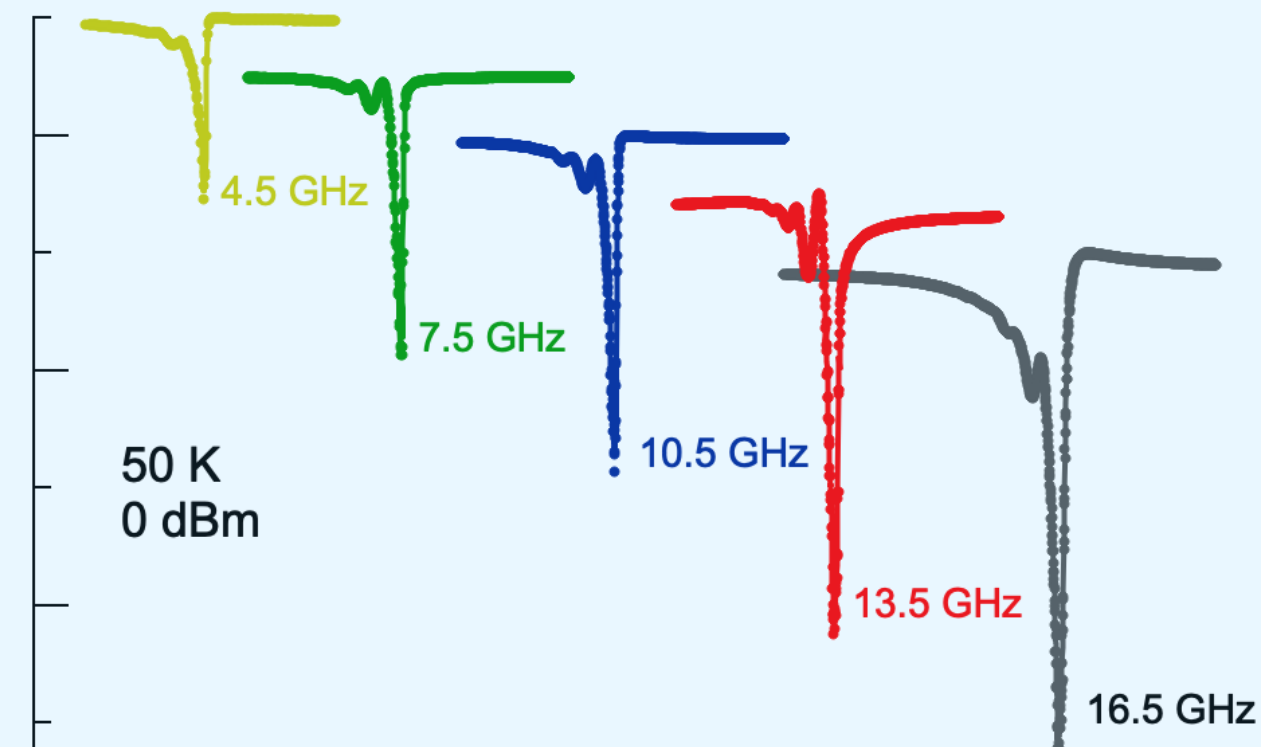


# Spin waves in YIG film: characterisation by broadband stripline

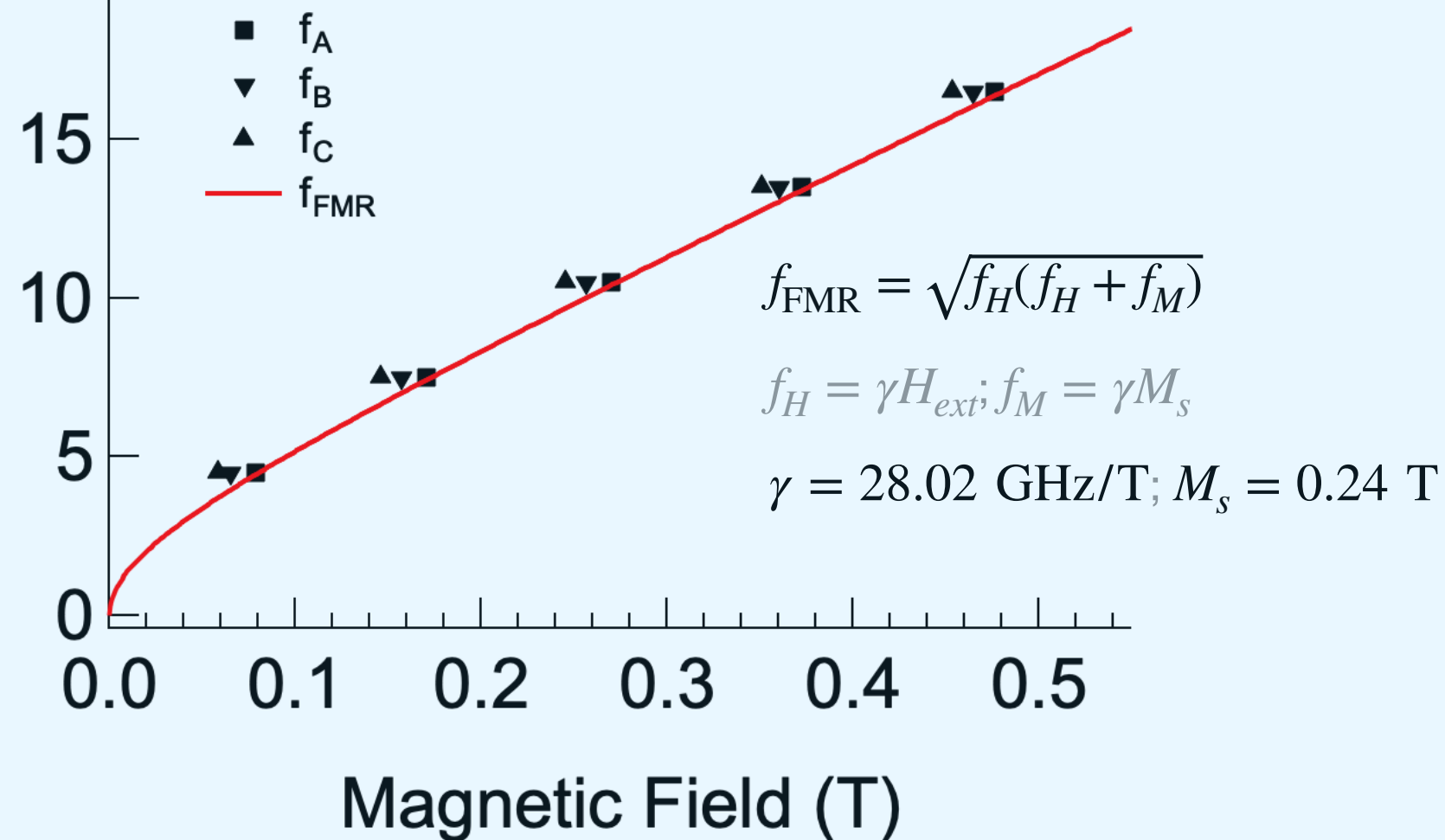
**Metallic (Ag) broadband Microstrip line**  
lateral width 500  $\mu\text{m}$



Normalised  
Transmission

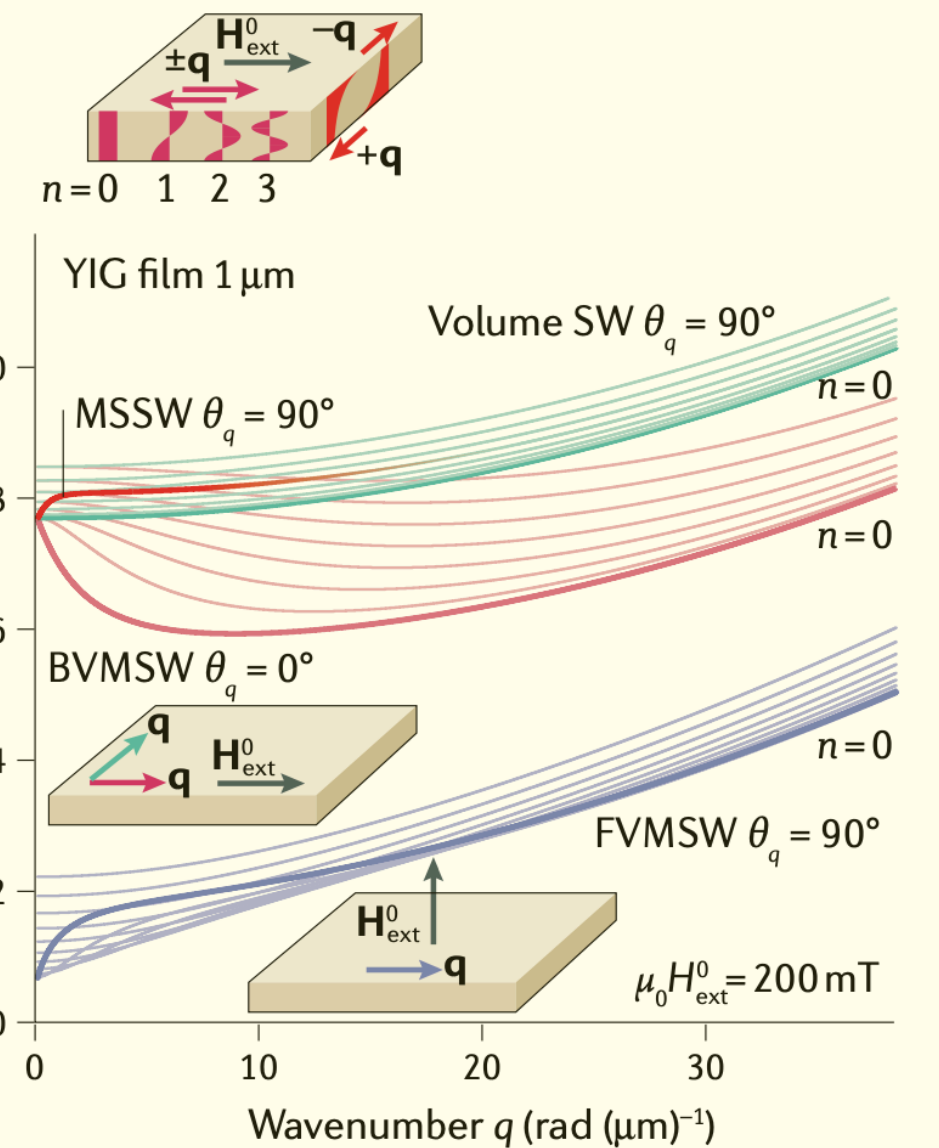
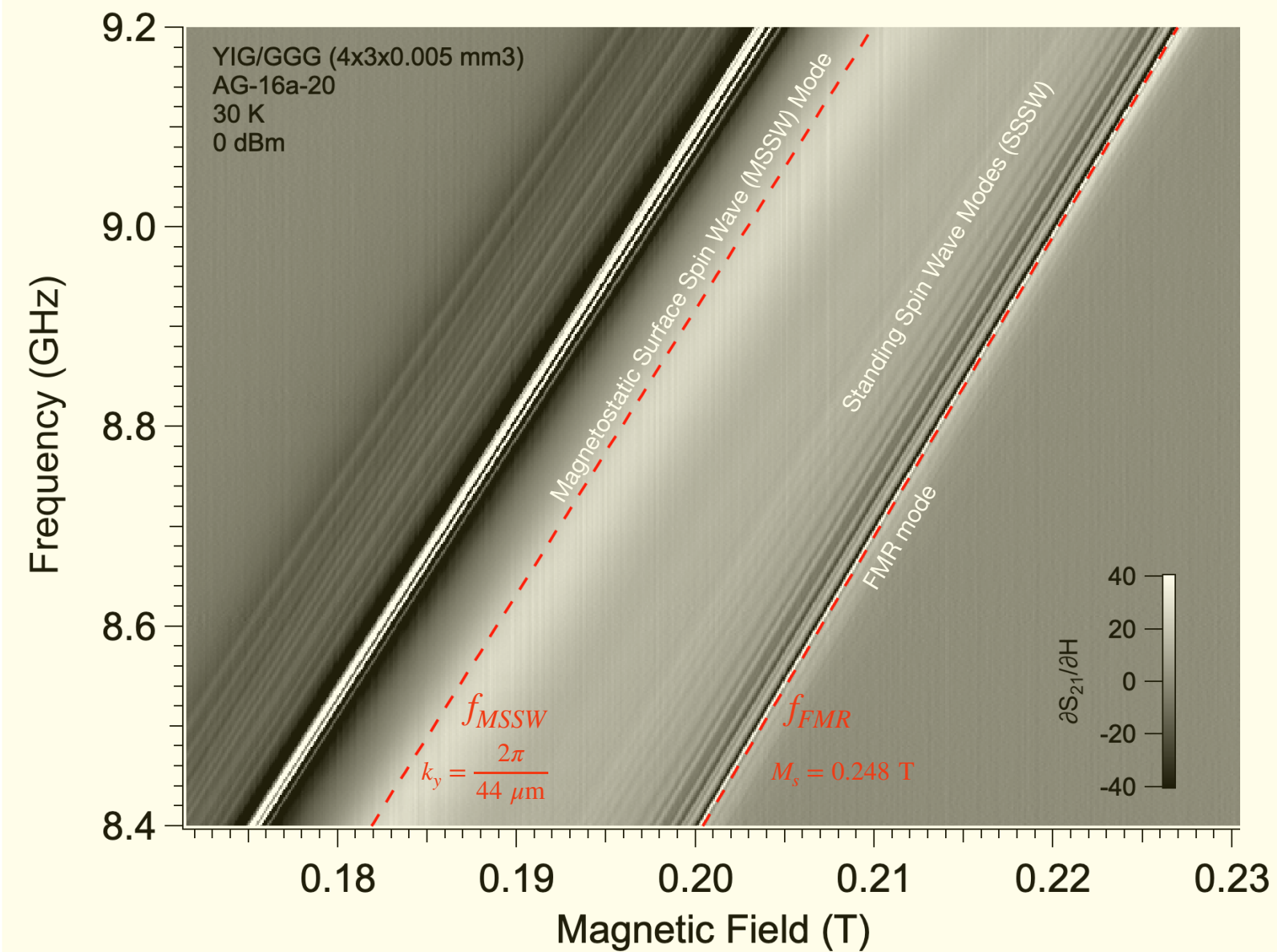
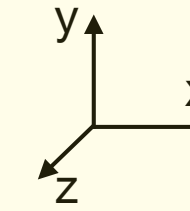
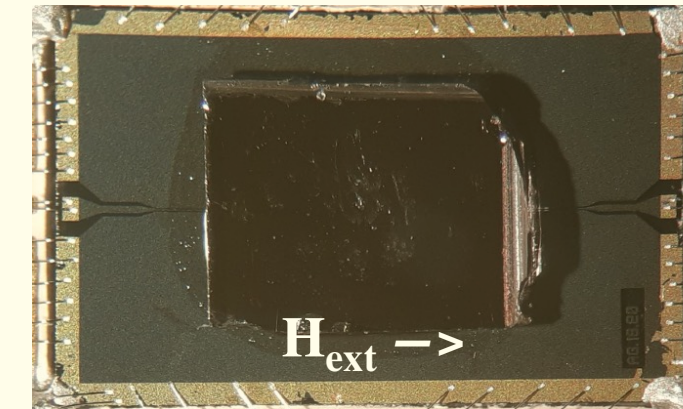


