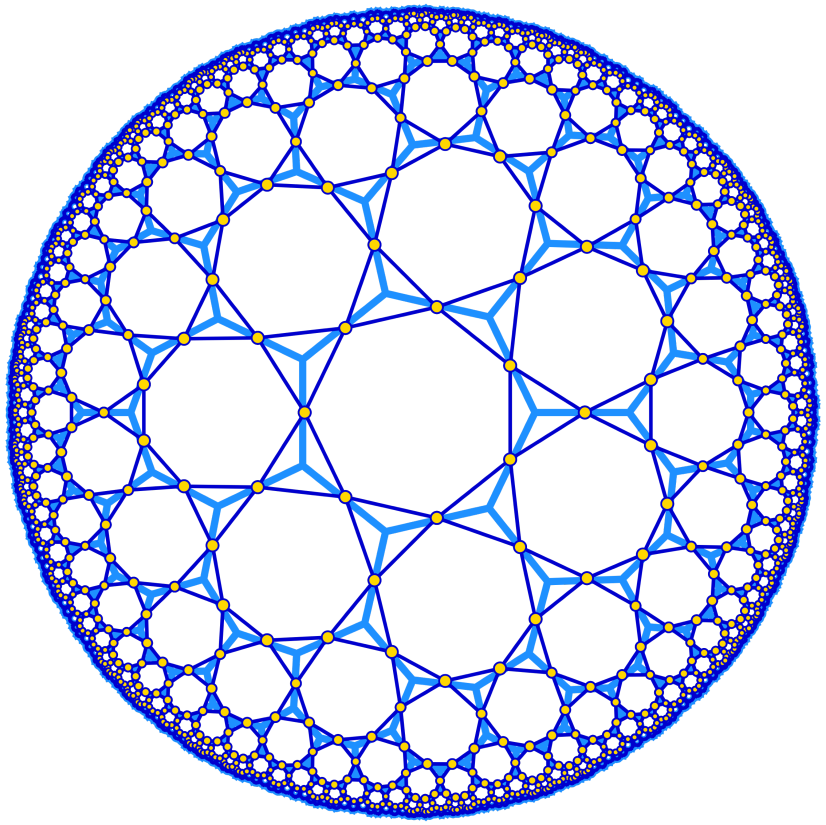
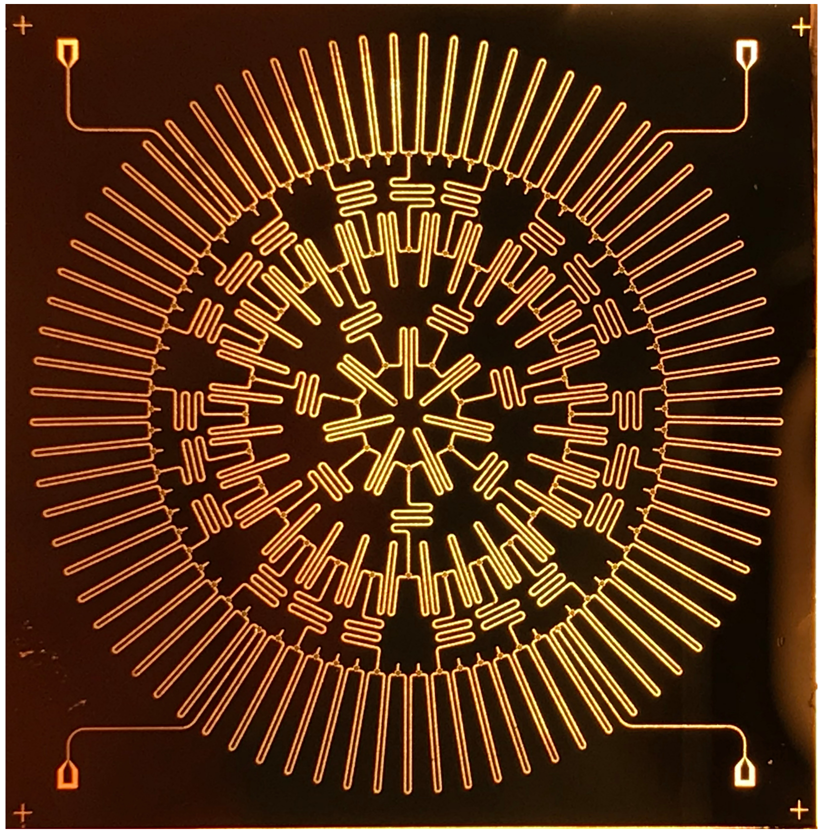
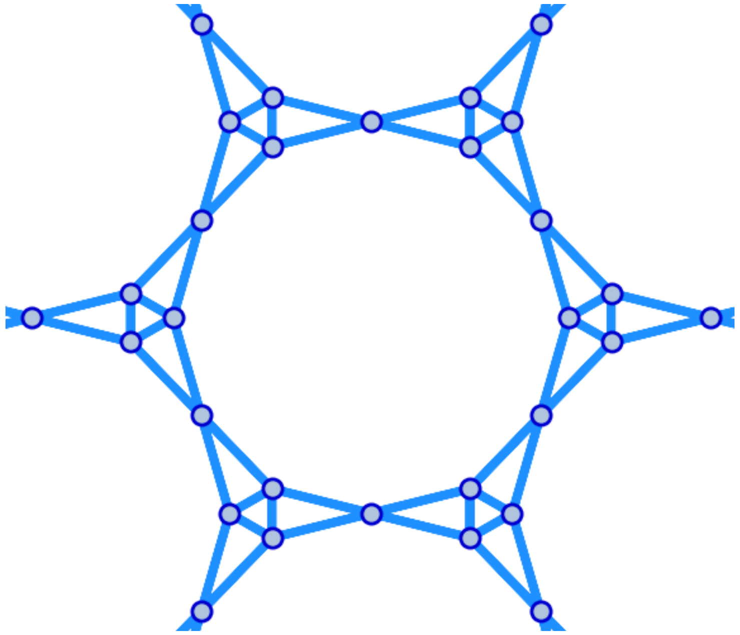
Circuit QED Lattices

Alicia Kollár

Department of Physics and JQI, University of Maryland







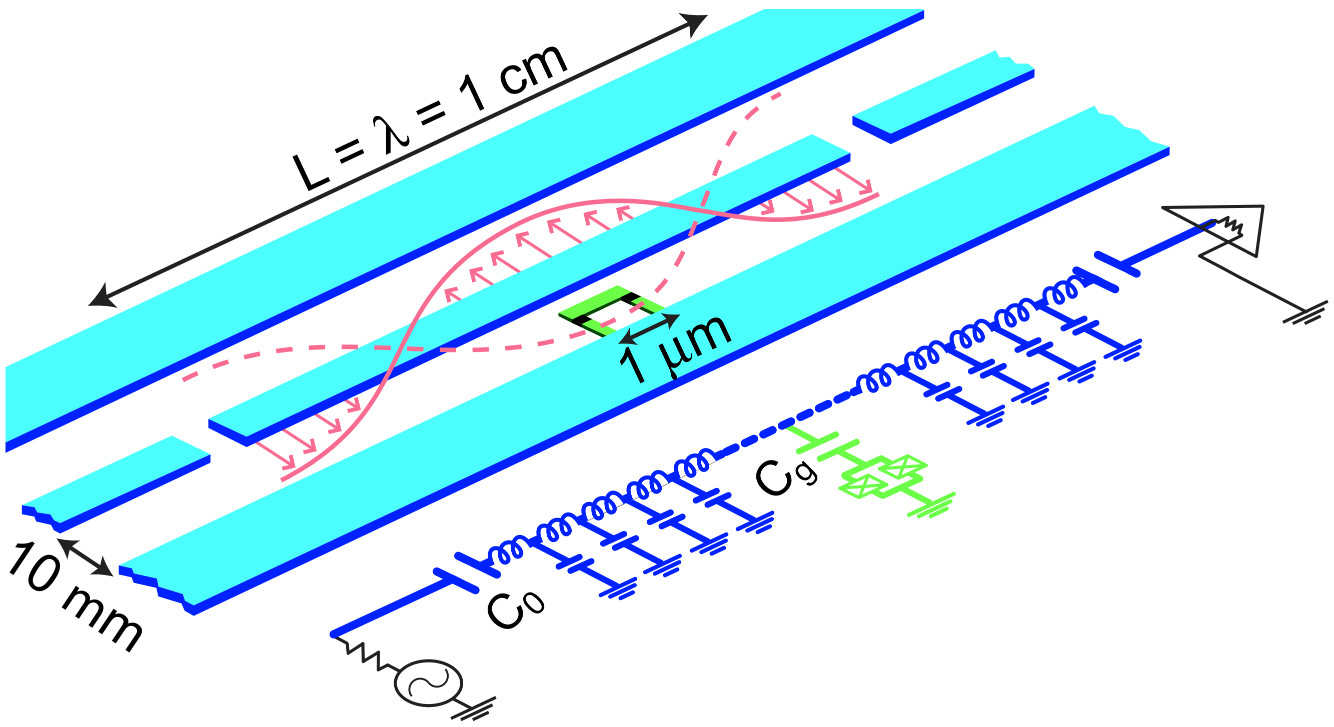
Trento, Oct 4, 2022

Outline

- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
  - Line-graph lattices
  - Interacting photons
- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
- Mathematical Connections
  - Planar Gaps
  - Maximal Gaps
  - Quantum Error Correcting Codes
- Experimental Developments

Microwave Coplanar Waveguide Resonators

- 2D analog of coaxial cable
- Cavity defined by cutting center pin
- Voltage antinode at “mirror”



Blais *et al.*, PRA **69**, 062320 (2014)

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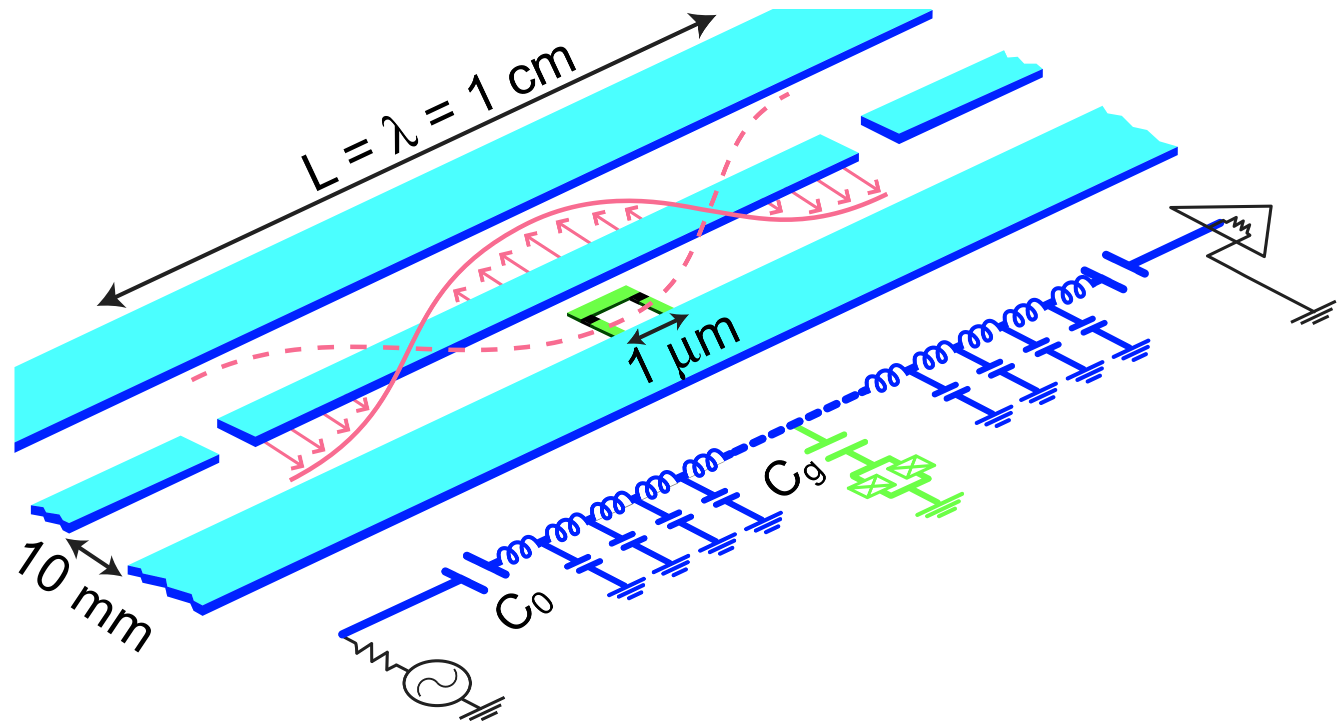

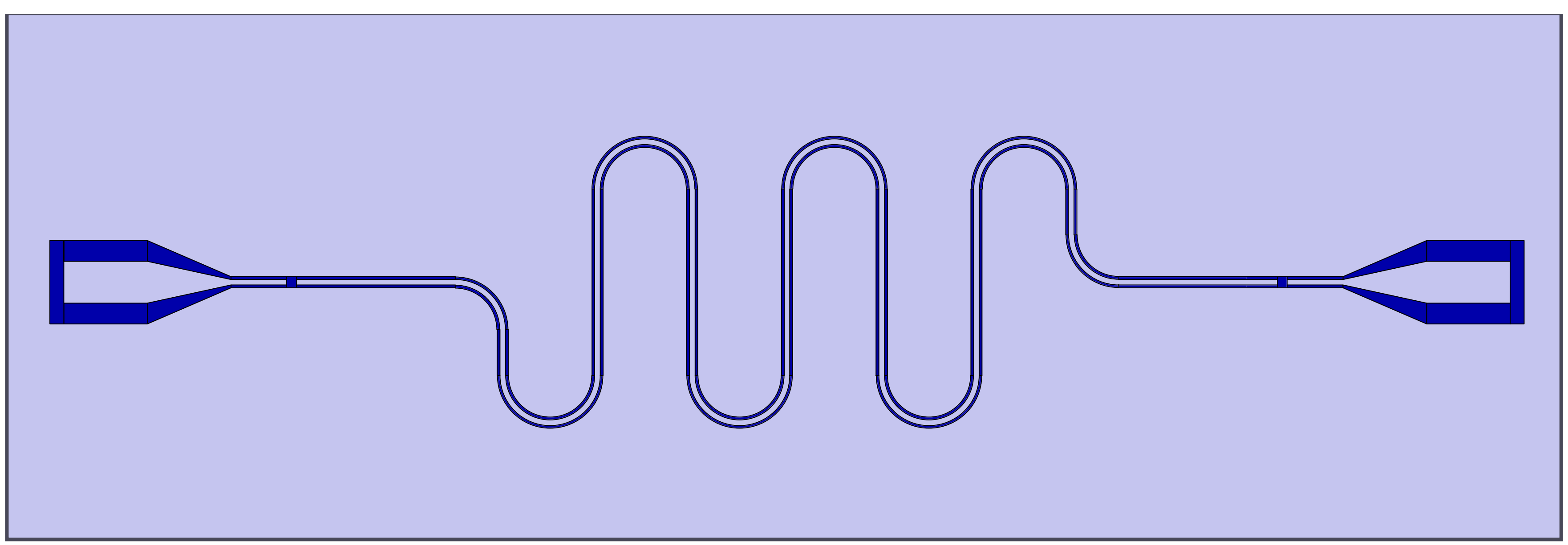

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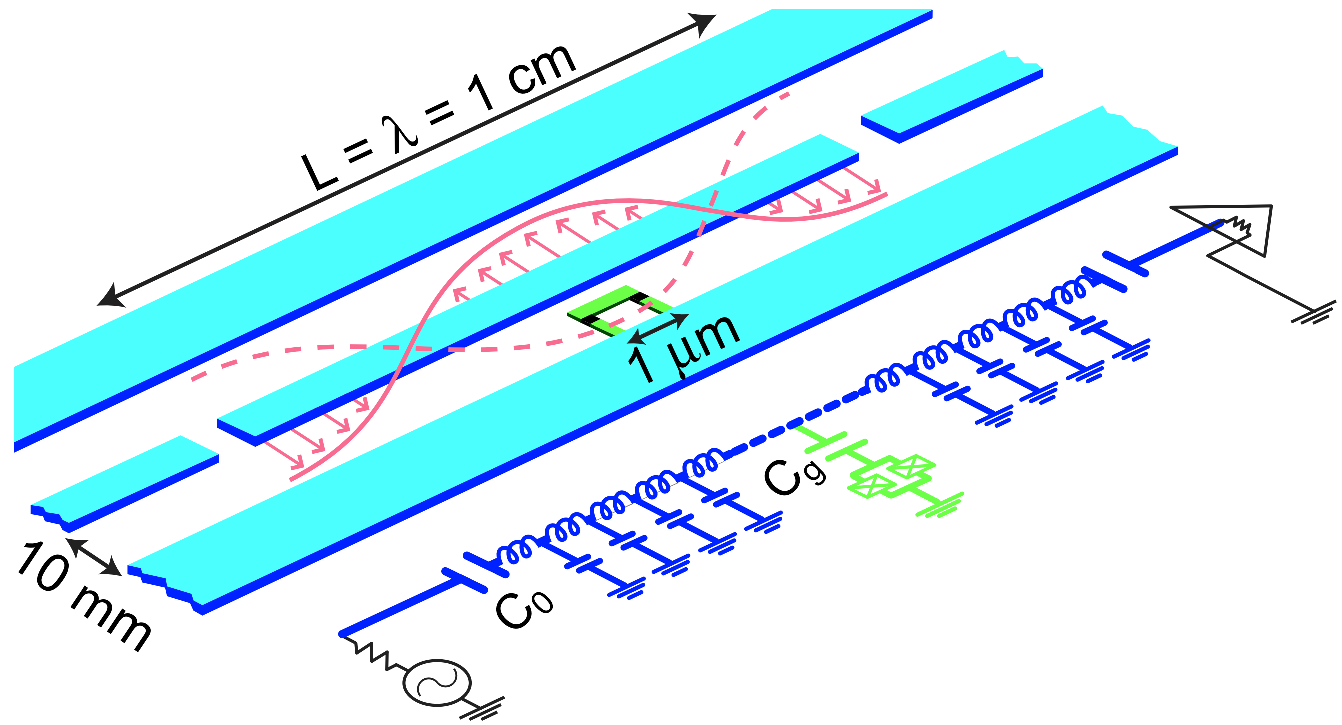
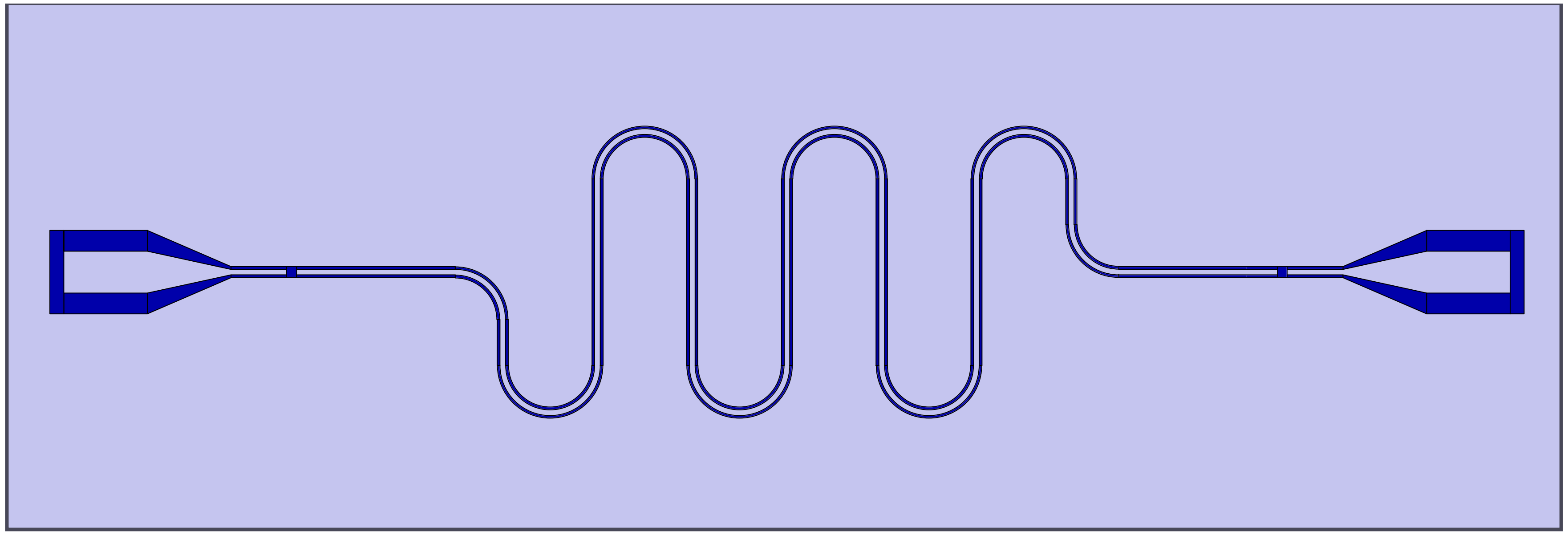


Harmonic oscillator

$$\hat{H} = \frac{1}{2C}\hat{n}^2 + \frac{1}{2L}\hat{\varphi}^2$$



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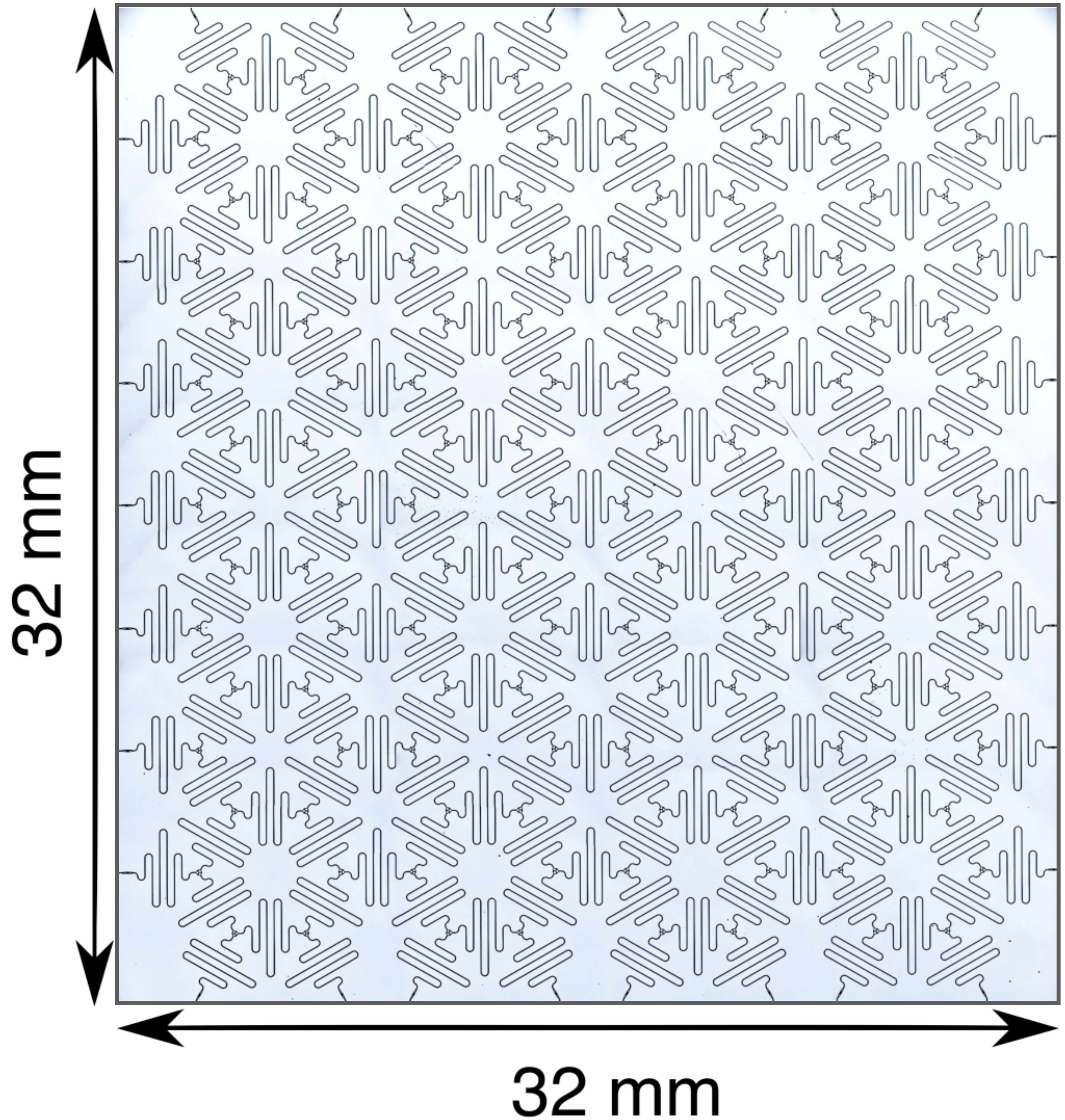
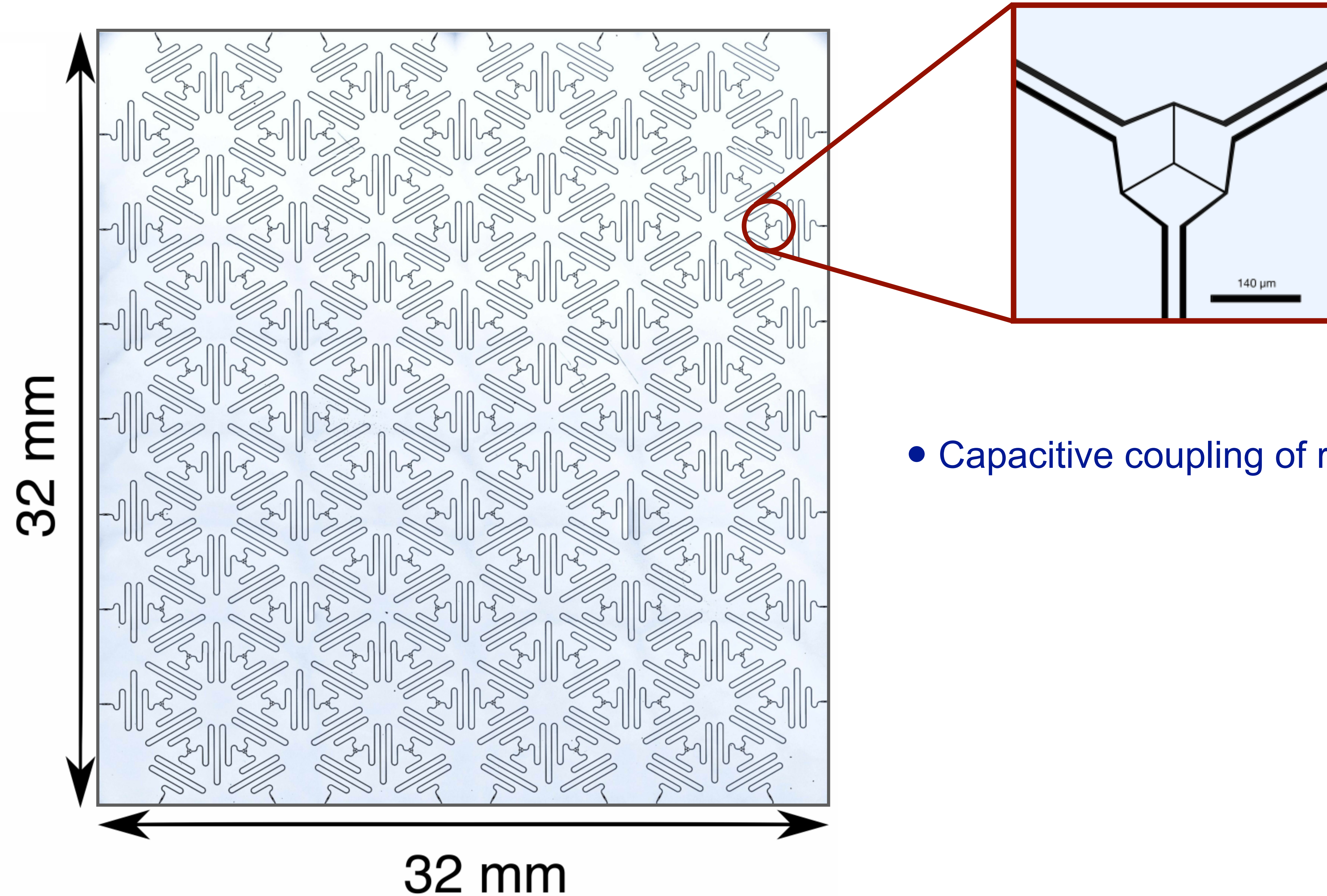
CPW Lattices



Houck *et al*. Nat Phys **8**, (2012)
Underwood *et al*. PRA **86**, 023837 (2012)

CPW Lattices



- Capacitive coupling of resonators

Houck *et al*. Nat Phys **8**, (2012)
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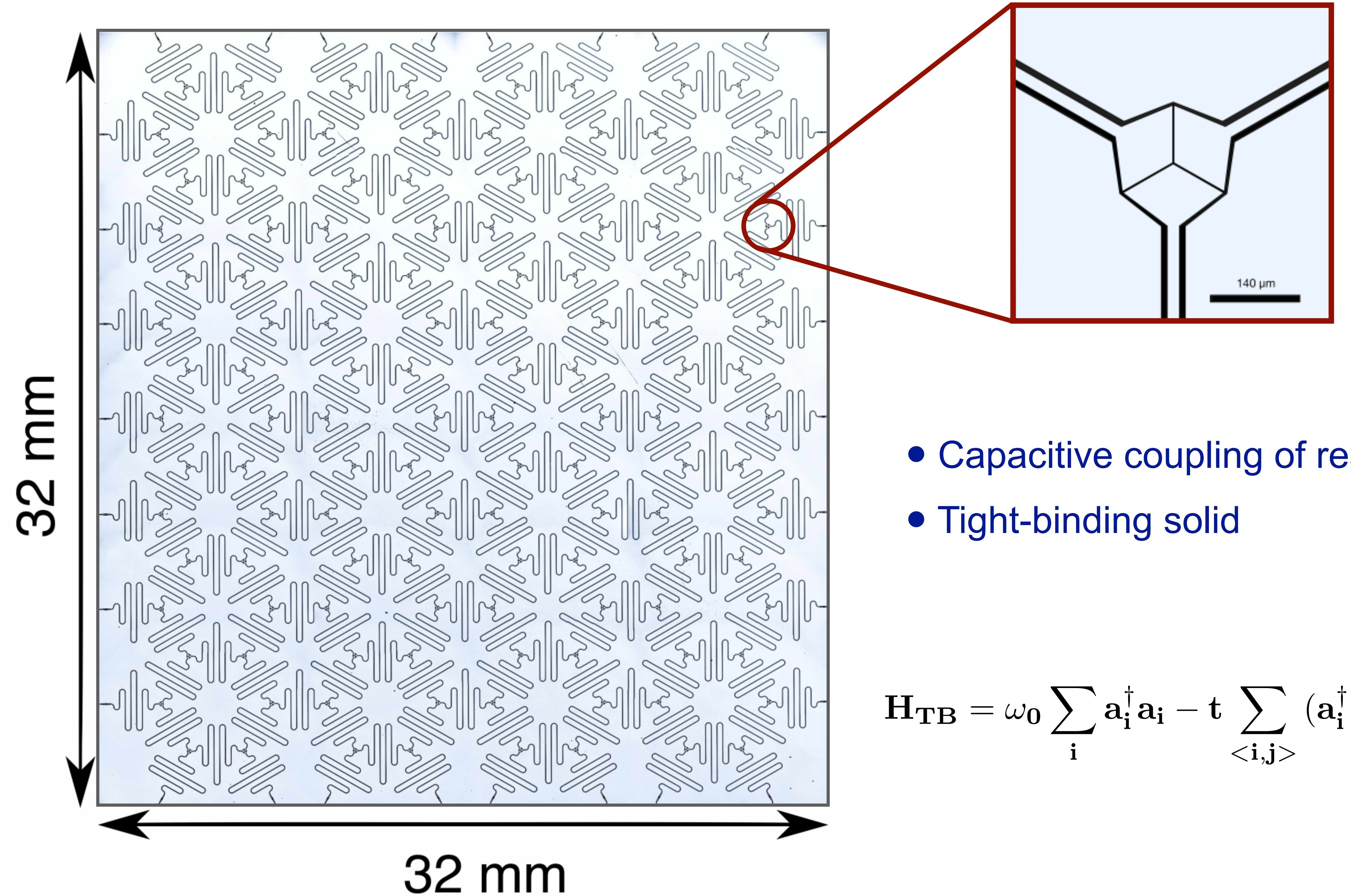
CPW Lattices



- Capacitive coupling of resonators
- Tight-binding solid

$$\mathbf{H_{TB}} = \omega_0 \sum_{\mathbf{i}} \mathbf{a_i^\dagger a_i} - \mathbf{t} \sum_{<\mathbf{i,j}>} (\mathbf{a_i^\dagger a_j} + \mathbf{a_j^\dagger a_i})$$

Houck *et al*. Nat Phys **8**, (2012)
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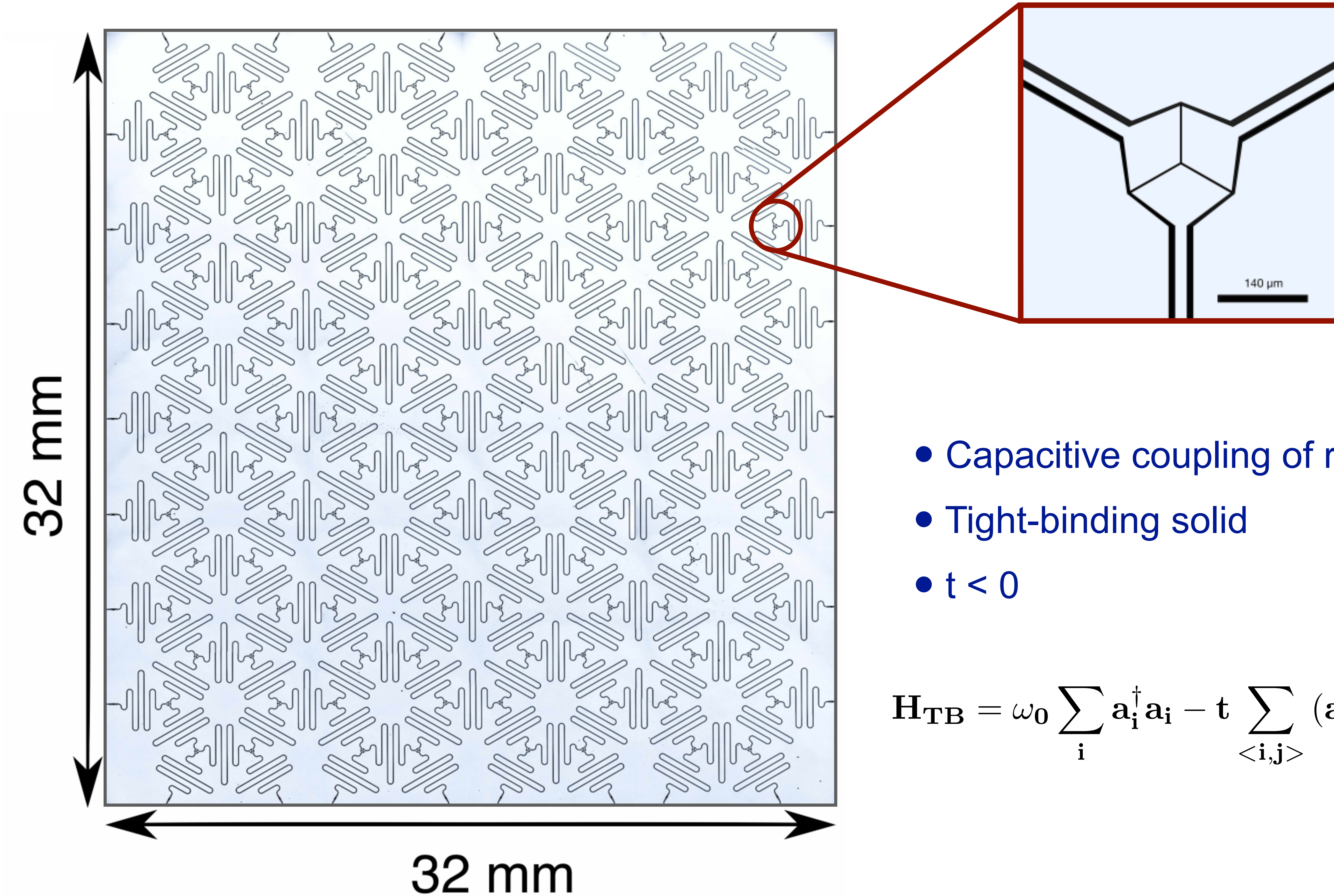
CPW Lattices



- Capacitive coupling of resonators
- Tight-binding solid
- $t < 0$

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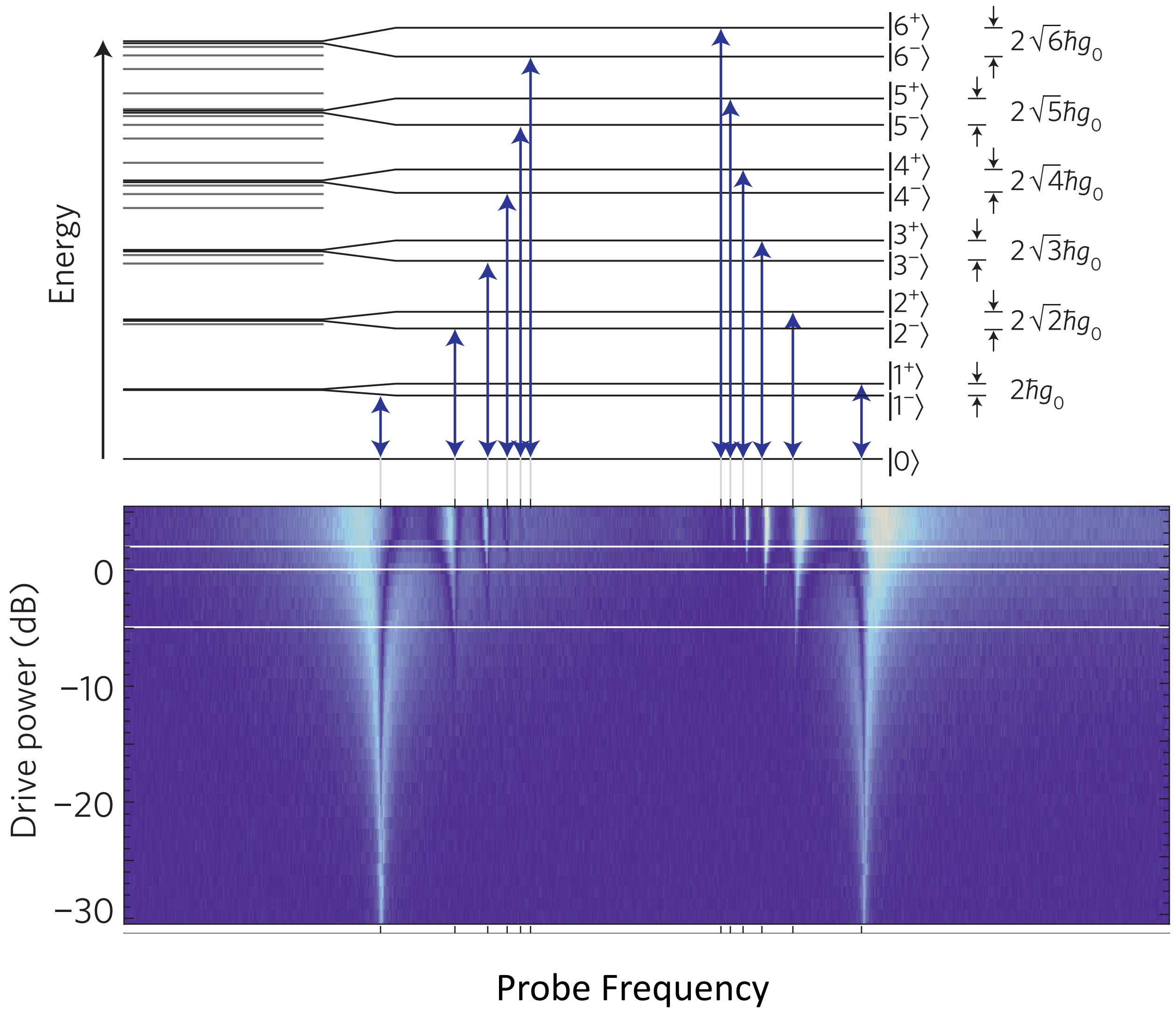
Combining Lattices and Qubits

Number-Resolved Transitions

$$H_{JC} = \omega_c a^\dagger a + \frac{1}{2}\omega_q \sigma_z + g_0(a^\dagger \sigma^- + a\,\sigma^+)$$



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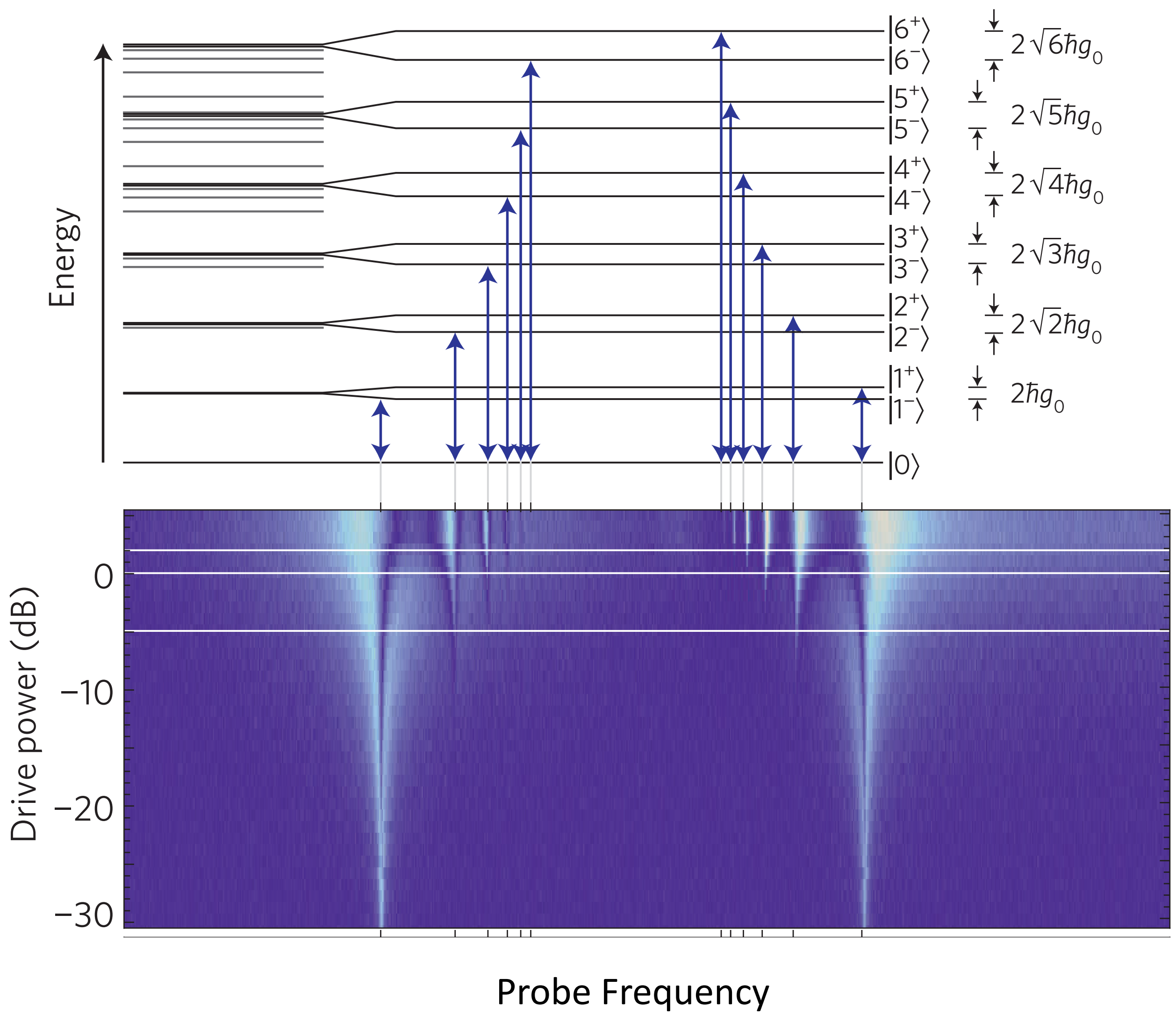

Qubits in Photonic Crystals

- Effective swap interaction between qubits
- All modes in parallel

$$H = \hbar\ \sigma_1^+ \sigma_2^- \ \sum_m \frac{g_m^2}{\Delta(m)}\ \psi_m(x_1)\psi_m^*(x_2) + h.c.$$

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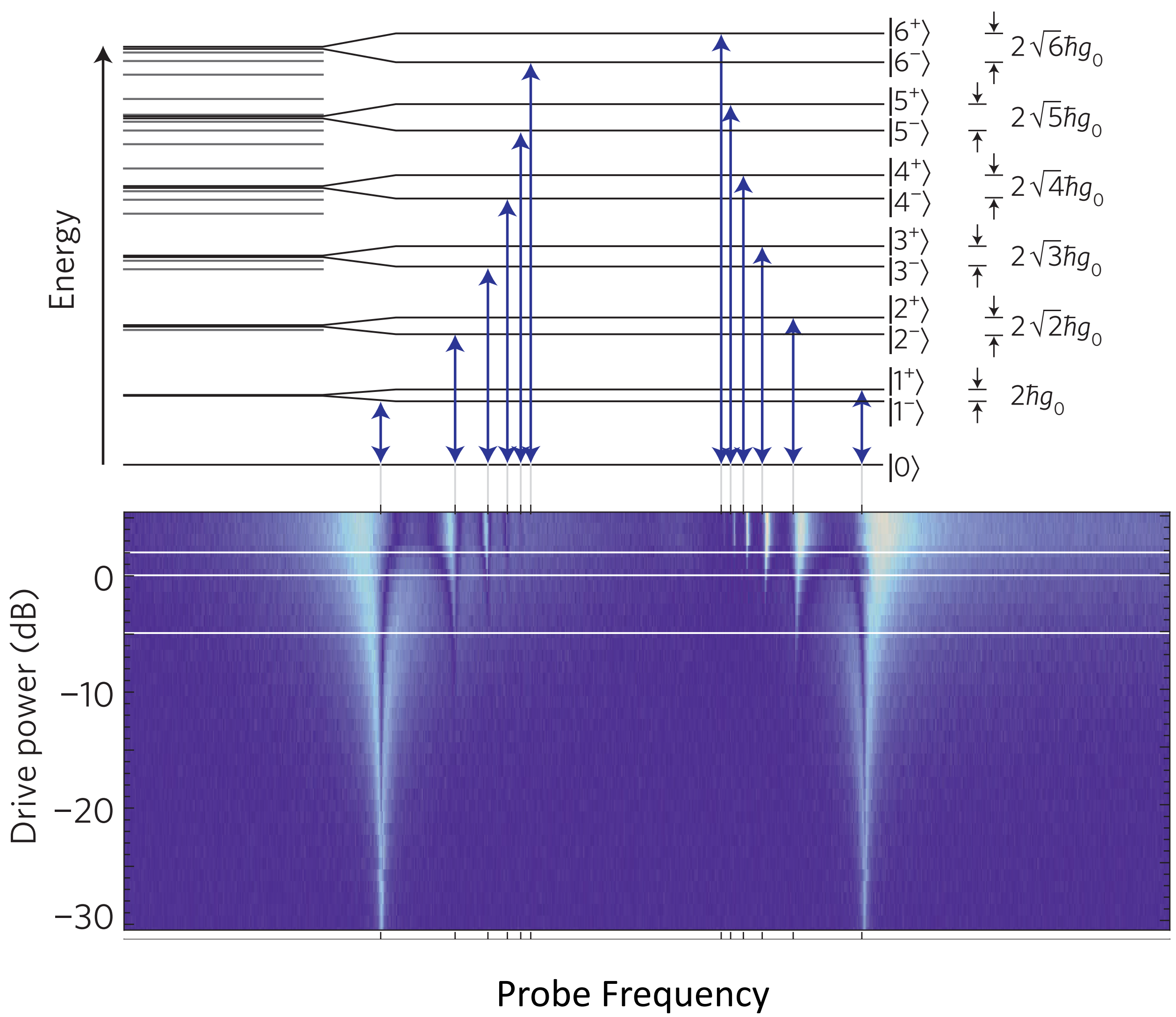

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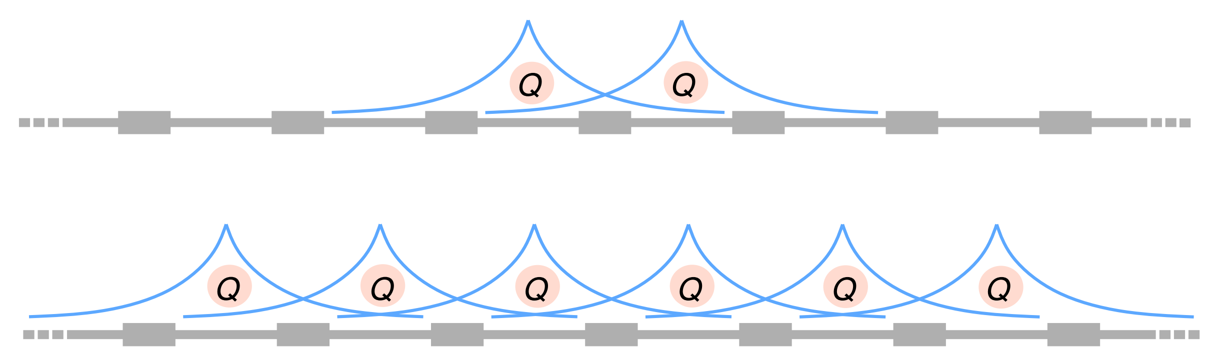
1D-Photonic Crystal

- Exponentially localized bound state



Douglas *et al*. Nat. Photon. (2015)
Calajó *et al*. PRA (2016)
Sundaresan *et al*. PRX (2019)
Ferreira *et al*. arXiv 2001.0324 (2020)

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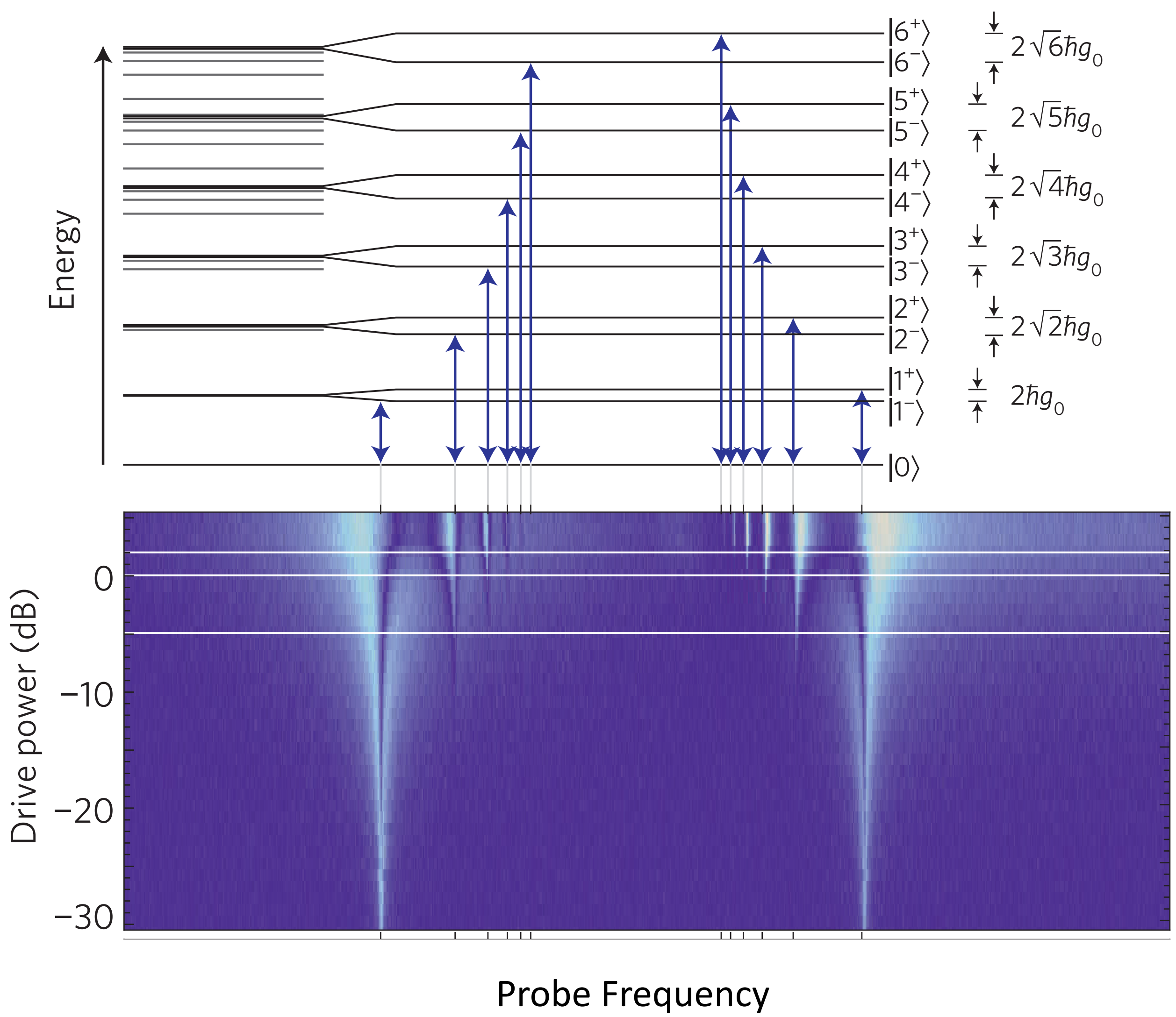

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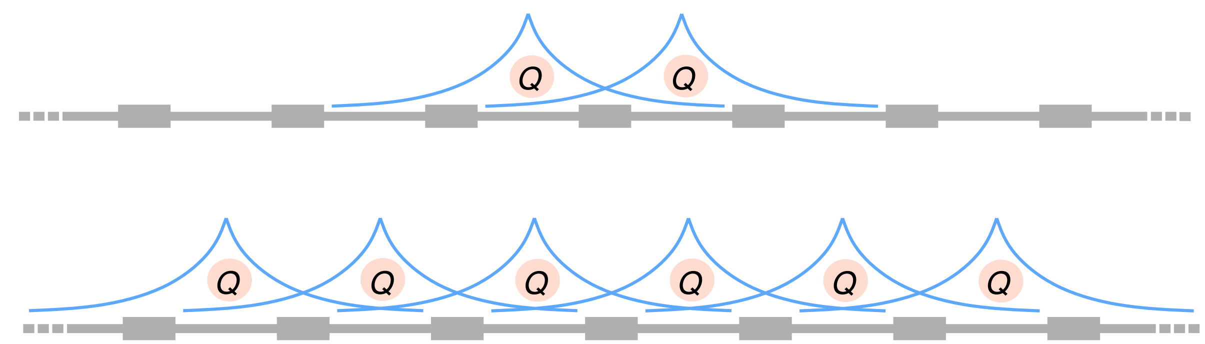

New Regimes:

- New lattices

Bishop *et al*. Nat Phys **5**, (2009) Houck *et al*. Nat Phys **8**, (2012)

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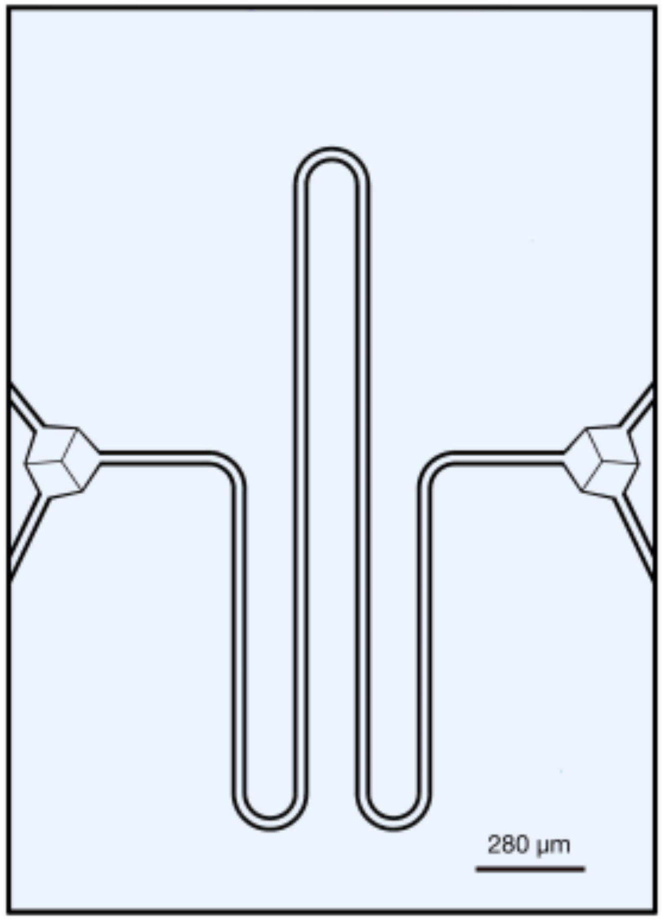
Deformable Resonators



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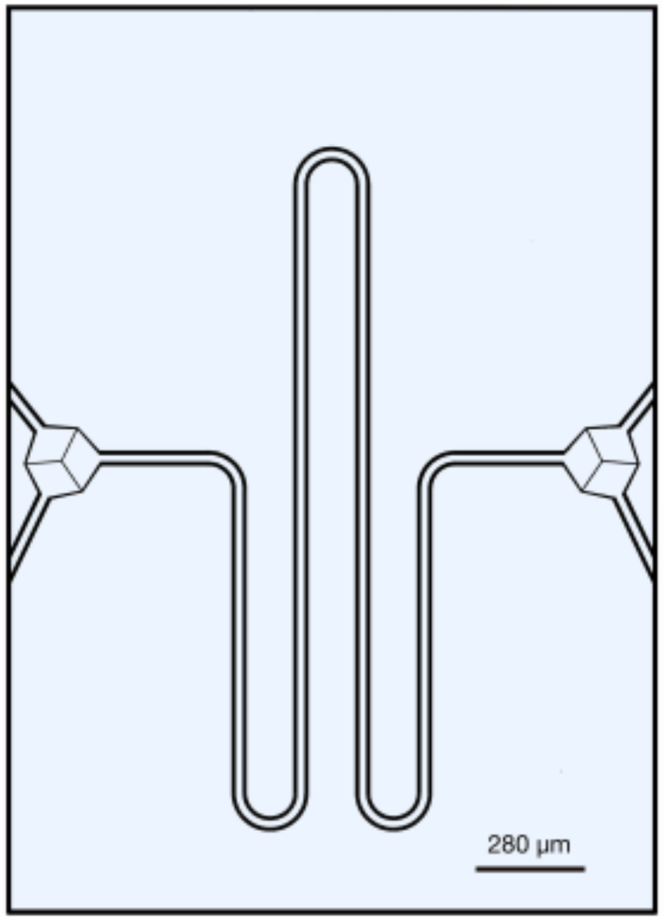
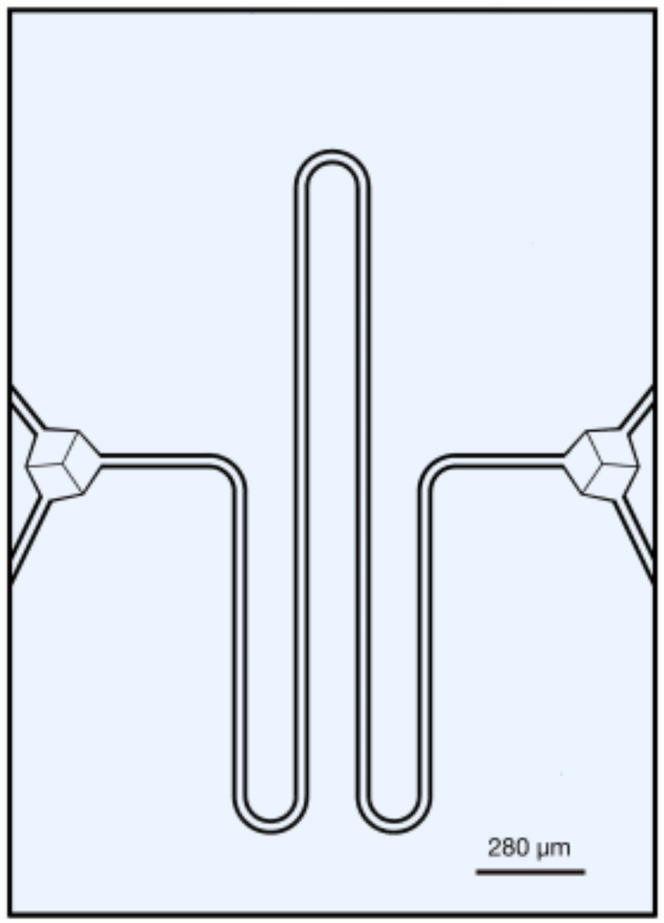


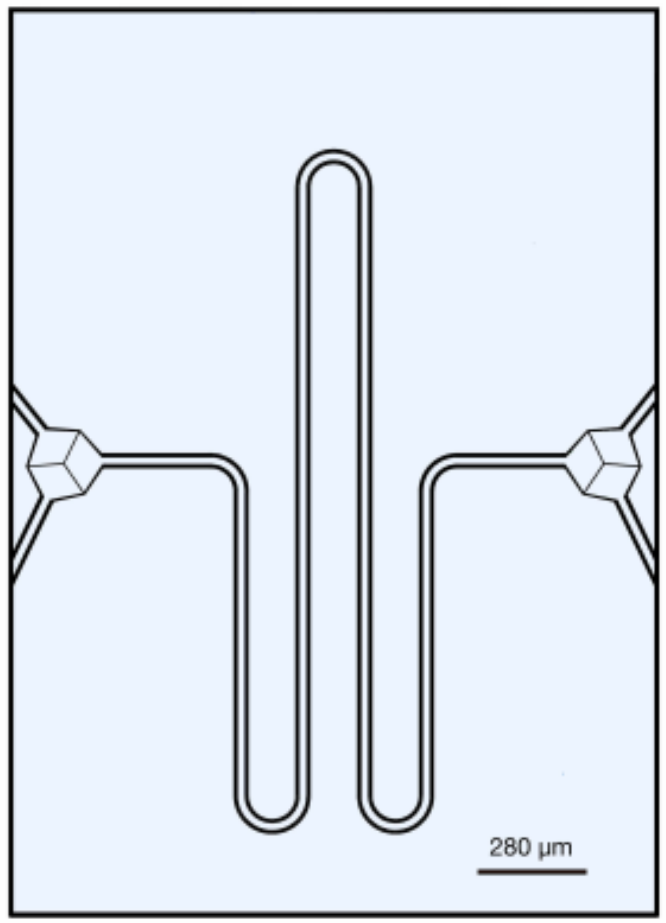
- Frequency depends only on length

Deformable Resonators



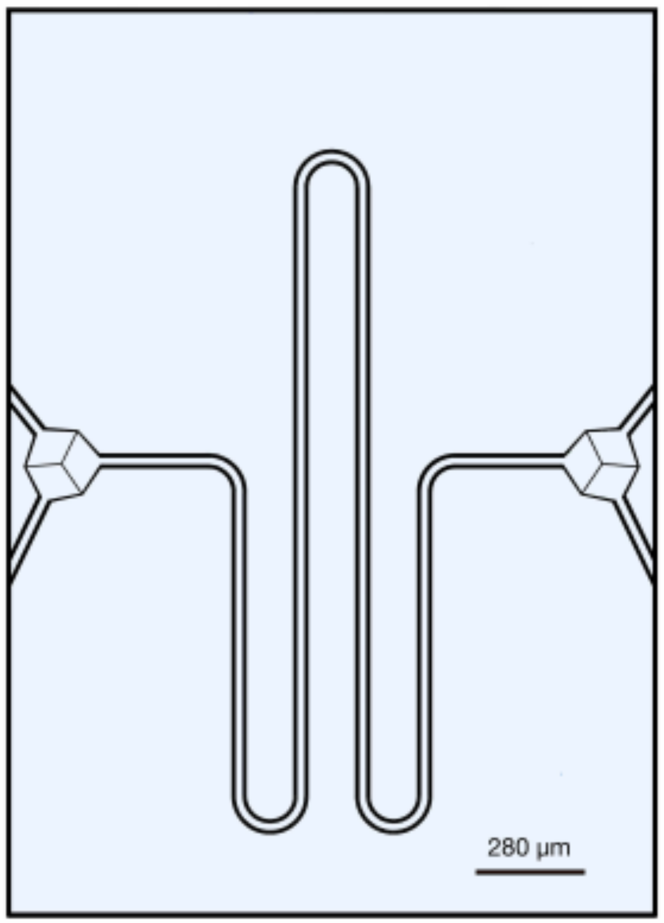
- Frequency depends only on length
- Coupling depends on ends

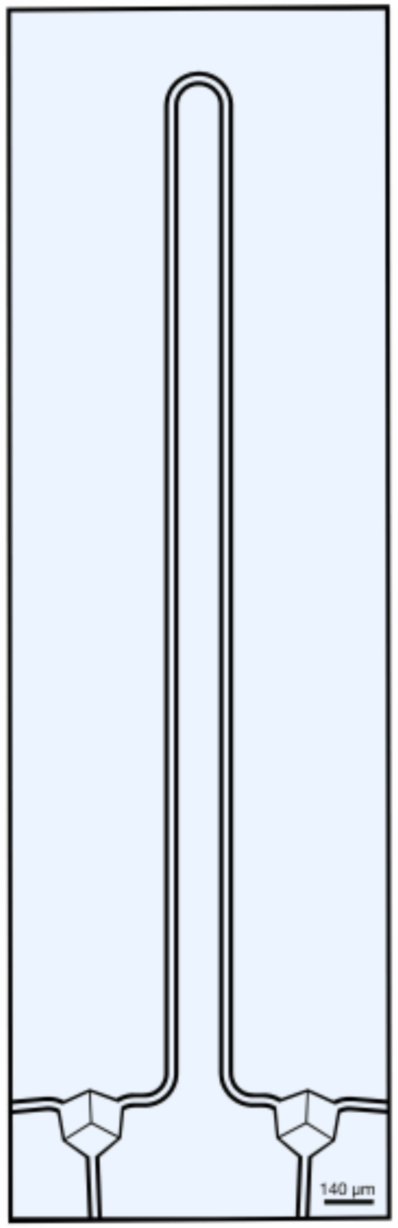
Deformable Resonators



- Frequency depends only on length
- Coupling depends on ends
- “Bendable”

Deformable Resonators





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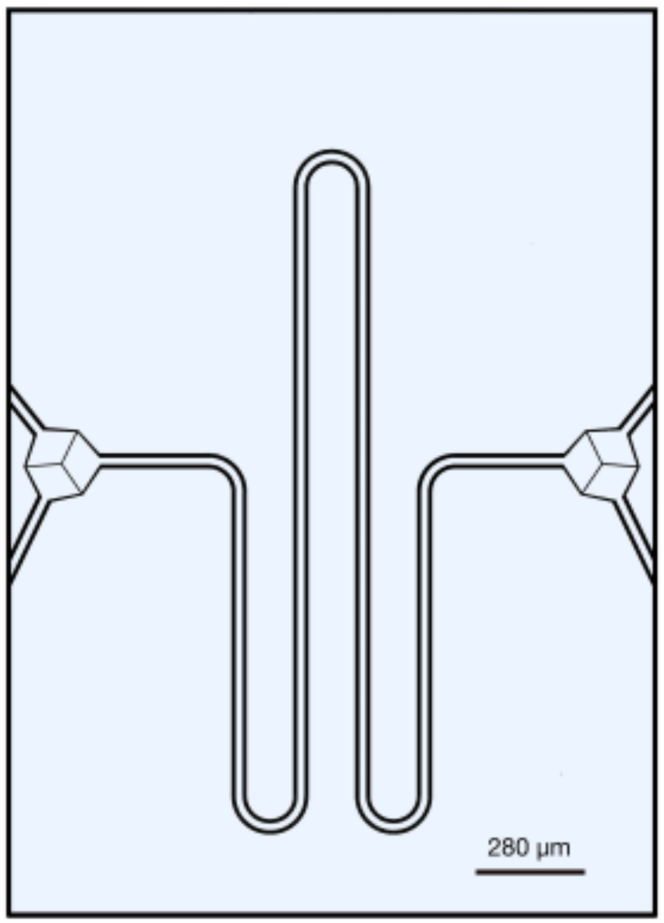

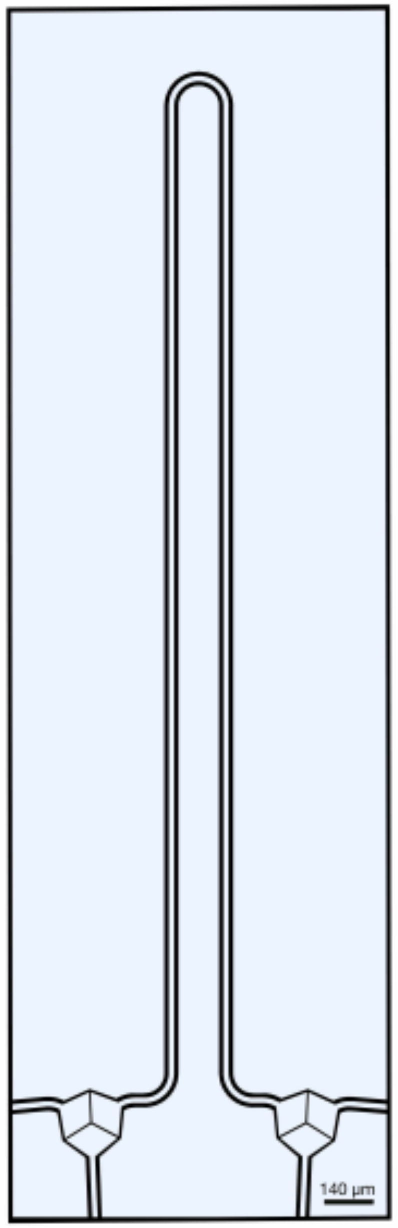

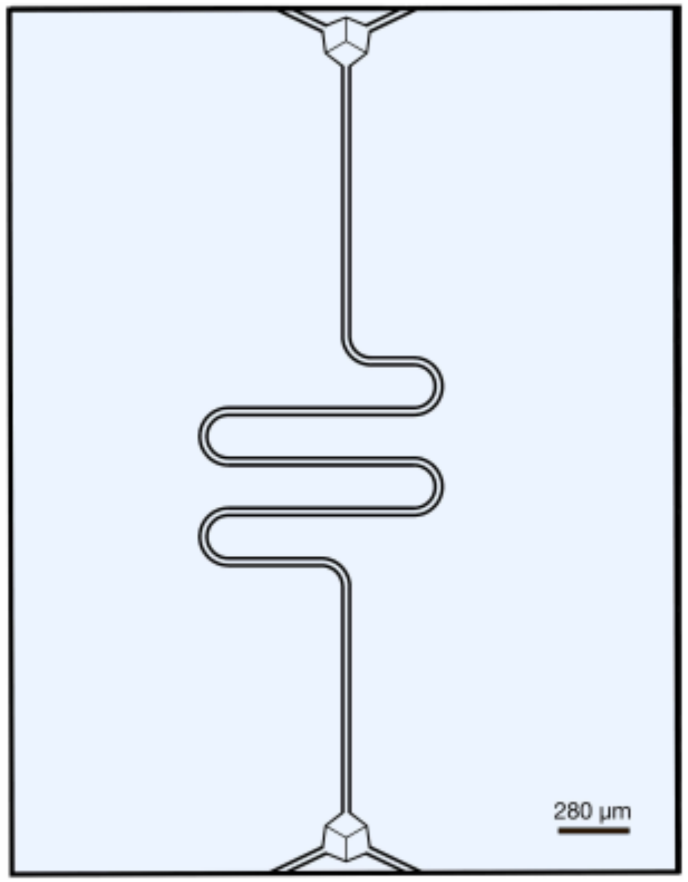

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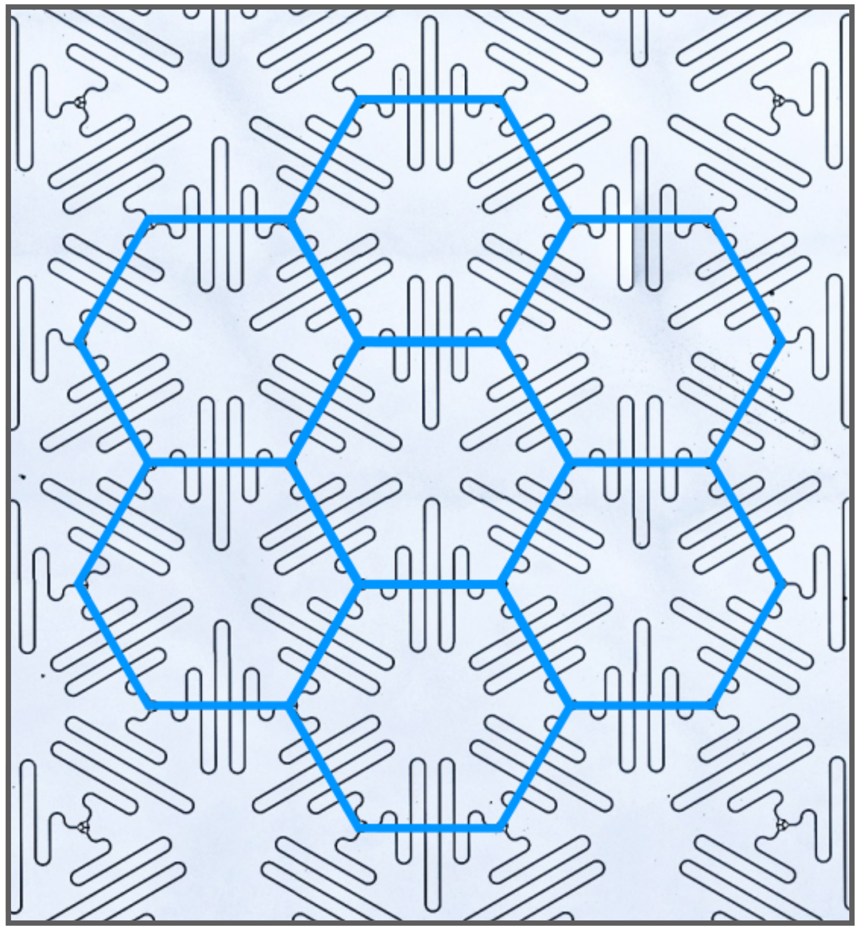
Layout and Effective Lattices

Resonator Lattice



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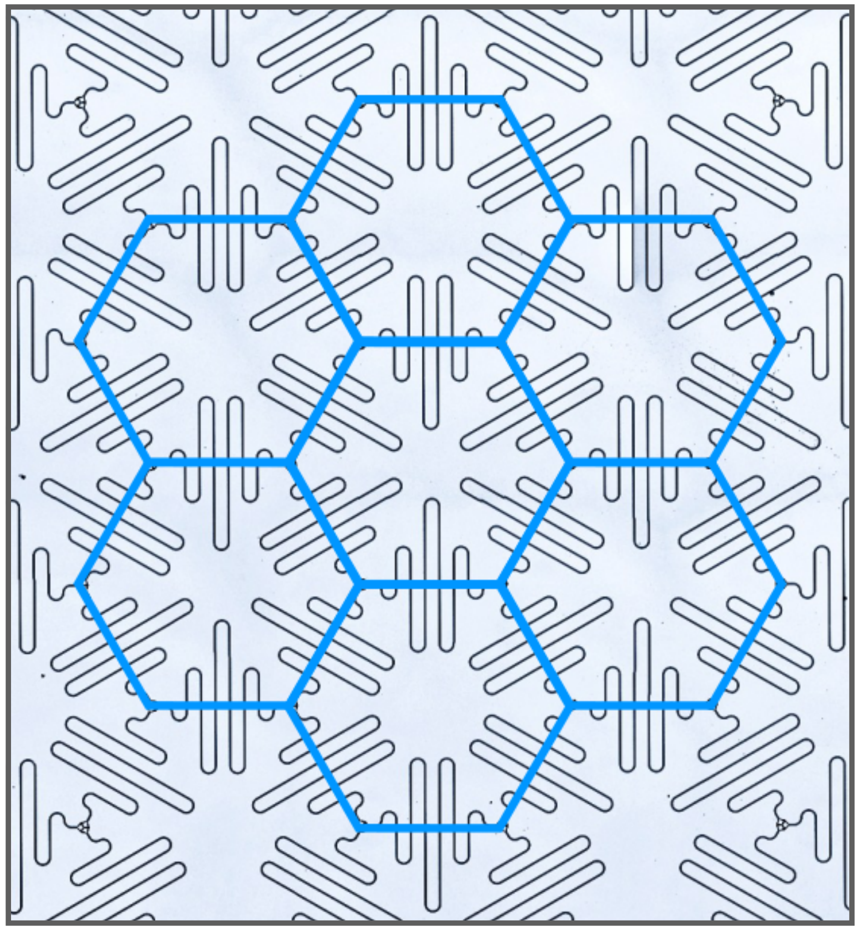

- An *edge* on each resonator

Layout and Effective Lattices

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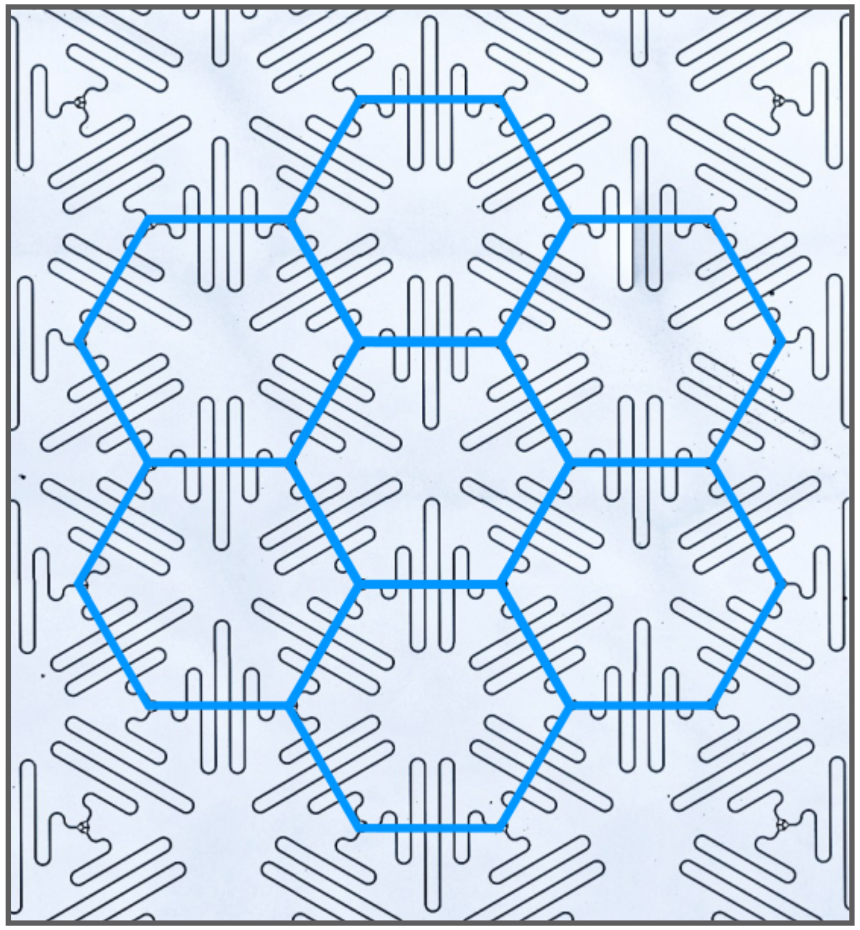
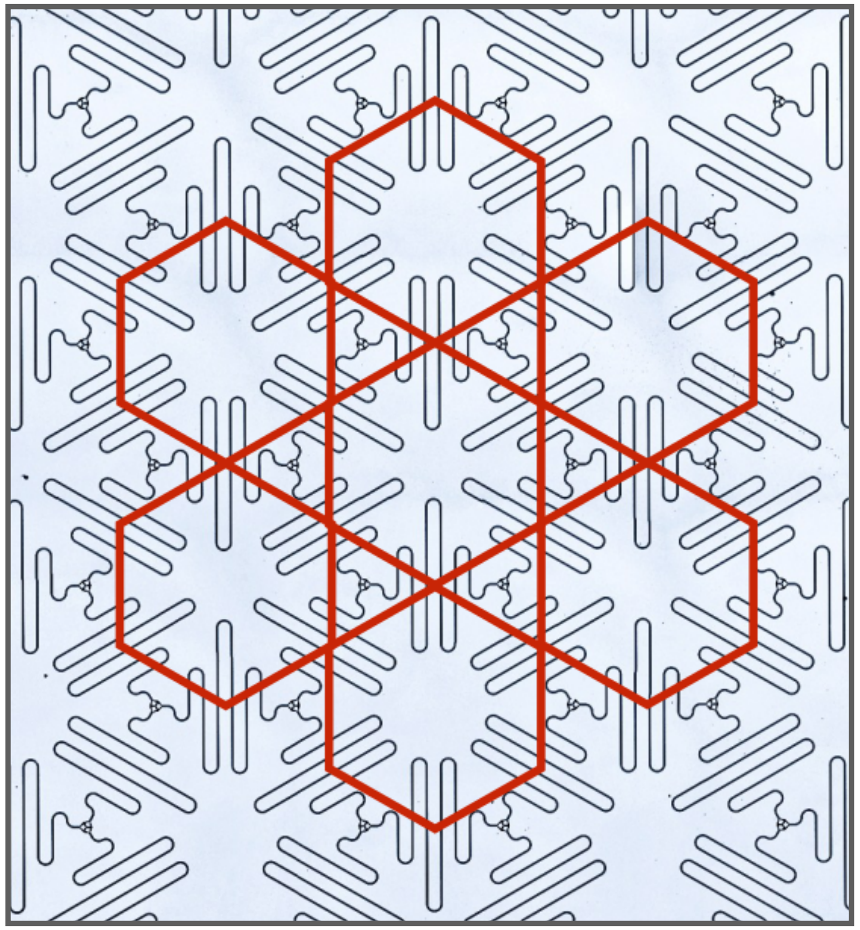


Effective Photonic Lattice



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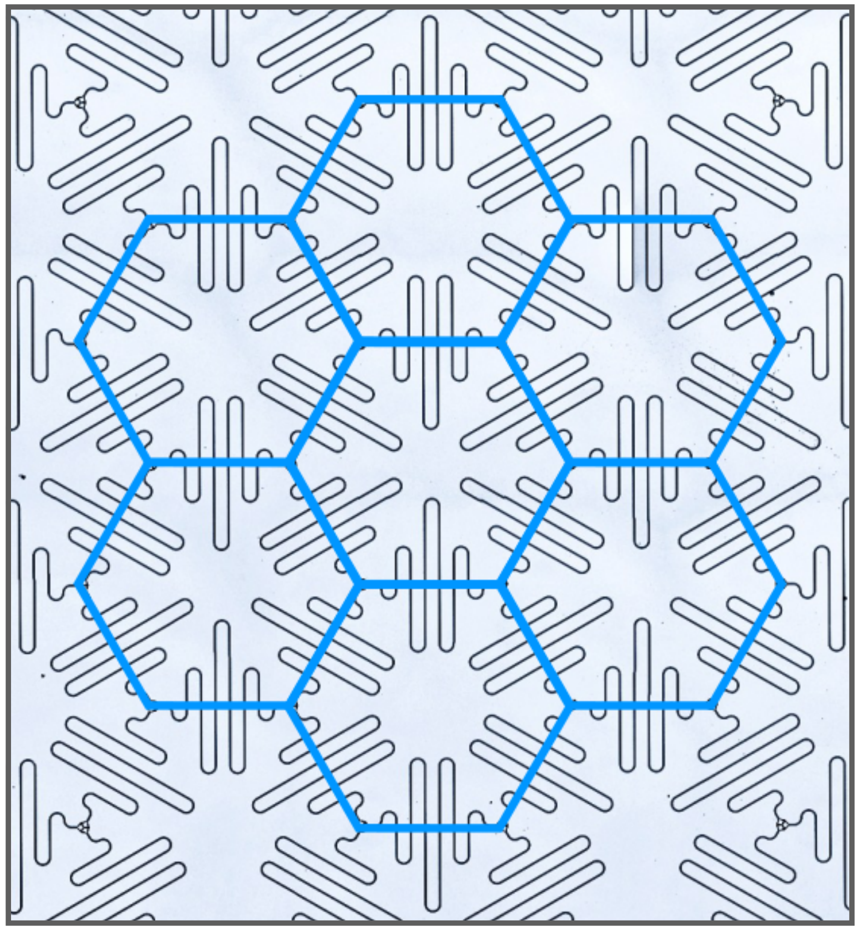

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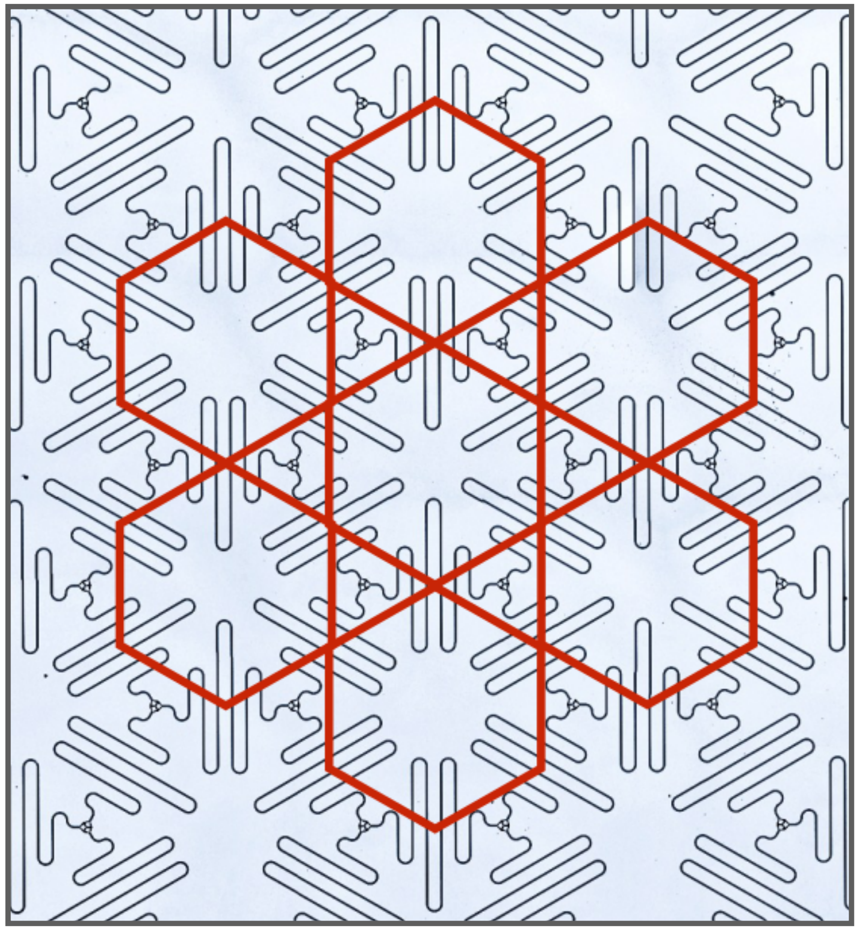
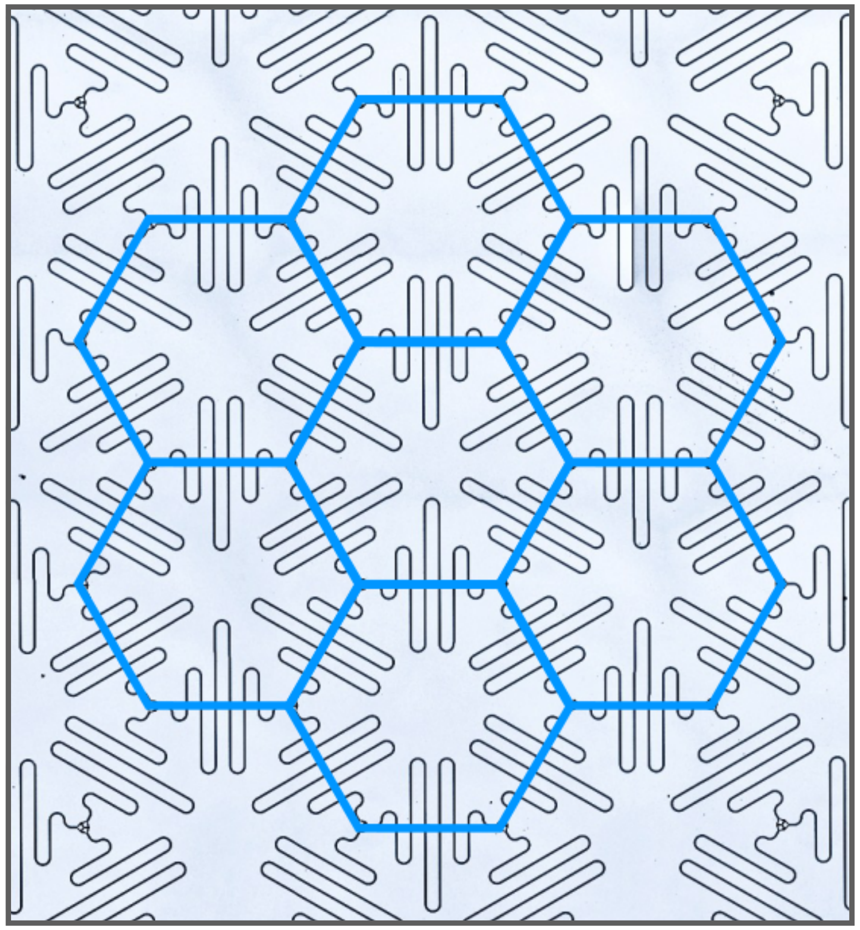


- A *vertex* on each resonator

Layout and Effective Lattices

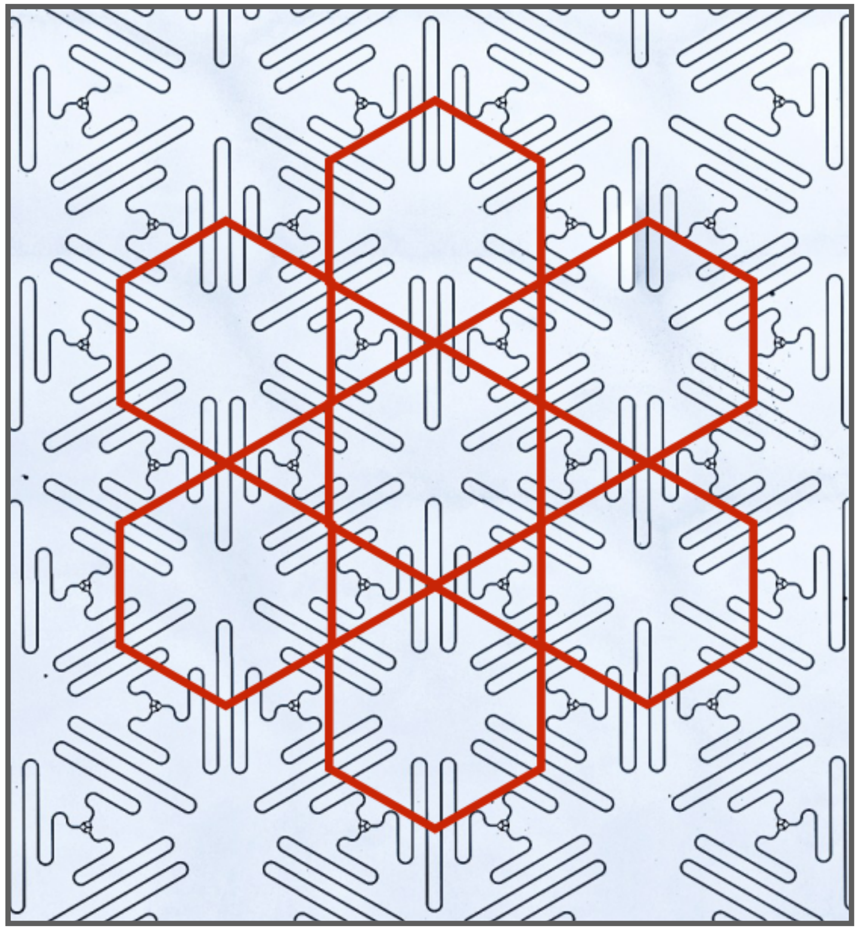
Resonator Lattice



- An *edge* on each resonator

Layout X

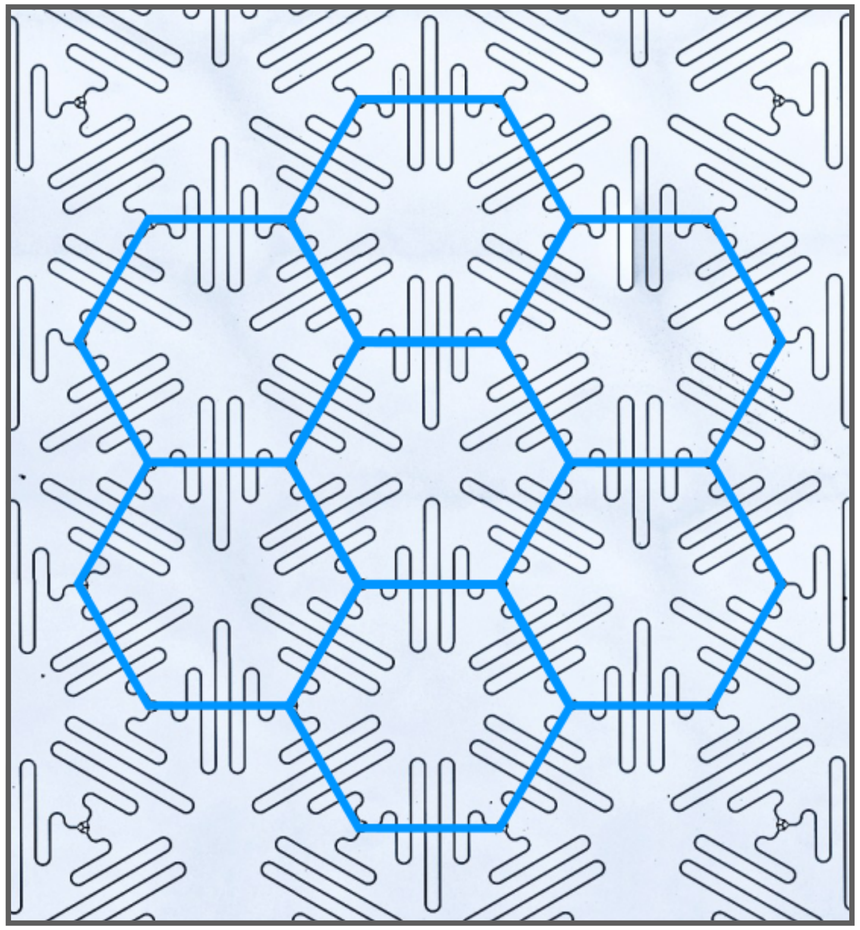
Effective Photonic Lattice



- A *vertex* on each resonator

Layout and Effective Lattices

Resonator Lattice



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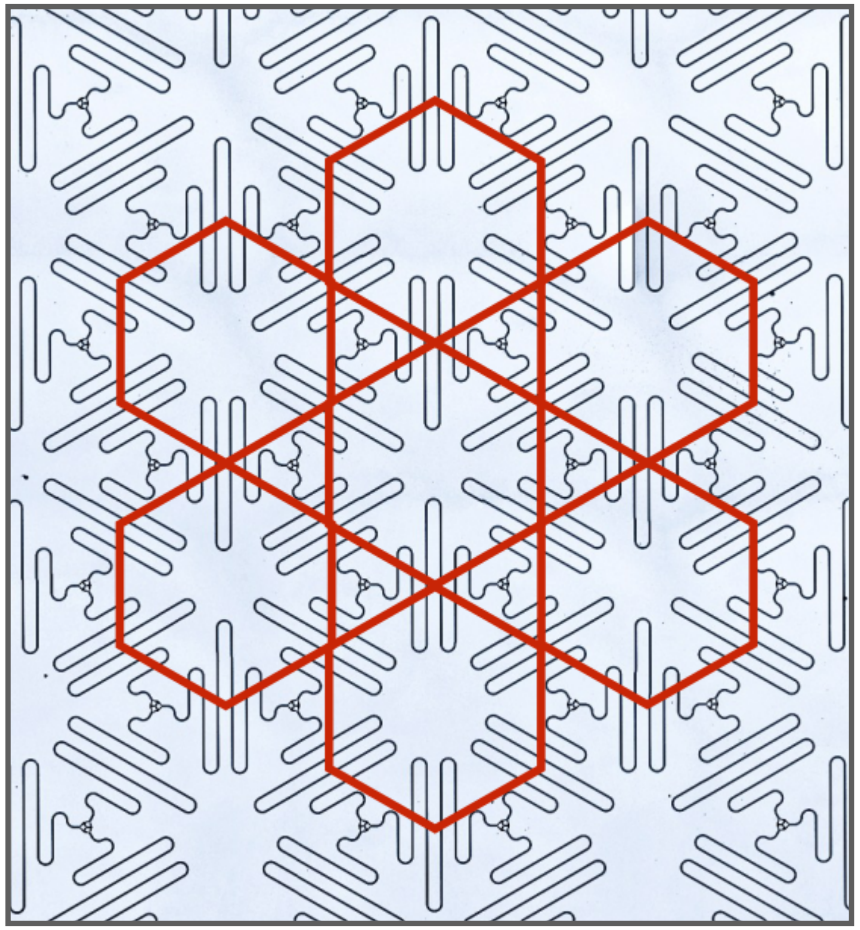
Effective Photonic Lattice



- A *vertex* on each resonator

Line Graph $L(X)$

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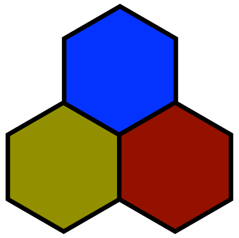
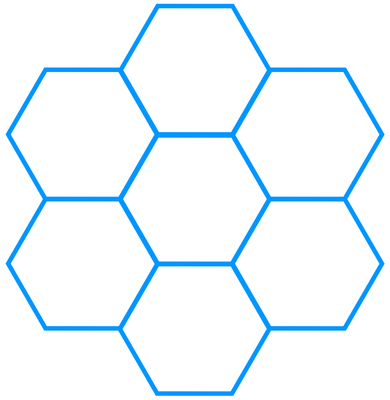
Projecting to Flat 2D

n = 6

flat





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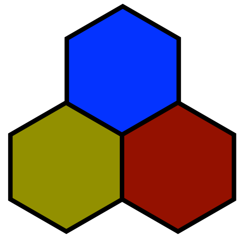
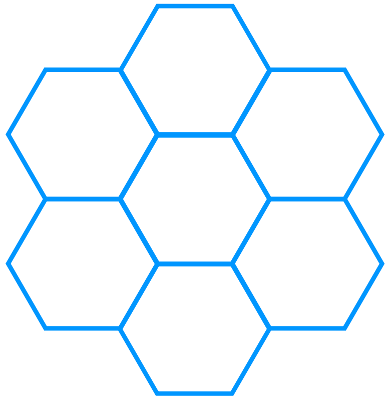
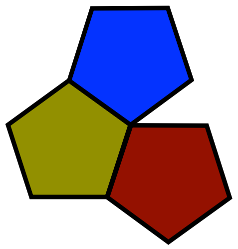
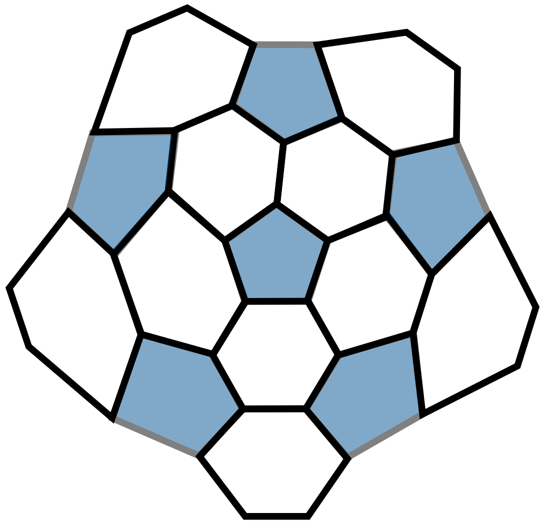




n = 5

spherical





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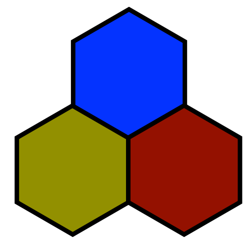

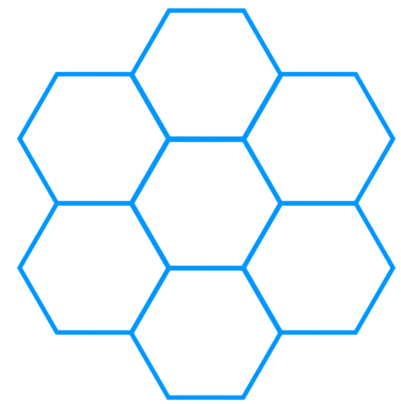

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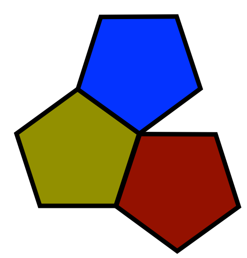

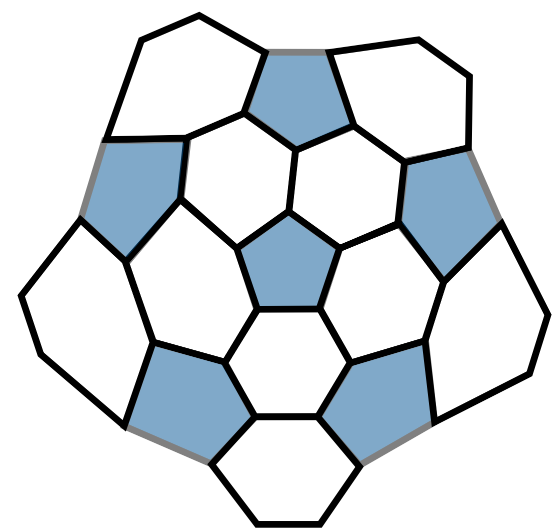

Distance is not preserved.