Frequency (GHz)



central conductor lateral width 20  $\mu\text{m}$

**sample #1**  
YIG/GGG film  
size: 4x3 mm<sup>2</sup>  
thickness: 5  $\mu\text{m}$



Pirro et al. Nat. Rev. Mater. 6, 1114 (2021)

$$\omega_n(\mathbf{k} = 0) = \omega_{\text{FMR}} + \gamma D \left( \frac{\pi n}{d} \right)^2$$

Film thickness  $d = 5 \mu\text{m}$

Exchange stiffness:  $D = 5.29 \times 10^{-17} \text{ T m}^2$

Saturation magnetisation  $M_s = 0.24 \text{ T}$

Damon Eshbach, J. Phys.Chem. Solidi 19,308 (1961)

Kalinikos, Sov. Phys. J. 24, 718 (1981)

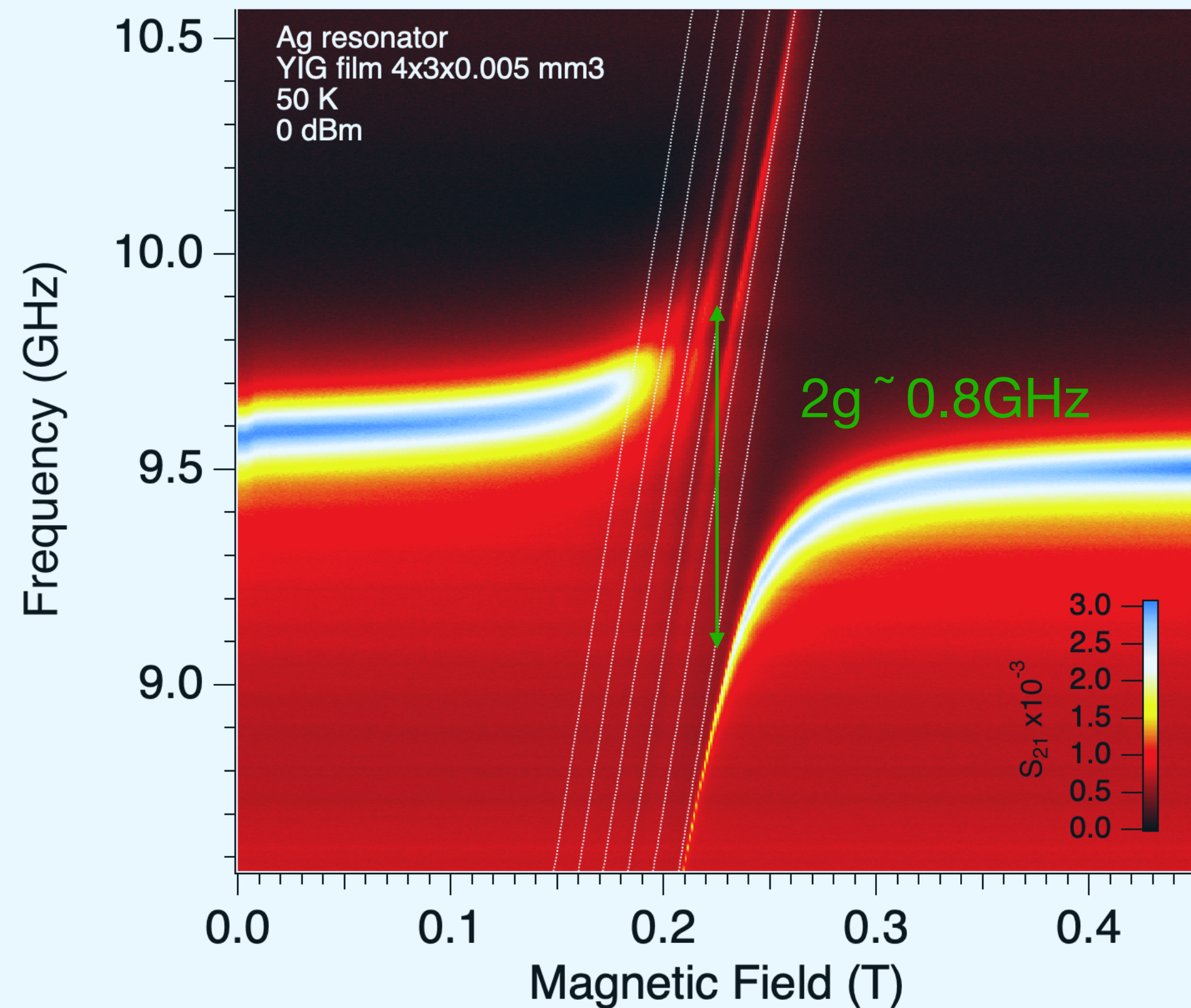
Harms and Duine, J. Magn. Magn. Mater. 567, 169426 (2022)

Kreisel et al., Eur. Phys. J. B 71, 59 (2009)

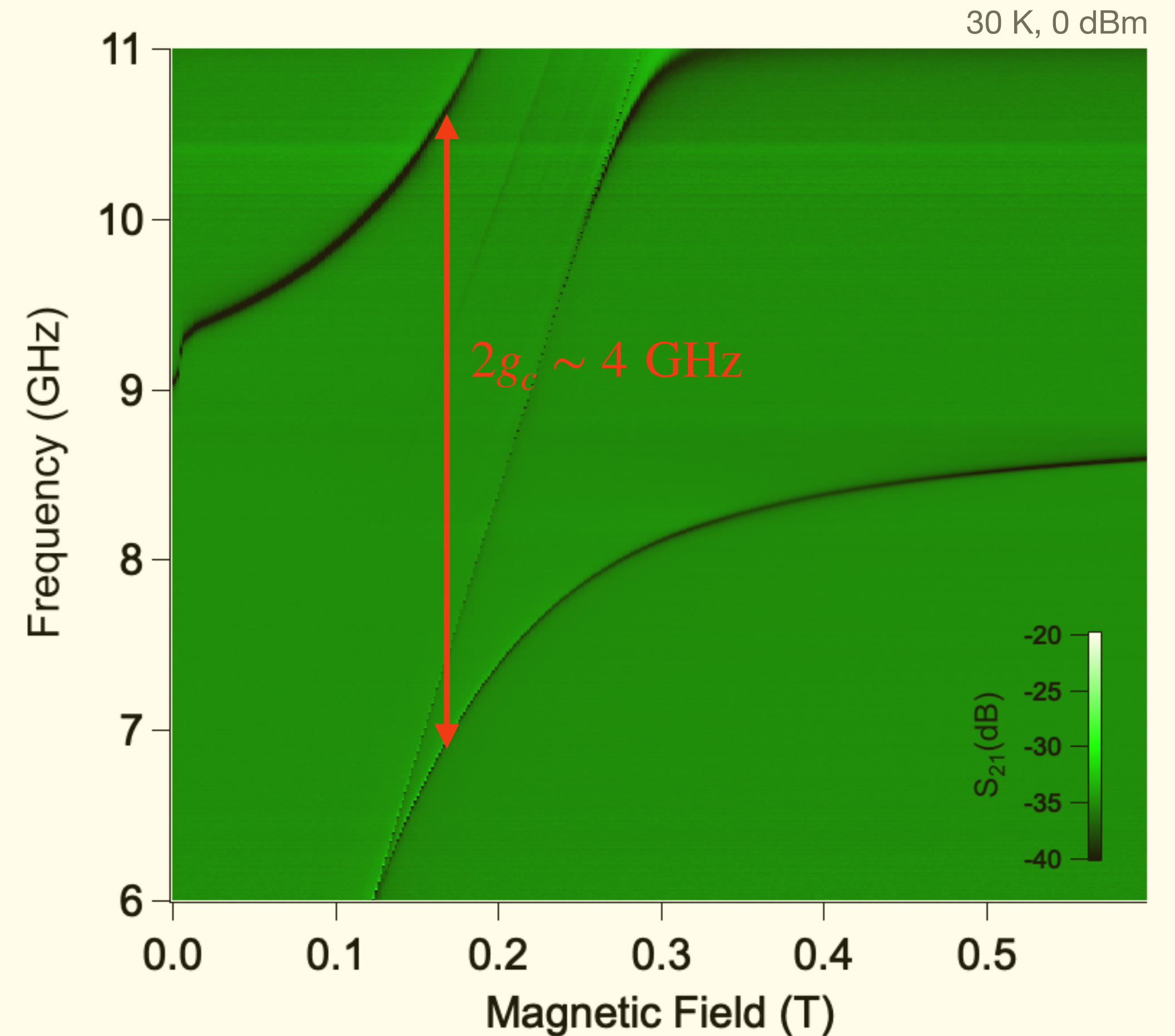
Klinger et al., J. Phys. D: Appl. Phys. 48 015001 (2015)



# Increasing coupling strength through the effective Volume



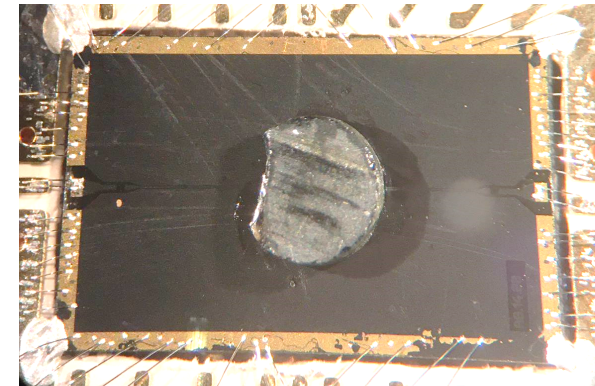
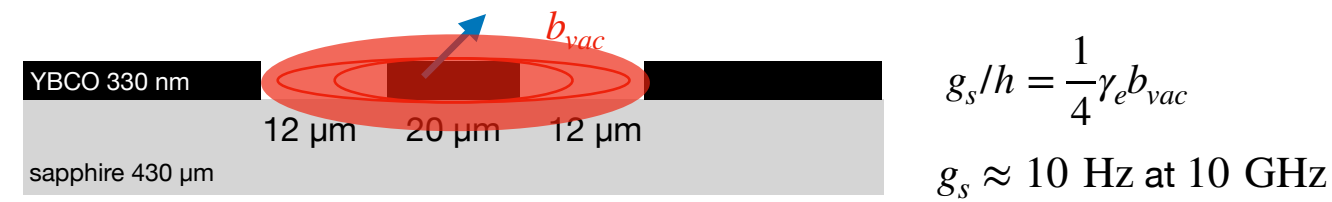
linear resonator, lateral size 500 $\mu\text{m}$   
YIG/GGG film  
thickness 5 $\mu\text{m}$   
area 12mm<sup>2</sup>



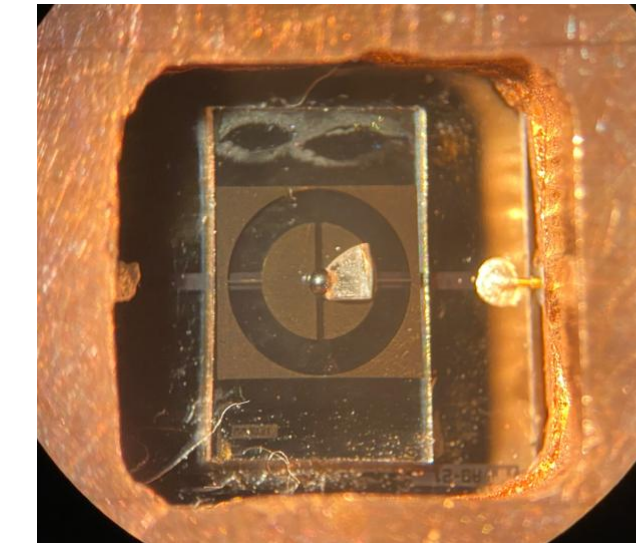
CPW resonator, lateral size 20  $\mu\text{m}$   
YIG/GGG film  
thickness 5  $\mu\text{m}$   
area 12 mm<sup>2</sup>