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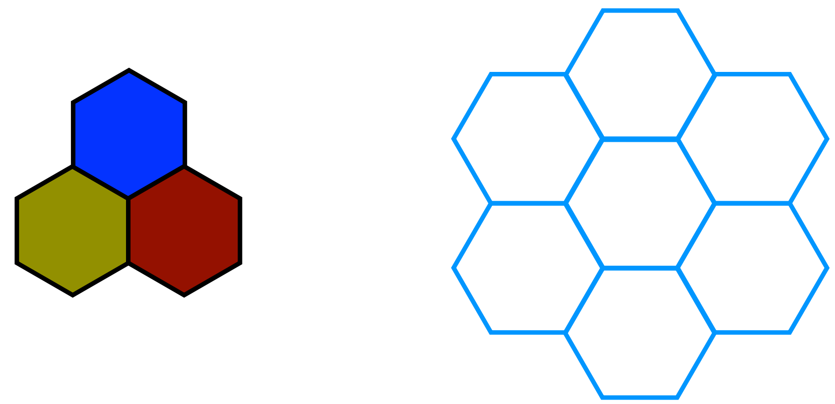

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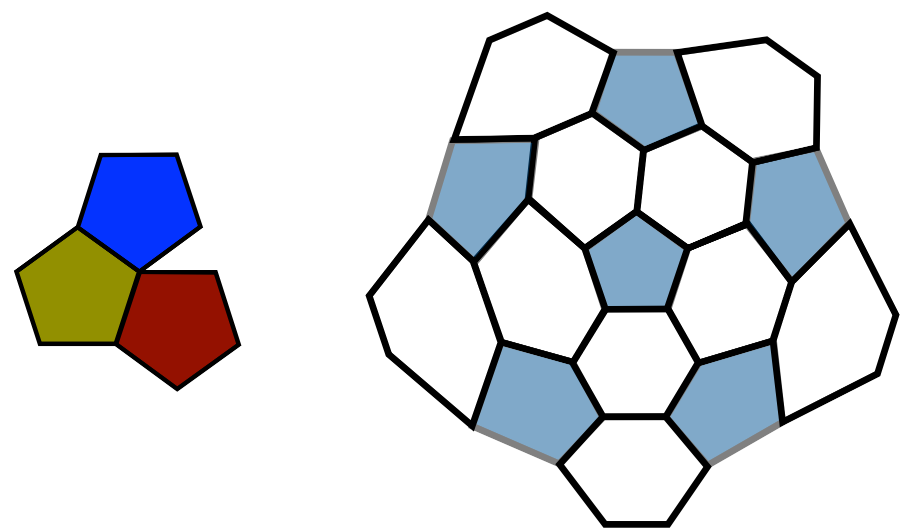
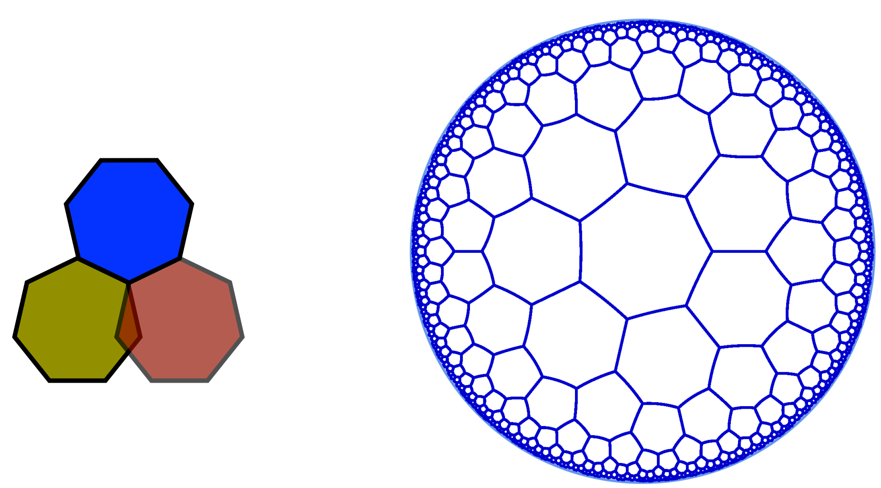


Distance is not preserved.

n = 7

hyperbolic



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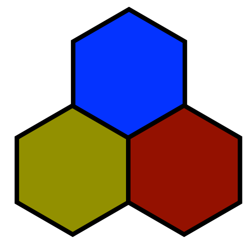

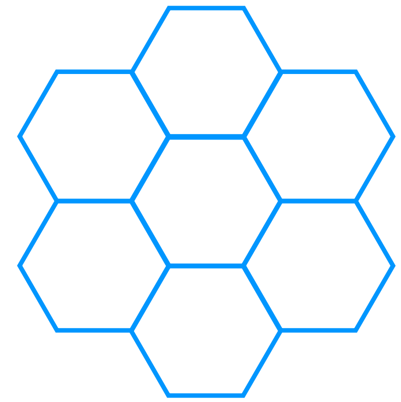

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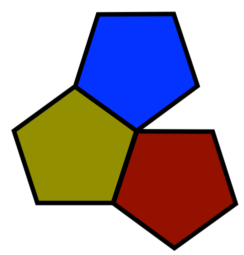

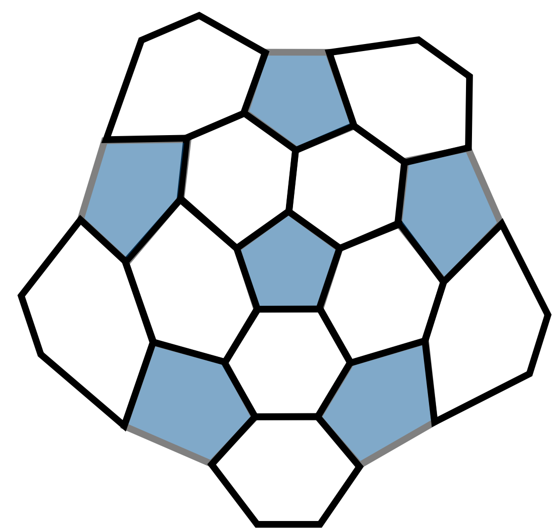

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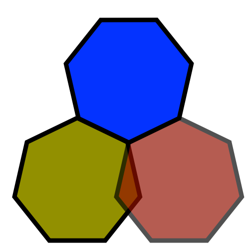

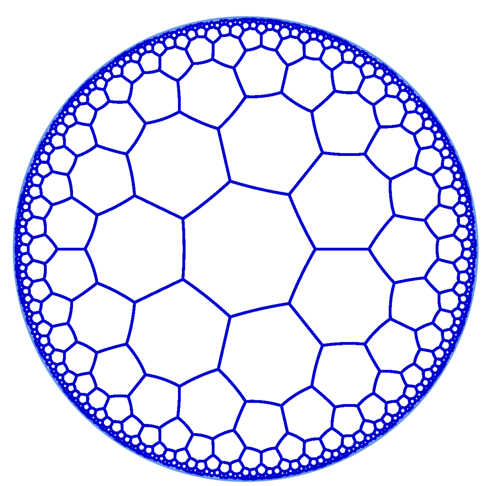

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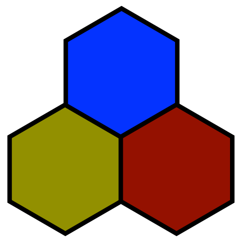
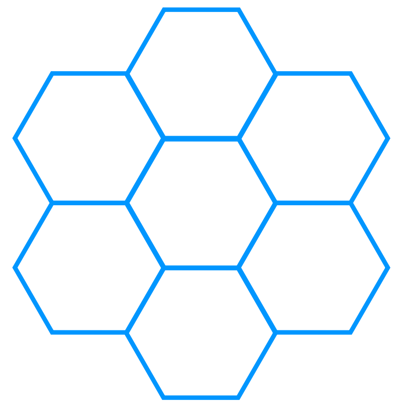
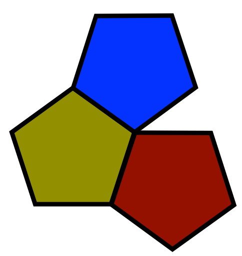
flat

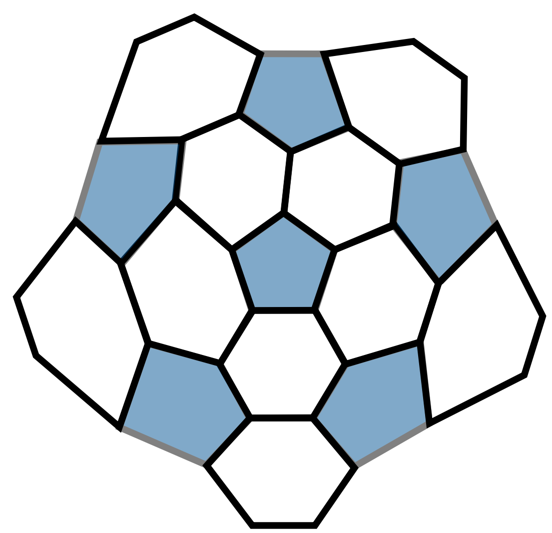




n = 5

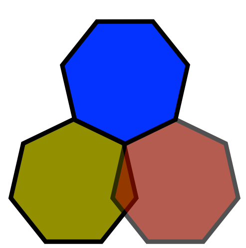
spherical

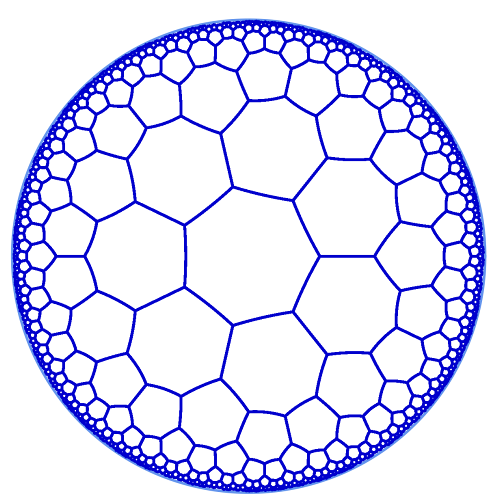




n = 7

hyperbolic



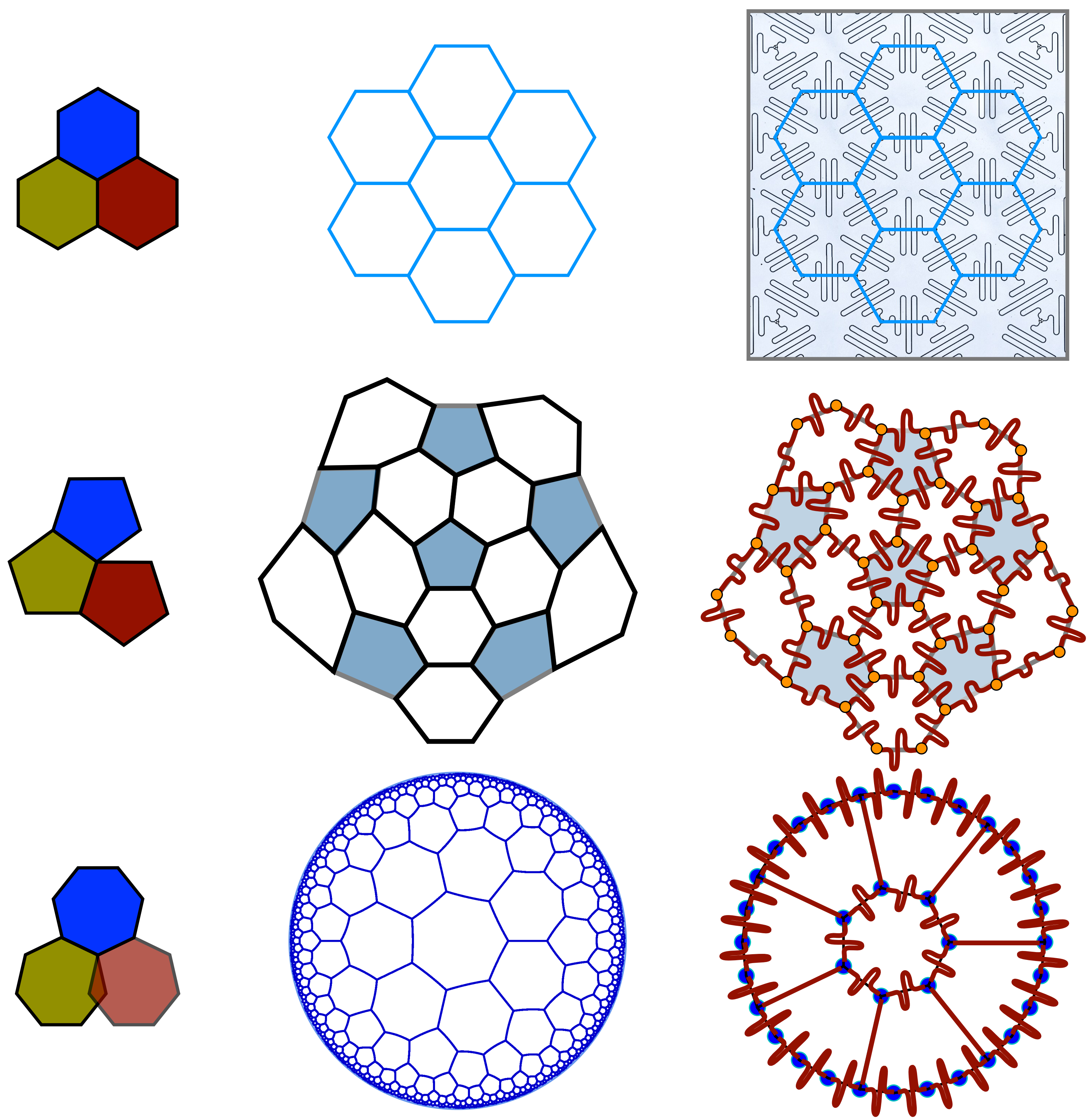


Projecting to Flat 2D

n = 6
flat

n = 5
spherical

n = 7
hyperbolic



Projecting to Flat 2D

n = 6

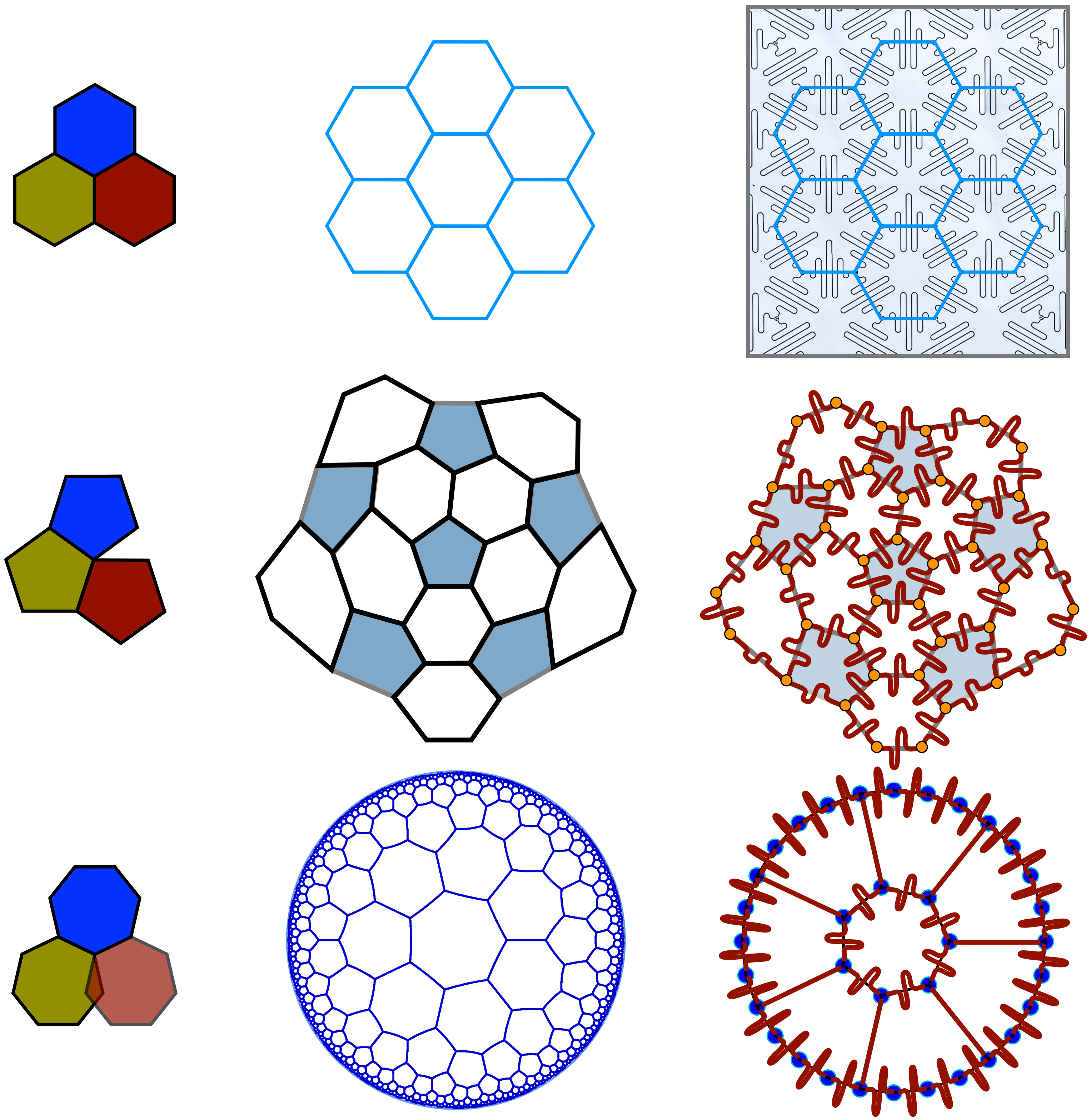
flat

n = 5

spherical

n = 7

hyperbolic



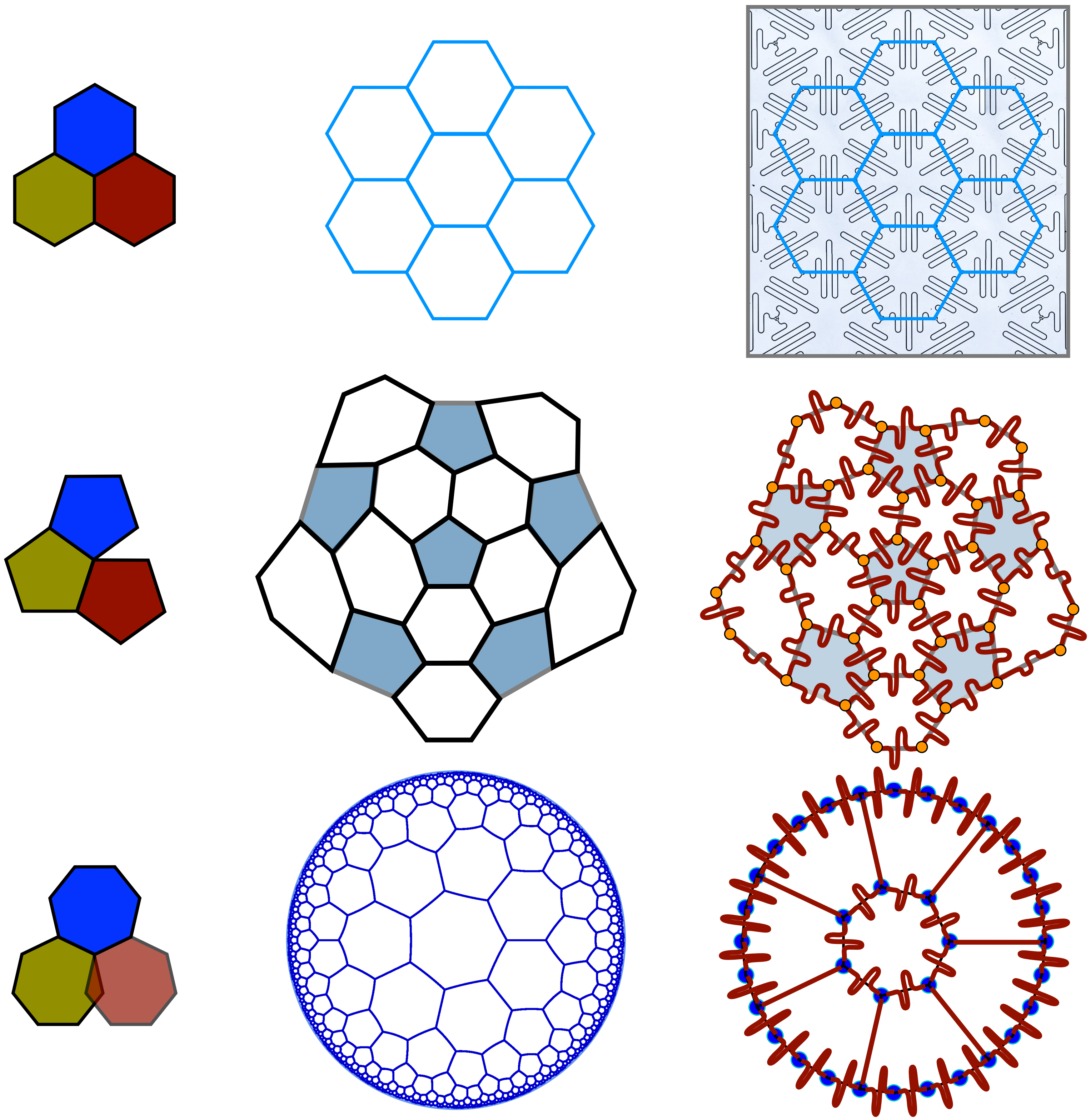
Distance is not preserved.

Projecting to Flat 2D

n = 6
flat

n = 5
spherical

n = 7
hyperbolic



Distance is not preserved.

t *is* preserved.

Projecting to Flat 2D

n = 6

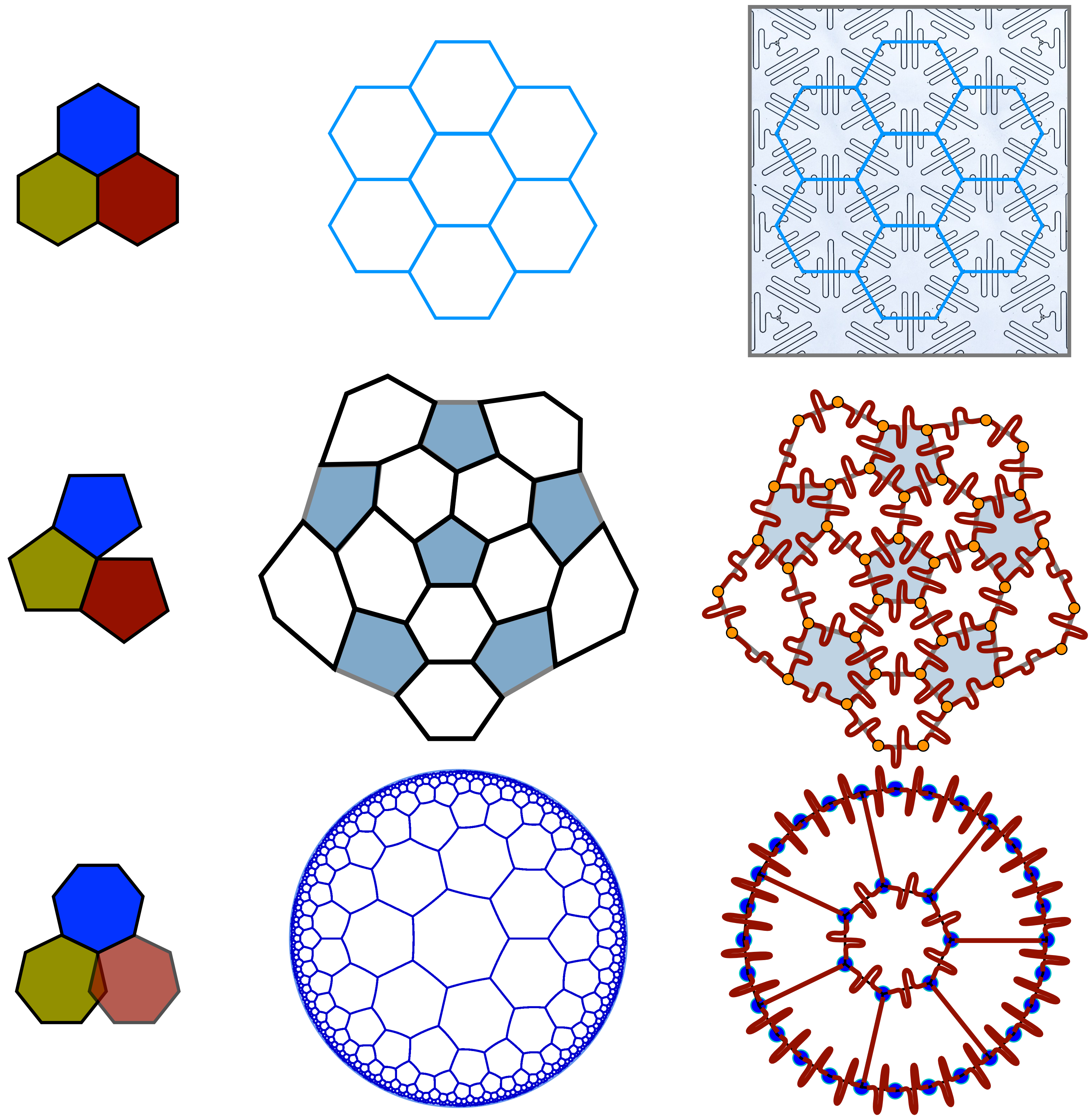
flat

n = 5

spherical

n = 7

hyperbolic

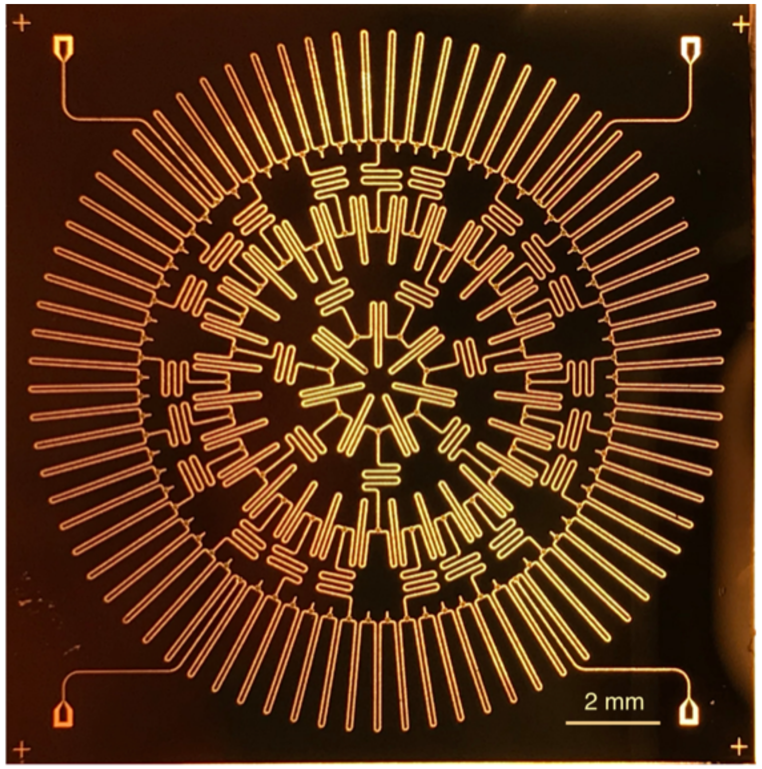


Distance is not preserved.

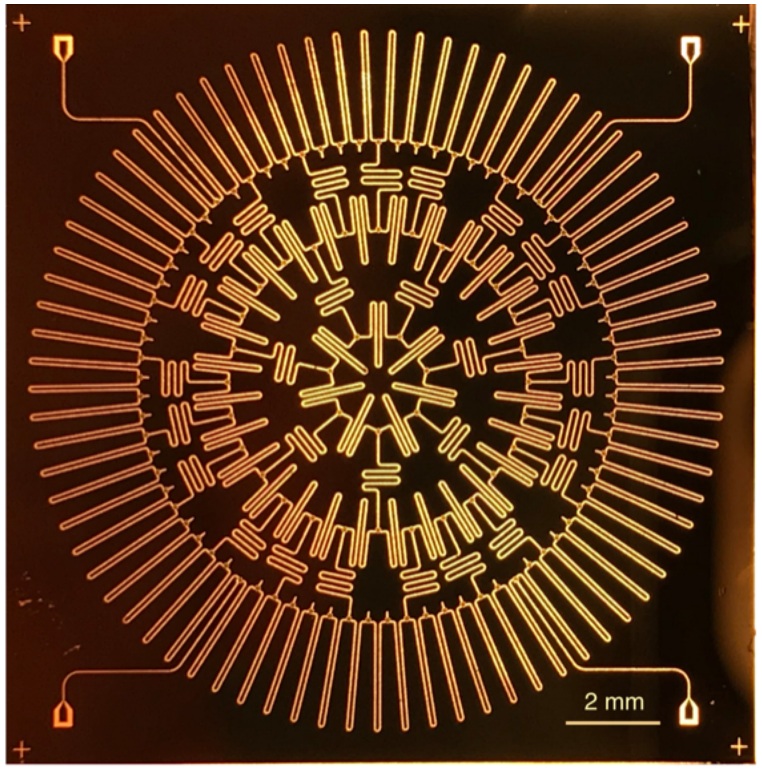
t *is* preserved.

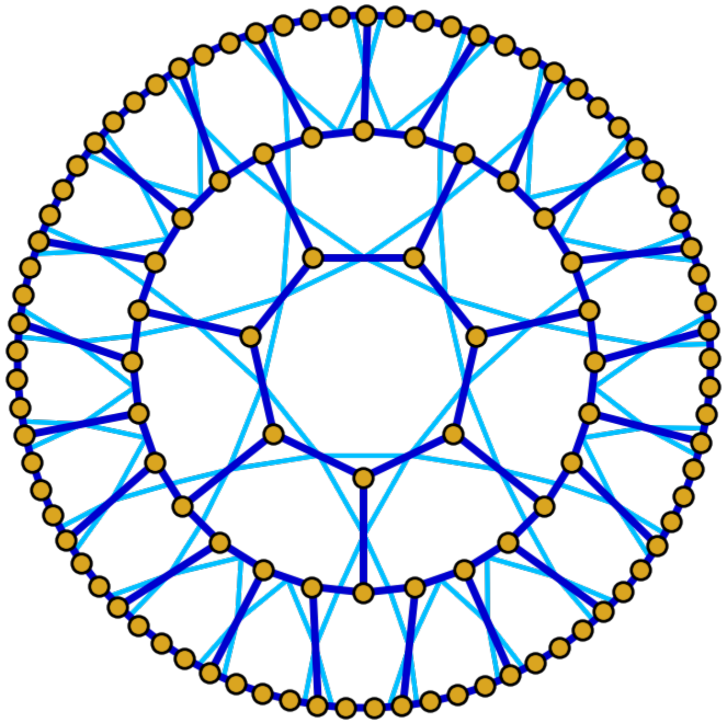
Graph *is* preserved.

Heptagon-Kagome Device



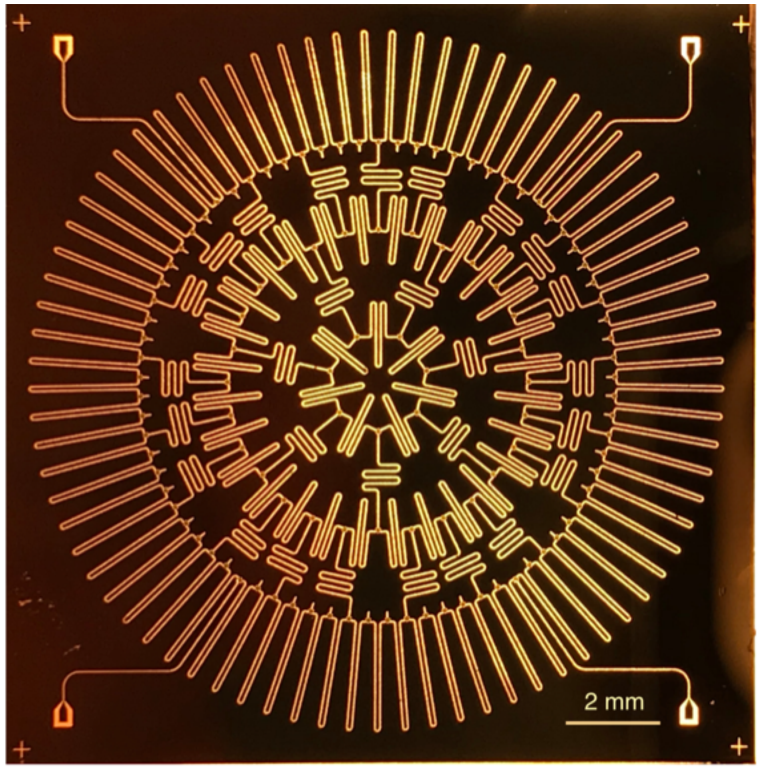
Heptagon-Kagome Device

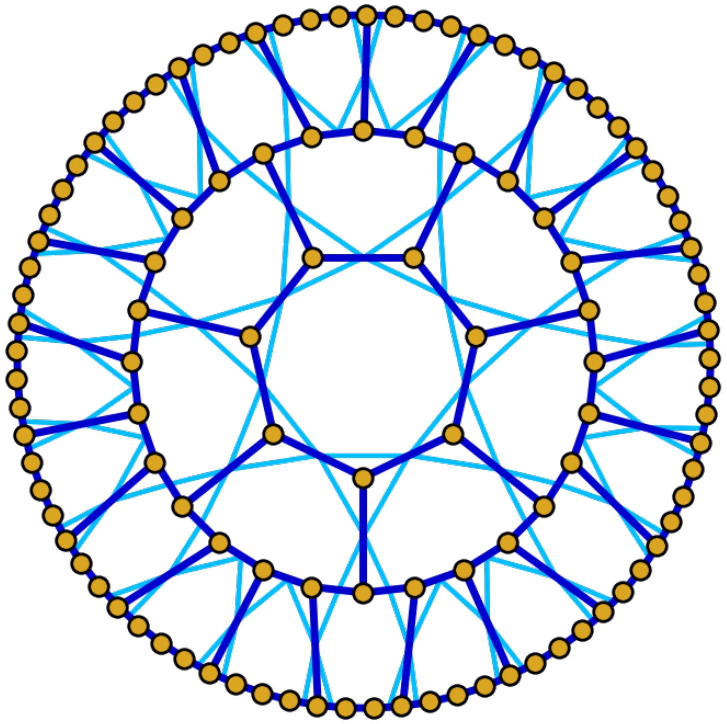




- 2 shells

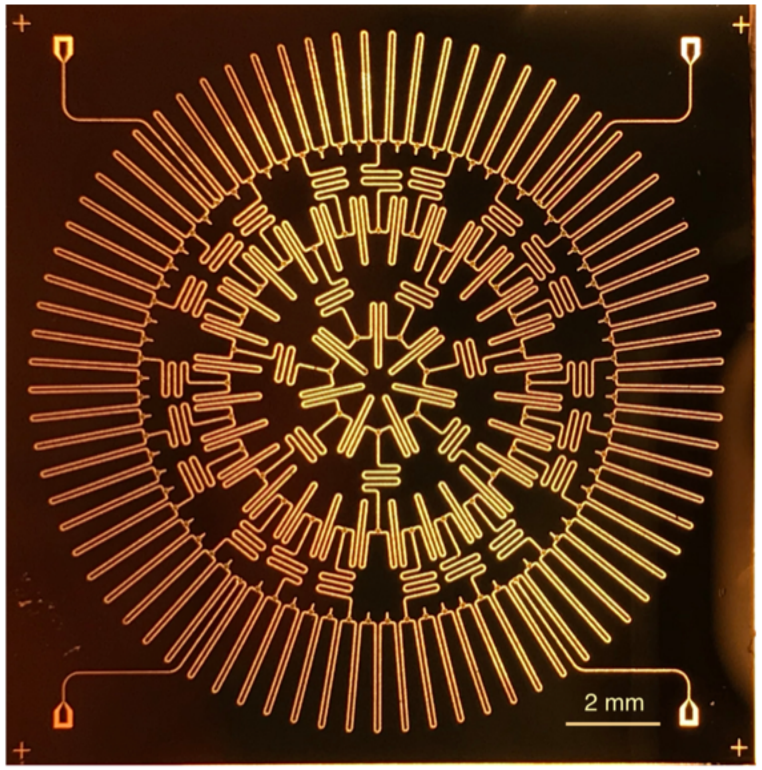
Heptagon-Kagome Device

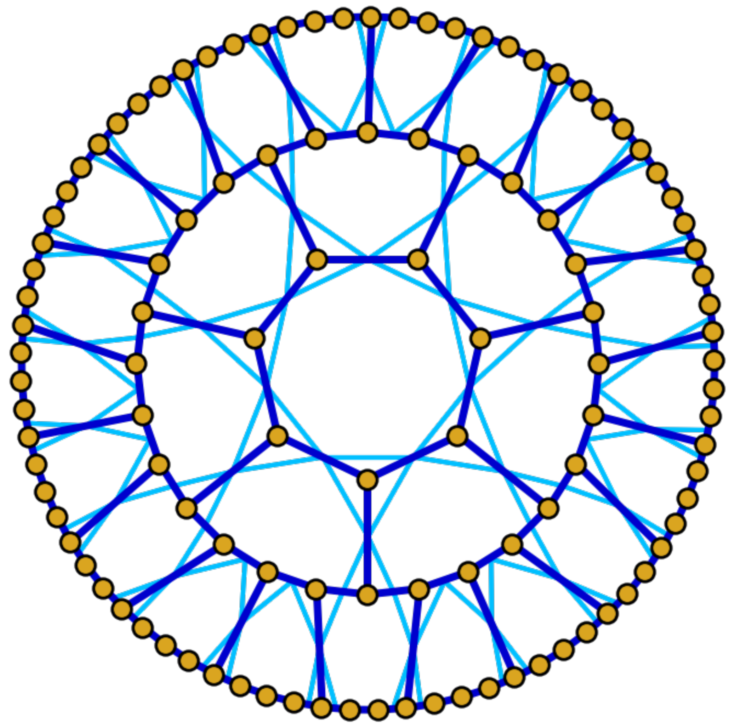




- 2 shells
- Operating frequency: 16 GHz

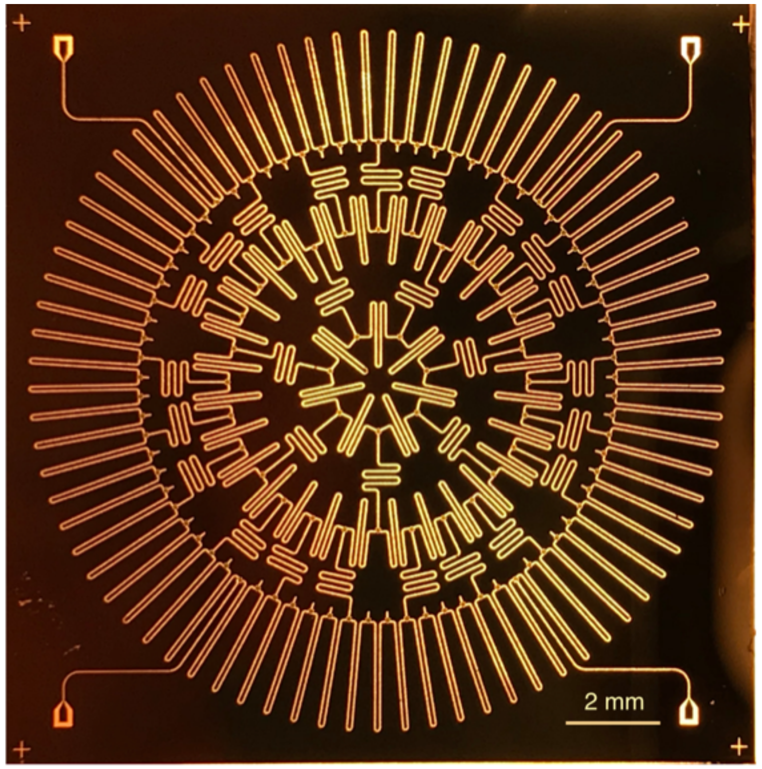
Heptagon-Kagome Device

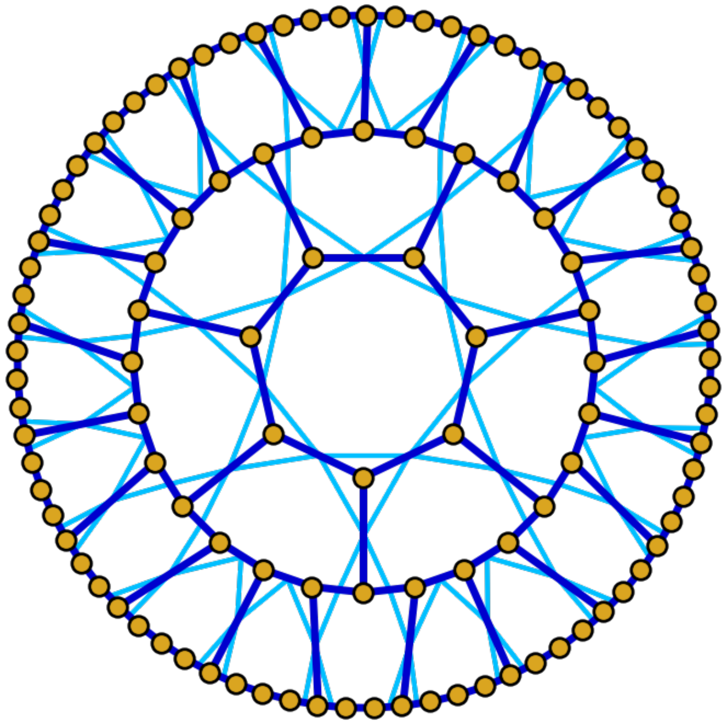




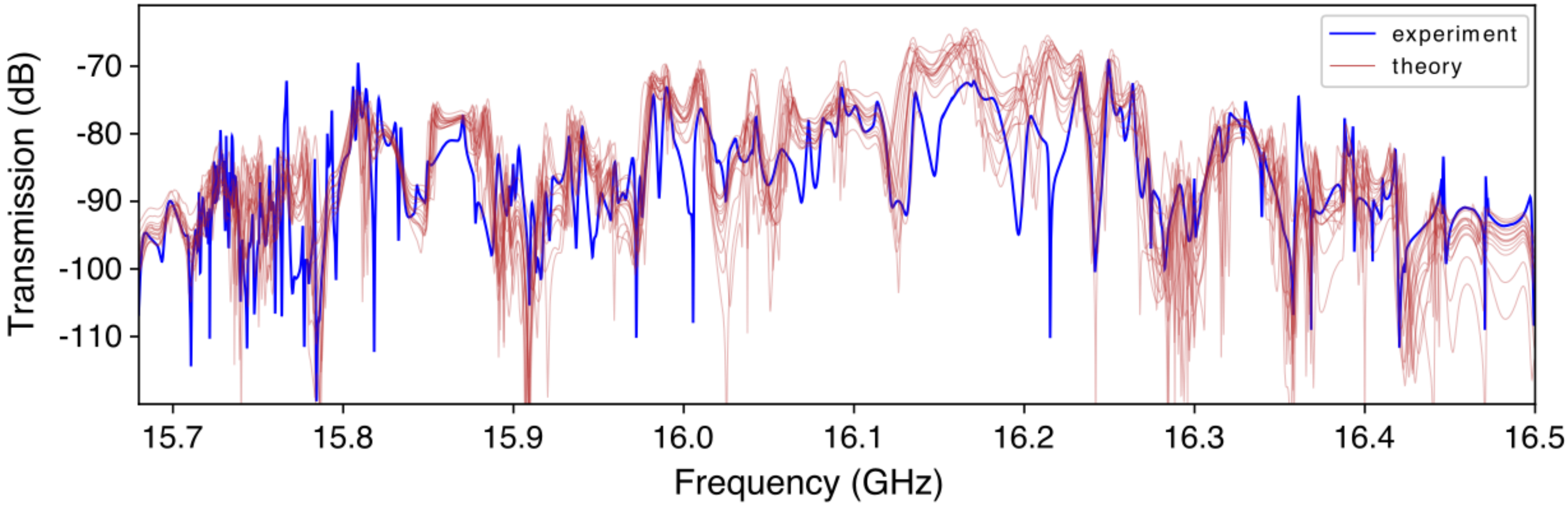
- 2 shells
- Operating frequency: 16 GHz
- 4 input-output ports

Heptagon-Kagome Device





- 2 shells
- Operating frequency: 16 GHz
- 4 input-output ports



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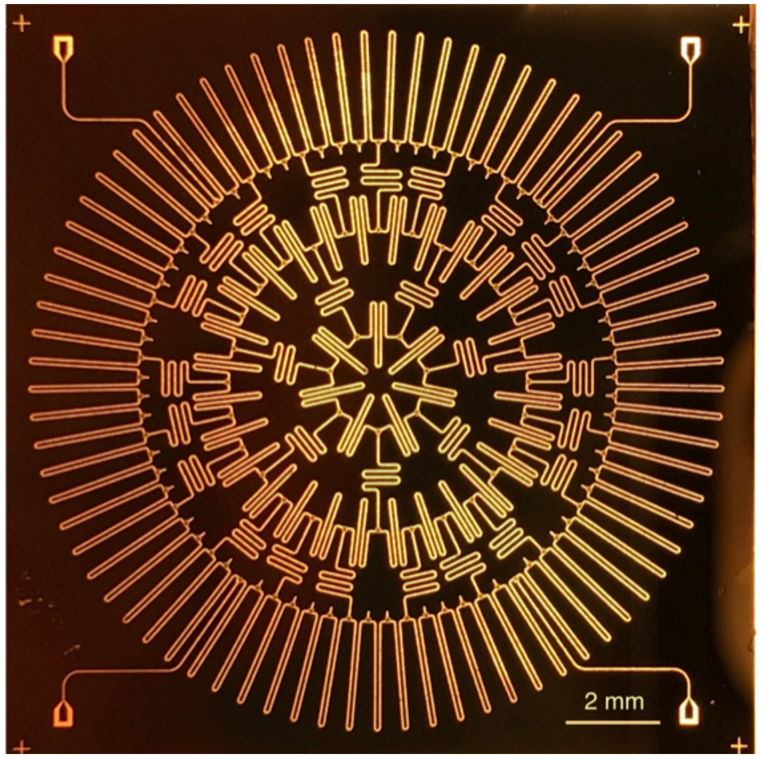
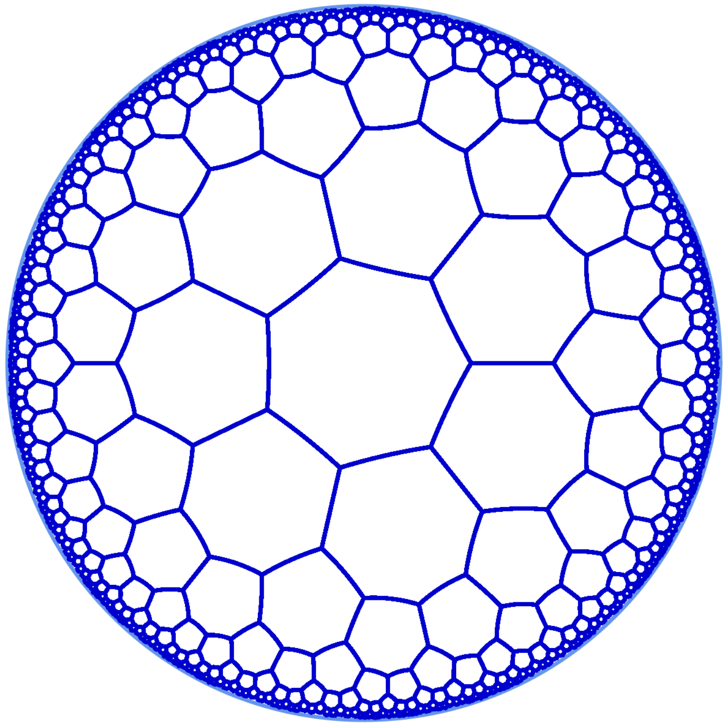
Spectrum Calculations

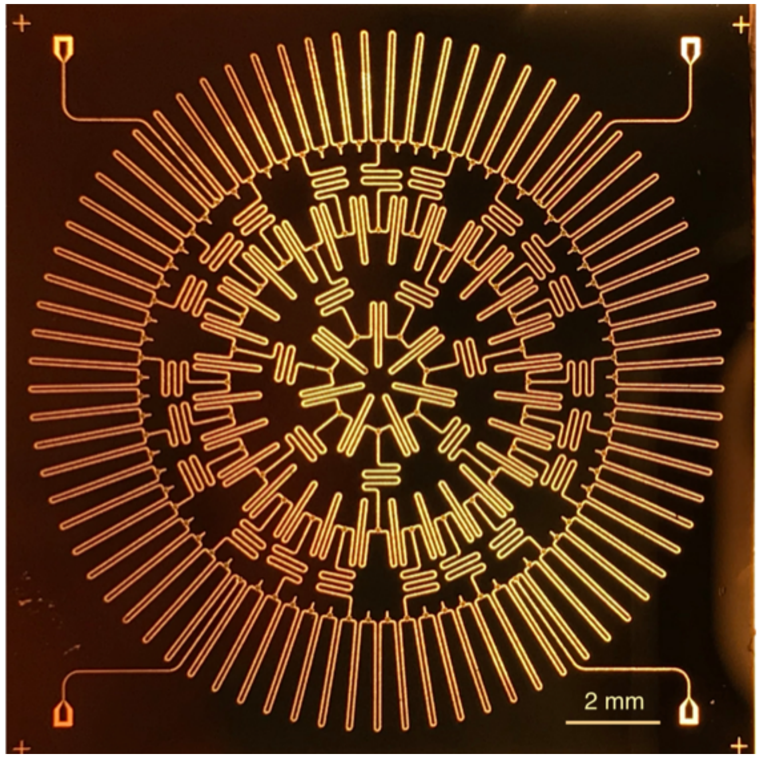
What is the spectrum of this?

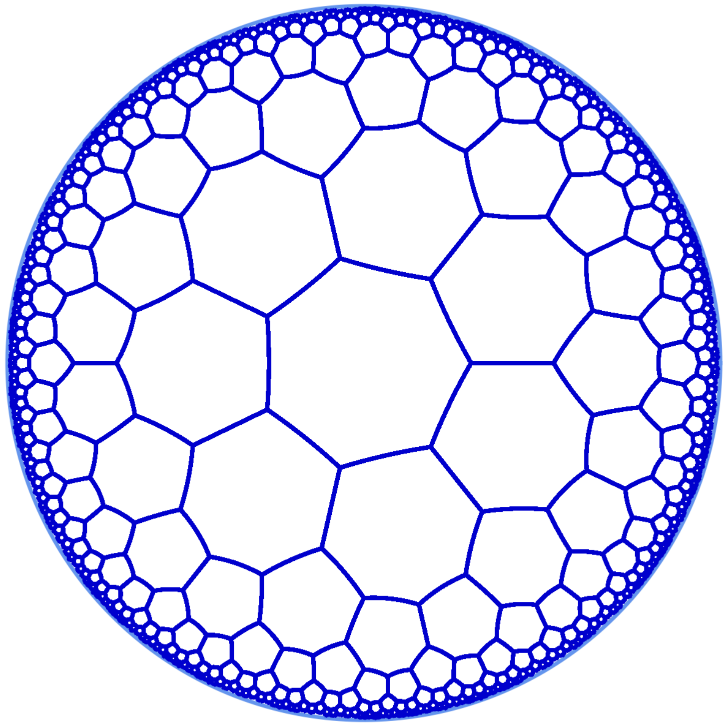


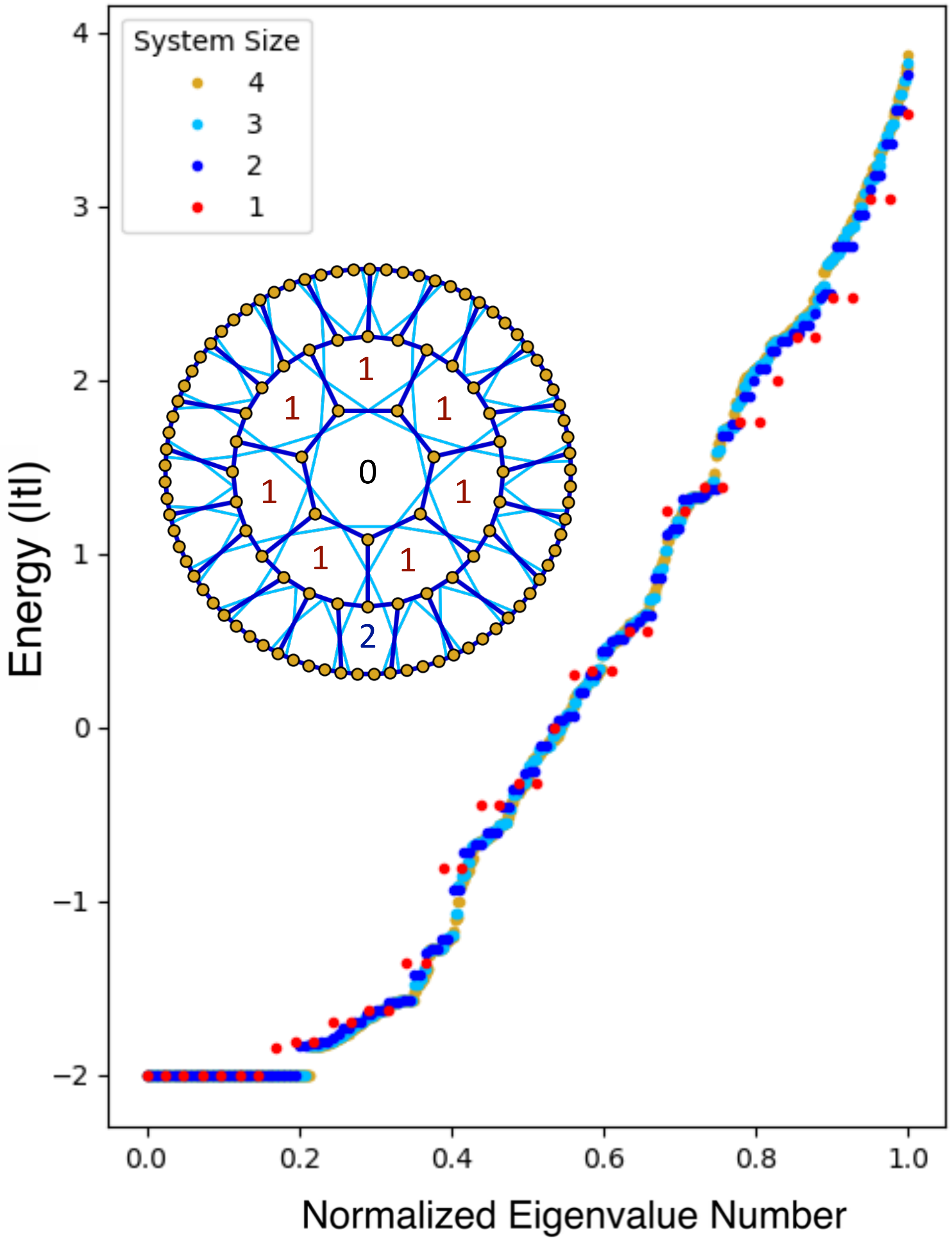


Spectrum Calculations

What is the spectrum of this?

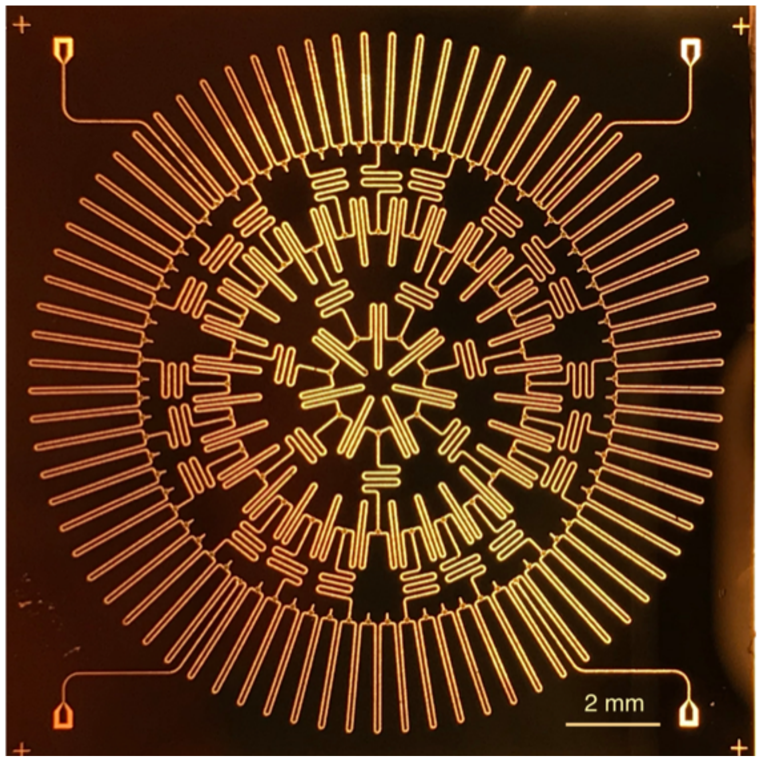


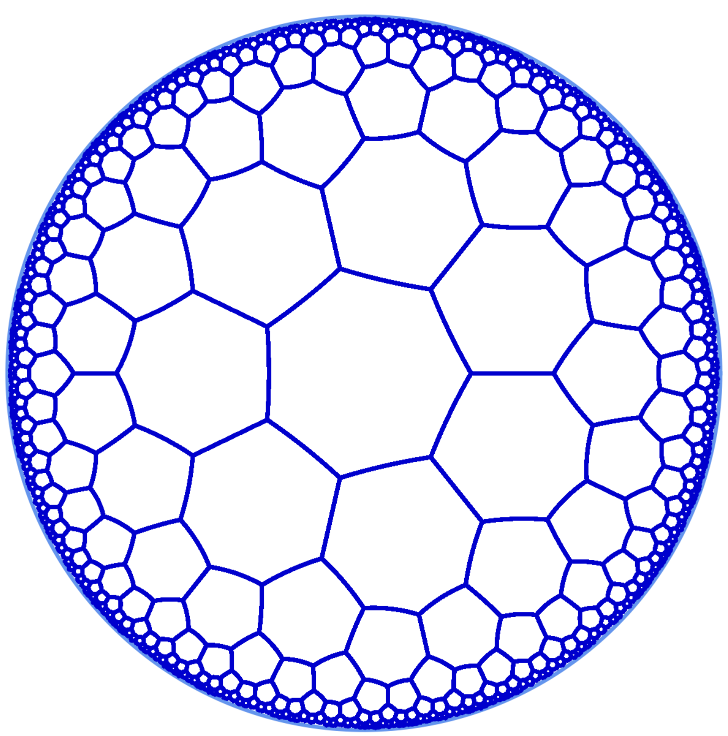


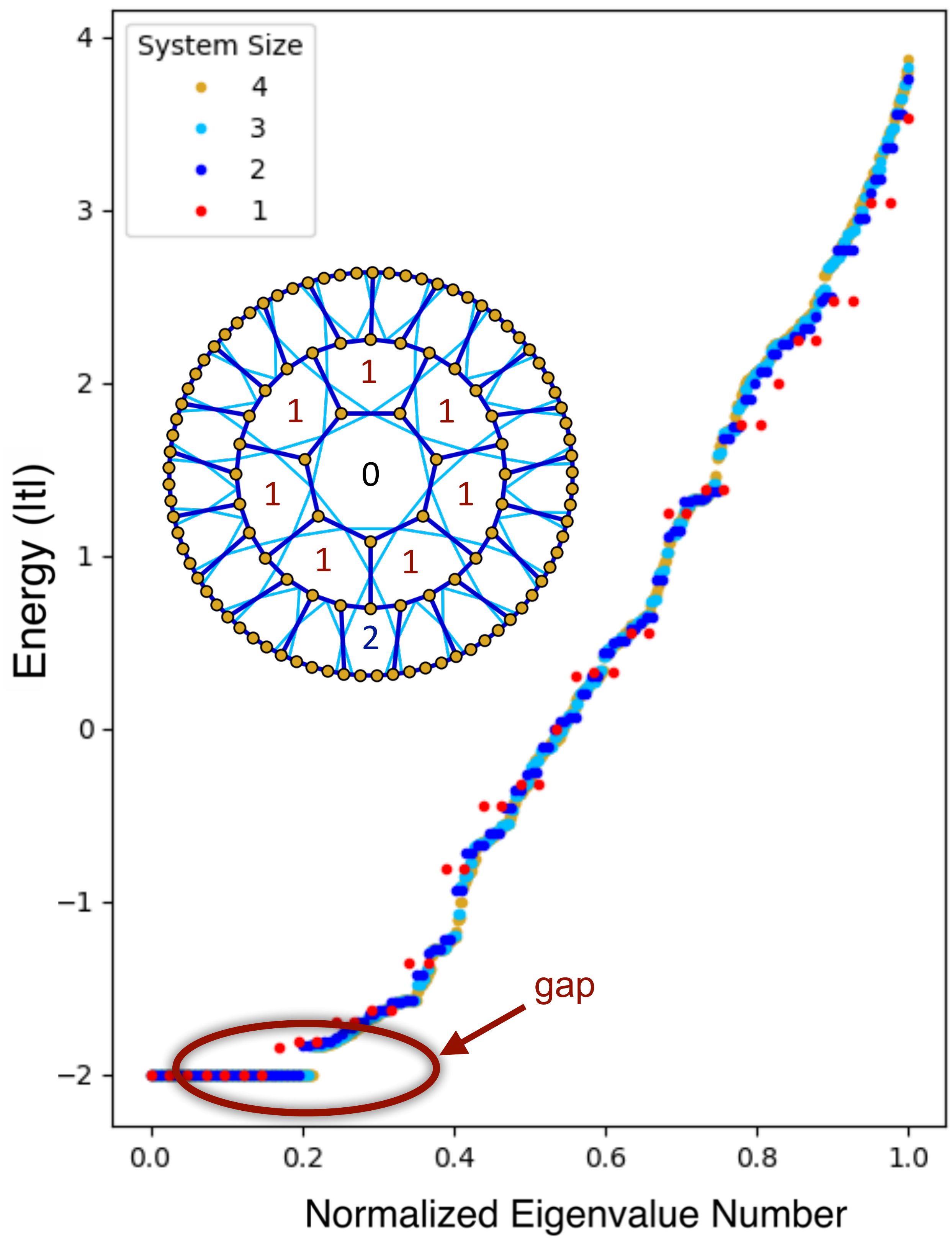


Spectrum Calculations

What is the spectrum of this?

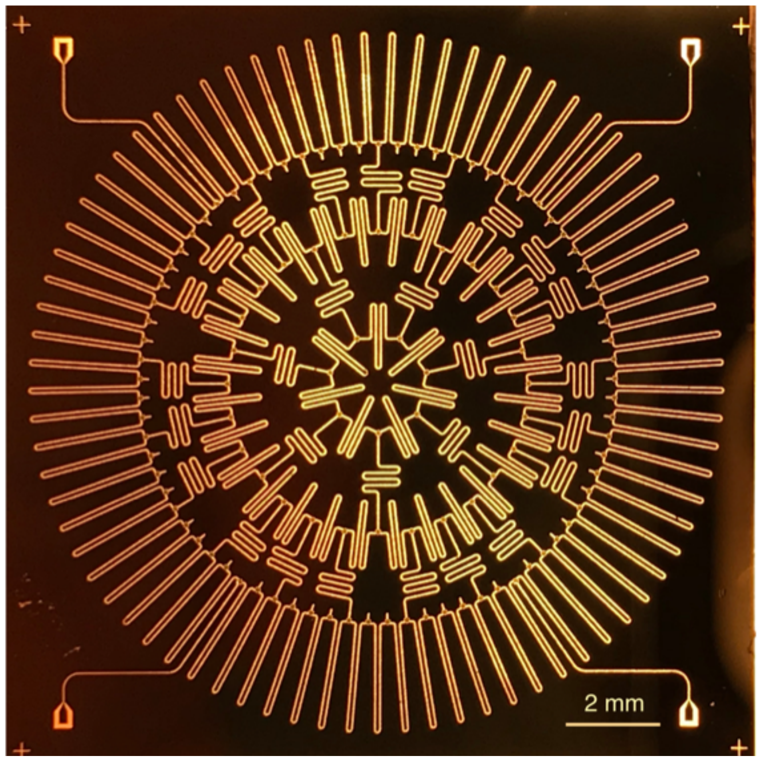


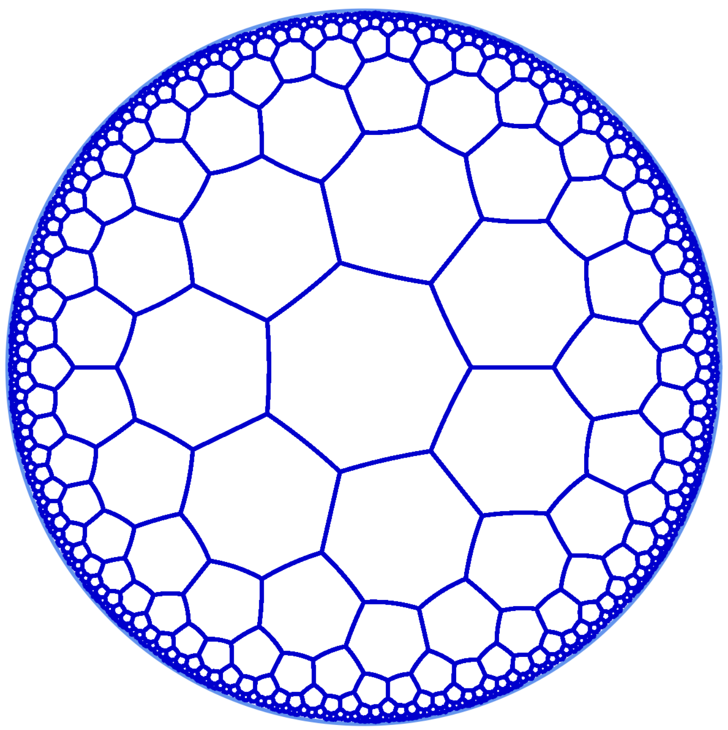


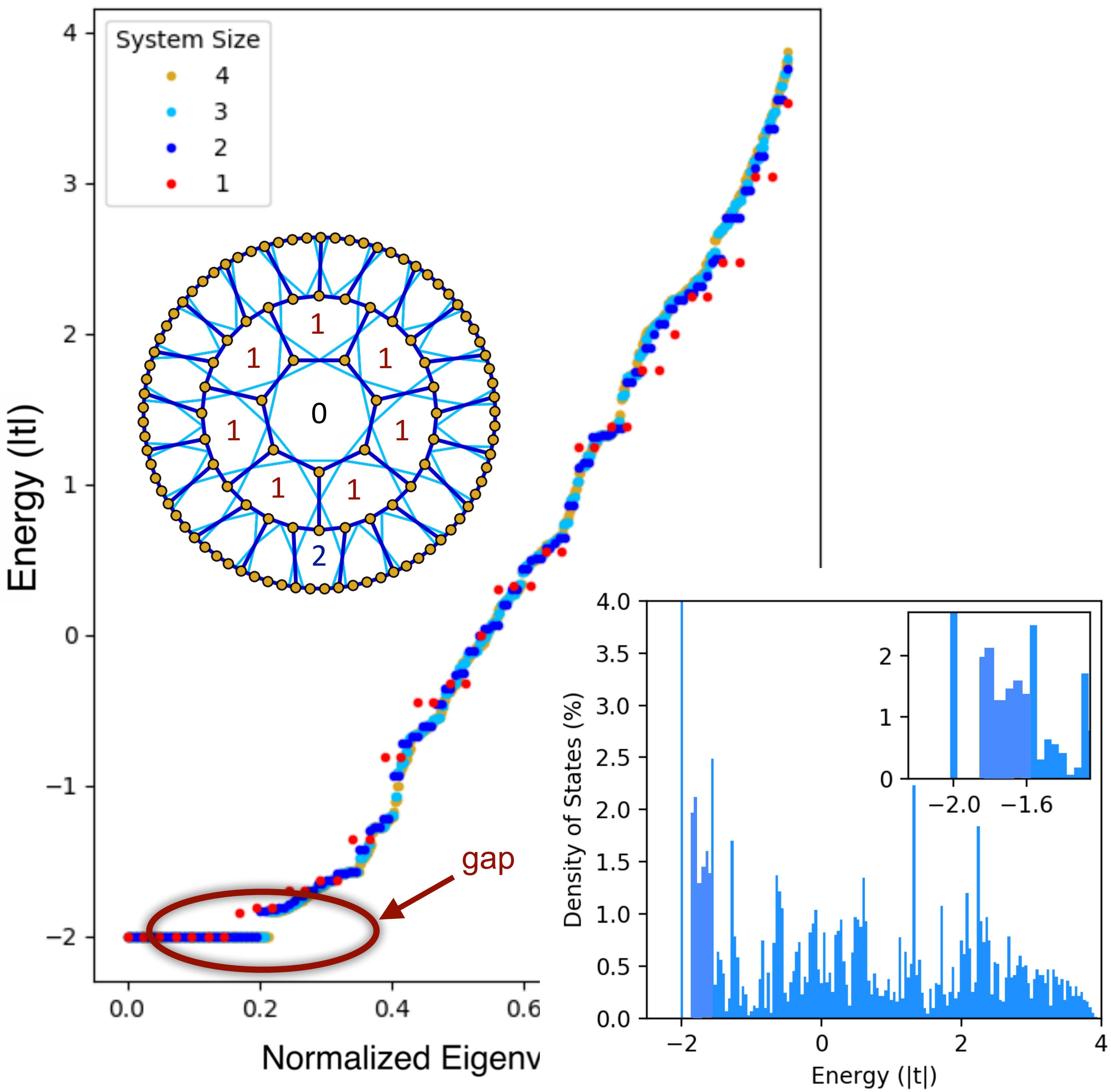


Spectrum Calculations

What is the spectrum of this?