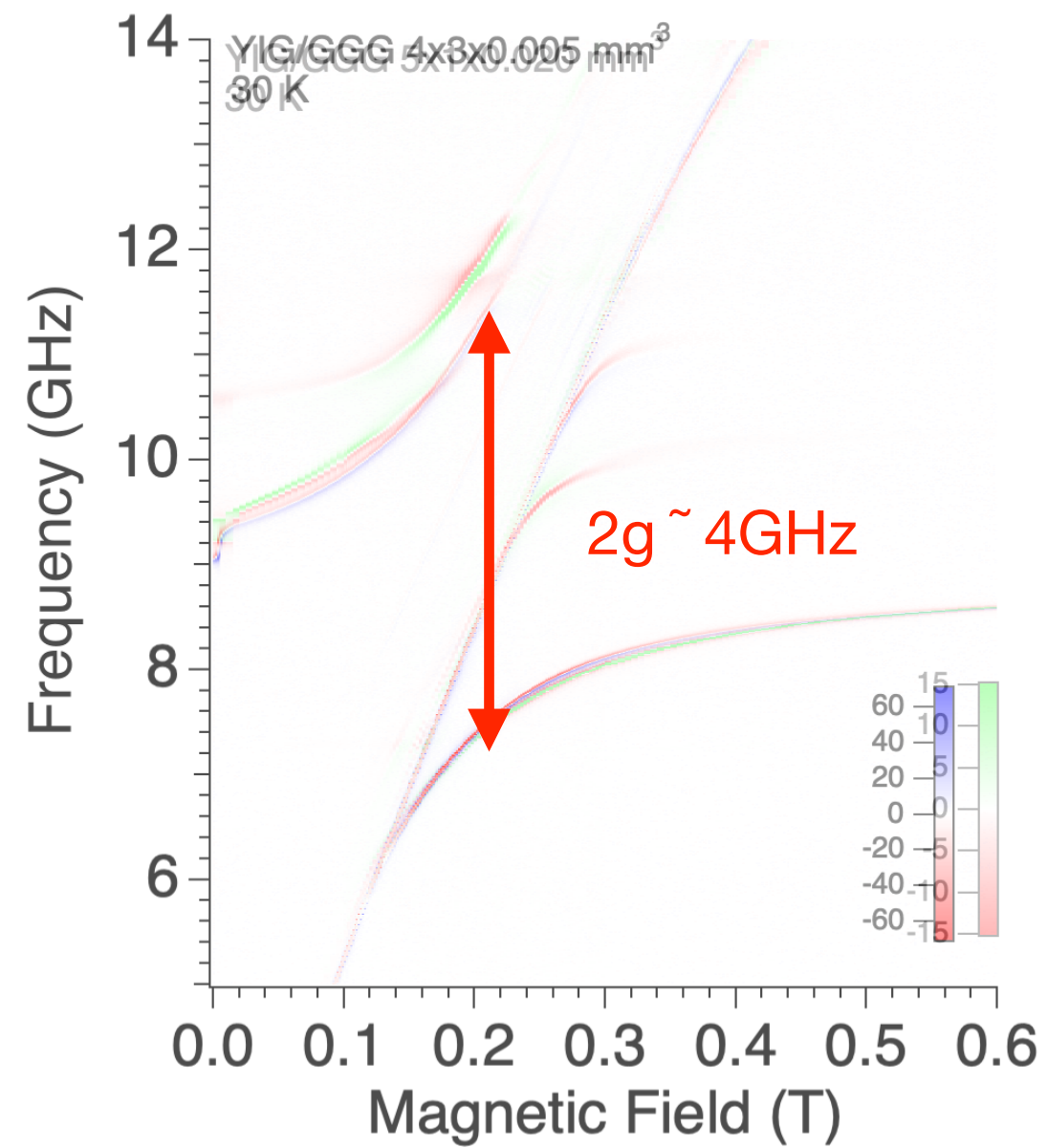
# further tests with bulk samples



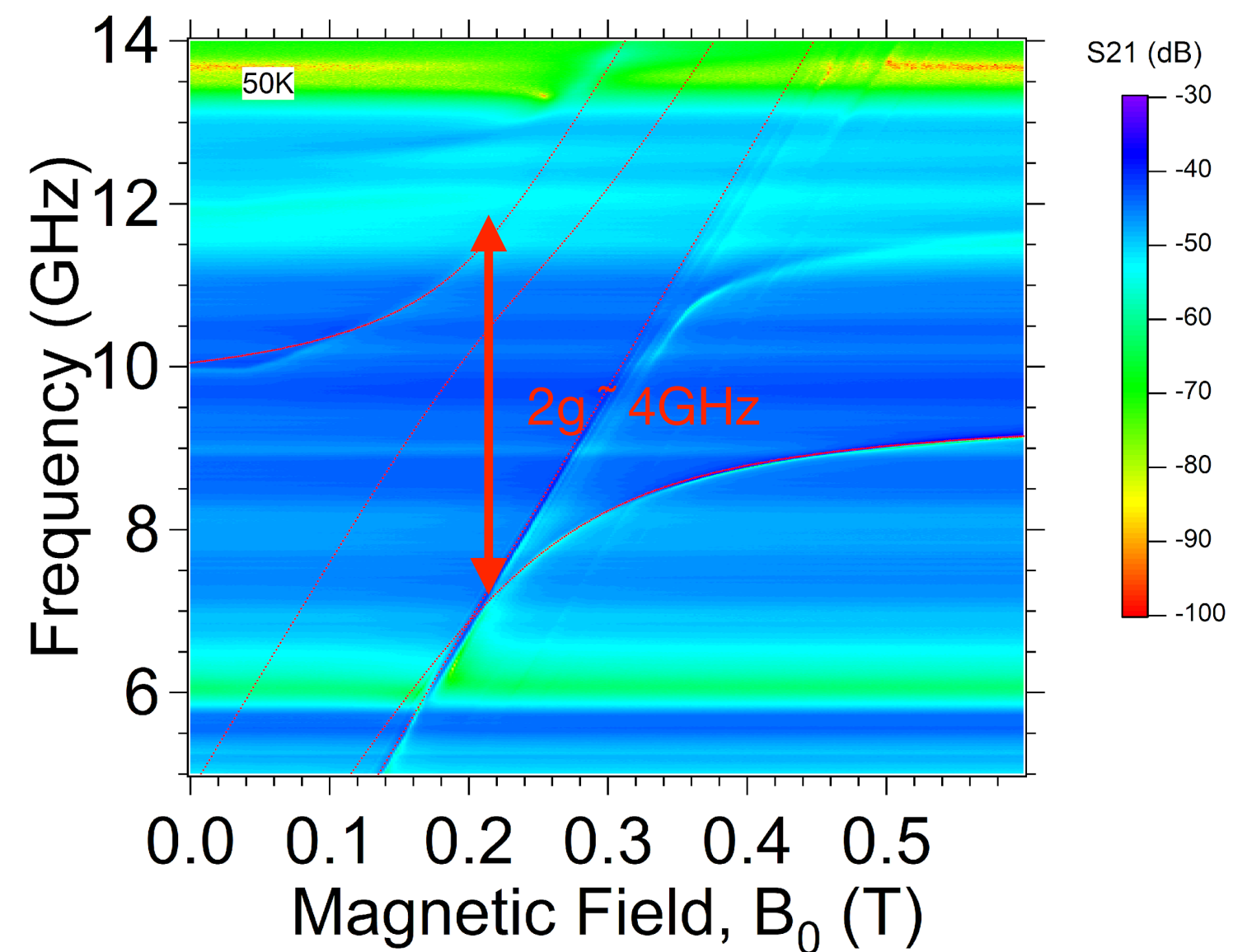
linear Resonator+ YIG disk



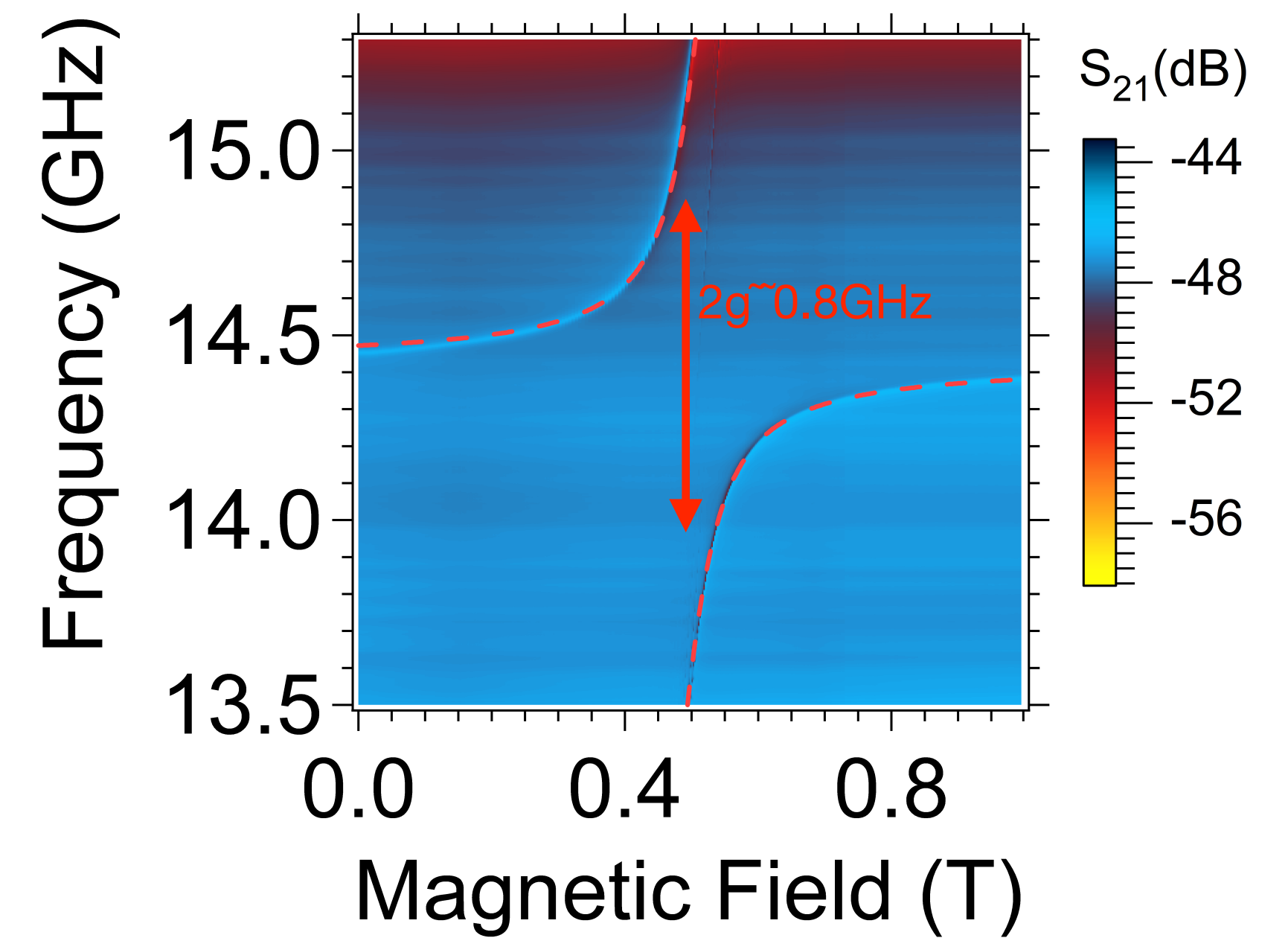
Inv. Anapole Resonator+ YIG sphere



USC achieved with thicker film (20μm)  
but significant increase of the coupling



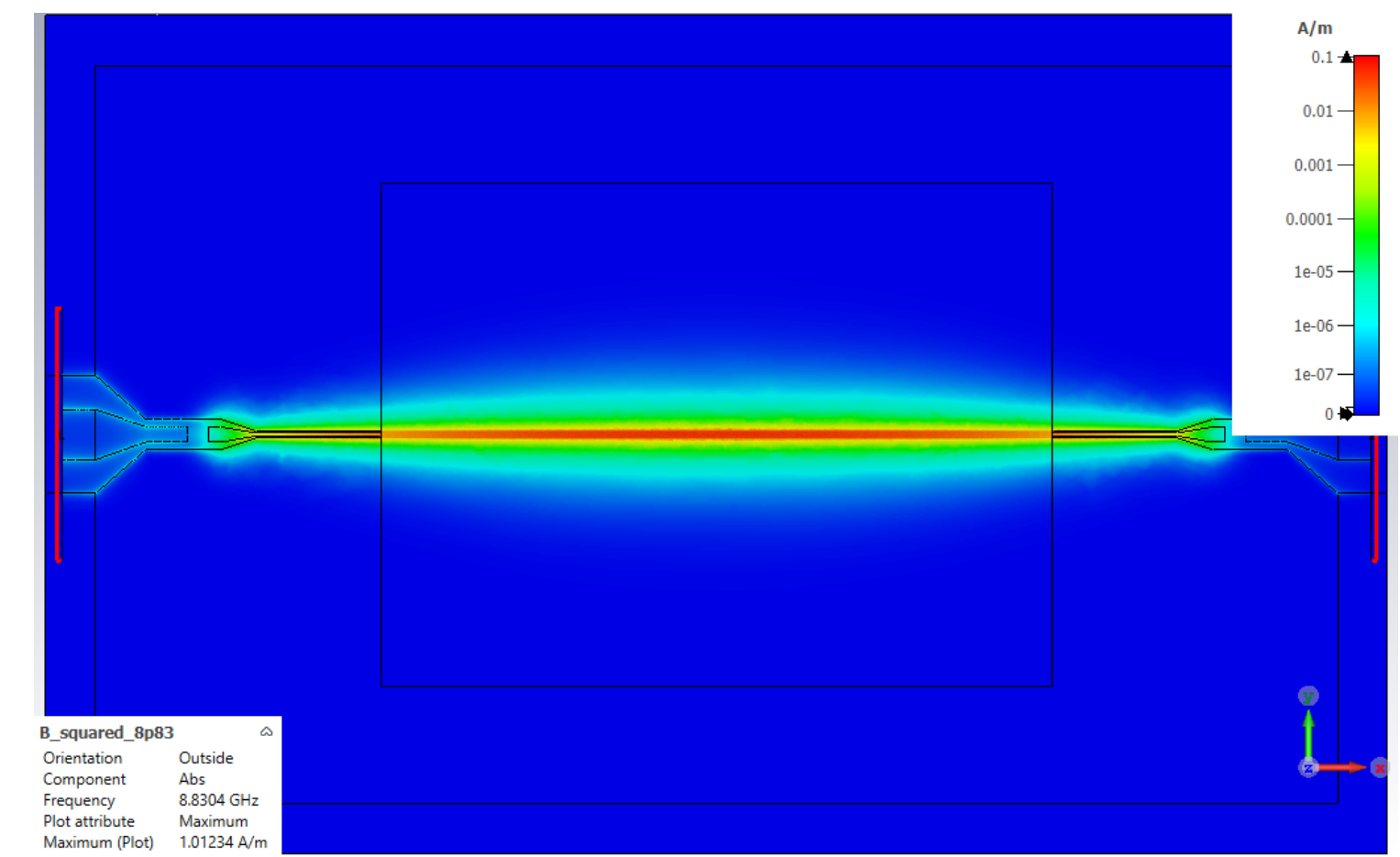
radiation field as CPW  
-> USC achieved but not larger than film



radiation field focused in small volume (5μm)<sup>3</sup>  
-> USC not achieved

# ESTIMATION OF NUMBER OF COUPLED SPINS

Number of spins is estimated starting from CST simulation of the resonant mode volume, using a 1:1 scale model filled with vacuum and with same sample dimension and position.



$$N_{eff} = N_0 p(T) = \rho V_{eff} P(T) \approx \rho V_{eff} \quad \rho = 4.22 \cdot 10^{27} \text{ spin/m}^3$$

From effective number of spins it is also possible to estimate the single spin coupling

Diameter (mm)	Resonator	$\nu_0$ (GHz)	$V_{eff}$ (m <sup>3</sup> )	$g_s$ (Hz)	$g = g_s \sqrt{N}$ (GHz)
#1	Meander	8.7	$2.2 \cdot 10^{-12}$	11	1.0
	Copl. 20 $\mu\text{m}$	9.55	$9.3 \cdot 10^{-11}$	6.7	4.2
	Copl. 600 $\mu\text{m}$	7.08	$3.97 \cdot 10^{-10}$	1.1	1.4
#2	Meander	8.7	$7.9 \cdot 10^{-12}$	11	2.0
	Copl. 20 $\mu\text{m}$	9.55	$1.9 \cdot 10^{-10}$	6.7	5.9
	Copl. 600 $\mu\text{m}$	7.08	$1.5 \cdot 10^{-9}$	1.1	2.8



# Modelling.

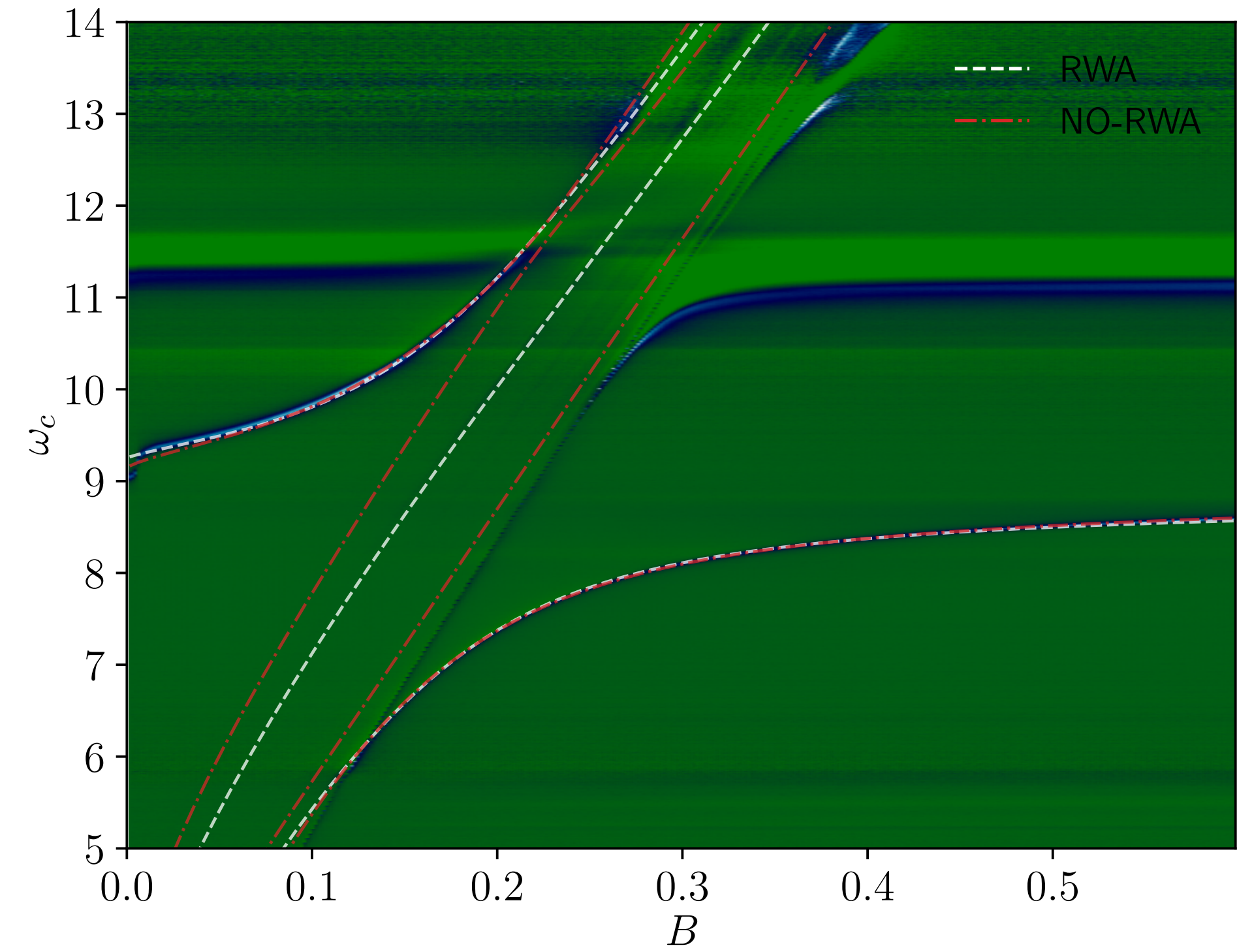
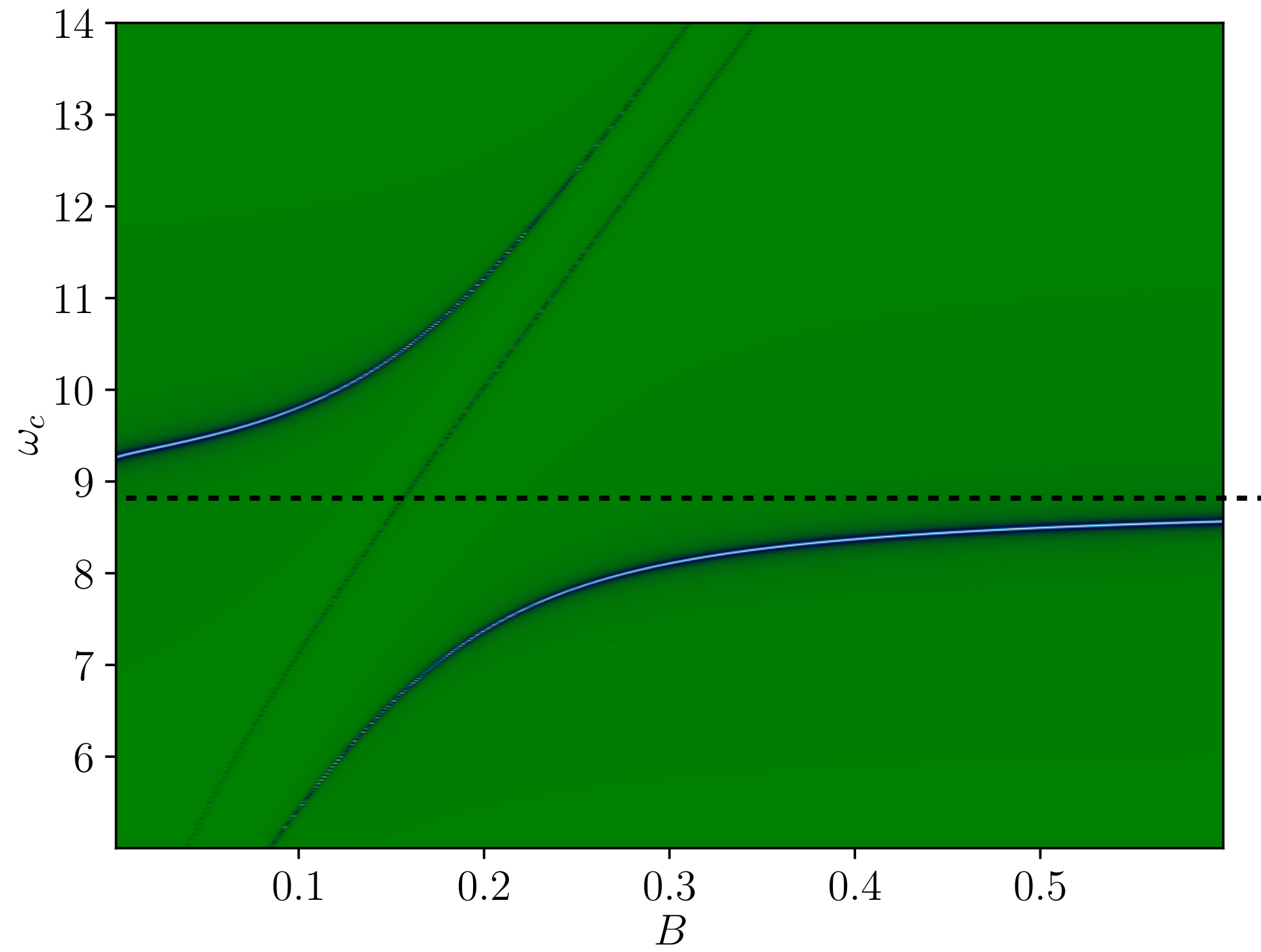
- Q-Rabi hamiltonian:

$$H = \omega_c \hat{a}^\dagger \hat{a} + \omega_1 \hat{b}_1^\dagger \hat{b}_1 + \omega_2 \hat{b}_2^\dagger \hat{b}_2 + g_1 (\hat{b}_1 + \hat{b}_1^\dagger) (\hat{a} + \hat{a}^\dagger) + g_2 (\hat{b}_2 + \hat{b}_2^\dagger) (\hat{a} + \hat{a}^\dagger) + \alpha (\hat{a} + \hat{a}^\dagger)^2$$