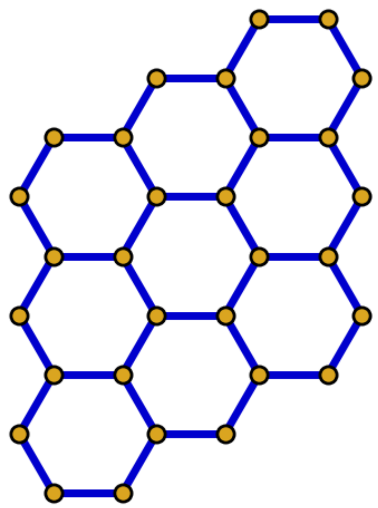




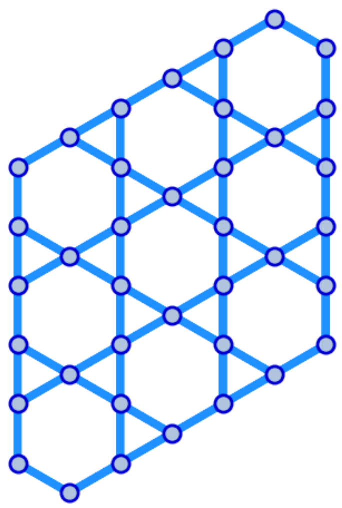
Line-Graph Lattices

Graphene

Layout X



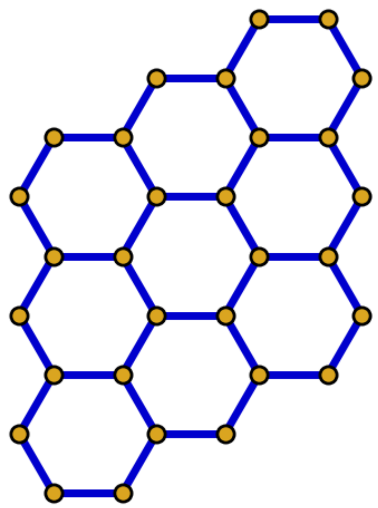
Line Graph $L(X)$



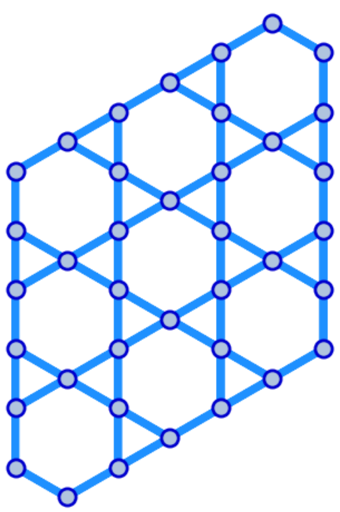
Line-Graph Lattices

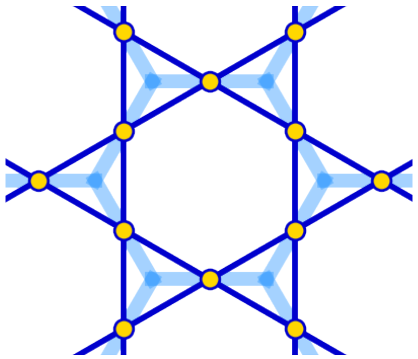
Graphene

Layout X



Line Graph $L(X)$

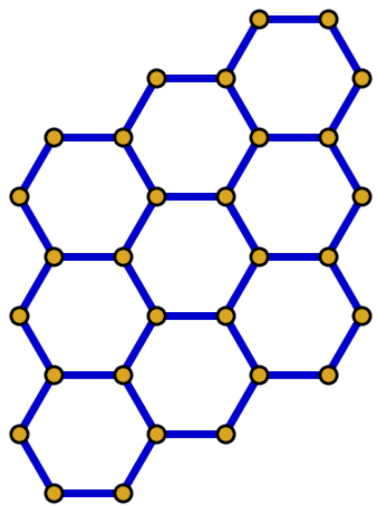


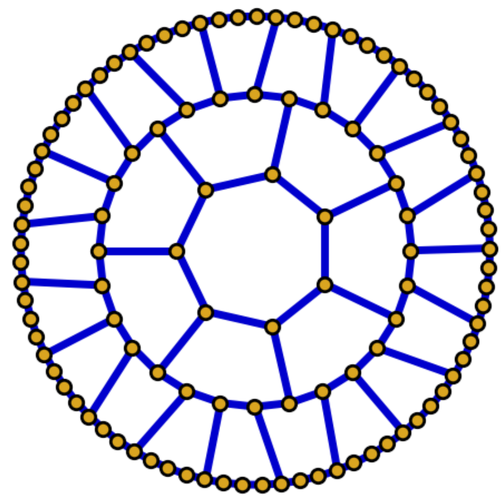


Line-Graph Lattices

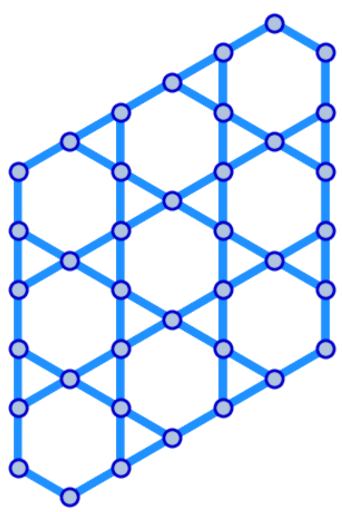
Graphene | **Heptagon-Graphene**

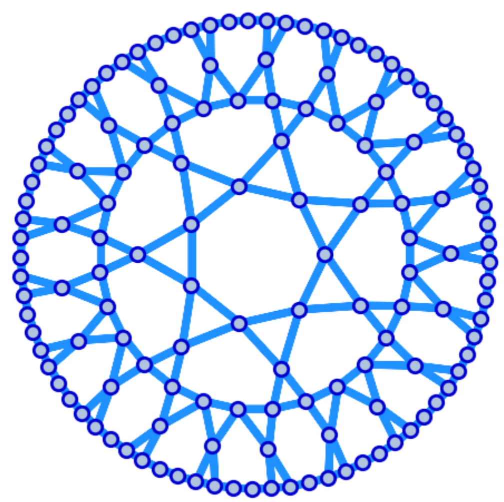
Layout X

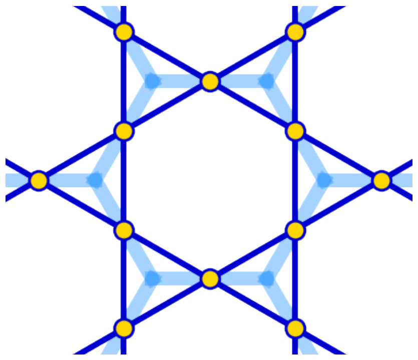




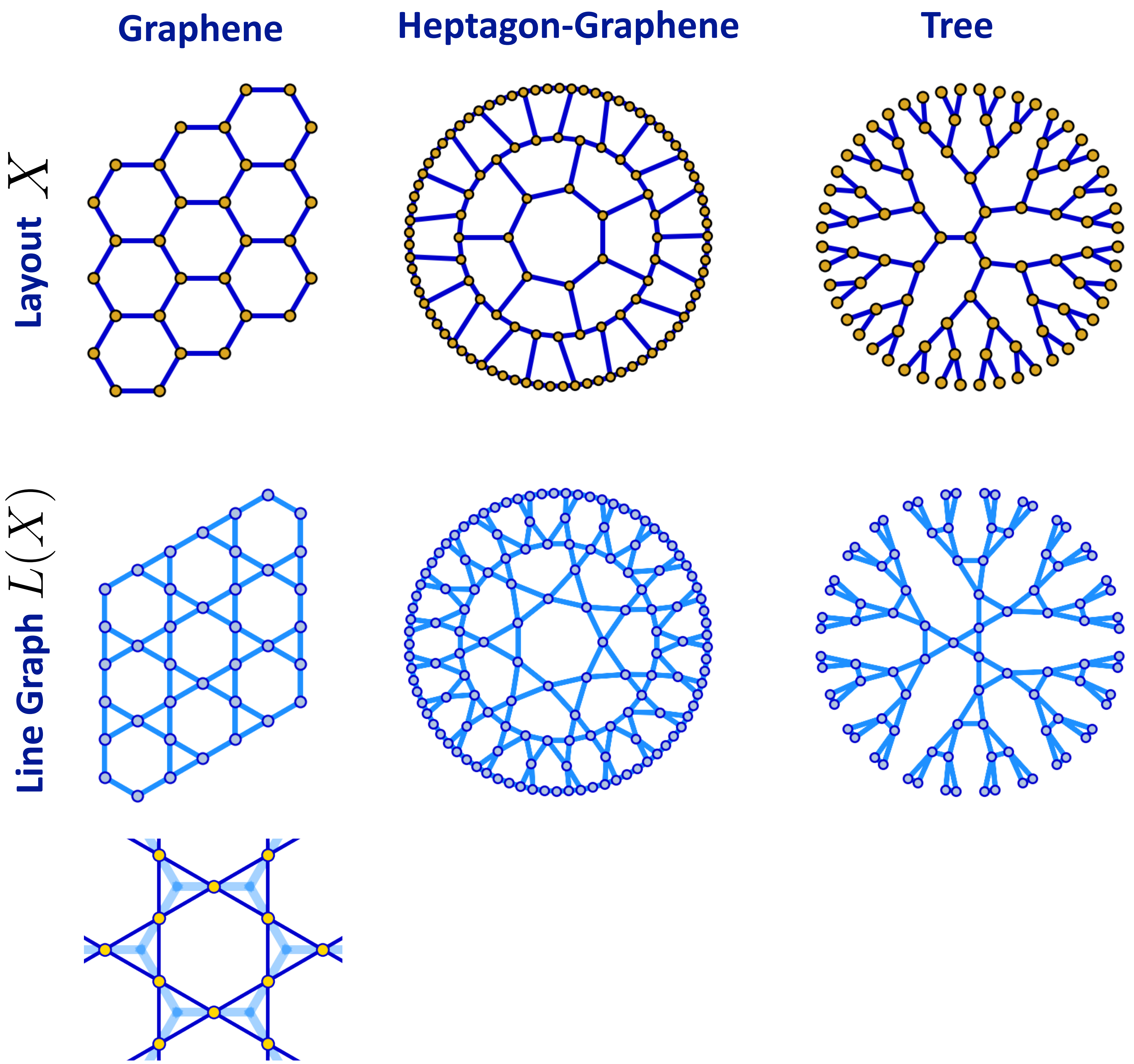
Line Graph $L(X)$



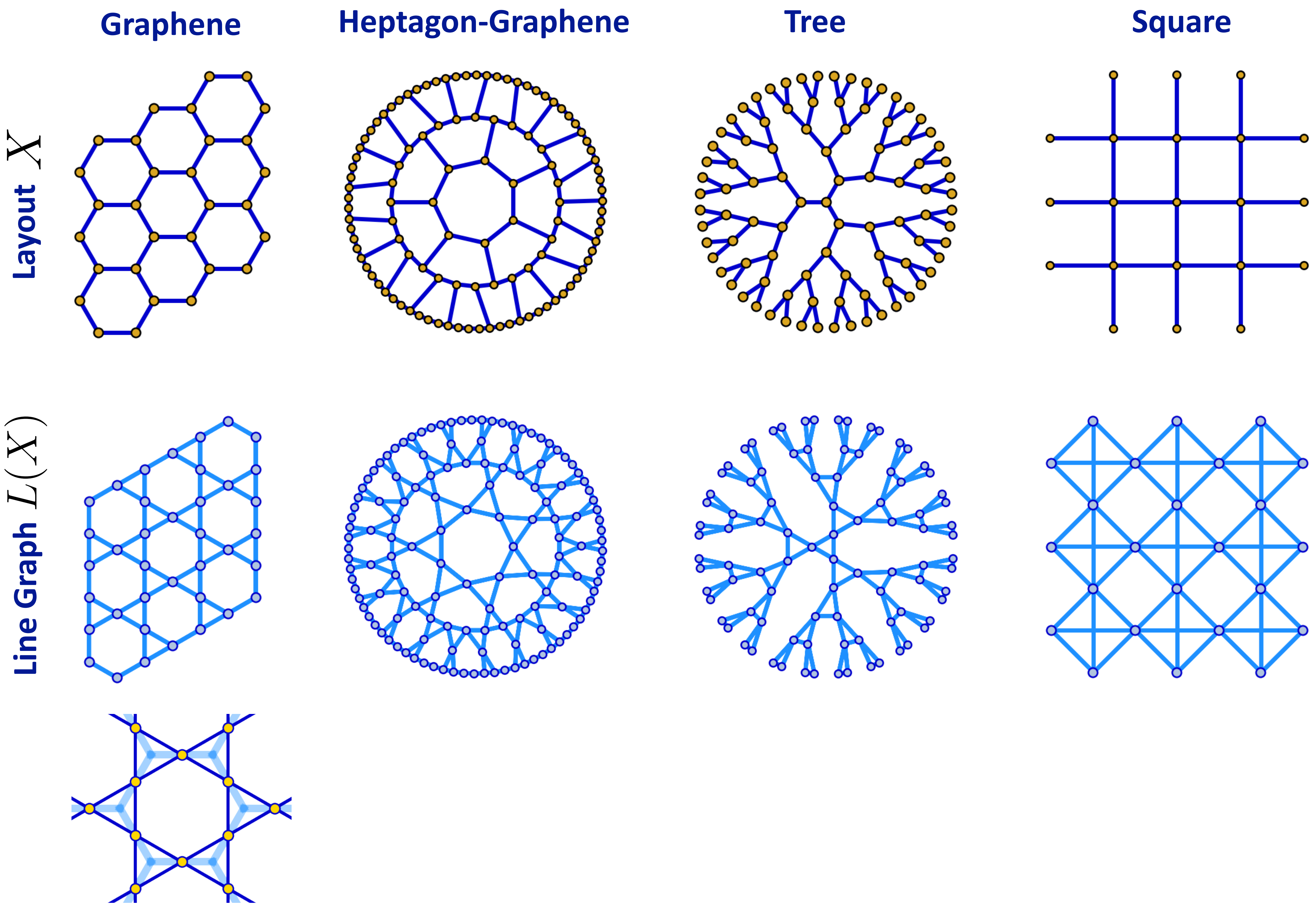




Line-Graph Lattices

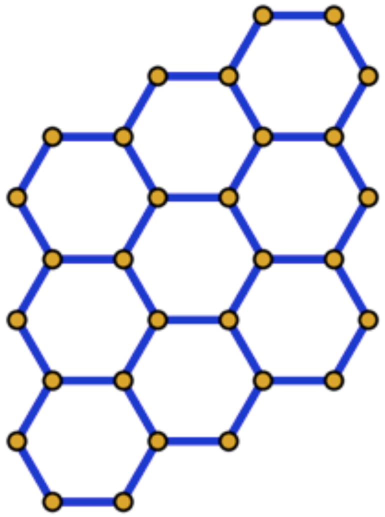


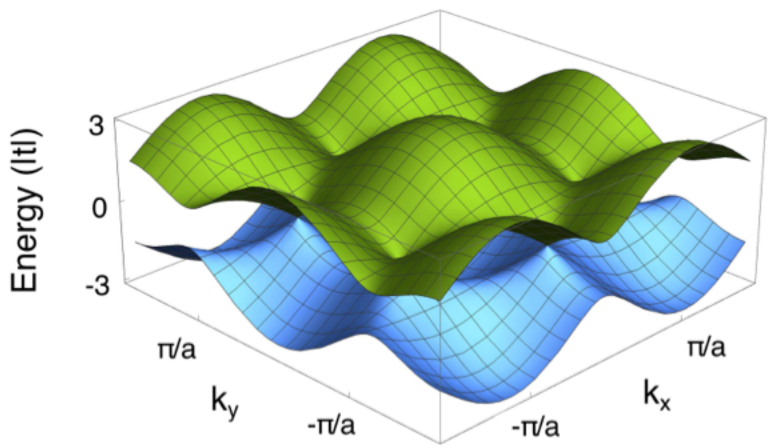
Line-Graph Lattices



Band Structure Correspondence

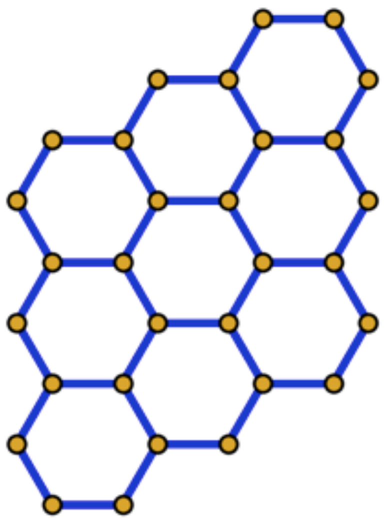
Layout X

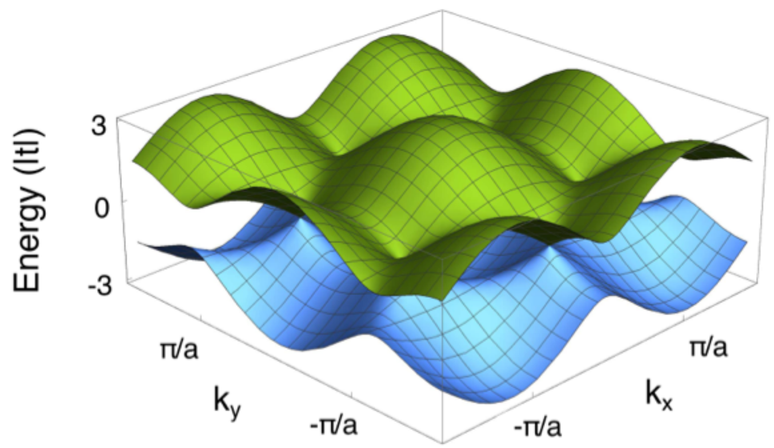




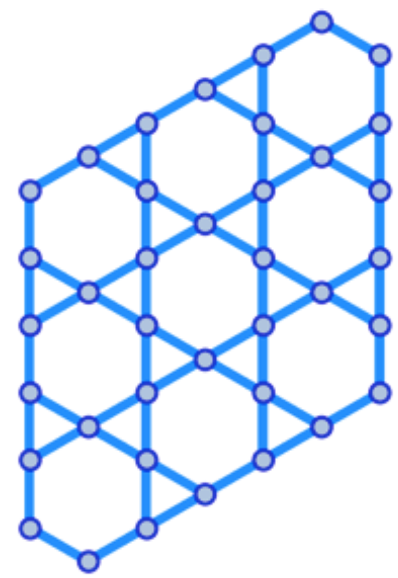
Band Structure Correspondence

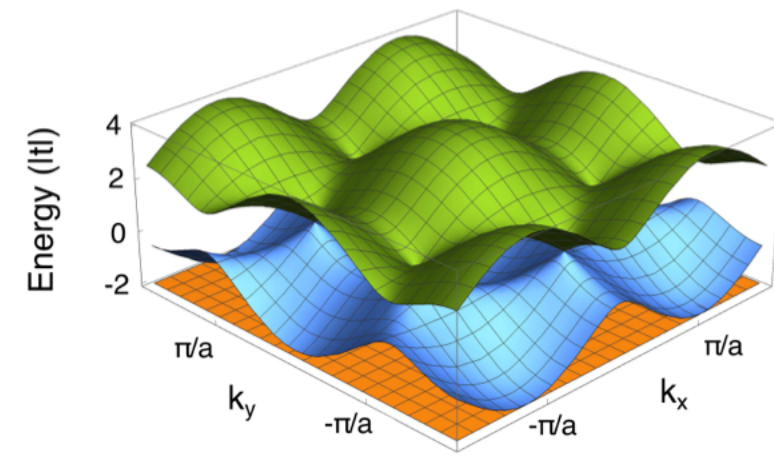
Layout X





Line Graph $L(X)$

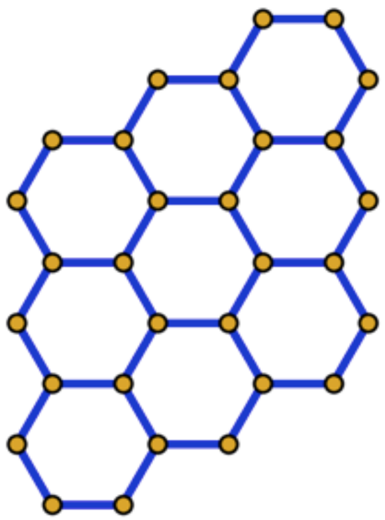


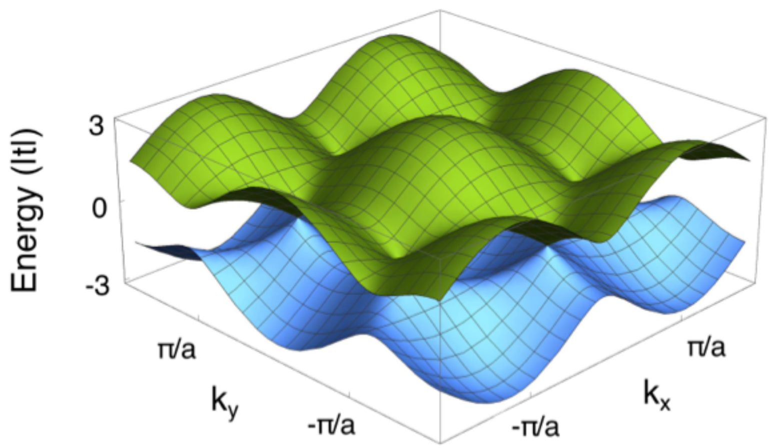


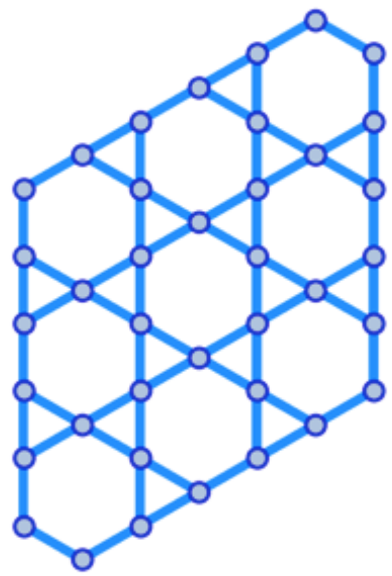
Band Structure Correspondence

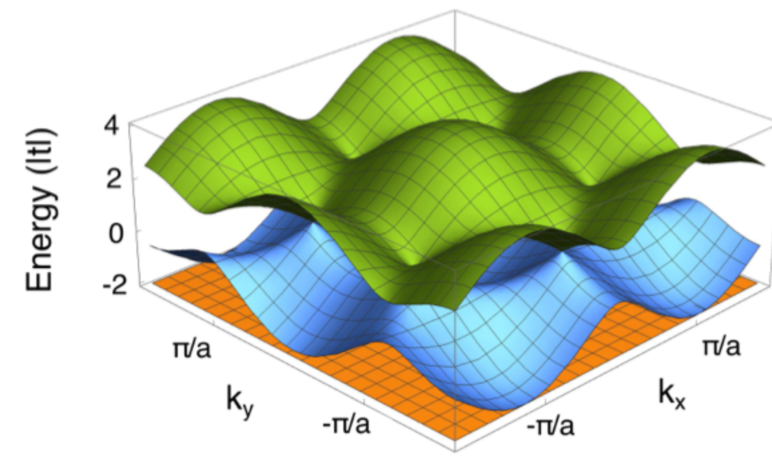
Layout X

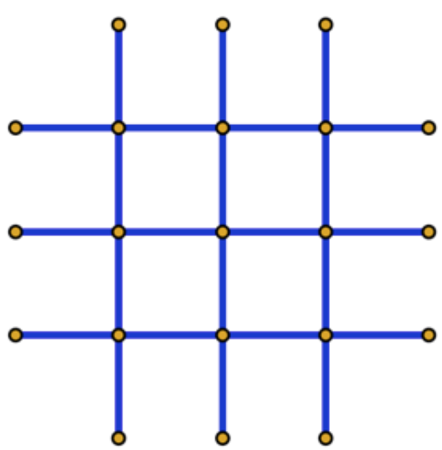
Line Graph $L(X)$

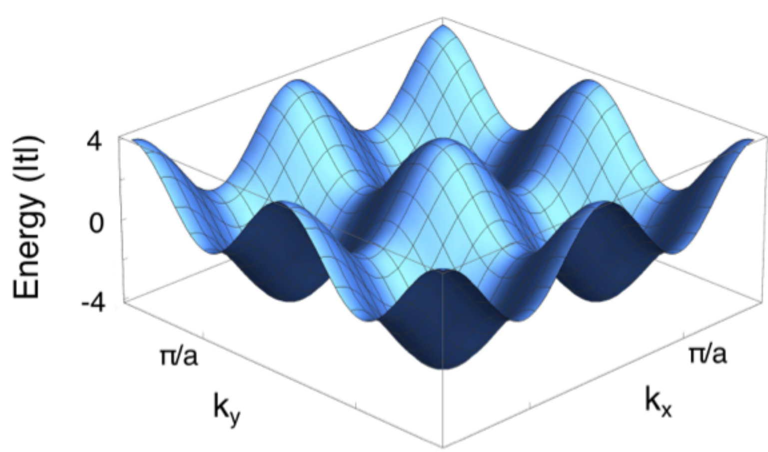






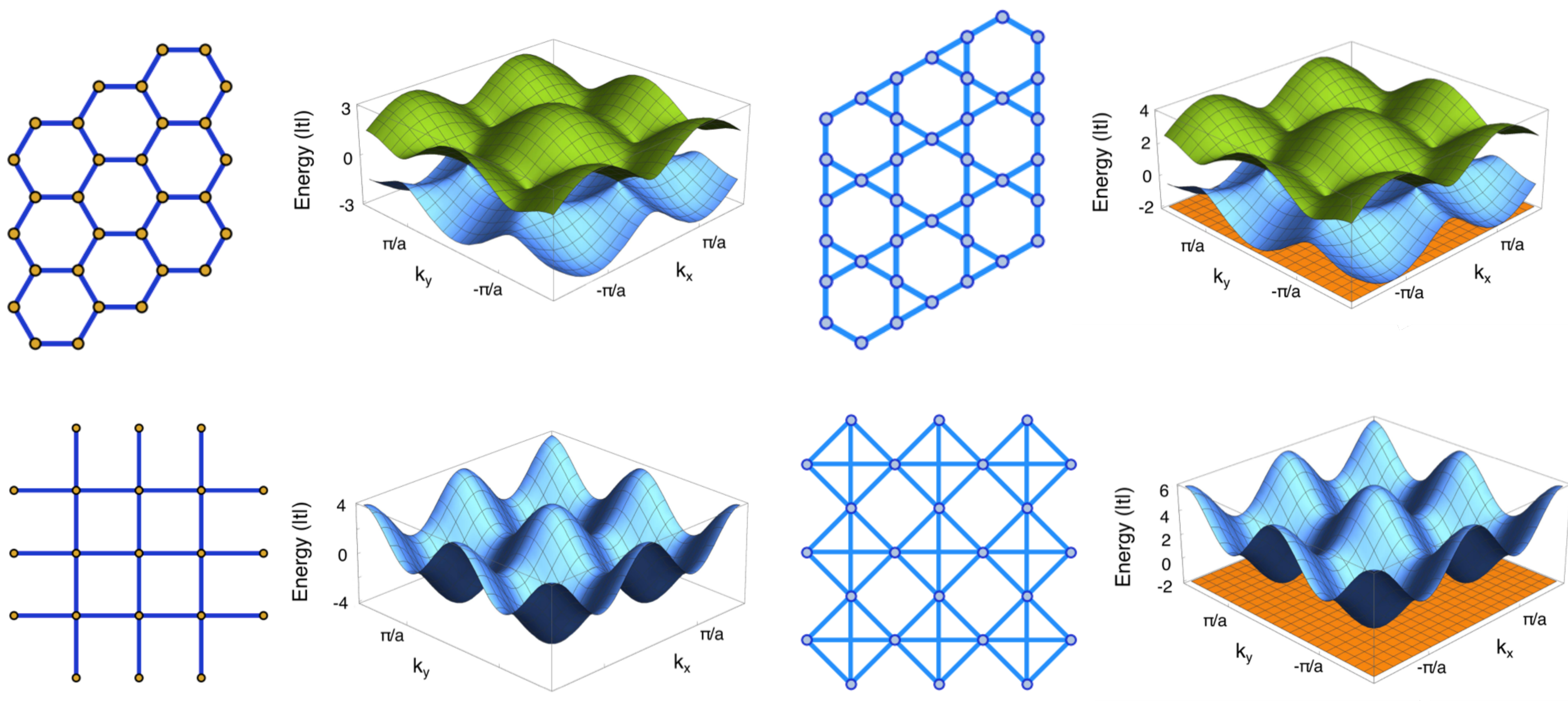






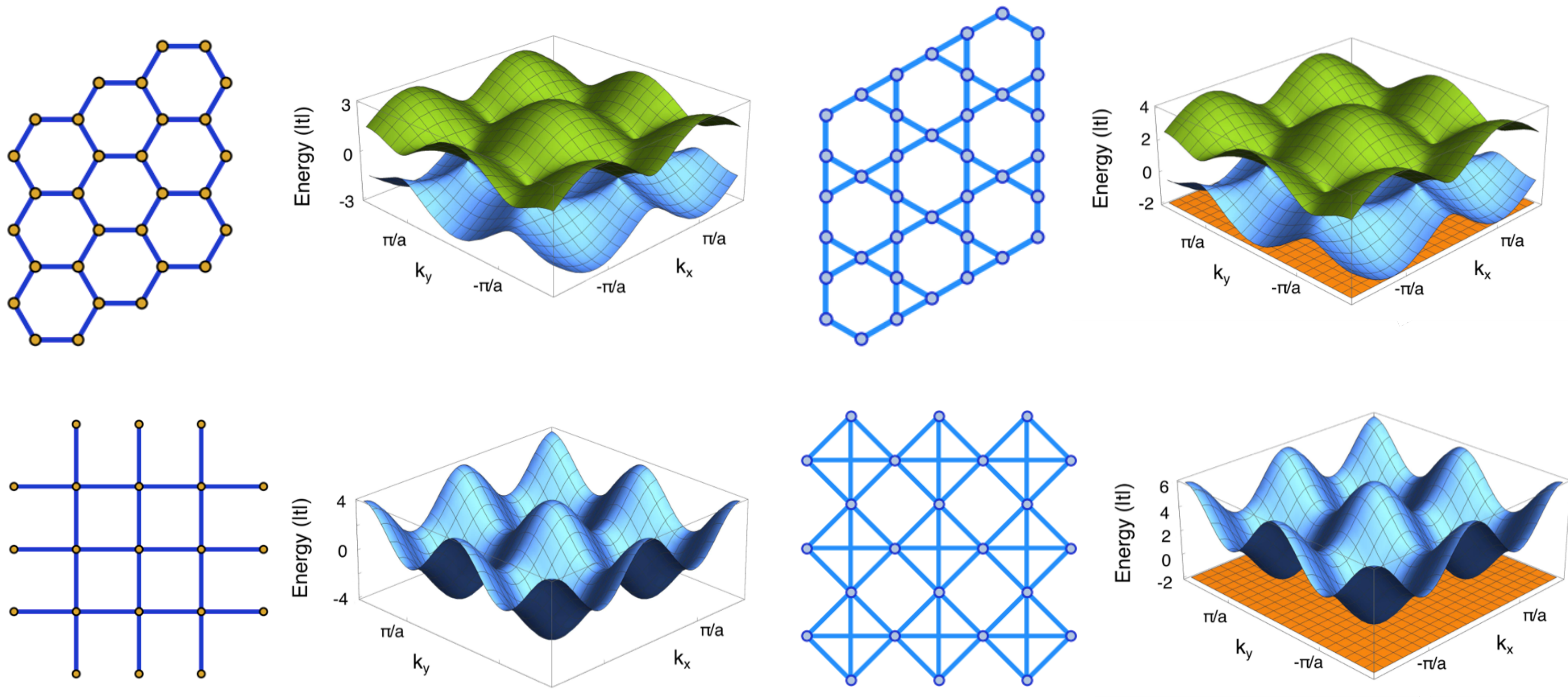
Band Structure Correspondence

Layout X **Line Graph** $L(X)$



Band Structure Correspondence

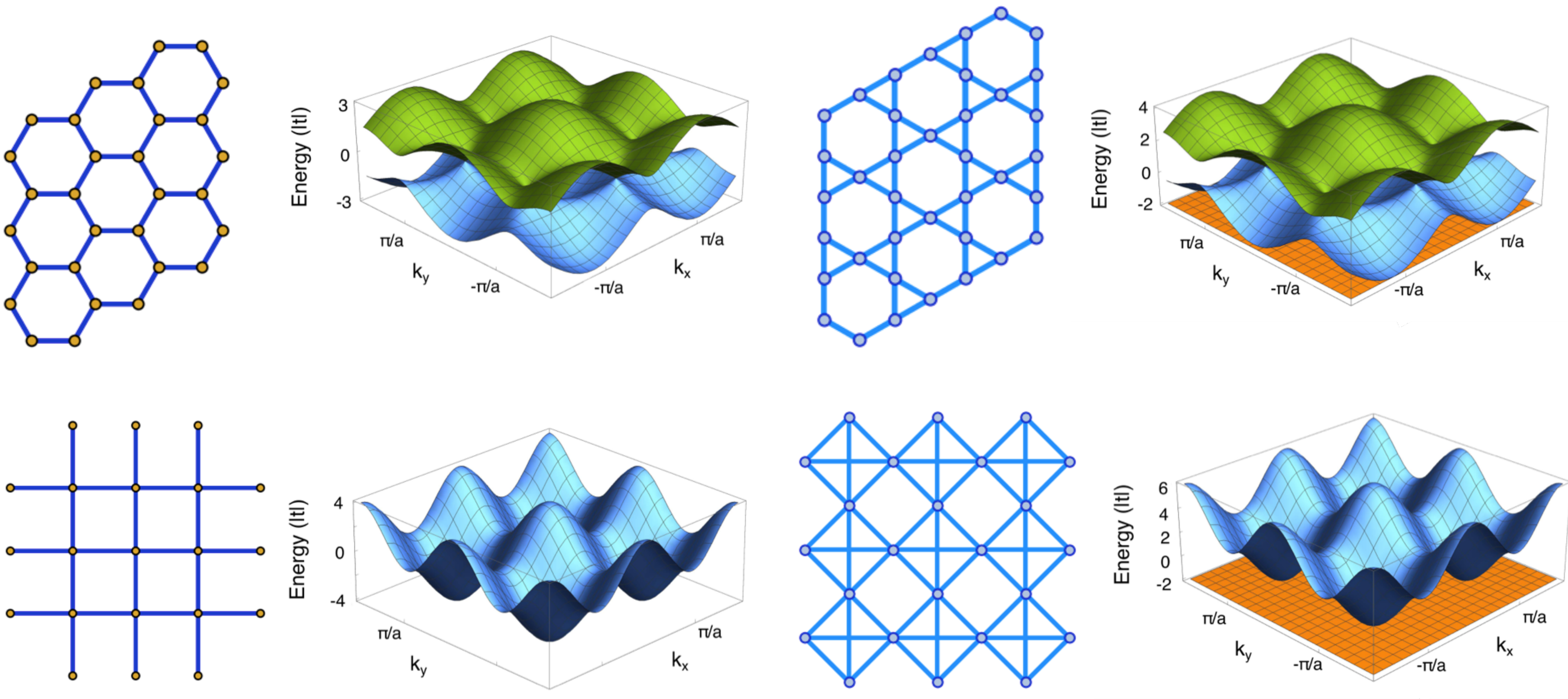
Layout X **Line Graph** $L(X)$



$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

Band Structure Correspondence

Layout X **Line Graph** $L(X)$

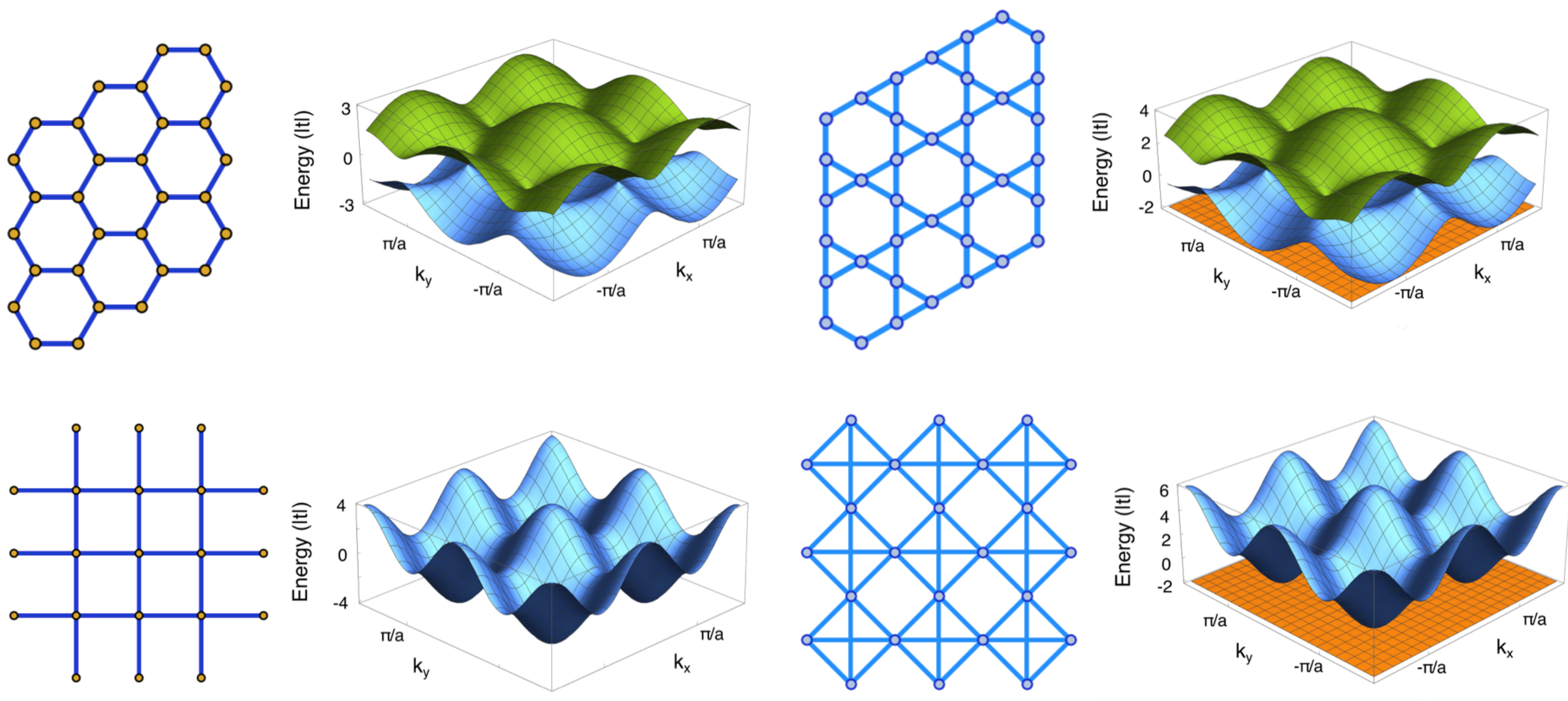


$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

Shifted Bands

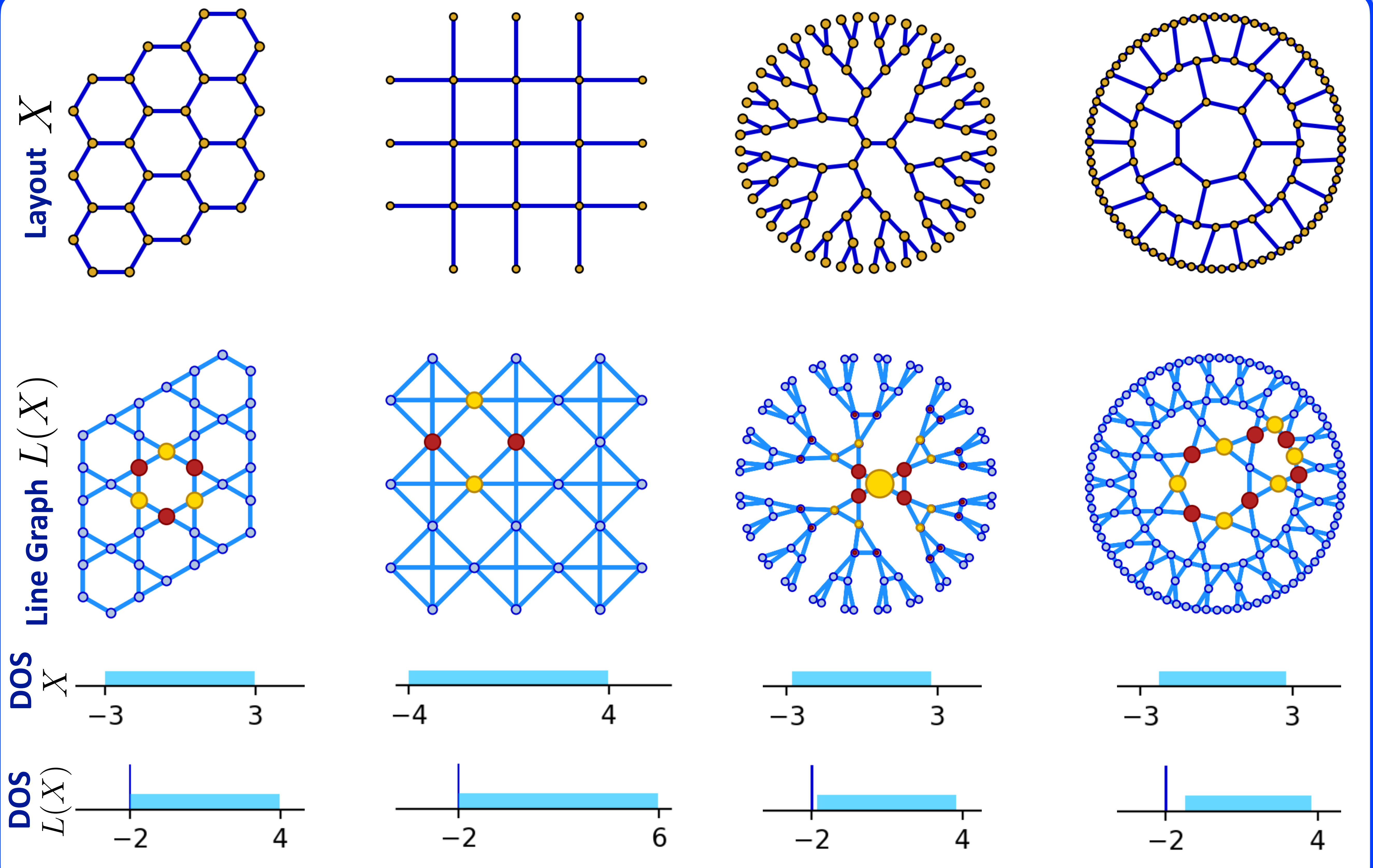
Band Structure Correspondence

Layout X **Line Graph** $L(X)$

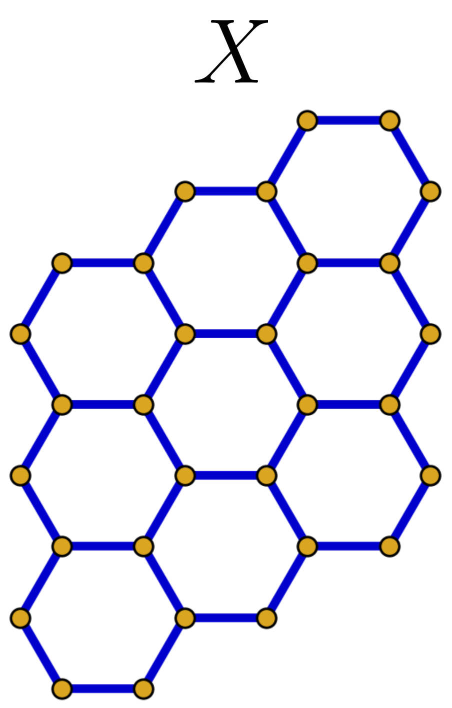


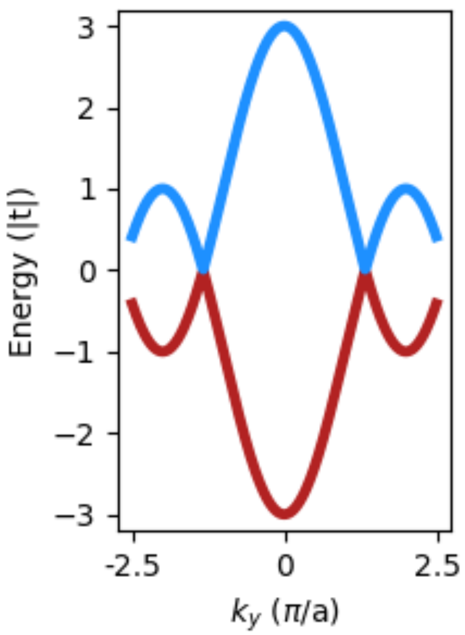
$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} & \leftarrow \text{Shifted Bands} \\ -2 & \leftarrow \text{Flat Band(s)} \end{cases}$$

Density of States and Flat-Band States



Subdivision Graphs and Optimally Gapped Flat Bands

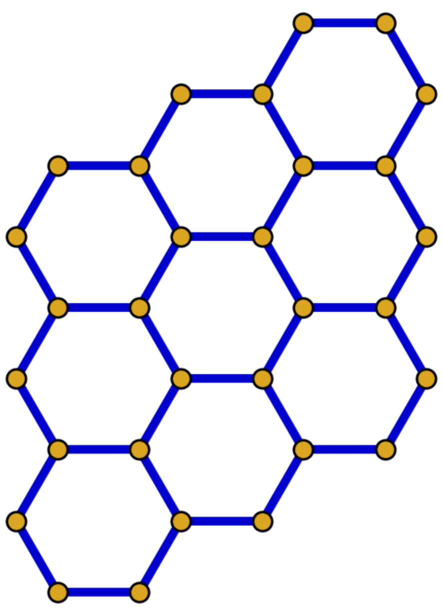


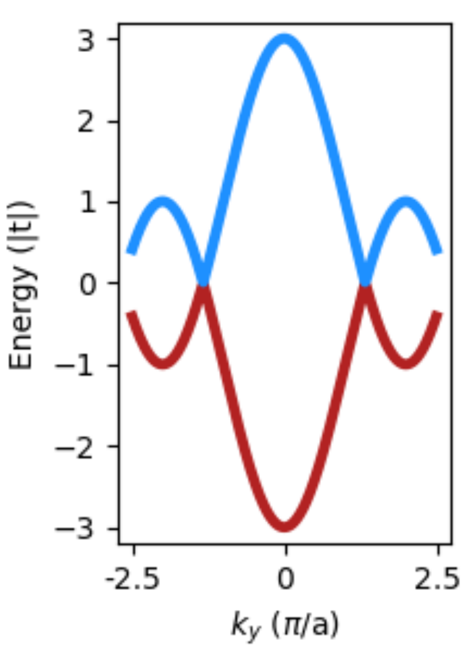


Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Subdivision Graphs and Optimally Gapped Flat Bands

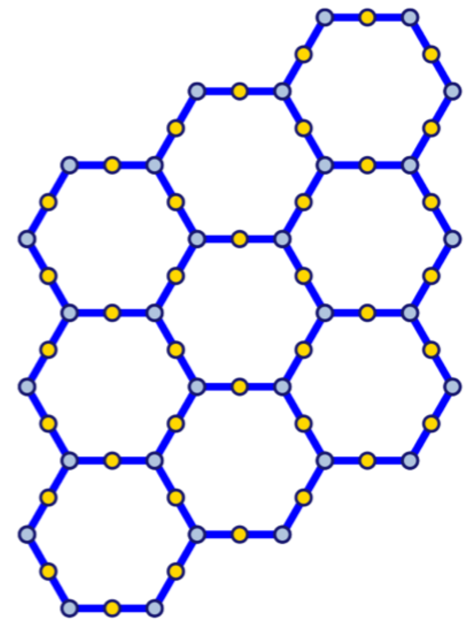
X

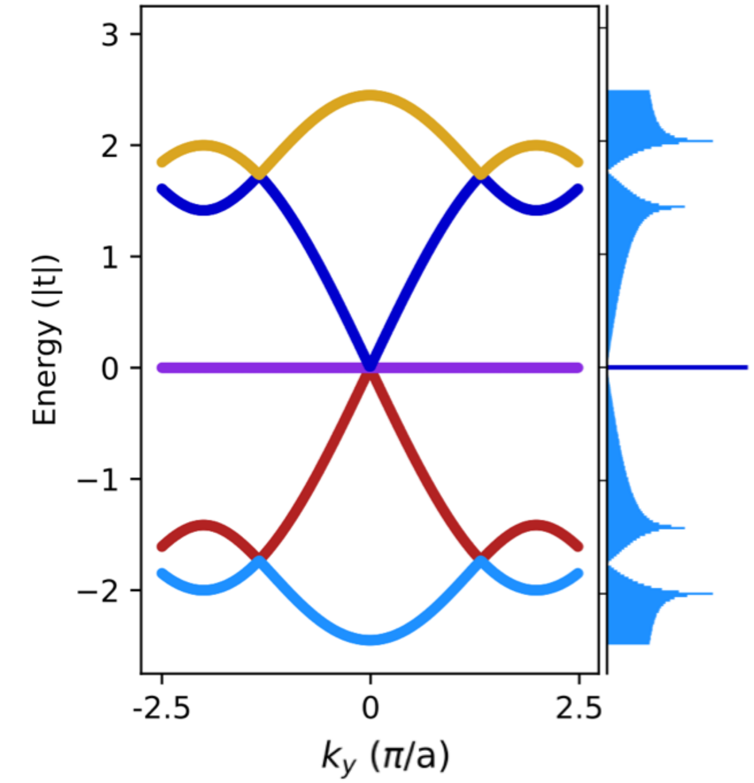




$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

$\mathbb{S}(X)$

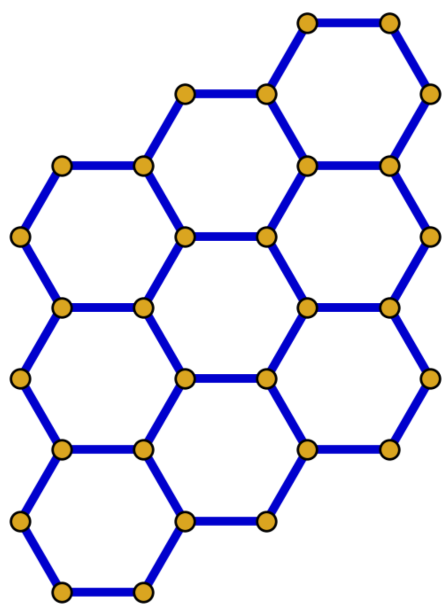


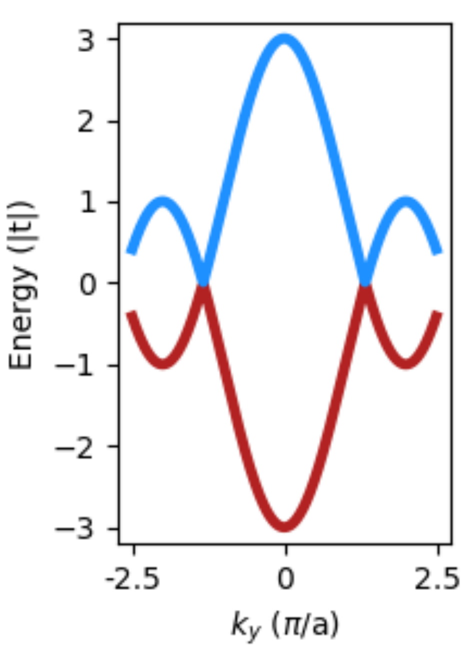


Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

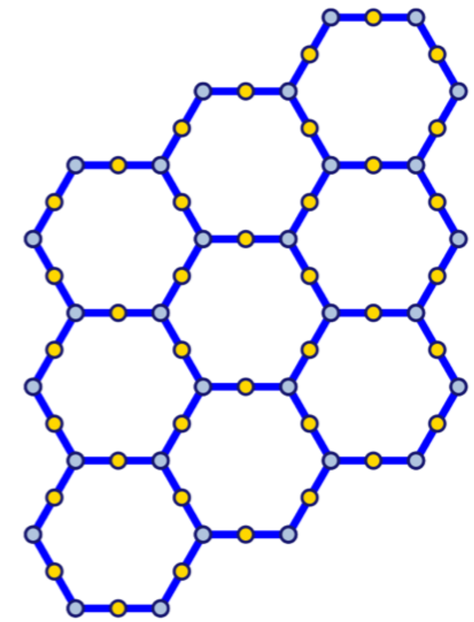
Subdivision Graphs and Optimally Gapped Flat Bands

X

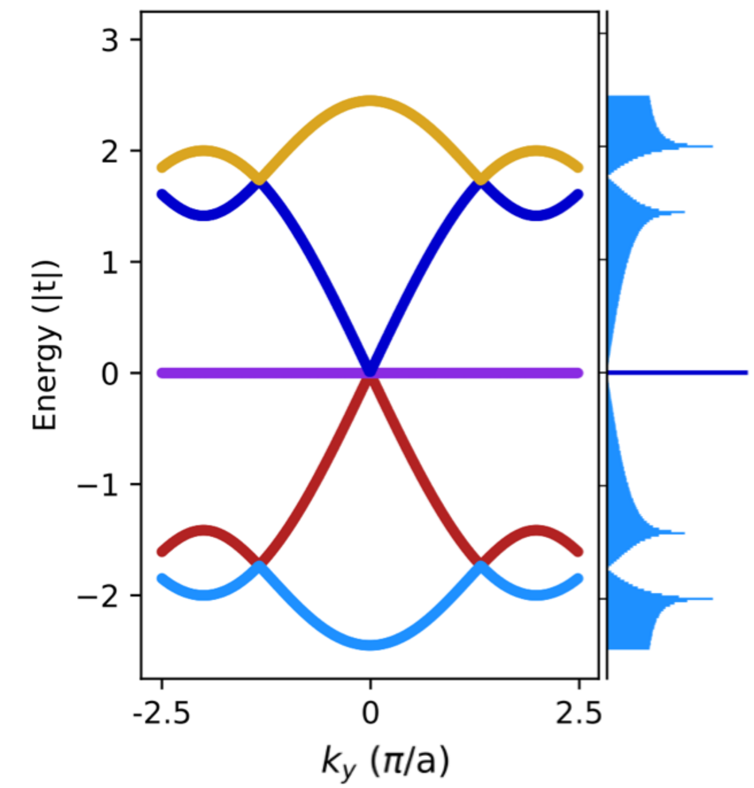




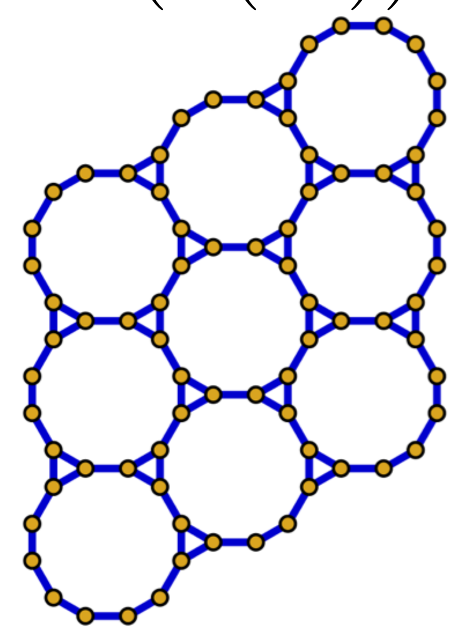
$\mathbb{S}(X)$



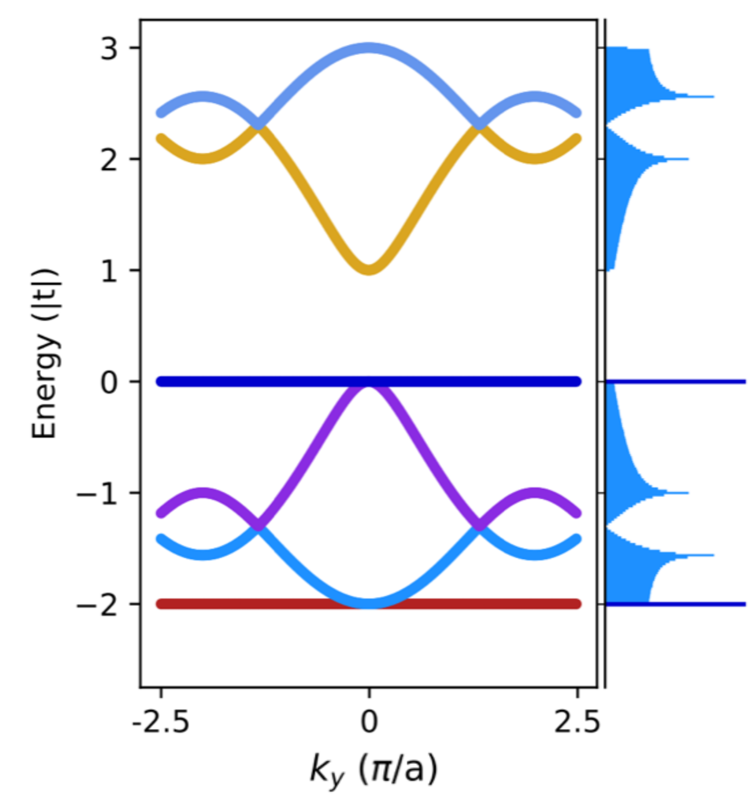
$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$



$L(\mathbb{S}(X))$

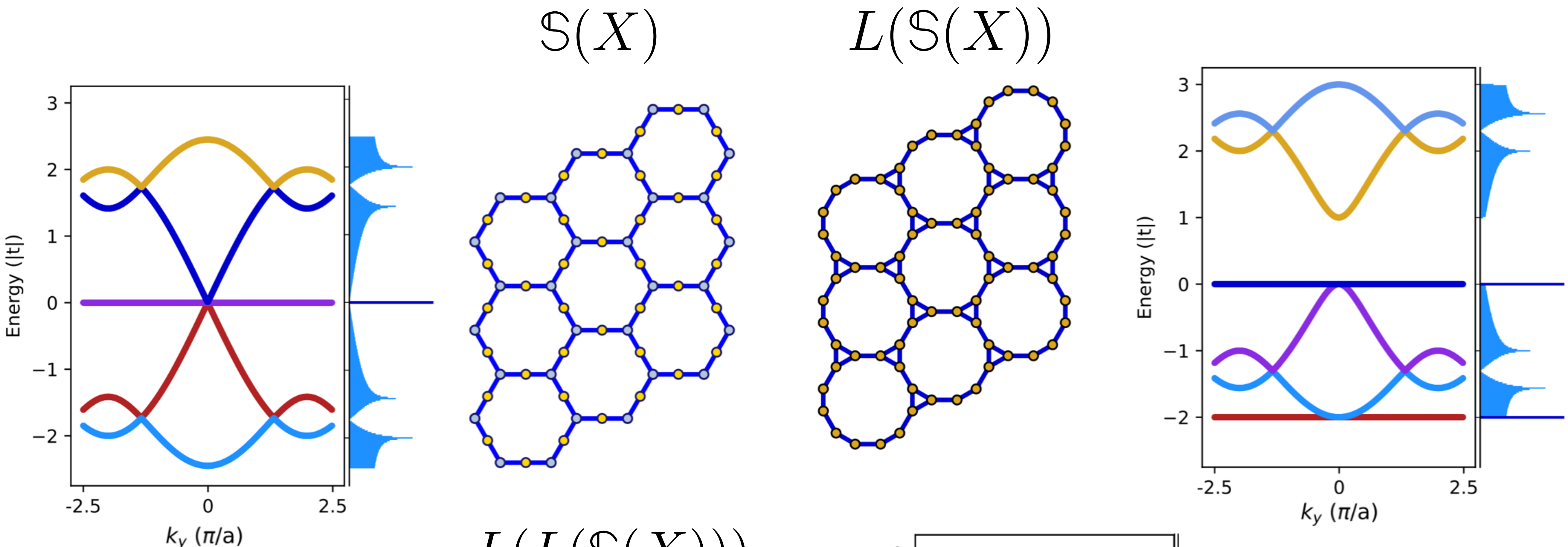


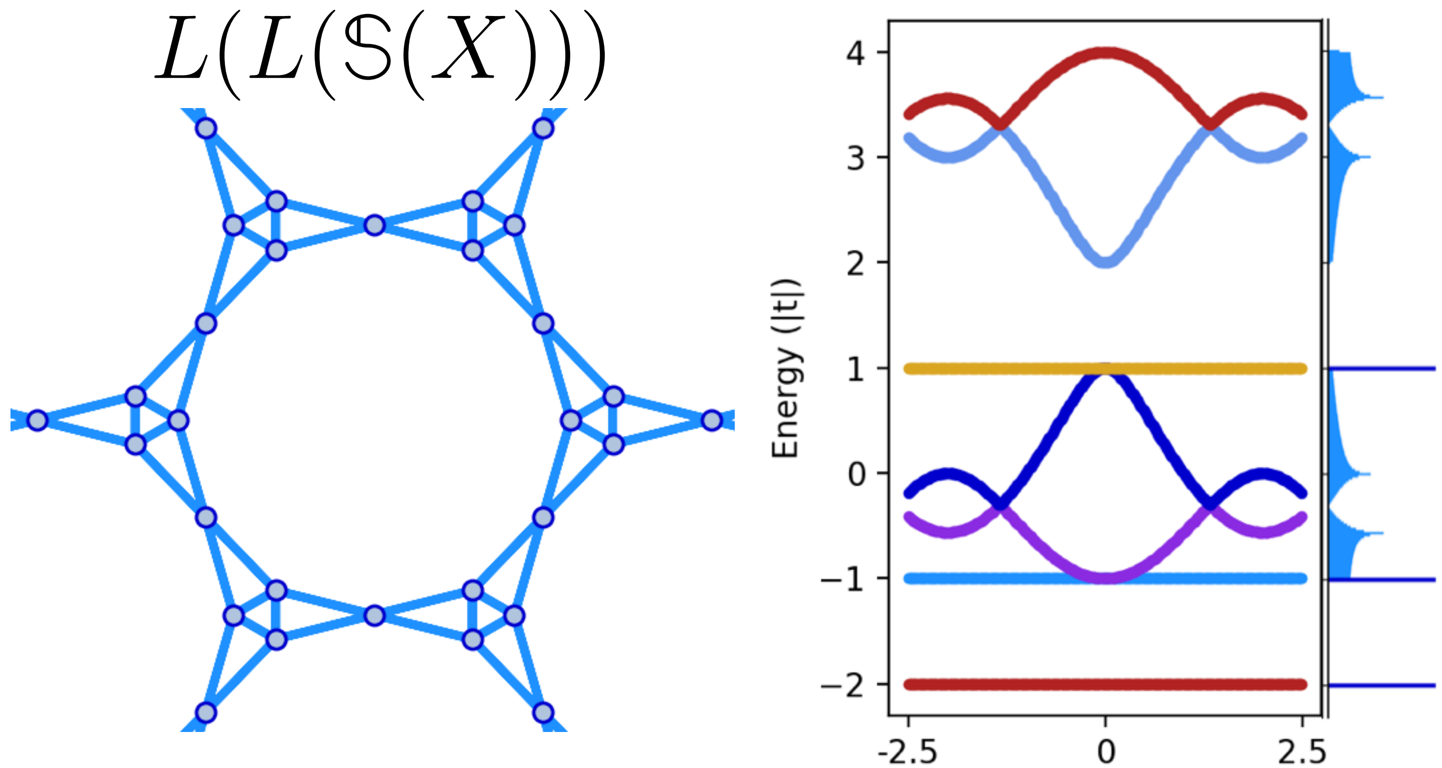
$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1 \pm \sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$



Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Subdivision Graphs and Optimally Gapped Flat Bands





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New Lattices for Photon-Mediated Interactions

Photonic Crystal + qubits

- Effective swap interaction
- All modes in parallel

$$H = \hbar\, \sigma_1^+ \sigma_2^- \sum_m \frac{g_m^2}{\Delta(m)}\, \psi_m(x_1)\psi_m^*(x_2) + h.c.$$

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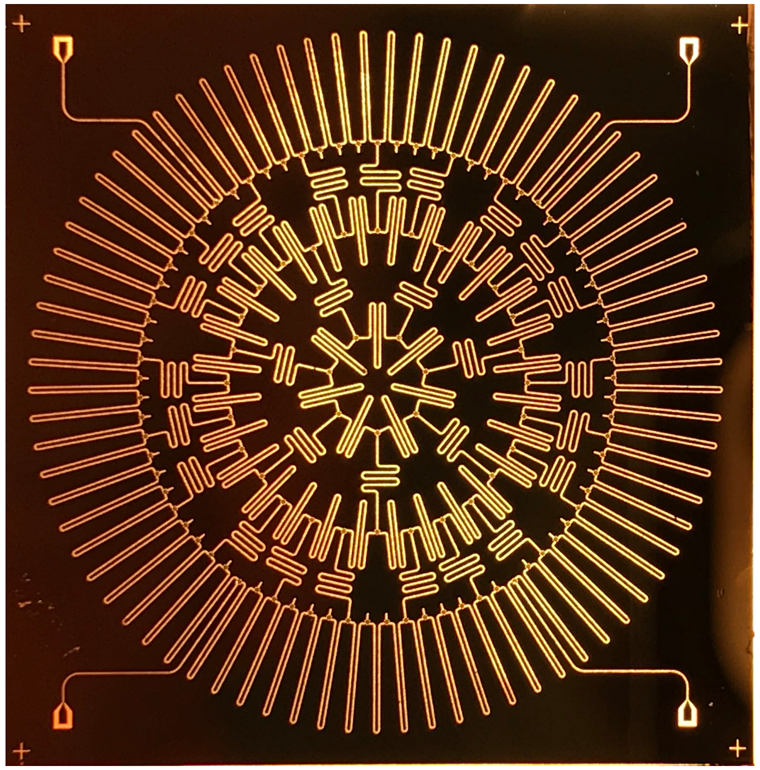

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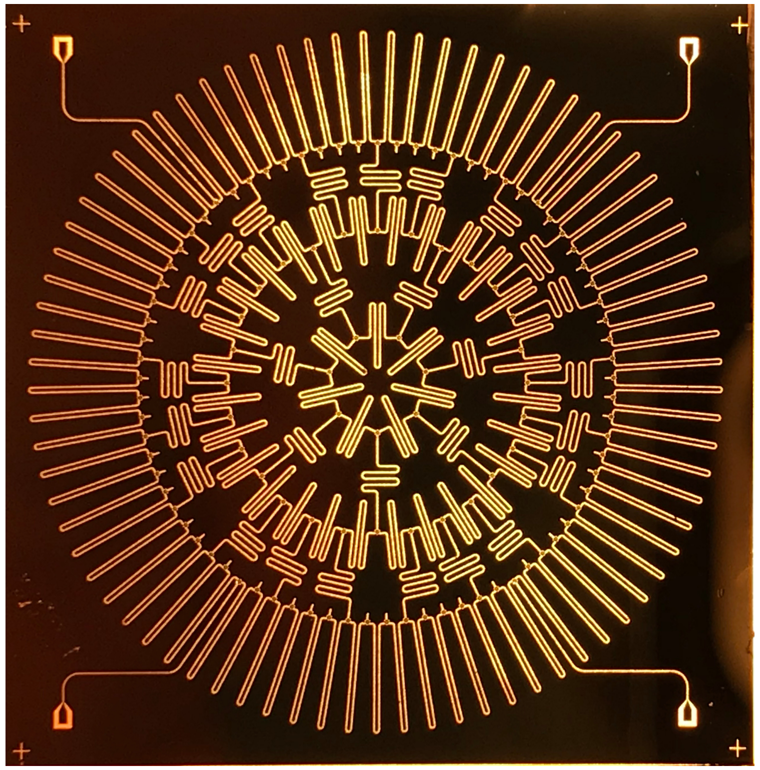

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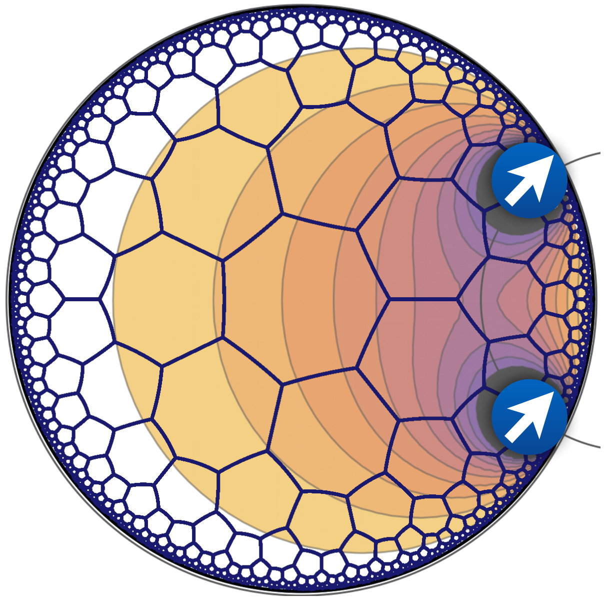

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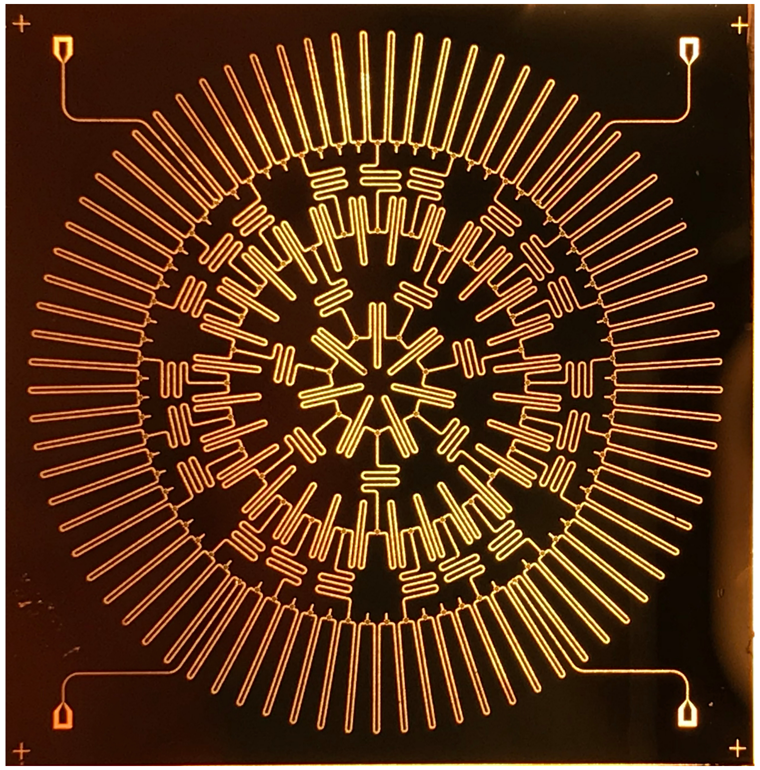

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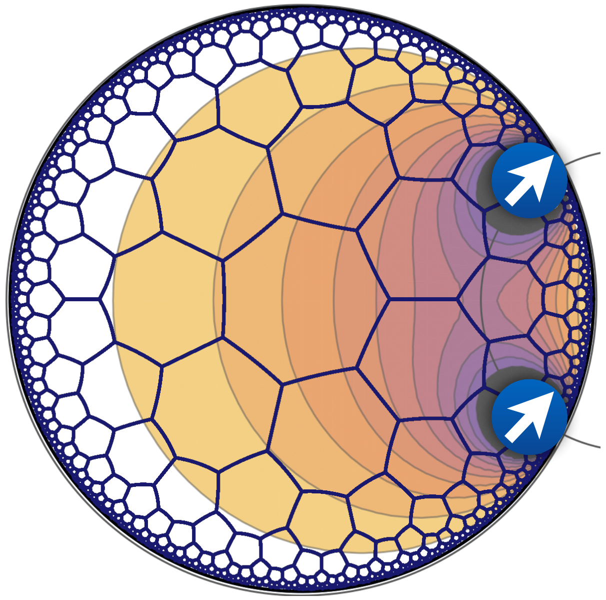

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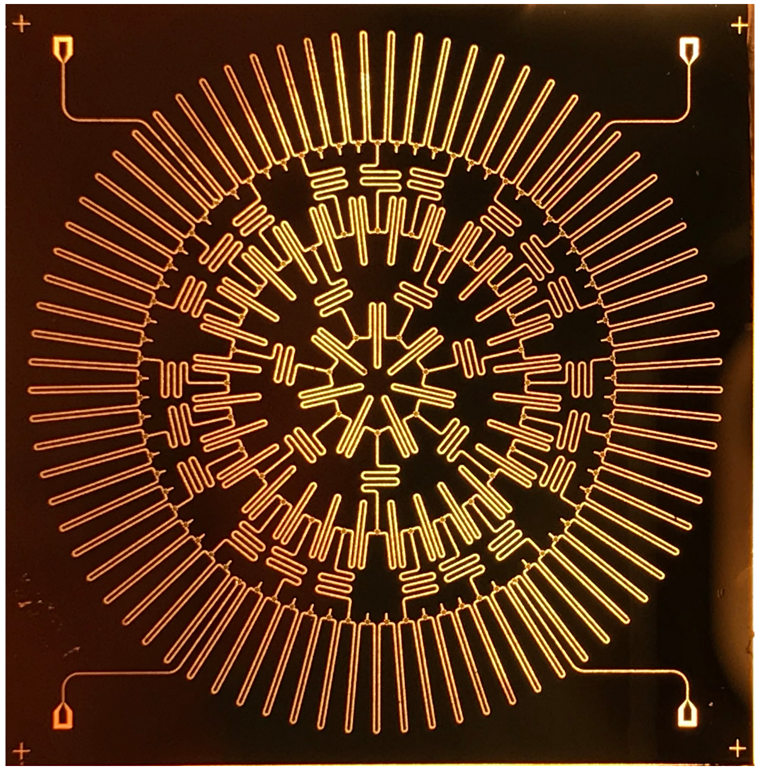

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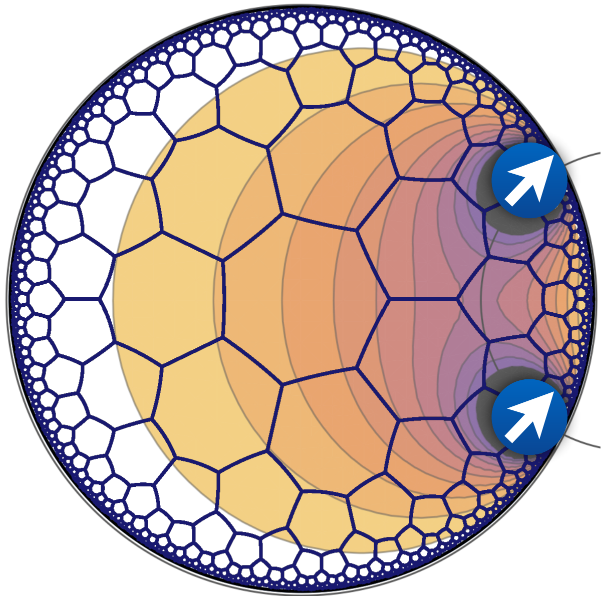

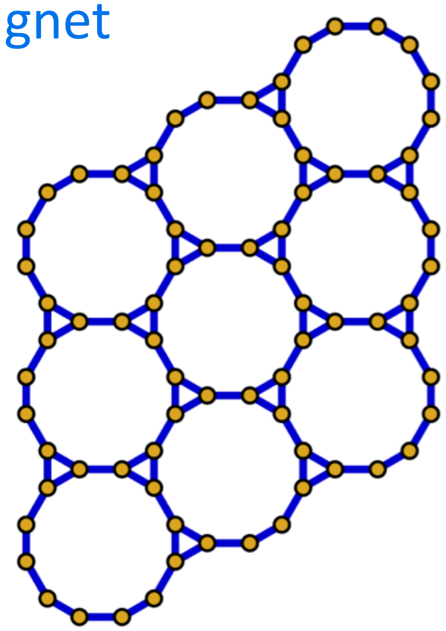

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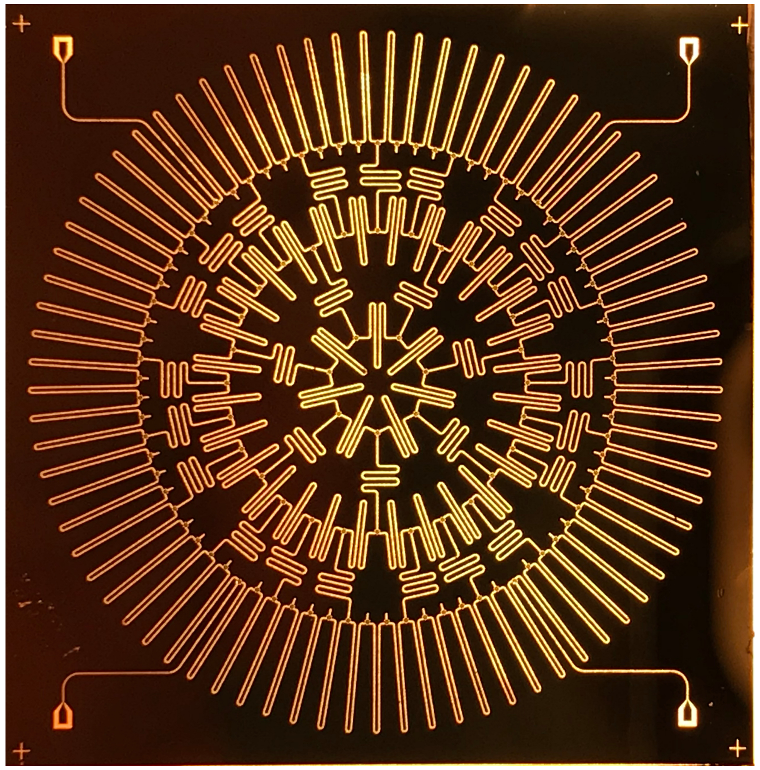

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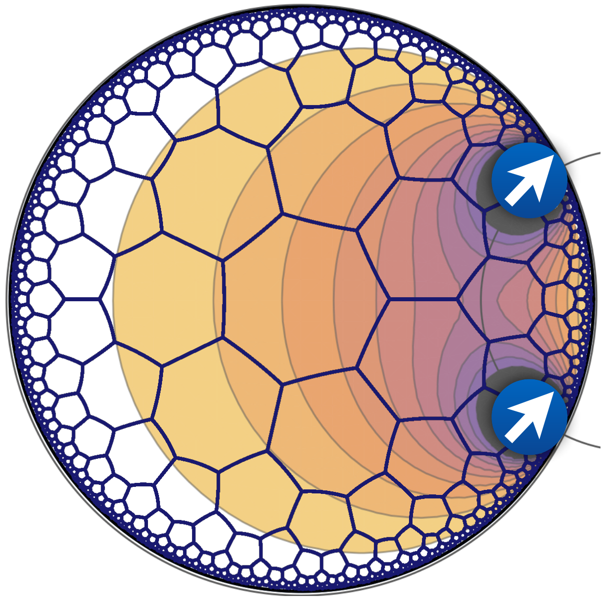

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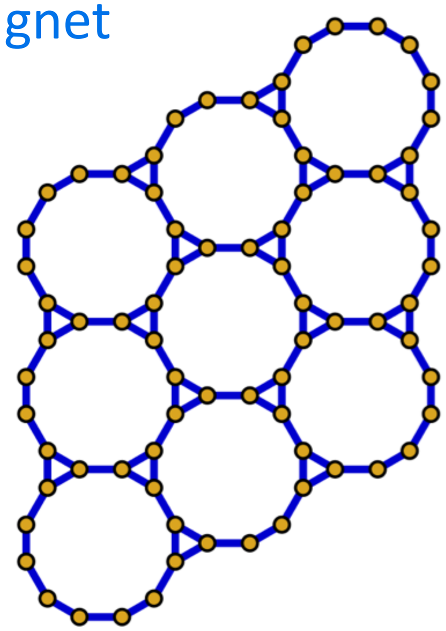

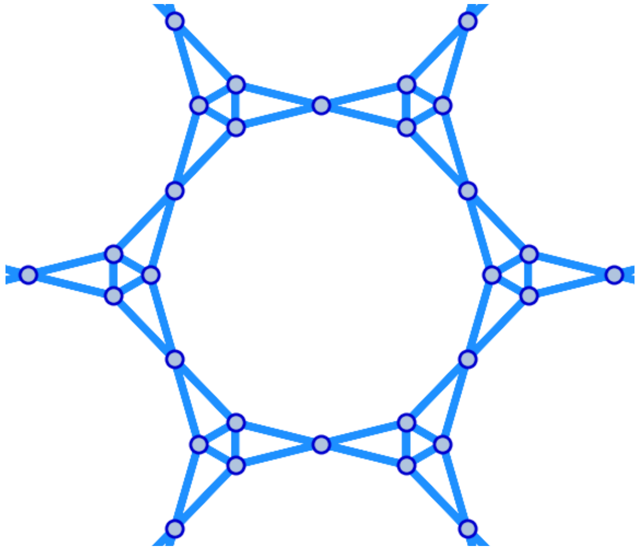

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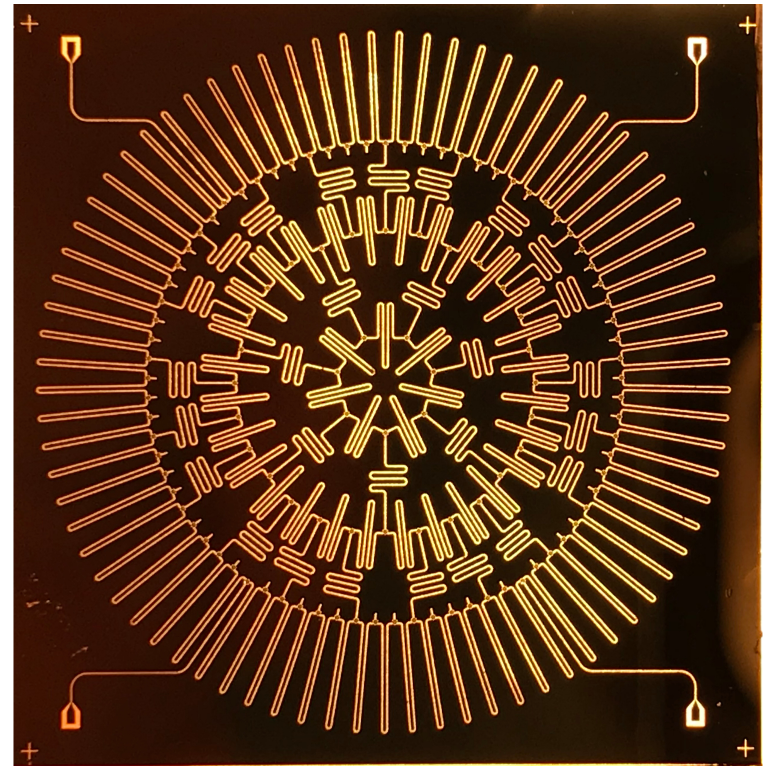

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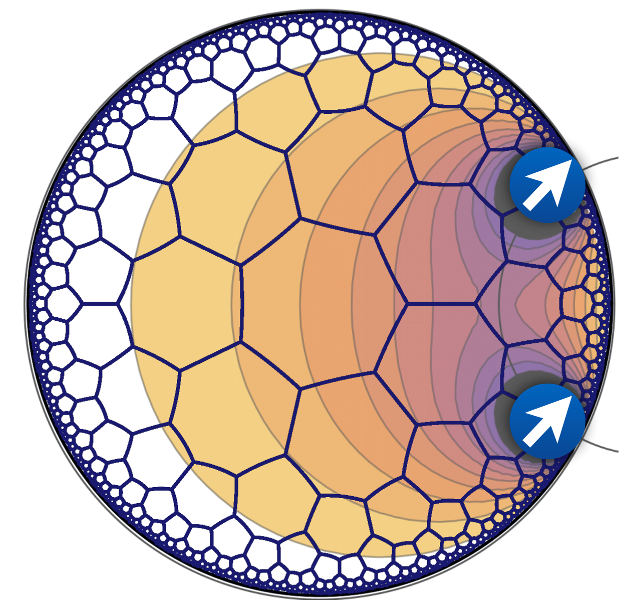

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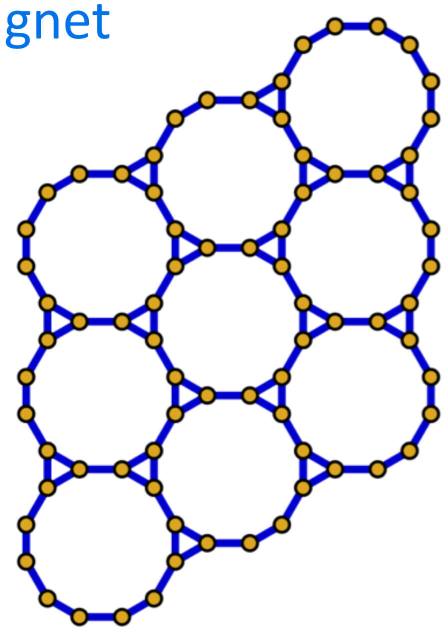

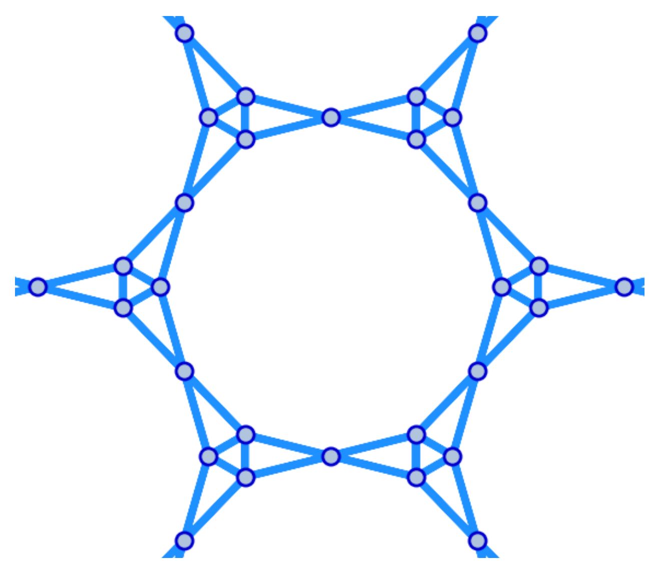

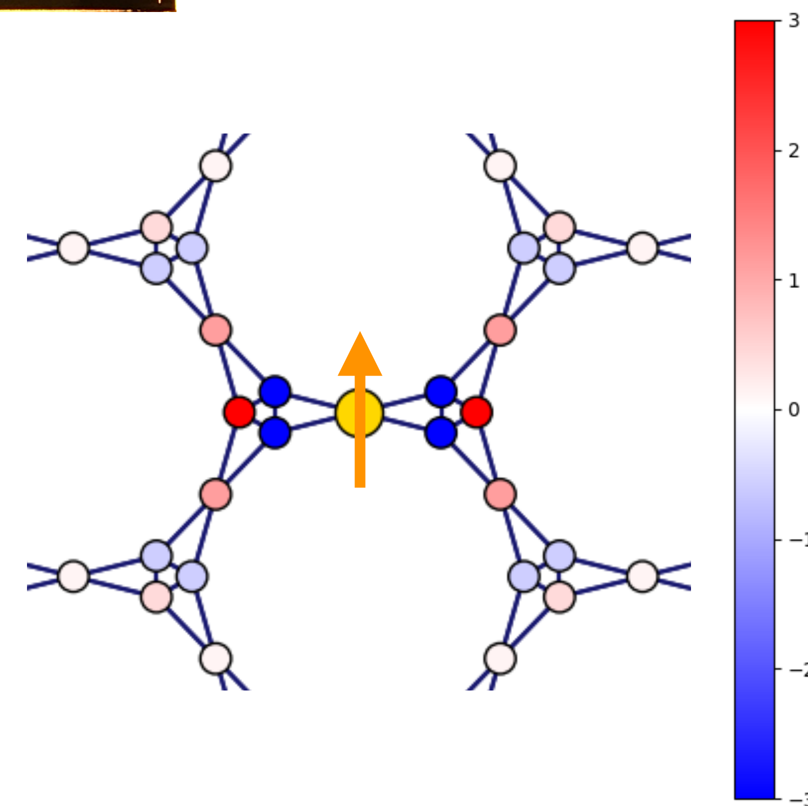

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Outline

- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
  - Line-graph lattices
  - Interacting photons
- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
- Mathematical Connections
  - Planar Gaps
  - Maximal Gaps
  - Quantum Error Correcting Codes
- Experimental Developments

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Other Maximal Gaps?

Two Driving Questions

- Even larger gaps possible at other energies?
- Where can planar graphs have gaps?

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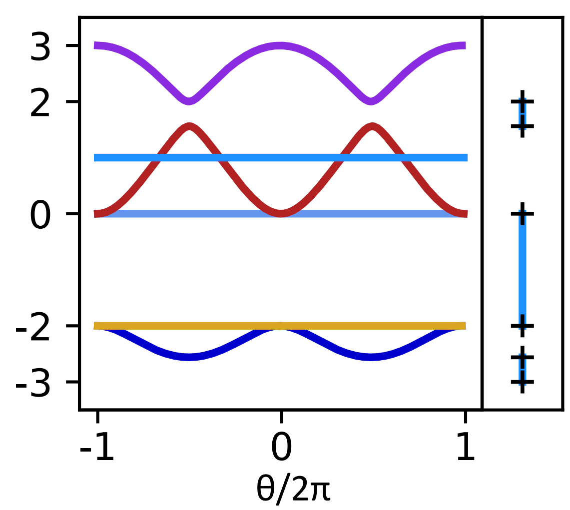

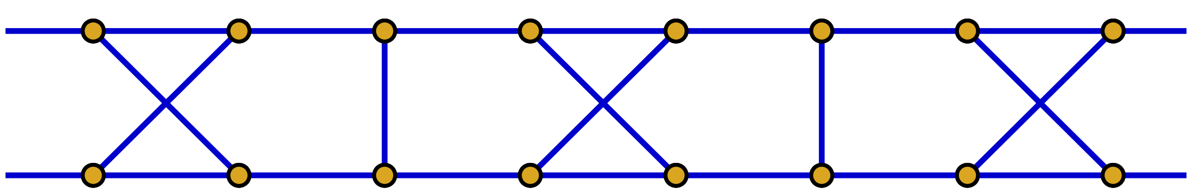

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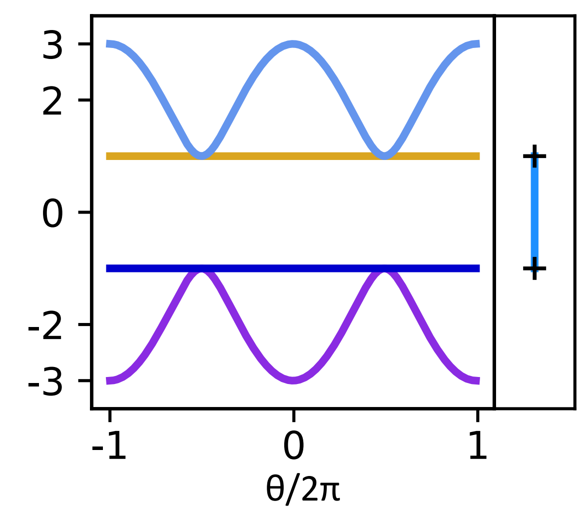

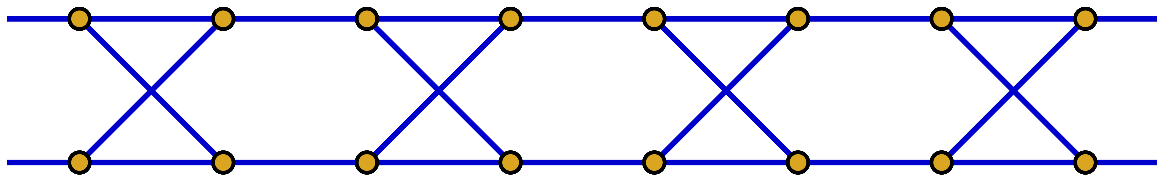

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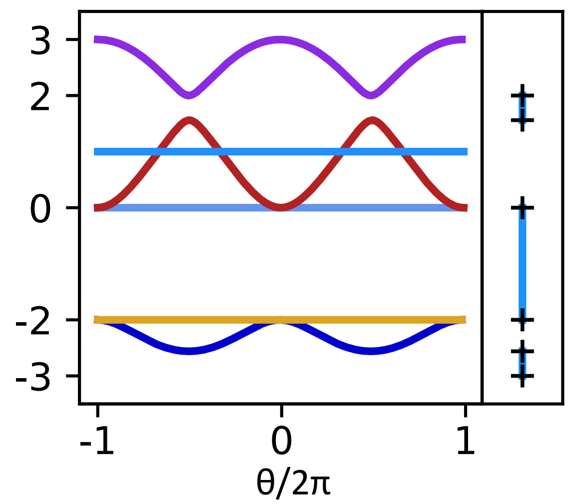

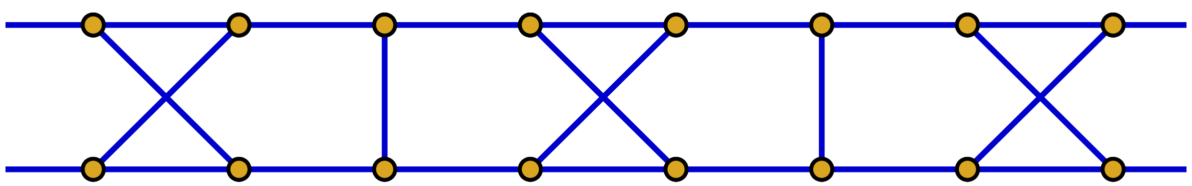

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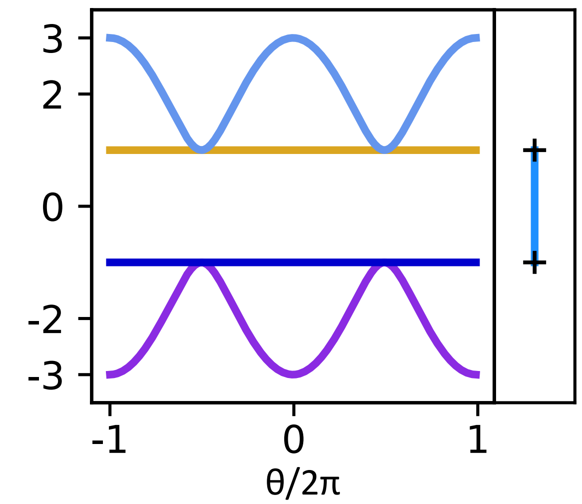

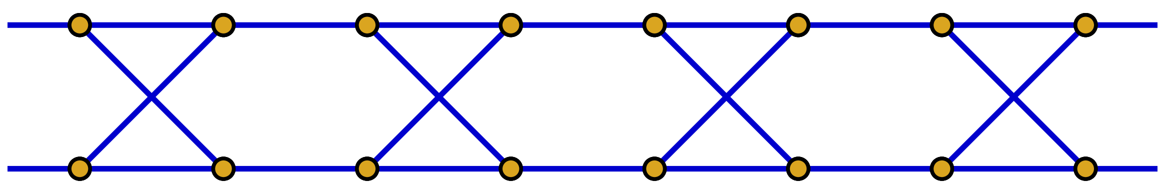

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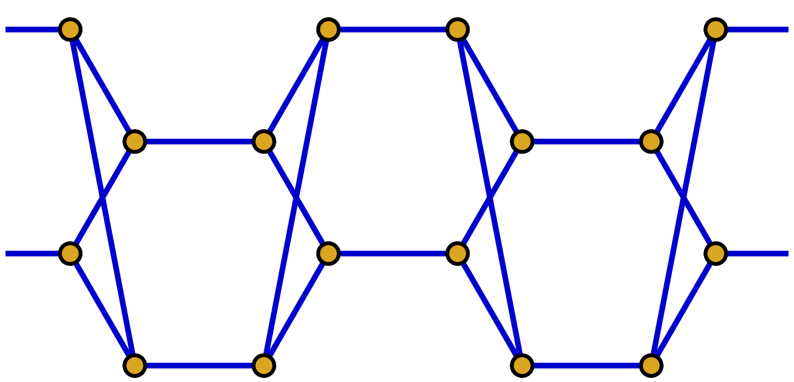
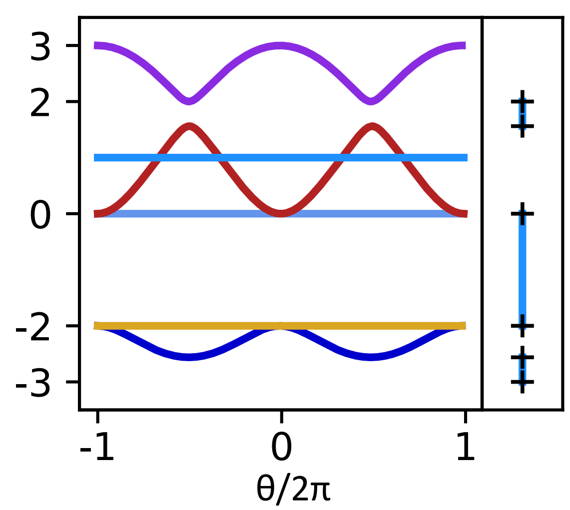
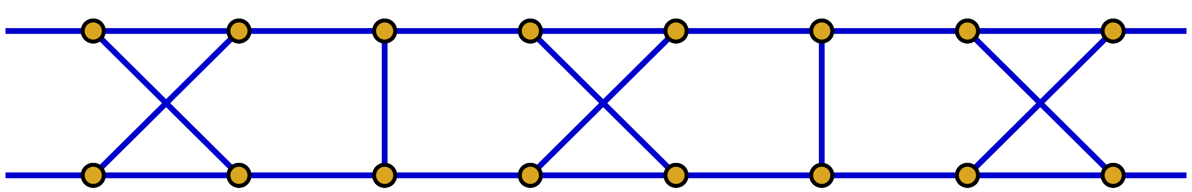
A.K.A.

n=2, m=0 carbon nanotube







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Abelian Covers and Planar Gaps

New Lattice Viewpoint

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- Use method of Abelian covers to construct examples.
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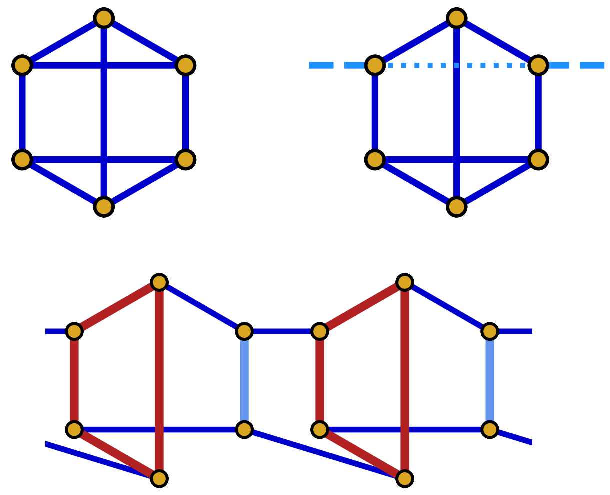

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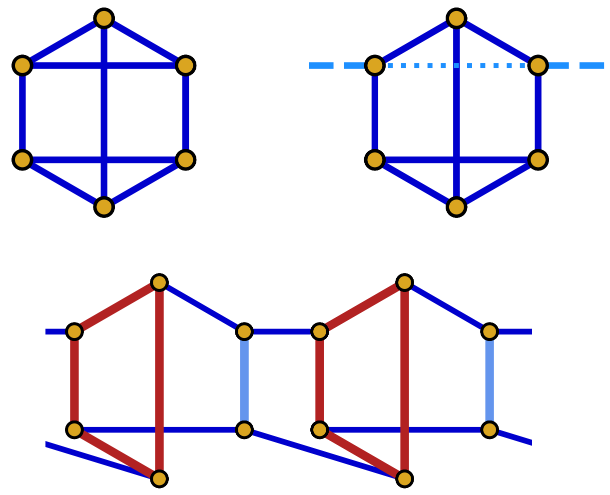

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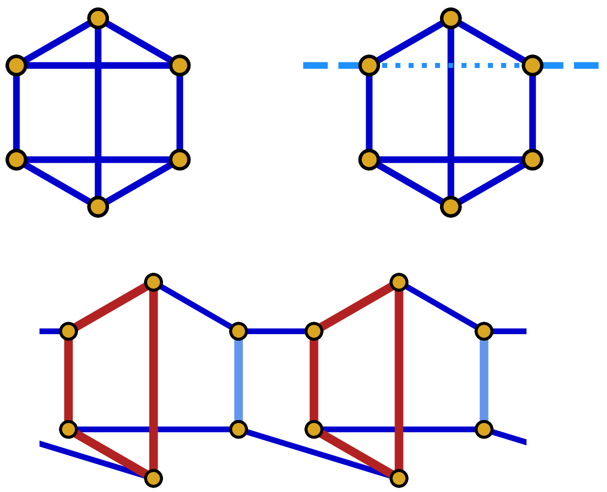

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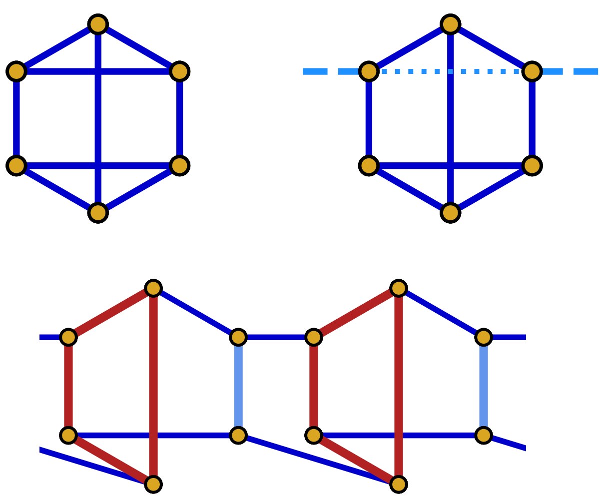

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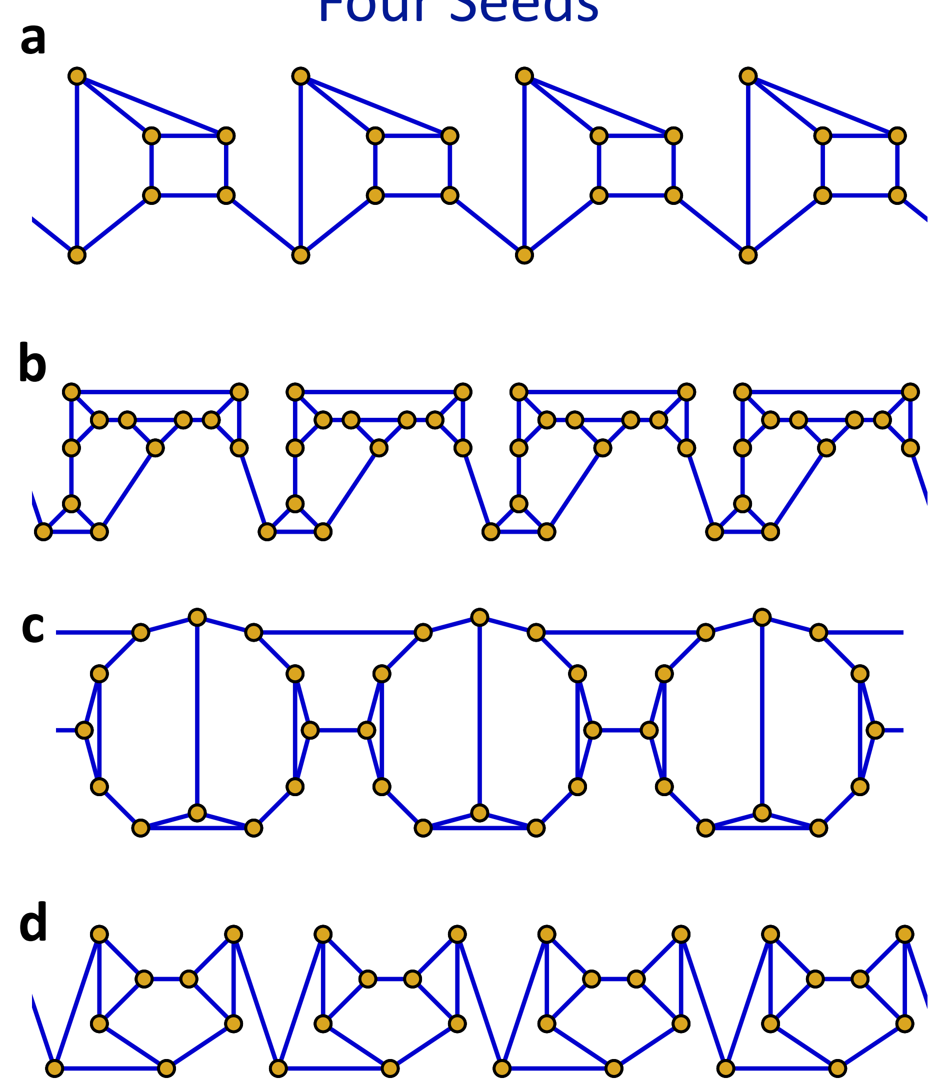
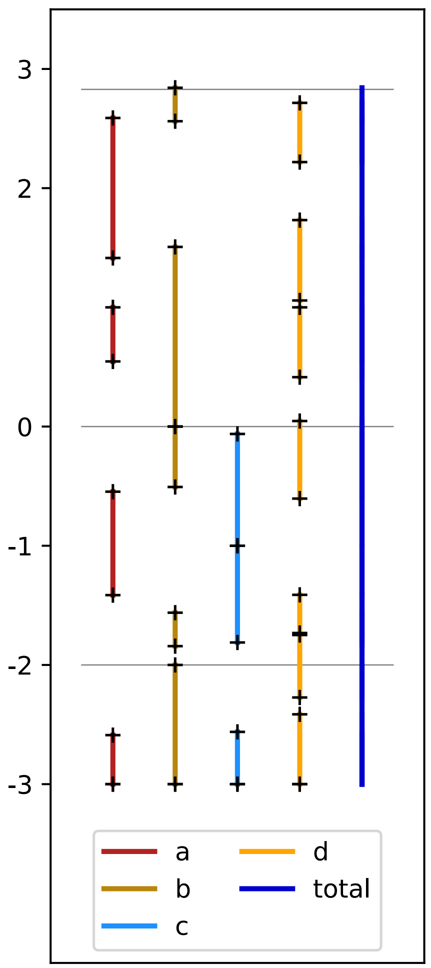
Four Seeds





- Combined gaps cover [-3,2√2].
- Iteration of L(S(X)) covers the rest.

Line-Graph Subsystem Codes

Thm: (Chapman and Flammia)
A spin model can be solved exactly by mapping to free fermions if and only the anticommutation relations of its terms have the structure of a line graph.

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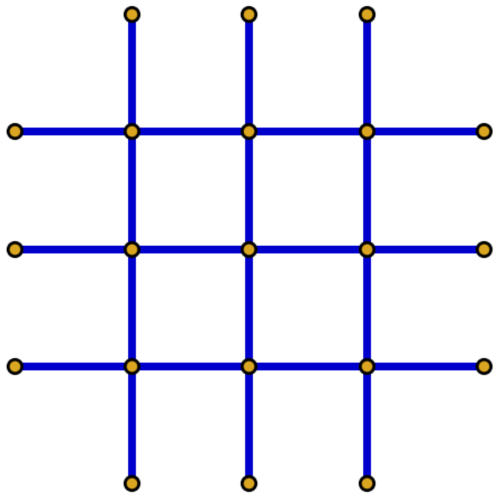
The Checkerboard-Lattice Code

- Built on the square lattice

Fermion Lattice



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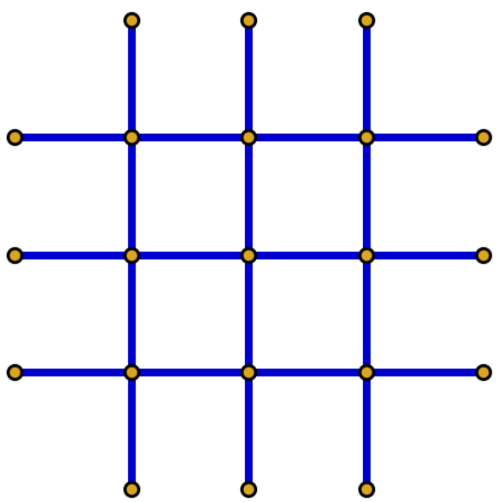
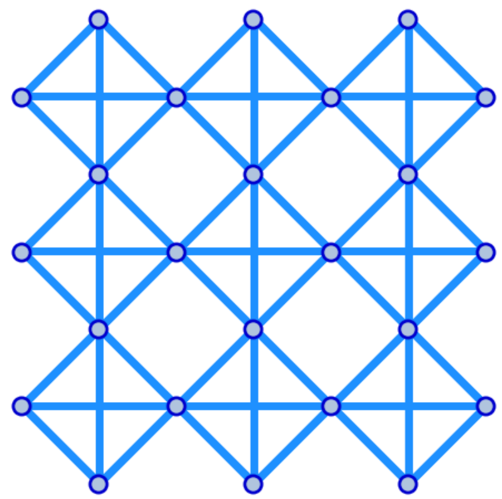


Anticommutation Relations



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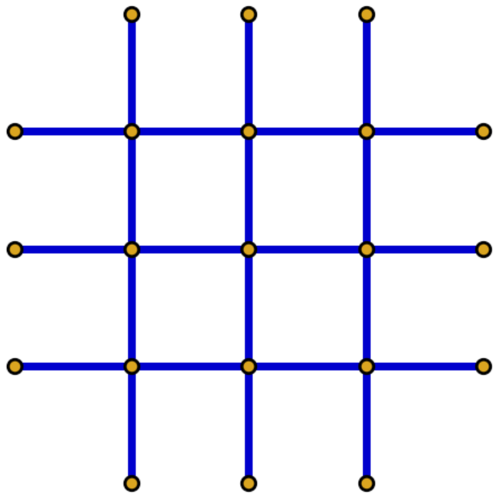

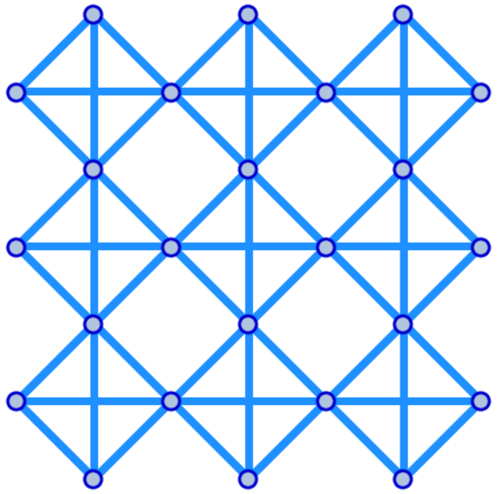

- Three Ingredients
  - Two commuting free-fermion models on the square lattice
  - Set of stabilizers

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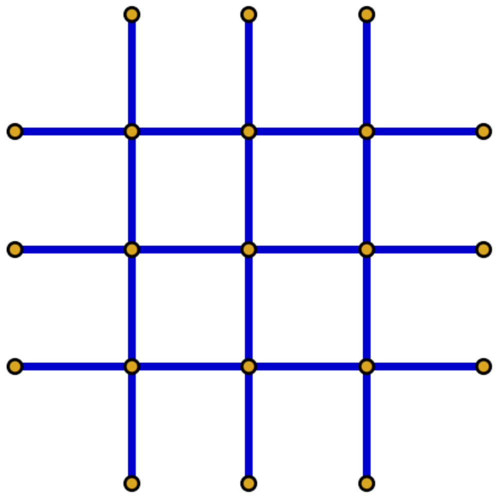

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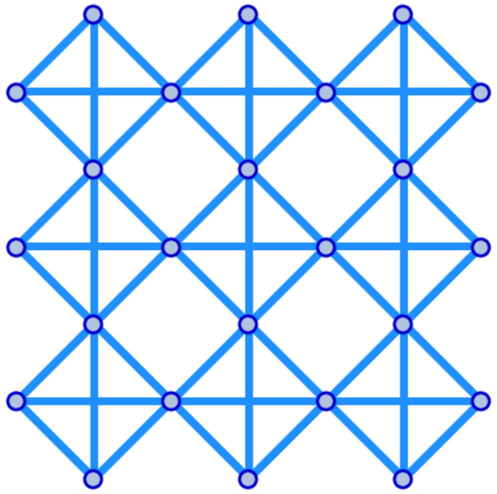
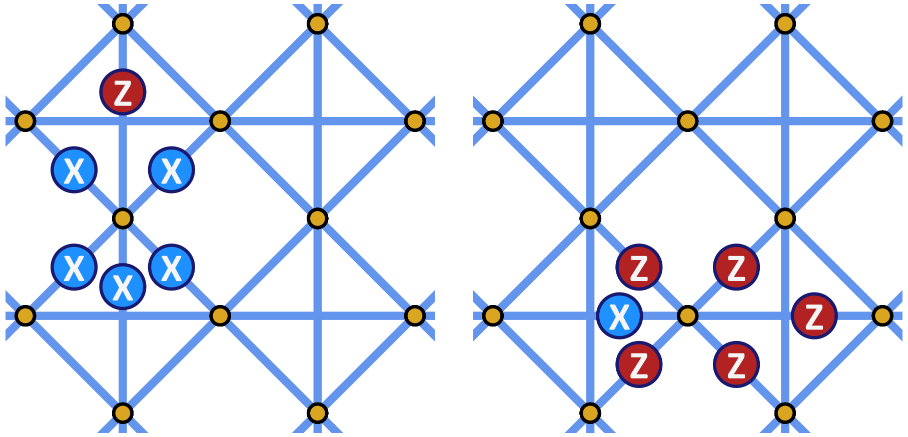


- Three Ingredients
  - Two commuting free-fermion models on the square lattice
  - Set of stabilizers

Free-Fermion 1



Chapman, Flammia, AJK, PRX Quantum **3**, 030321 (2022)

Line-Graph Subsystem Codes

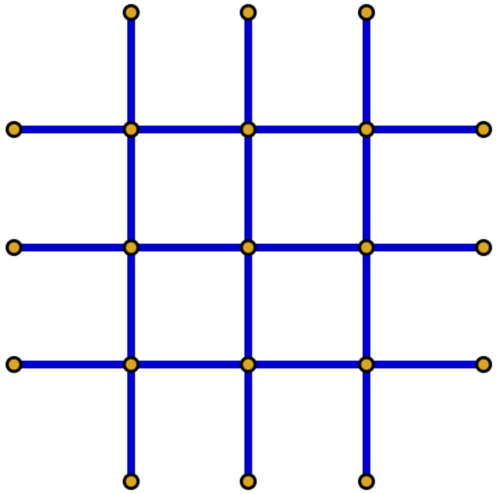
Thm: (Chapman and Flammia)
A spin model can be solved exactly by mapping to free fermions if and only the anticommutation relations of its terms have the structure of a line graph.

Chapman *et al*. Quantum **4**, 278 (2020)

The Checkerboard-Lattice Code

- Built on the square lattice

Fermion Lattice



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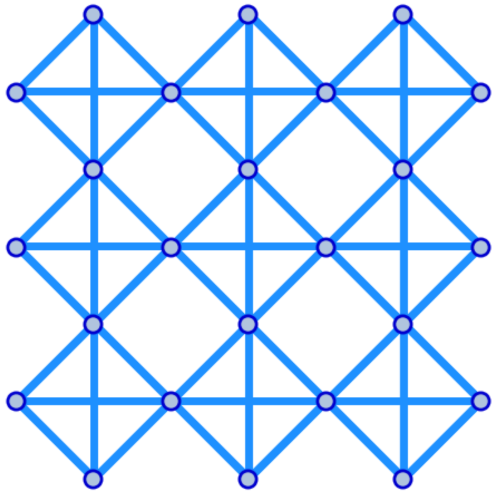

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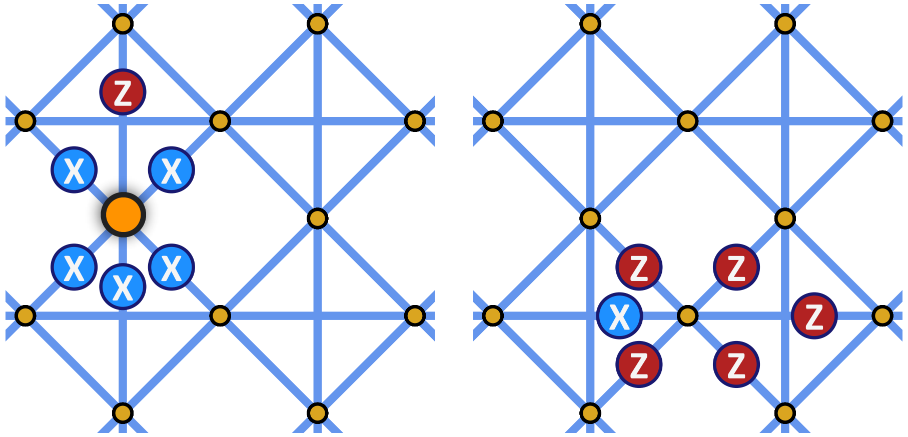

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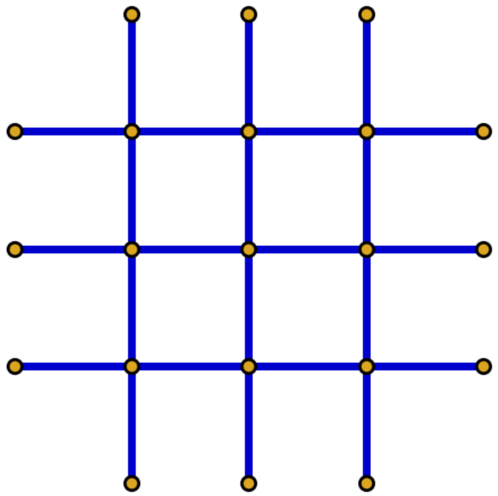

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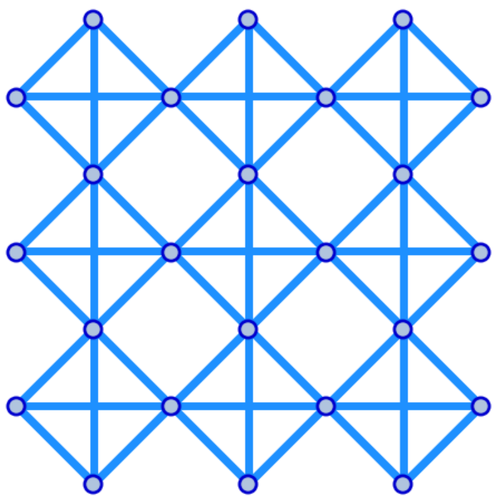

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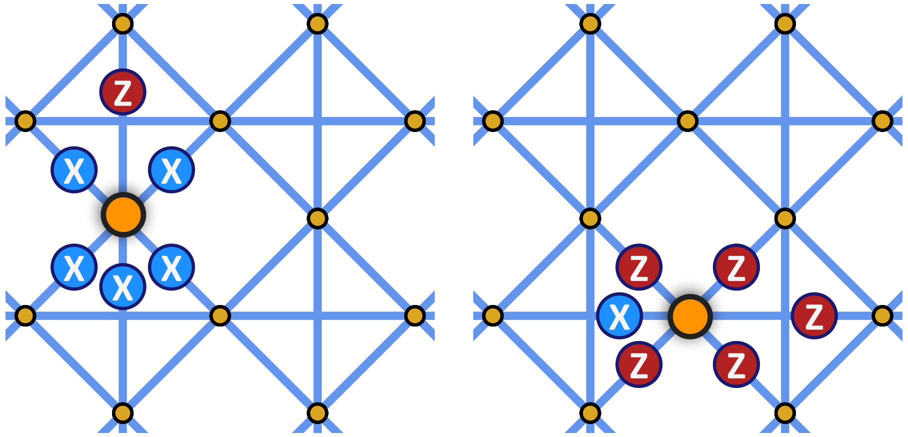

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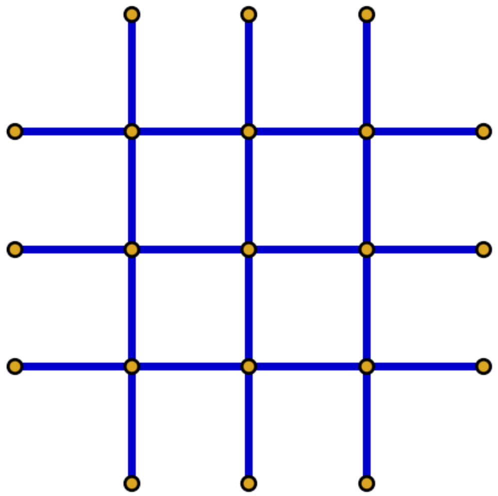

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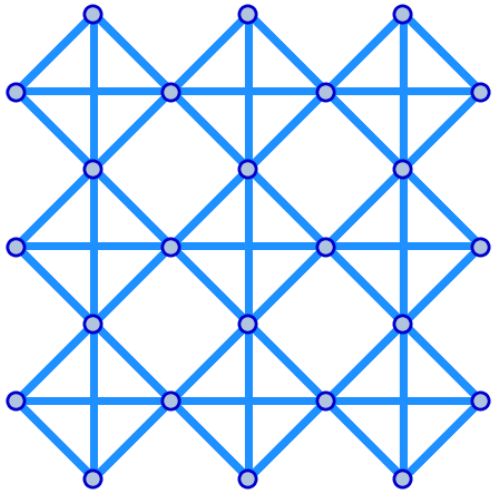

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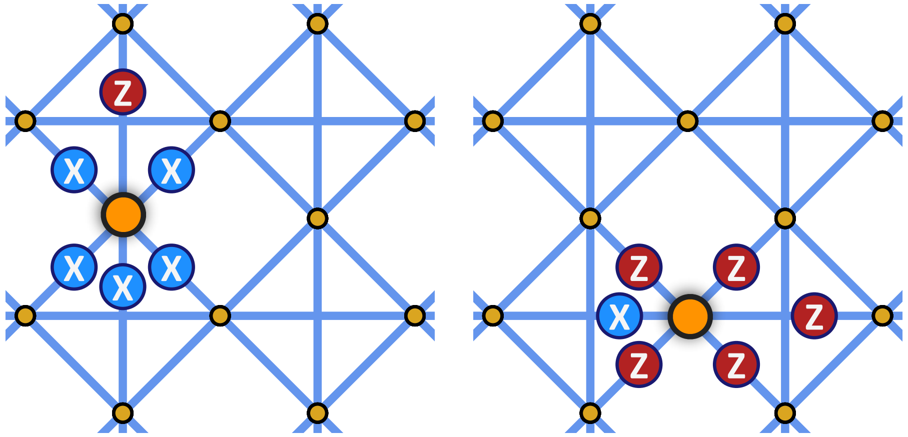
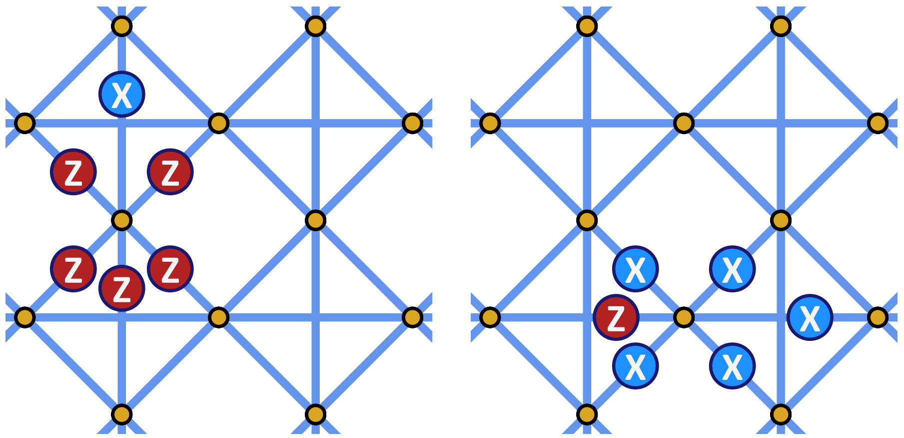


Free-Fermion 2



Chapman, Flammia, AJK, PRX Quantum **3**, 030321 (2022)

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Chapman *et al*. Quantum **4**, 278 (2020)

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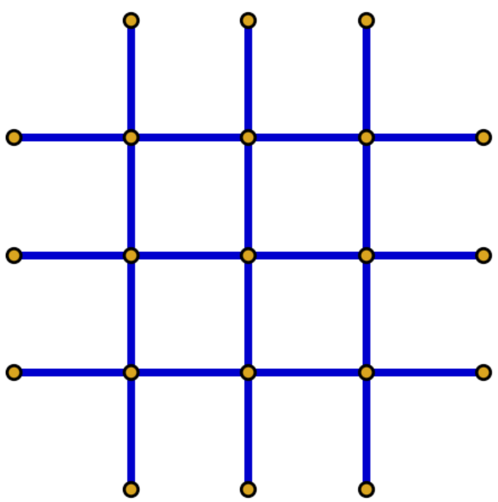

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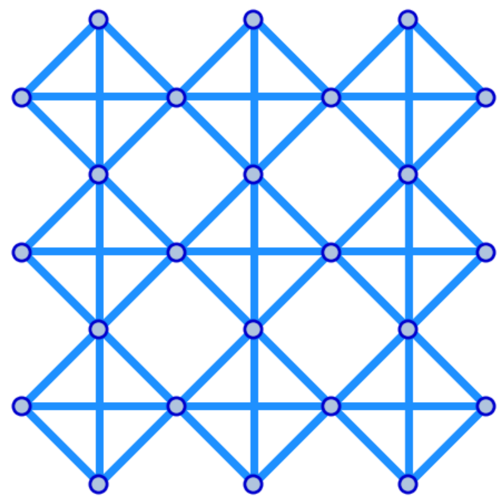

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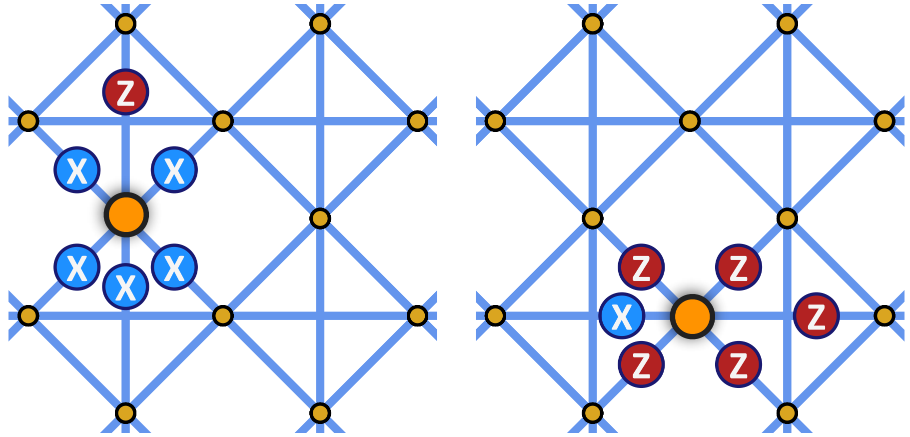

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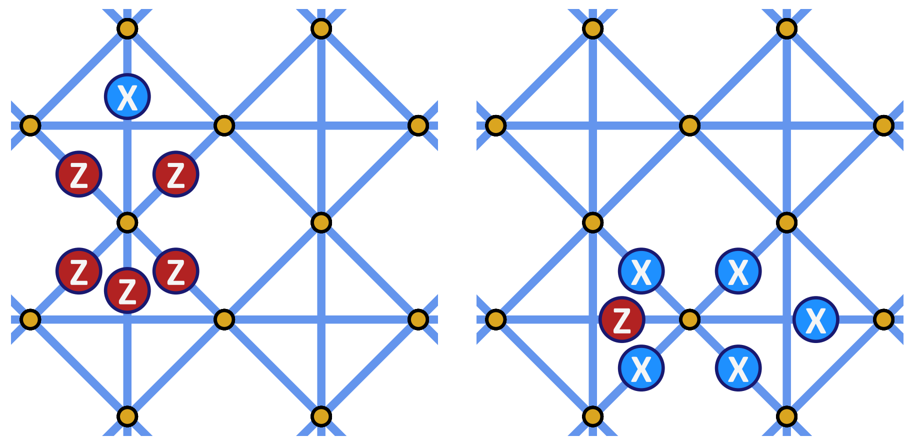
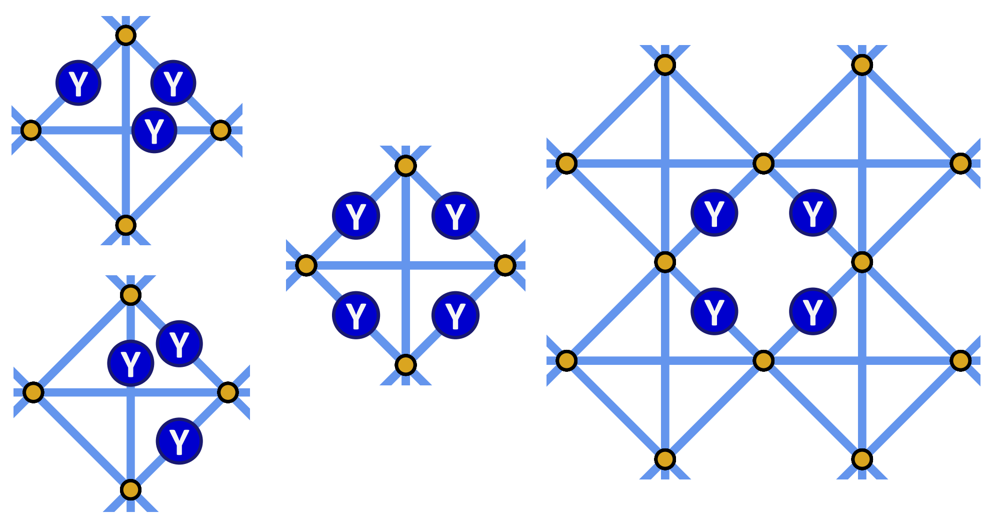


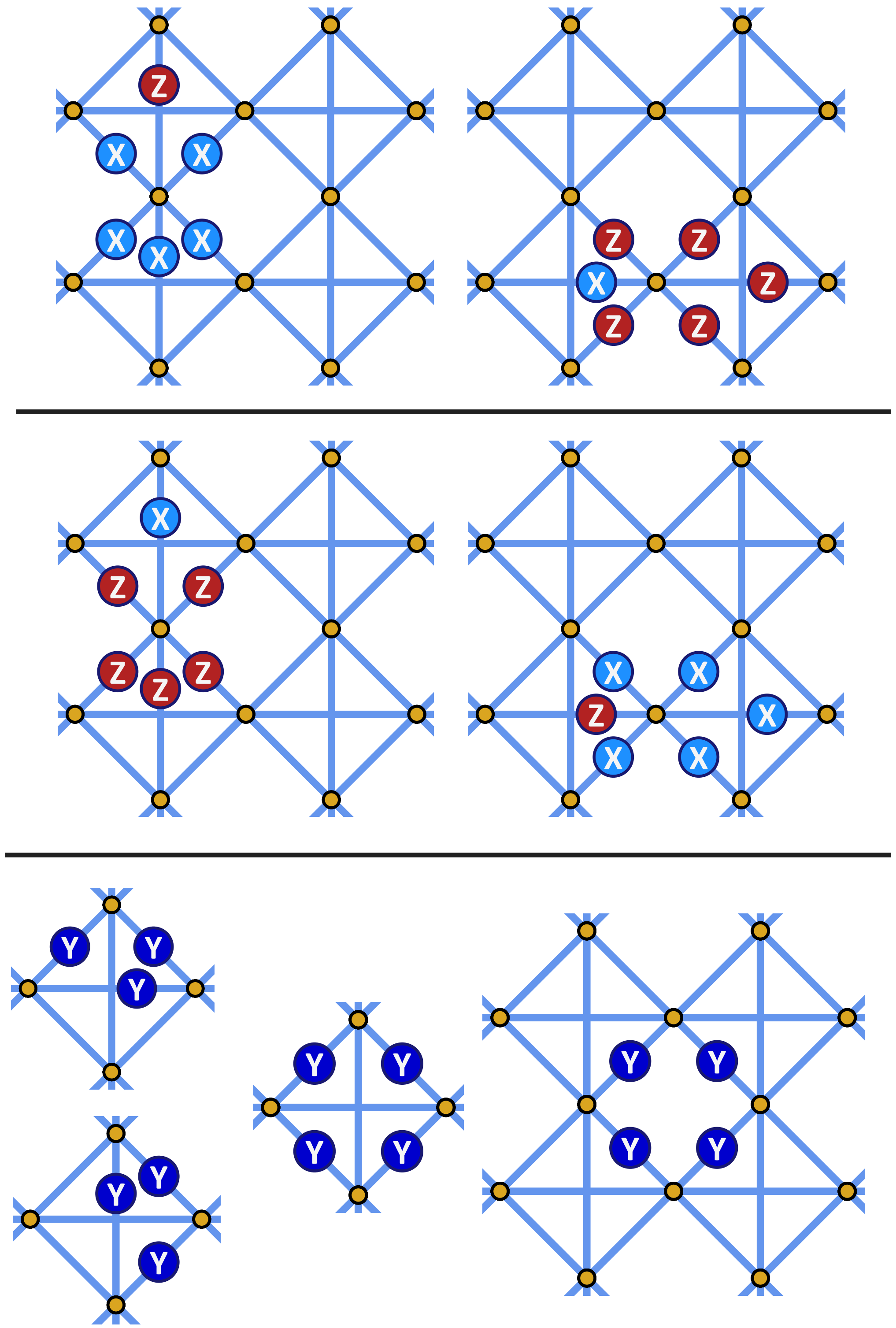
Stabilizers



Chapman, Flammia, AJK, PRX Quantum **3**, 030321 (2022)

Checkerboard Lattice Code

Hamiltonian Terms/Gauge Generators



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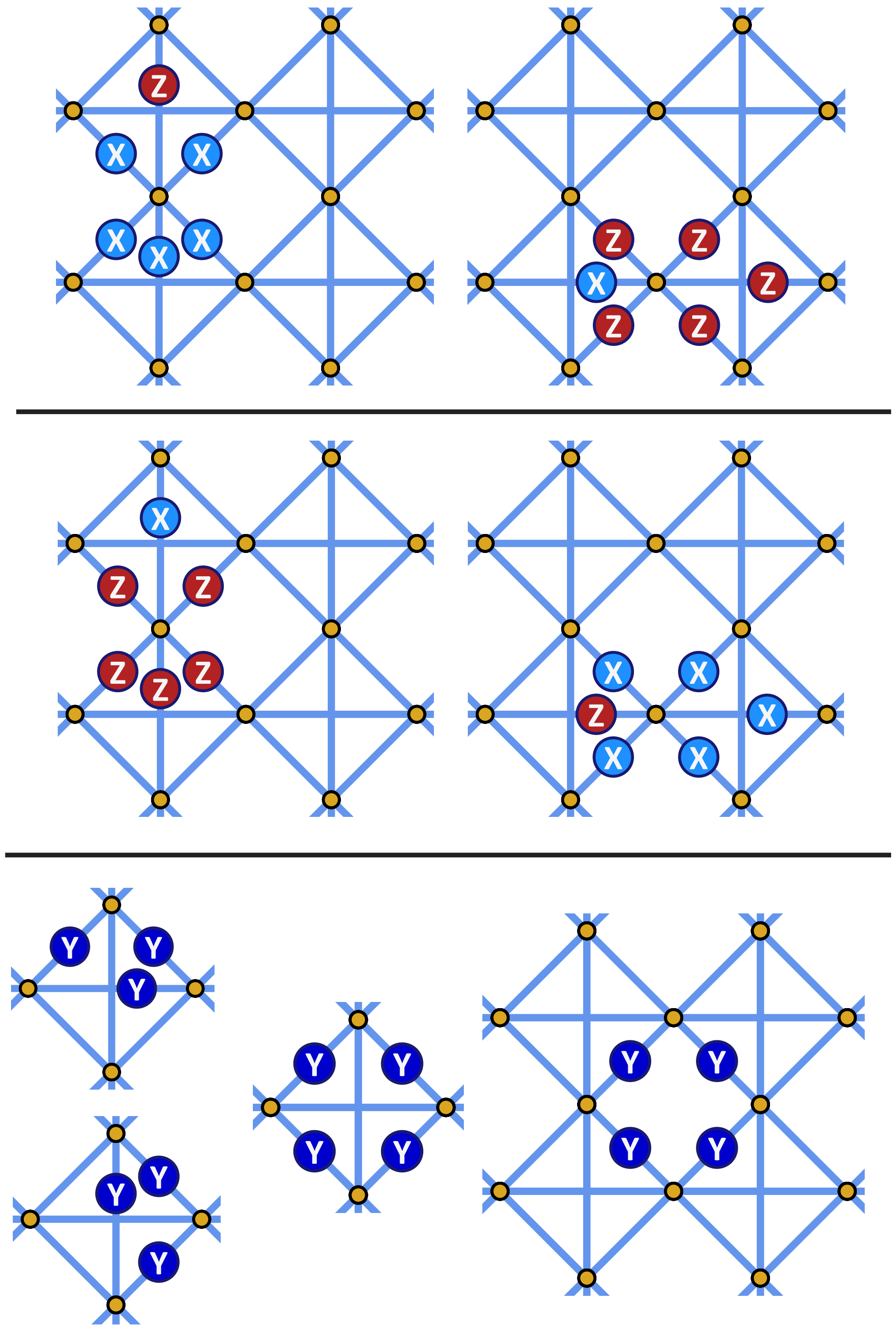
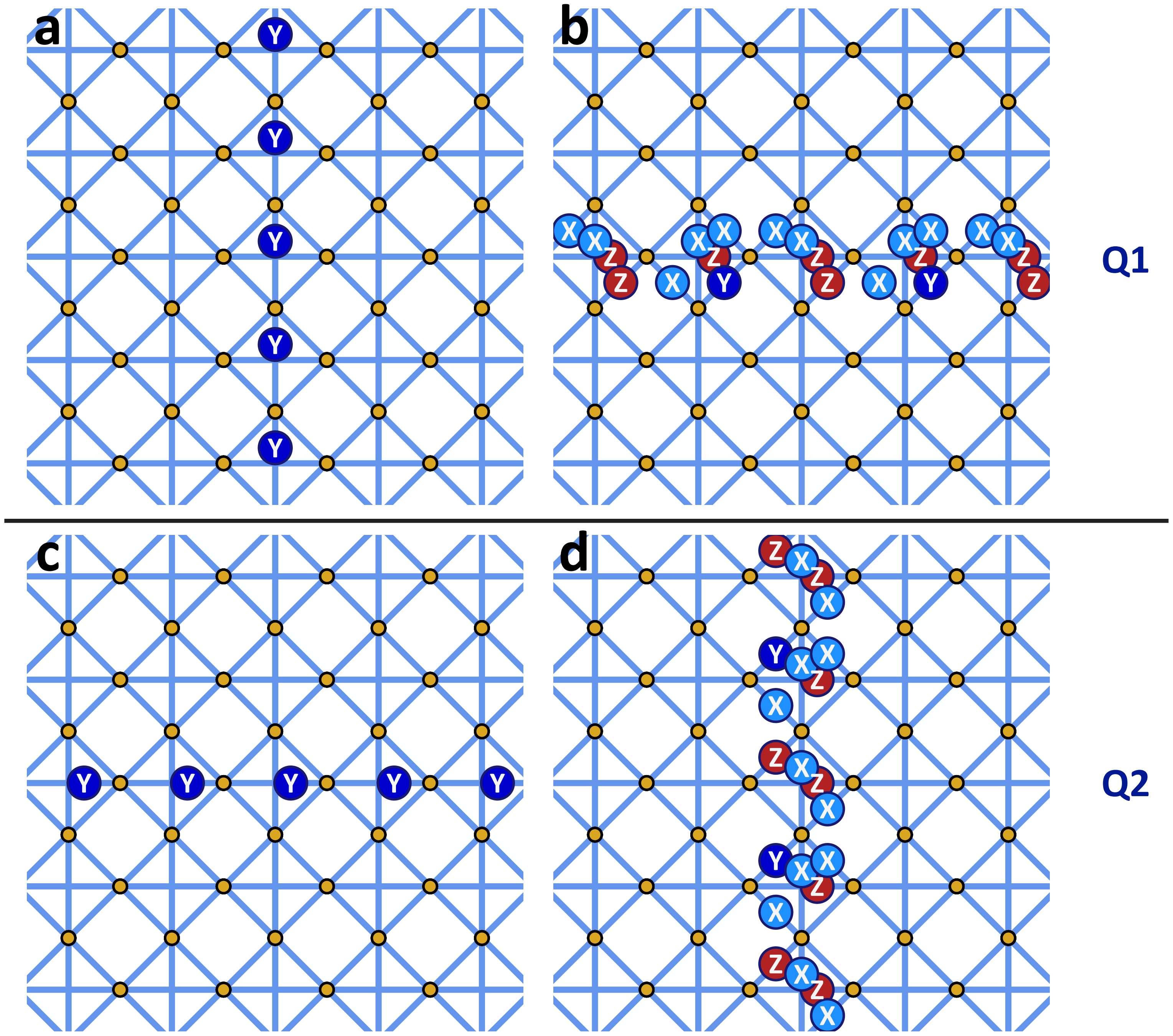


Exact Logical Qubits



Chapman, Flammia, AJK, PRX Quantum **3**, 030321 (2022)

Outline

- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
  - Line-graph lattices
  - Interacting photons
- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
- Mathematical Connections
  - Planar Gaps
  - Maximal Gaps
  - Quantum Error Correcting Codes
- Experimental Developments

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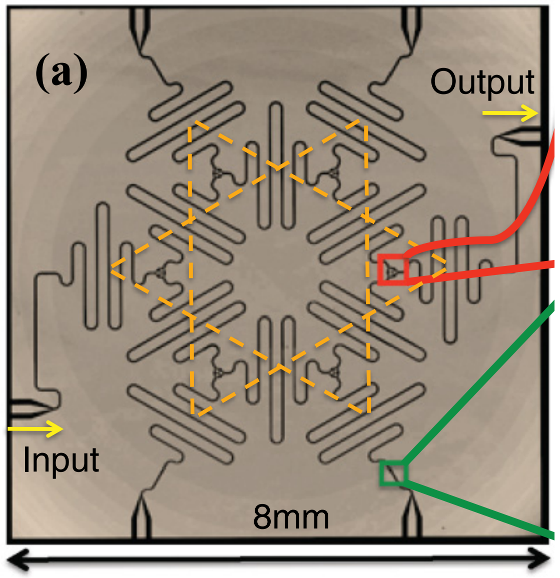
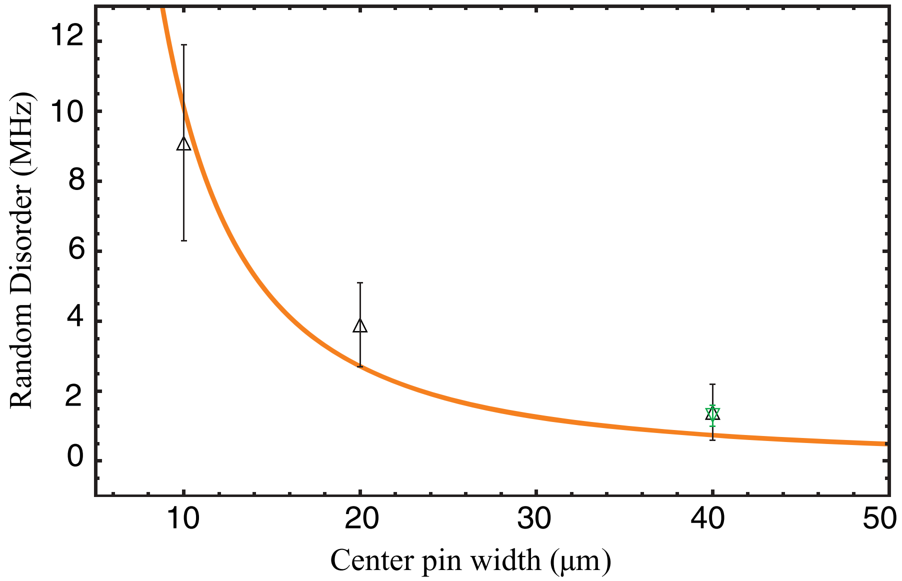
Intrinsic Fabrication Disorder

Previous Benchmarks

- Kagome star normal modes
- Fabricated at Princeton
- Fabrication disorder ~3e-4





Underwood *et al*. PRA **86**, 023837 (2012)

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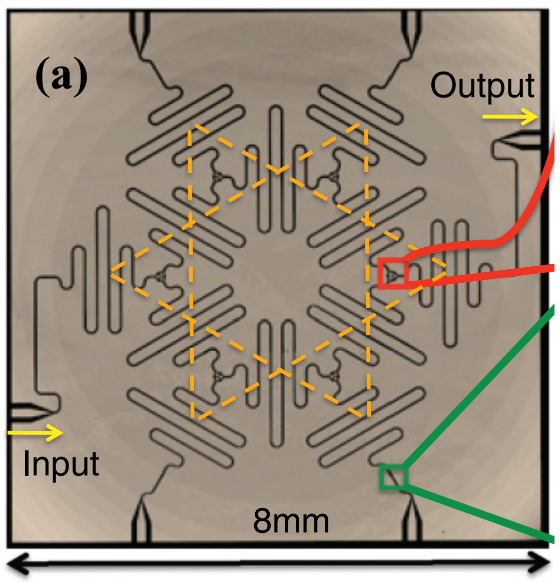

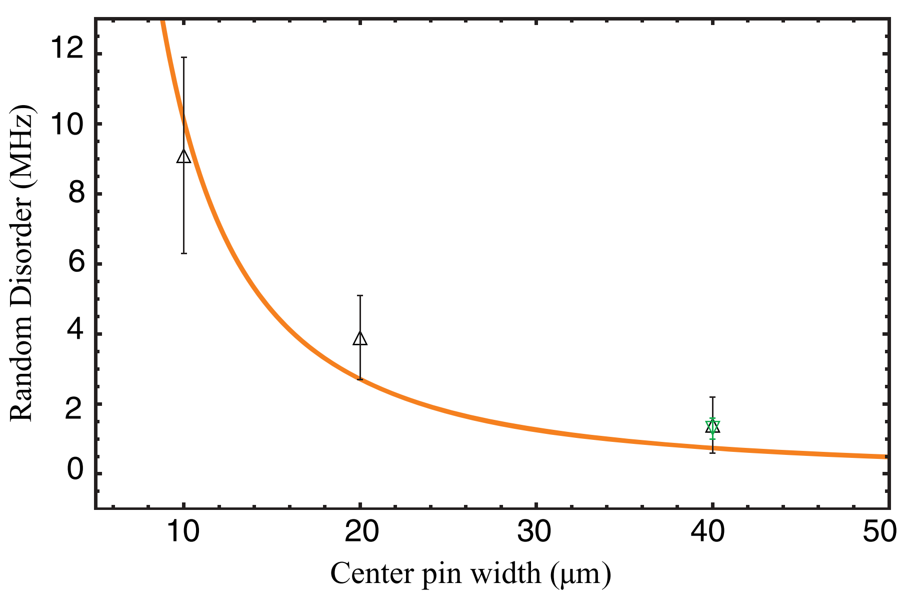

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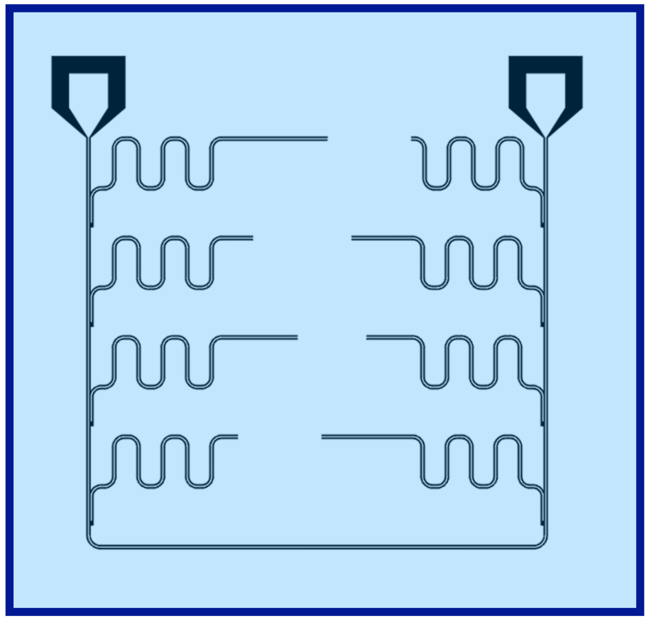

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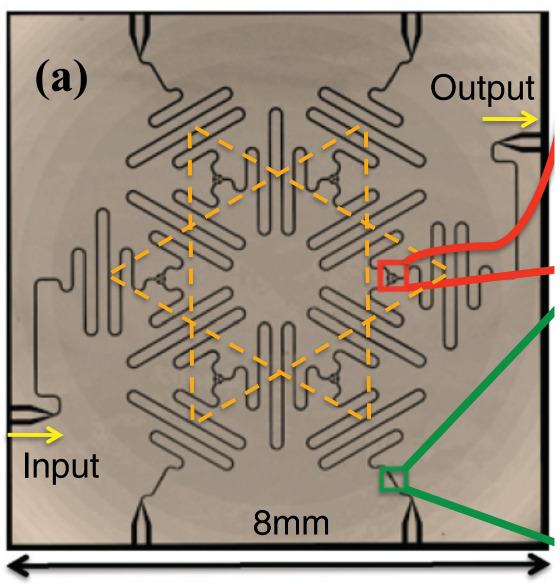

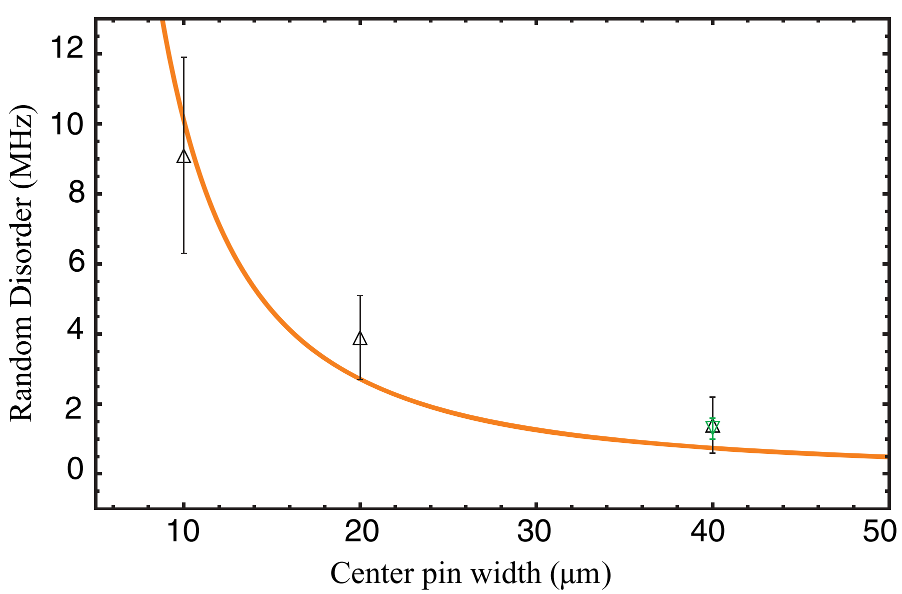

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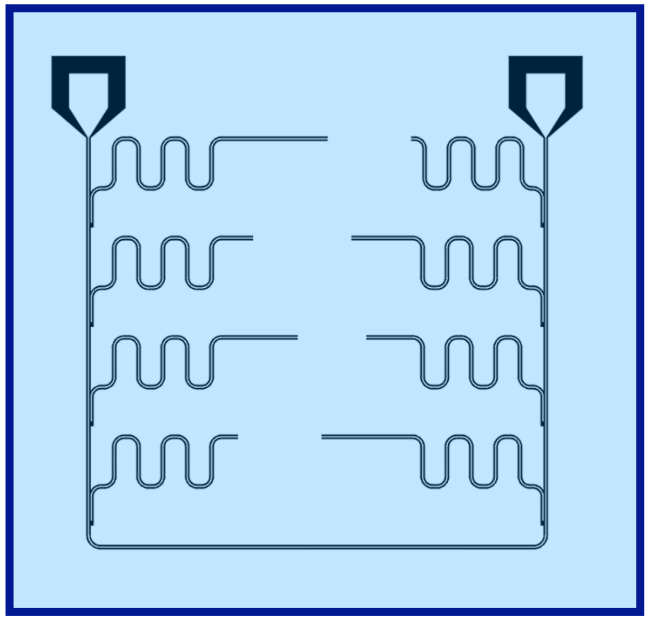

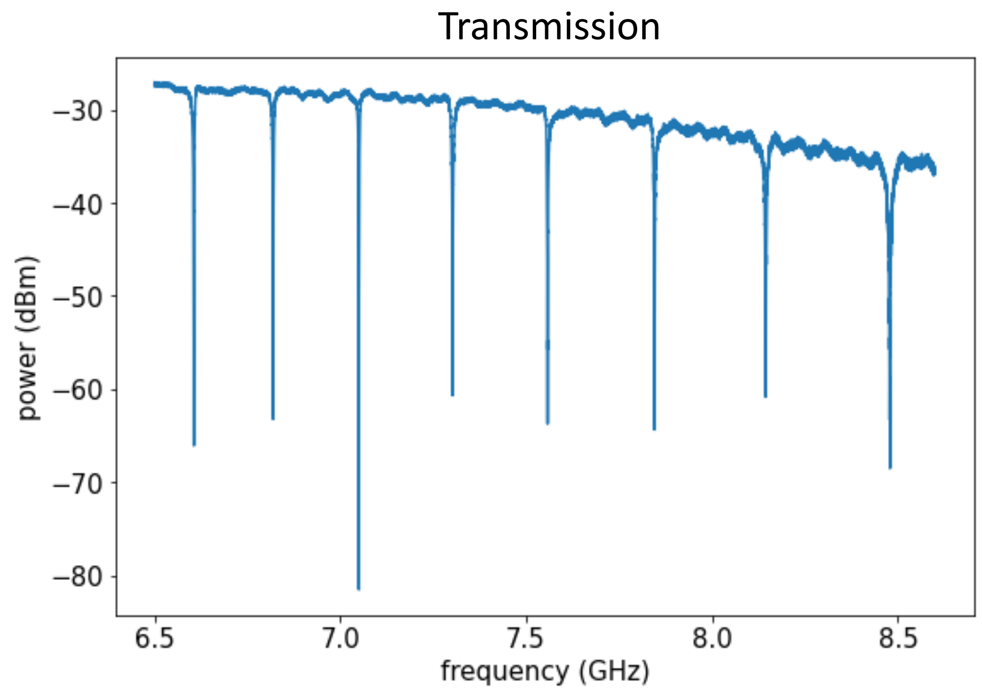

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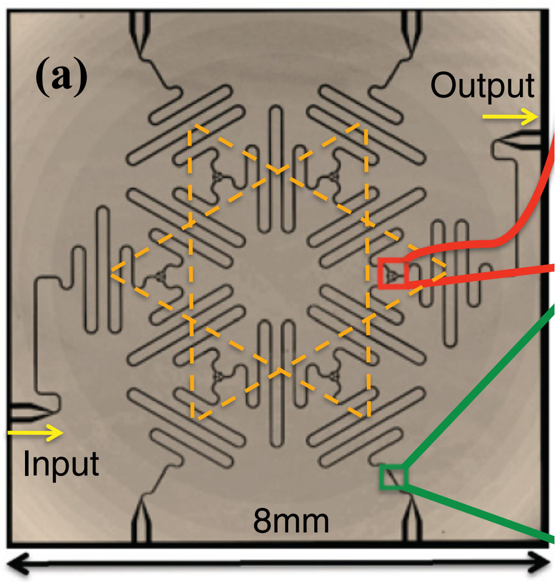

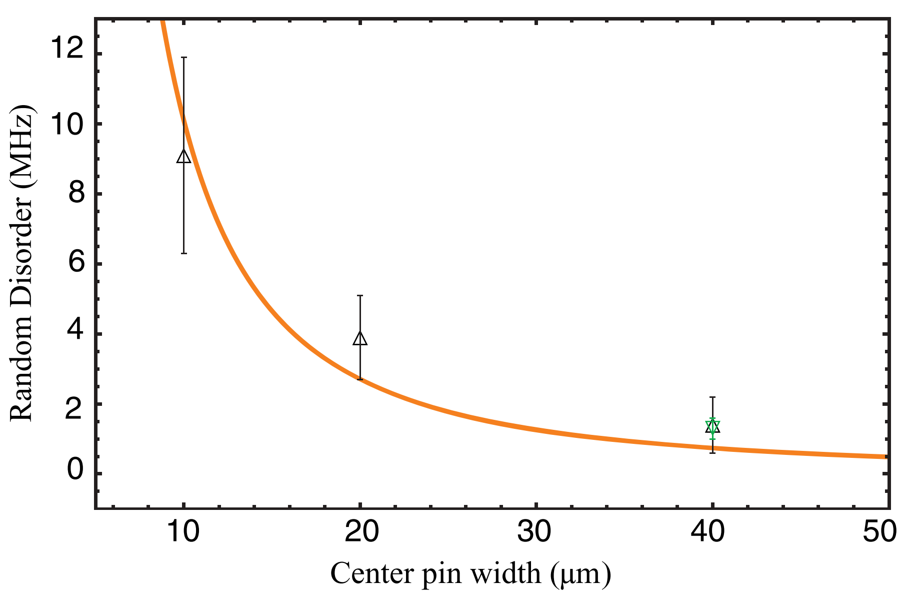

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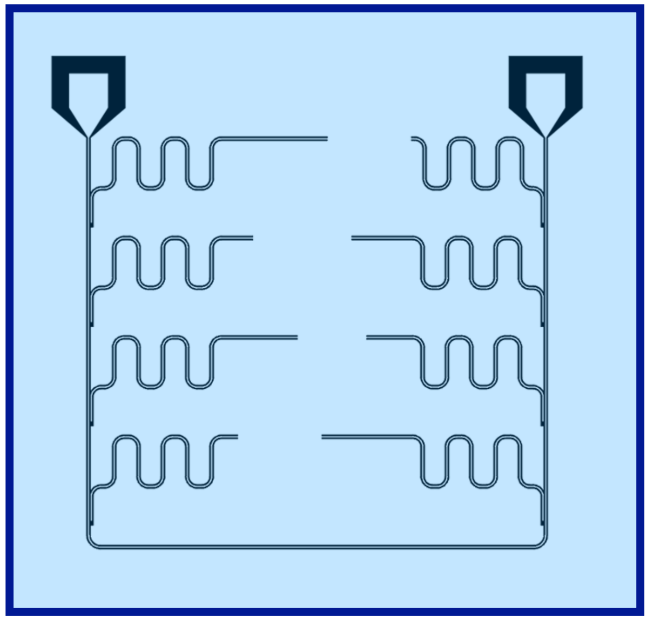

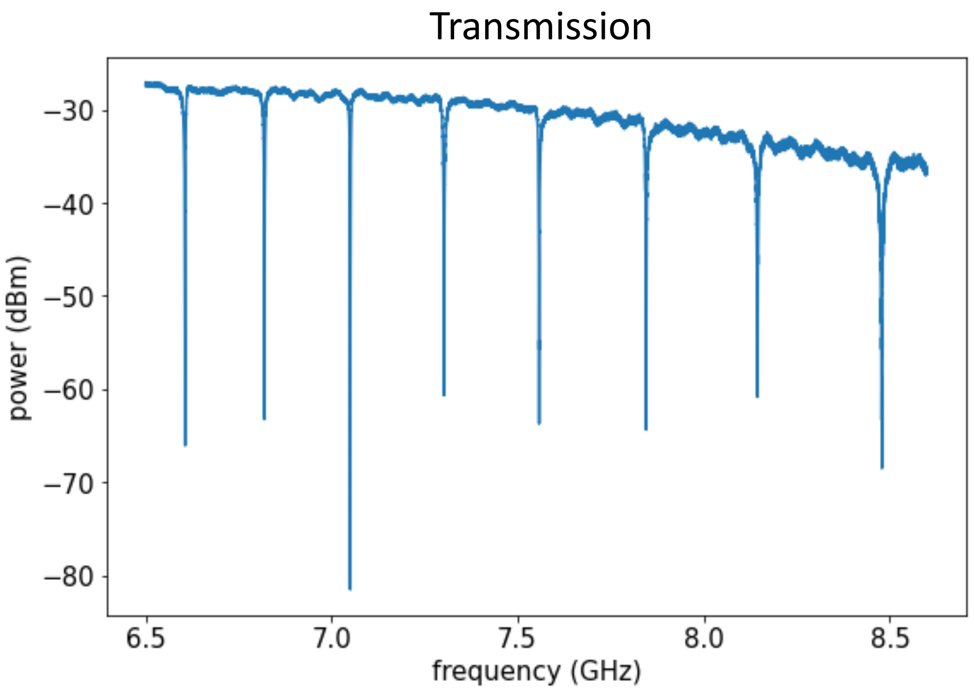

- Parallel measurement

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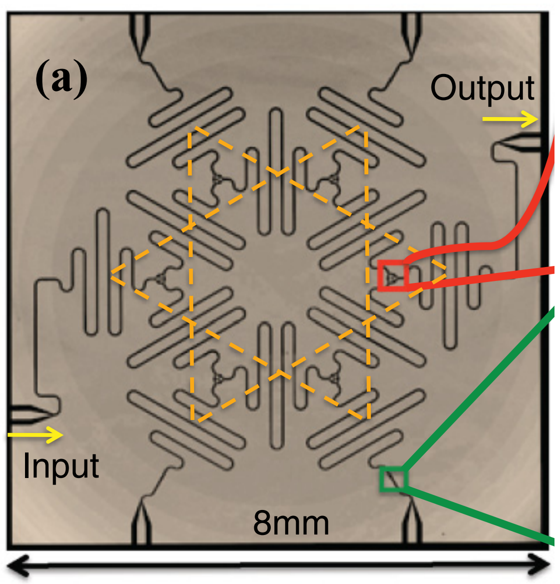

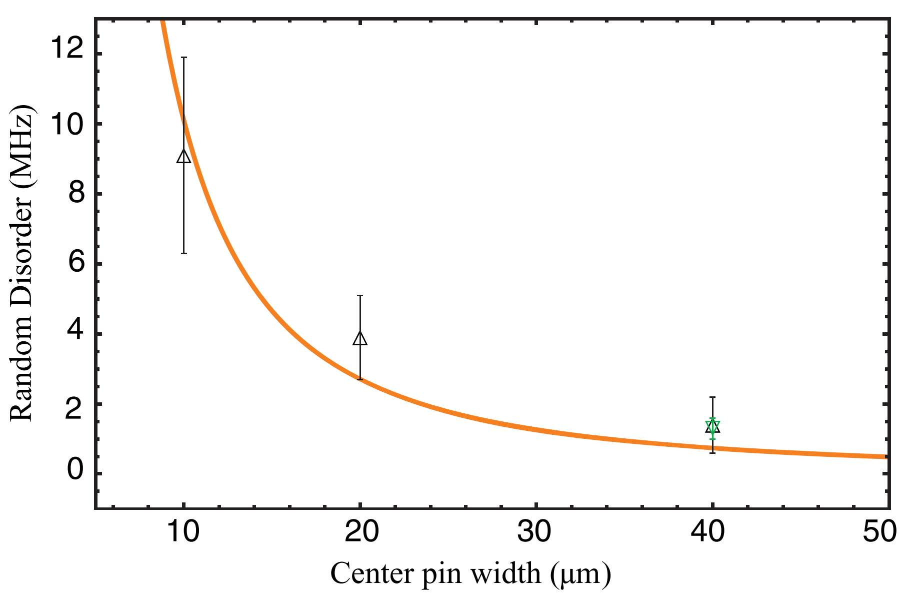

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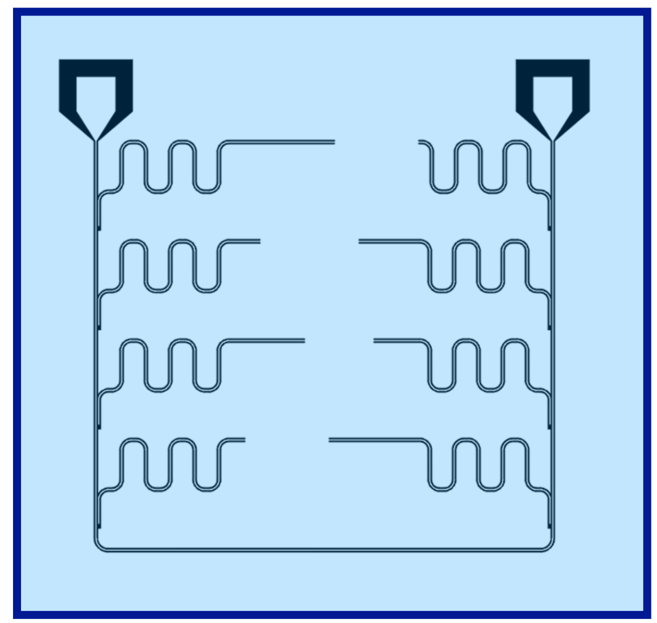

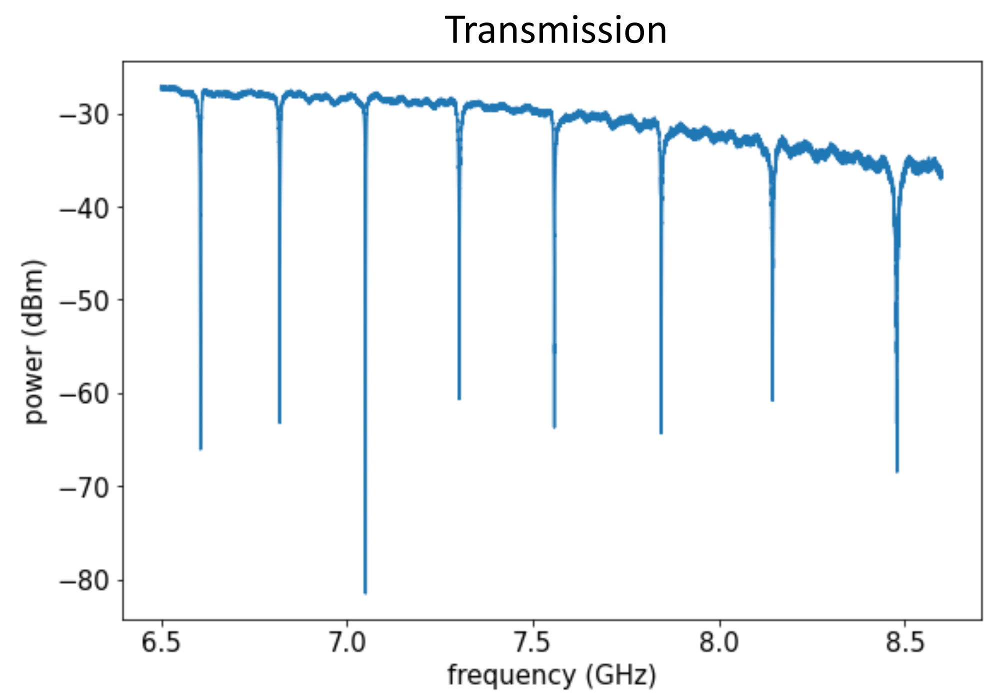

- Parallel measurement
- Disorder extracted from comb spacing

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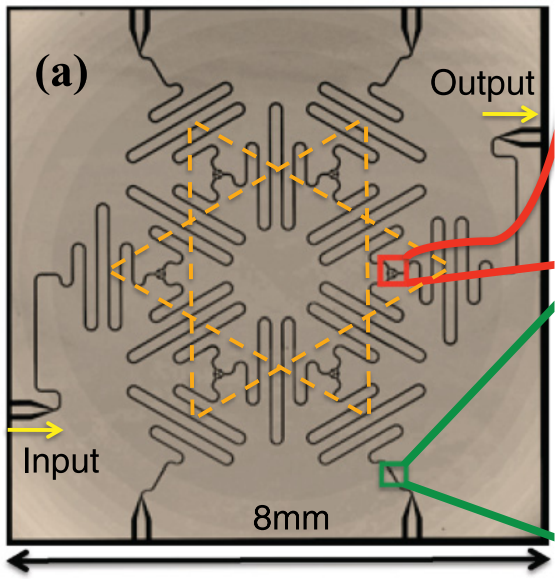

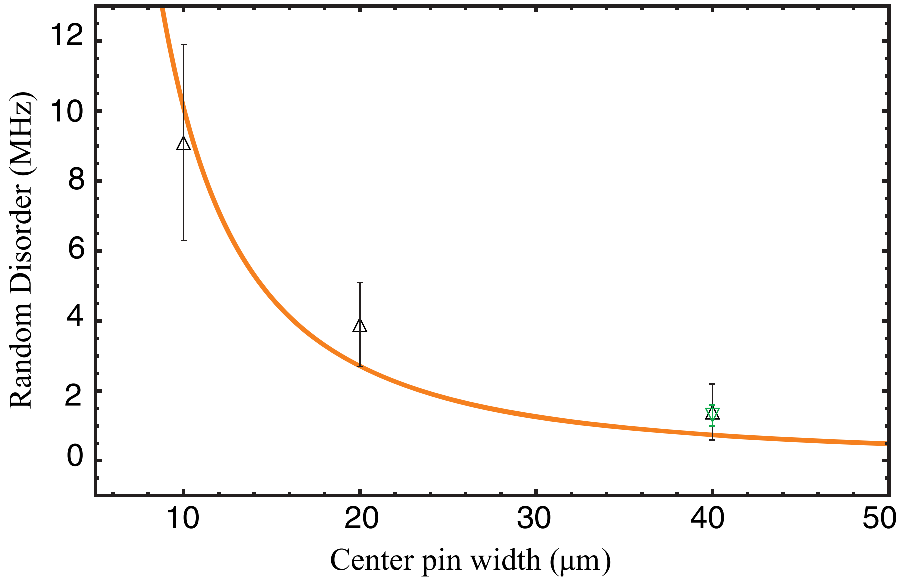

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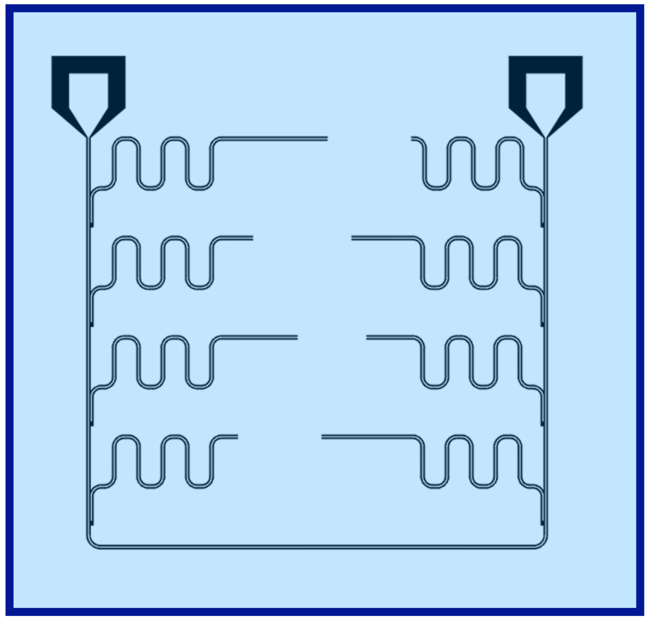

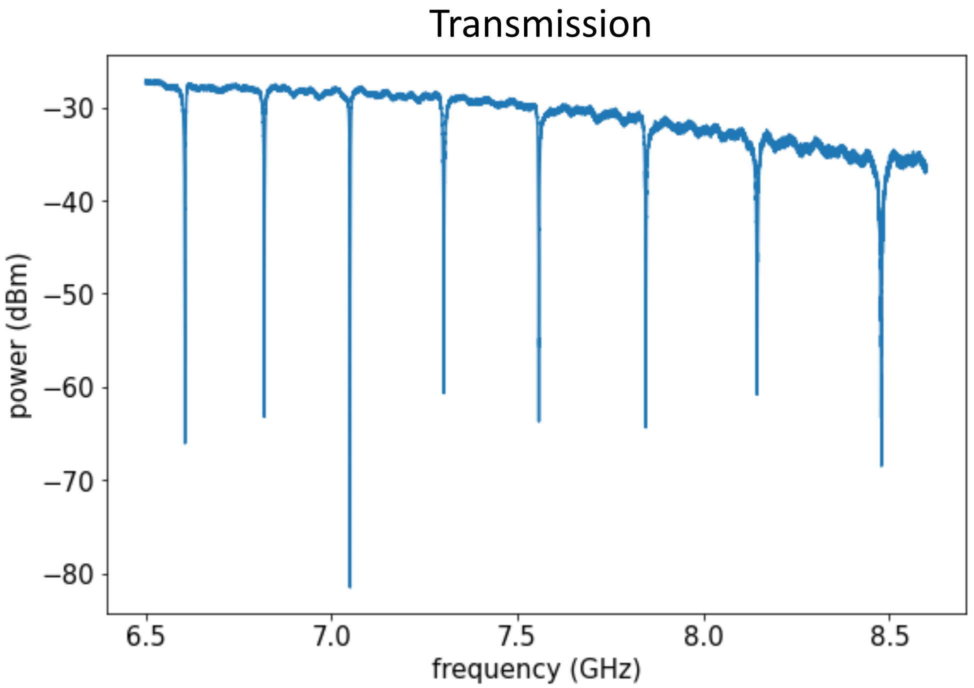

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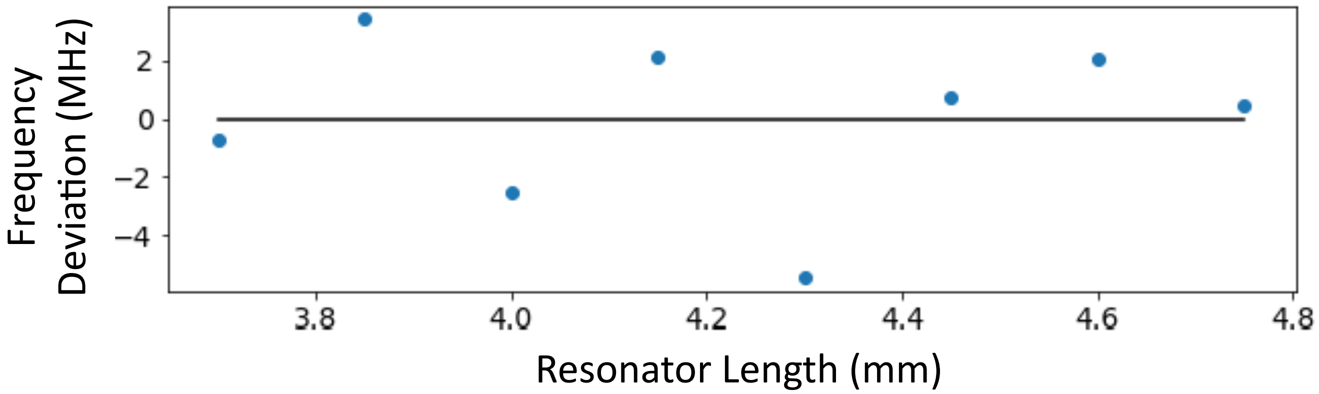

Disorder Mitigation

Systematic v. Random Disorder

- Fabrication disorder ~3e-4
- Shape-dependent disorder ~2-3e-3

Underwood *et al*. PRA **86**, 023837 (2012)   Kollár *et al*. Nature **571** (2019)

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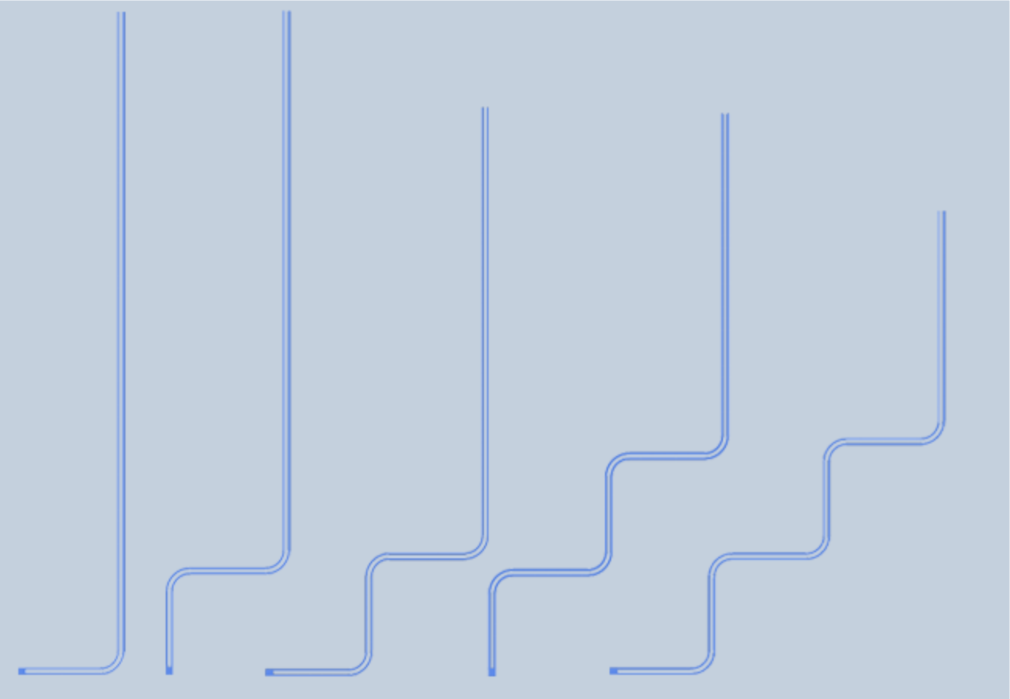

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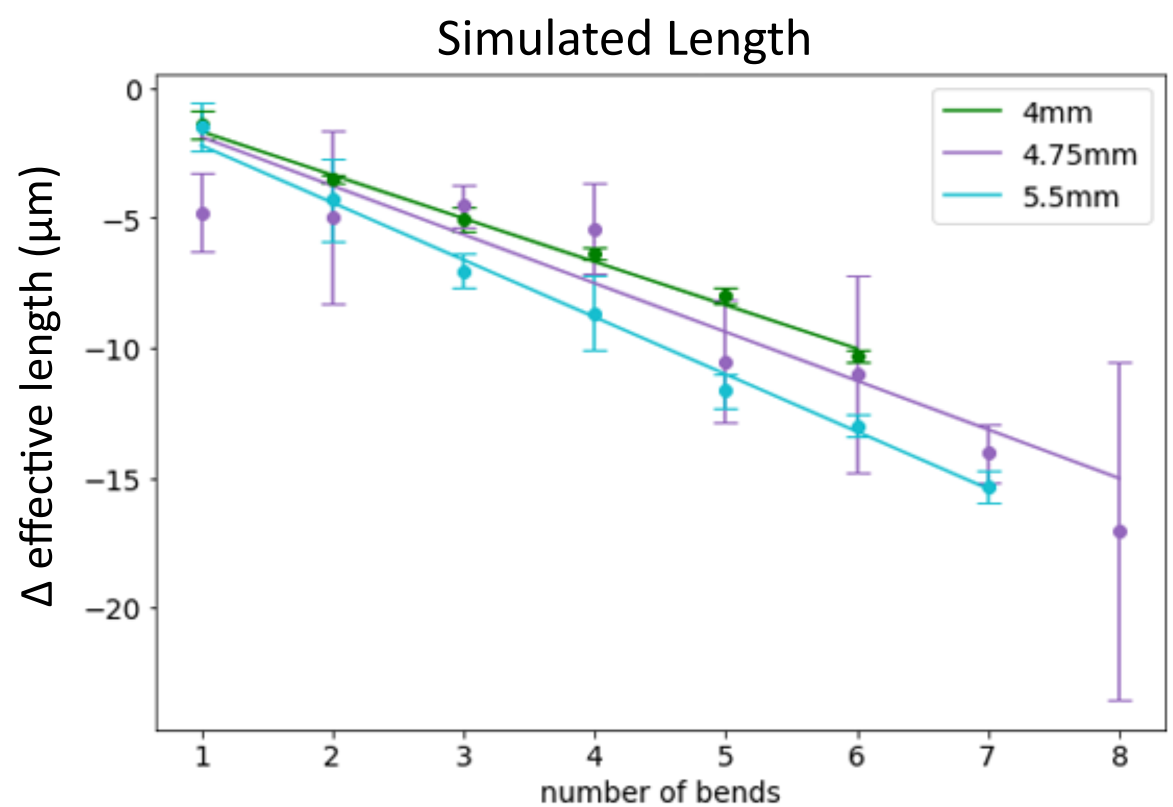

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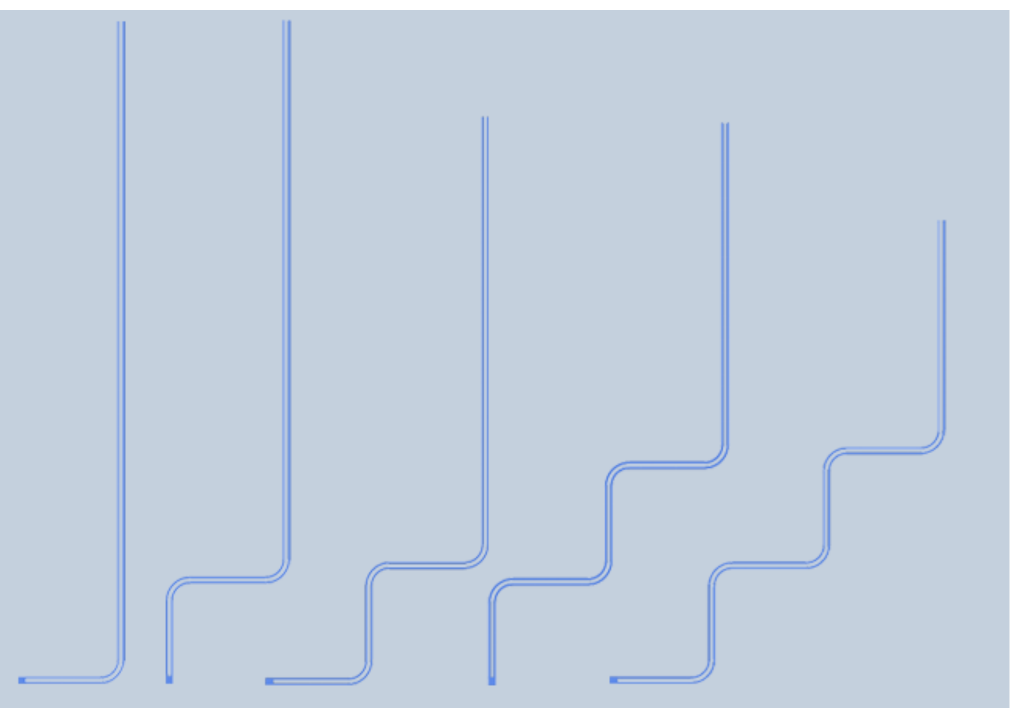

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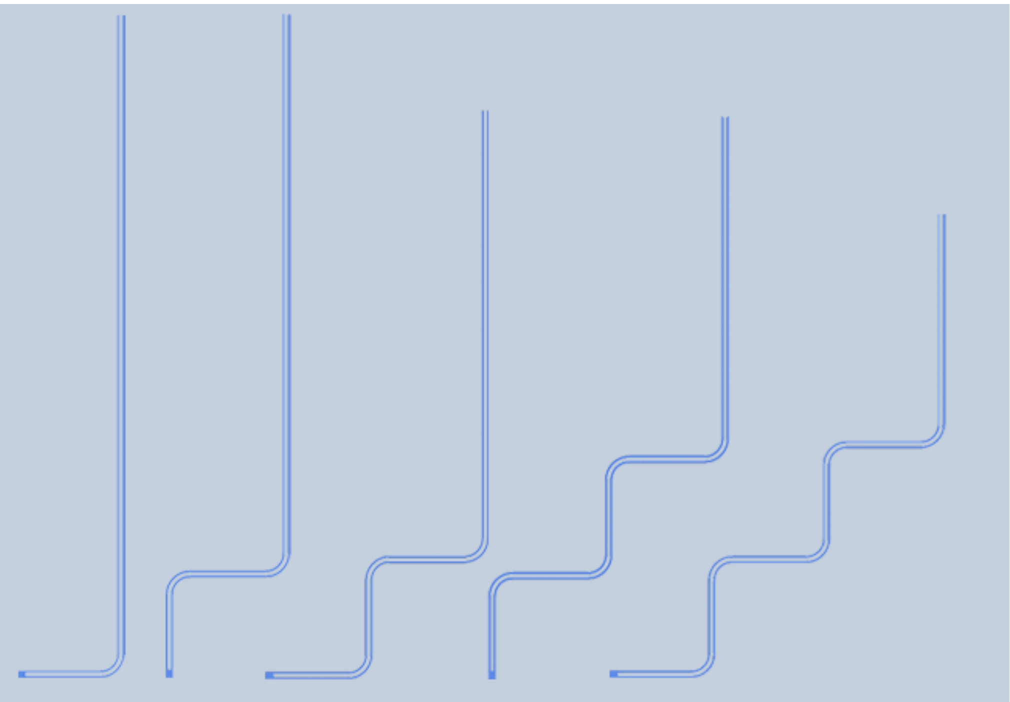

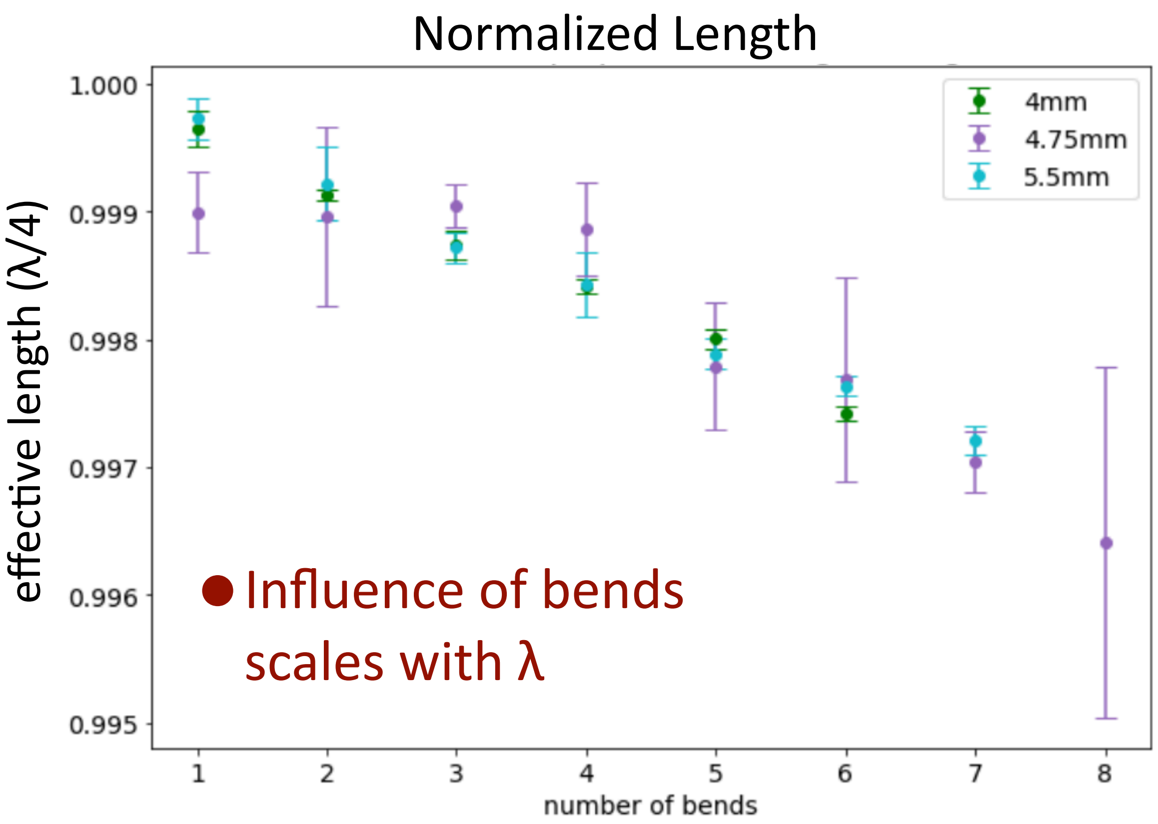

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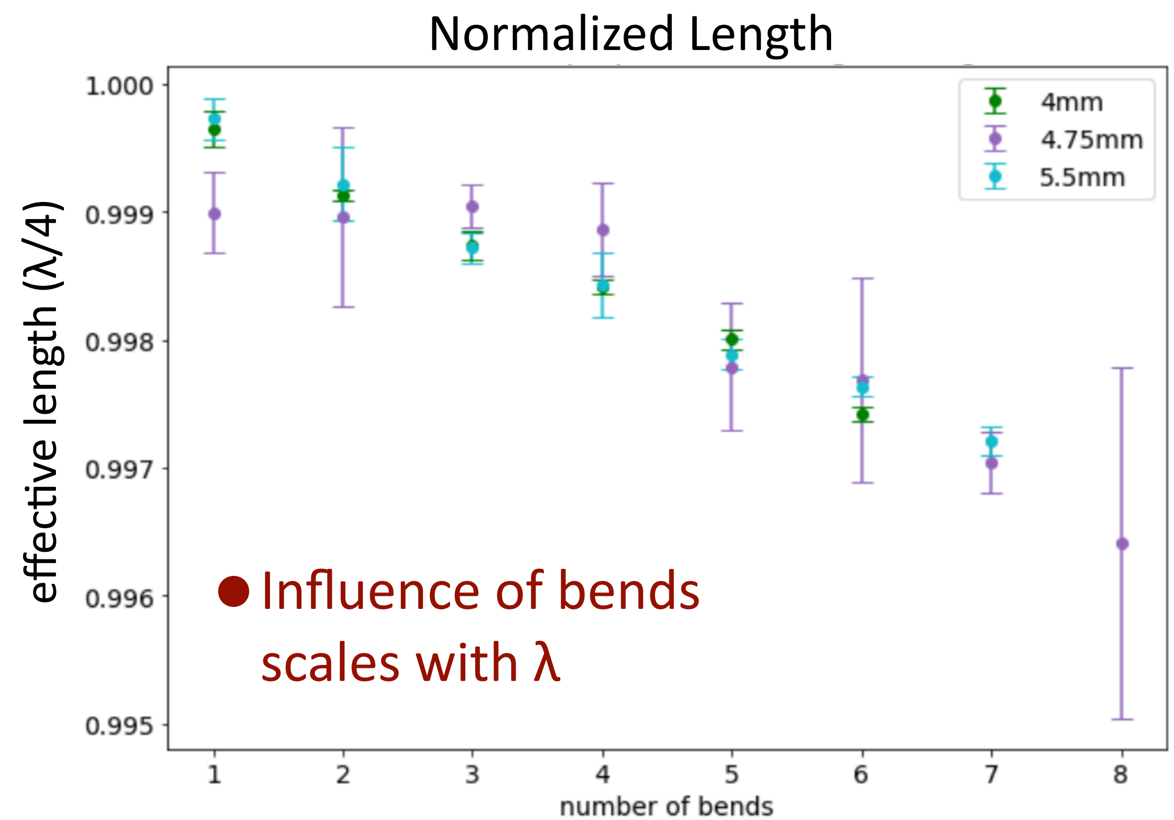

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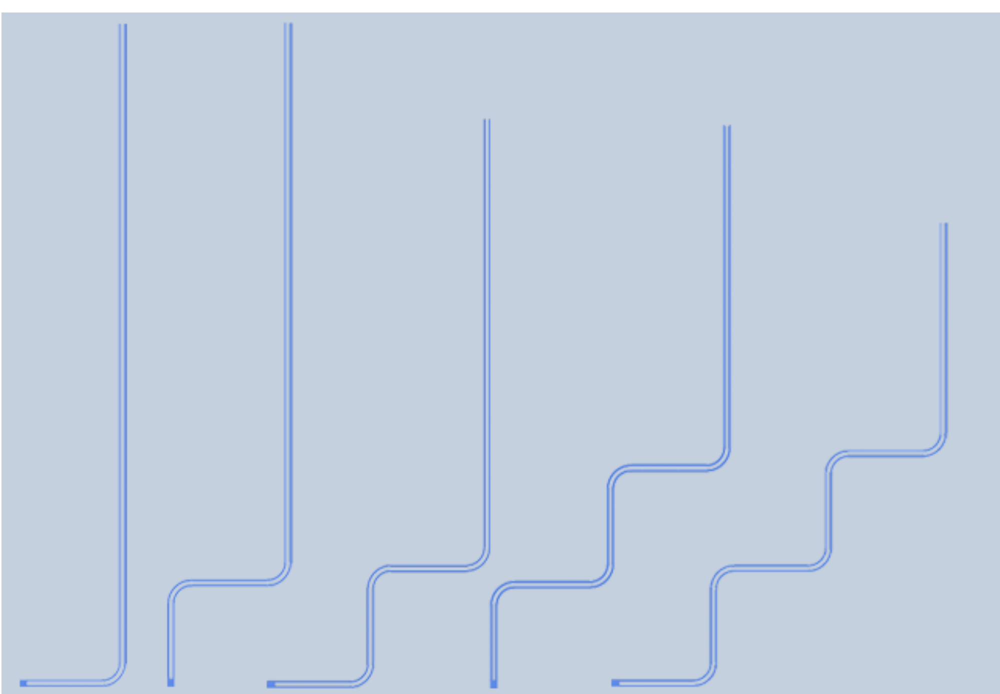

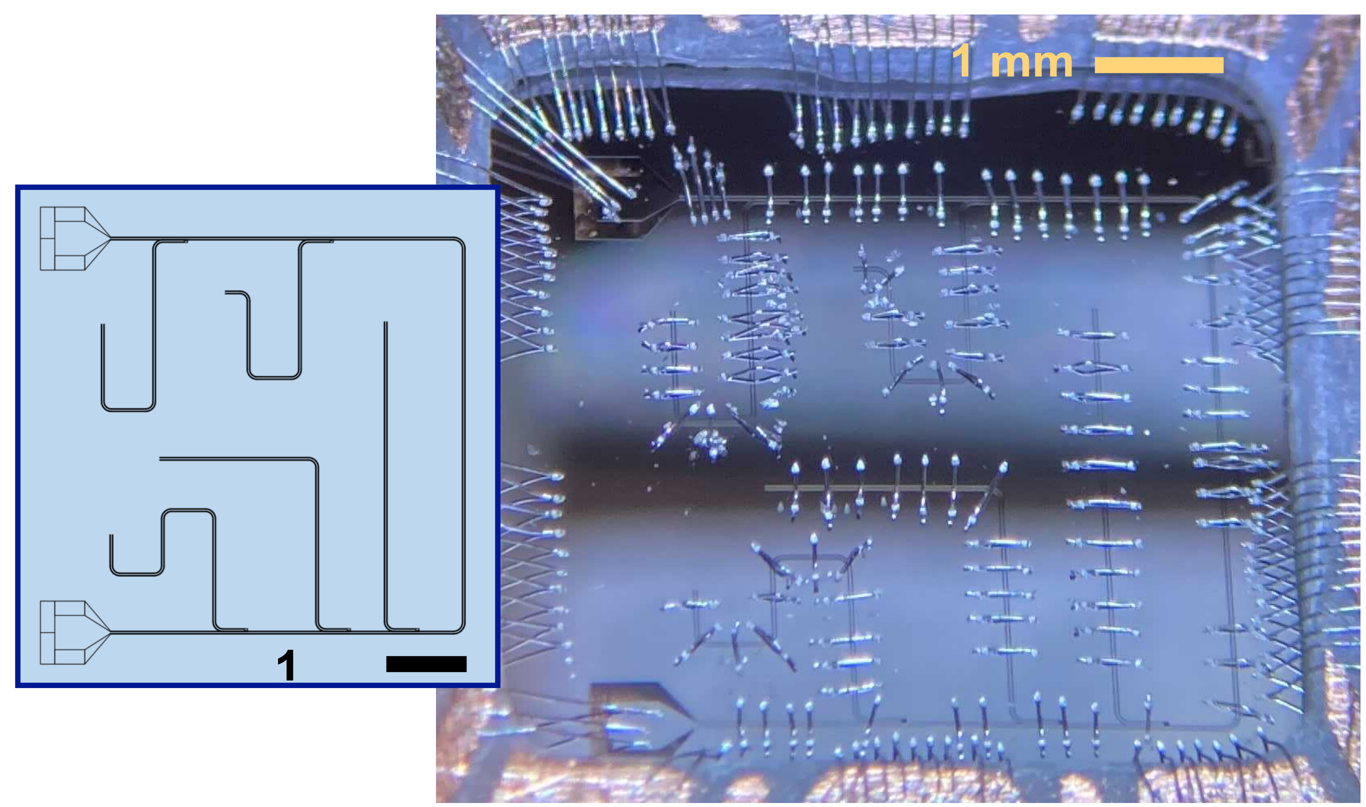

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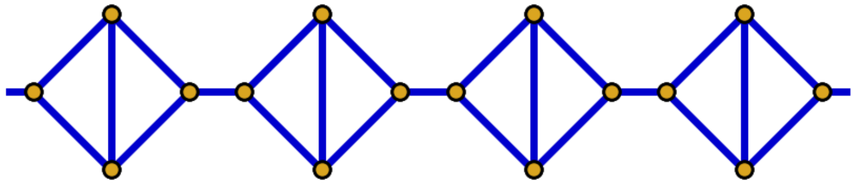
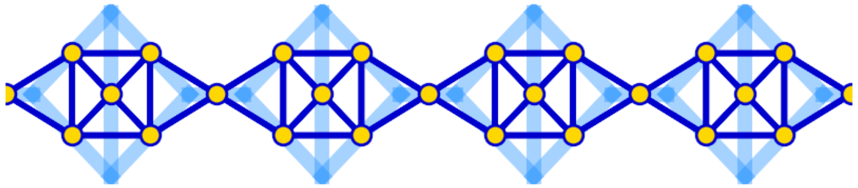
Quasi-1D Lattice Device

Hardware Layout



Effective Lattice



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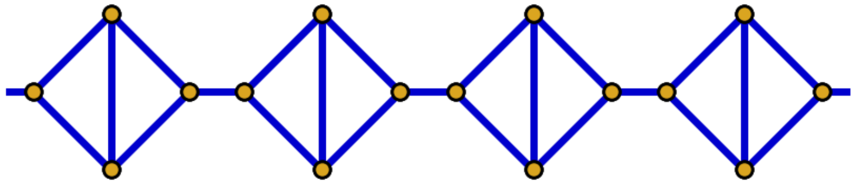

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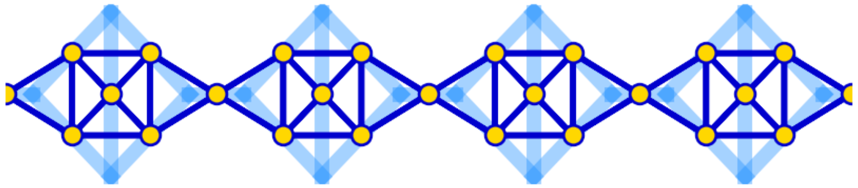
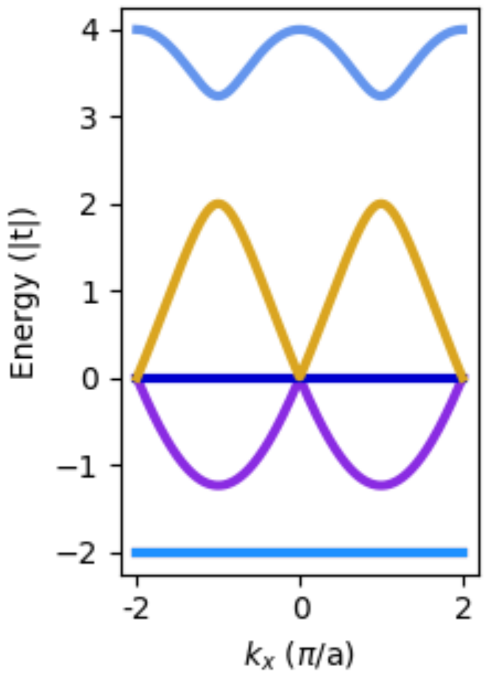


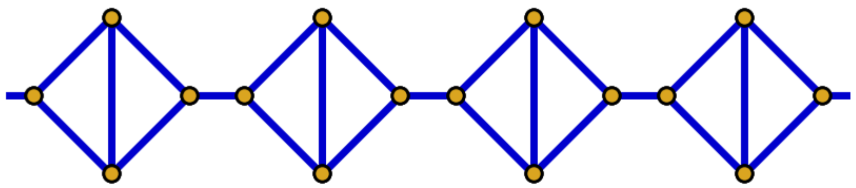
Band Structure



- Flat bands
  - Gapped
  - Ungapped
- Linear bands
- Quadratic bands

Quasi-1D Lattice Device

Hardware Layout



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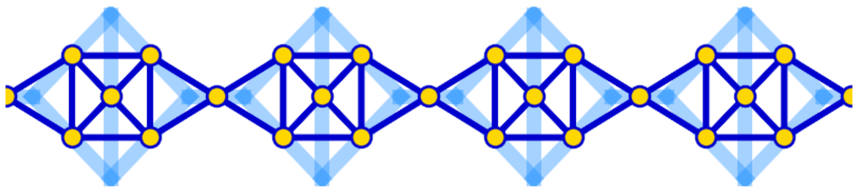
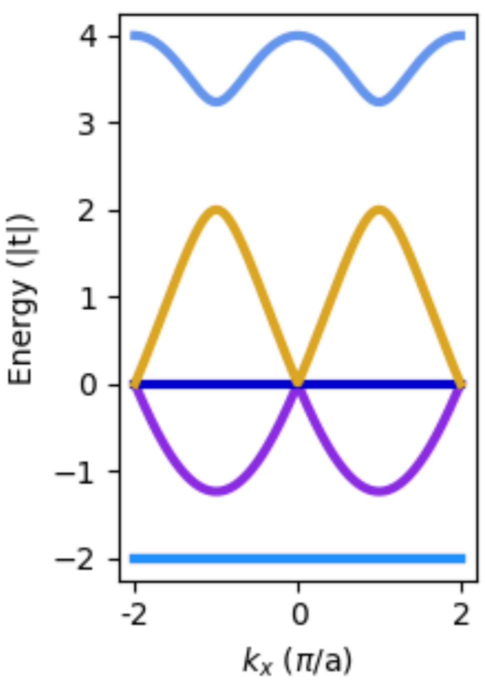


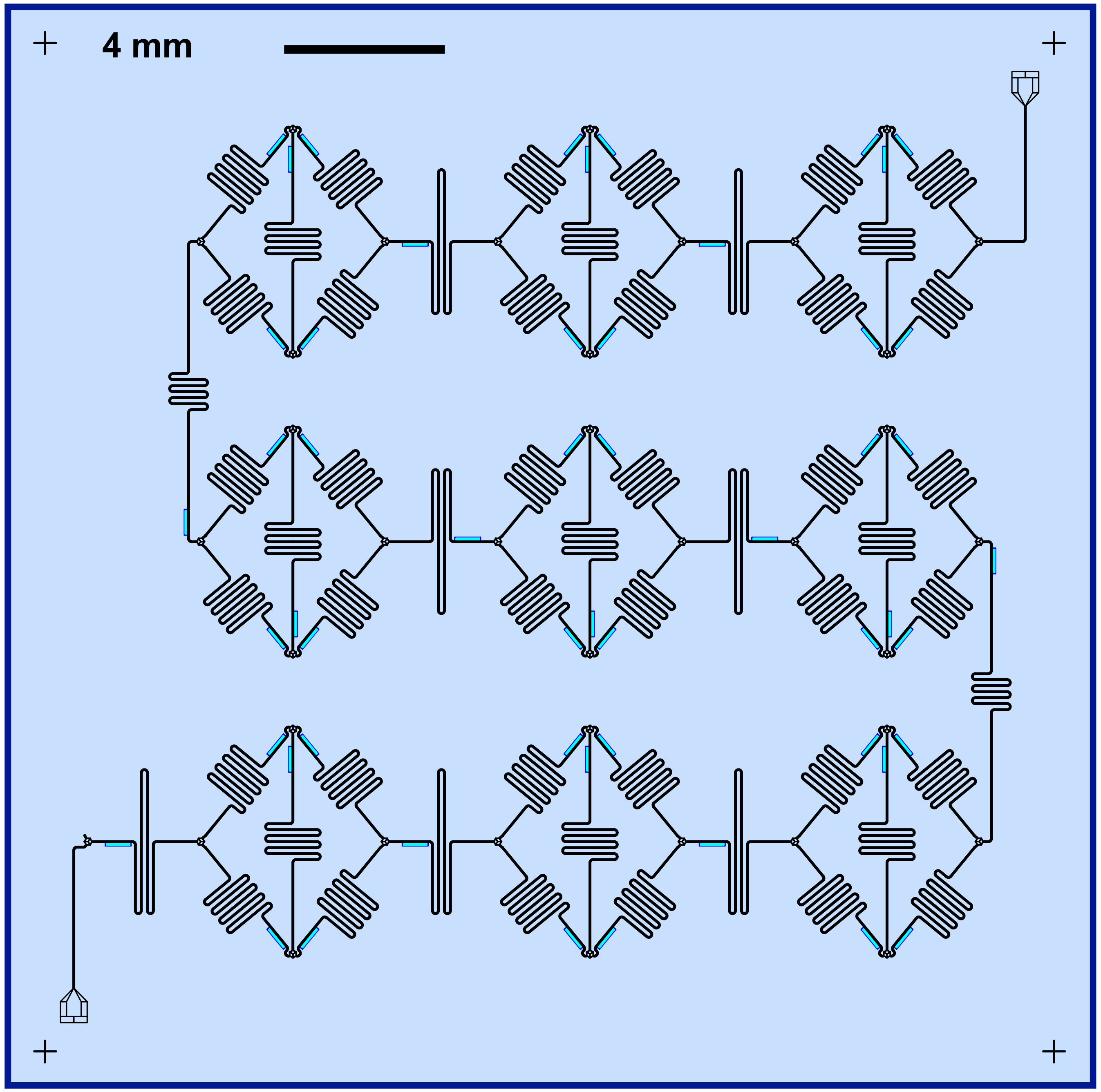
Band Structure



- Flat bands
  - Gapped
  - Ungapped
- Linear bands
- Quadratic bands

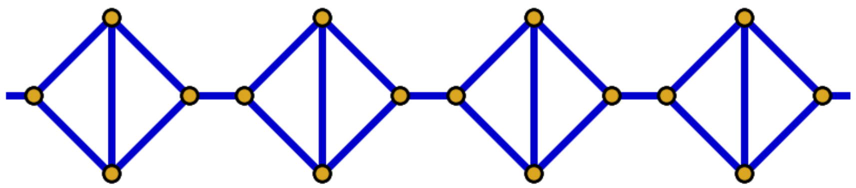
Device Design

(preliminary)

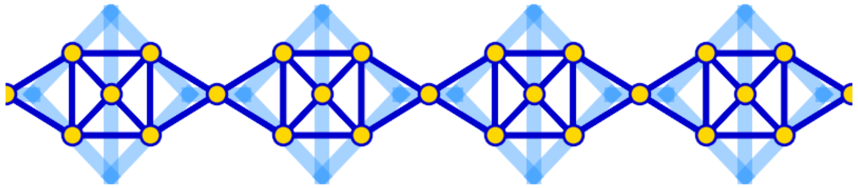


Quasi-1D Lattice Device

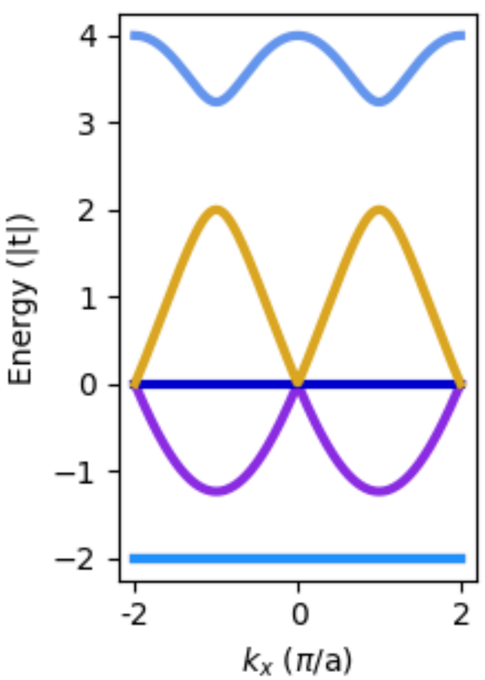
Hardware Layout



Effective Lattice



Band Structure



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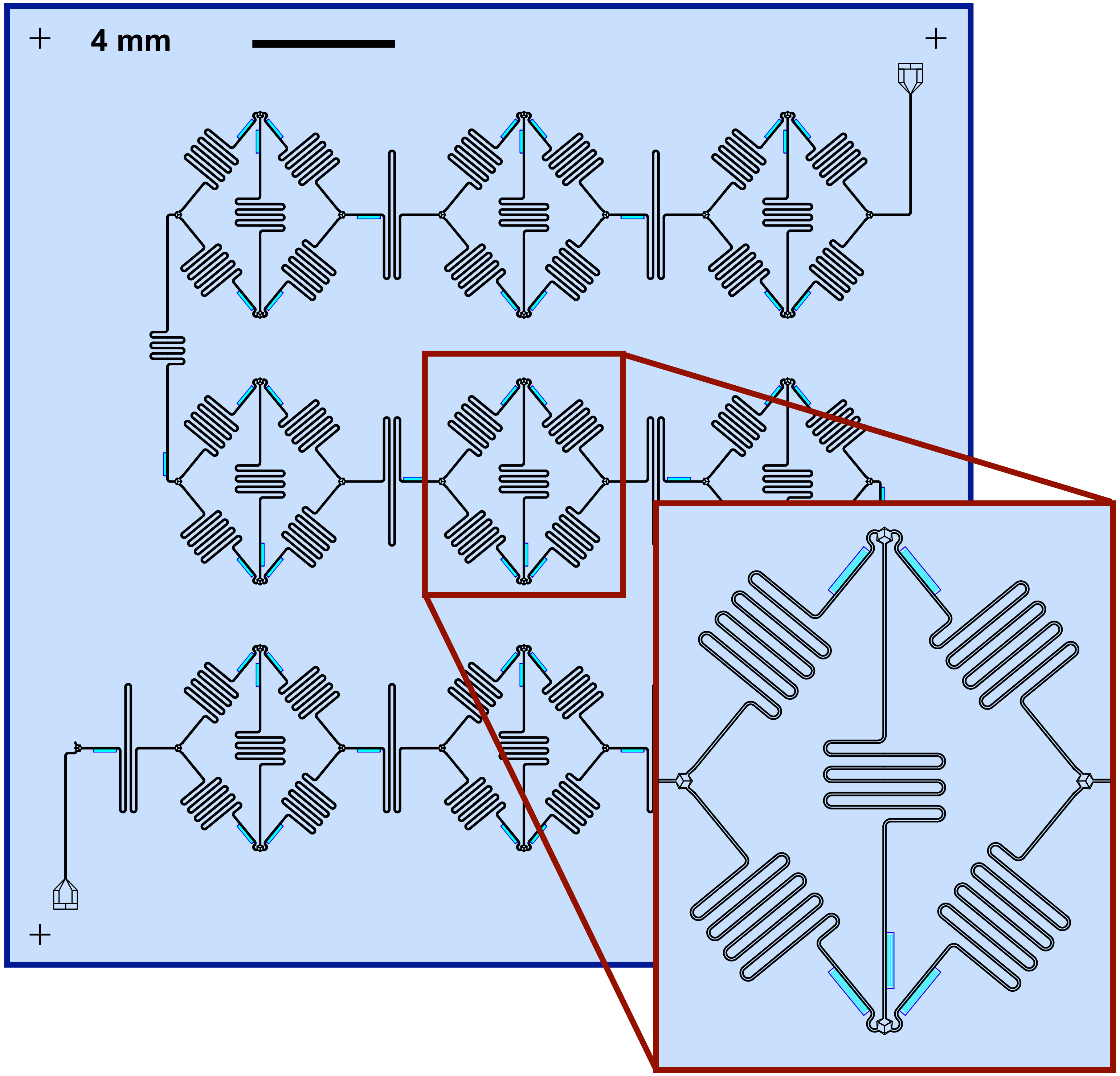

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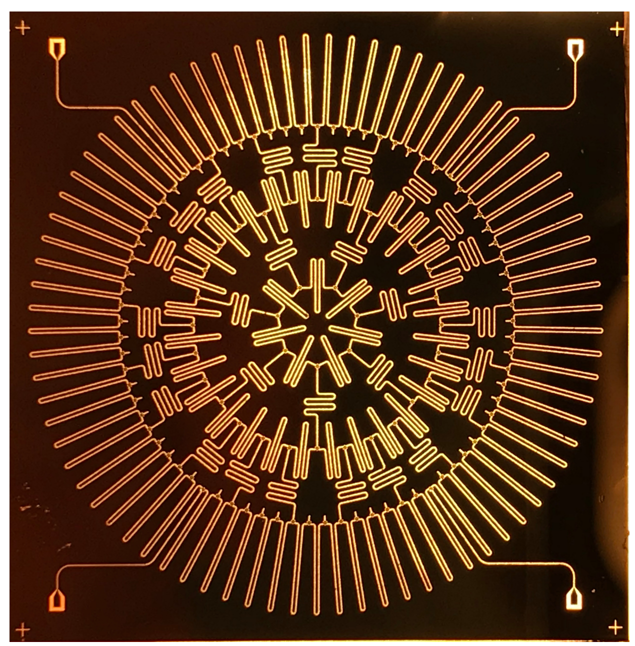

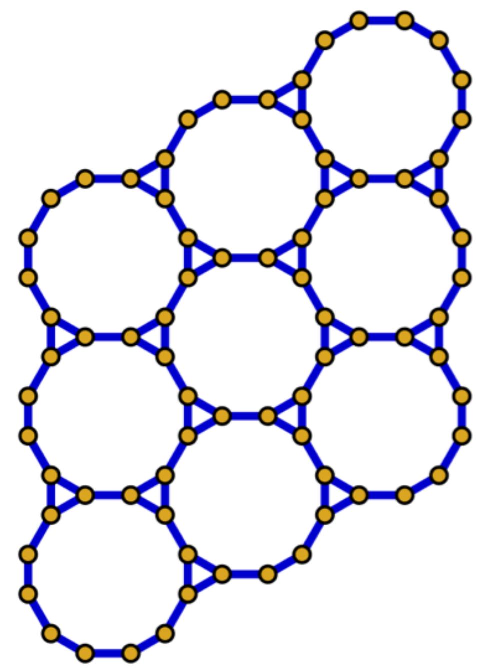

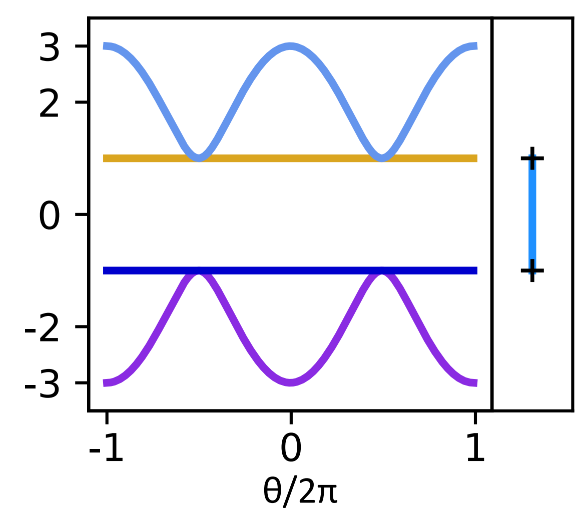

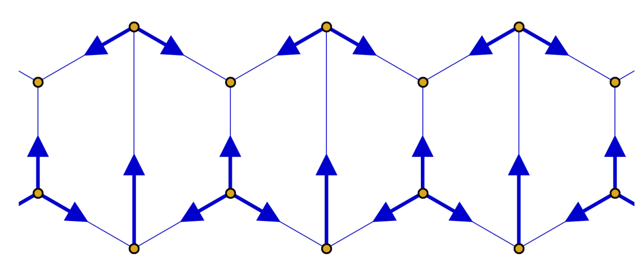

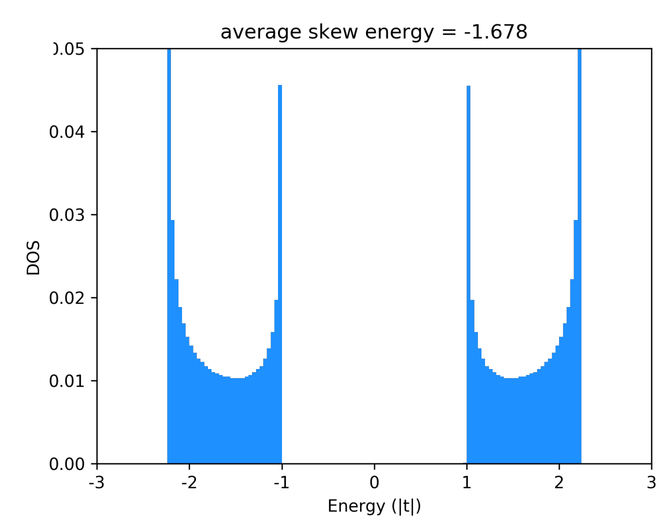

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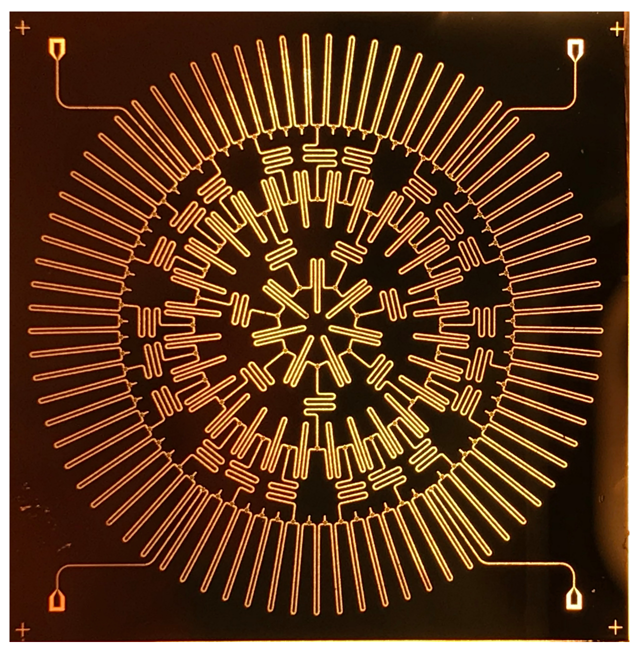