- direct numerical diagonalization with  $n$  magnetic modes
- Analytical solution without RWA:

$$\omega_{\pm} = \frac{\sqrt{\omega b^2 + 4 \alpha \omega c + \omega c^2} \pm \sqrt{\omega b^4 + 16 \lambda^2 \omega b \omega c - 2 \omega b^2 \omega c (4 \alpha + \omega c) + \omega c^2 (4 \alpha + \omega c)^2}}{\sqrt{2}}$$

# transmission spectra & best fit



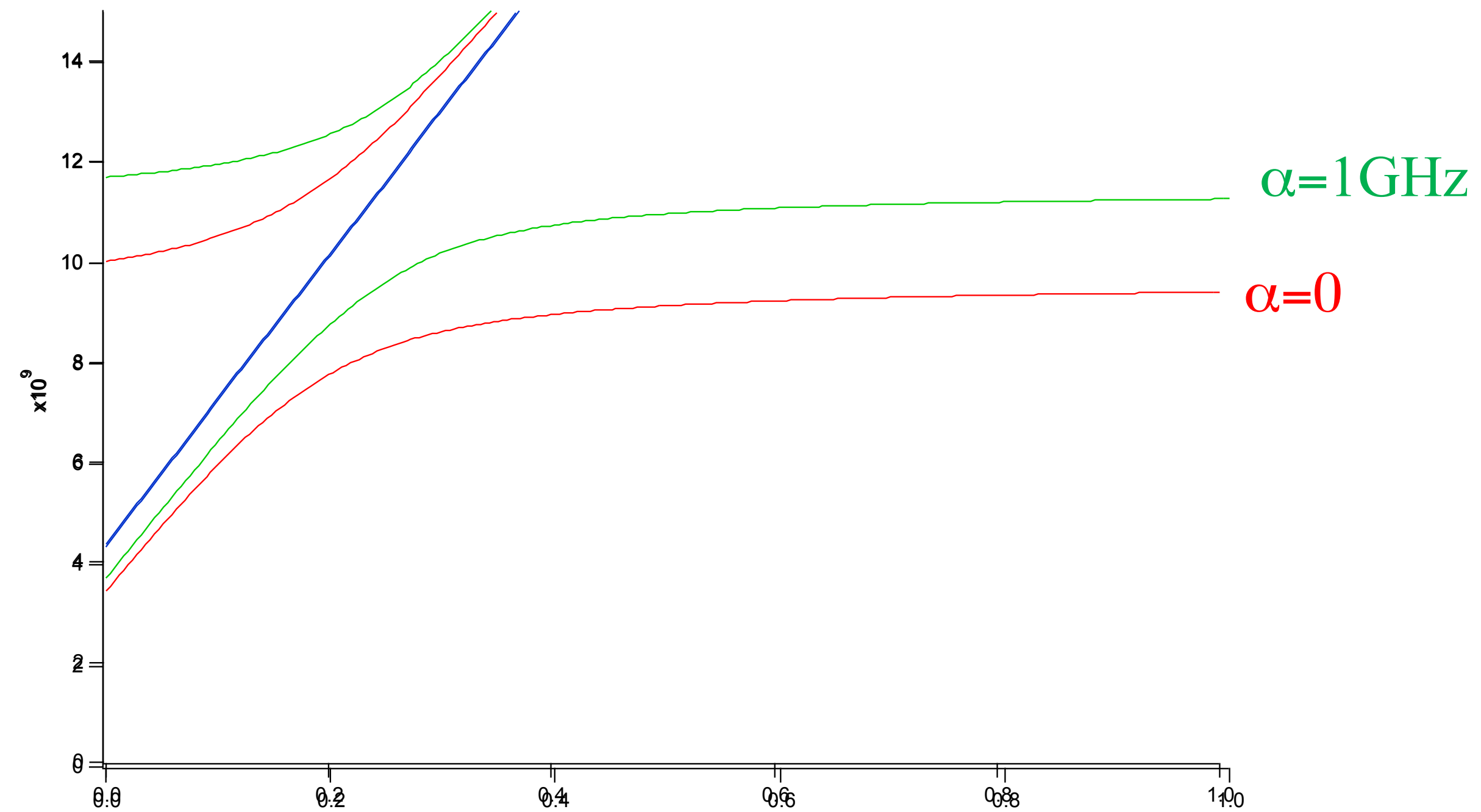
RWA:			
$\Delta_1$	=	0.99	GHz
$\Delta_2$	=	2.13	GHz
$g_1$	1,51	GHz	
$g_2$	0,946	GHz	
$\omega_c$	=	8.818	GHz

$$g_1/\omega_c > 0.1$$

USC achieved!

NO-RWA:			
$\Delta_1$	=	0.33	GHz
$\Delta_2$	=	1.63	GHz
$\Delta_3$	=	2.58	GHz
$g_1$	0,283	GHz	
$g_2$	1.76	GHz	
$g_3$	0,283	GHz	
$\omega_c$	=	8.963	GHz

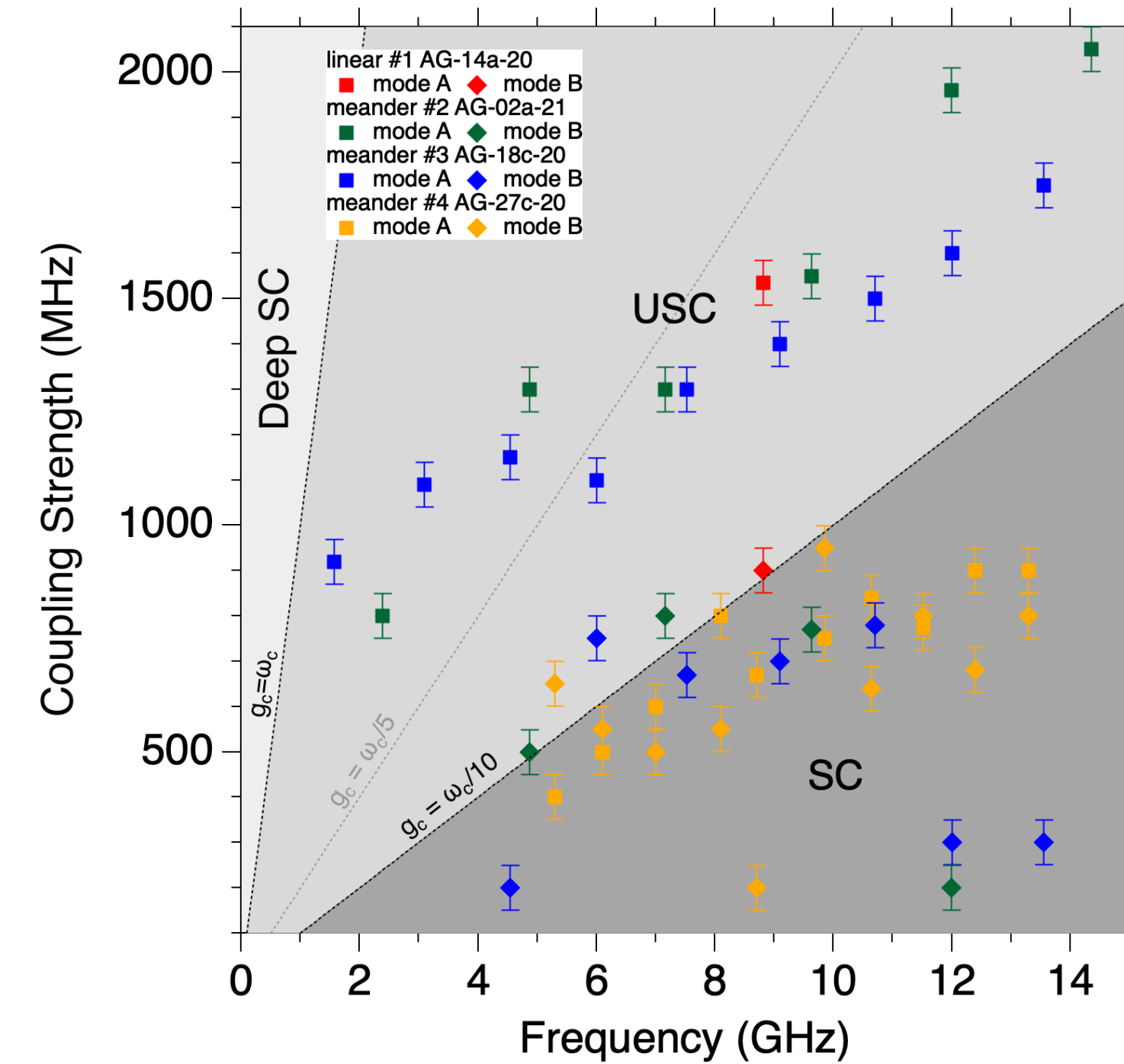
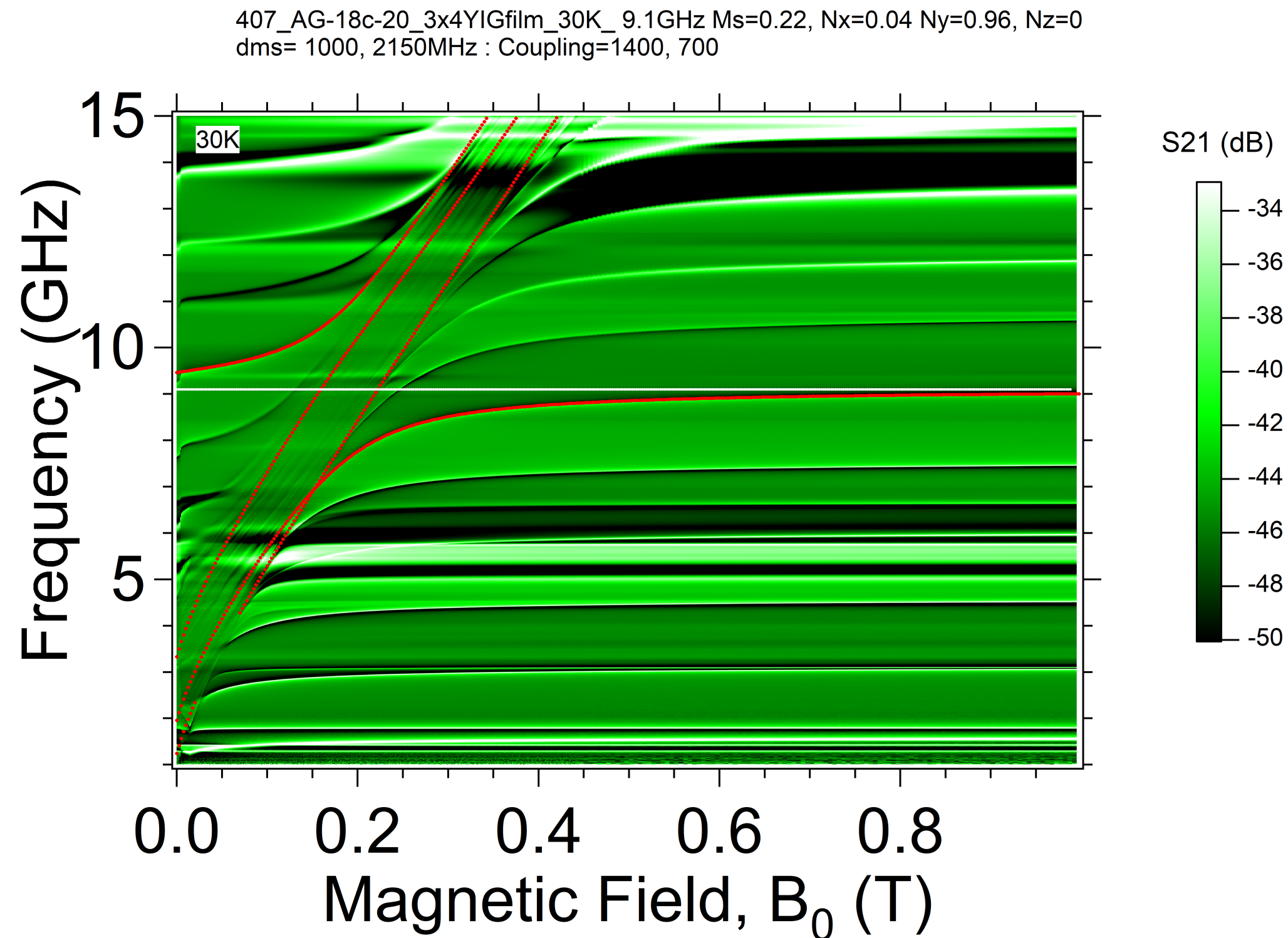
# Effect of diamagnetic term



Ag14Op4x2 ( $\omega_c=9.65\text{GHz}$ ) with YIGfilm 20micron on top  
Rabi( $\gamma=0.24$ ,  $N_{xx}=0$ ,  $N_{yy}=1-0.4$ ,  $N_{zz}=0.4$ ,  $\Delta=1\text{GHz}$ , 0,  
 $\lambda=1.9\text{e}9$ )



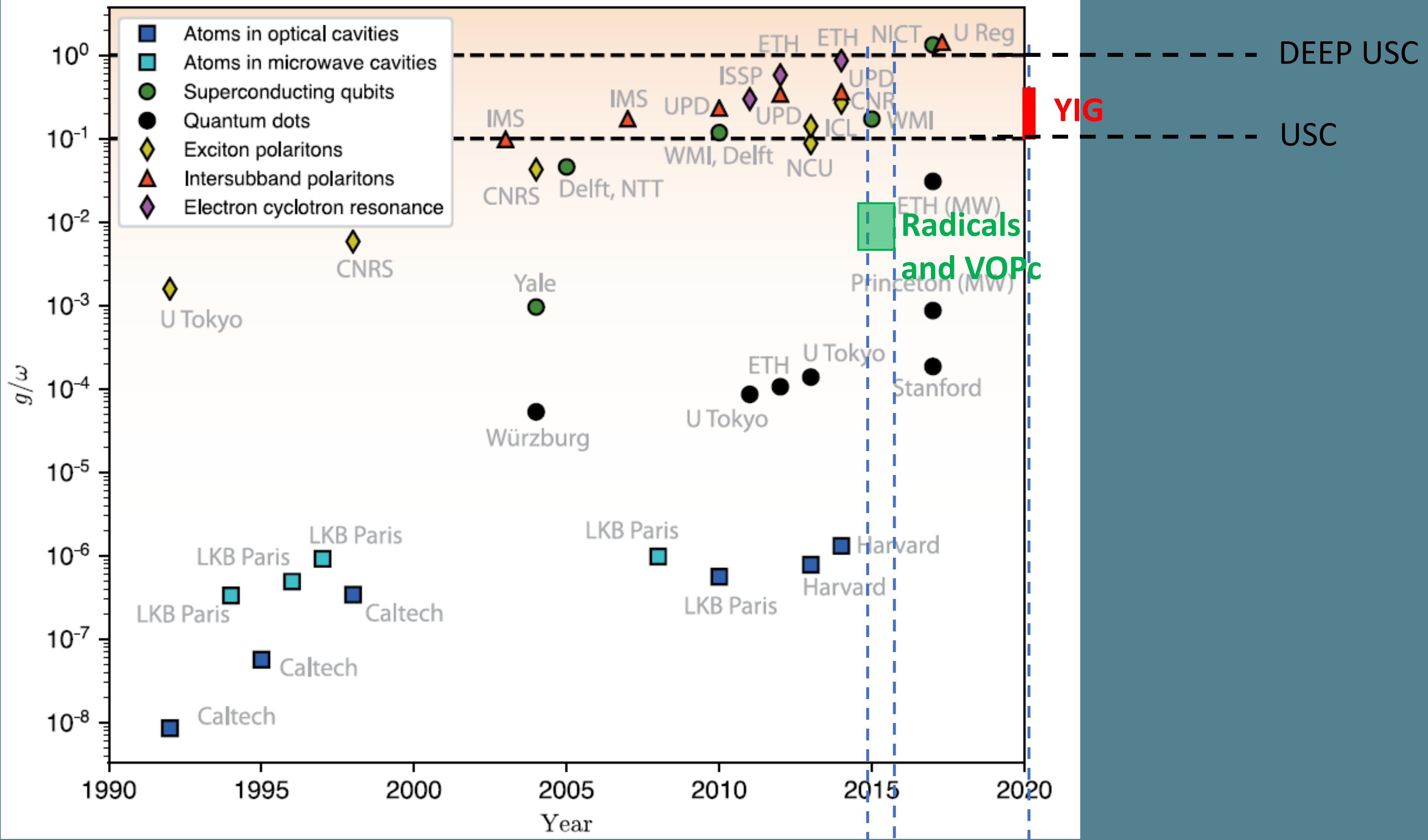
# Multi-mode coupling using meander resonator



- Input-output formalism:

$$S_{21} = \frac{\kappa_c}{i(\omega - \omega_c) - \frac{1}{2}(2\kappa_c + \kappa_{\text{int}}) + \sum_j \frac{|g_j|^2}{-\frac{1}{2}\gamma_j + i(\omega - \omega_j)}},$$

# Evolution of $g/\omega$ over last 20 years



adapted from Forn-Diaz et al Rev. Mod. Phys. 91, 025005 (2019)

# New Phenomena & Applications in USC regime

🌐 fast and protected QIP

🌐 nonlinear optics

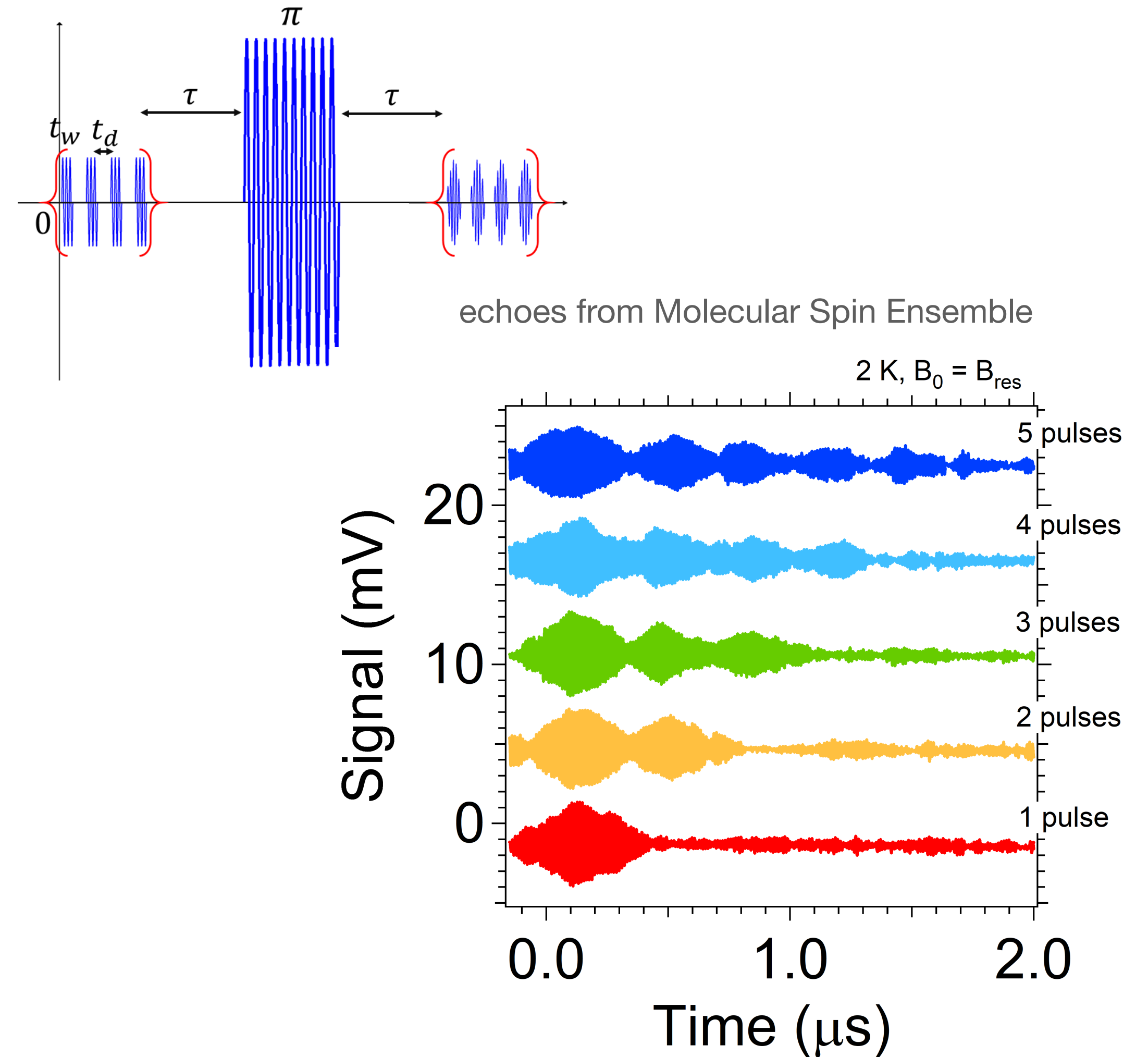
🌐 superradiance

🌐 enhancement of quantum phenomena



# New Phenomena & Applications

- fast and protected QIP
  - storage & retrieval of MW pulses
- nonlinear optics
- superradiance
- enhancement of quantum phenomena