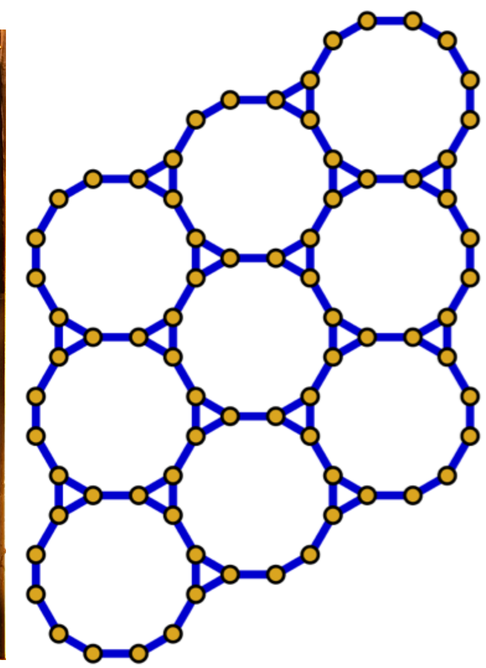

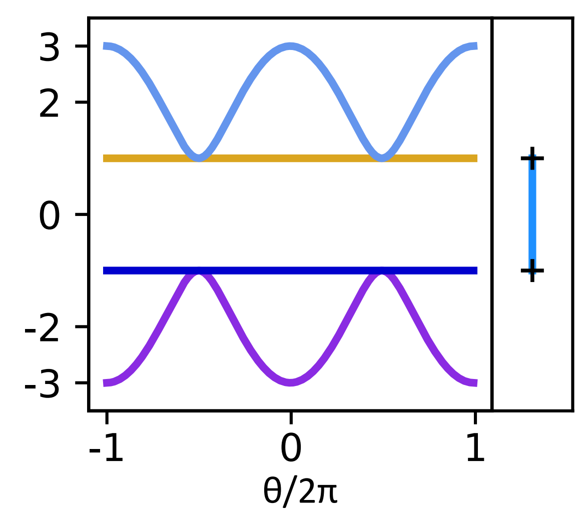

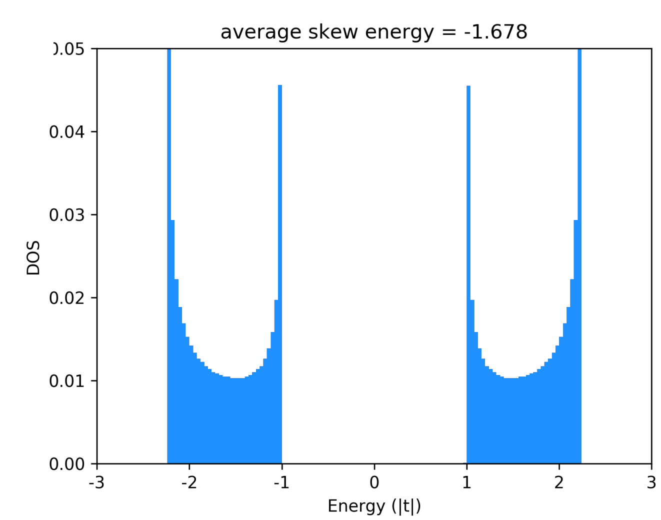

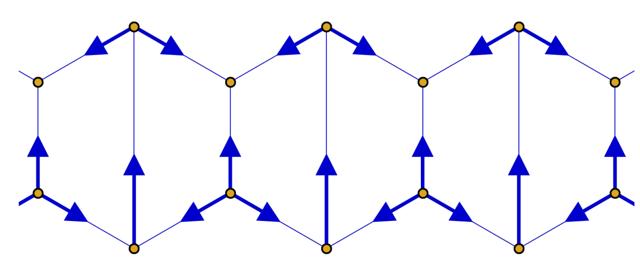

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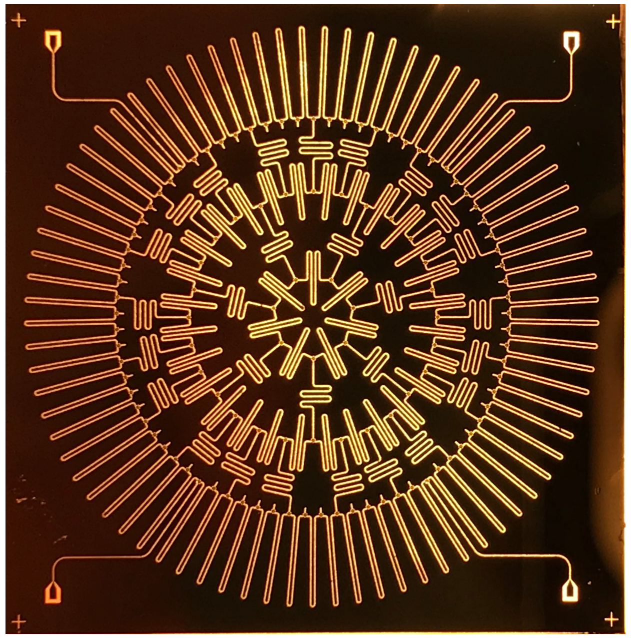

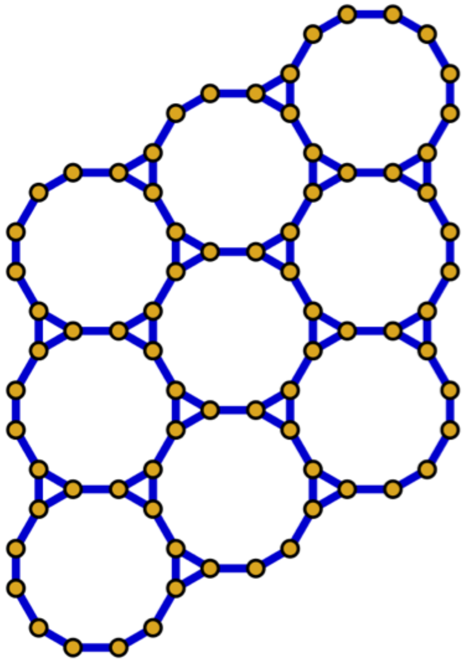

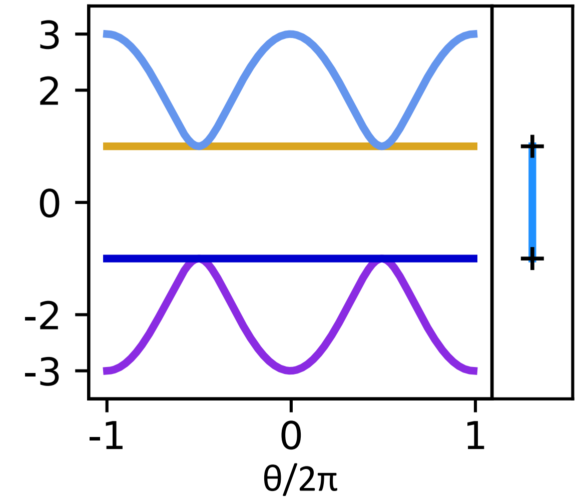

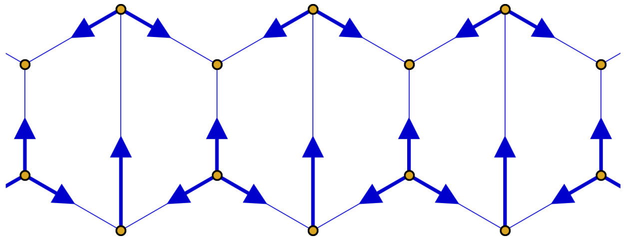

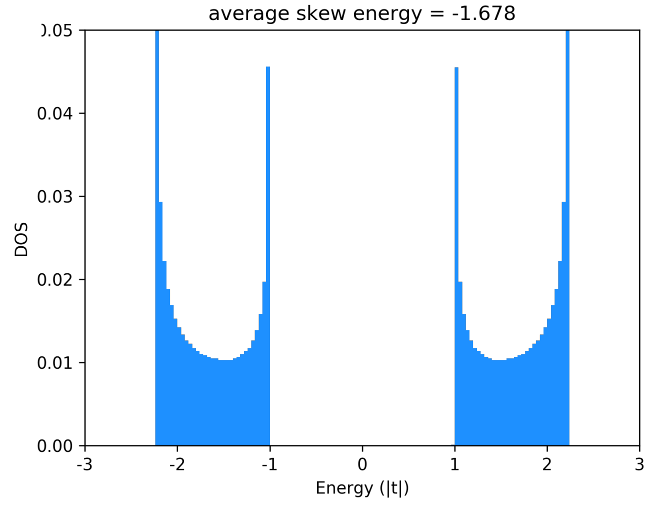

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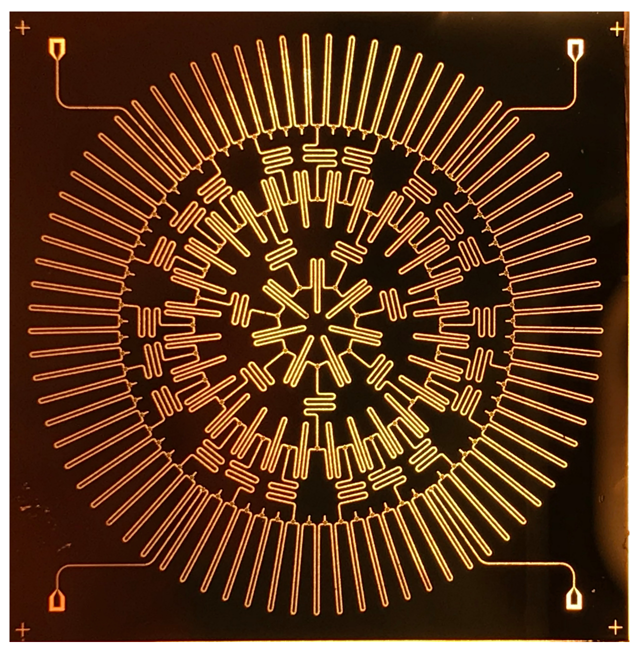

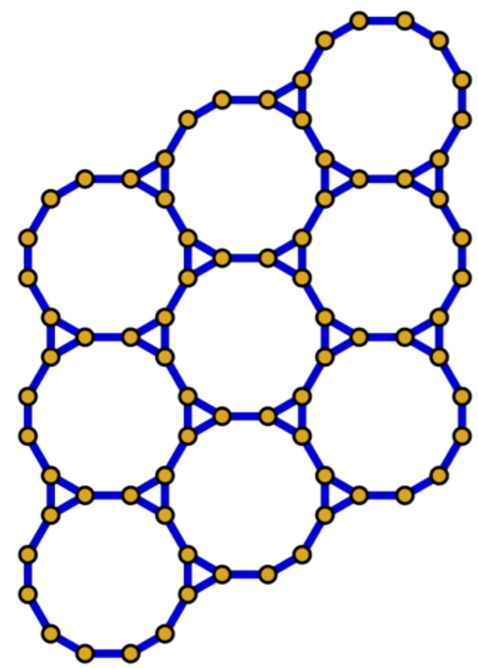

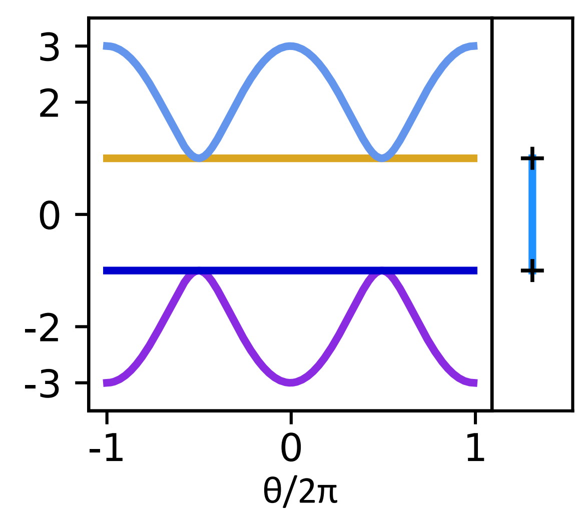

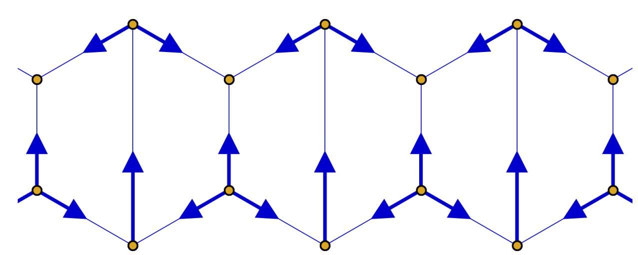

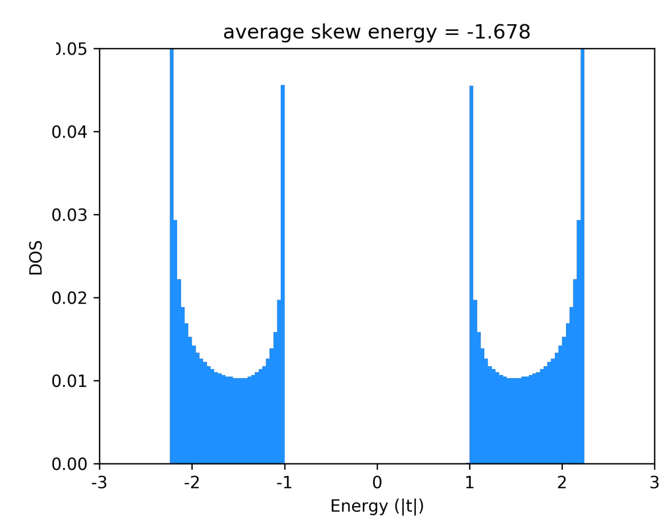

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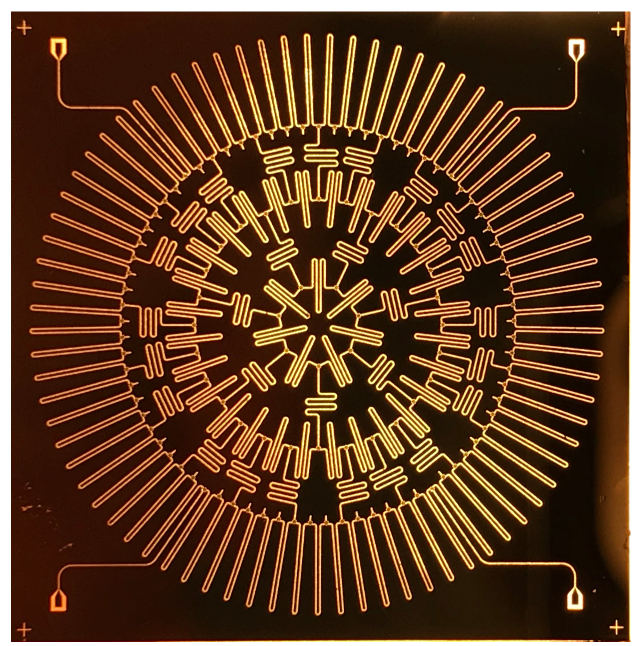

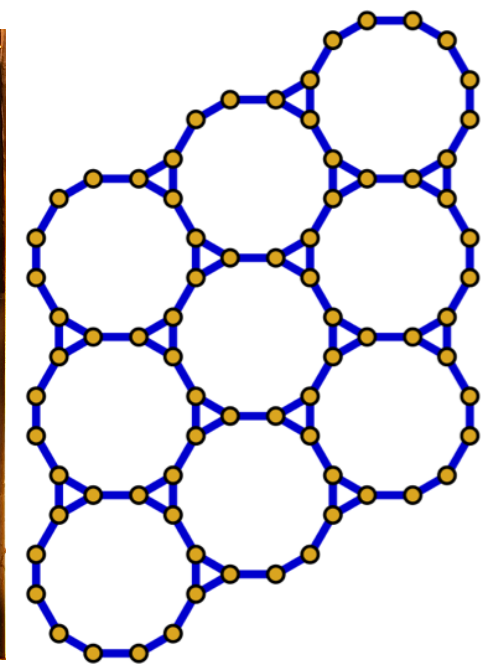

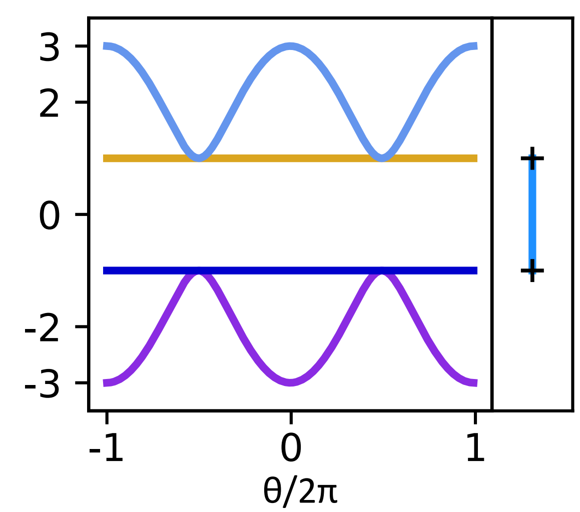

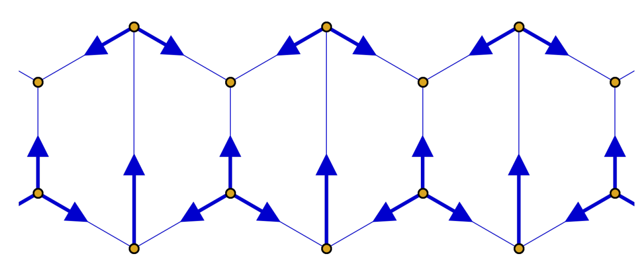

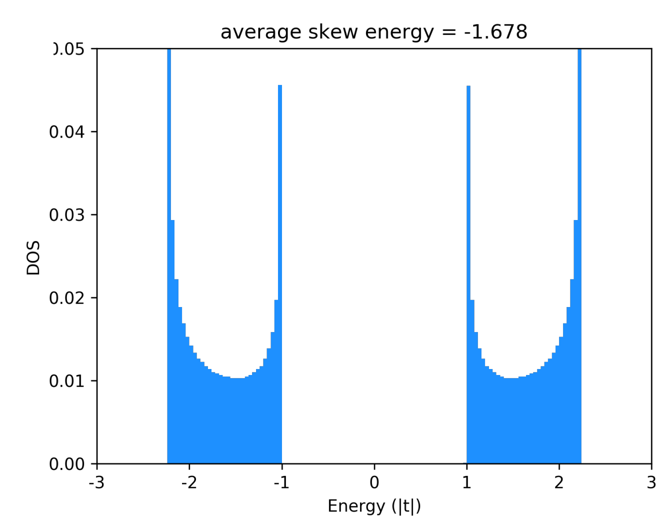

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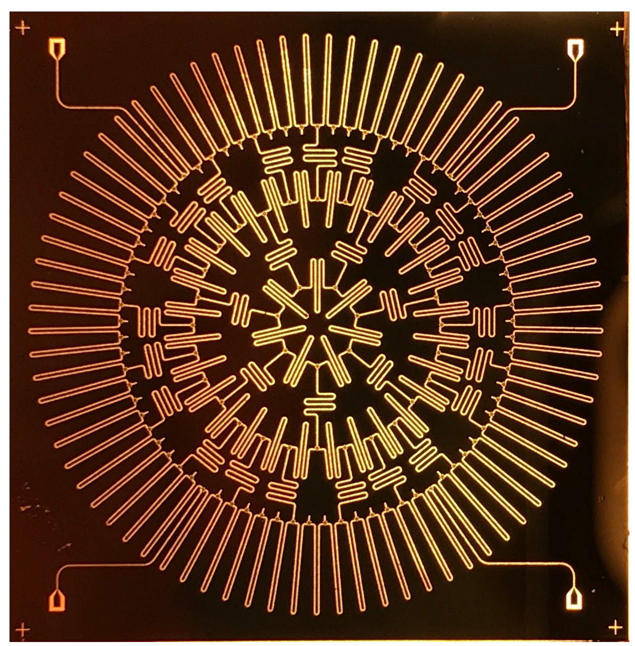

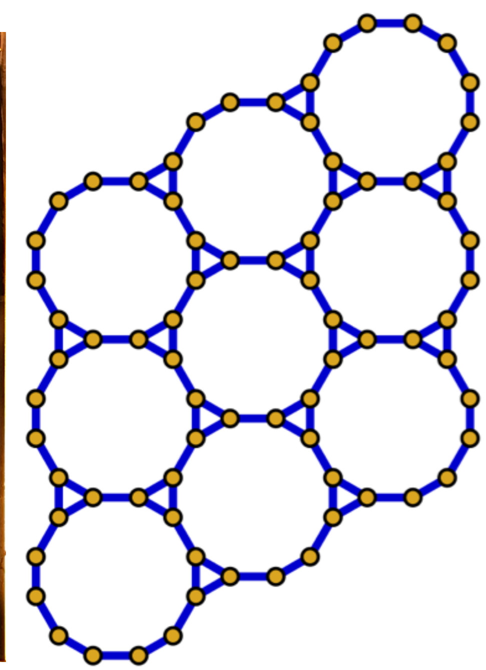

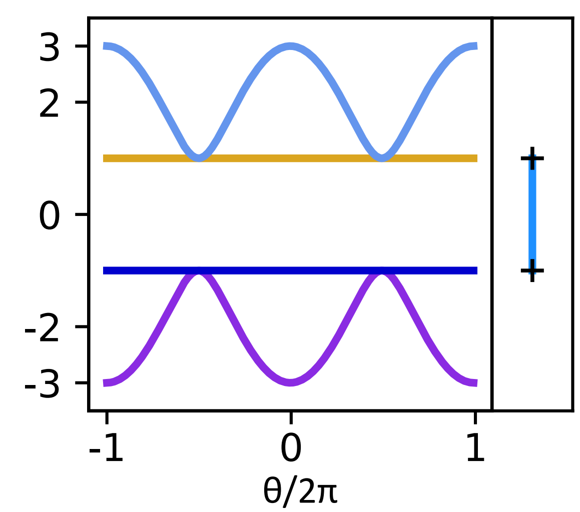

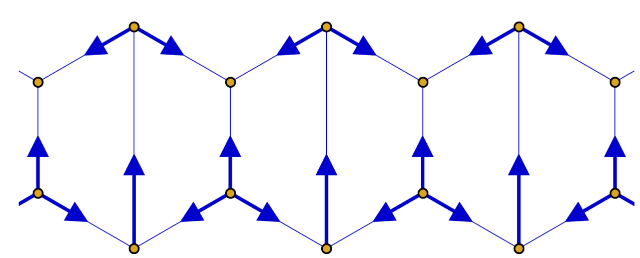

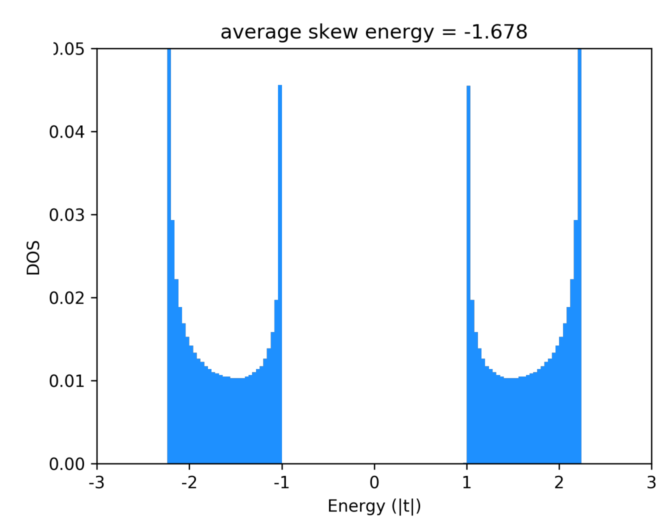

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Circuit QED Lattices

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Peter Sarnak
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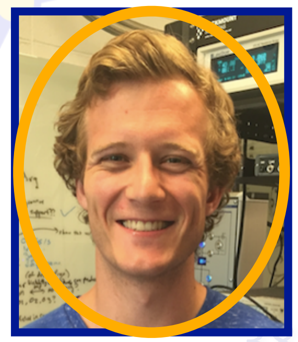
Mattias Fitzpatrick



Maya Amouzegar

Martin Ritter

Jeffrey Wack

Adrian Chapman

Przemislaw Bienias

Igor Boettcher

David Long

ALFRED P. SLOAN FOUNDATION

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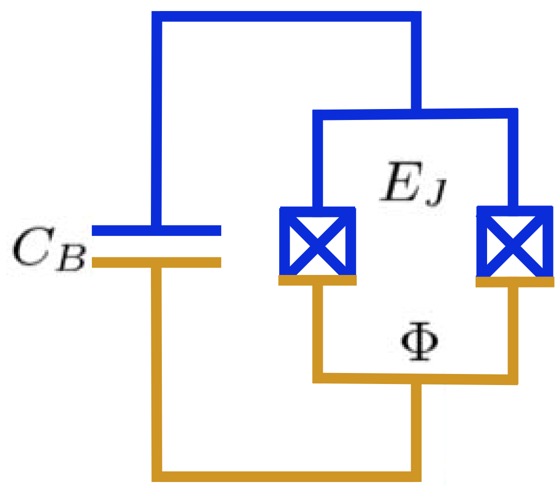
Quantum Materials Center

Transmon Qubit



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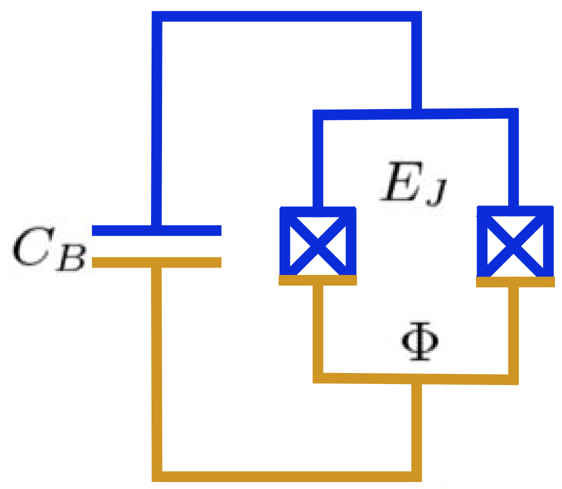

Anharmonic oscillator

$$\hat{H} = 4E_C\,\hat{n}^2 - E_J\cos\hat{\varphi}$$

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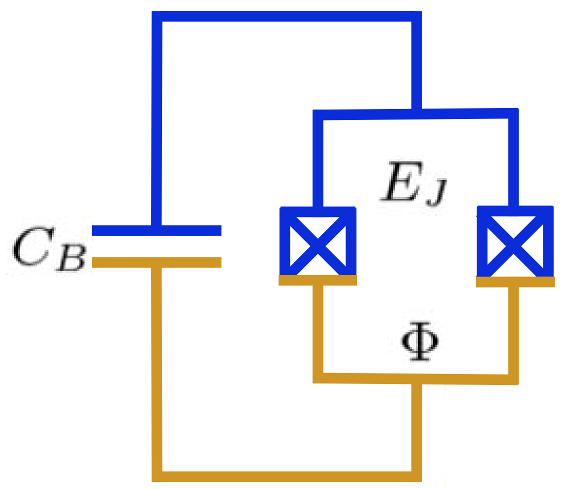

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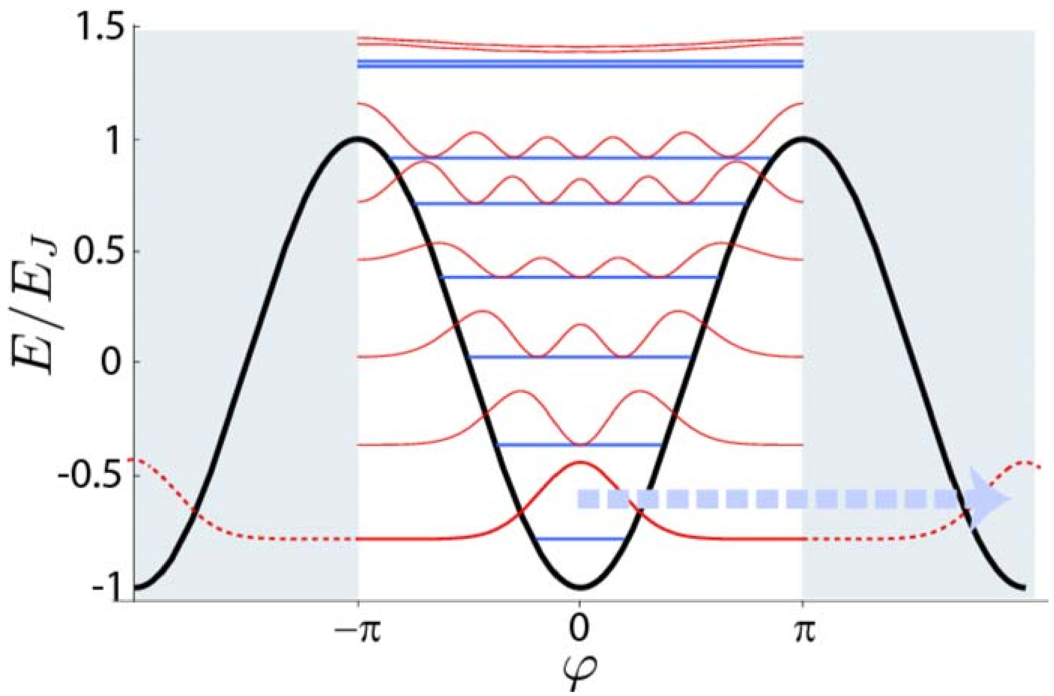

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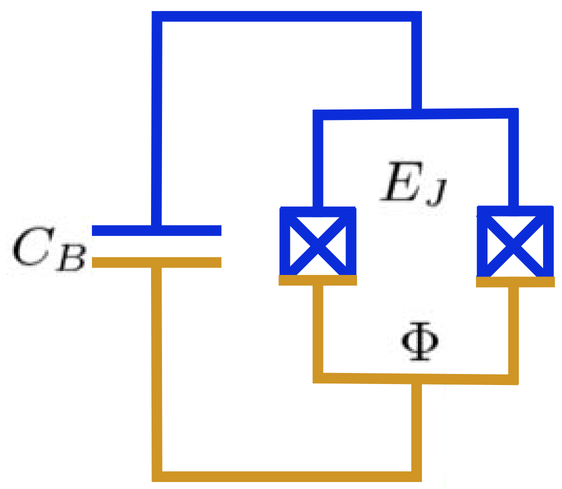

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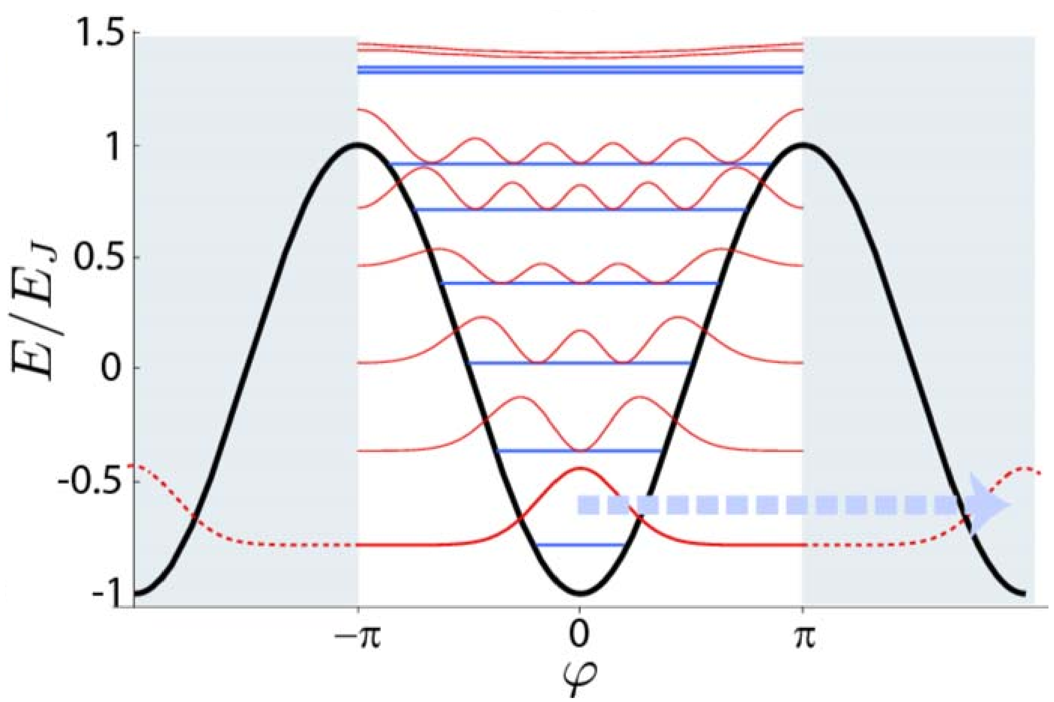

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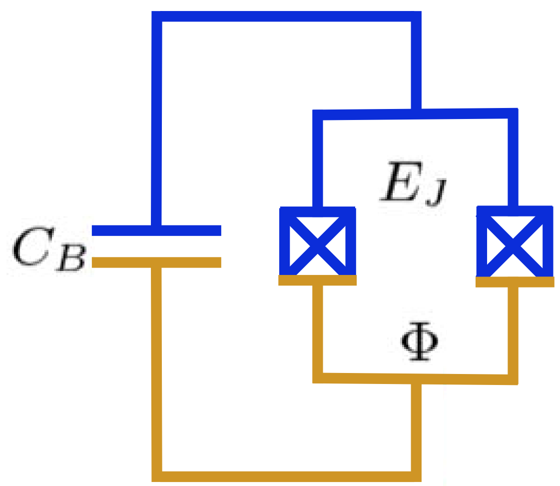
Qubit-Cavity

(Jaynes-Cummings Model)

$$H_{JC} = \omega_c a^\dagger a + \frac{1}{2}\omega_q \sigma_z + g_0(a^\dagger \sigma^- + a\,\sigma^+)$$

Transmon Qubit



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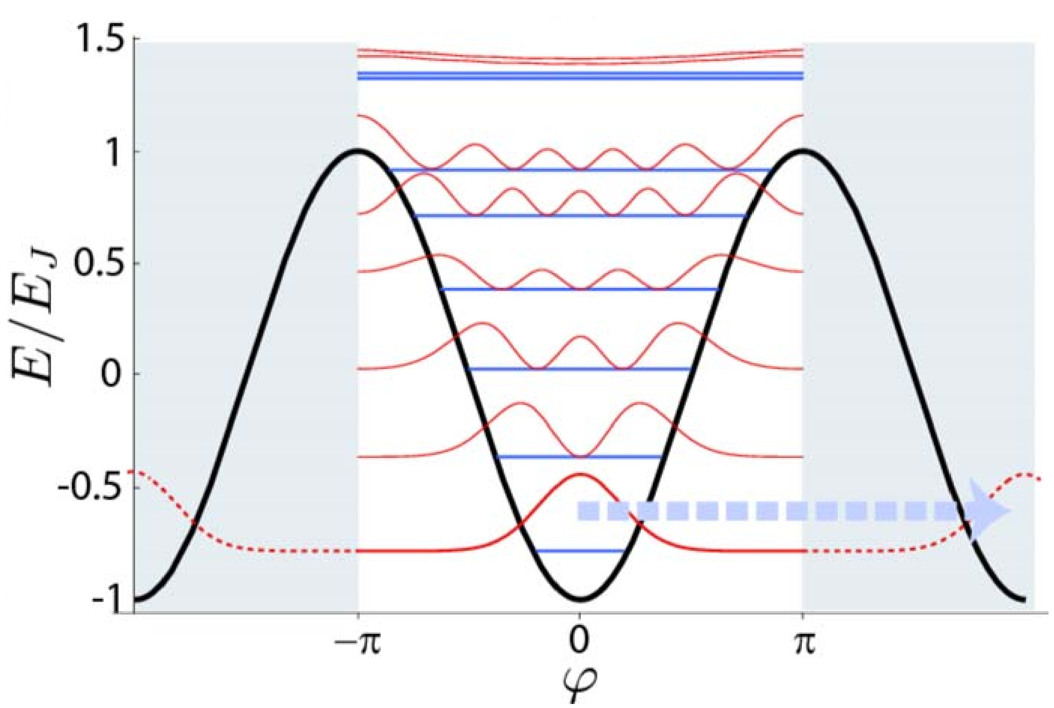

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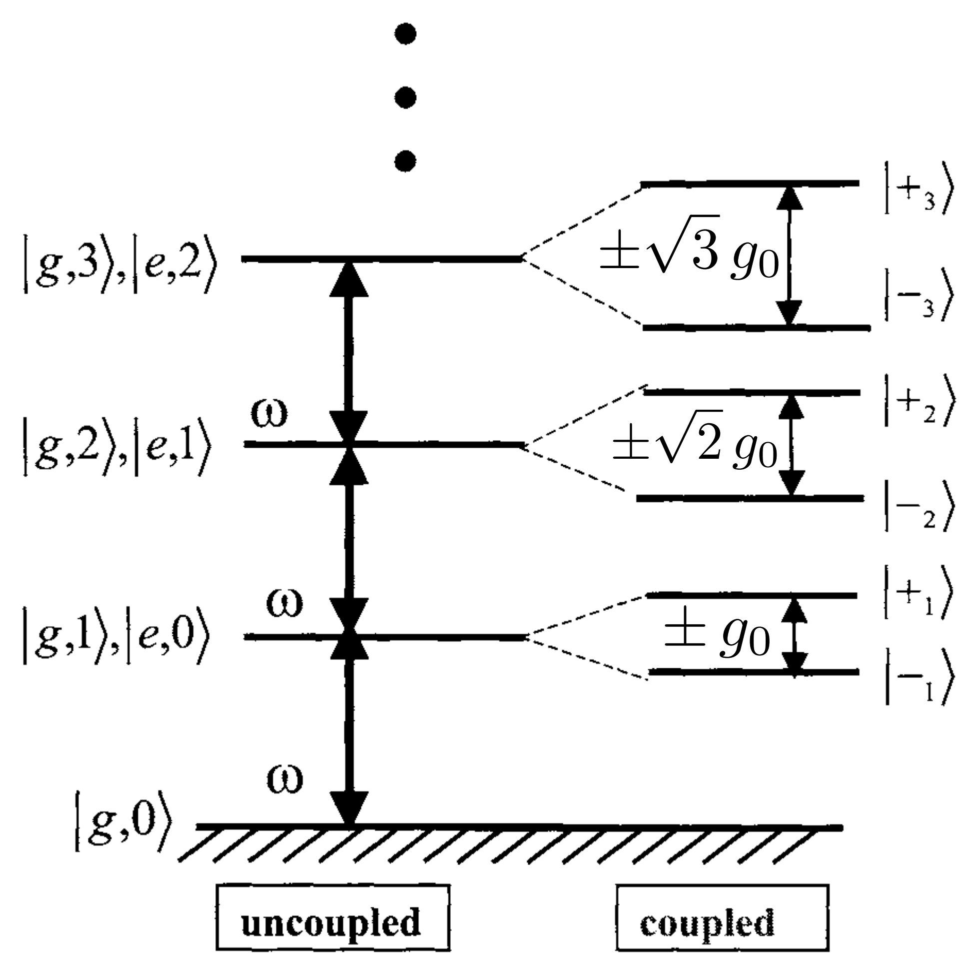
$$|\pm_n\rangle = \frac{1}{\sqrt{2}}\left(|g,n\rangle \pm |e,n-1\rangle\right),$$



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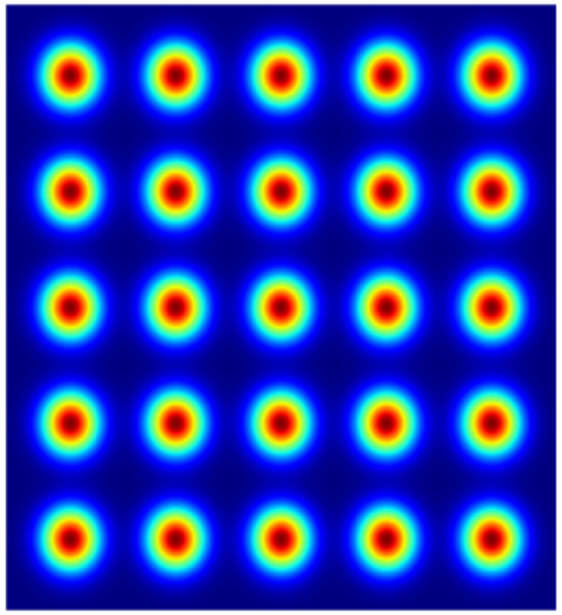
Ye *et al*. Advances in AMO Physics **49** (2003)

The Graph is Everything

Regular Lattice



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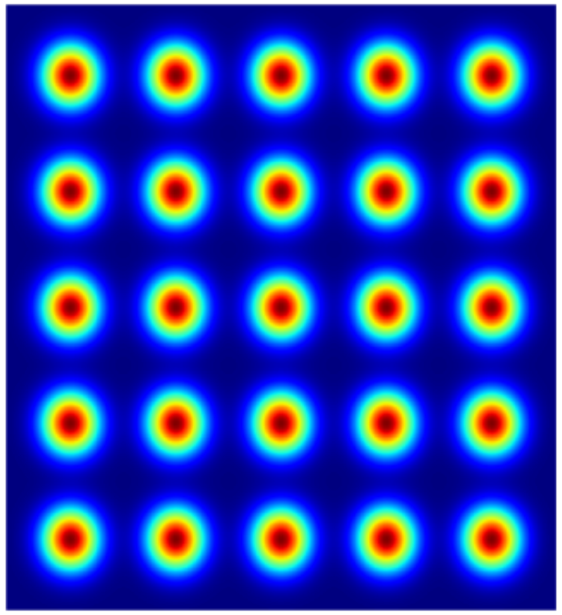
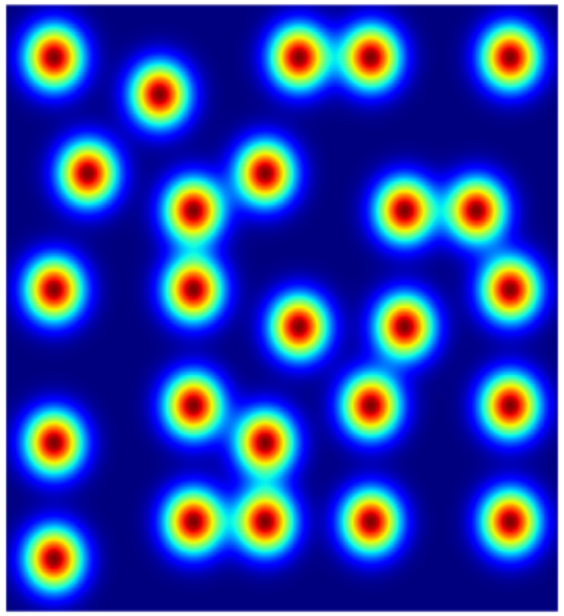


Disordered Lattice



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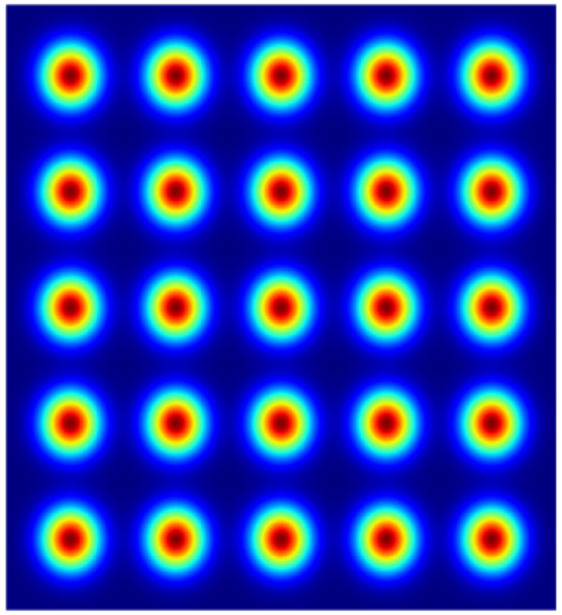

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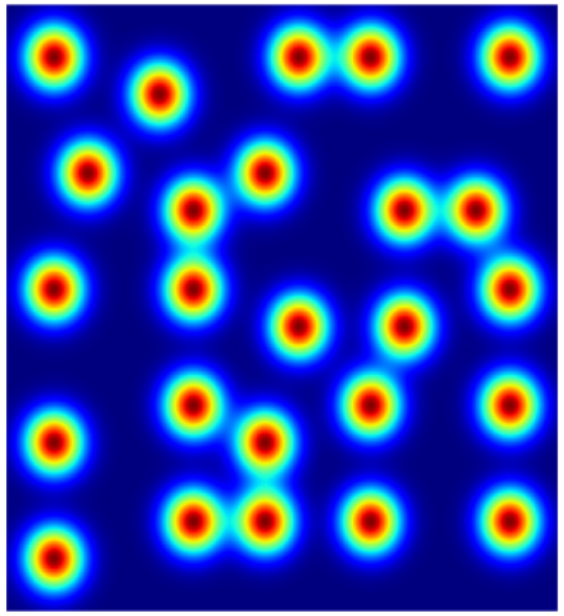
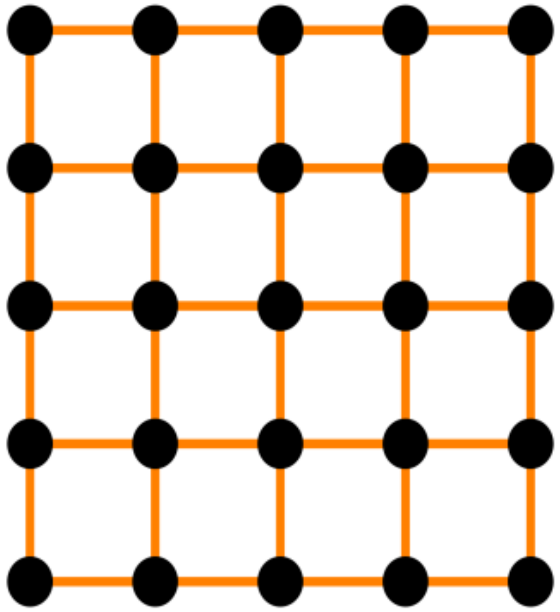


Regular Tight-Binding Graph



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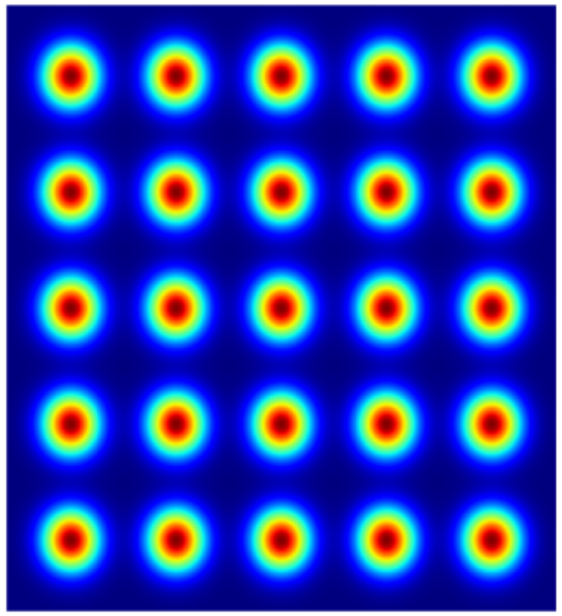

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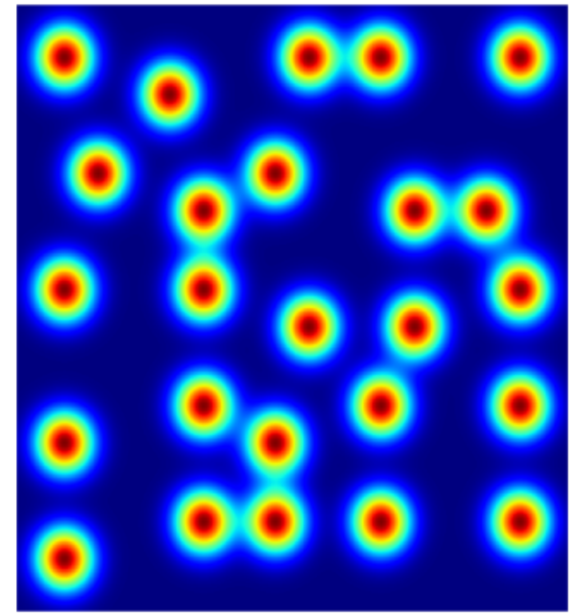
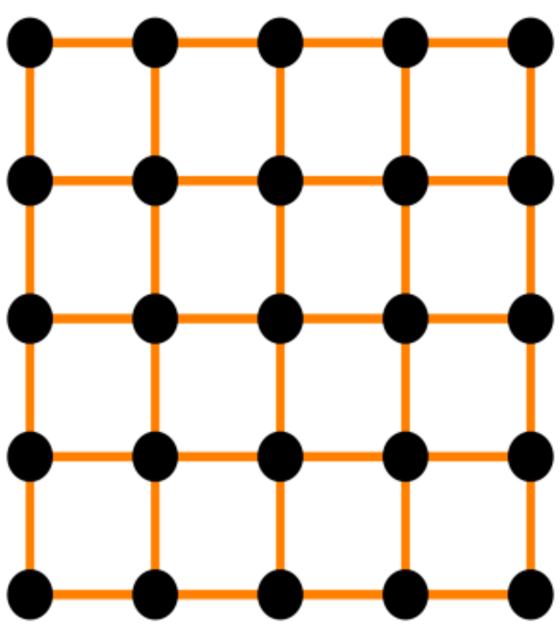
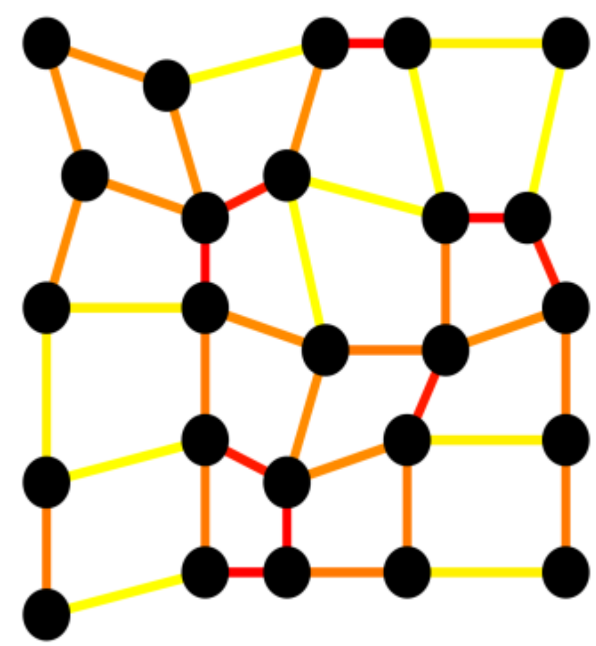


Regular Tight-Binding Graph

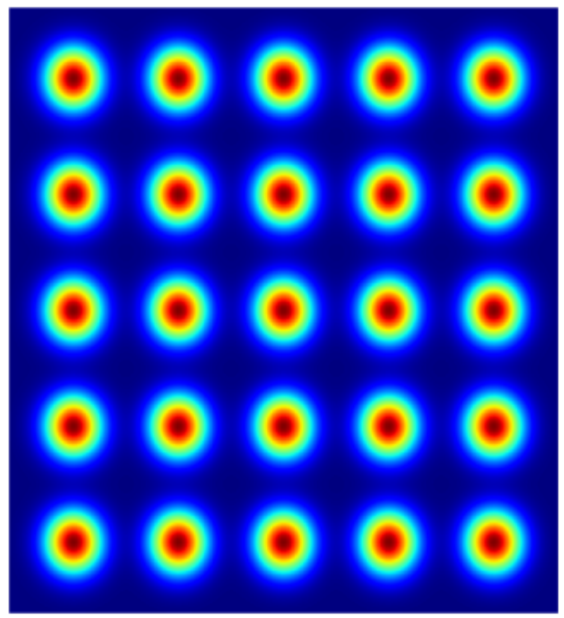


Disordered TB Graph



The Graph is Everything

Regular Lattice



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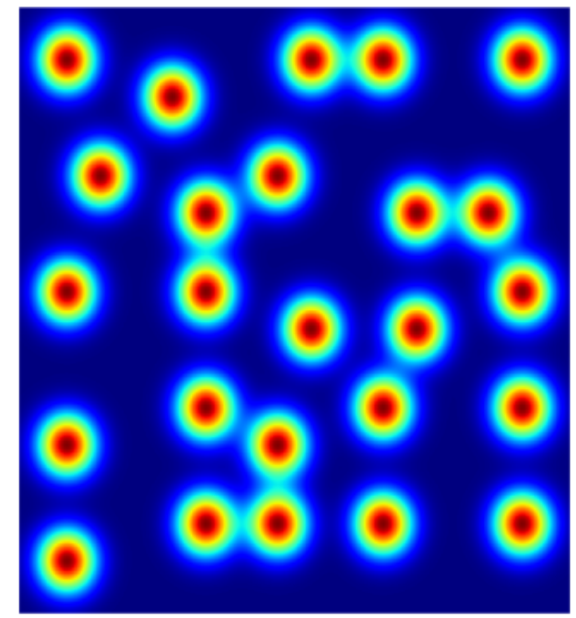
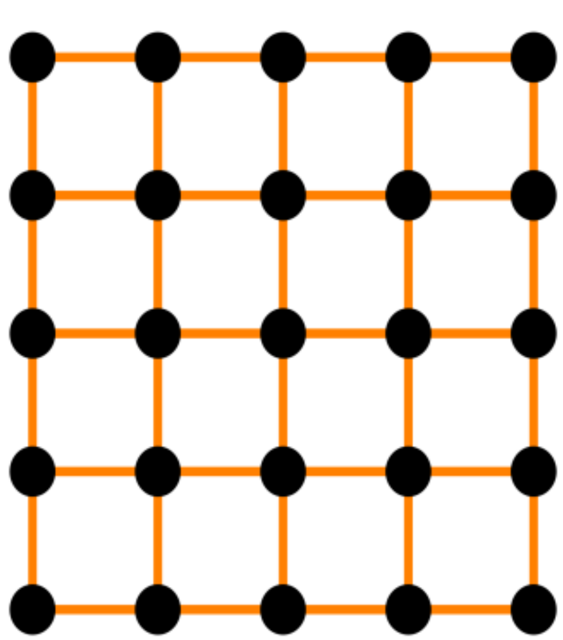
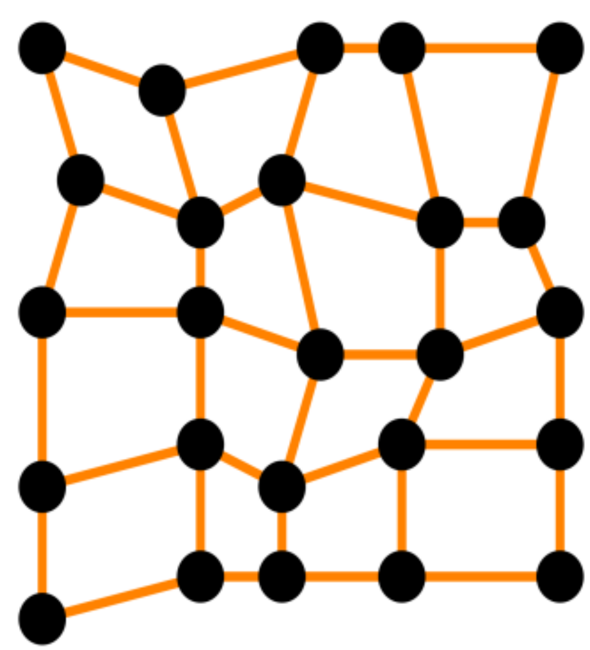


Regular Tight-Binding Graph





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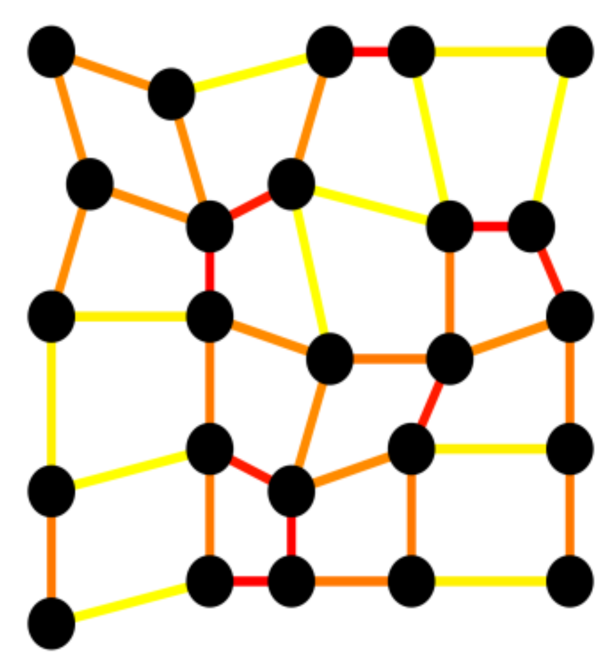

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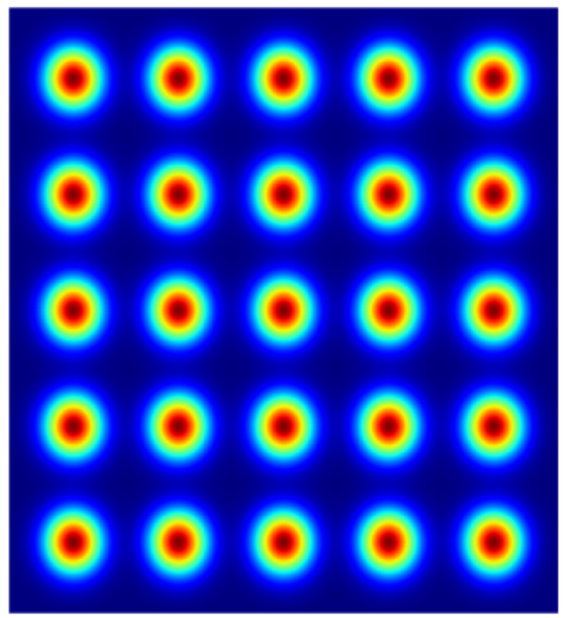

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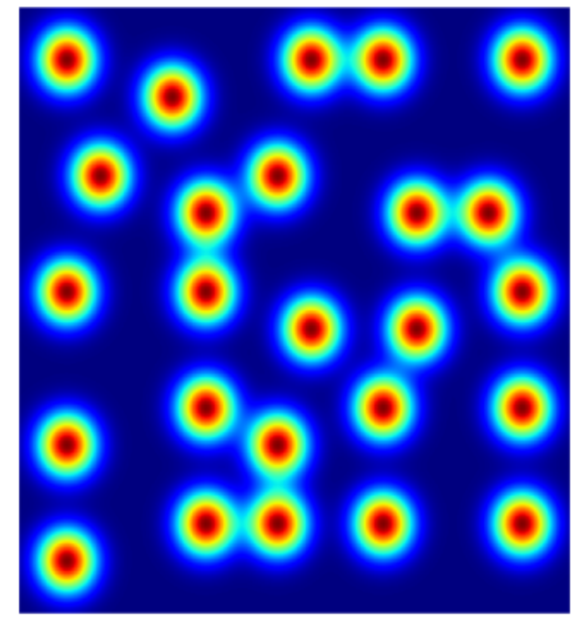

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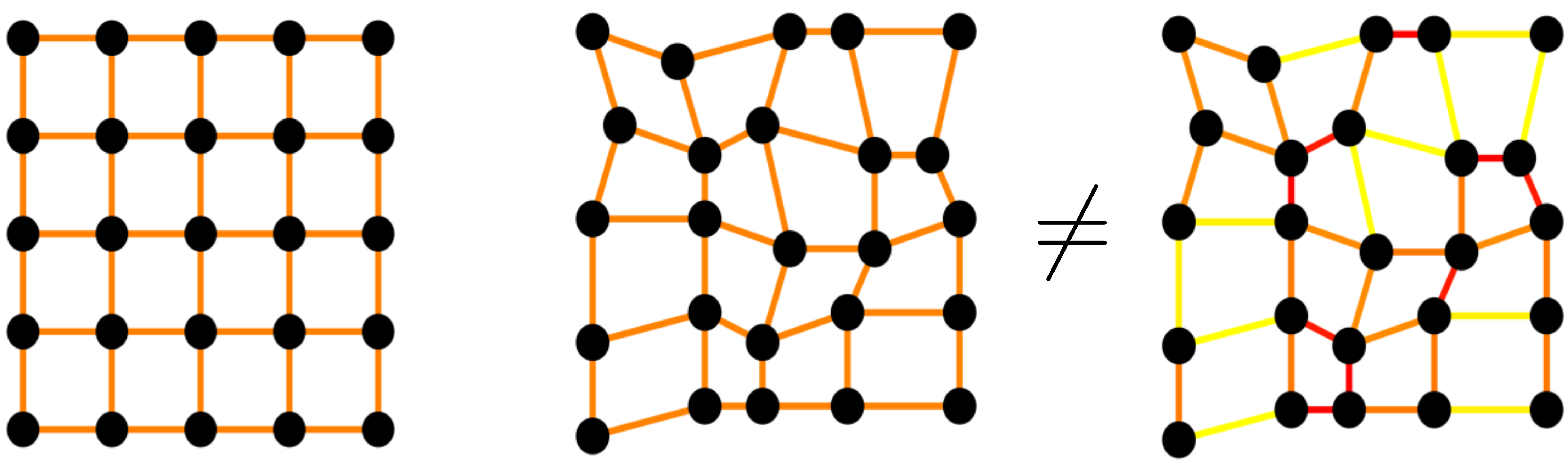

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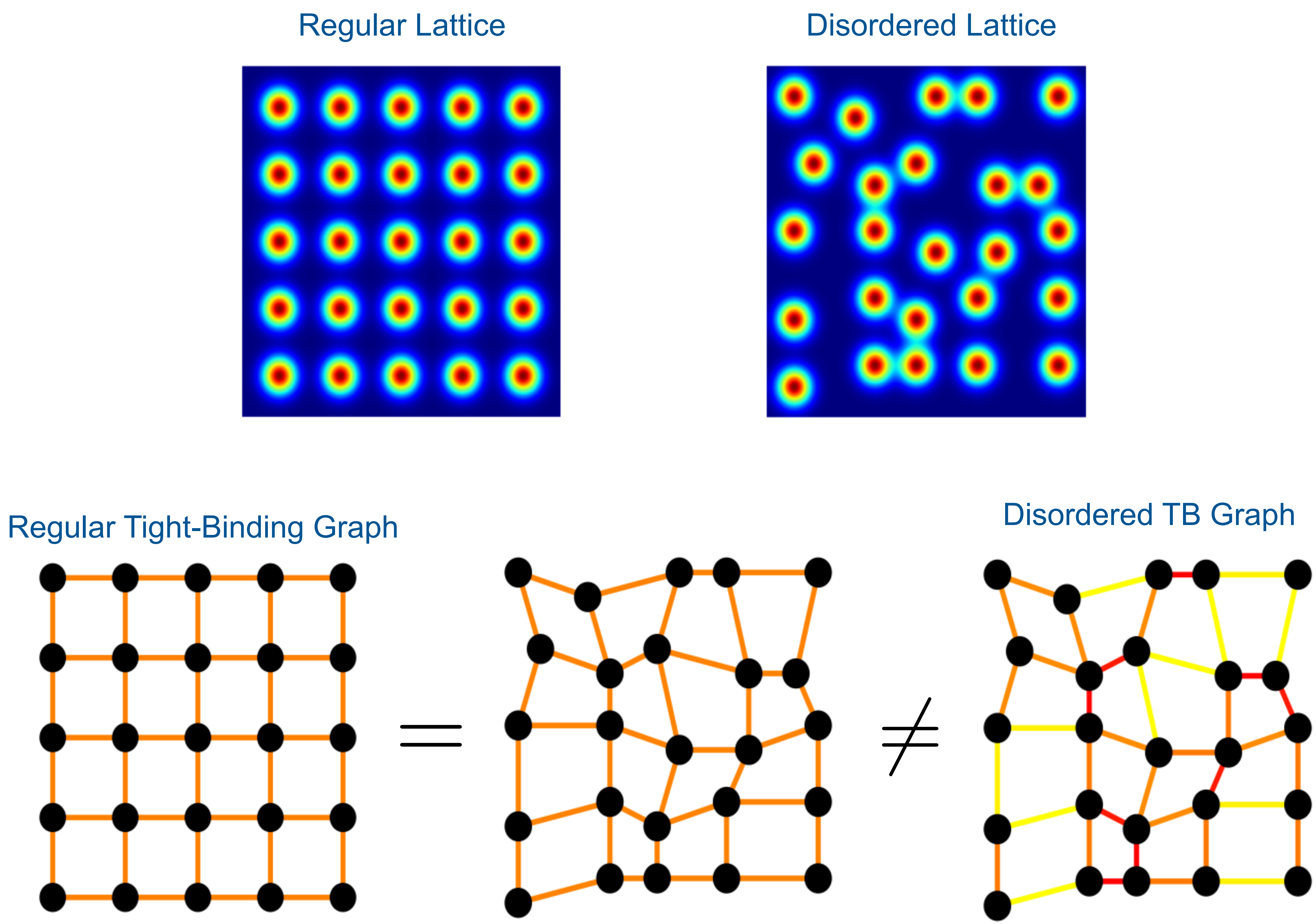
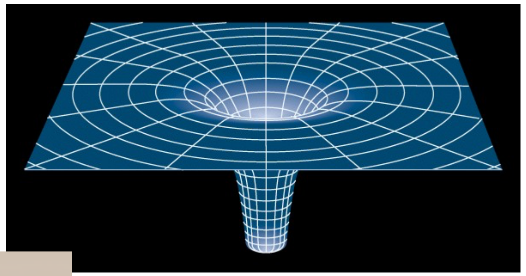
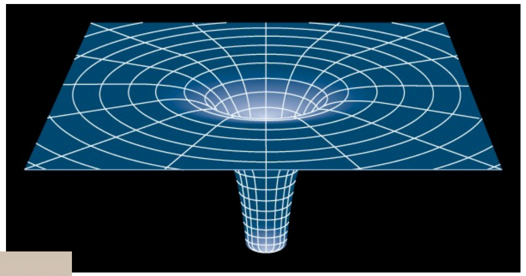


Applications of Hyperbolic Systems

- General relativity
  - Curved space-time
- 2D materials
  - graphene, fullerenes



Applications of Hyperbolic Systems



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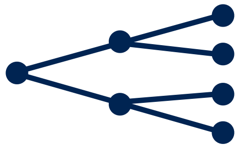
- Mathematics
  - Trees 
  - Cayley graphs of non-commutative groups
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Applications of Hyperbolic Systems

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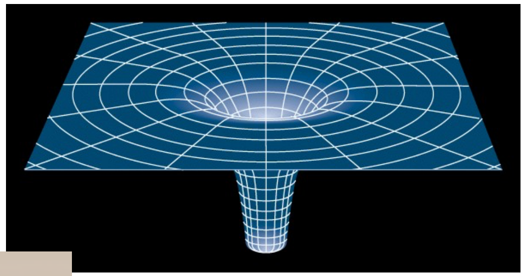

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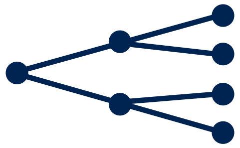
  - Trees 
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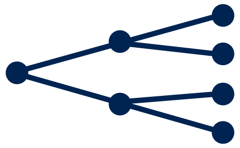
  - Trees 
  - Efficient communication networks
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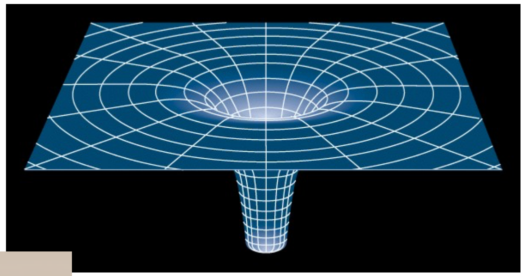
Applications of Hyperbolic Systems



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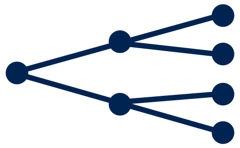
  - Trees 
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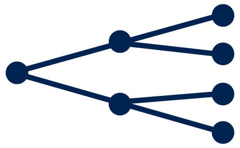
  - Trees 
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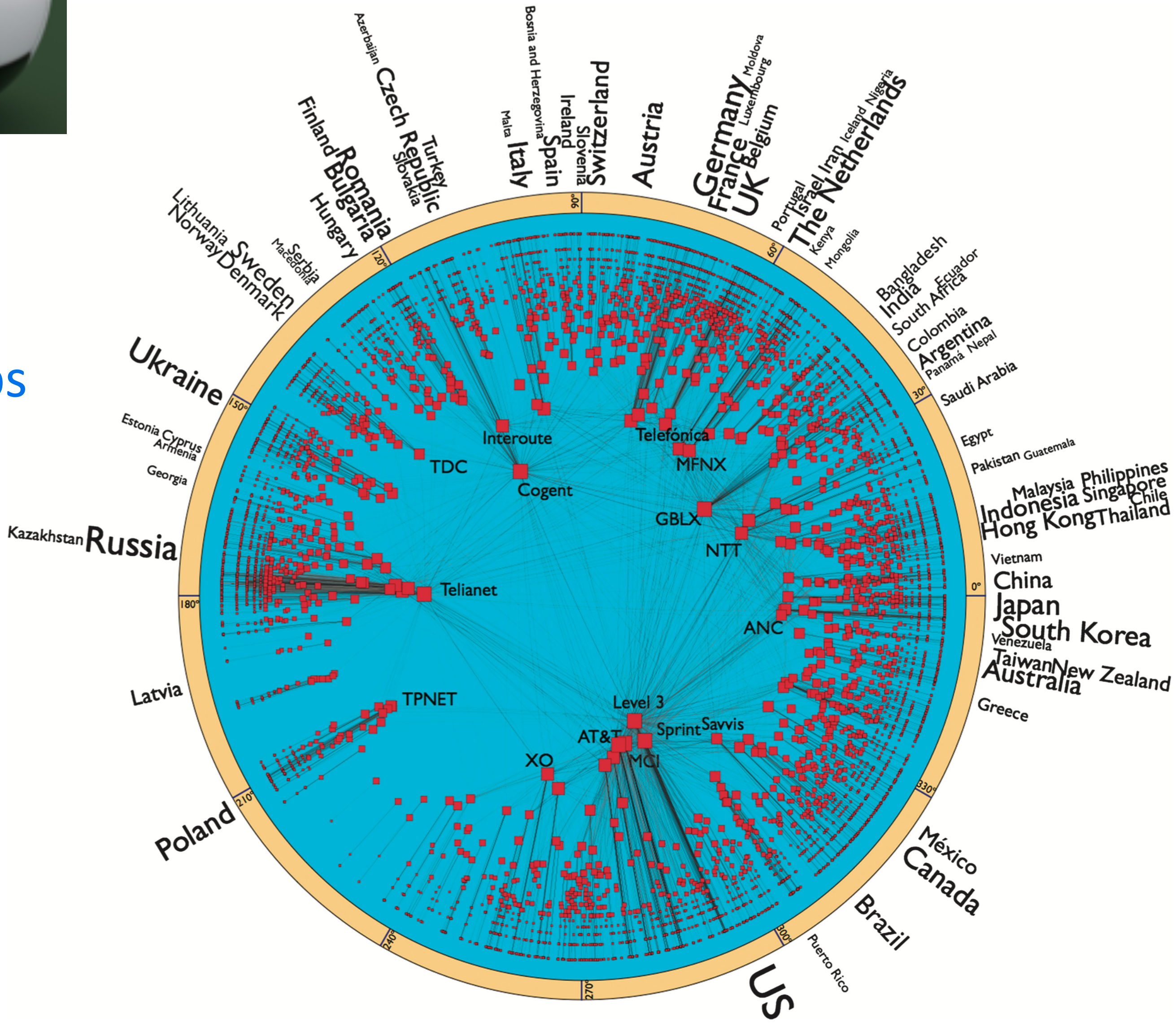

Bogu *et al.* Nat Comms **1063** (2010)

Continuum Limit and Green's Function

High Energy Limit of The Spectrum

Continuum Limit and Green's Function

High Energy Limit of The Spectrum

- Long-wavelength modes

Continuum Limit and Green's Function

High Energy Limit of The Spectrum

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Continuum Limit and Green’s Function

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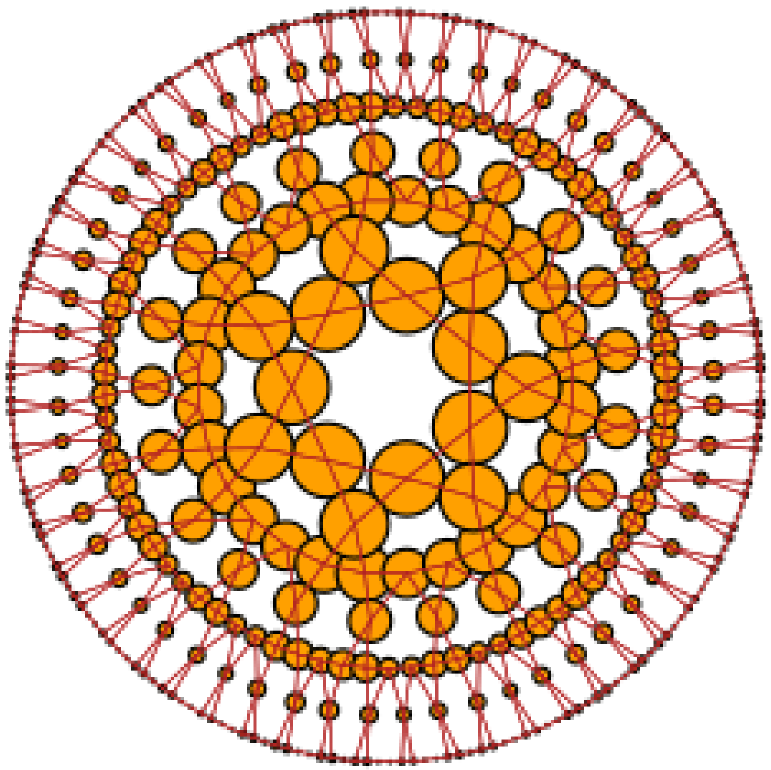

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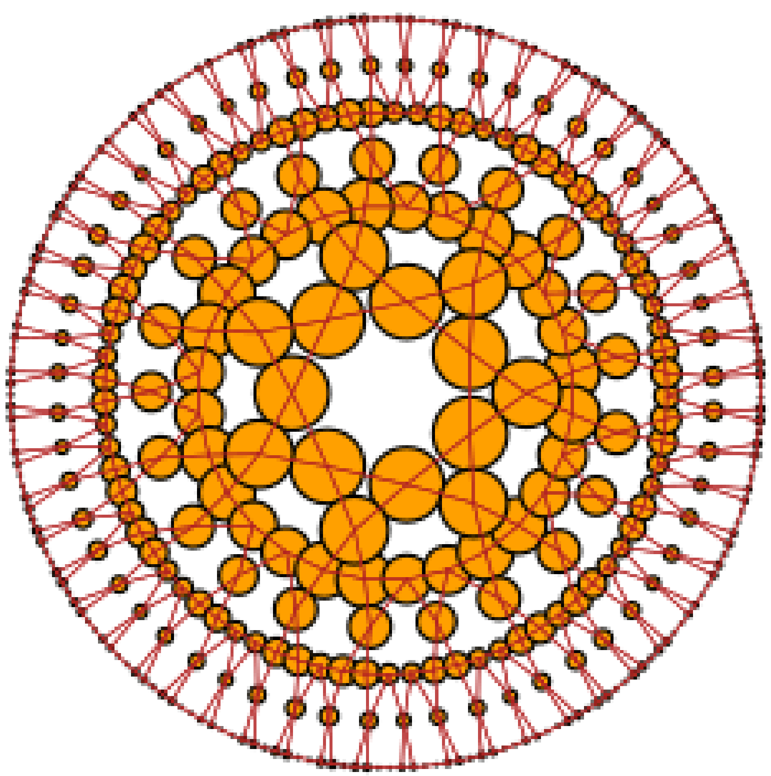

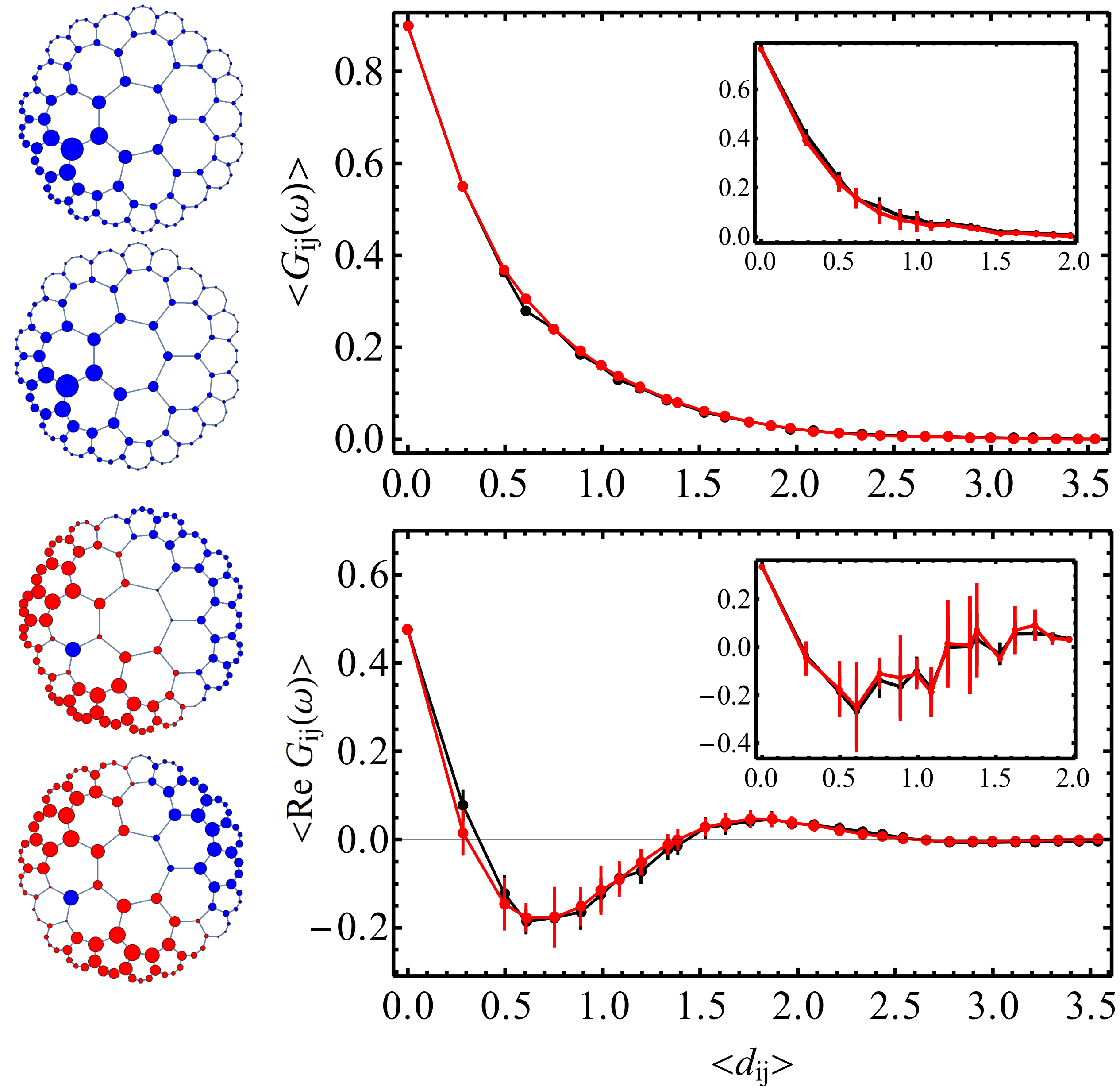

Boettcher *et al*. Phys. Rev. A **102**, 032208 (2020)

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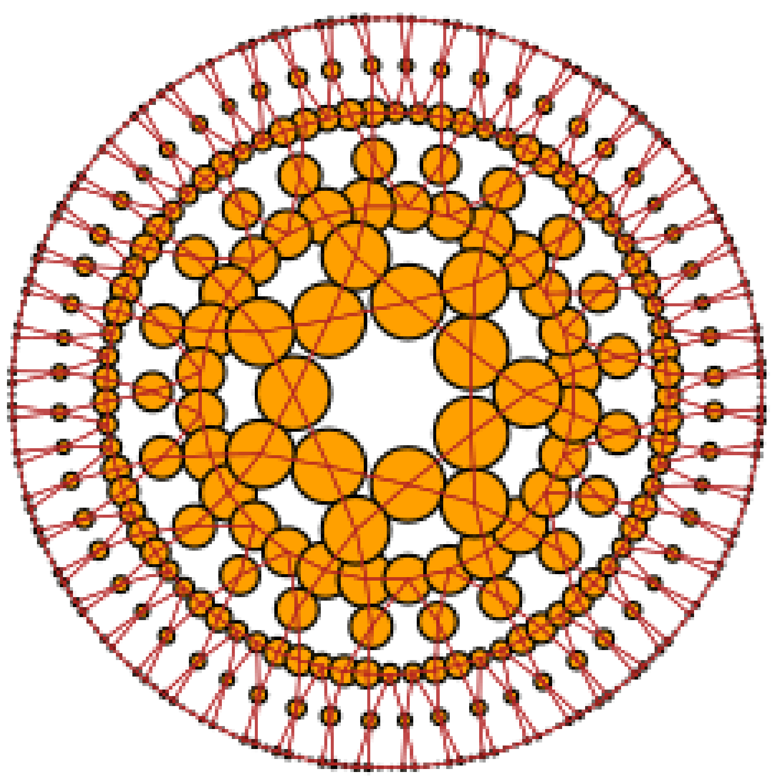

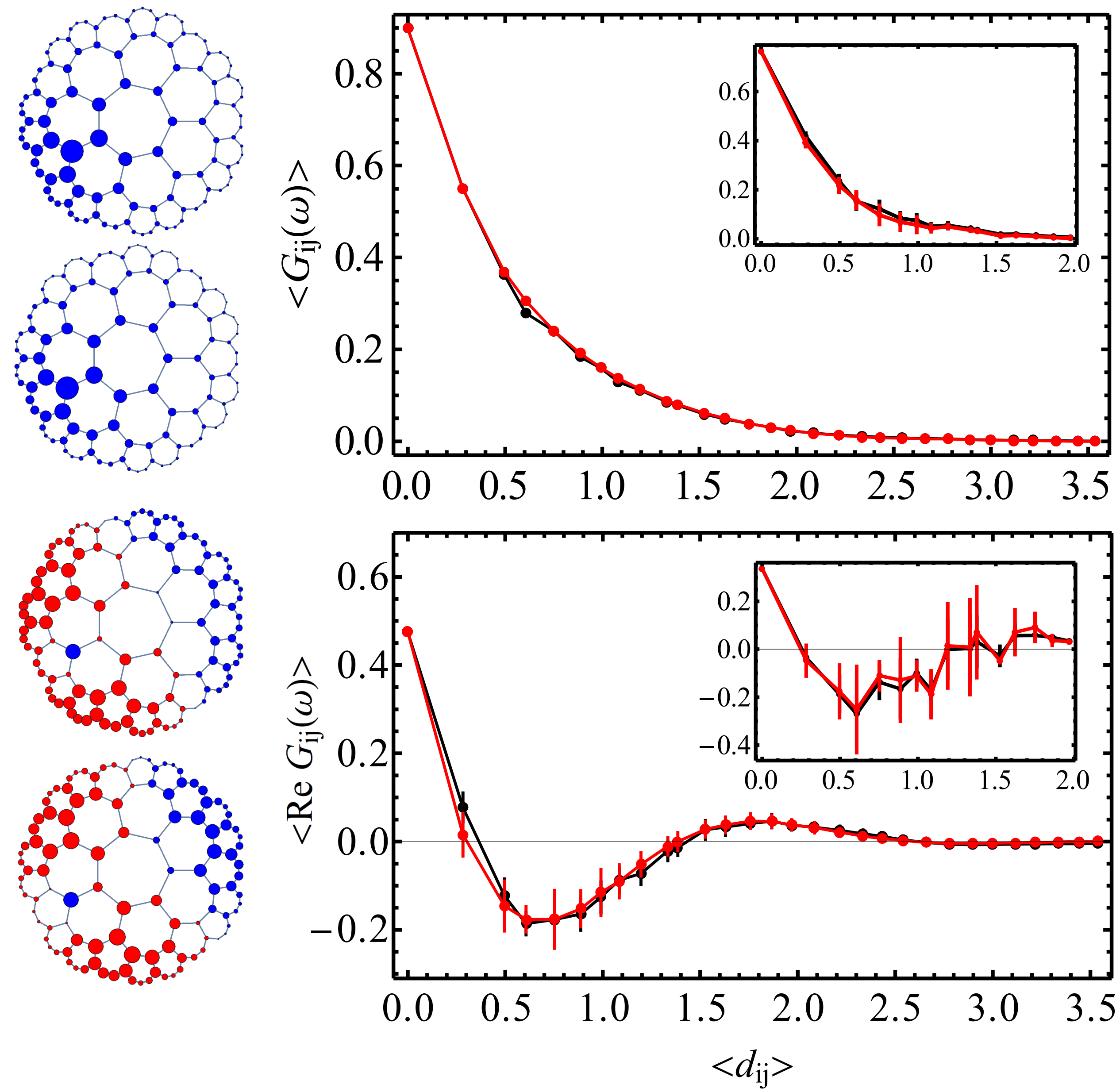

Quantitive Match for Large System Sizes

- Green's function

Boettcher *et al*. Phys. Rev. A **102**, 032208 (2020)

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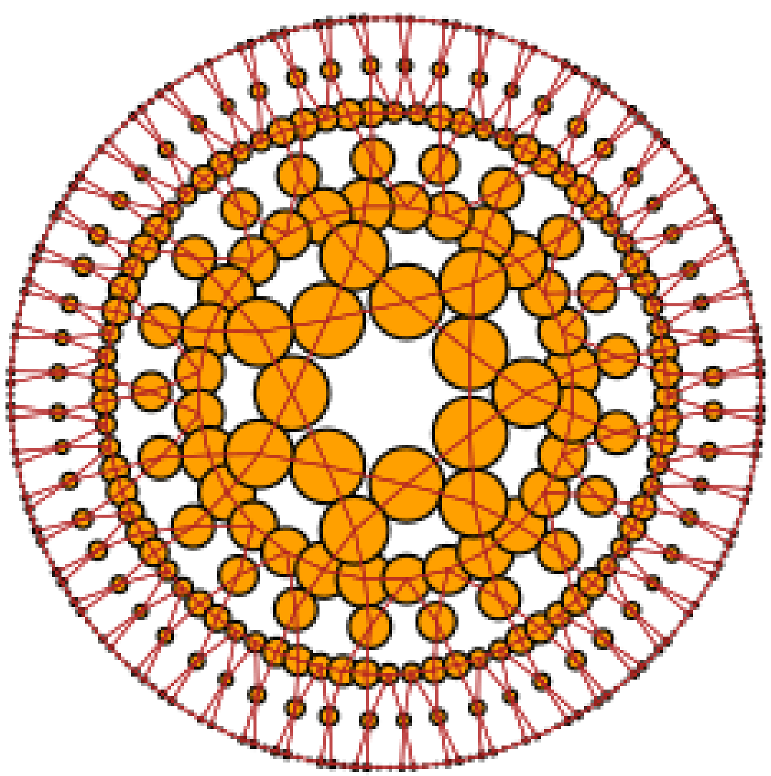

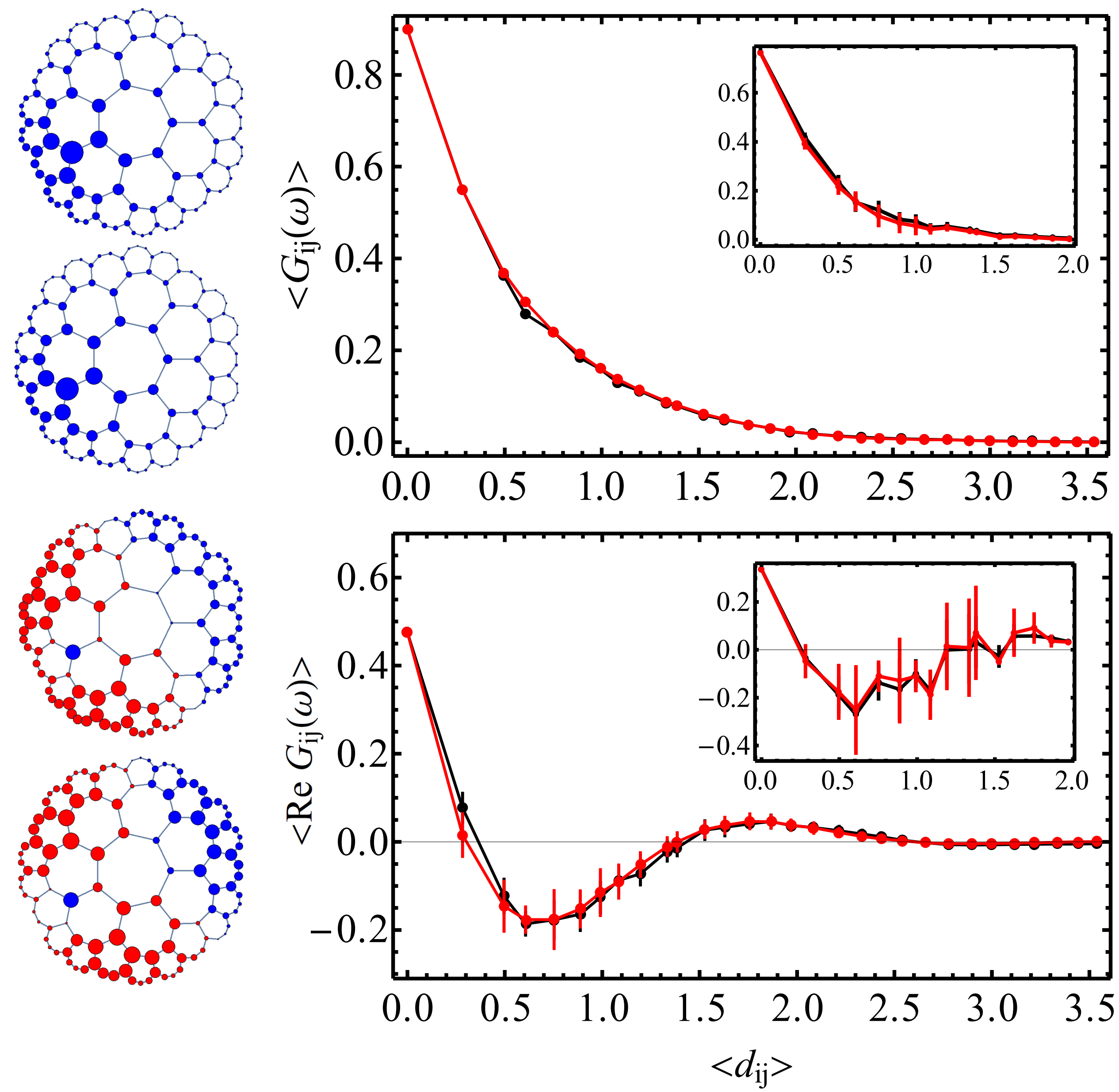

Quantitive Match for Large System Sizes

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- "Ground" state energy

Boettcher *et al*. Phys. Rev. A **102**, 032208 (2020)

Continuum Limit and Green's Function

High Energy Limit of The Spectrum

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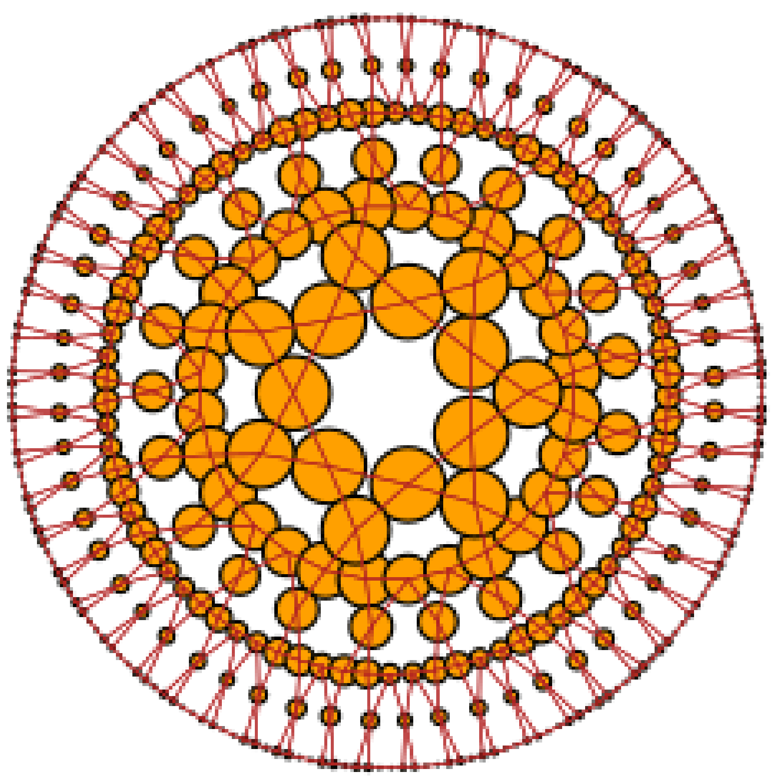
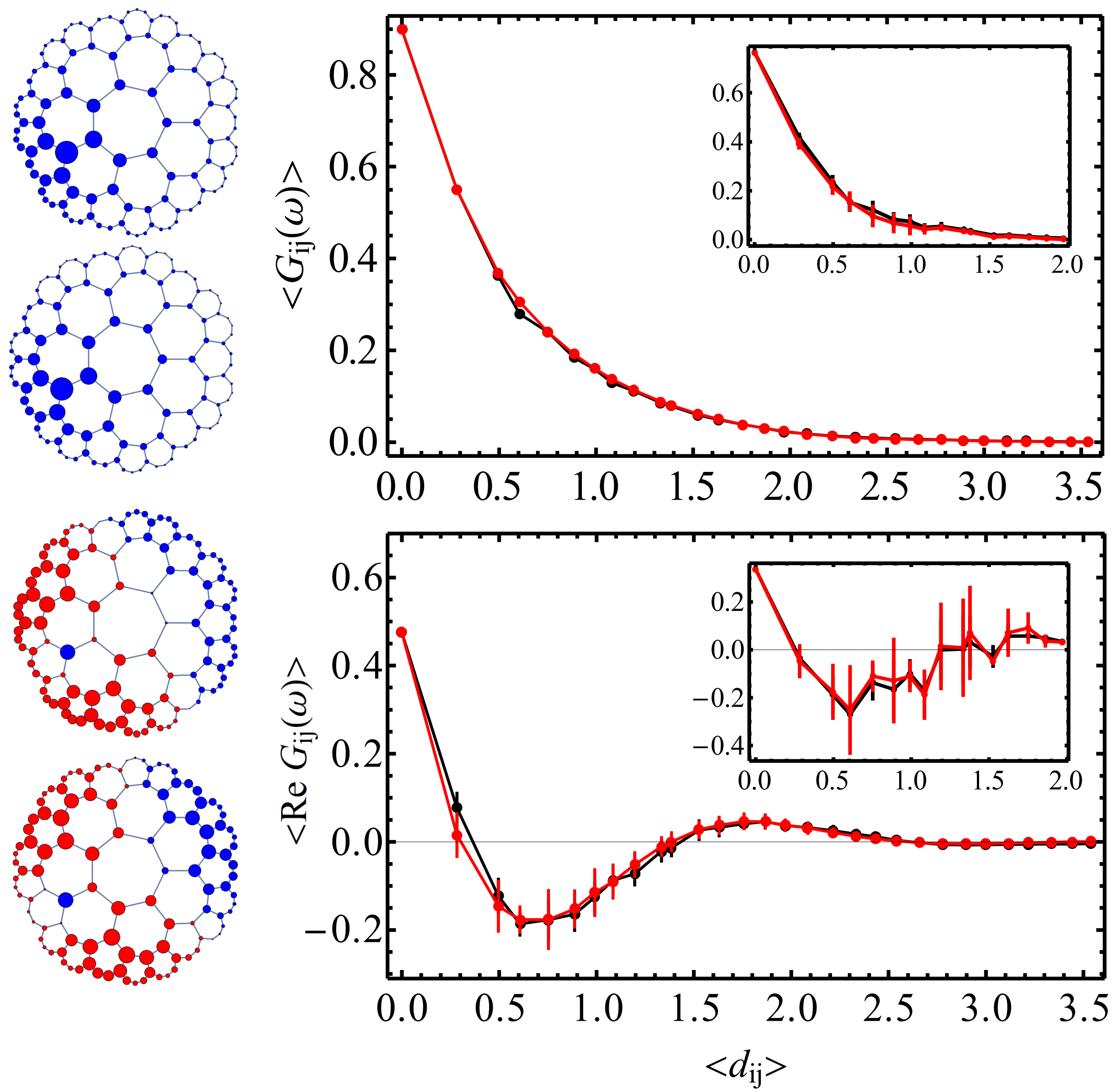
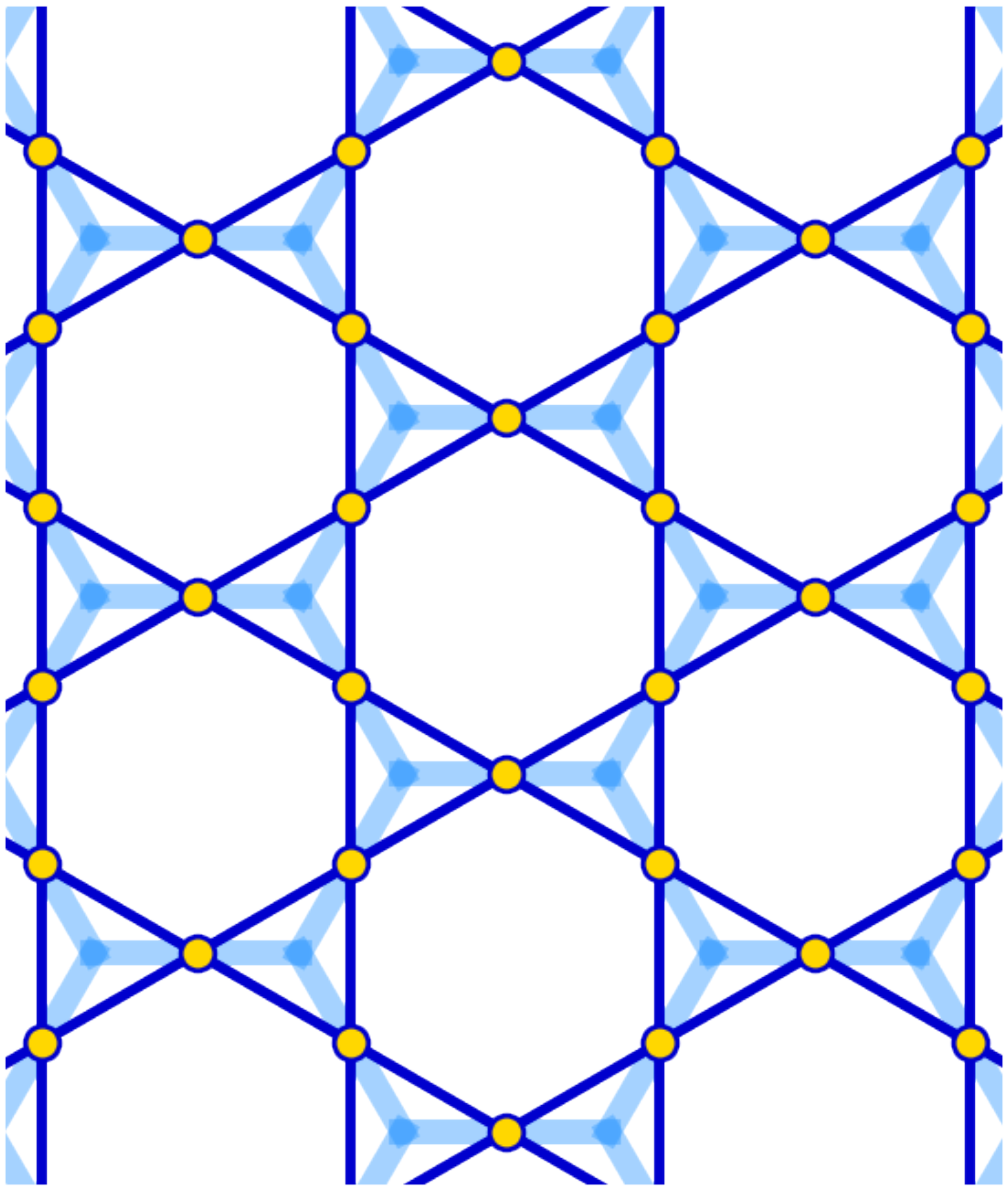


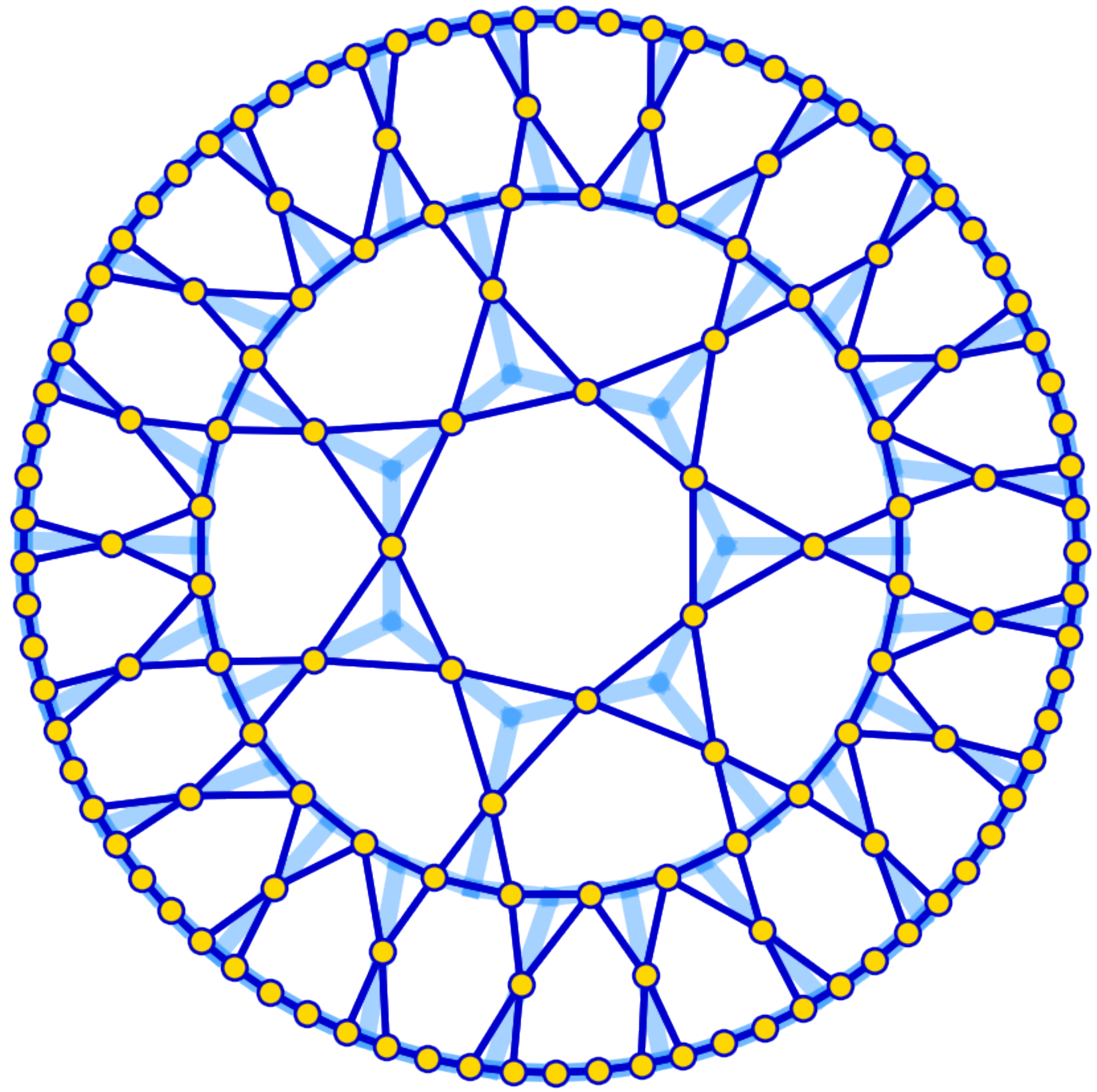


Quantitive Match for Large System Sizes

- Green's function
- "Ground" state energy
- "First" excited state energies.