# New Phenomena & Applications

fast and protected QIP

nonlinear optics

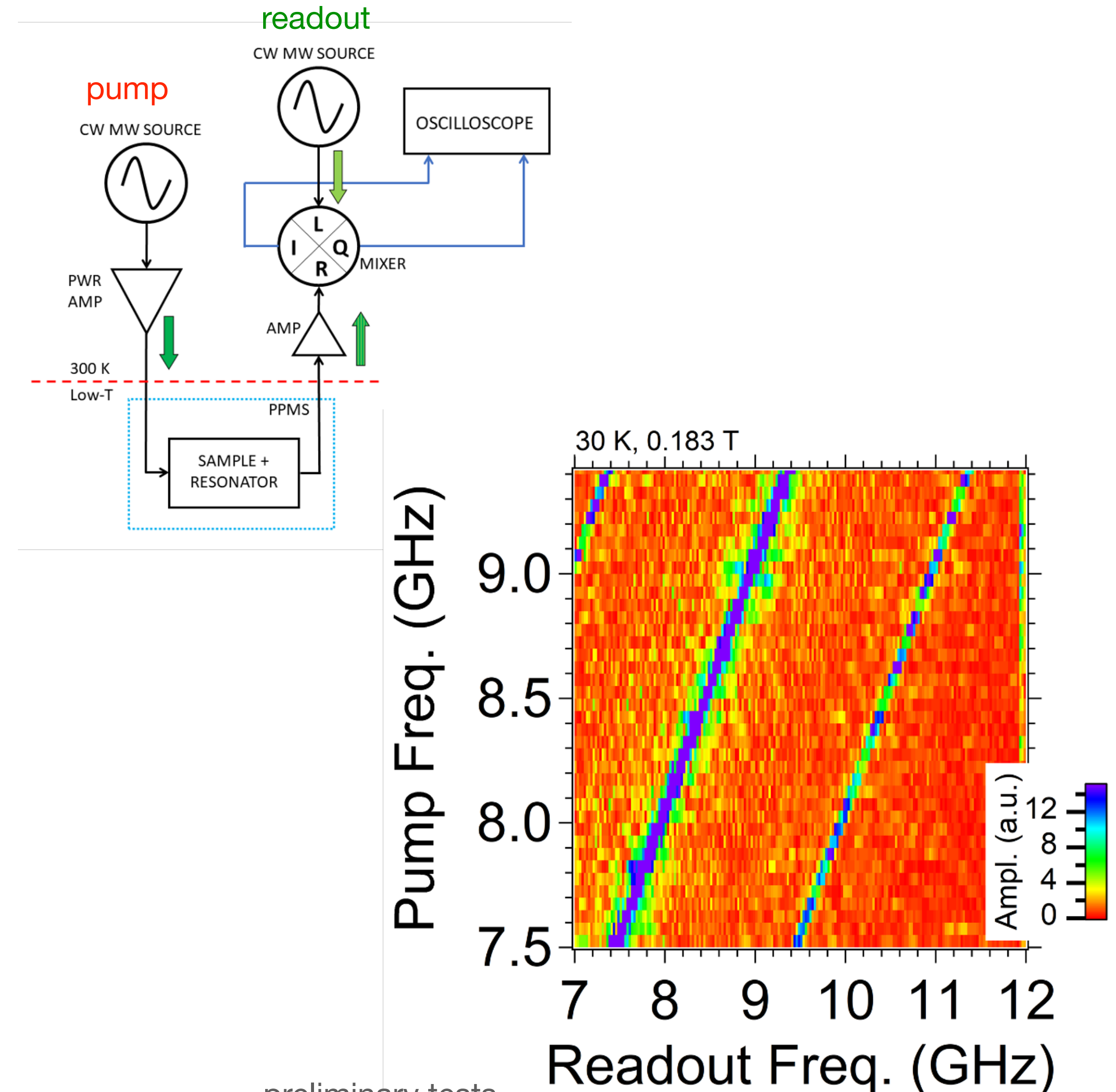
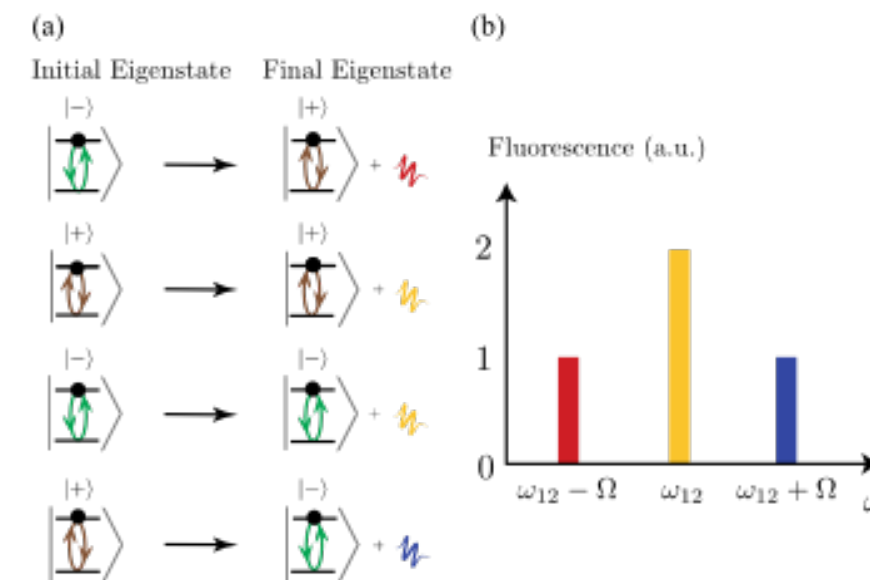
fluorescence

superradiance

enhancement of quantum phenomena

PHYSICAL REVIEW B 92, 201402(R) (2015)  
**Theory of intersubband resonance fluorescence**

Nathan Shammah and Simone De Liberato



preliminary tests

(C. Bonizzoni, A. Ghirri, M. Maksutoglu, M. Affronte)



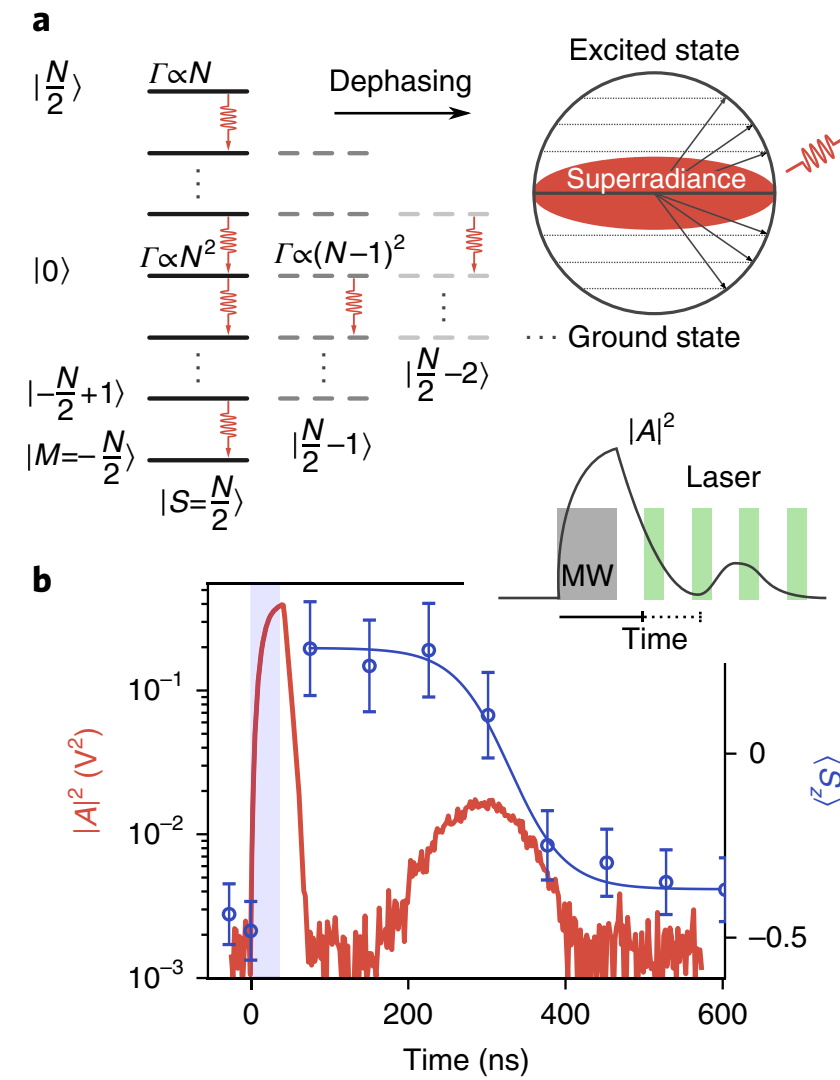
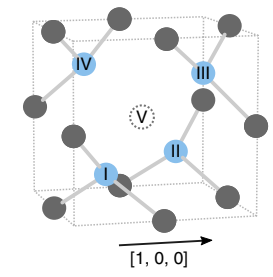
# New Phenomena & Applications

fast and protected QIP

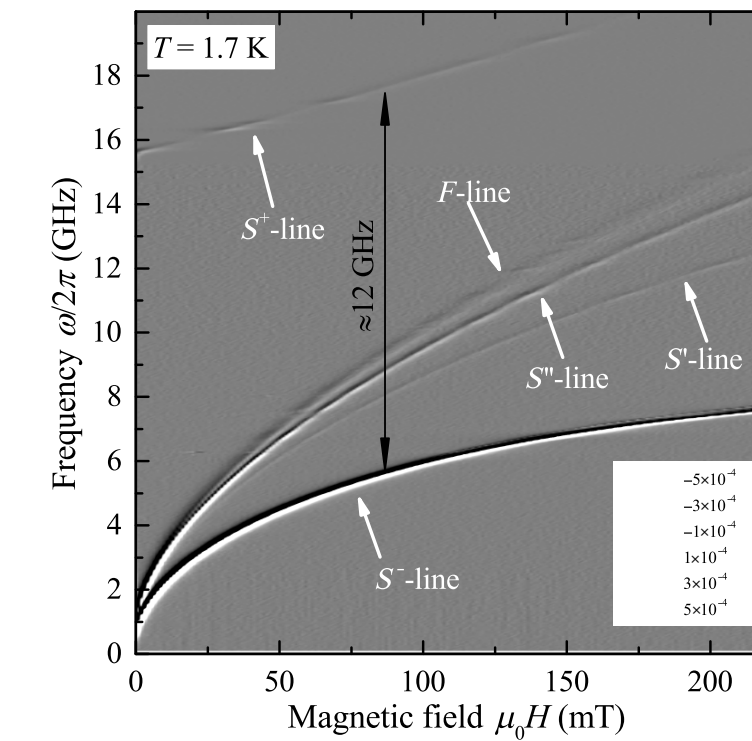
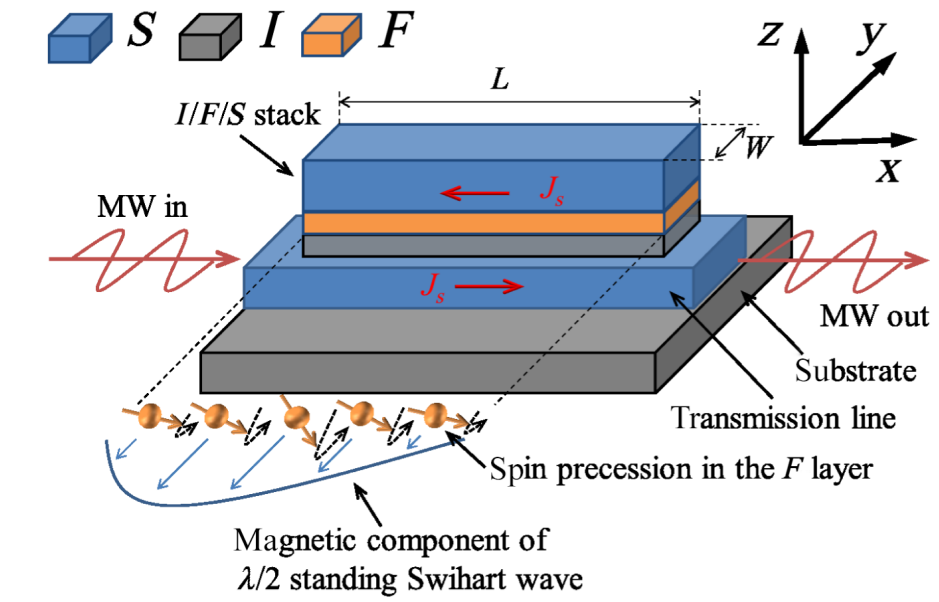
nonlinear optics

superradiance

enhancement of quantum phenomena

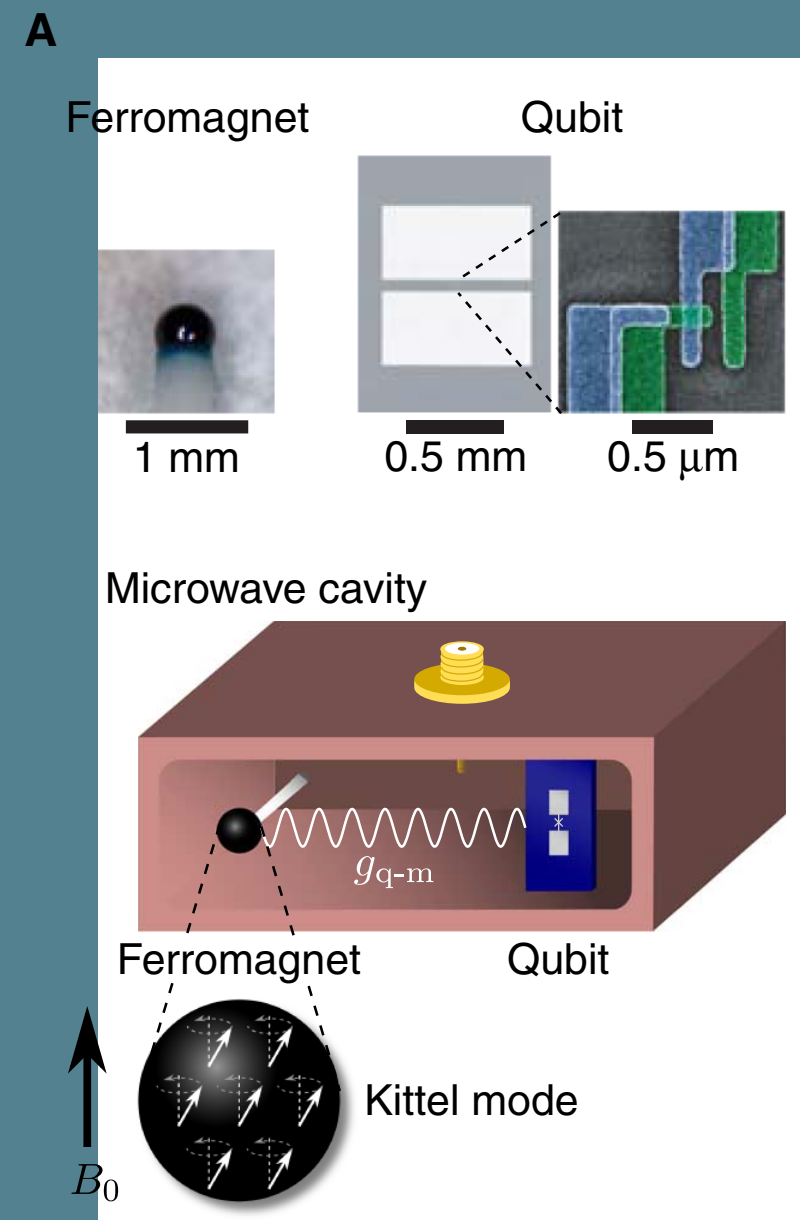


PHYS. REV. APPLIED 16, 034029 (2021)