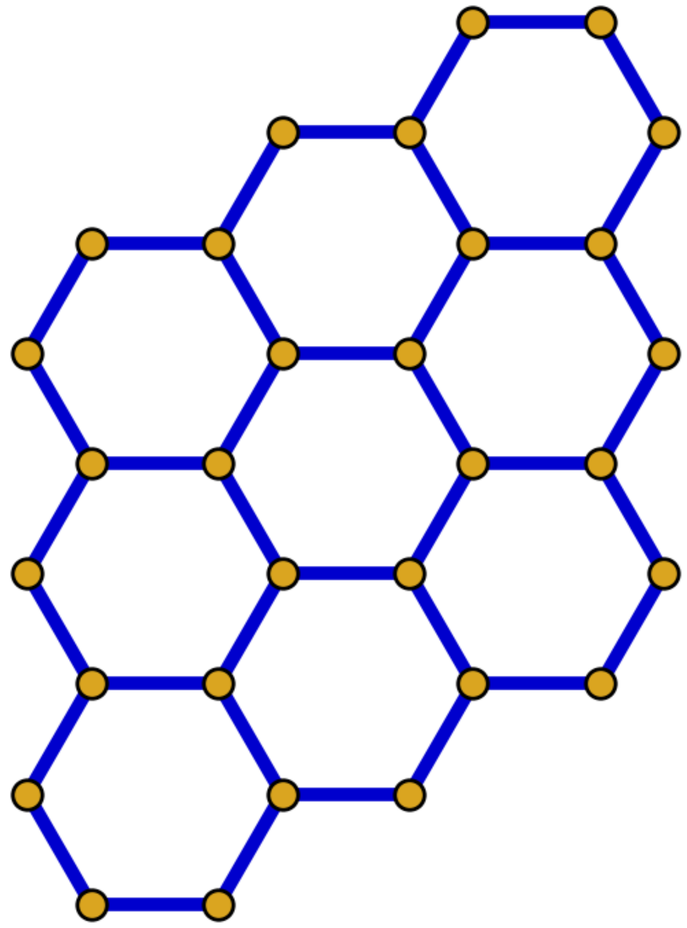
Boettcher *et al*. Phys. Rev. A **102**, 032208 (2020)





Bipartite and Non-Bipartite Graphs

Bipartite



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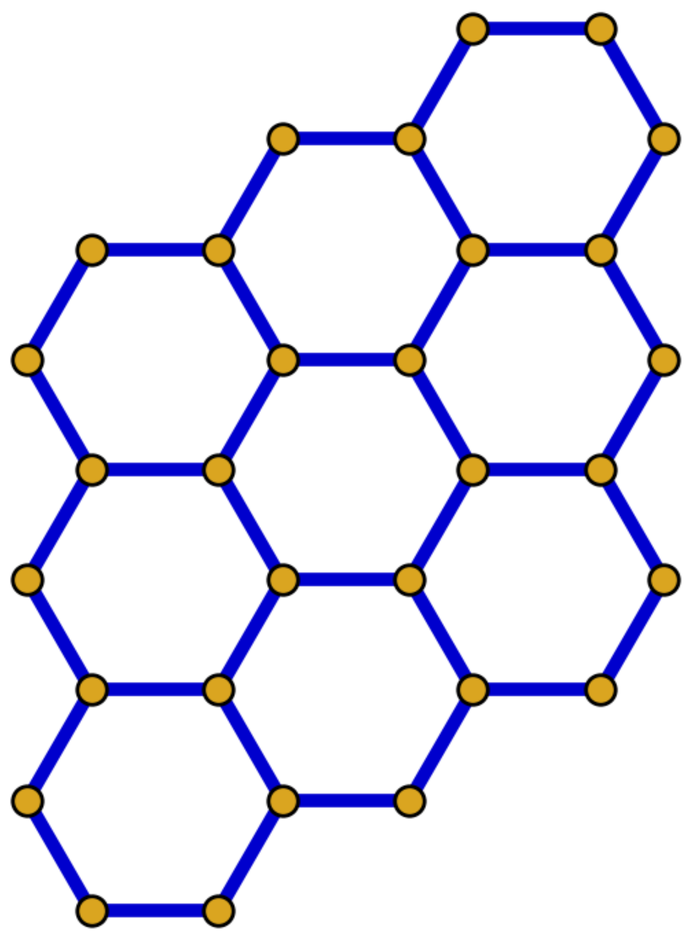
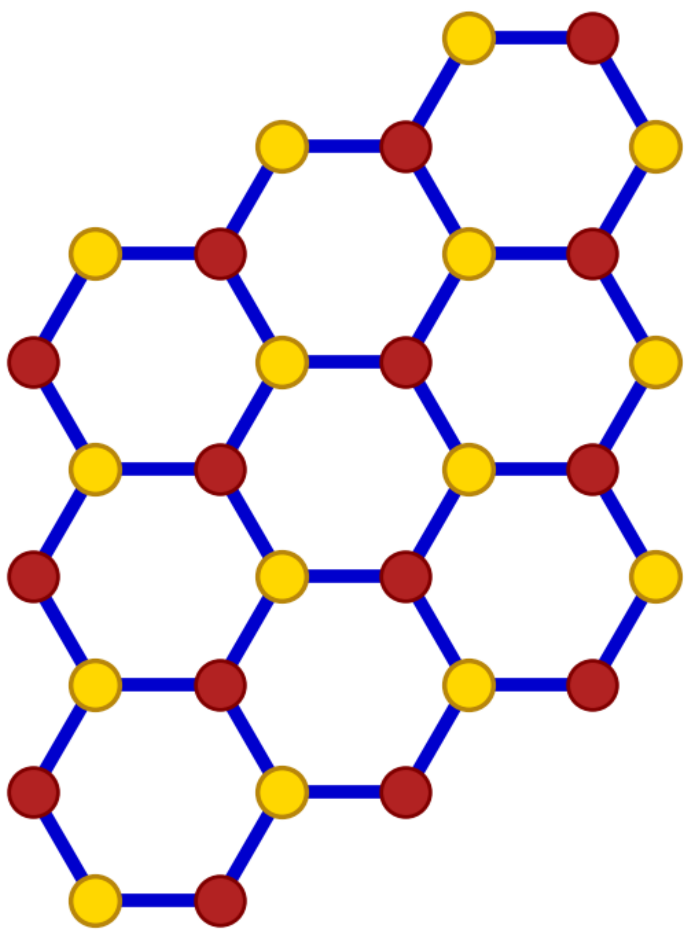


E = -3



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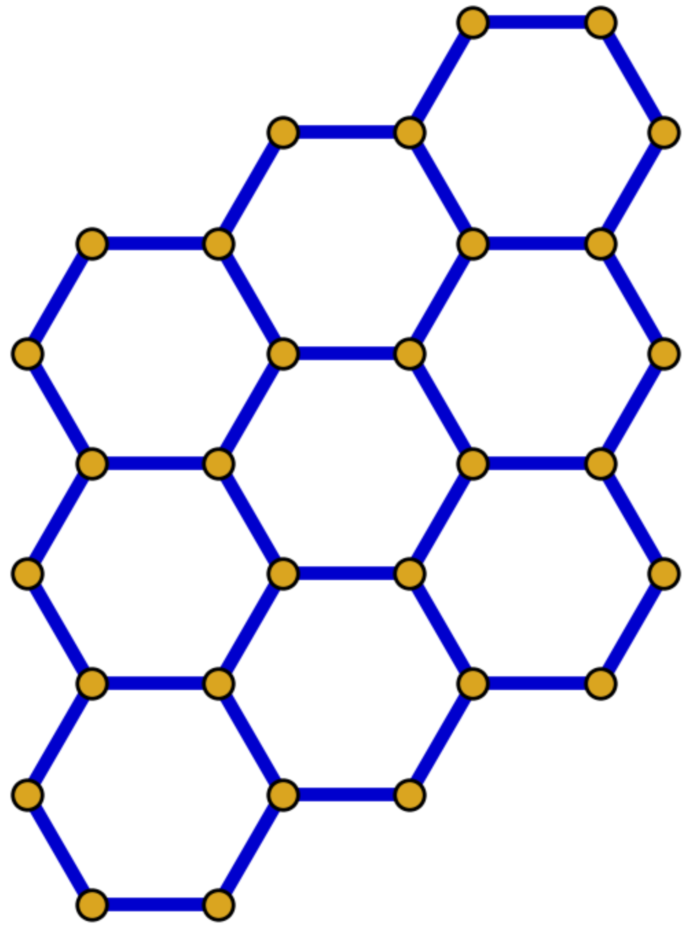

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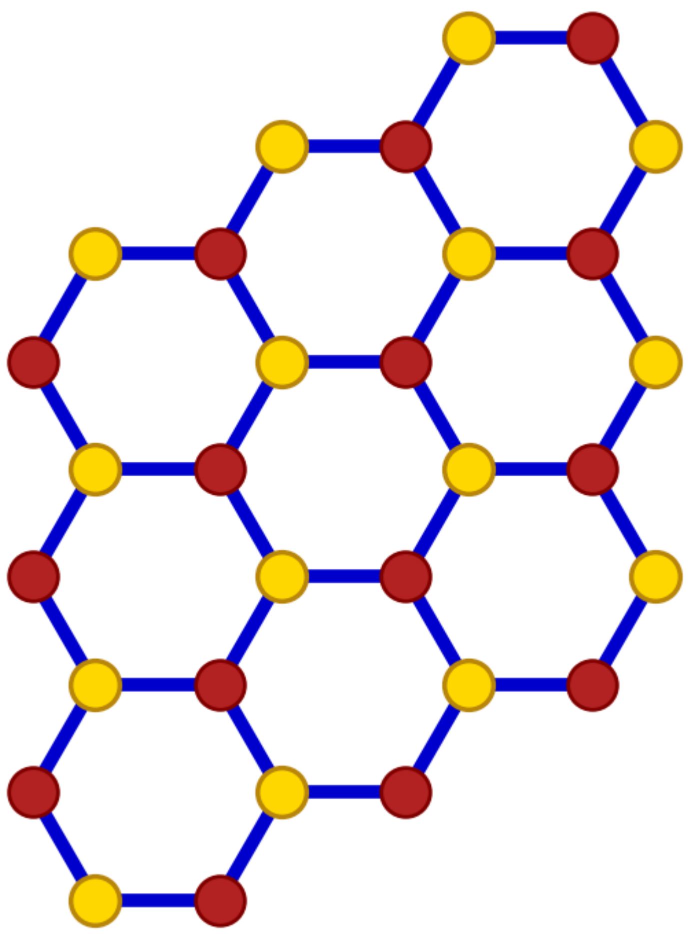
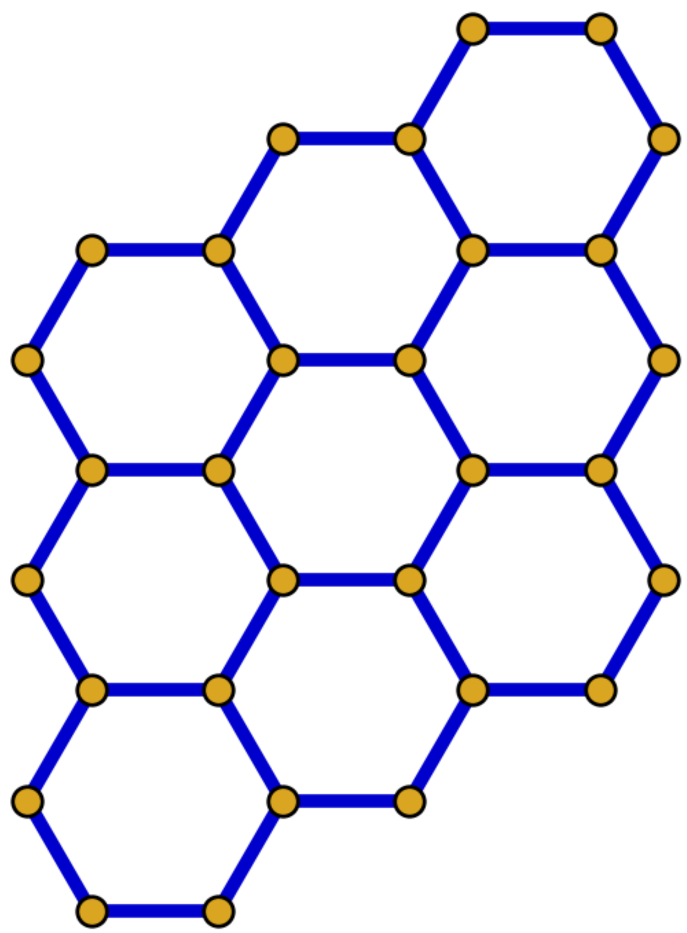


- All neighbors opposite sign

Bipartite and Non-Bipartite Graphs

Bipartite



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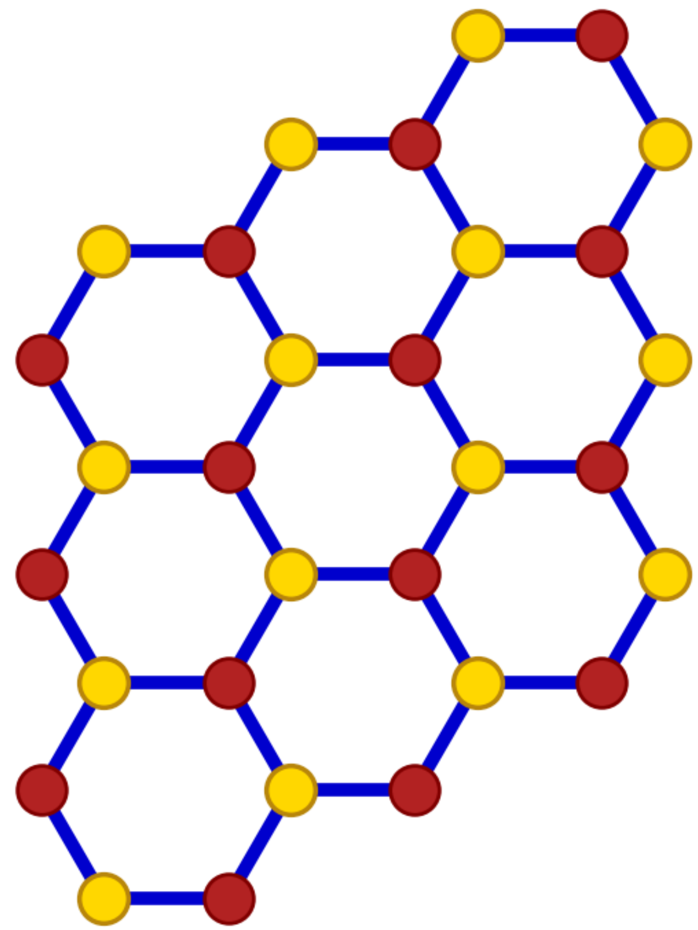
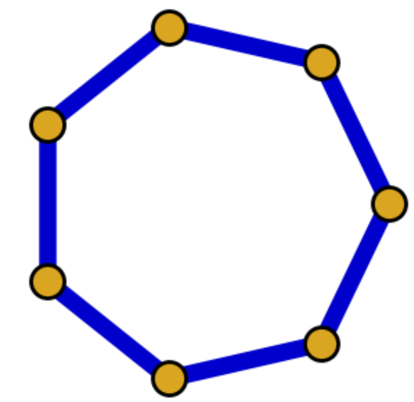


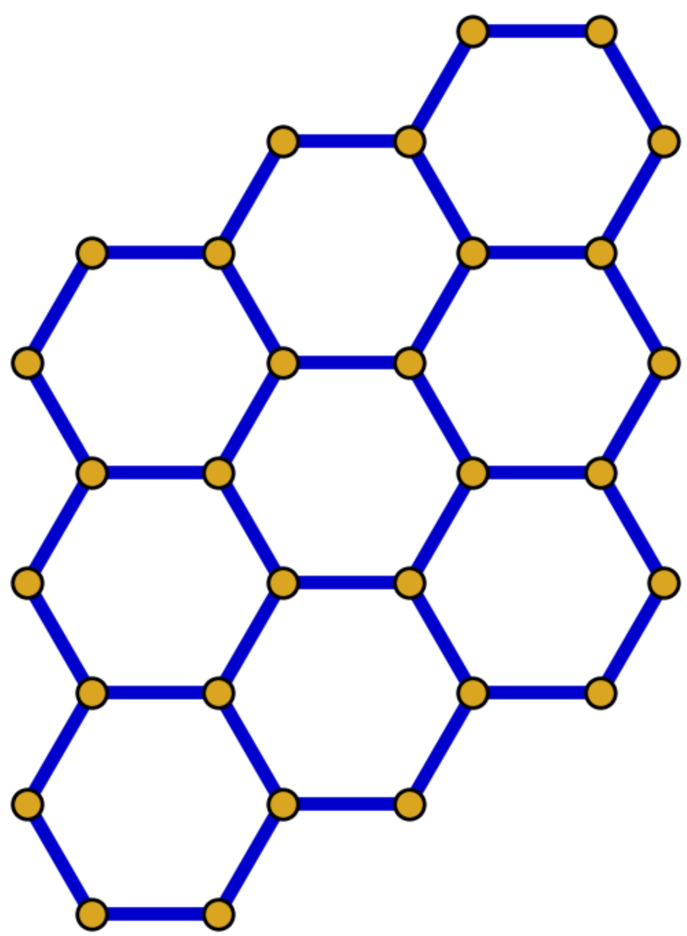
Non-Bipartite



- All neighbors opposite sign

Bipartite and Non-Bipartite Graphs

Bipartite



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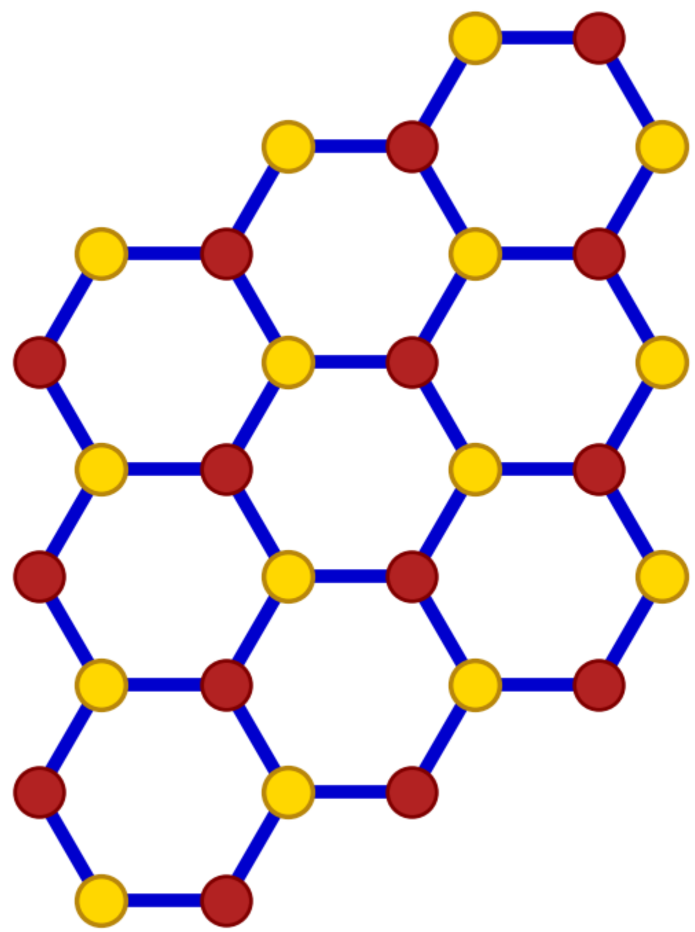

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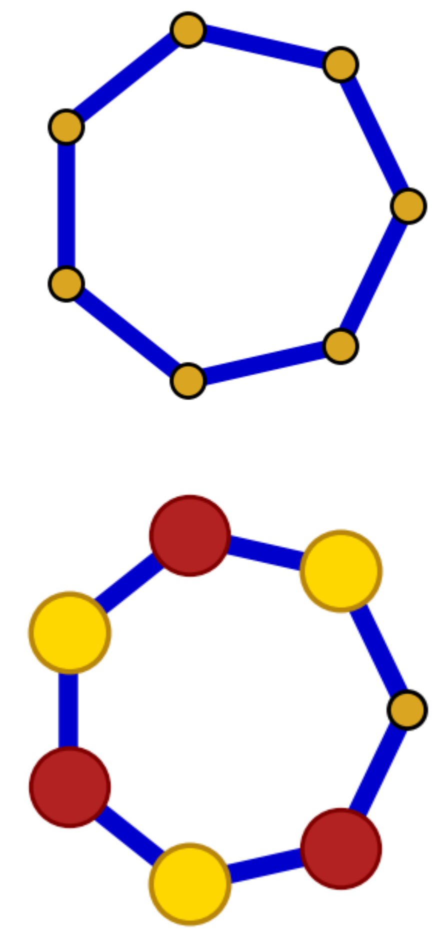

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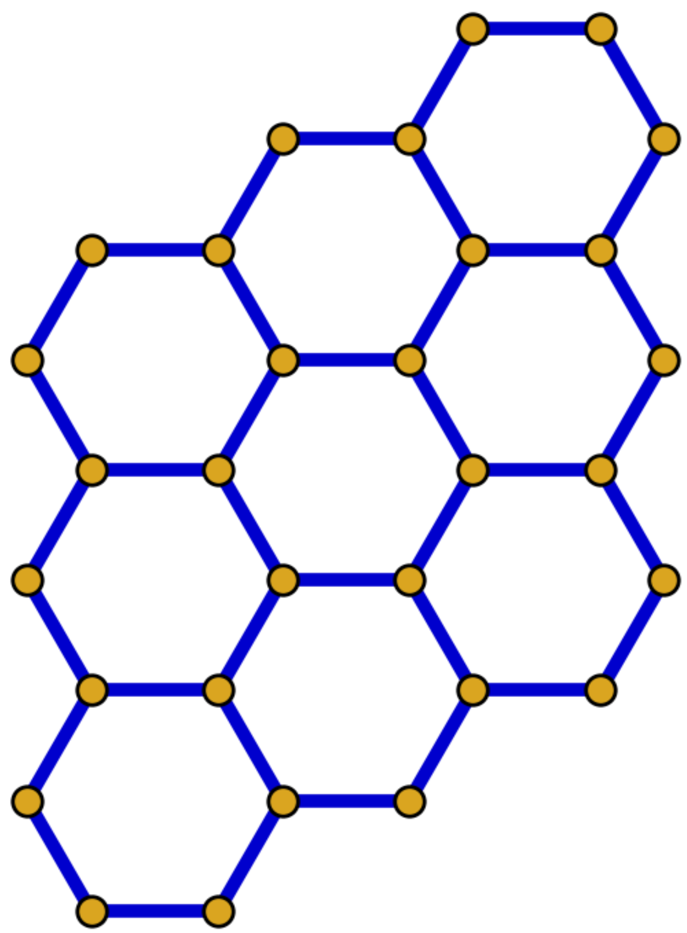

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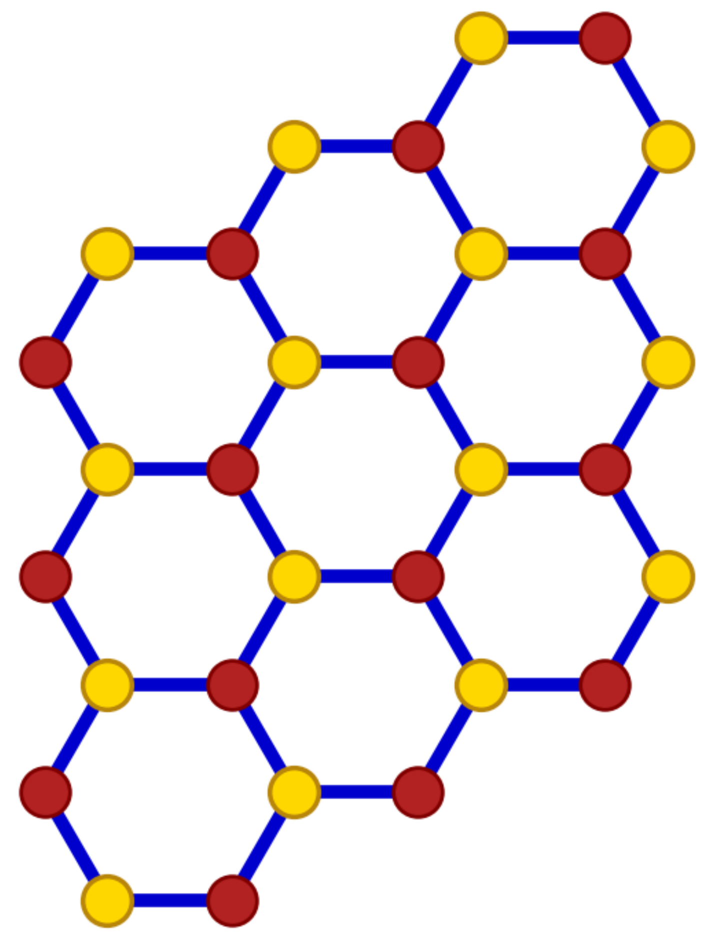

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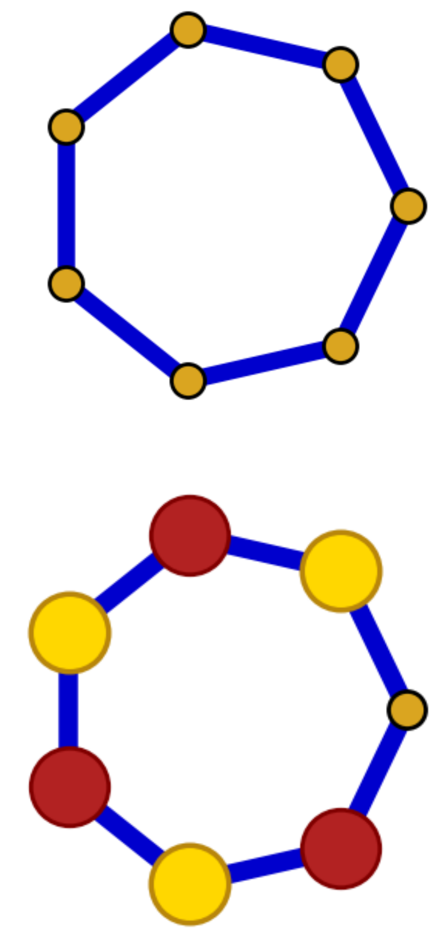

- All neighbors opposite sign
- Not all neighbors can be opposite sign

Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Band Structure Correspondence

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Effective Hamiltonian

- Bounded self-adjoint operator on L(X)

$$\bar{H}_s(X) = H_{L(X)}$$

Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Band Structure Correspondence

Layout Tight-Binding Hamiltonian

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Incidence Operator

- From X to L(X)

$$M : \ell^2(X) \to \ell^2(L(X))$$

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Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

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$$D_X + H_X \simeq 2I + \bar{H}_s(X)$$

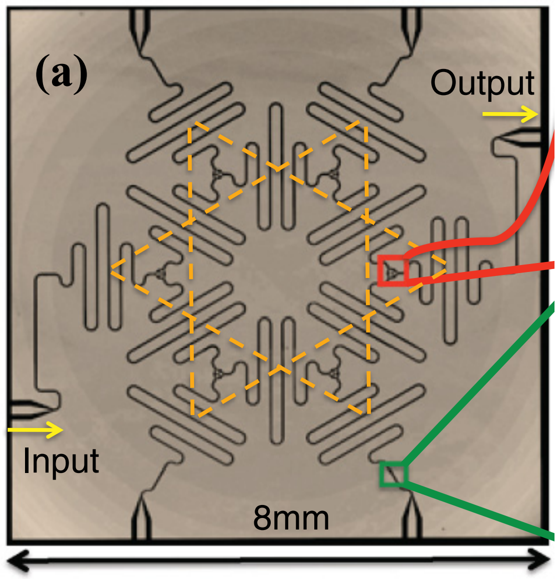
$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

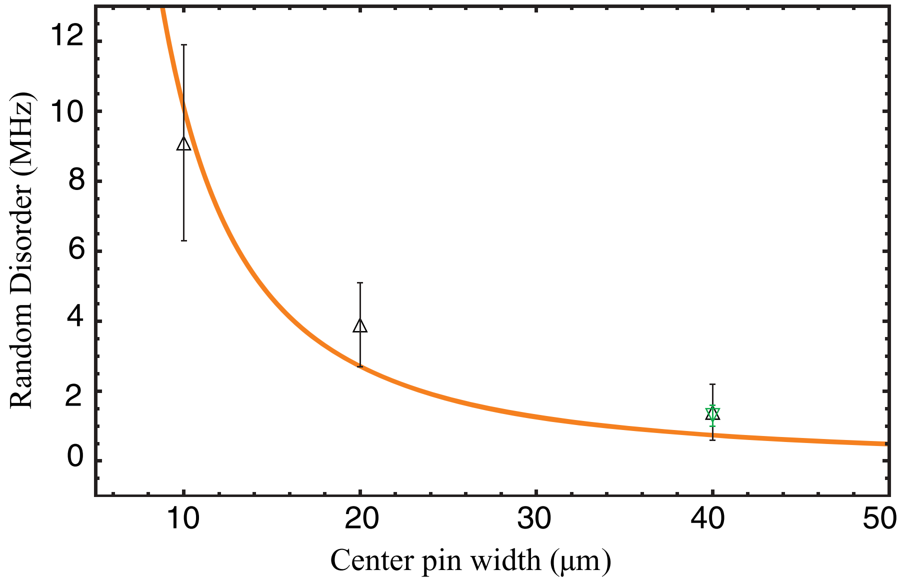
Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Intrinsic Fabrication Disorder

Previous Benchmarks

- Kagome star normal modes
- Fabricated at Princeton
- Fabrication disorder ~3e-4





Underwood *et al*. PRA **86**, 023837 (2012)

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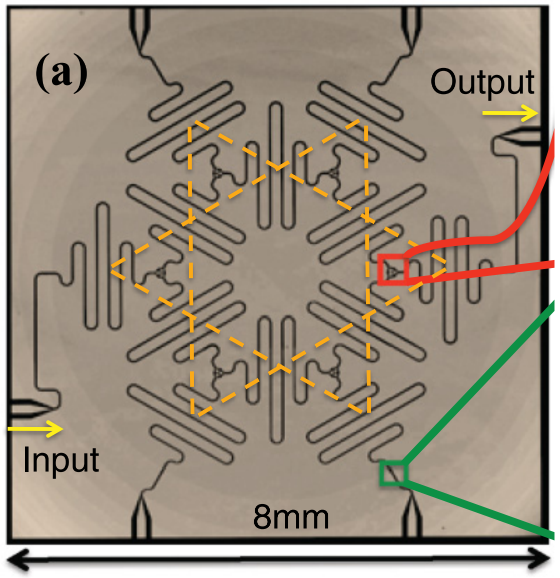
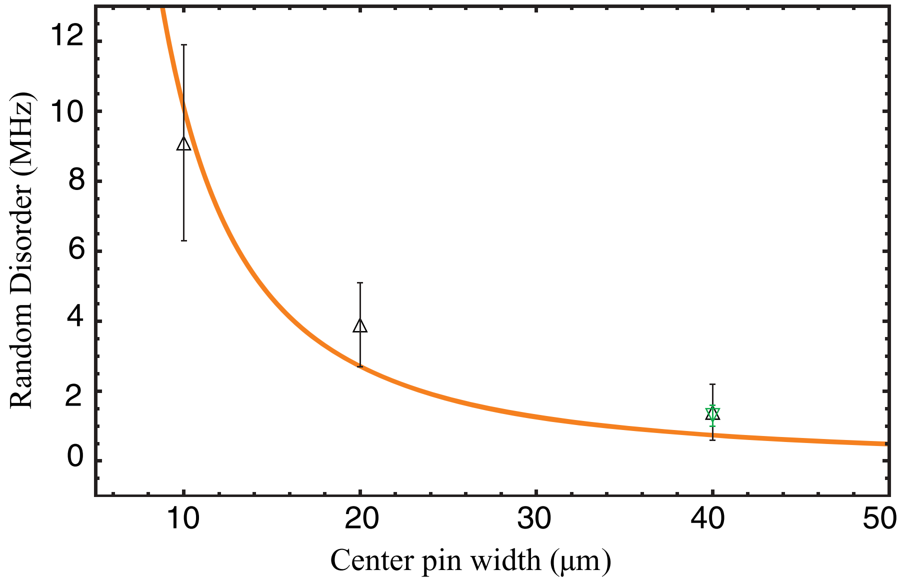
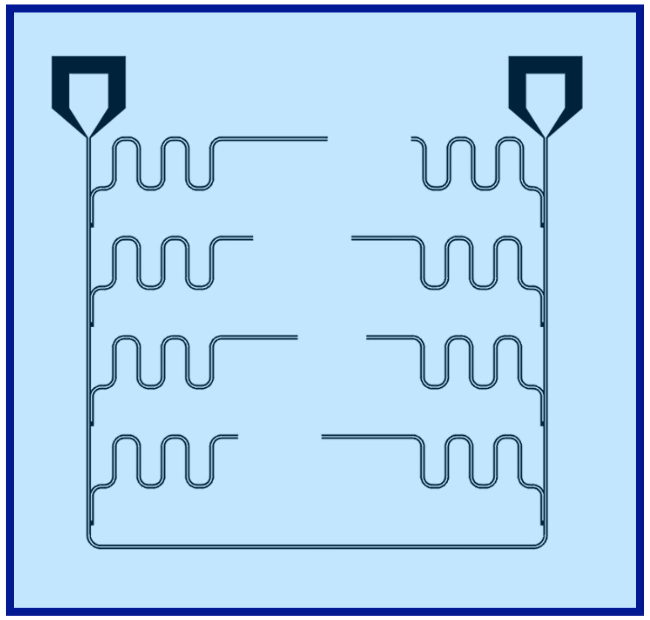




Underwood *et al*. PRA **86**, 023837 (2012)

Current Devices

- Fabricated at UMD
- Fabrication disorder ~3e-4



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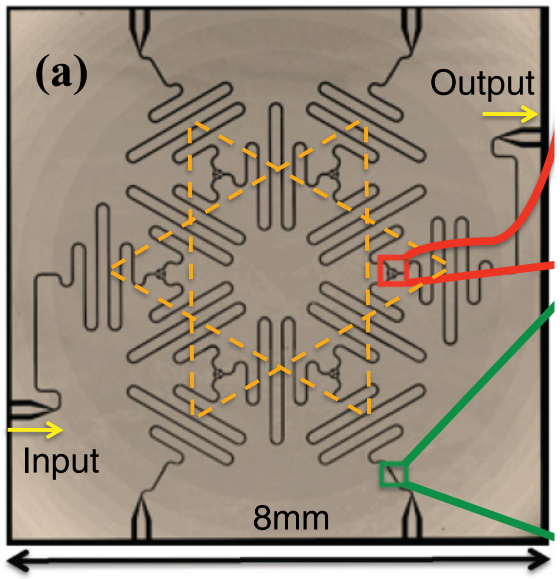

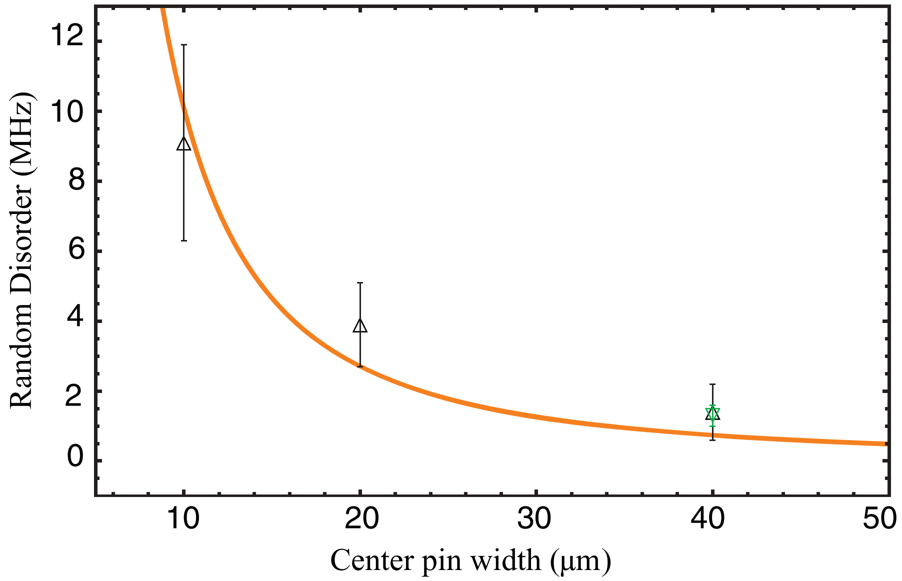

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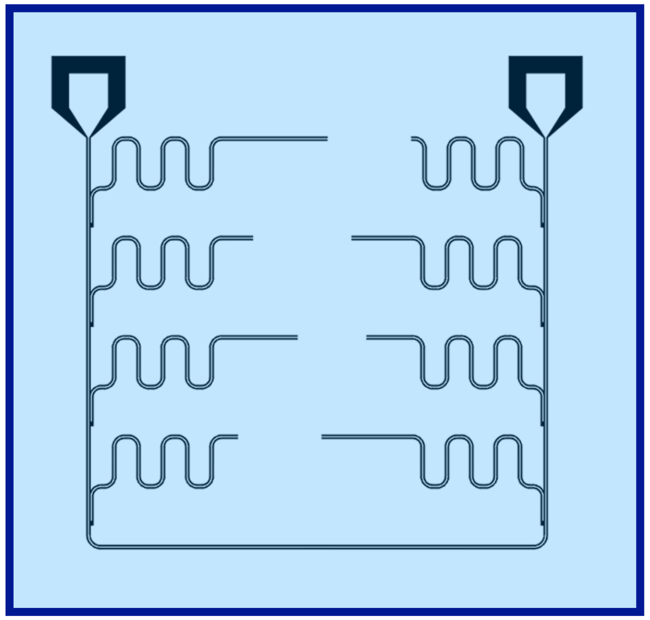
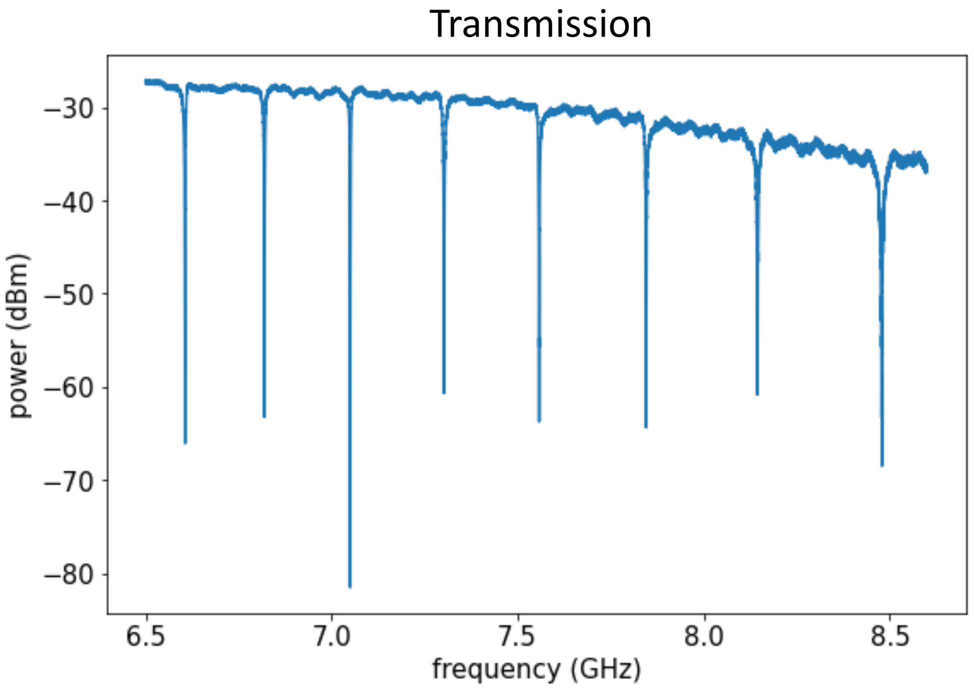
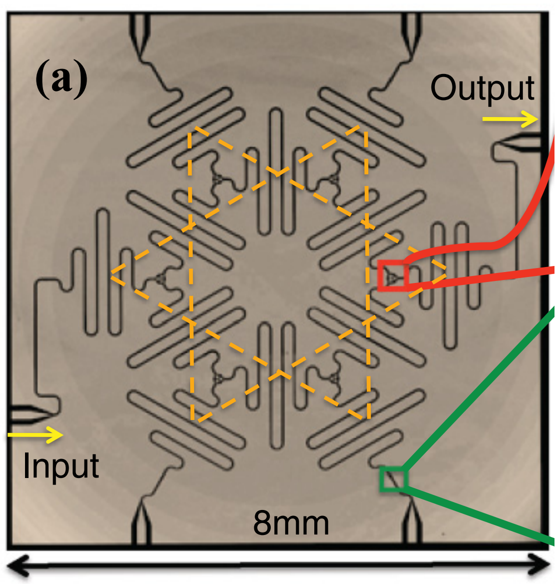


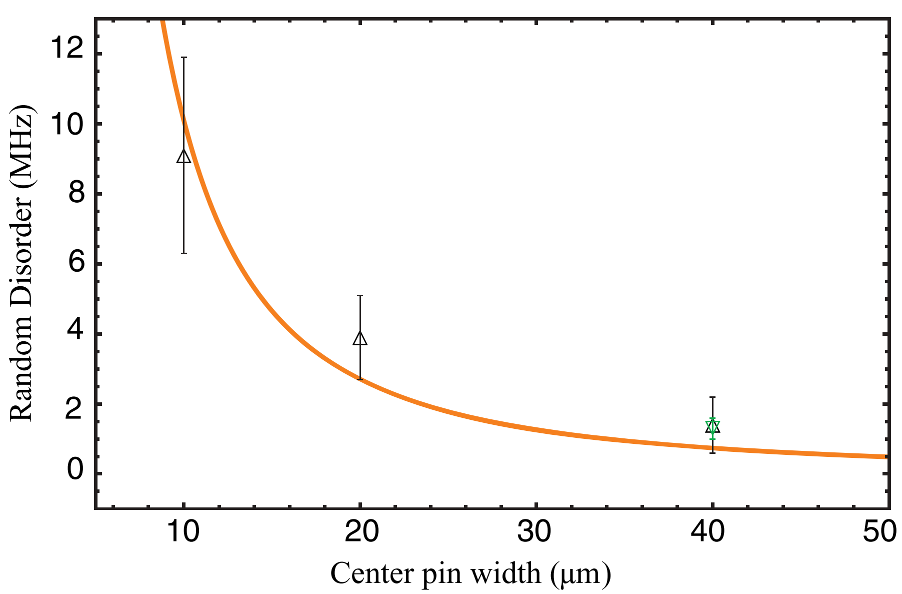


Intrinsic Fabrication Disorder

Previous Benchmarks

- Kagome star normal modes
- Fabricated at Princeton
- Fabrication disorder ~3e-4





Underwood *et al*. PRA **86**, 023837 (2012)

Current Devices

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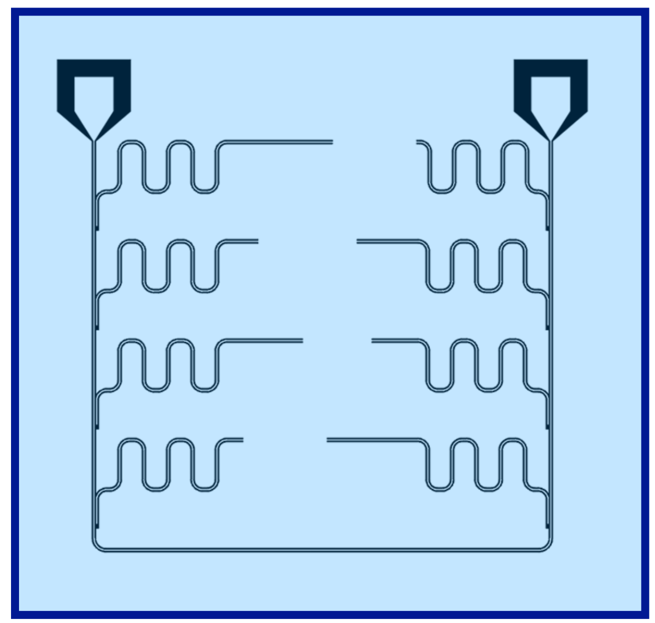
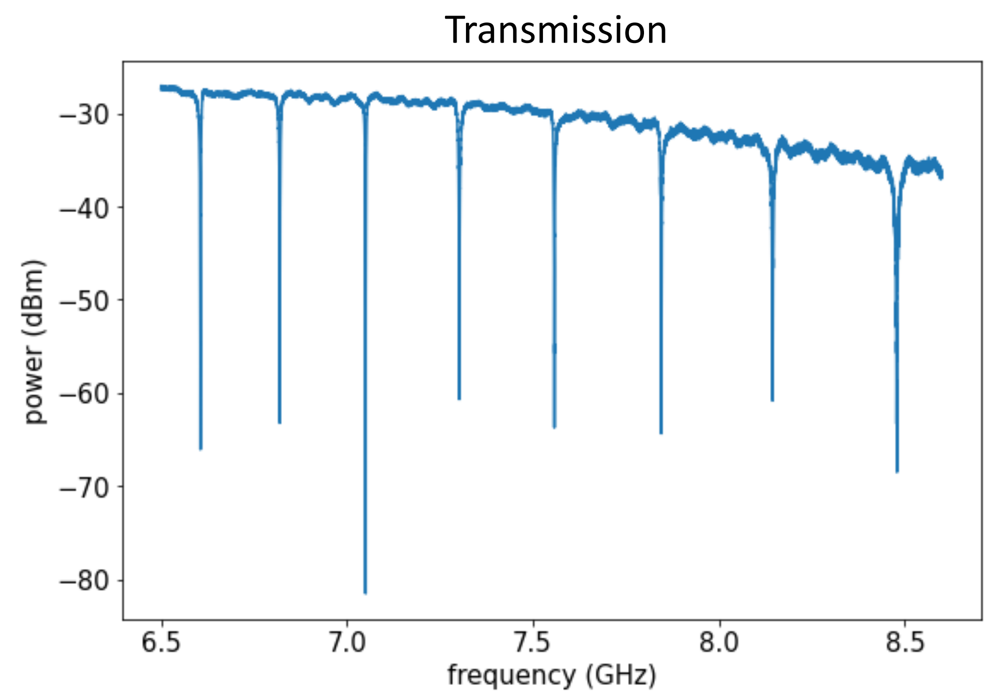
- Fabrication disorder ~3e-4





- Parallel measurement

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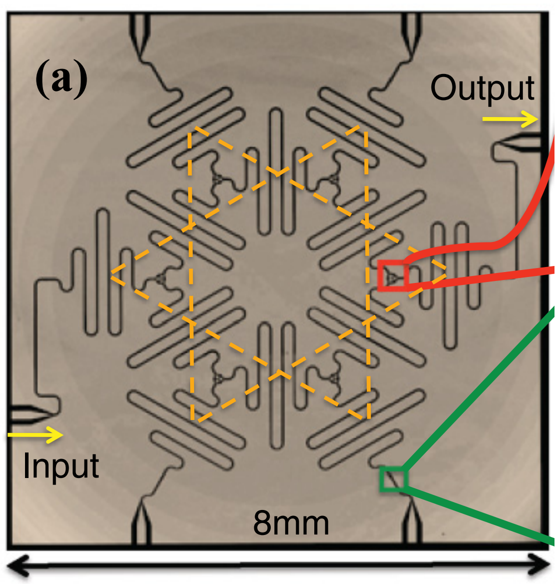

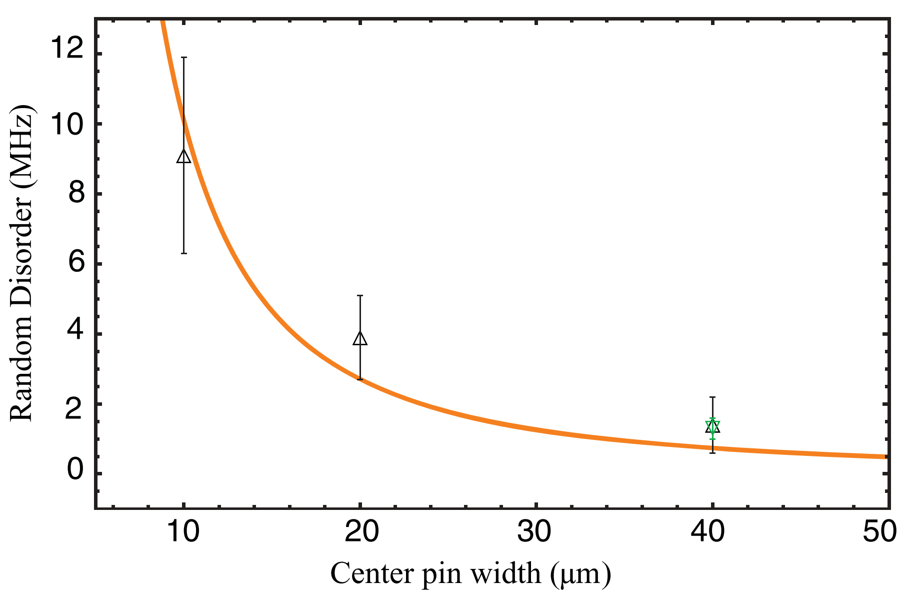

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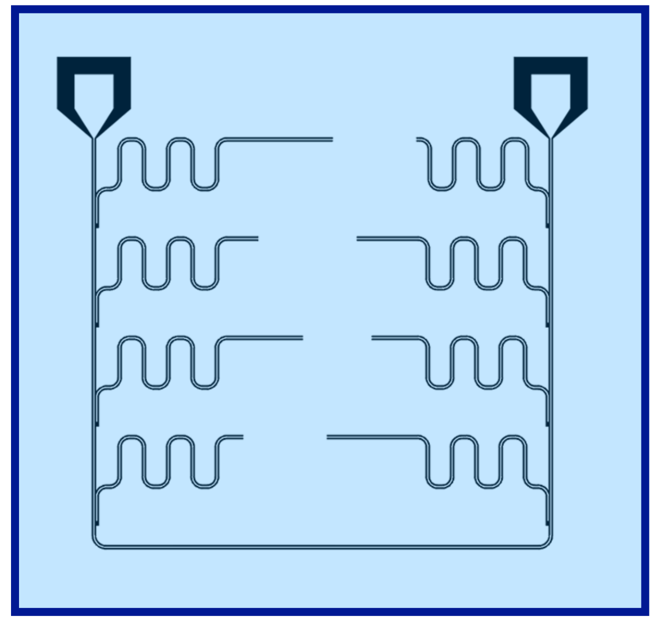

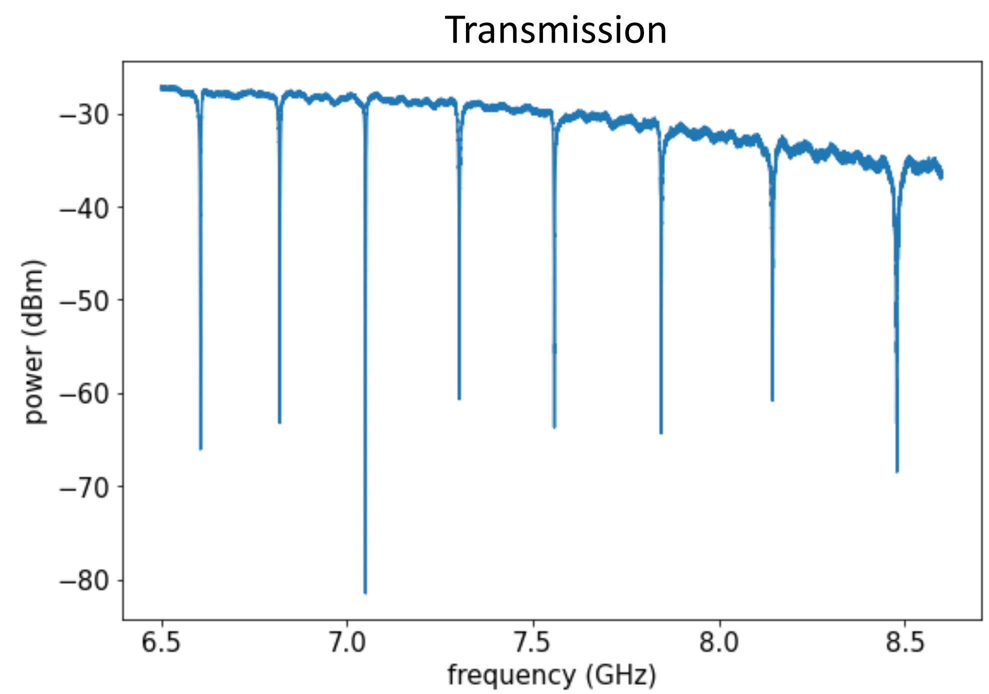

- Parallel measurement
- Disorder extracted from comb spacing

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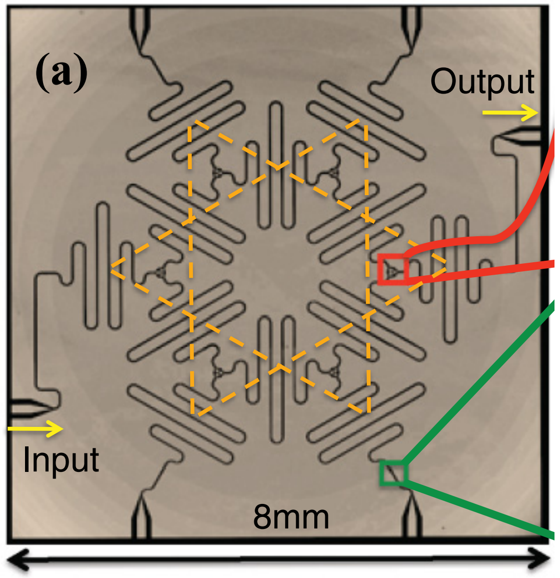

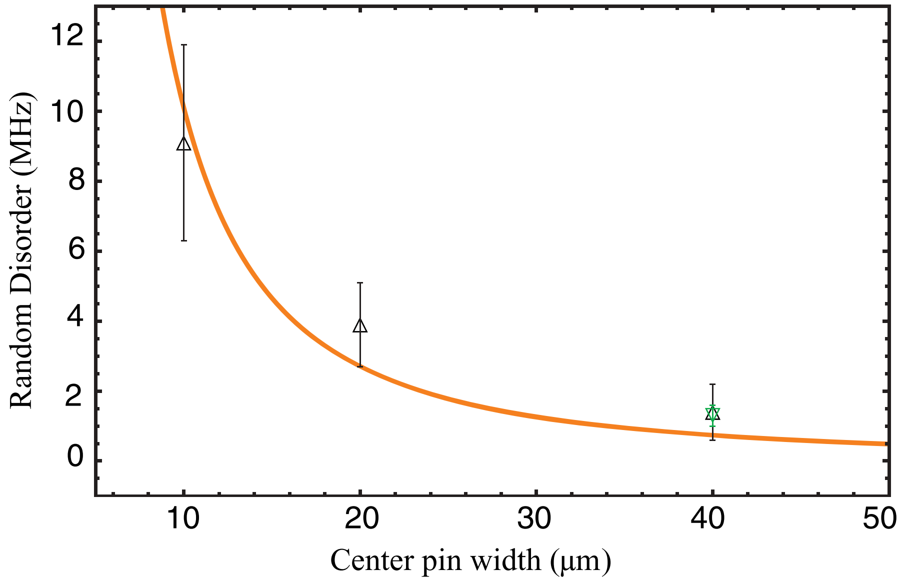

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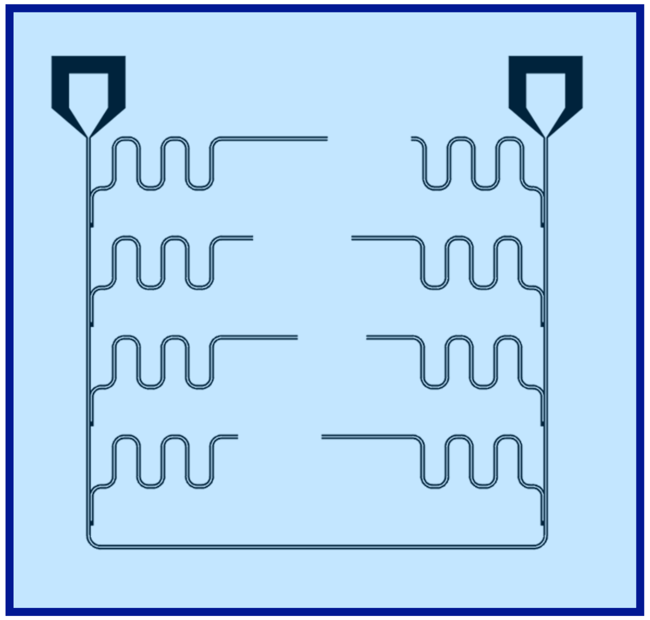

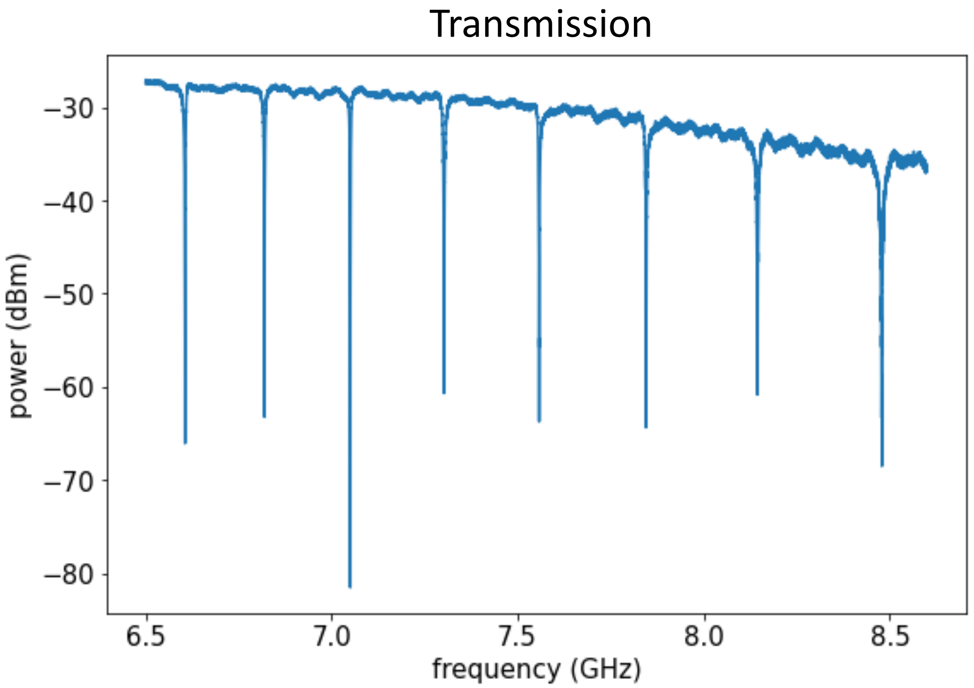

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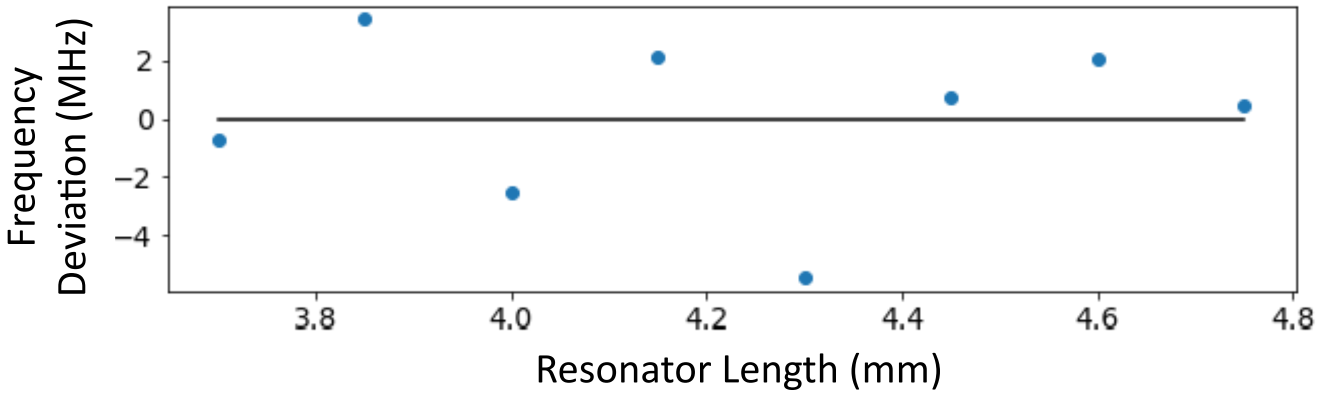

Disorder Mitigation

Systematic v. Random Disorder

- Fabrication disorder ~3e-4
- Shape-dependent disorder ~2-3e-3

Underwood *et al*. PRA **86**, 023837 (2012)   Kollár *et al*. Nature **571** (2019)

Disorder Mitigation

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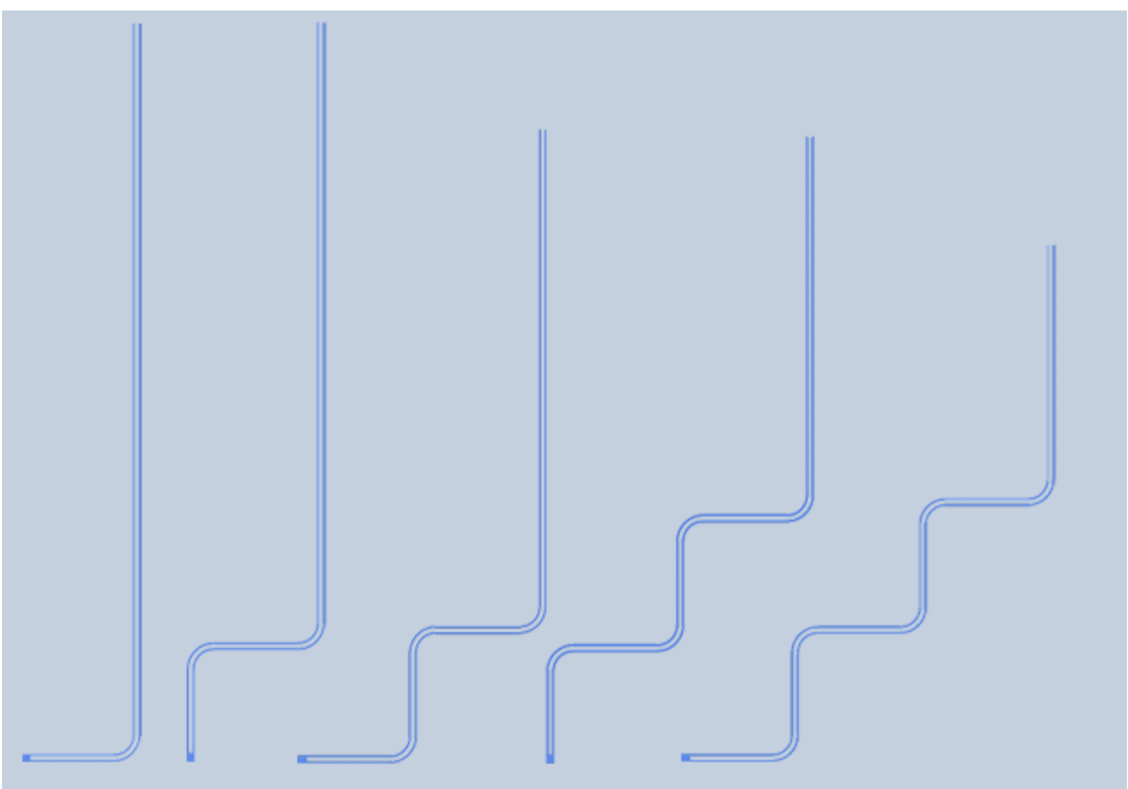
Numerical Test Geometries



Underwood *et al*. PRA **86**, 023837 (2012) Kollár *et al*. Nature **571** (2019)

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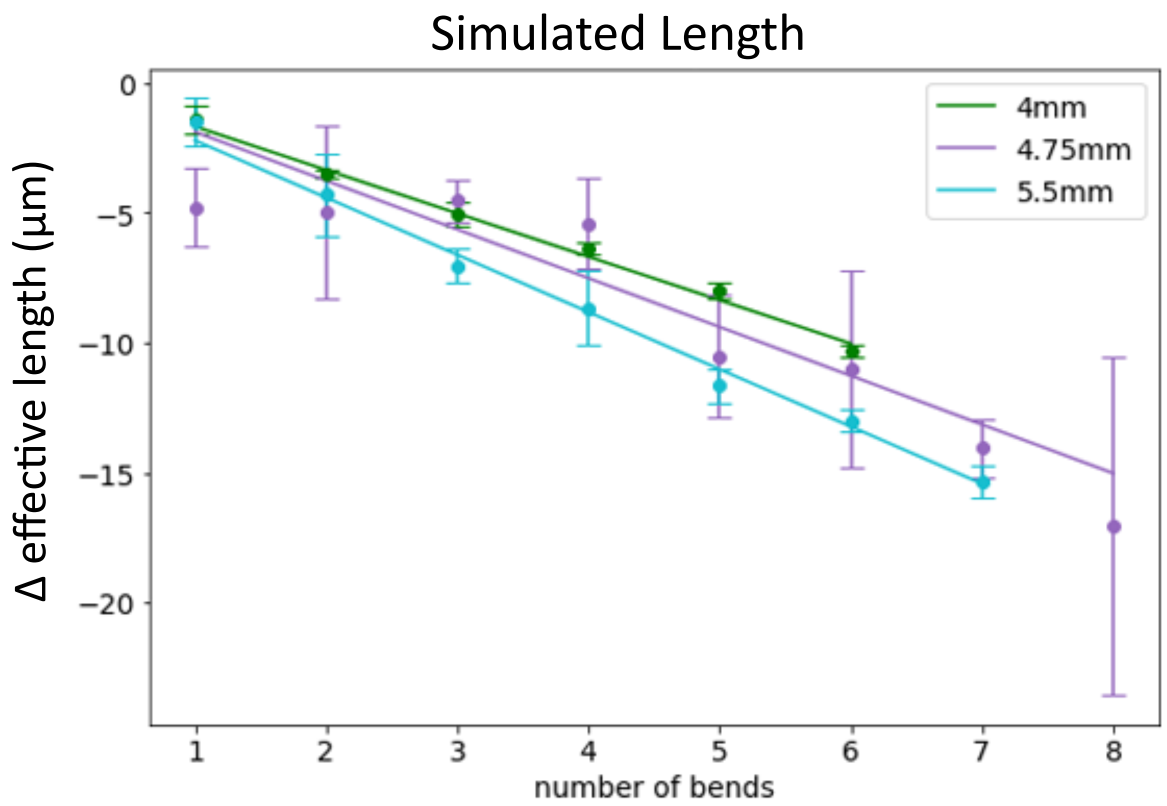

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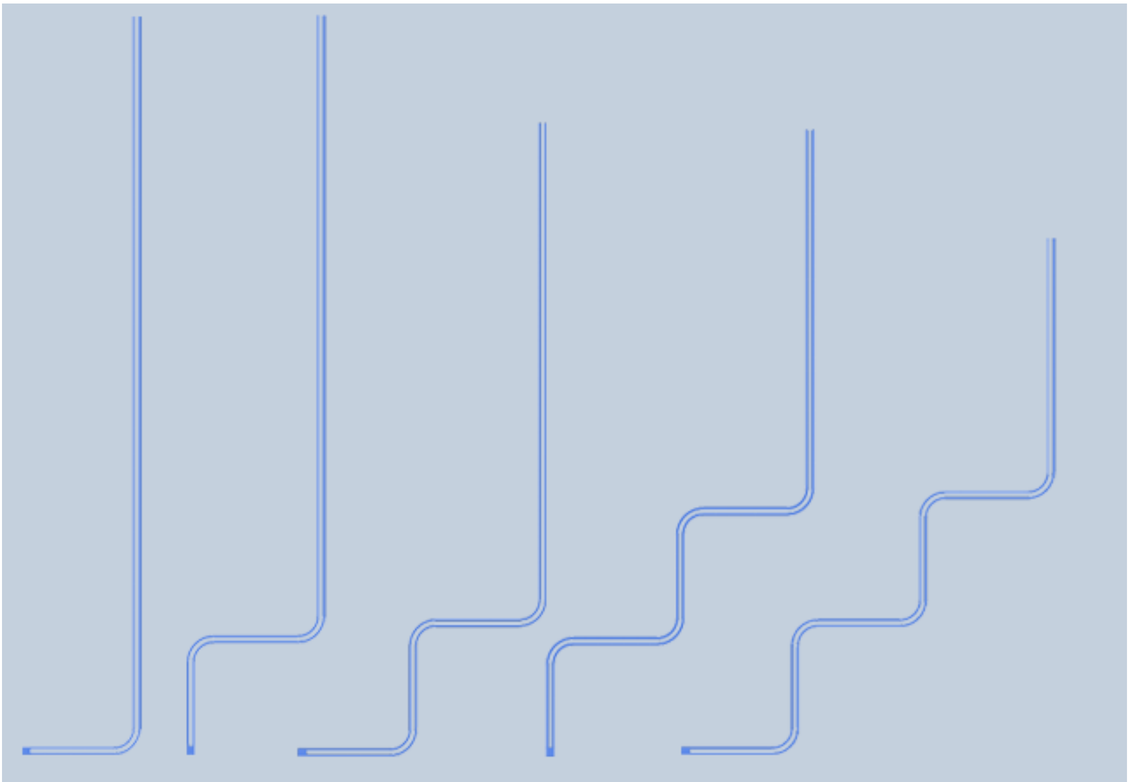

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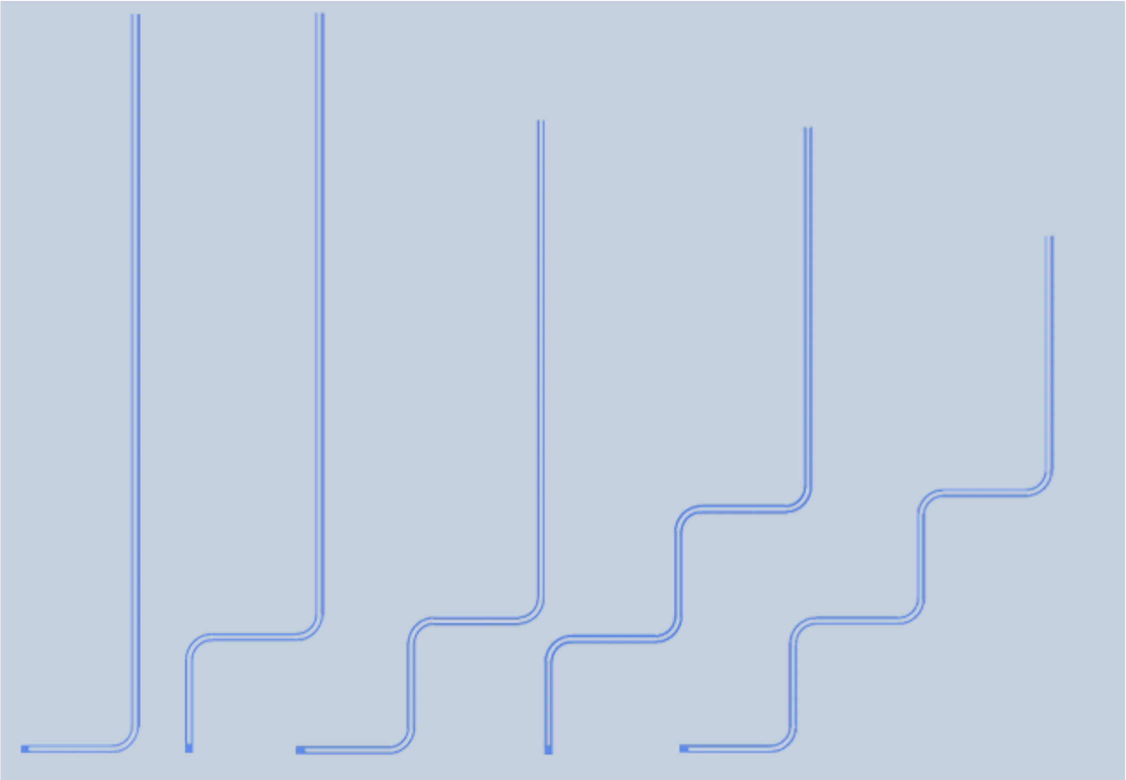

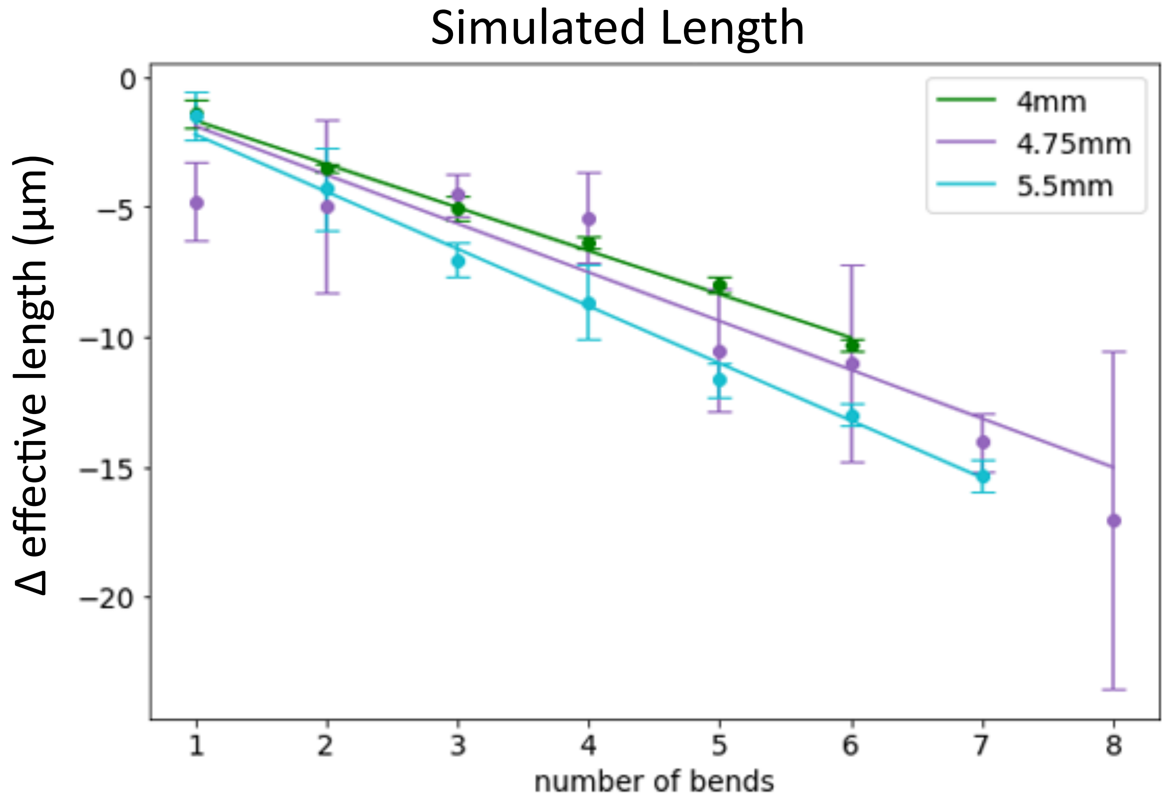

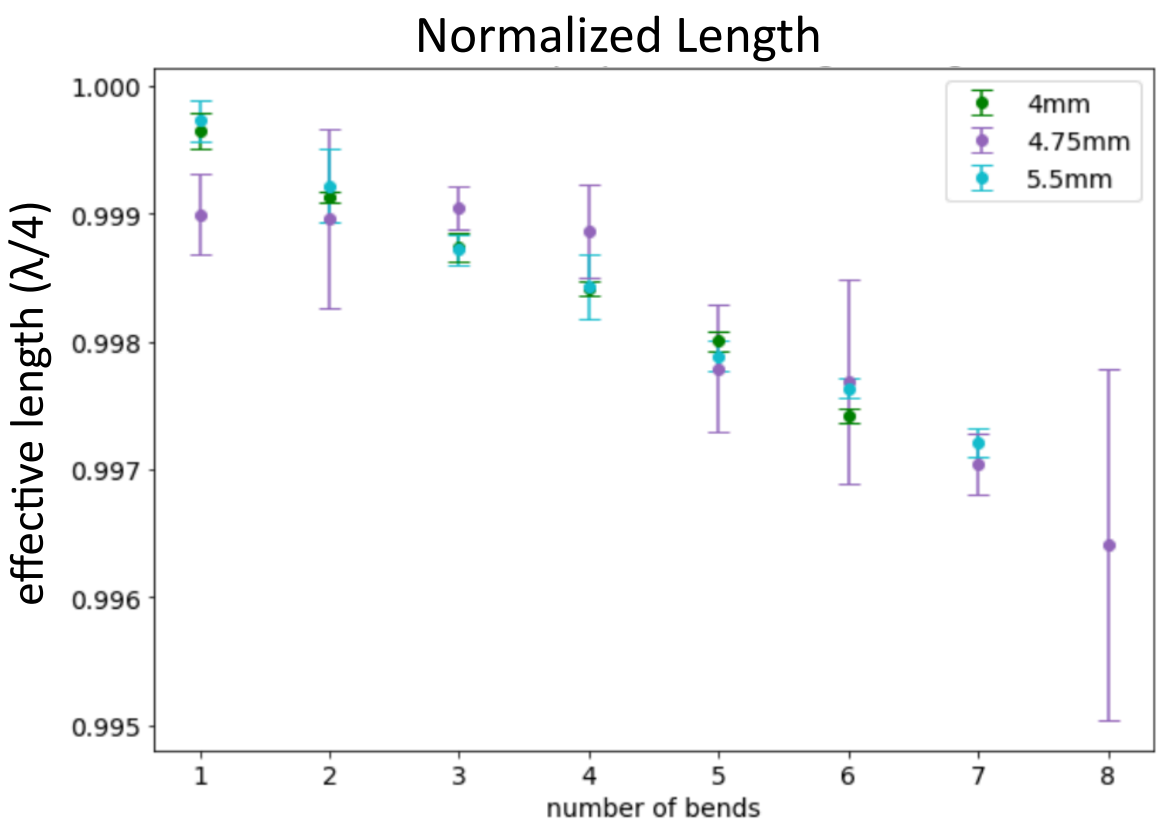

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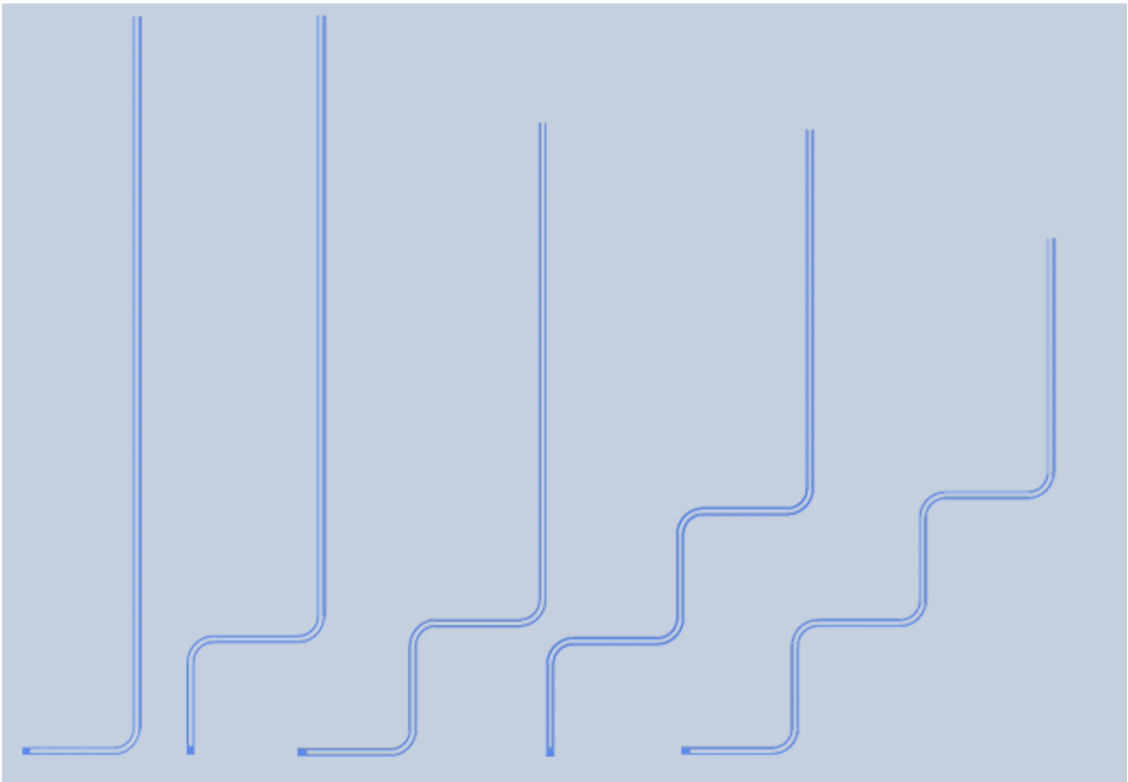
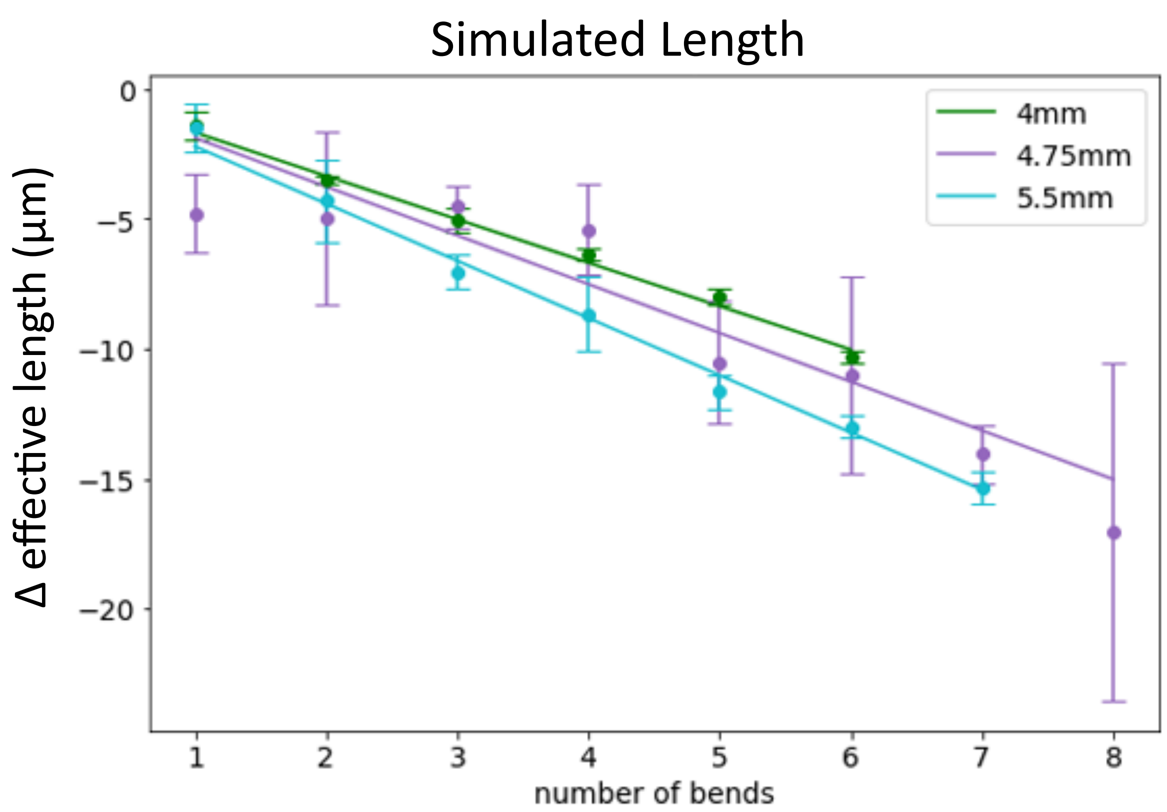
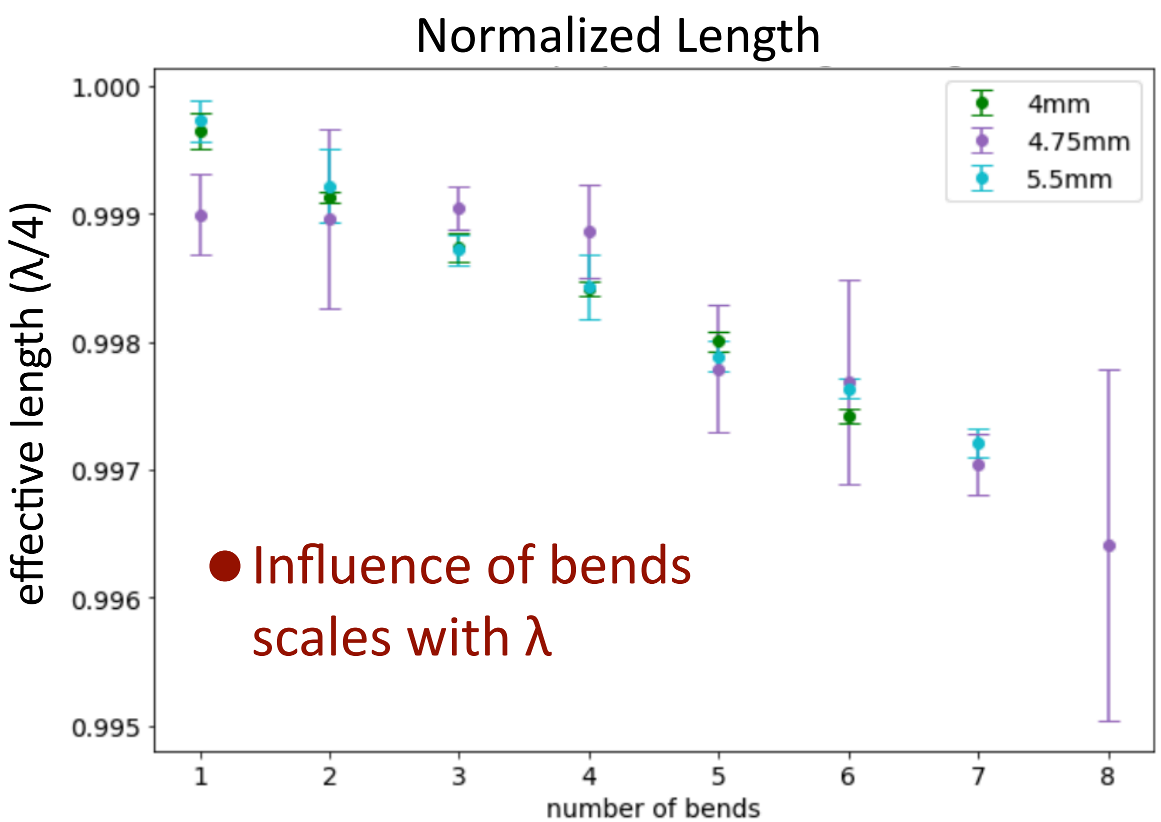




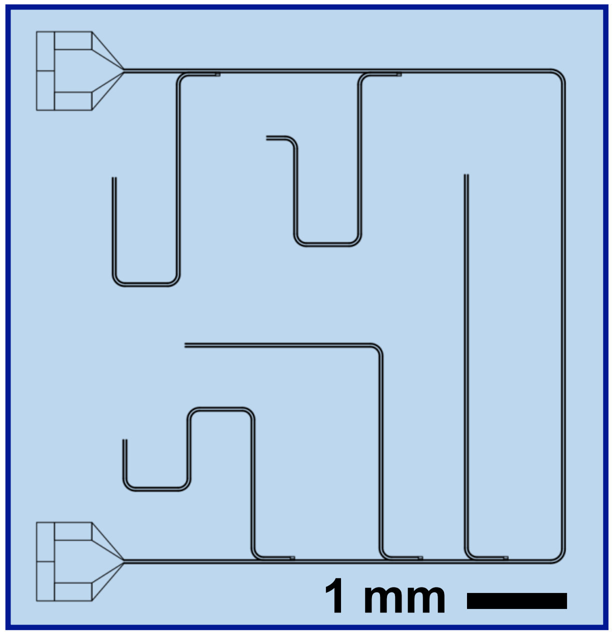


- Influence of bends scales with λ

Underwood *et al*. PRA **86**, 023837 (2012)   Kollár *et al*. Nature **571** (2019)

Disorder Mitigation

First Generation Test Device



- Varied number of bends

Disorder Mitigation

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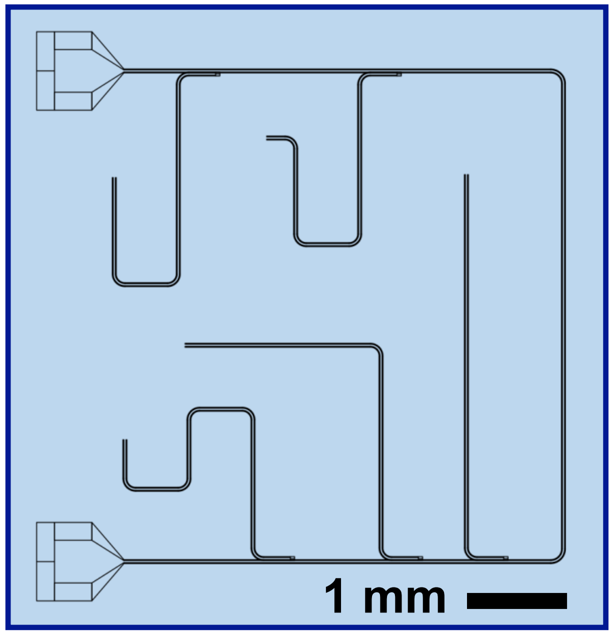

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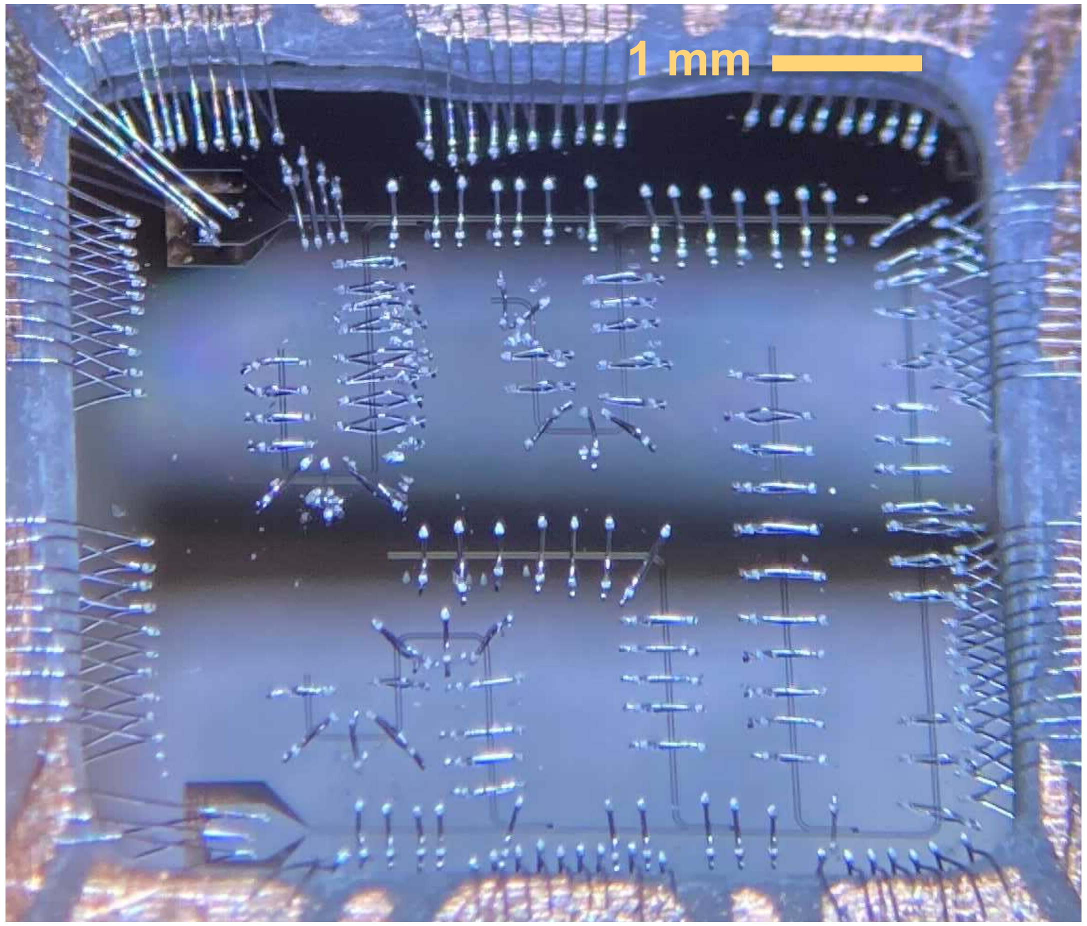