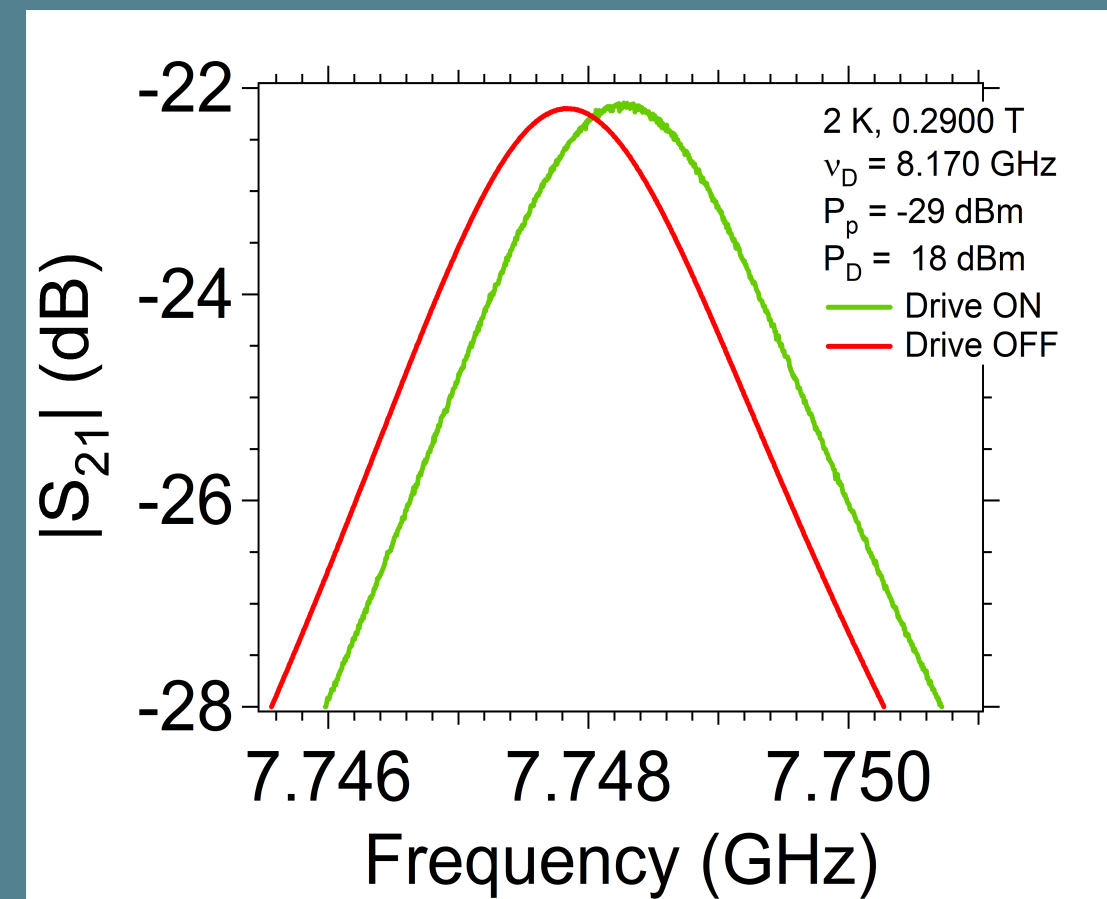
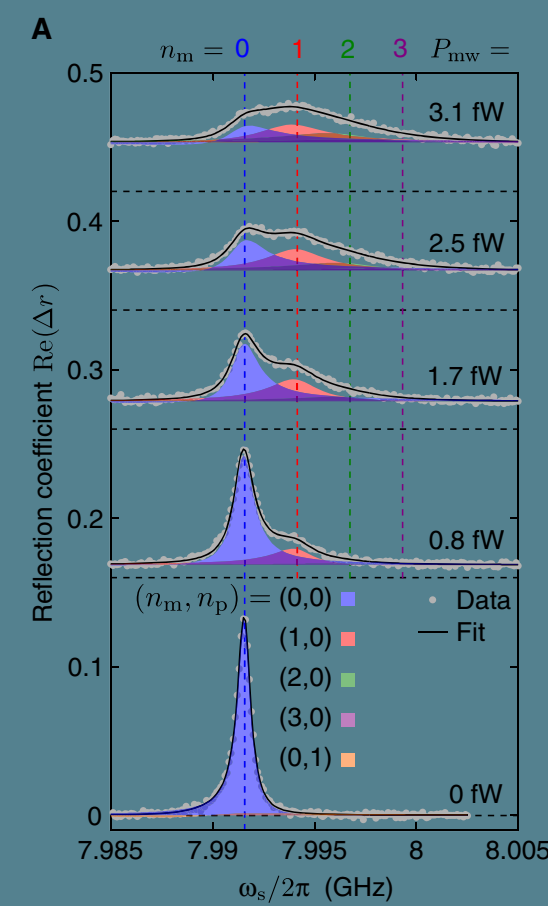


$g/\omega = 0.6 !$

# PERSPECTIVES FOR QUANTUM SENSING



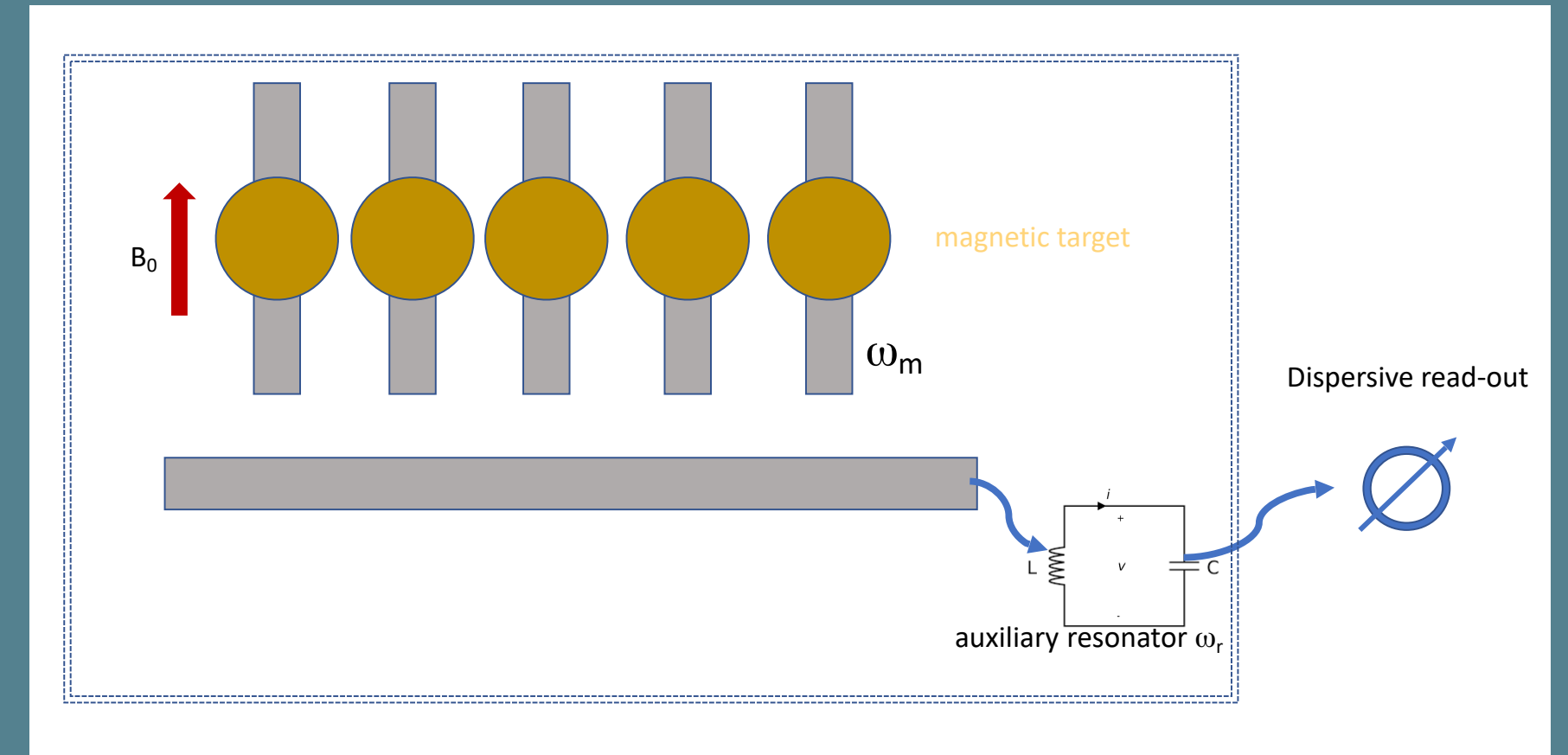
## Dispersive Read out of spin states



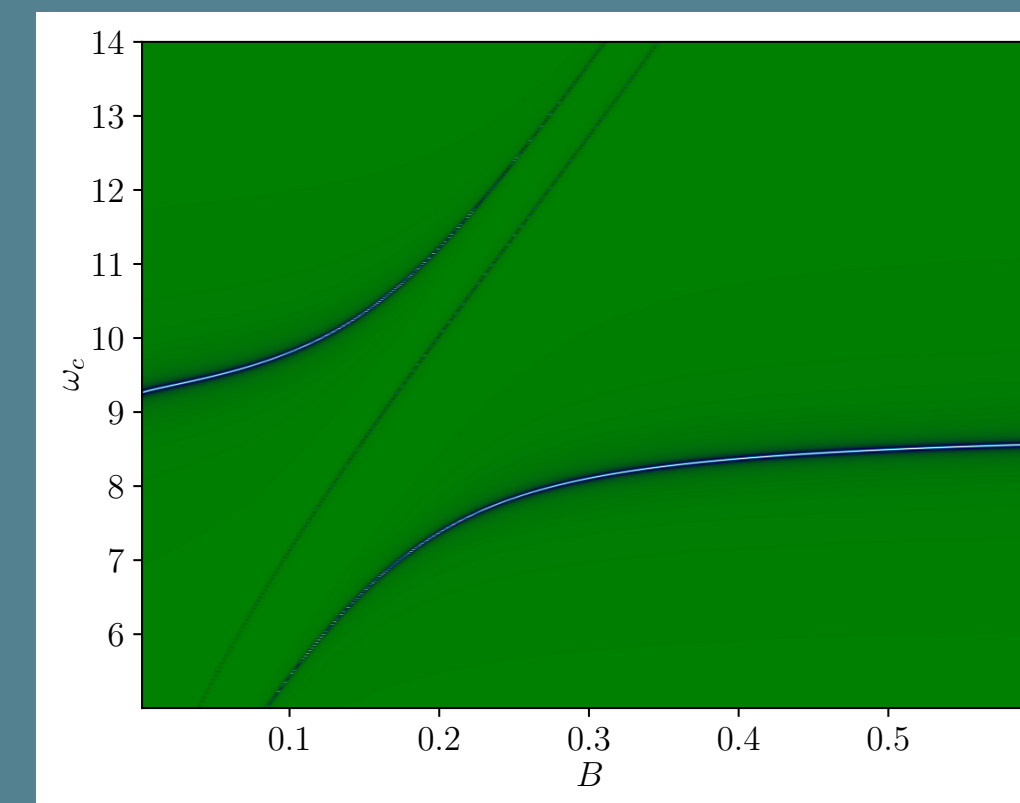
C. Bonizzoni et al. *Adv. Quantum Tech.* 4, 2100039 (2021)

## Scalability of hybrid s/c-spin architectures

Possible cQED scheme for dispersive read out of single magnetic excitation with planar architectures



## Ultra-Strong spin-photon coupling



Lachance-Quirion et al., *Sci. Adv.* 2017;3:e1603150



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