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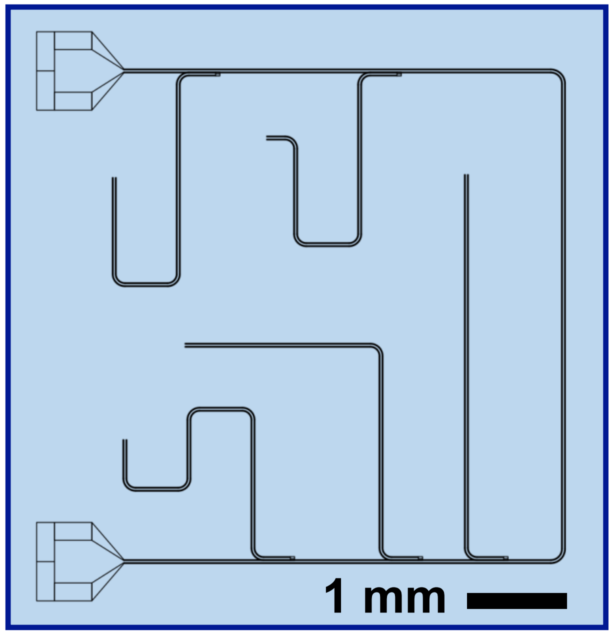

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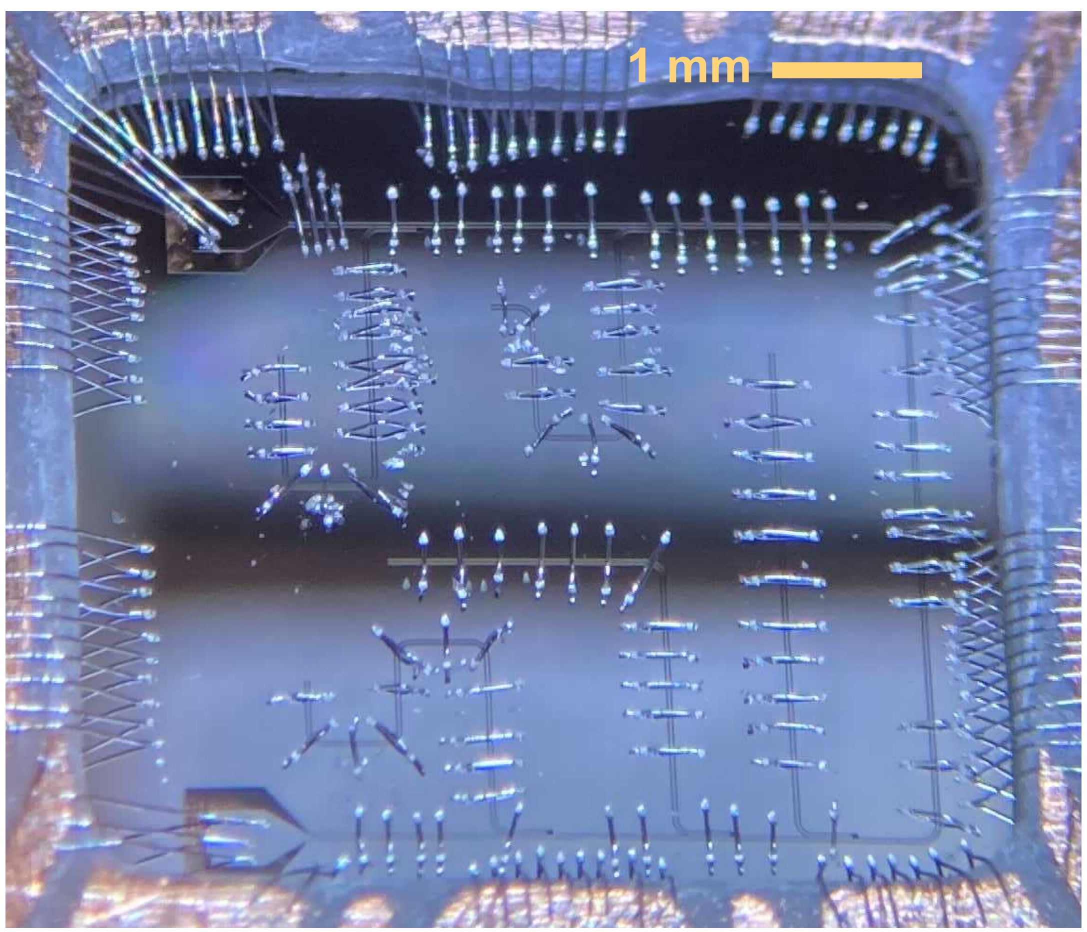
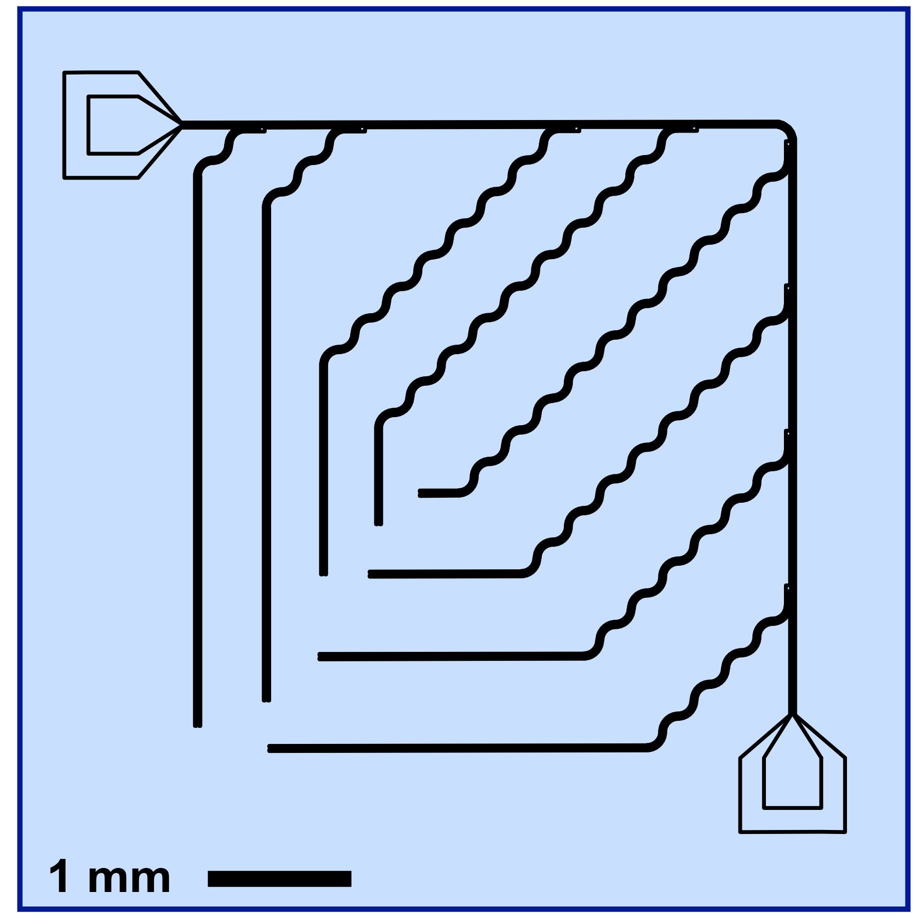


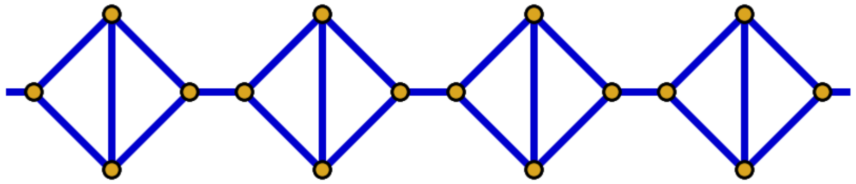
Second Generation Device

- Higher dynamic range (in progress)

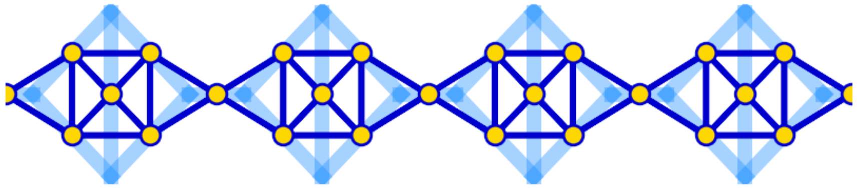


Quasi-1D Lattice Device

Hardware Layout



Effective Lattice



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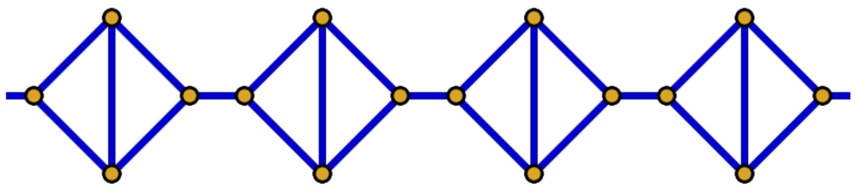

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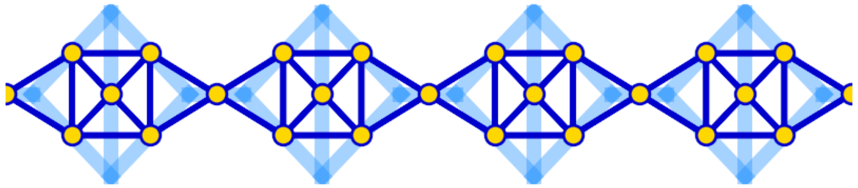
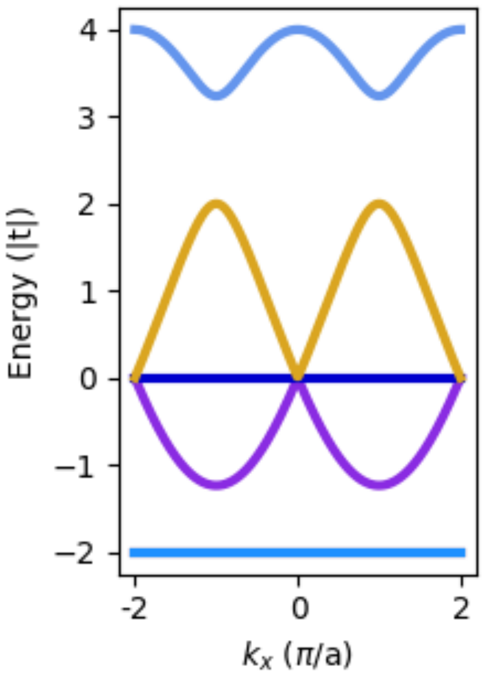


Band Structure



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    - Gapped
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- Quadratic bands

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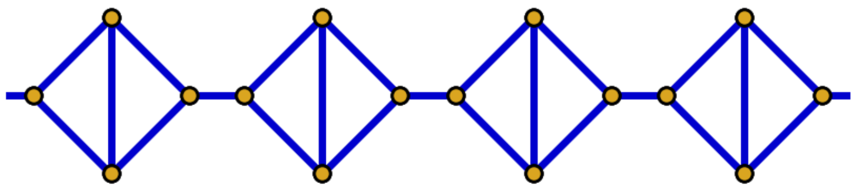

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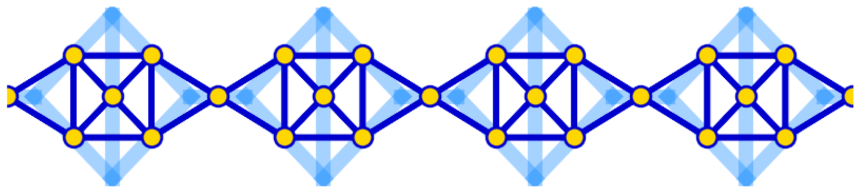

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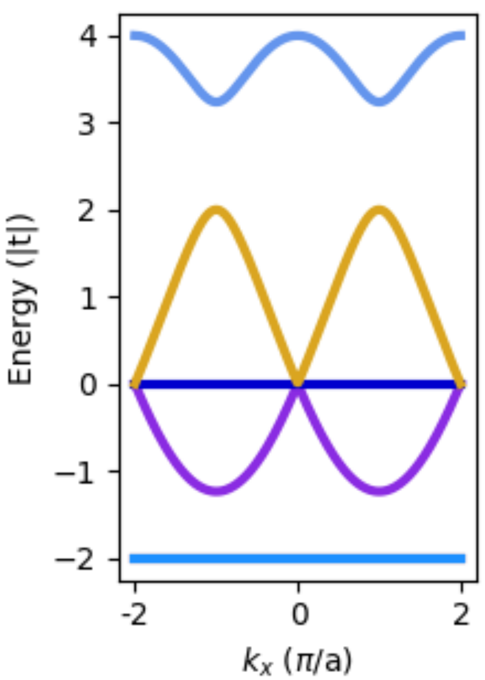

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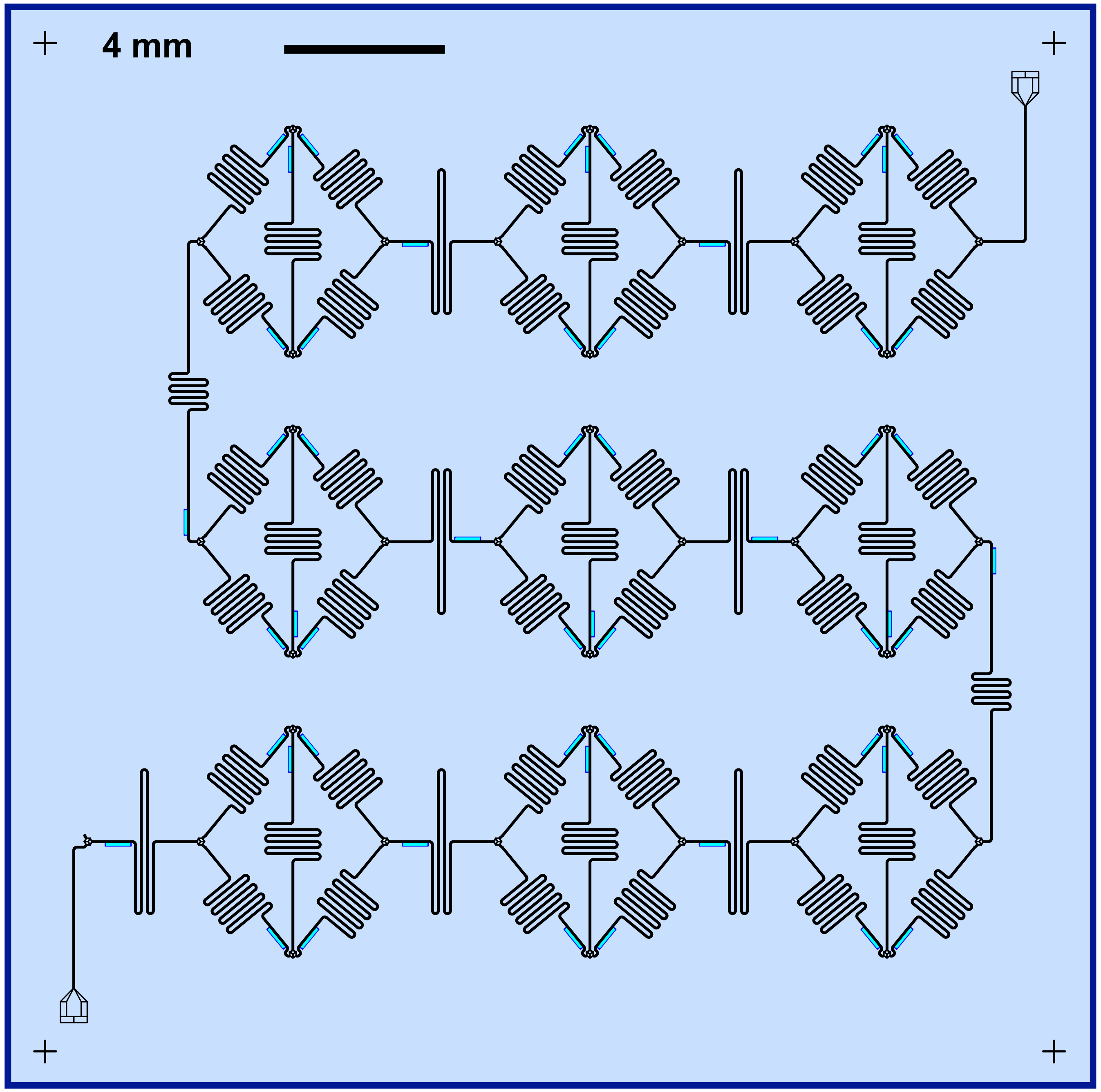
Device Design

(preliminary)



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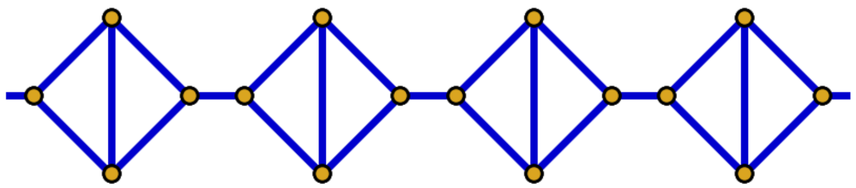

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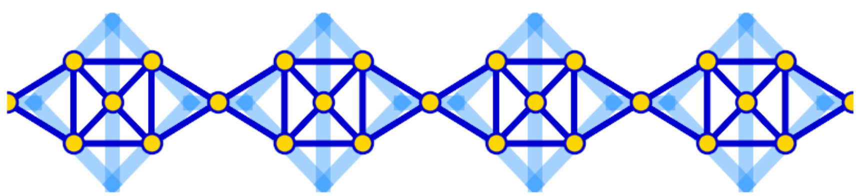

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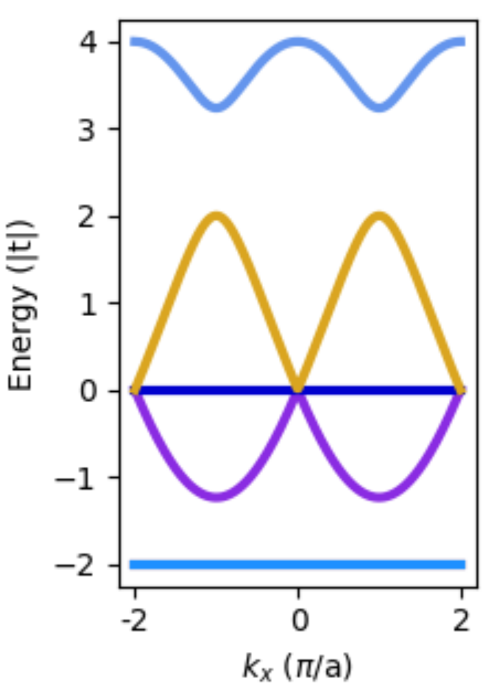

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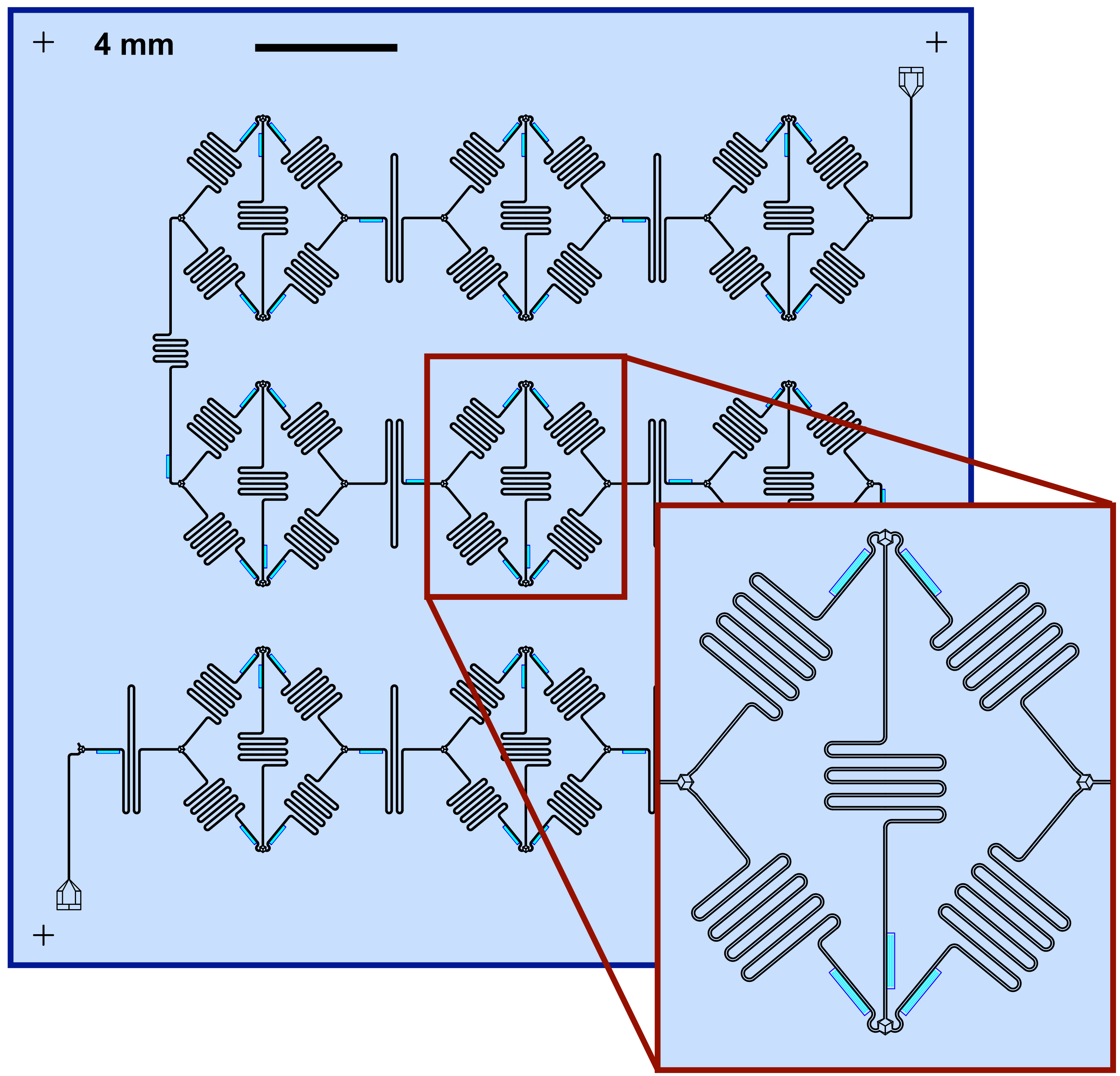

Line Graphs and Quantum Error Correction

Thm: (Chapman and Flammia)
A spin model can be solved exactly by mapping to free fermions if and only if the anticommutation relations of its terms have the structure of a line graph.

- Spin-model energies found from half-filling of magnetic models on the root graph.
- Gaps in and between these spectra dictate robustness of the code.

Numerical Phenomenology

→ **Error suppression is limited by energy differences between orientations, not single-particle gaps**

Chapman *et al*. Quantum **4**, 278 (2020)

Chapman, Flammia, AJK, arXiv:2201.07254 (2022)

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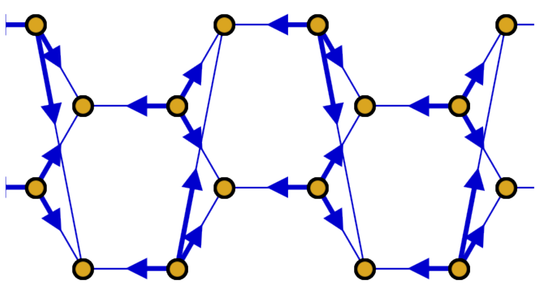
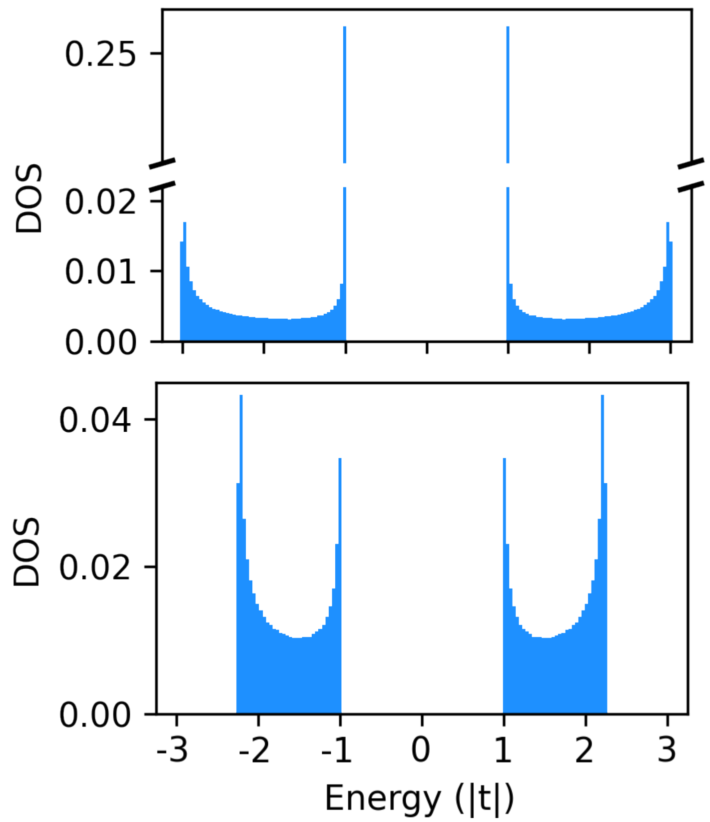
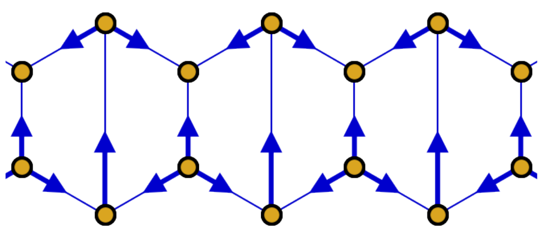




- Large gap
- But not the ground state



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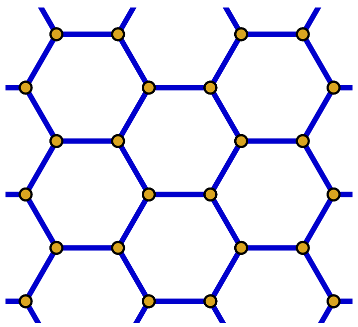
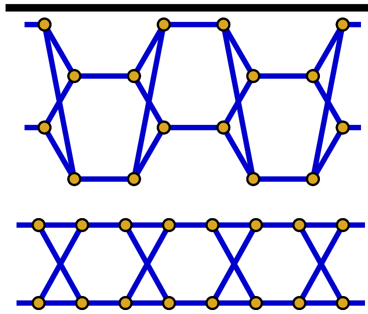
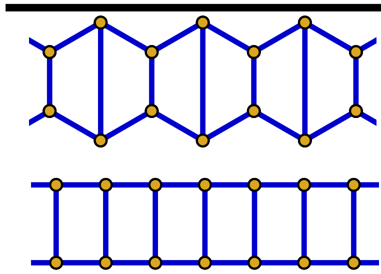
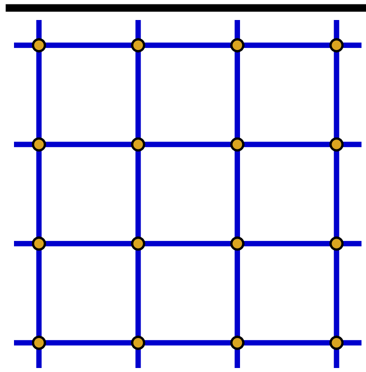
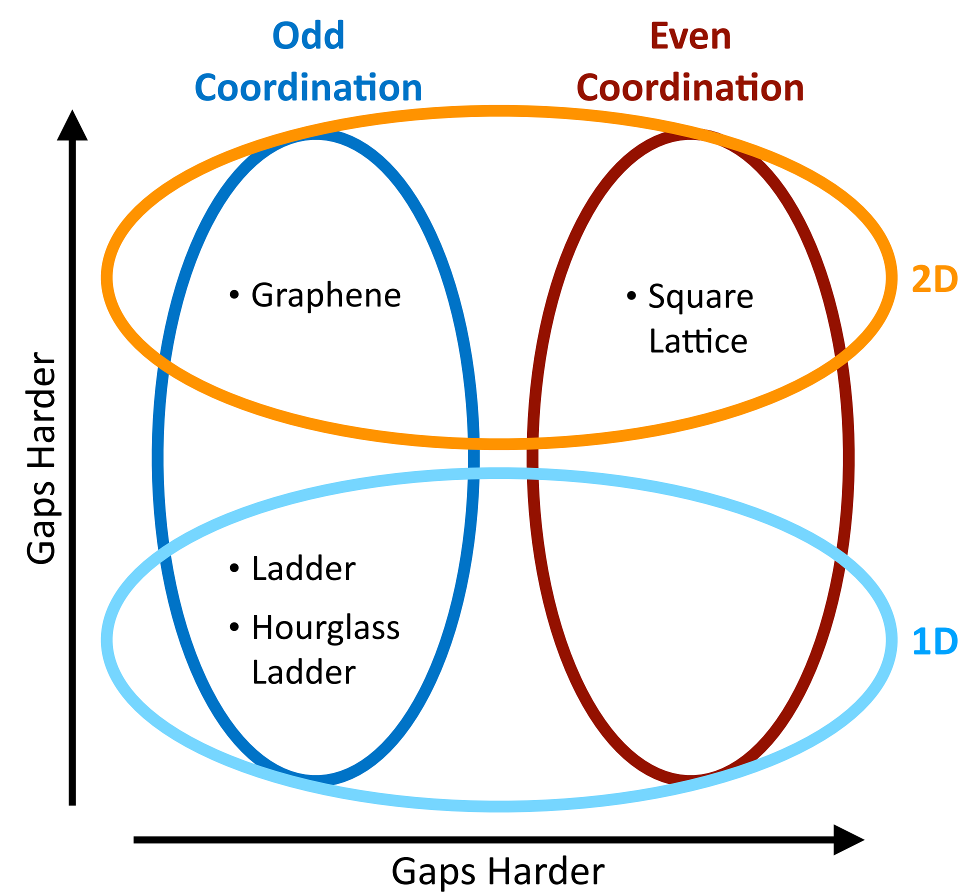
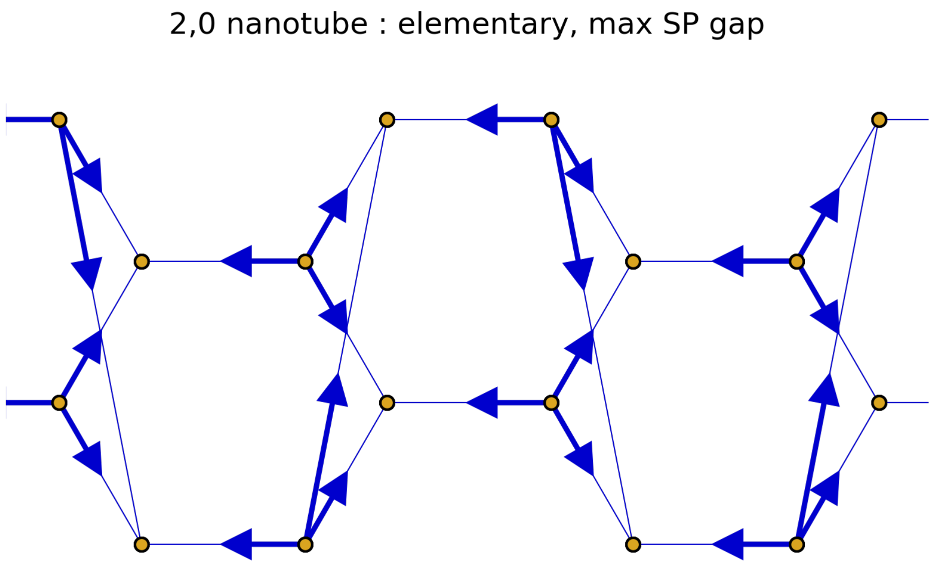
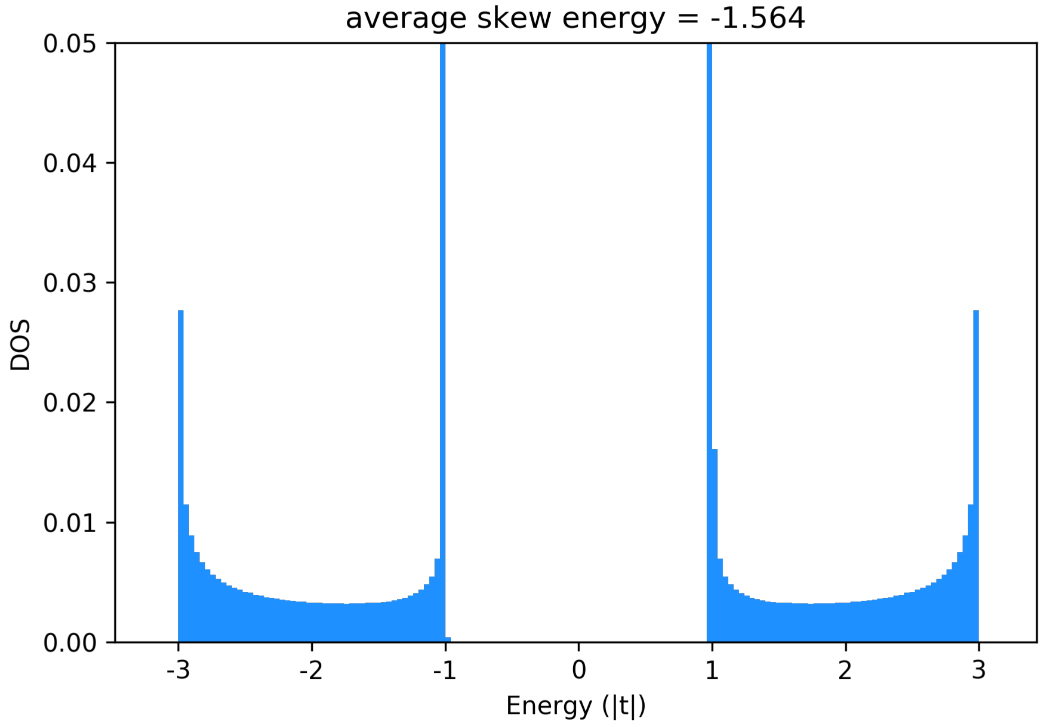
Lattice Gap Examples

	Orientation	Known	Odd	Fluxes	Single-particle gap	Distinct
a 	Elementary		No	0	No	No
	Ground State	Yes	No	0	No	
b 	Elementary		No	0	Yes, Large	Yes
	Ground State	No	—	—	—	
c 	Elementary		No	0	No	Yes
	Ground State	Yes	Yes	π	Yes, Large	
d 	Elementary		No	0	No	Yes
	Ground State	Yes	No	π	No	



Free-Fermion Solutions





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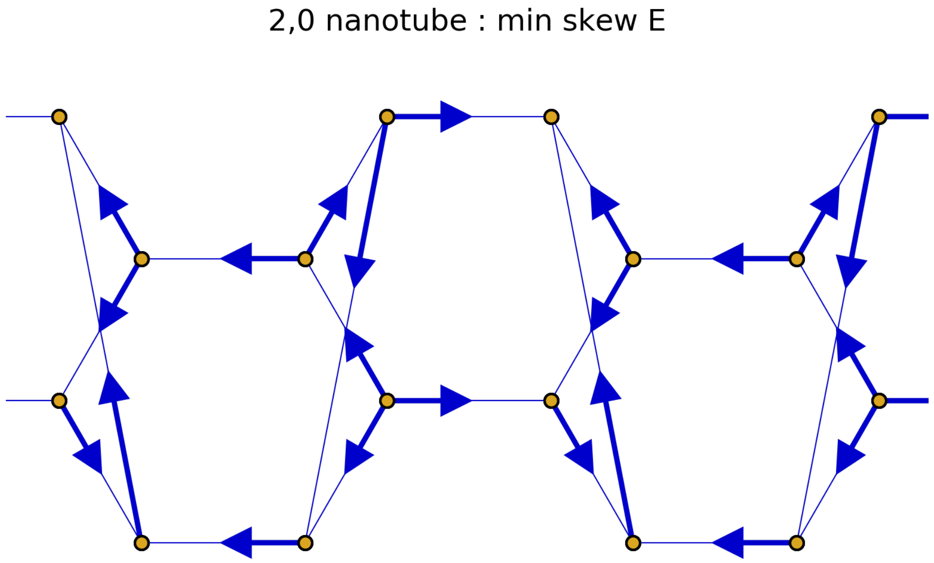
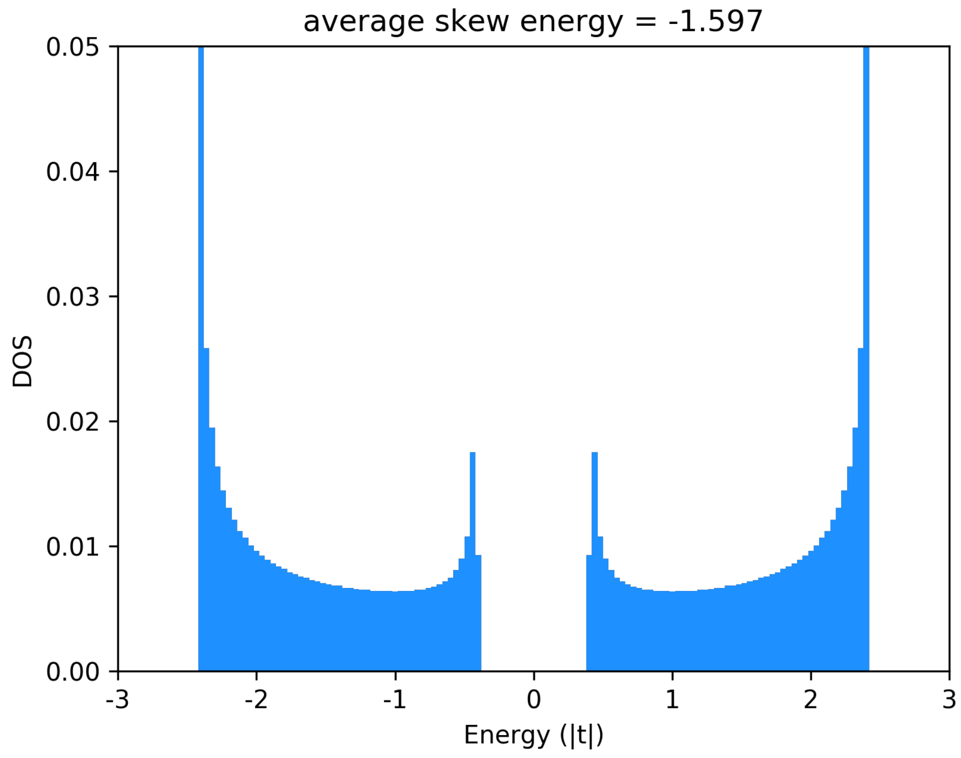
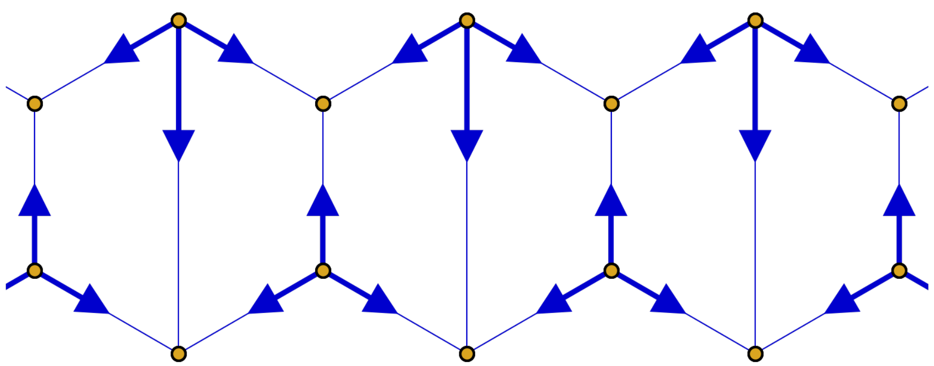


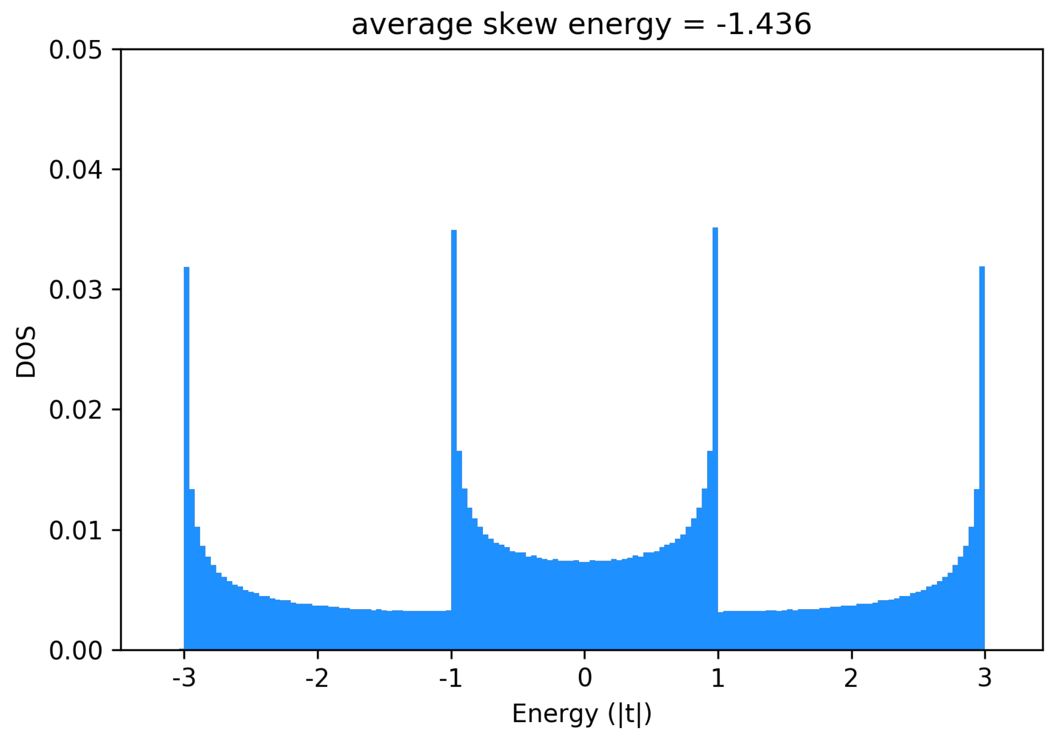


- Modest gap
- Ground state

Free-Fermion Solutions

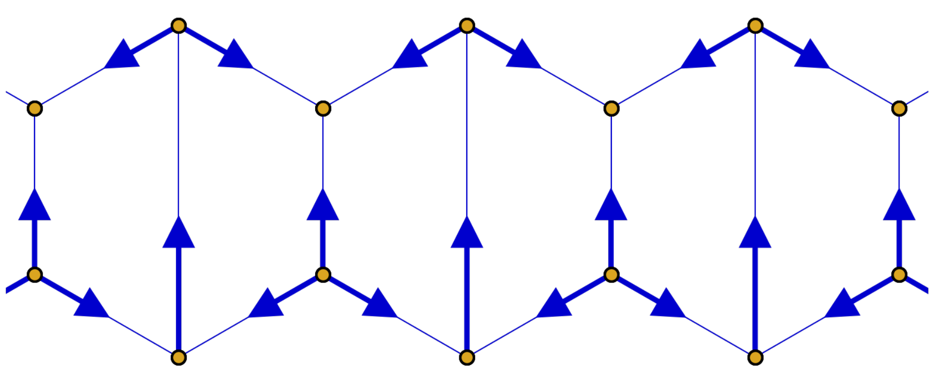
1,1 nanotube : elementary orientation

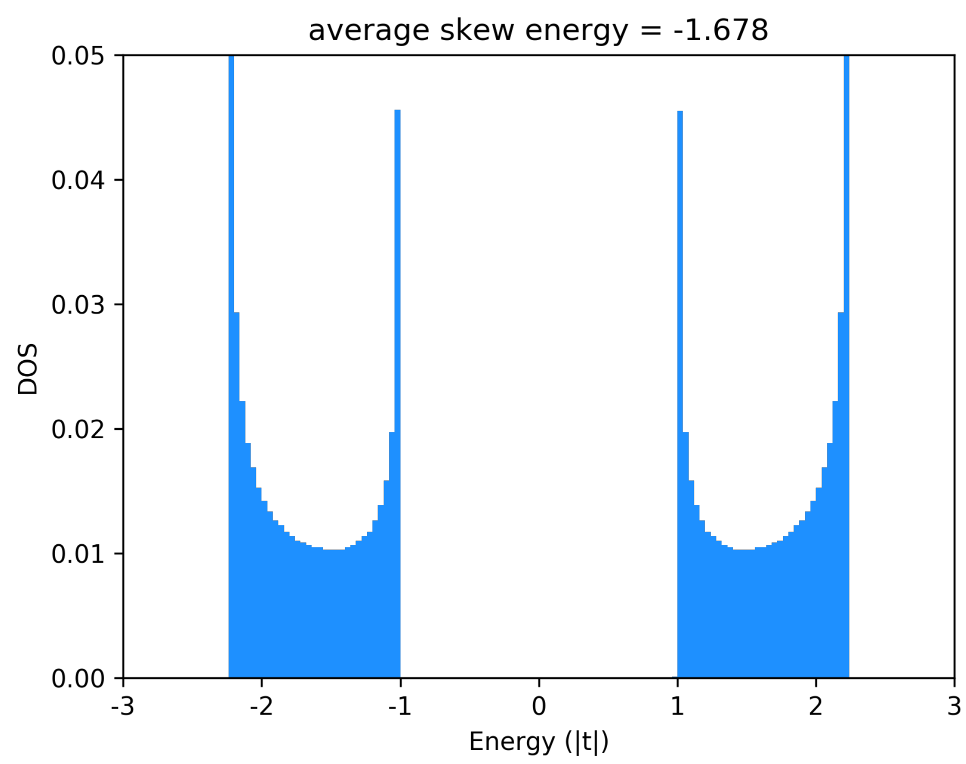




- Non-magnetic orientation
- No gap

1,1 nanotube : max SP gap, min skew E





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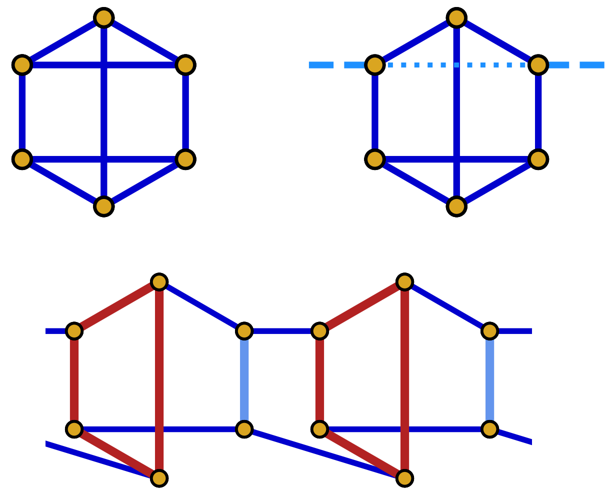
Mathematical Outlook: Abelian Covers and Error Correction

New Lattice Viewpoint

- Use method of Abelian covers to construct examples.
  - “Unwrap” small graph to form lattice



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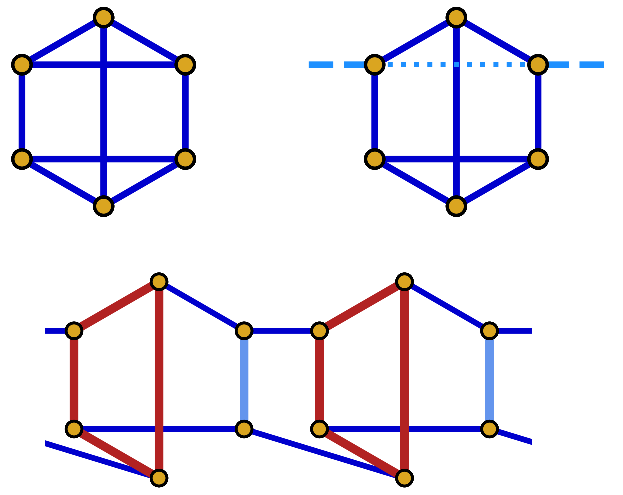

- Initial energies are k=0 energies of the lattice
  - Small graphs and their spectra tabulated.
  - "Periodic table" of unit cells to start from.

Kollár *et al*. arXiv:2005.05379 (2020) Chapman *et al*. Quantum **4**, 278 (2020) Chapman, Flammia, AJK, In Preparation (2021)

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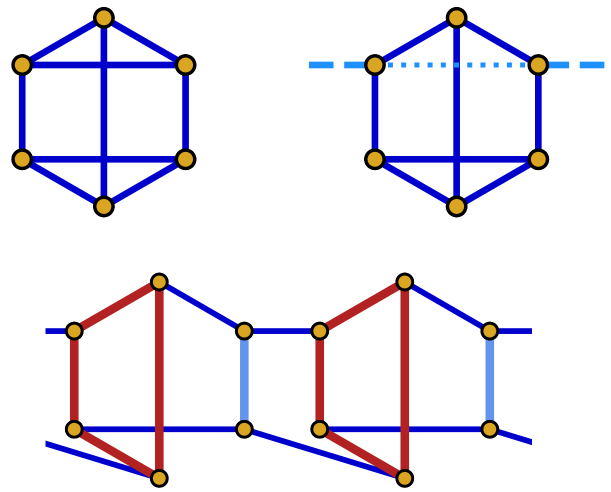

- Initial energies are k=0 energies of the lattice
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Kollár *et al*. arXiv:2005.05379 (2020)

Chapman *et al*. Quantum **4**, 278 (2020)

Chapman, Flammia, AJK, In Preparation (2021)

Mathematical Outlook: Abelian Covers and Error Correction

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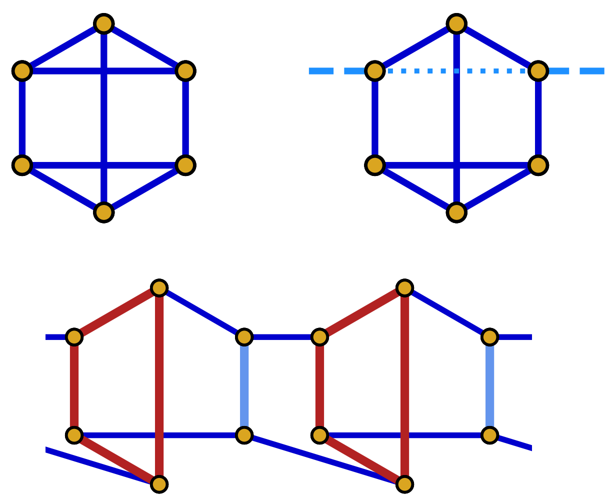

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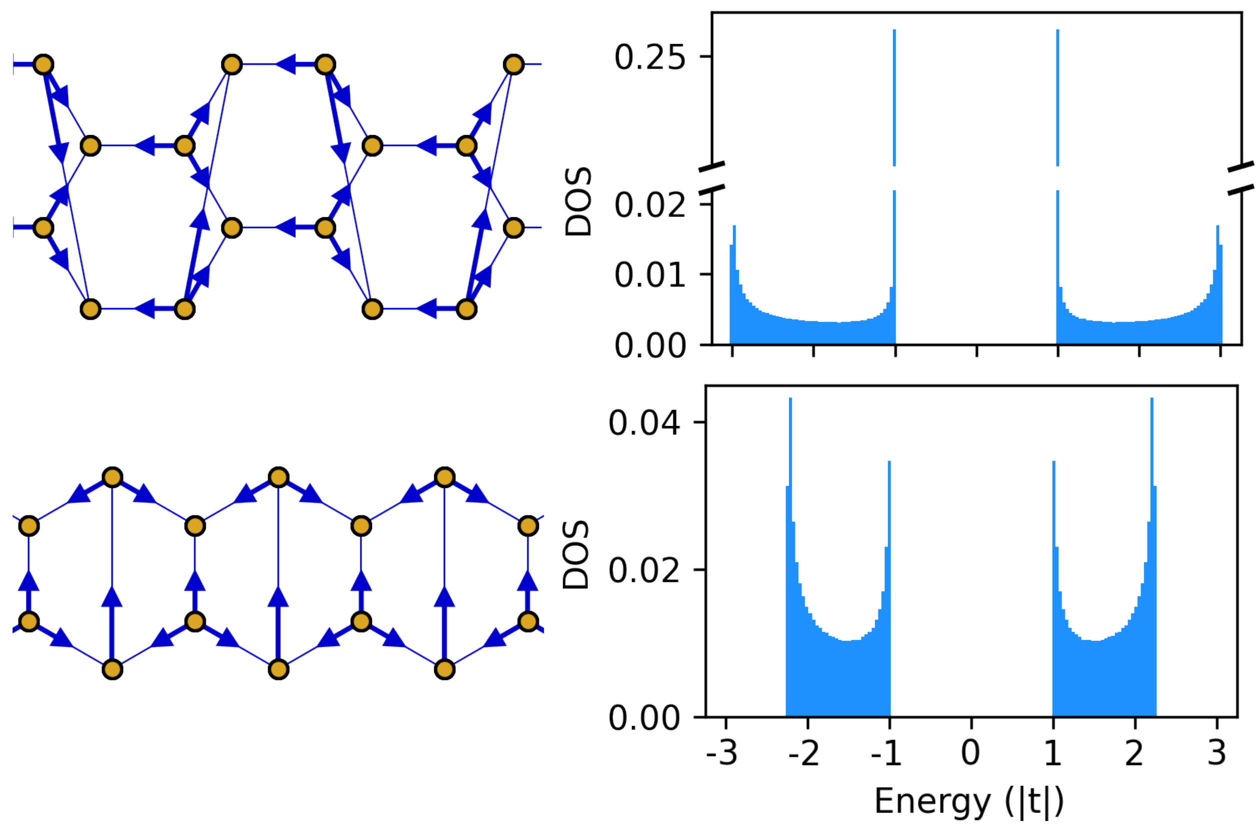

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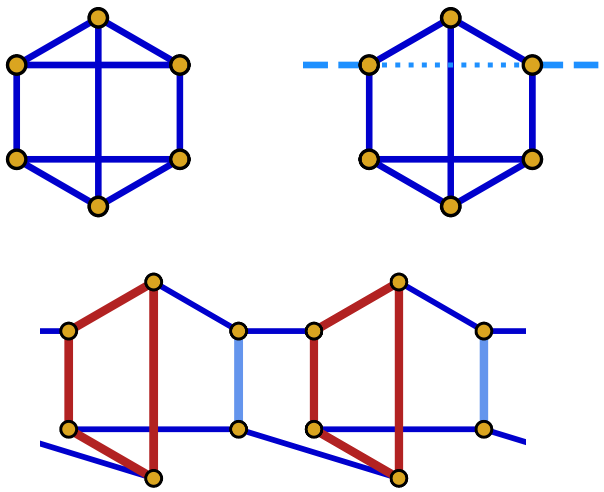

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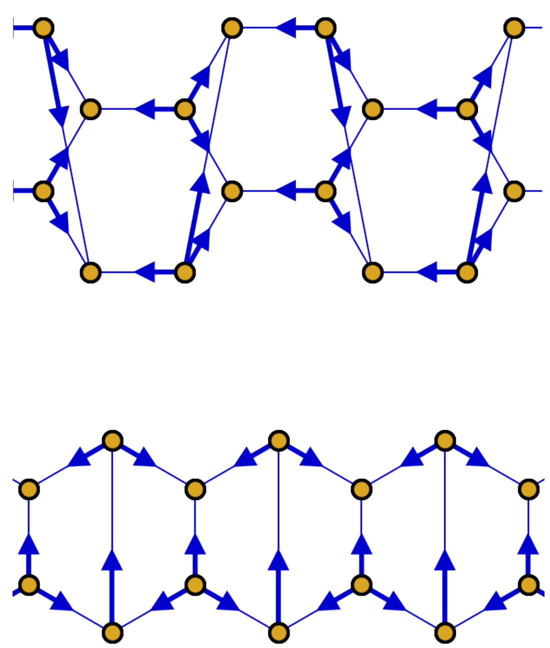

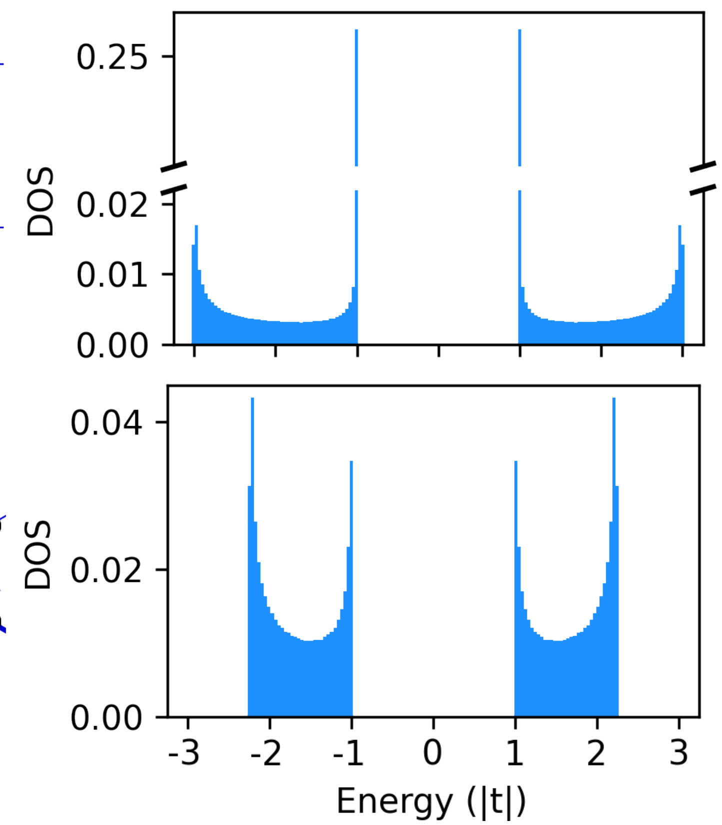

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So far, error suppression is limited by energy differences between orientations, not single-particle gaps

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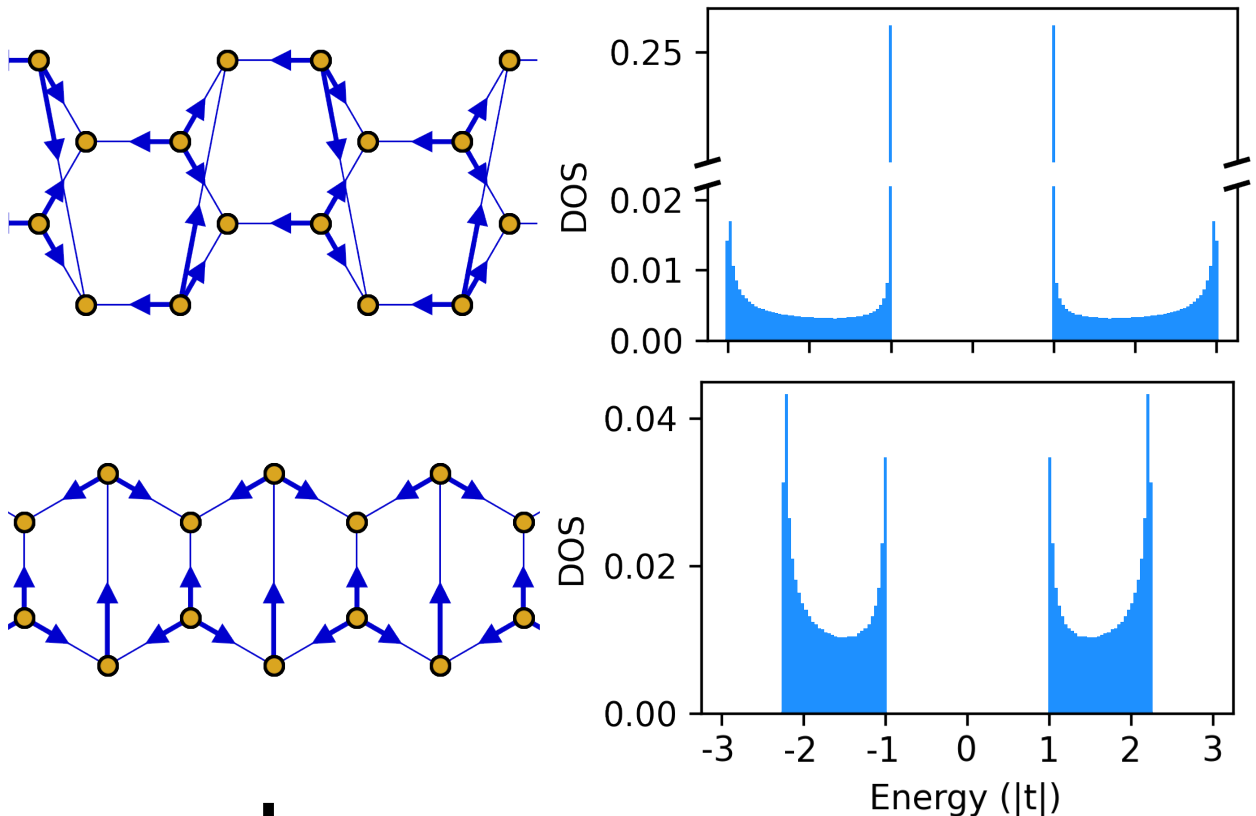

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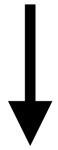

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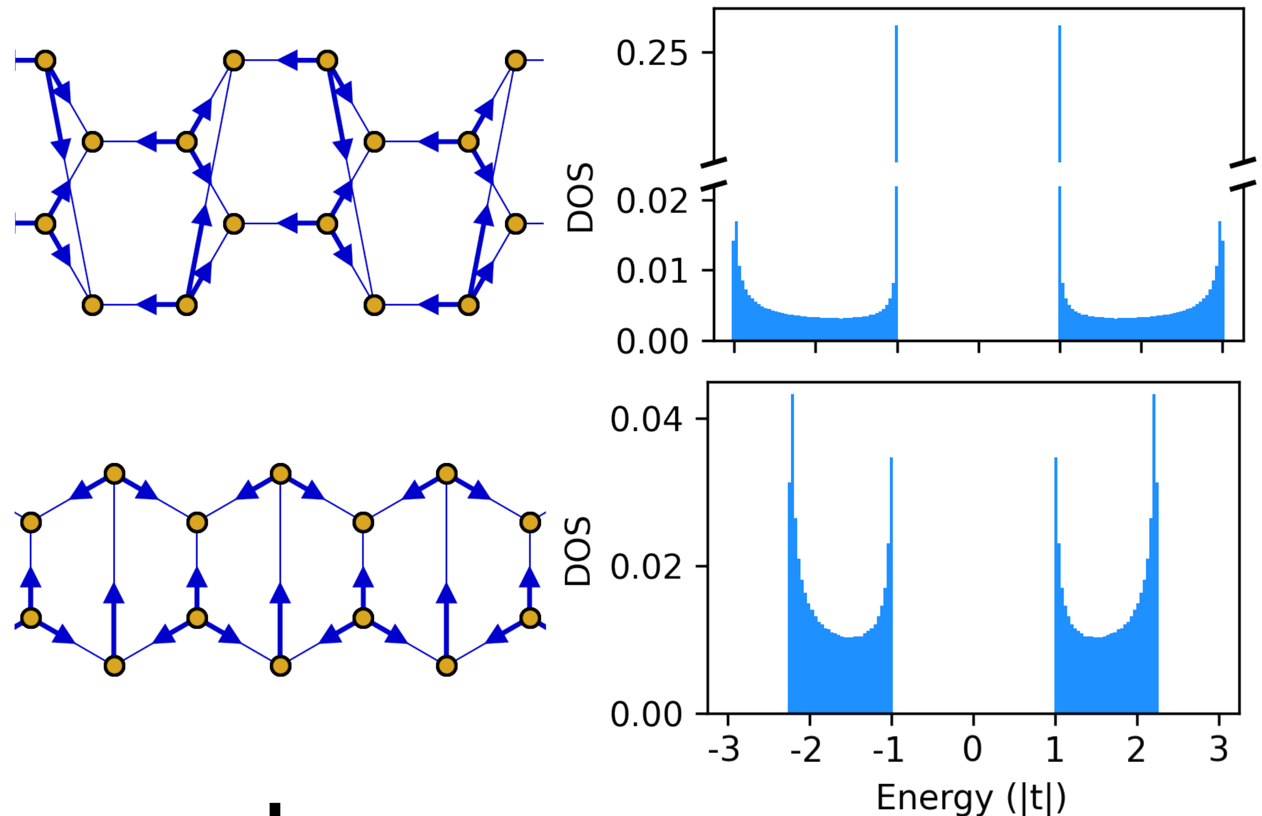

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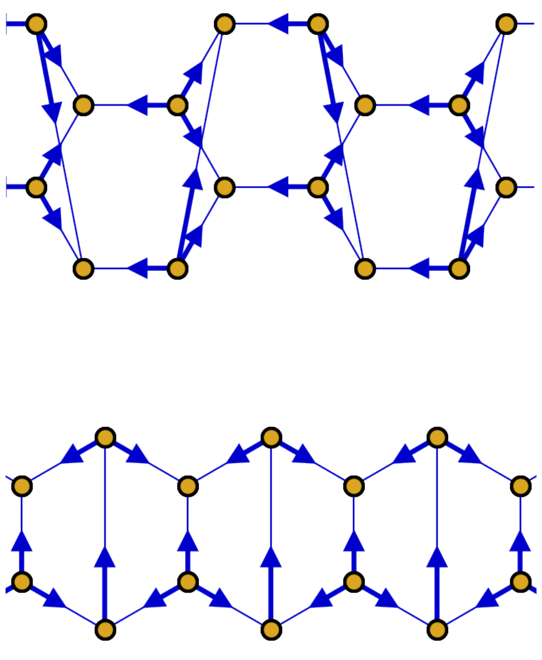

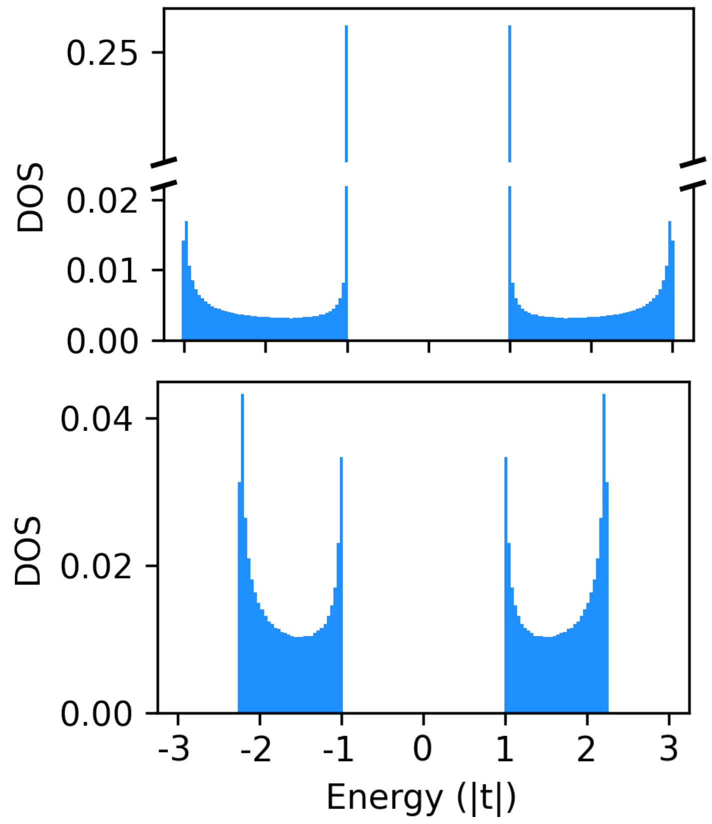

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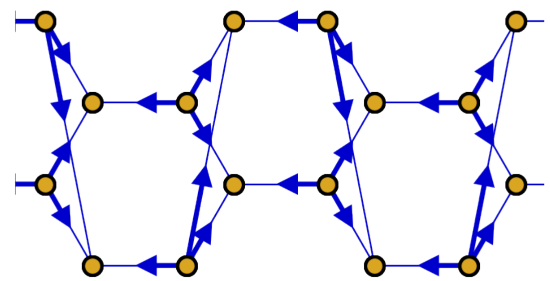

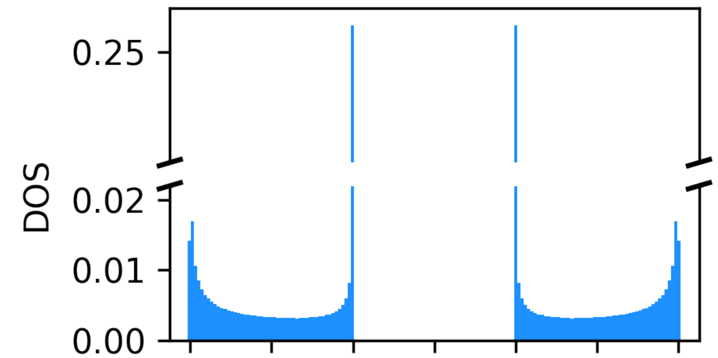

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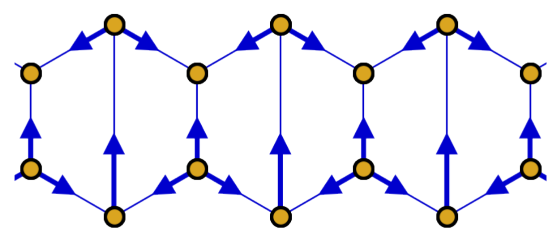

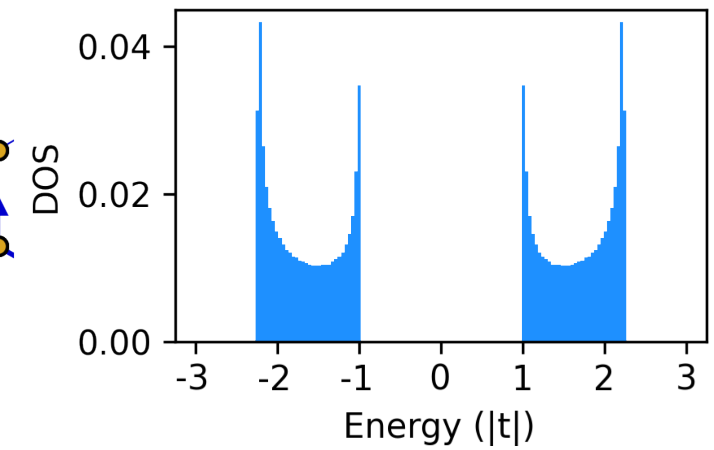

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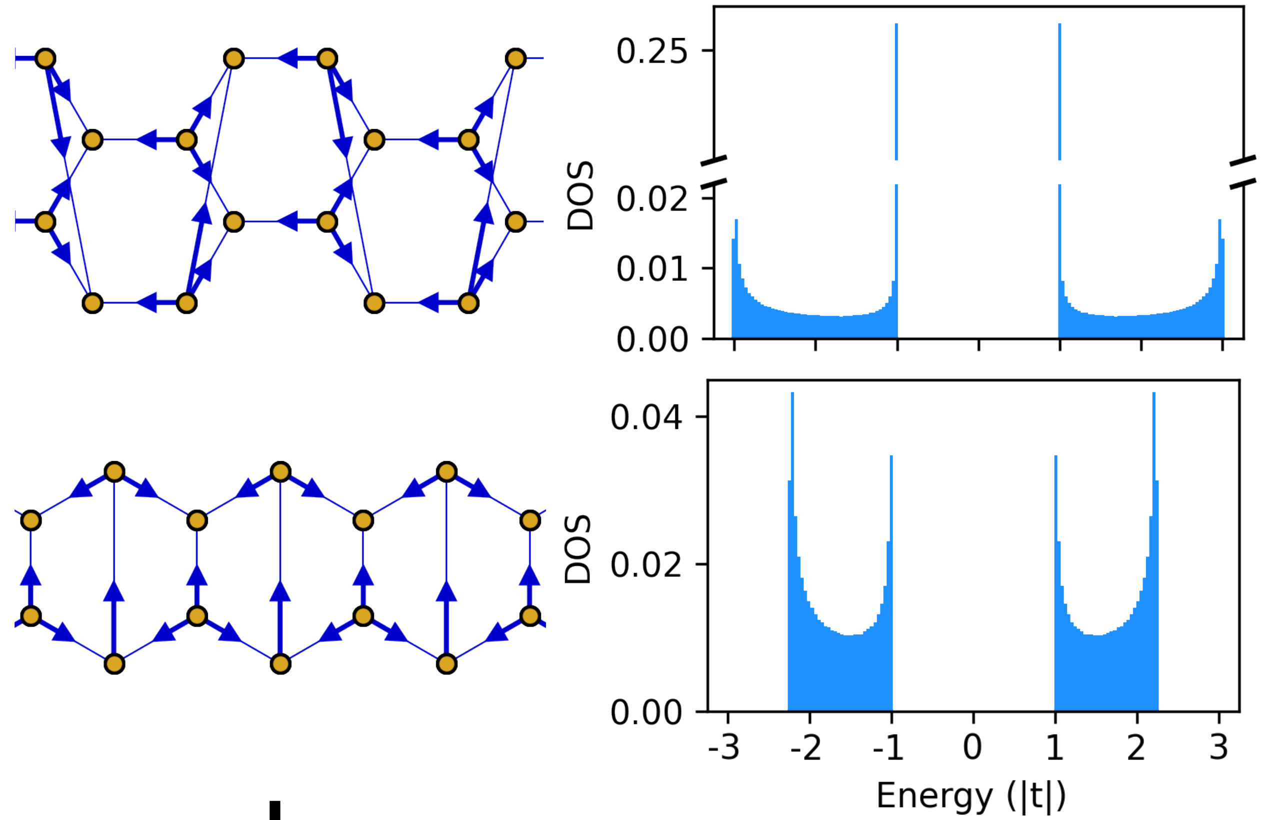

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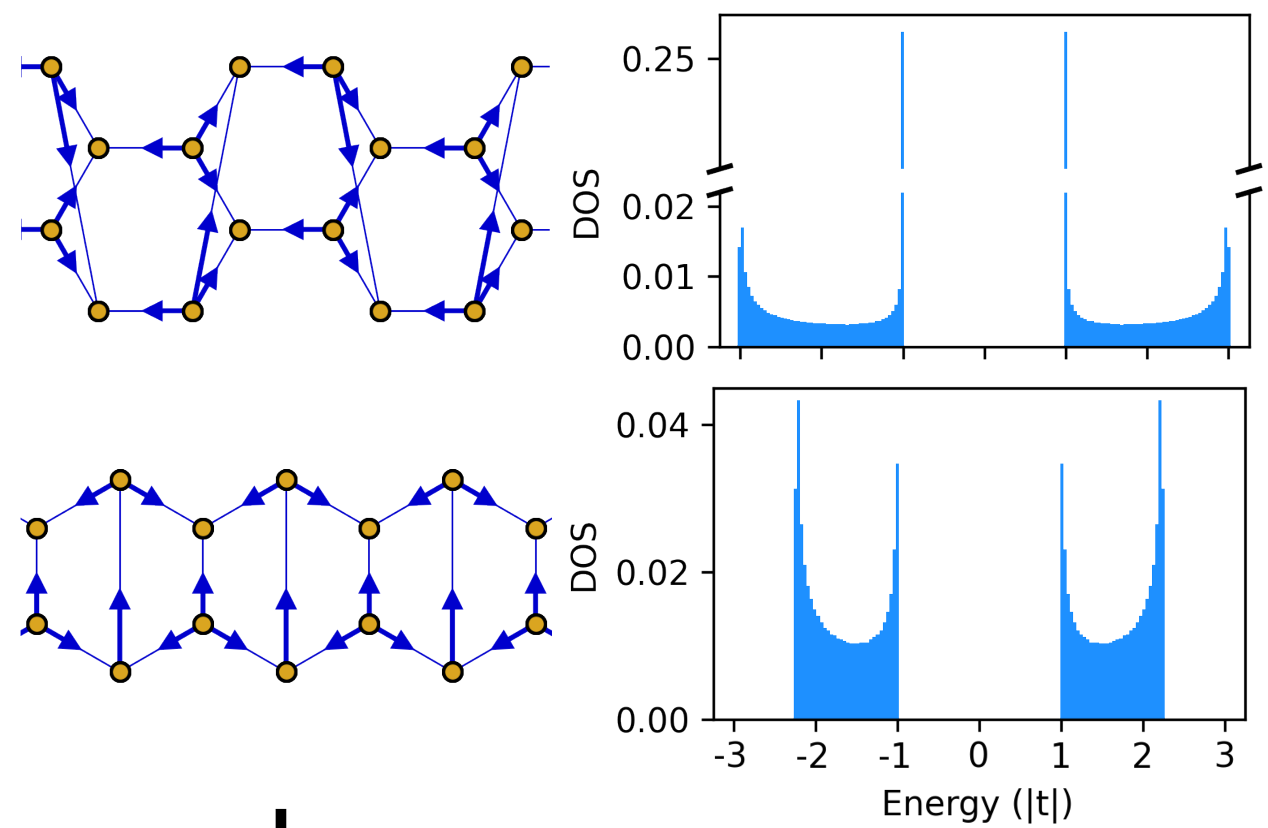

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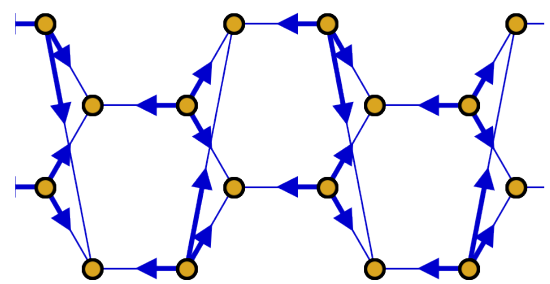

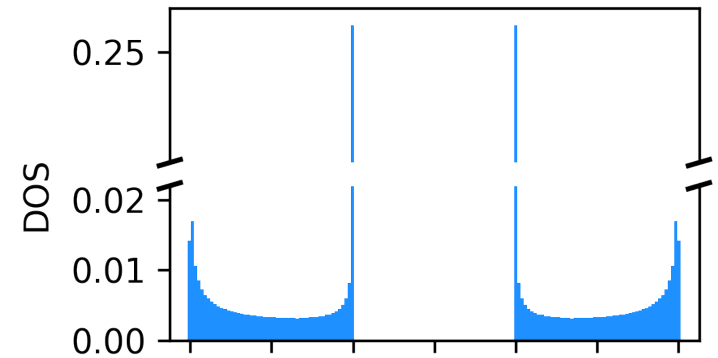

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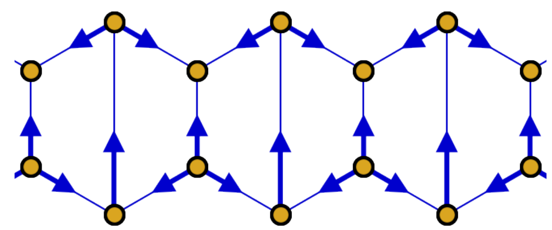

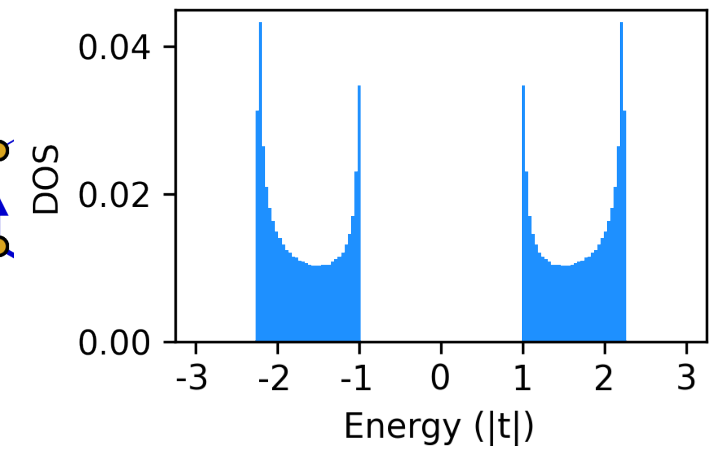

- Large gap
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- Abelian covers of small regular graphs yield examples with a large gap within the ground state orientation
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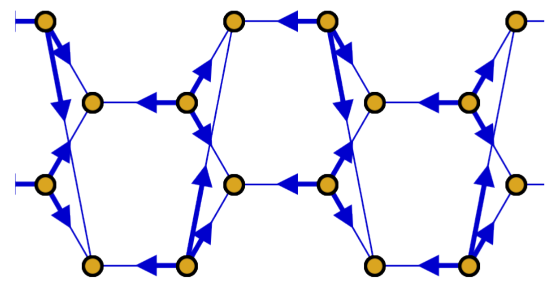

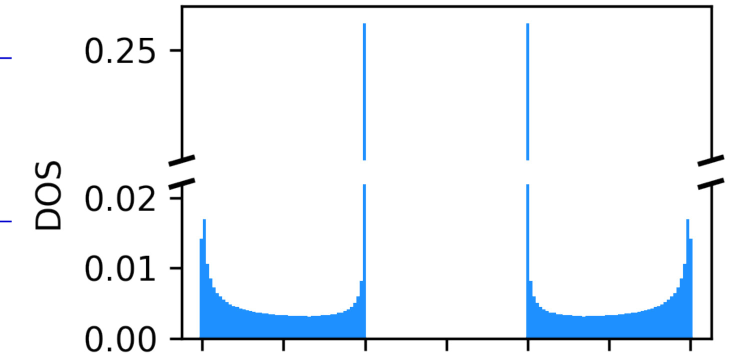

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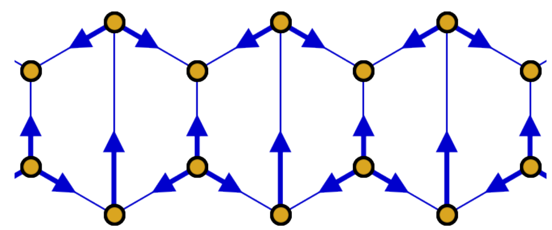

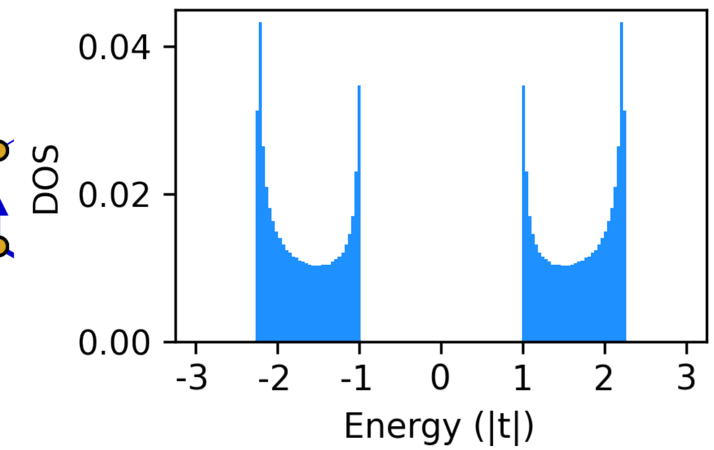

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- Abelian covers of small regular graphs yield examples with a large gap within the ground state orientation
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- Error suppression limited by skew energy, so far

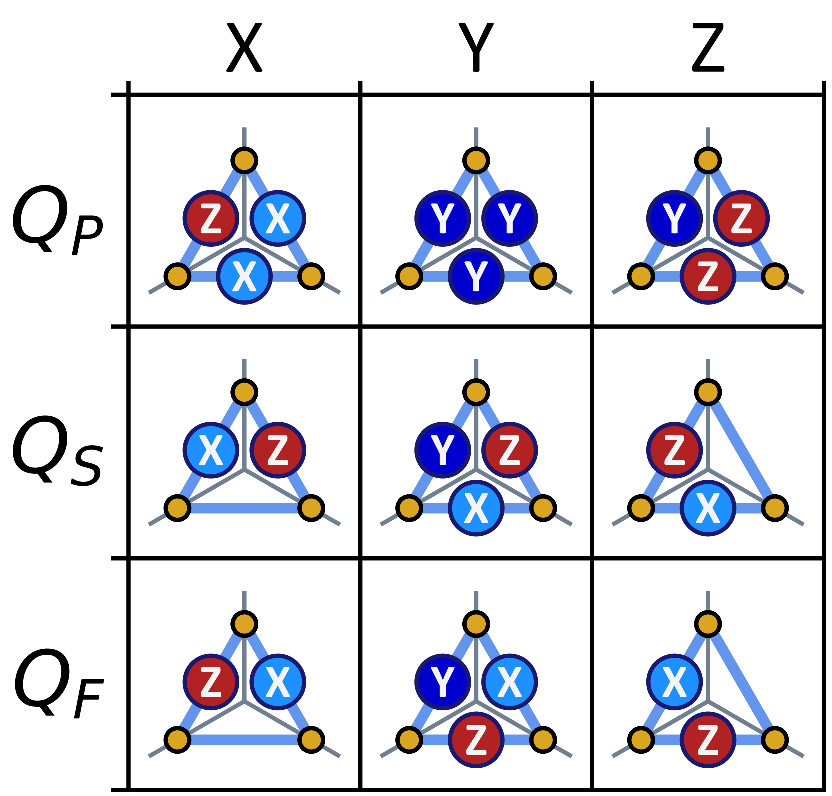
Chapman *et al*. Quantum **4**, 278 (2020)   AJK, Sarnak arXiv:2005.05379 (2020)   Chapman, Flammia, AJK, In Preparation (2021)

The Triangle Models

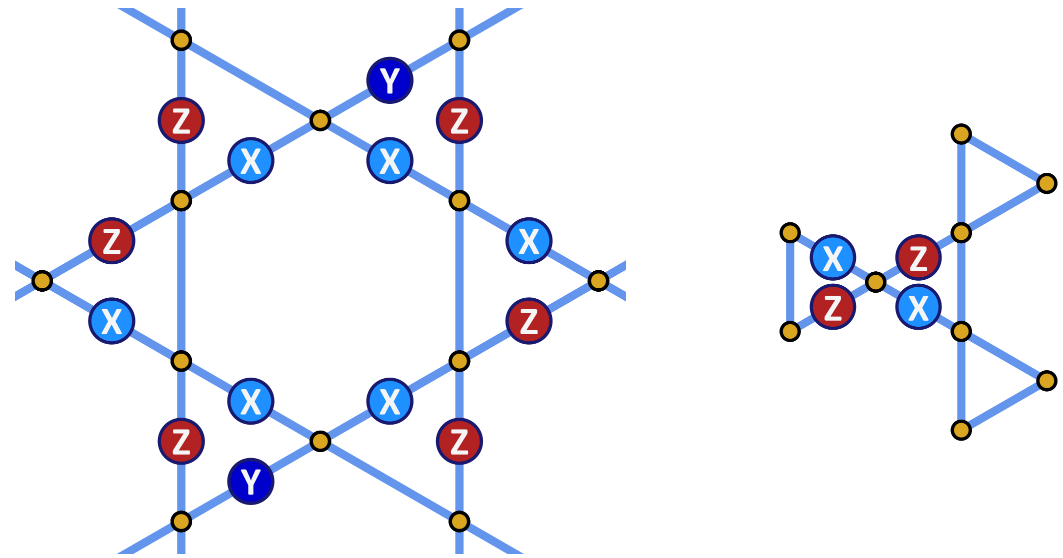
Three Combined Models

- Free-fermion model : Kitaev Honeycomb
- Stabilizer code : Wen Plaquette
- Paramagnet to couple the two

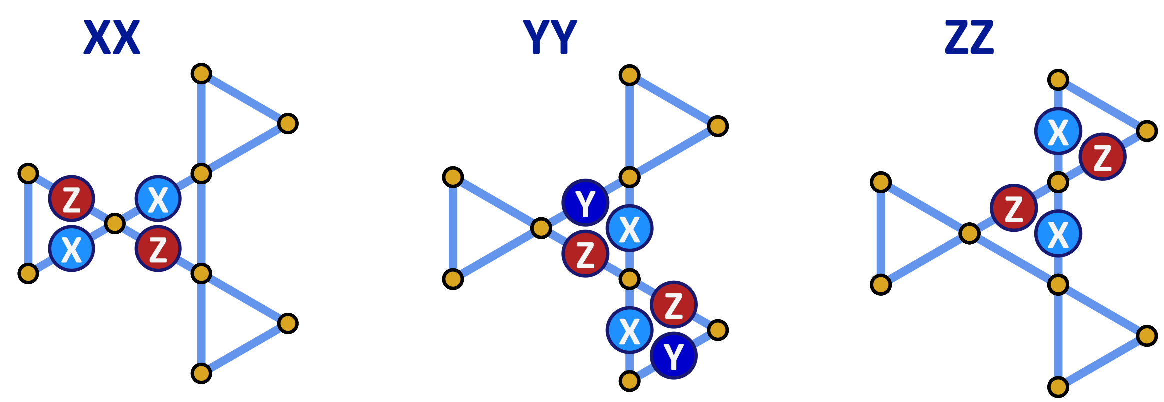
Effective Qubits



Wen Plaquette Model



Kitaev Honeycomb Model



Exact logicals **without** fermion participation

Chapman *et al*. Quantum **4**, 278 (2020)

Chapman, Flammia, AJK, arXiv:2201.07254 (2022)

Photon-Mediated Interactions

Photonic Crystal + qubits

- Effective swap interaction
- All modes in parallel

$$H = \hbar\, \sigma_1^+ \sigma_2^- \sum_m \frac{g_m^2}{\Delta(m)}\, \psi_m(x_1)\psi_m^*(x_2) + h.c.$$

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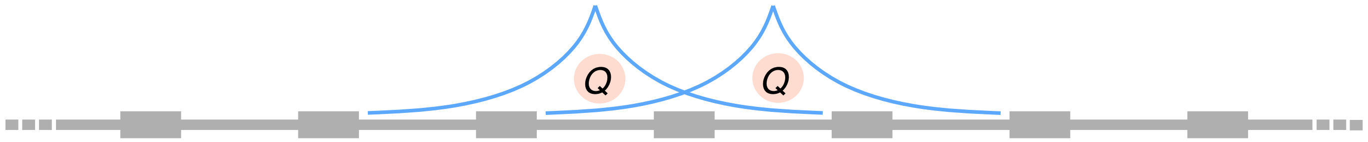
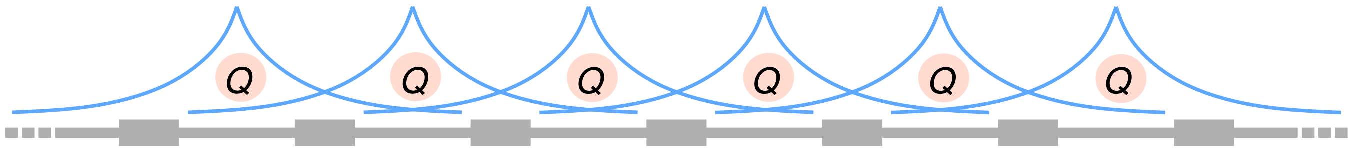
- Exponentially localized bound state





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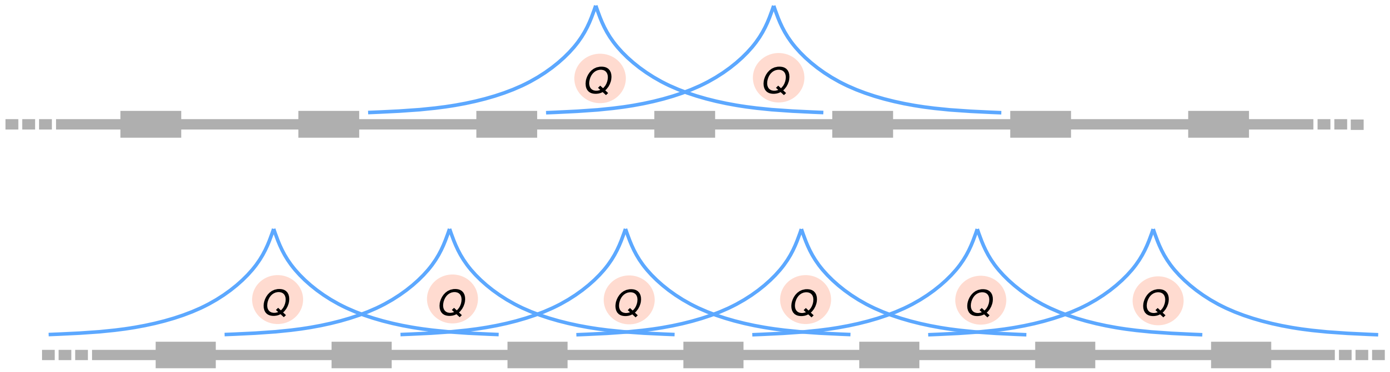

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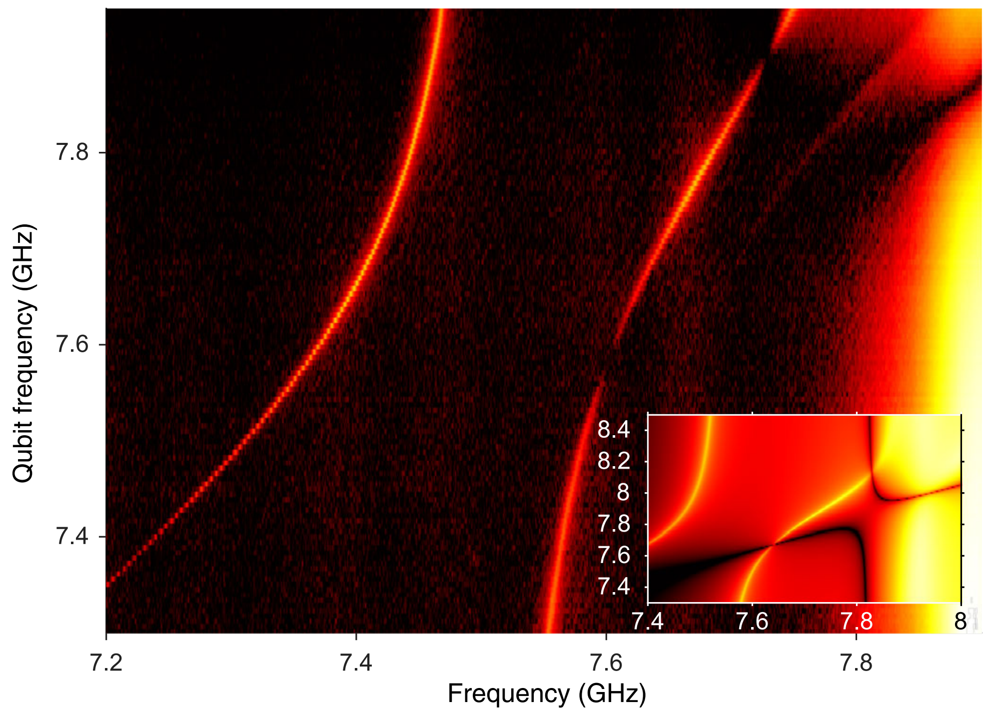

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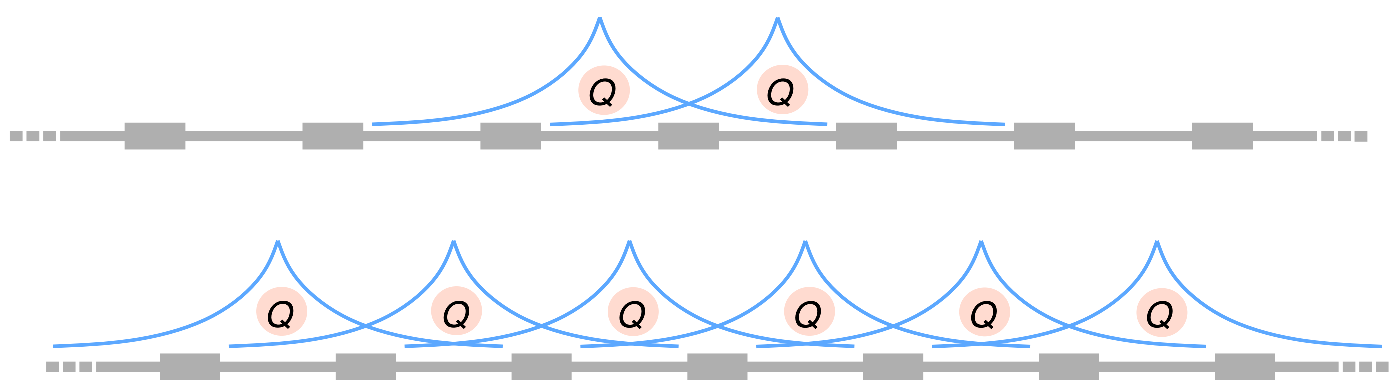

New Regimes:

- New lattices
- Different coupling scheme

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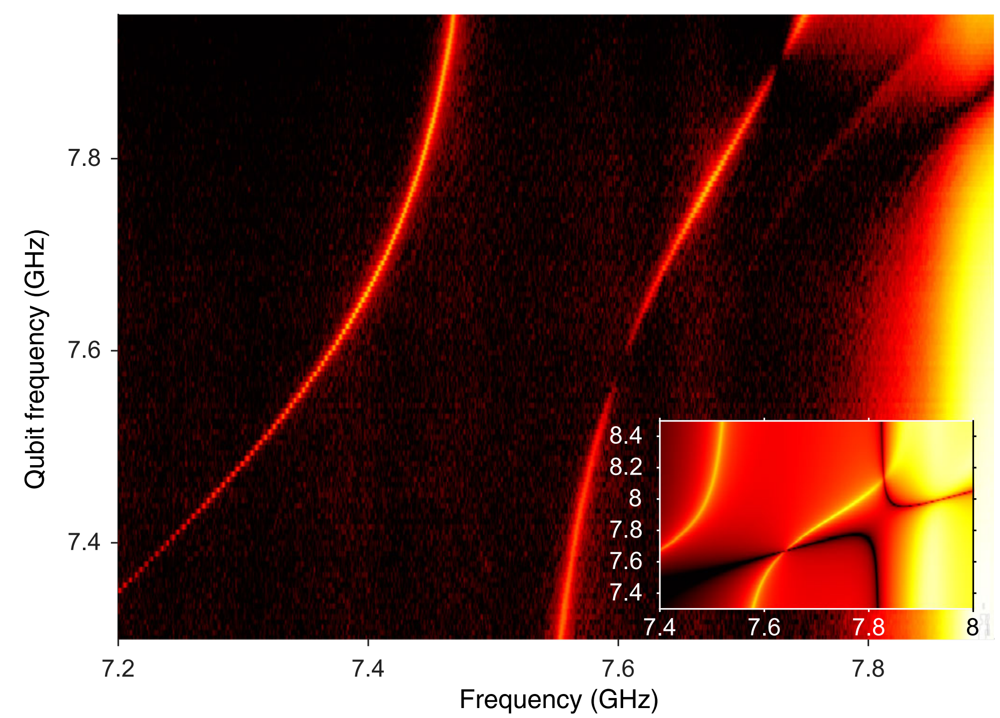

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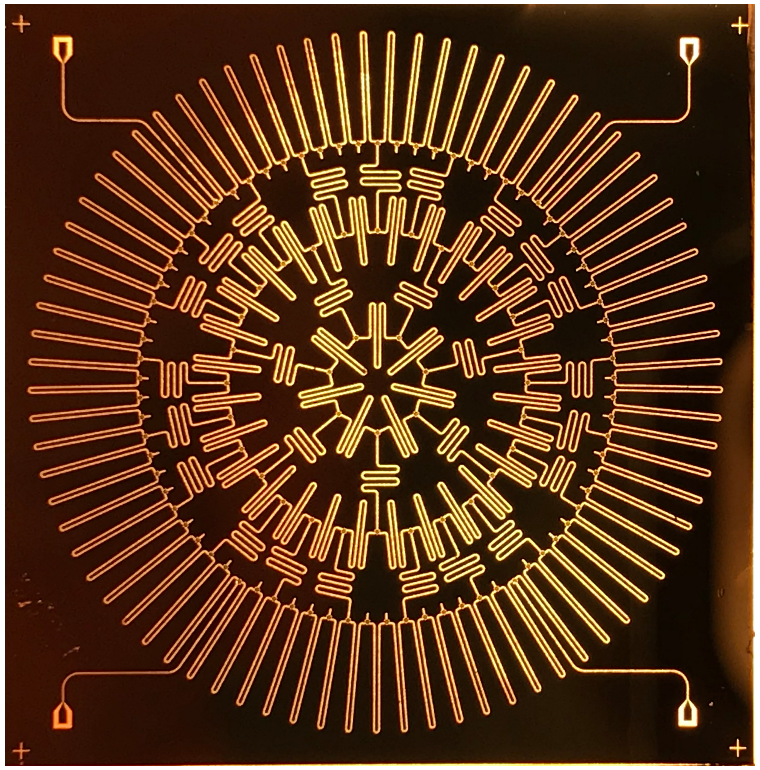
New Lattices for Photon-Mediated Interactions

AJK *et al*. Nature **571** (2019)



Hyperbolic Lattice

- Follows hyperbolic metric

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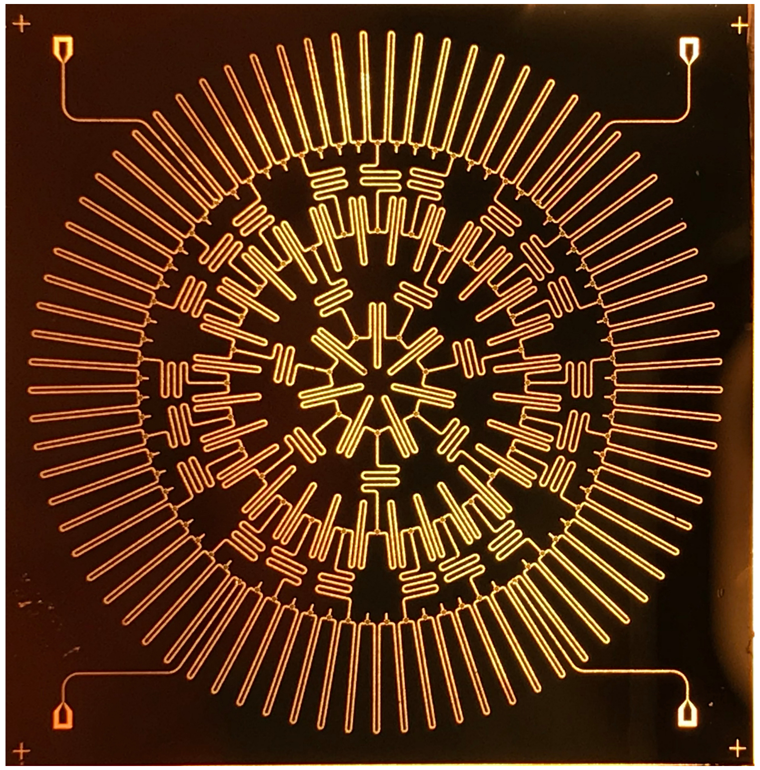

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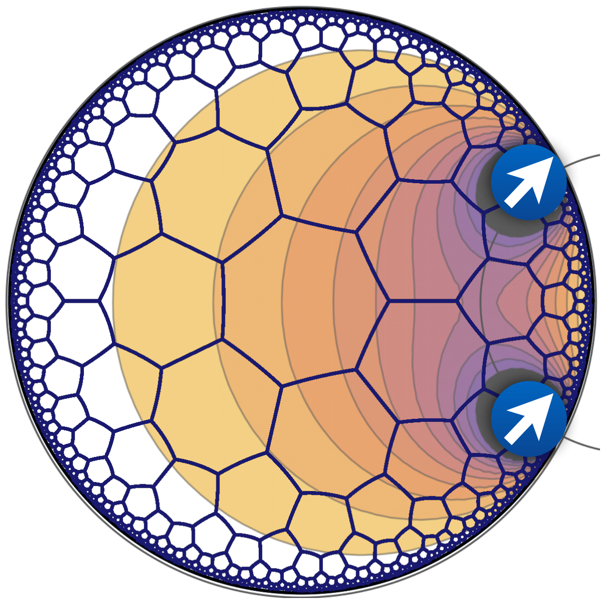

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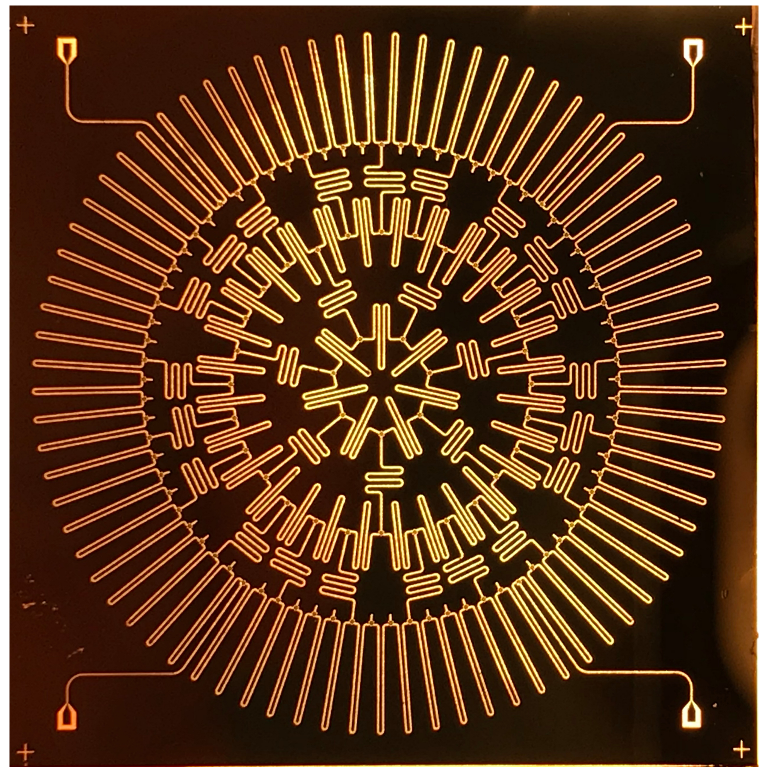

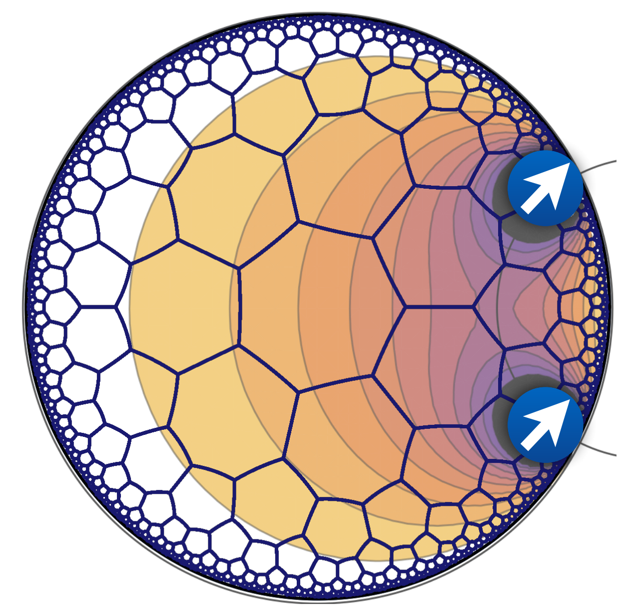

Flat-Band Lattice

- Frustrated Magnet

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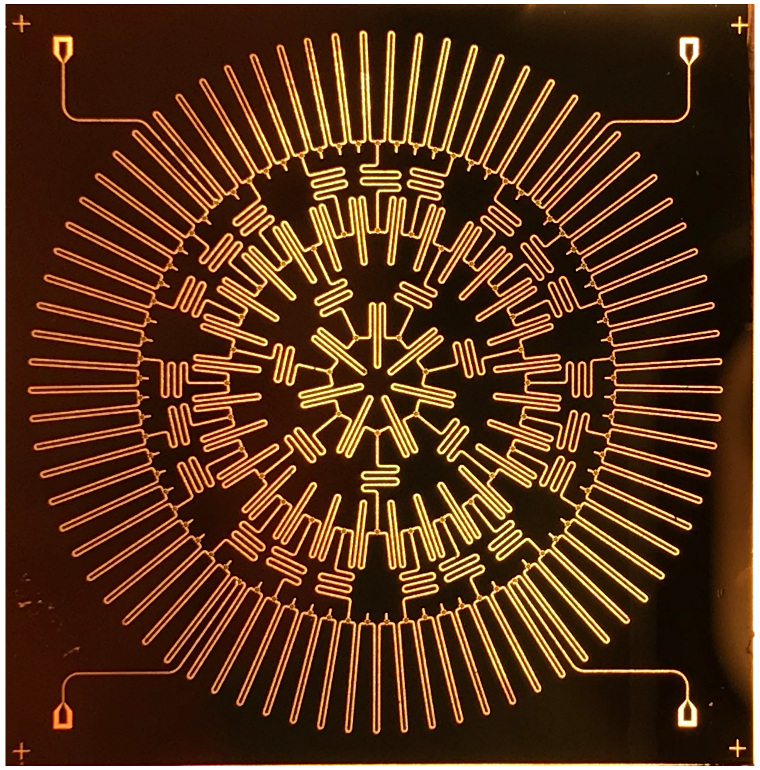

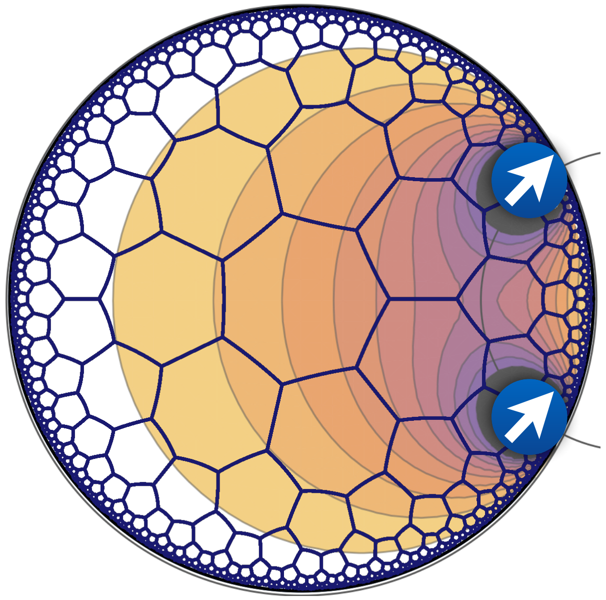

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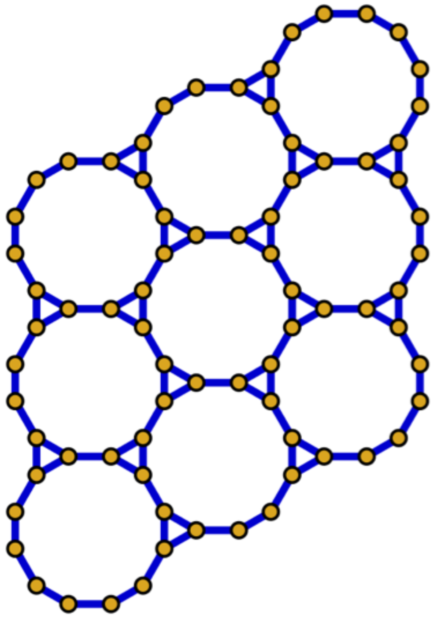

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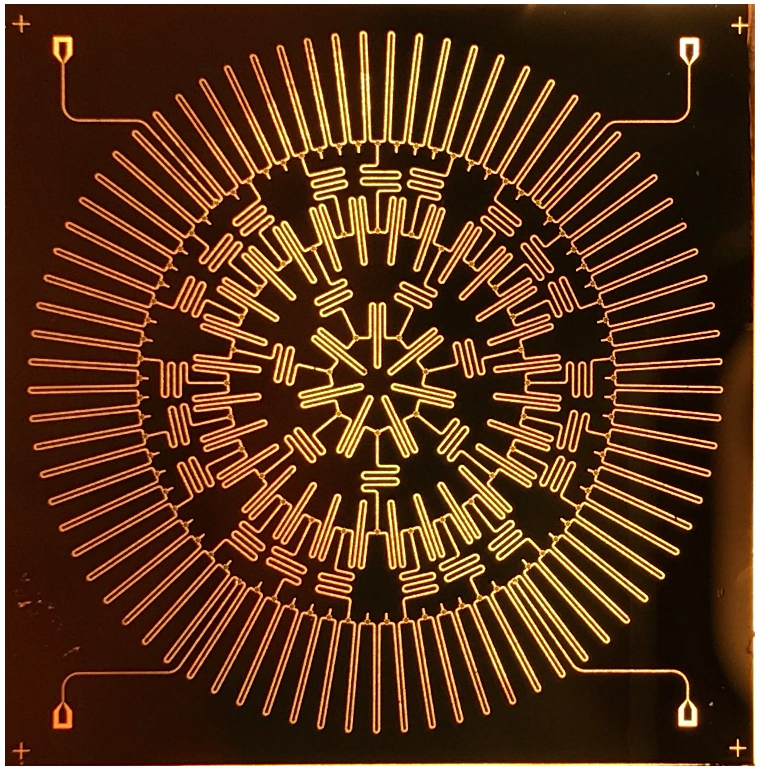

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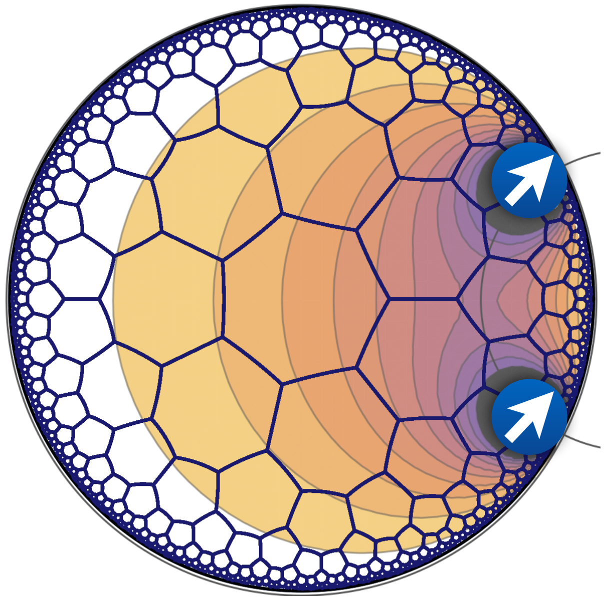

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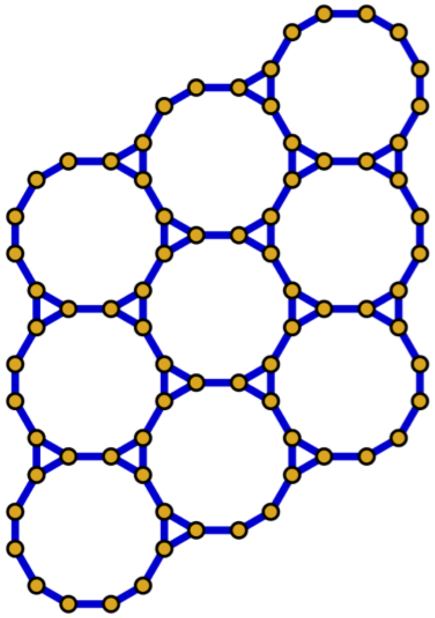
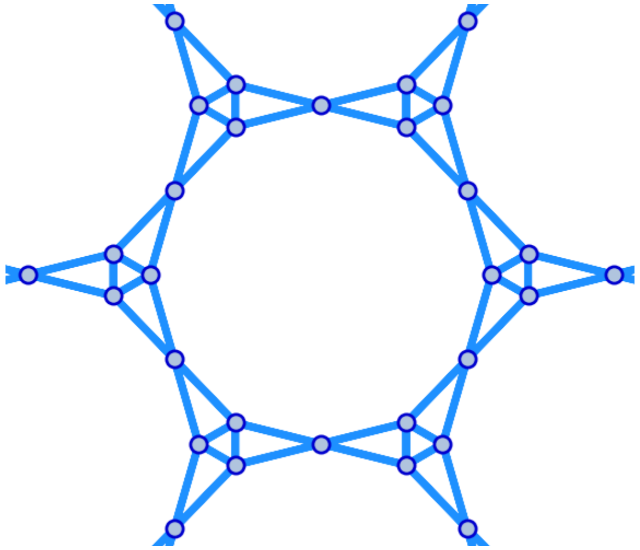




AJK *et al*. Comm. Math. Phys. **376**, 1909 (2020)

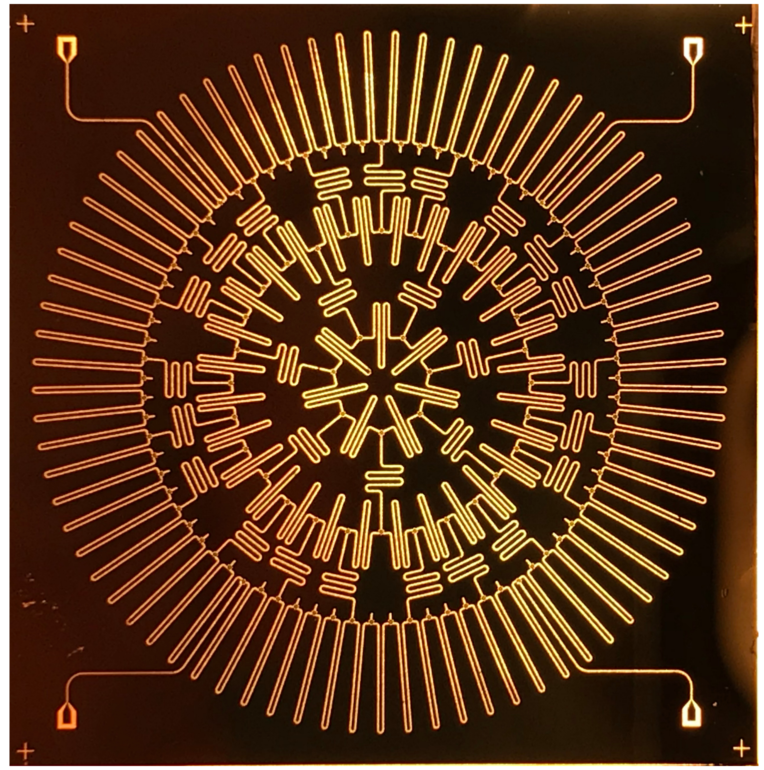
New Lattices for Photon-Mediated Interactions

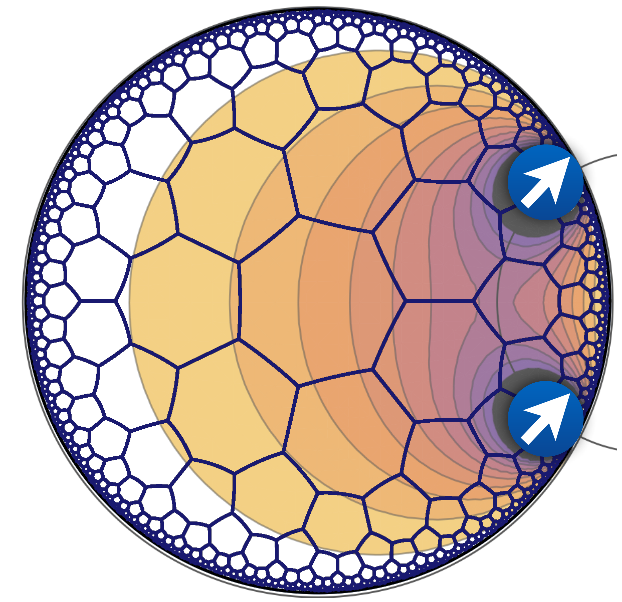
AJK *et al*. Nature **571** (2019)

Bienias, AJK *et al*. arXiv:2105.06490 (2021)

Hyperbolic Lattice

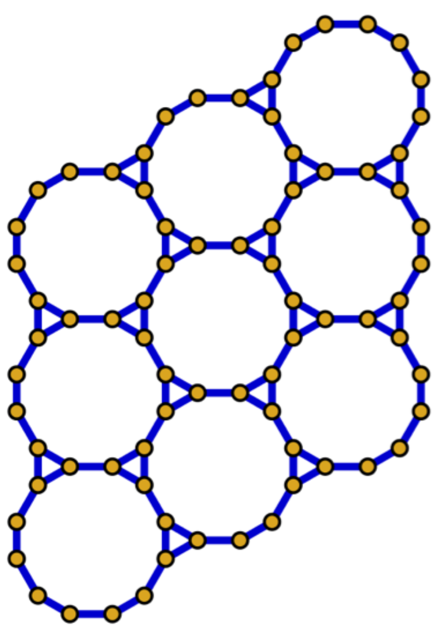
- Follows hyperbolic metric

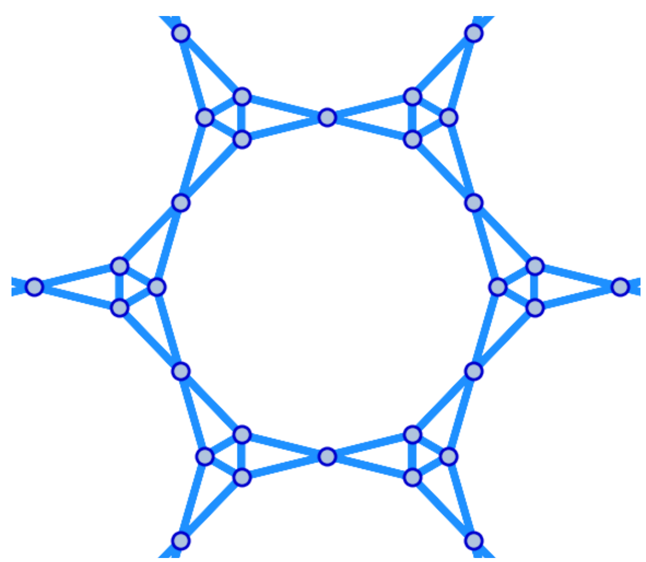


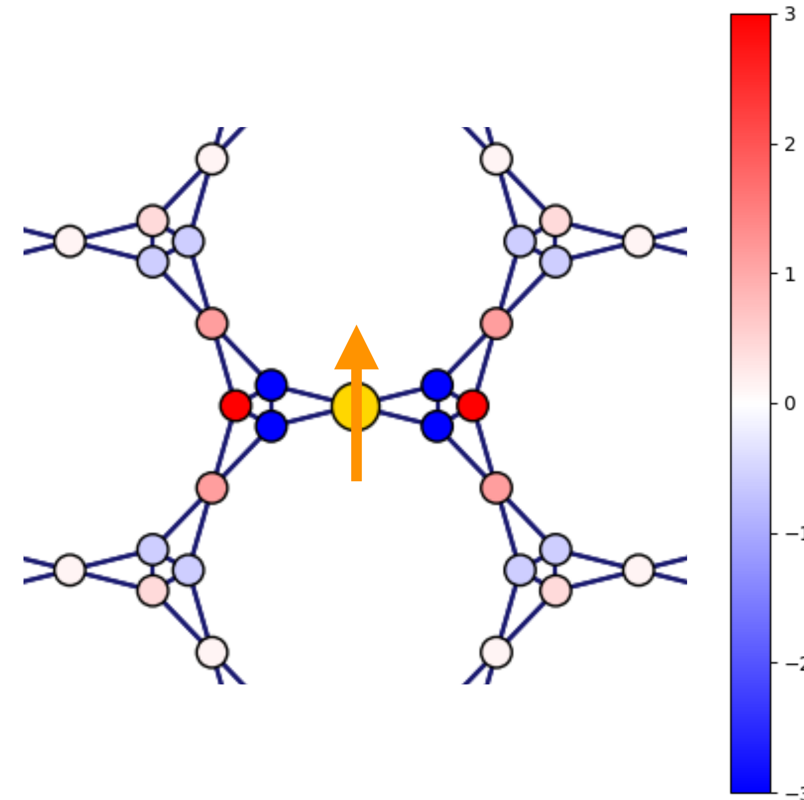


Flat-Band Lattice

- Frustrated Magnet

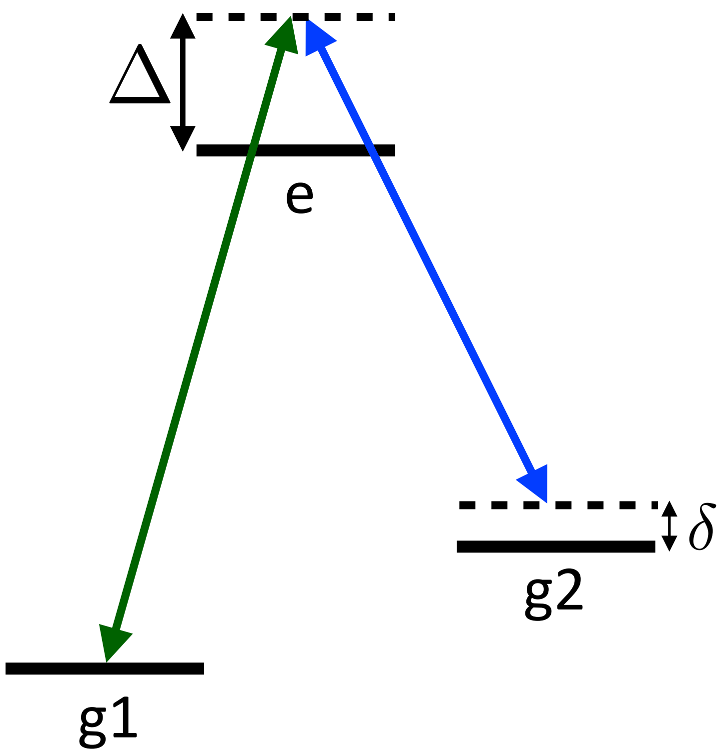






AJK *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Raman-Coupled Spin Models



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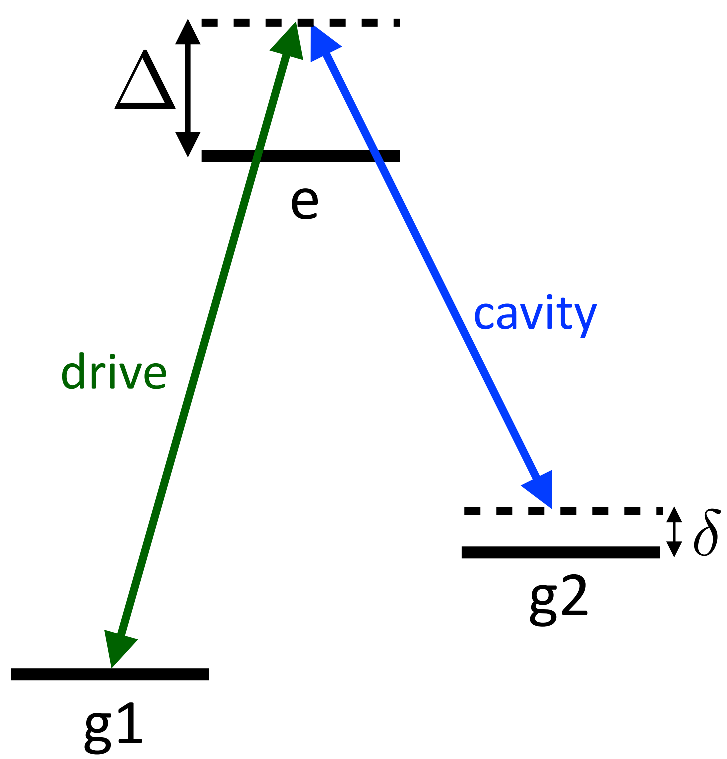

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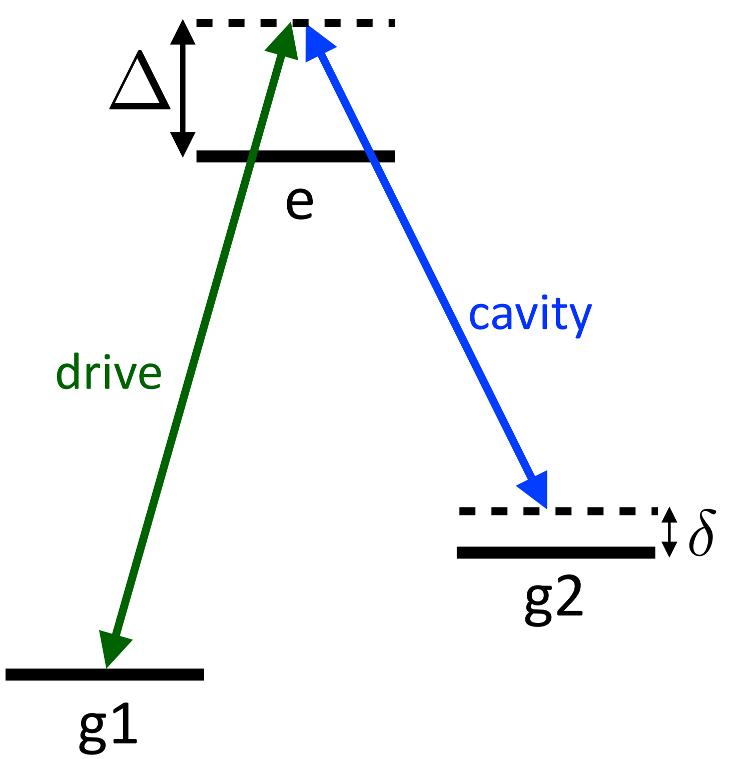
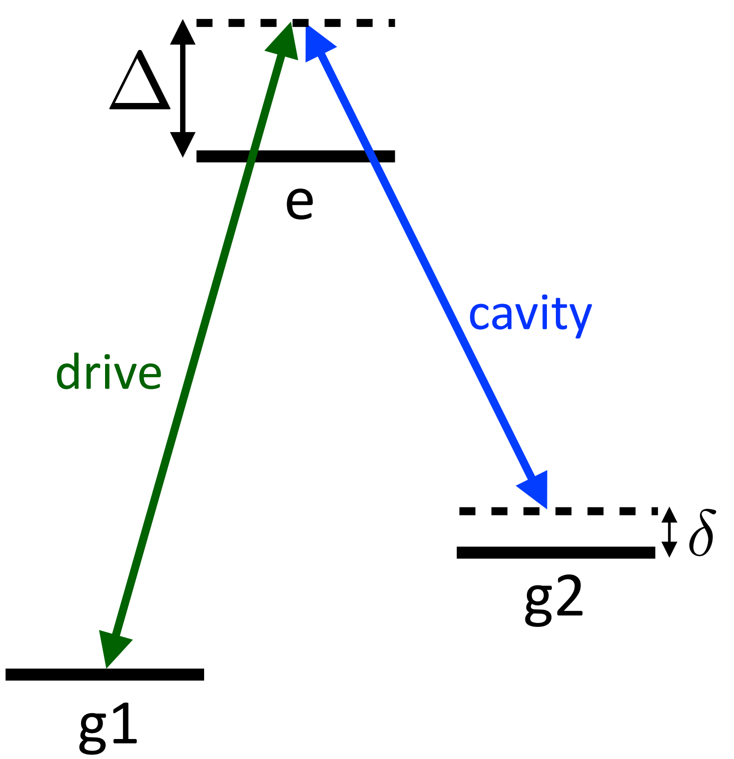


- Microwave-activated coupling

Raman-Coupled Spin Models



- Microwave-activated coupling
- Two relevant detunings

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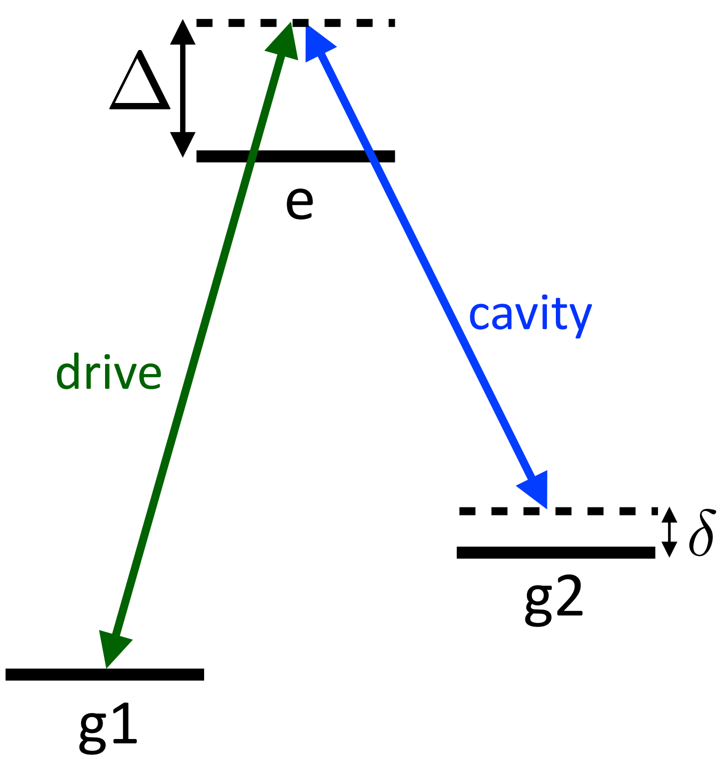

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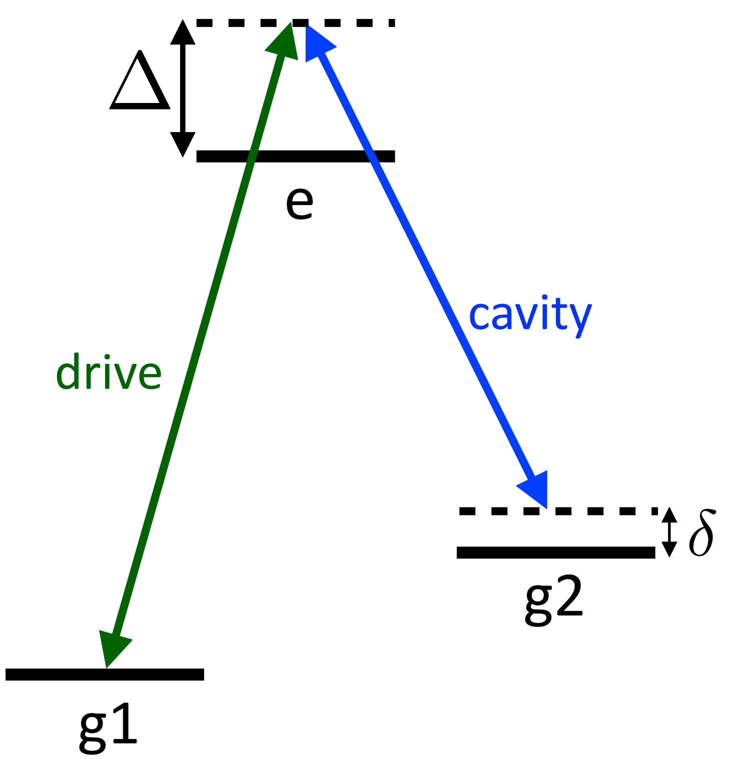

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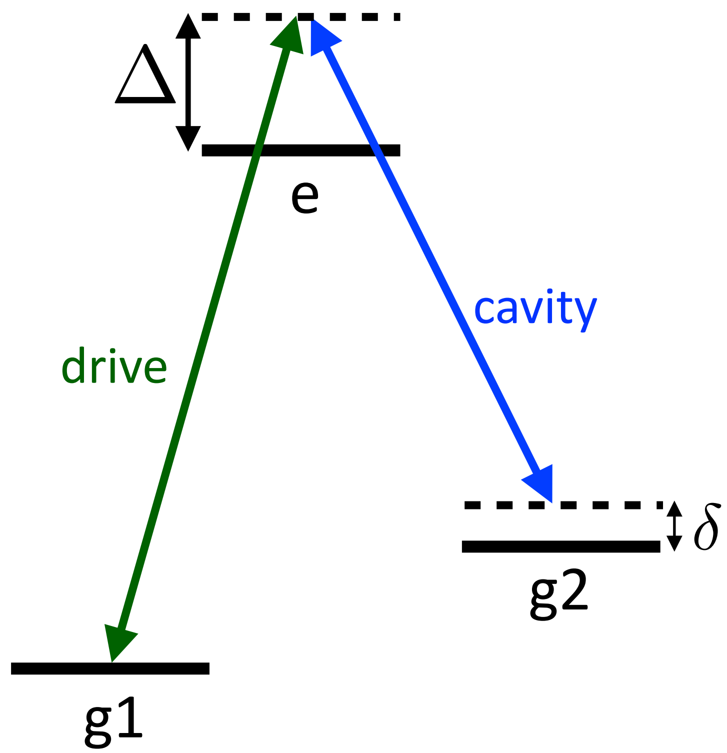
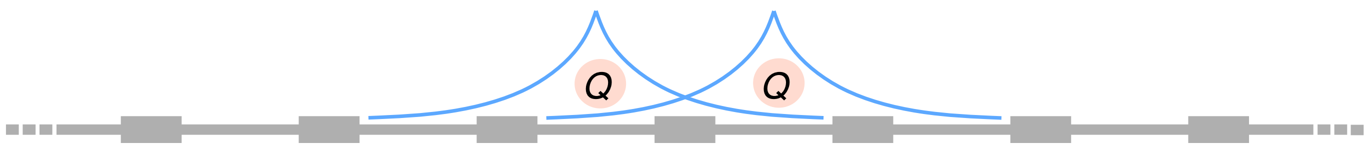
$$H_{Raman} = \hbar \frac{g^2\Omega^2}{\Delta^2\delta}\ \sigma_1^+\sigma_2^- + h.c.$$

Raman-Coupled Spin Models



1D-Photonic Crystal + Single Drive

- Exponentially localized interaction



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Douglas *et al*. Nat. Photon. 9 (2015)

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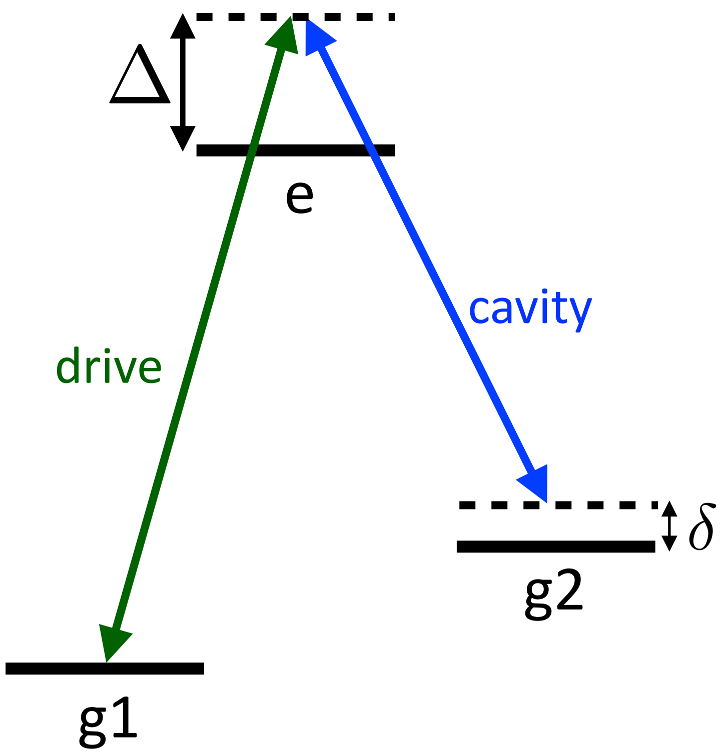

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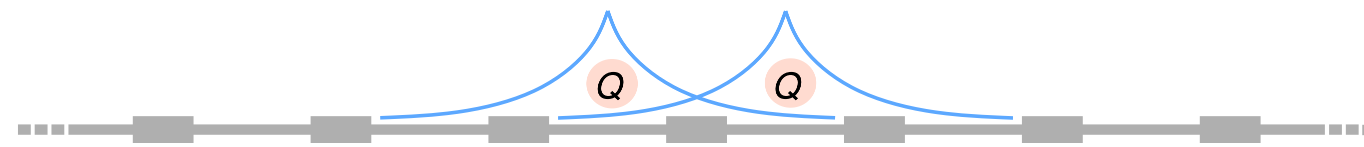

1D-Photonic Crystal + Multiple Drives

Douglas *et al*. Nat. Photon. 9 (2015)

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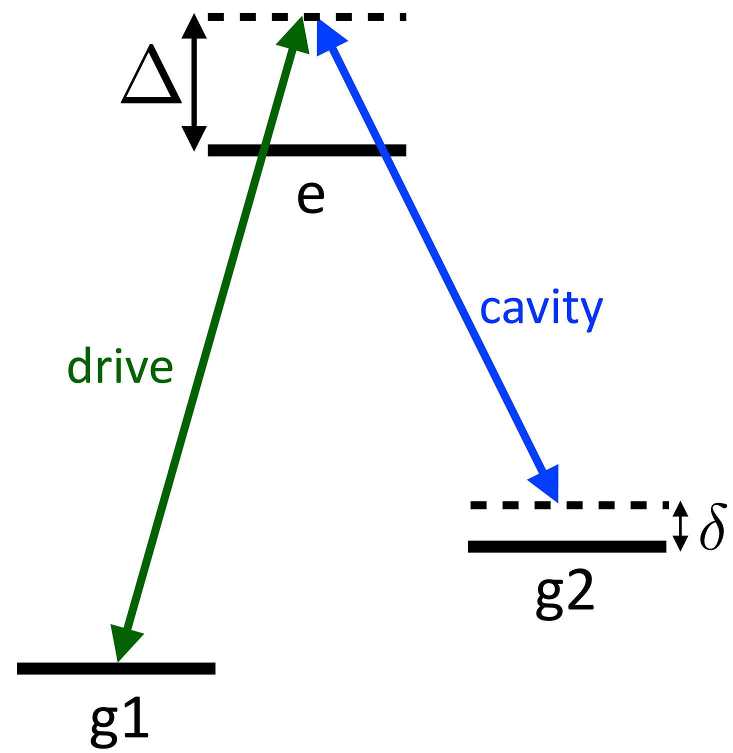

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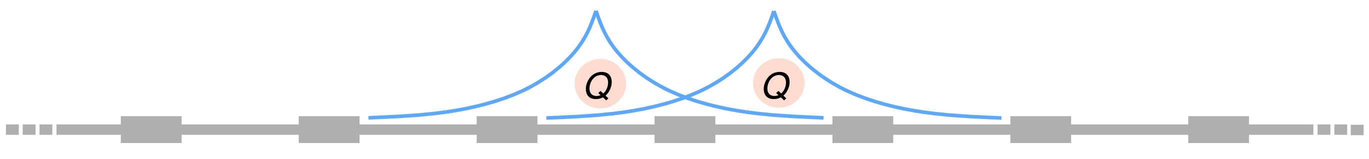

1D-Photonic Crystal + Multiple Drives

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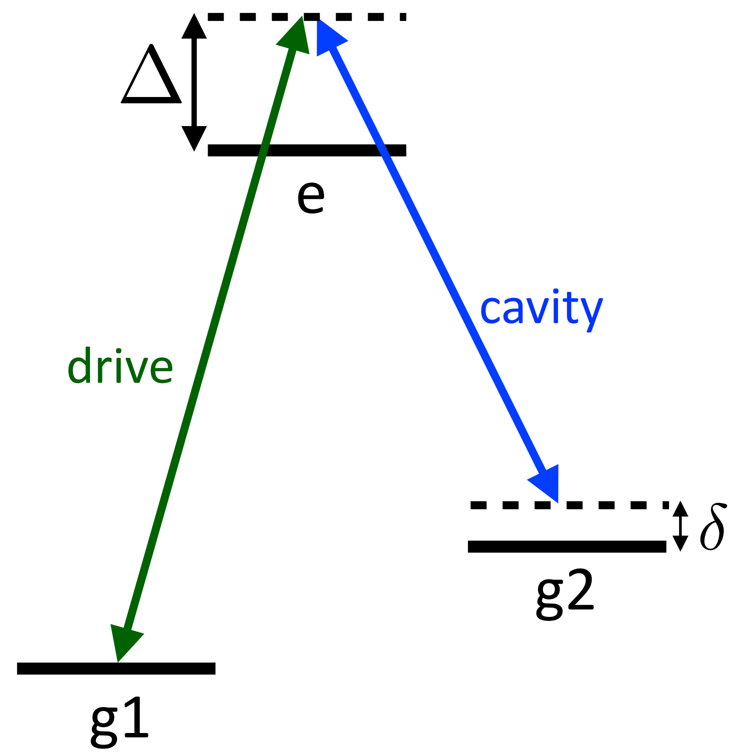

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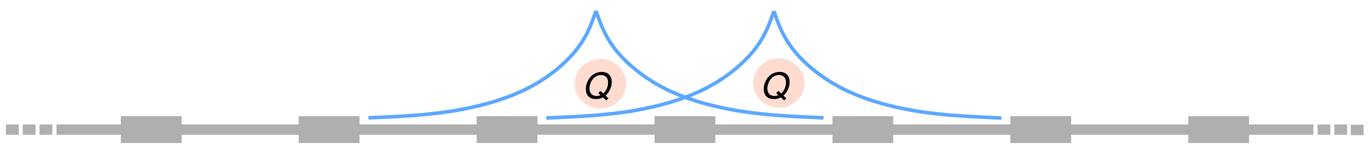

1D-Photonic Crystal + Multiple Drives

- Superposition of exponentials
- Approximate power-law interaction

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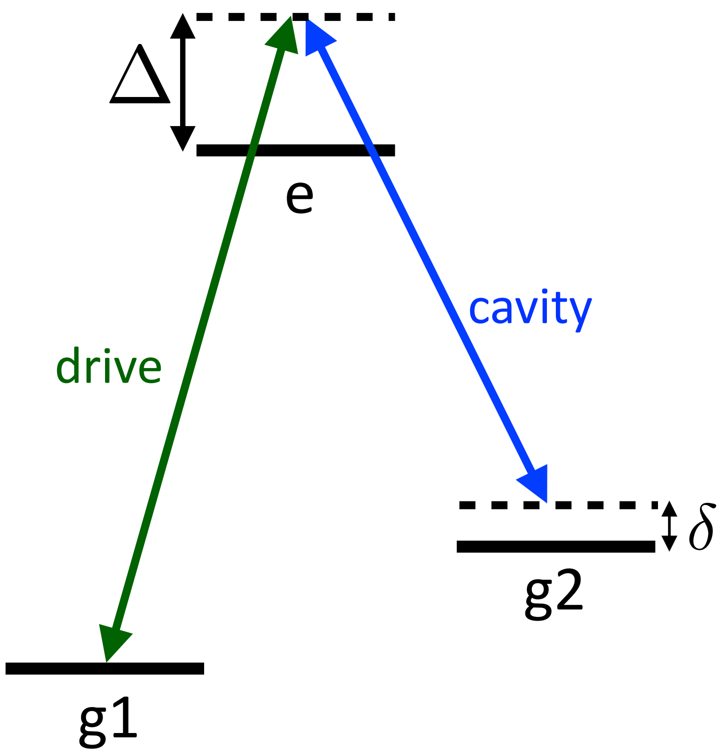

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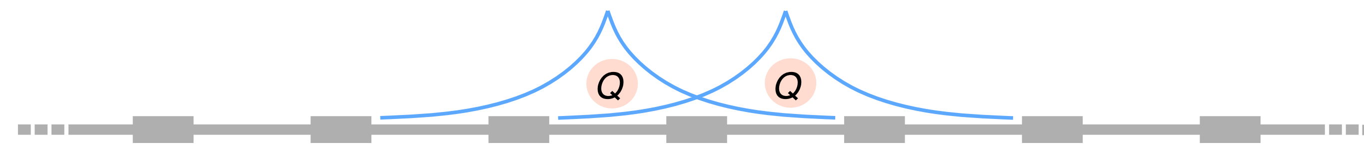

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↓

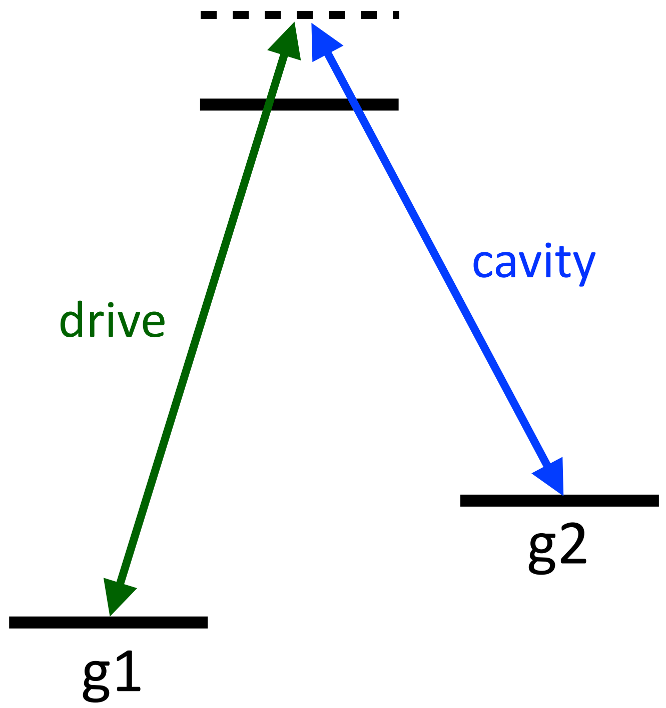
Need 3-level qubit

Douglas *et al*. Nat. Photon. 9 (2015)

Raman Transitions in Fluxonium

Rabi oscillation

- Gaussian pulse off-resonant of plasmon
- Vacuum Rabi rate of fluxon



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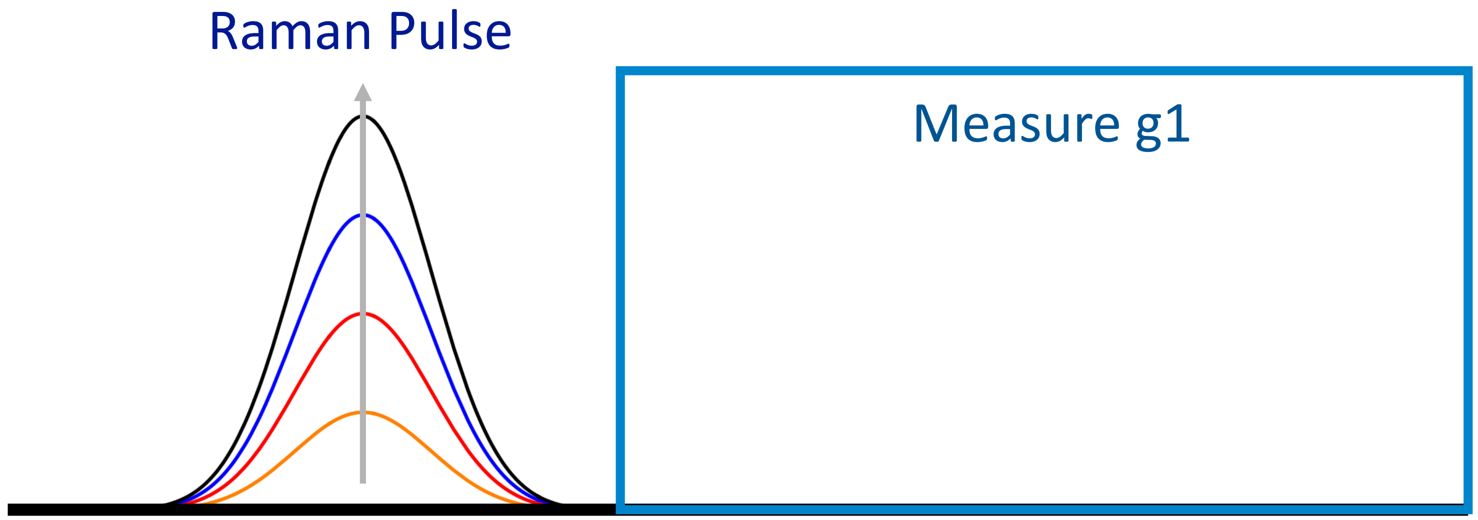

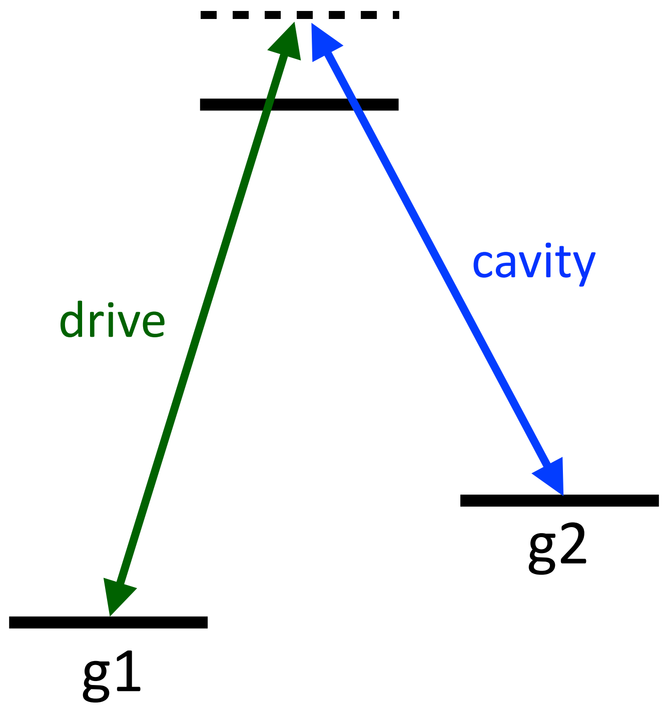

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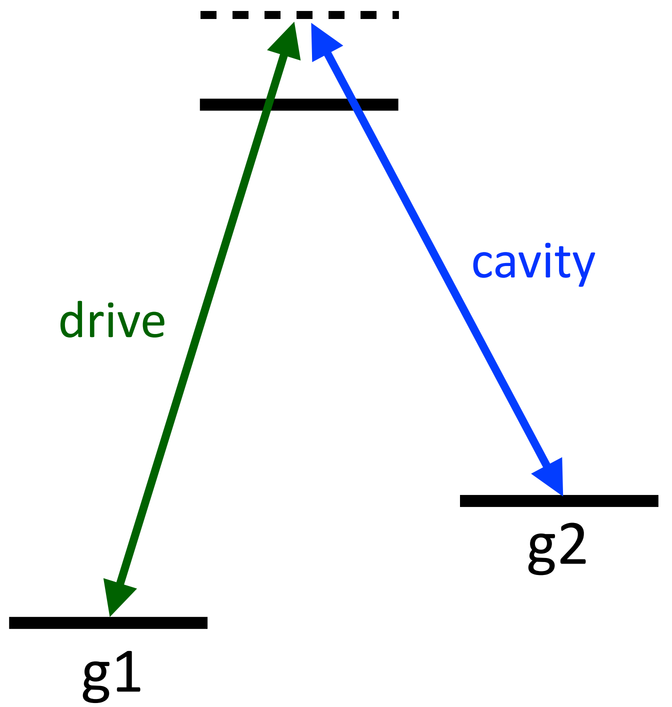
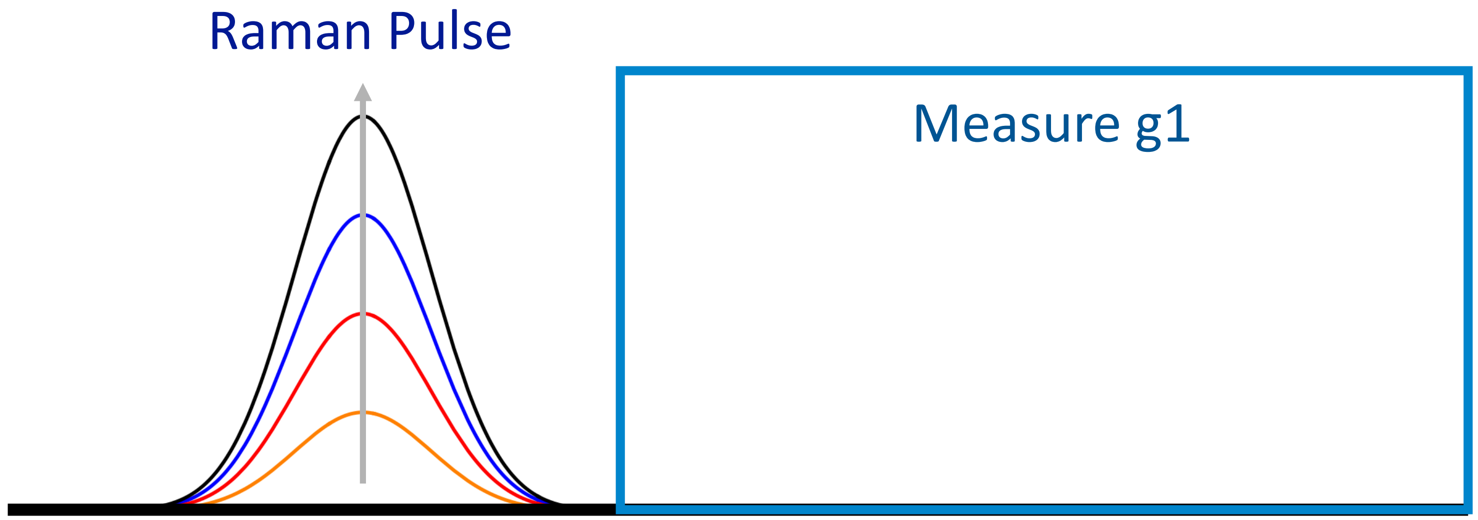
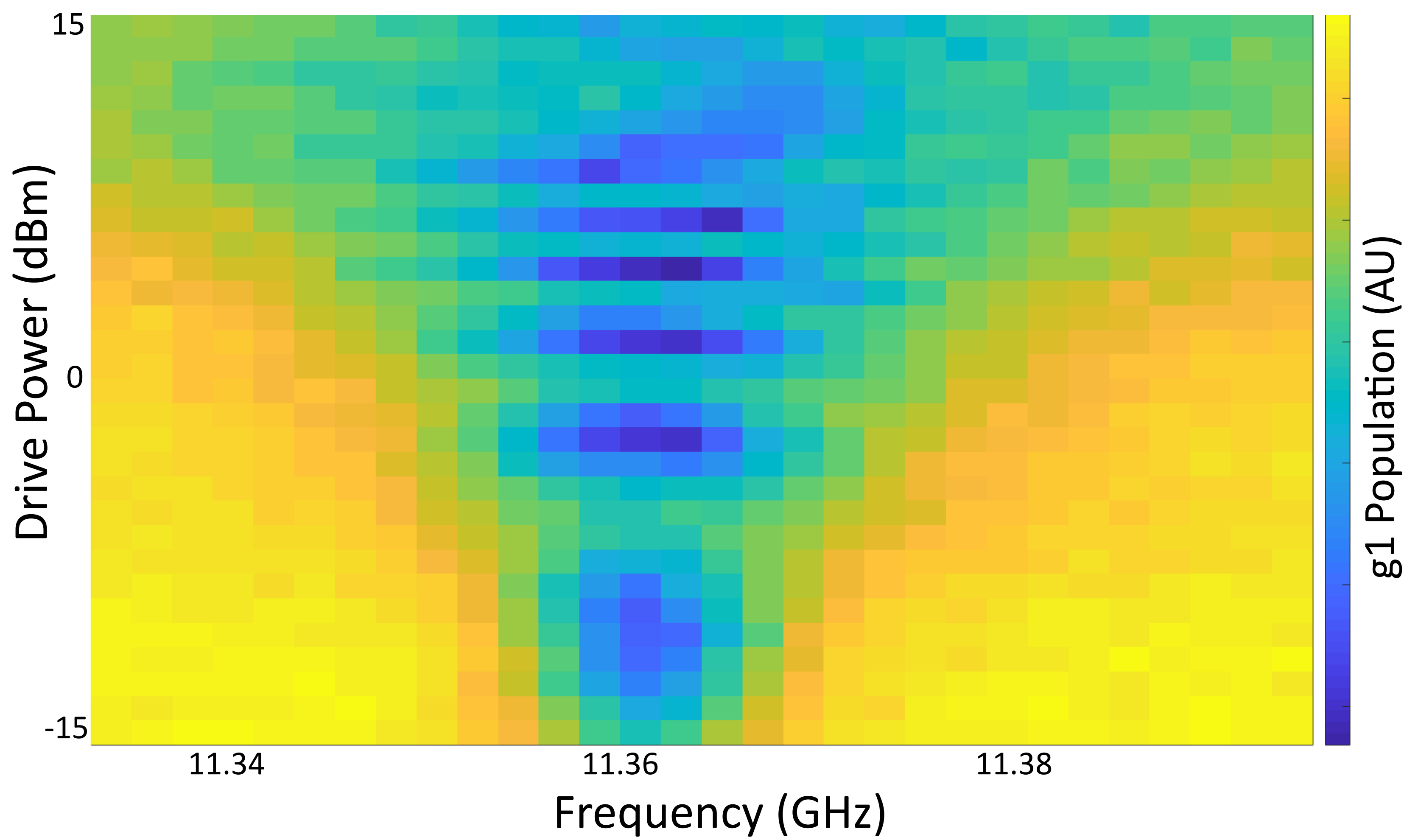


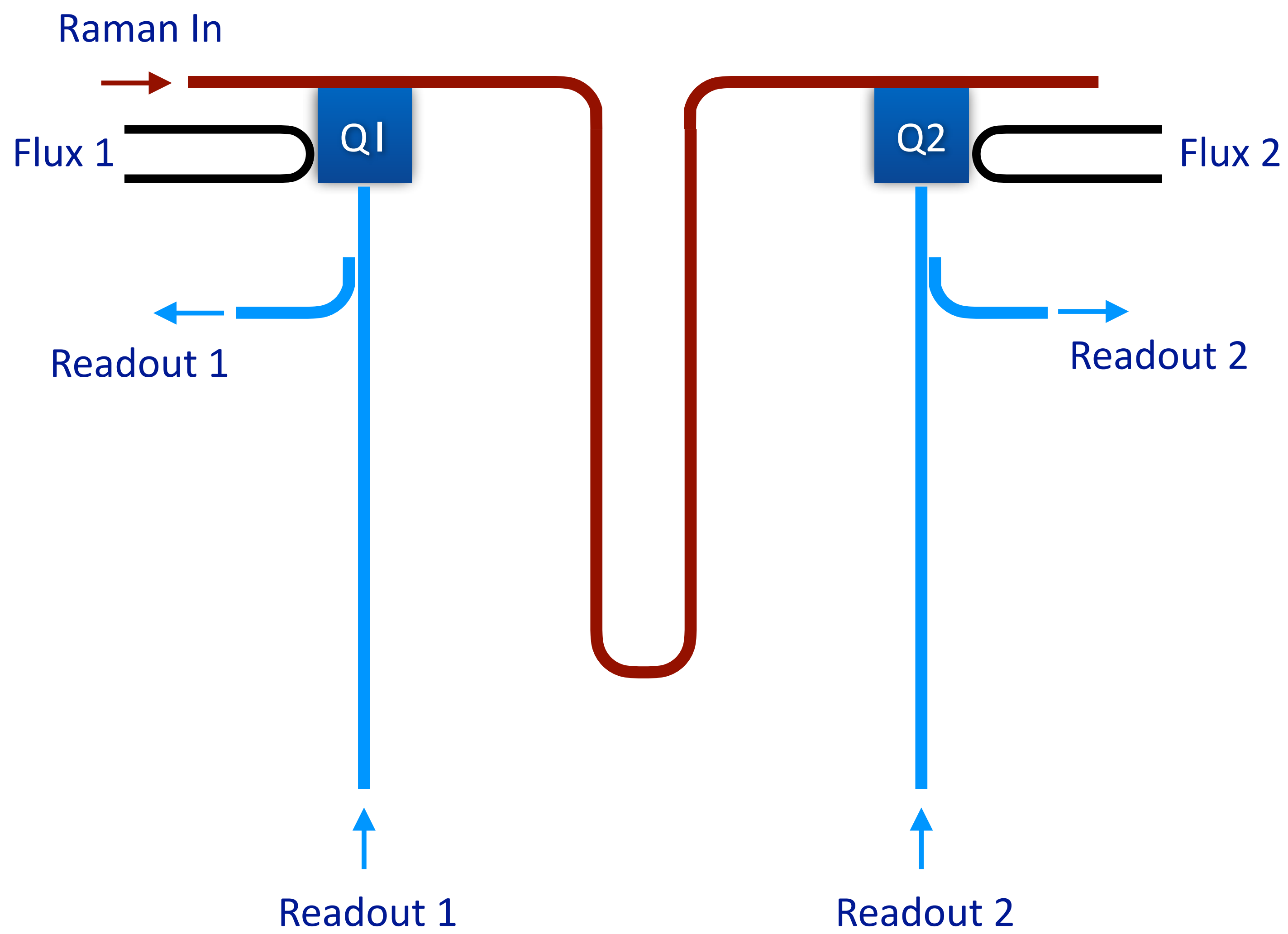




Second-Generation Raman Device

Redesigned Device

- 3-cavities
- Separate resonators allow
  - Optimized readout
  - Parallel readout and coupling



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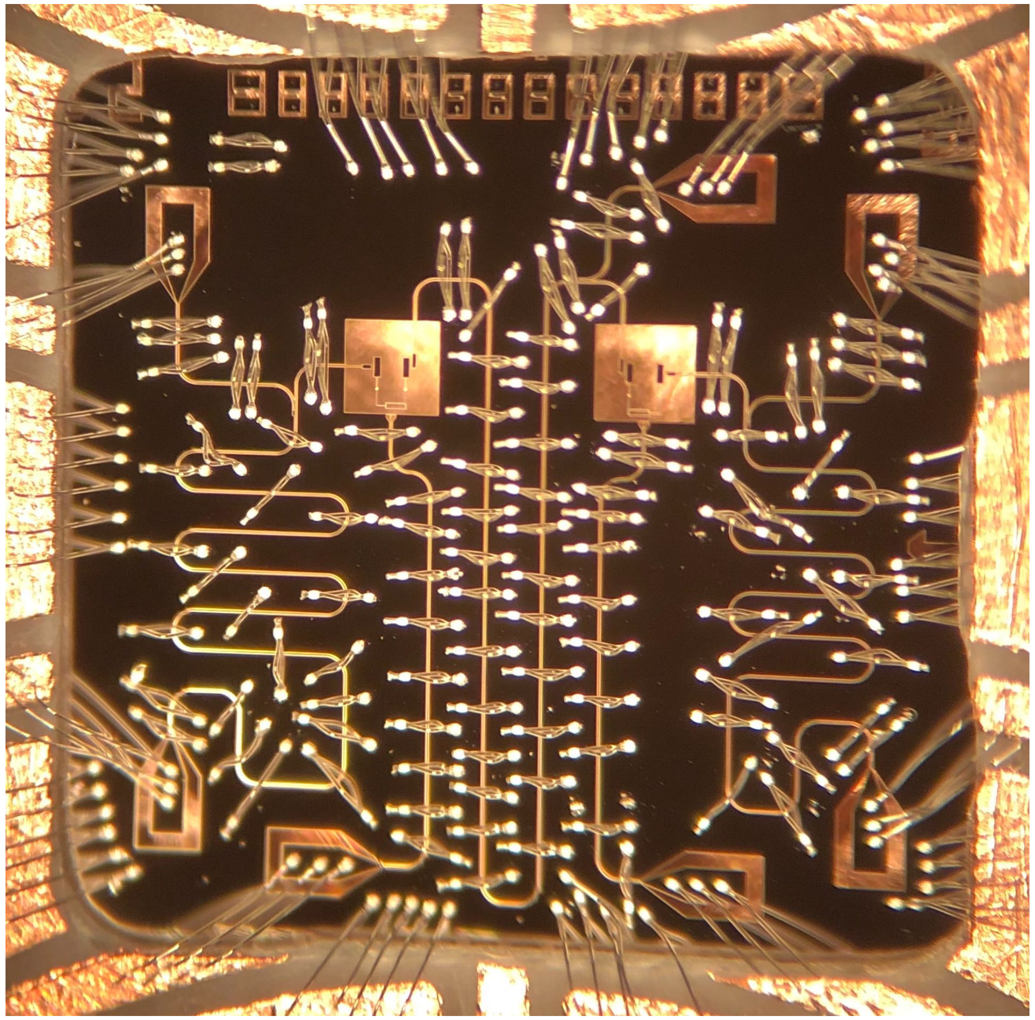

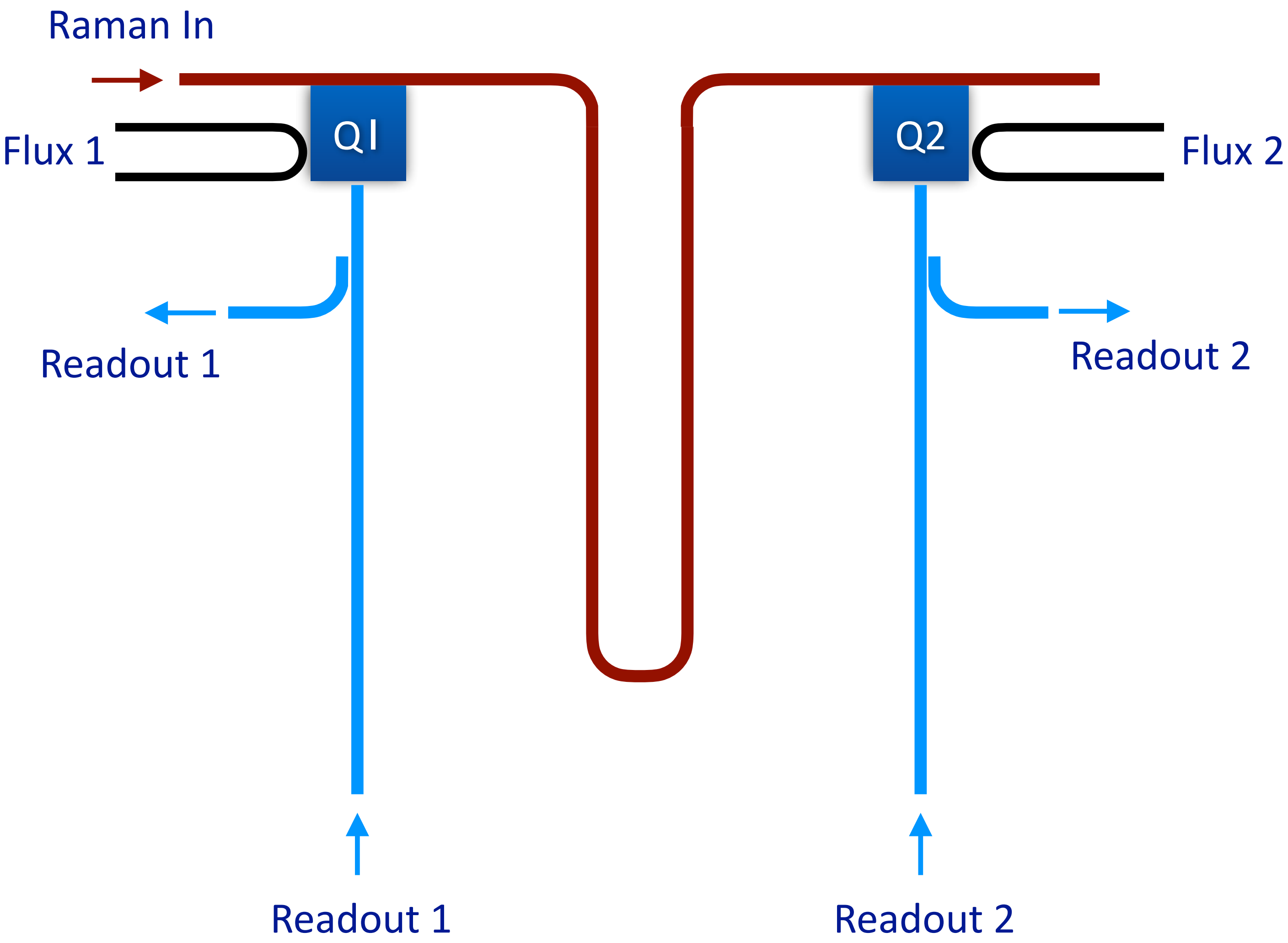

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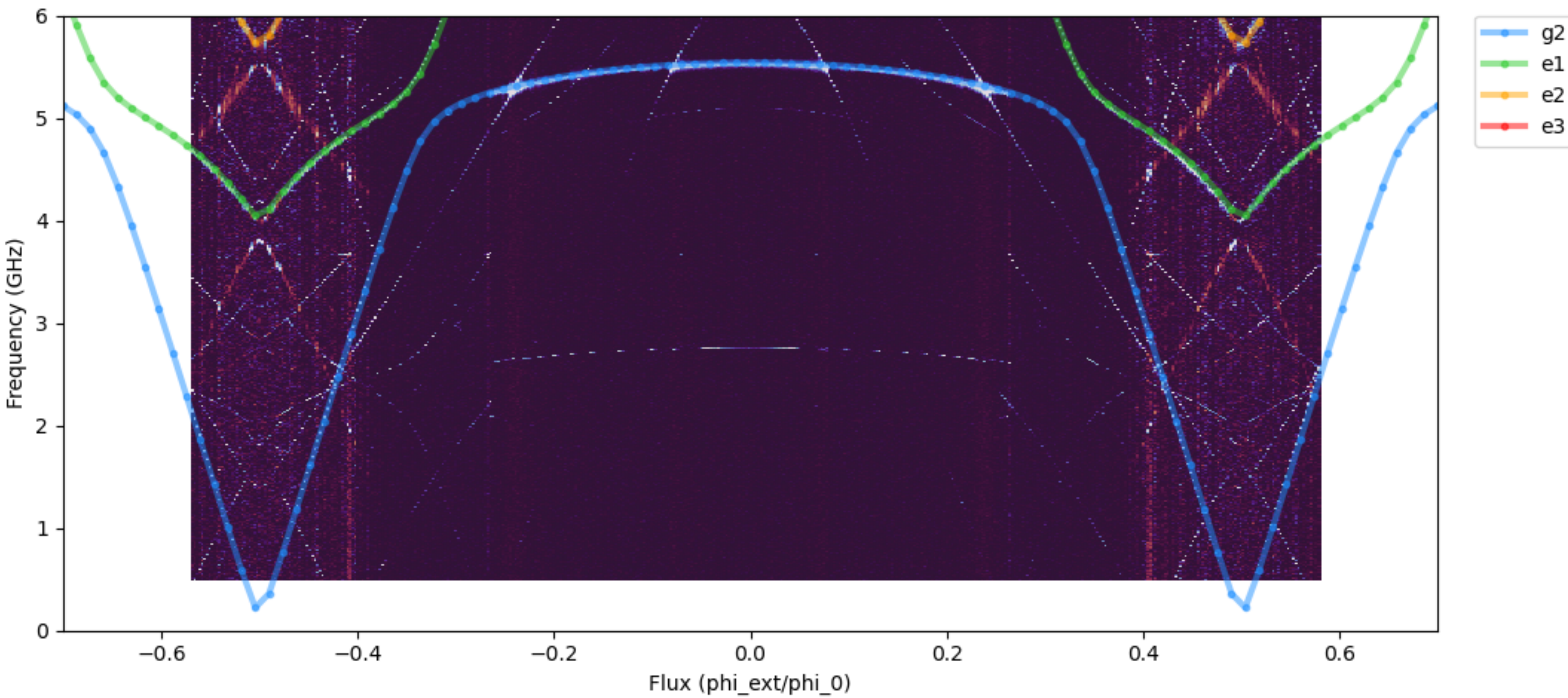
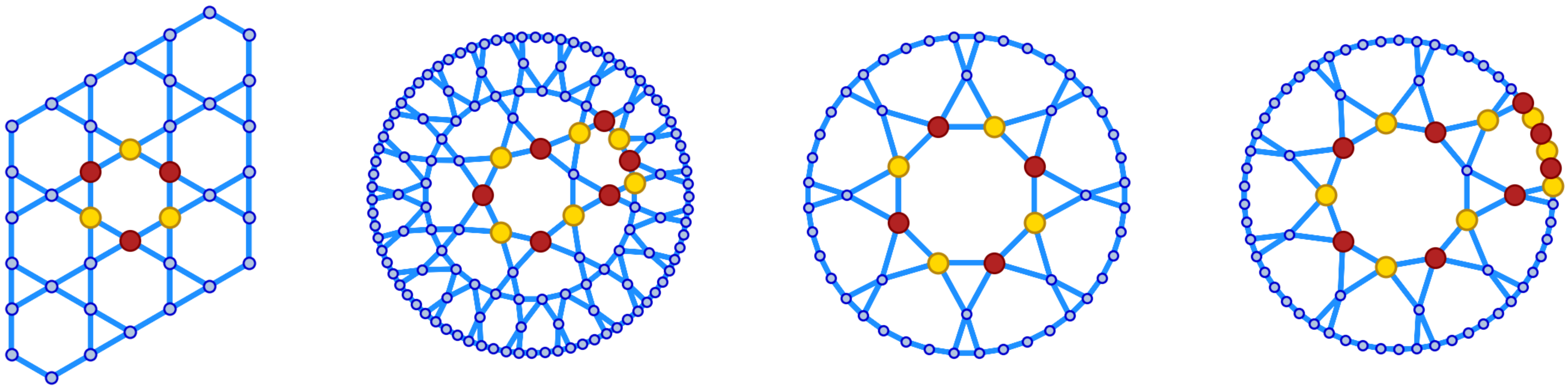
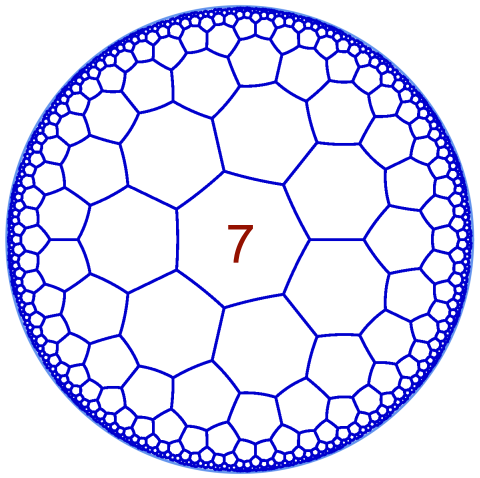
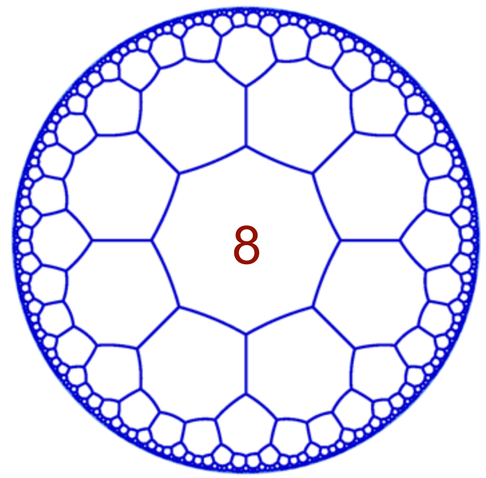


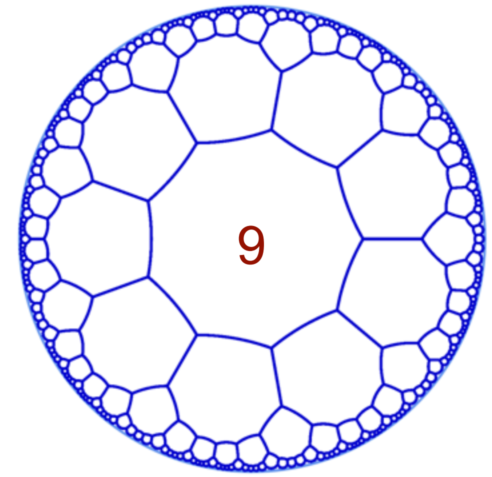
Full-Wave Flat-Band States

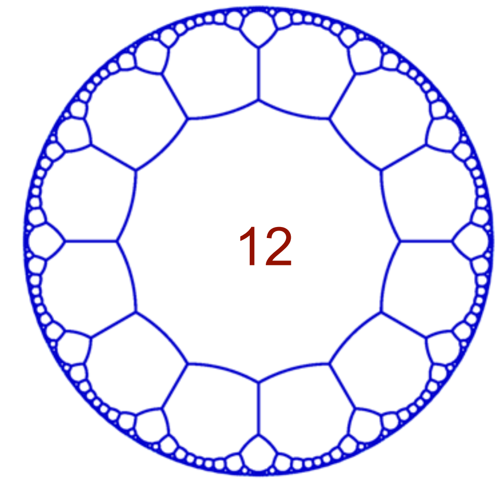


Hyperbolic Lattices and Curvature









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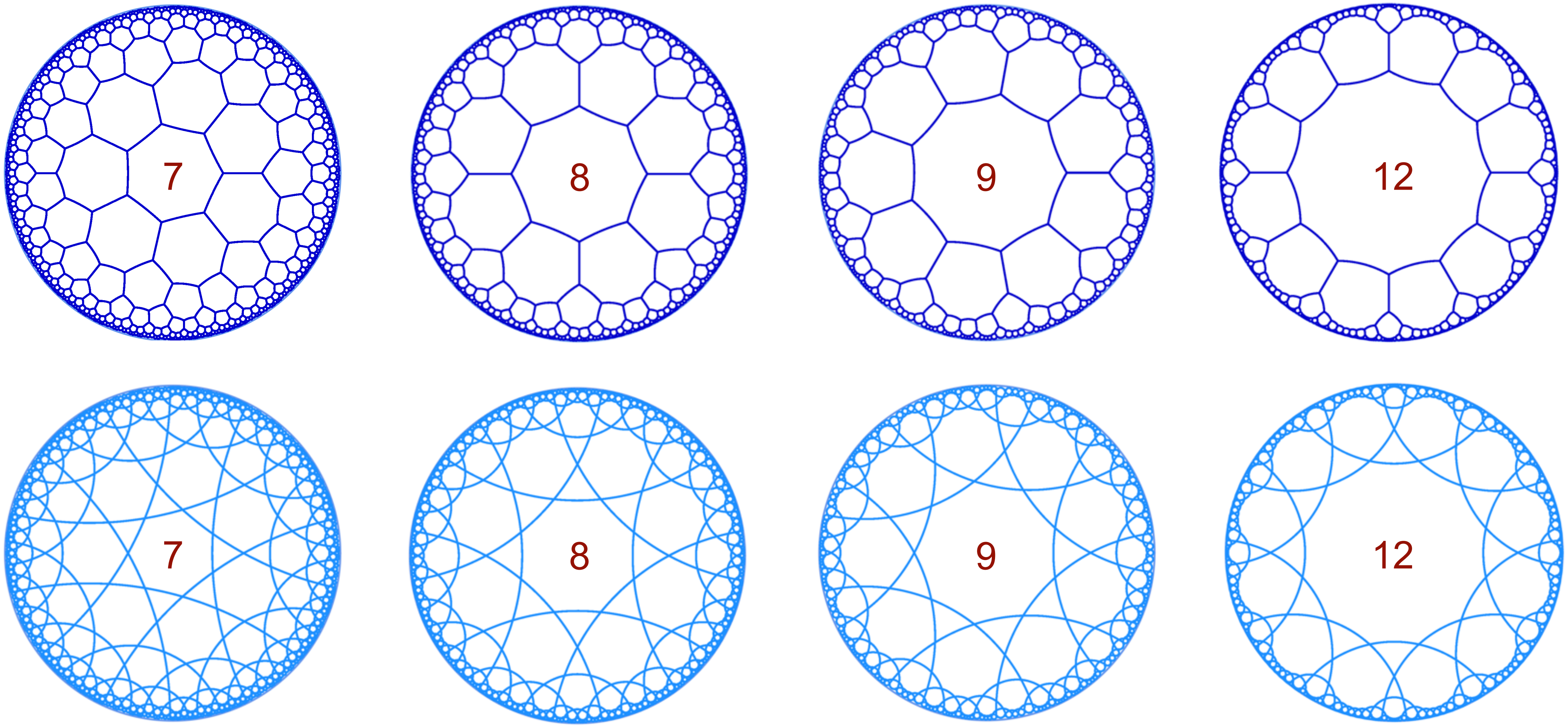

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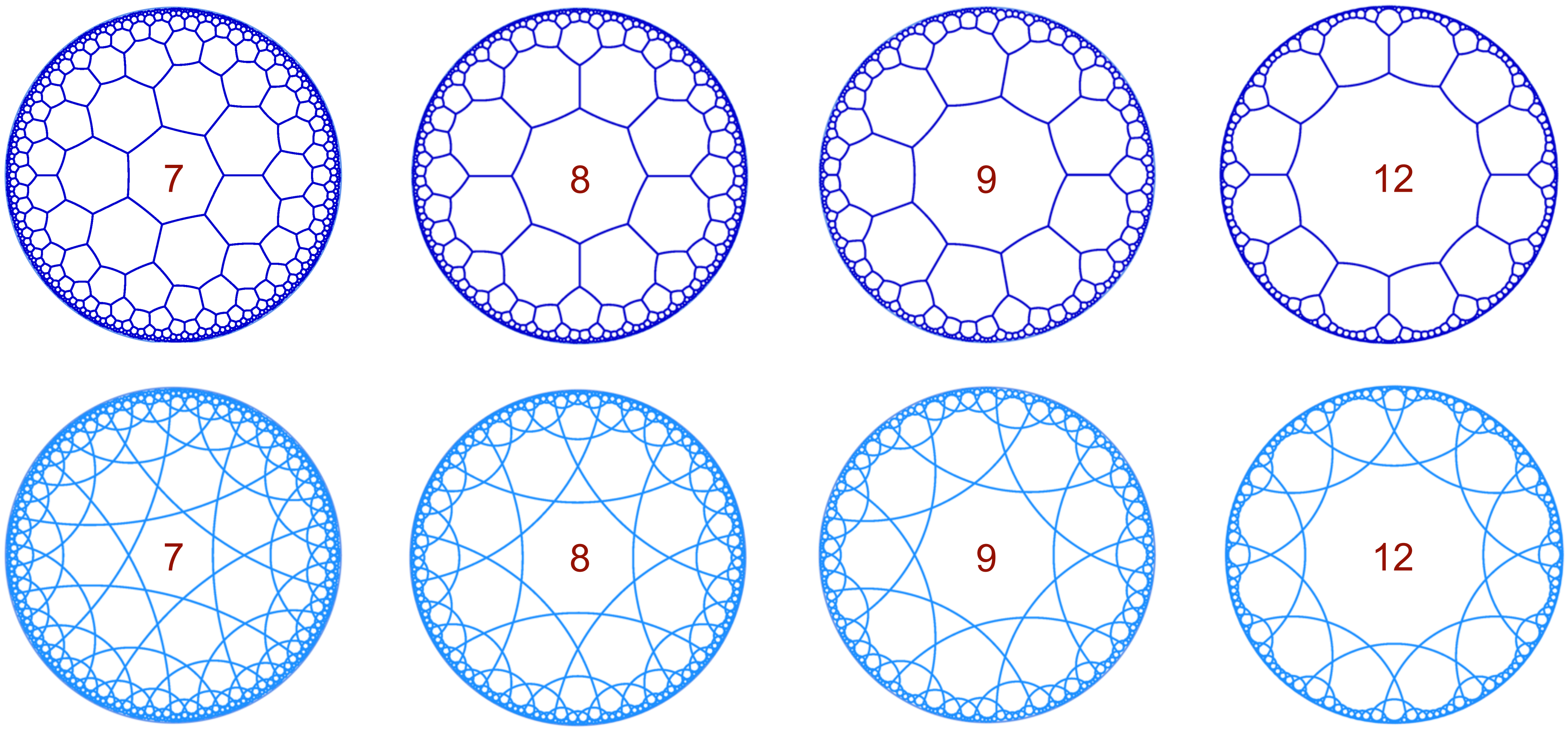
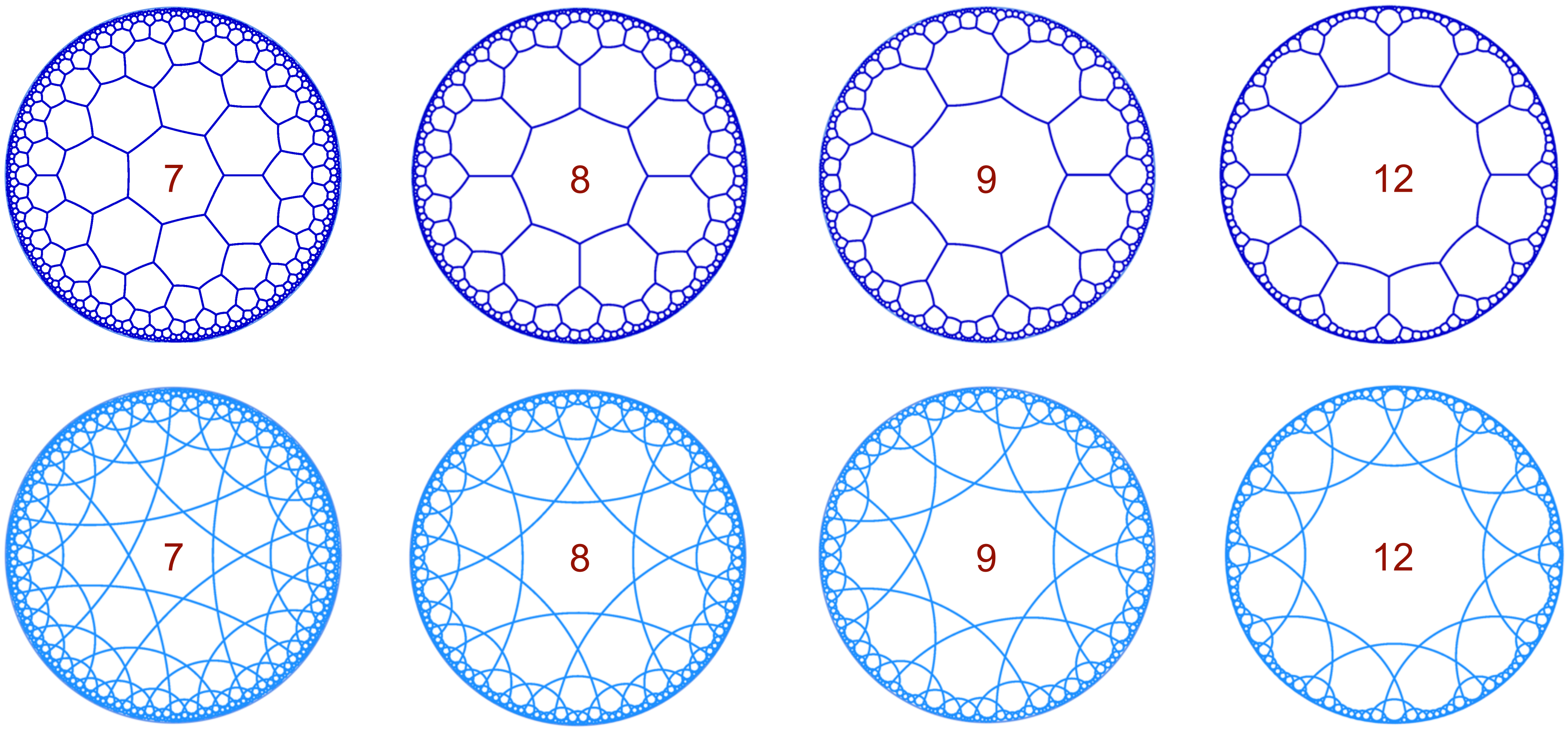


Gaussian Curvature

$$K = -\frac{1}{R^2}$$

Hyperbolic Lattices and Curvature



Gaussian Curvature

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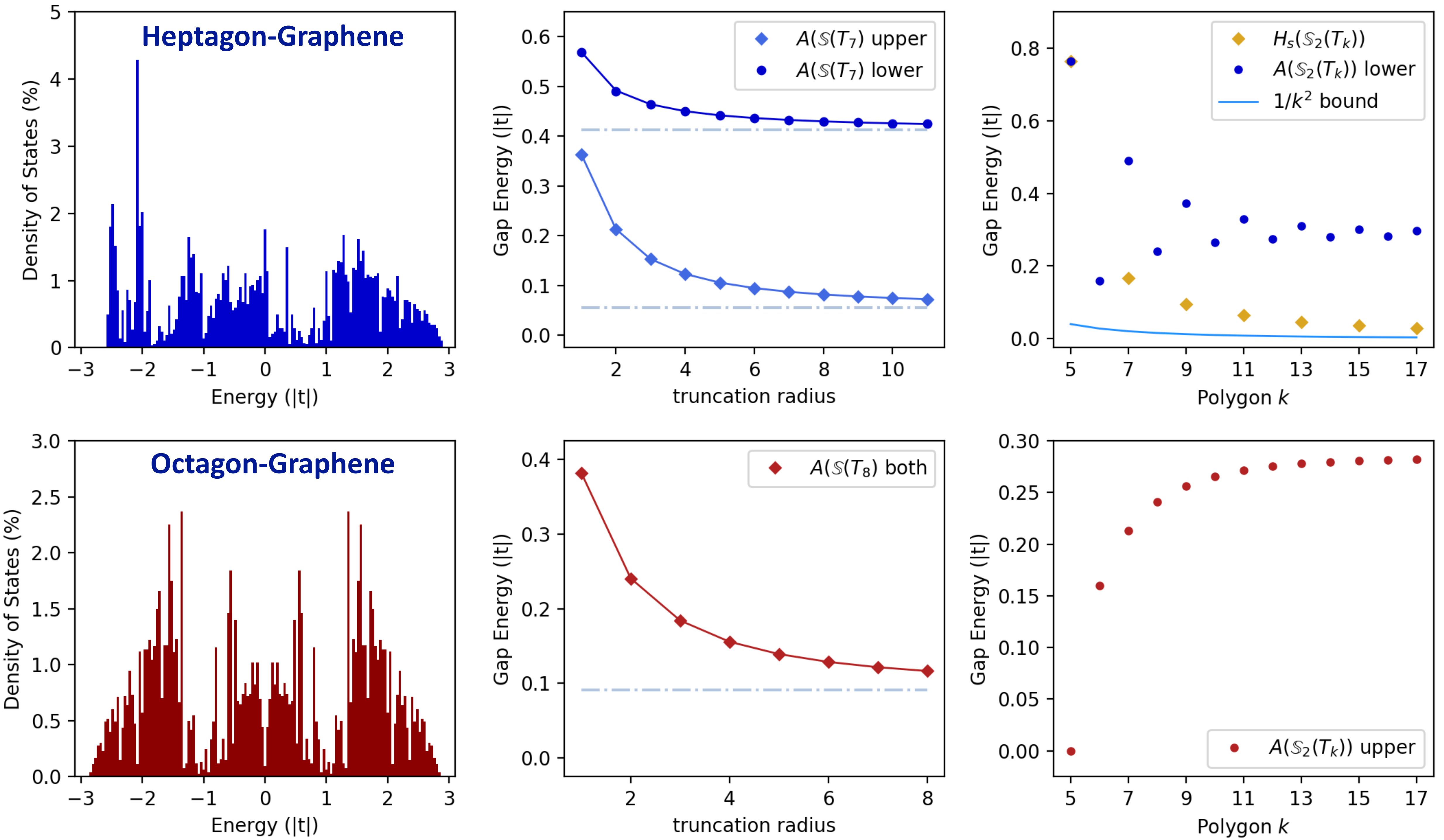
Tiling Polygon (n)	Lattice Constant	Medial Lattice Constant
7	0.566	0.492
8	0.727	0.633
9	0.819	0.714
10	0.879	0.767
11	0.921	0.804
12	0.952	0.831

Hyperbolic Numerics



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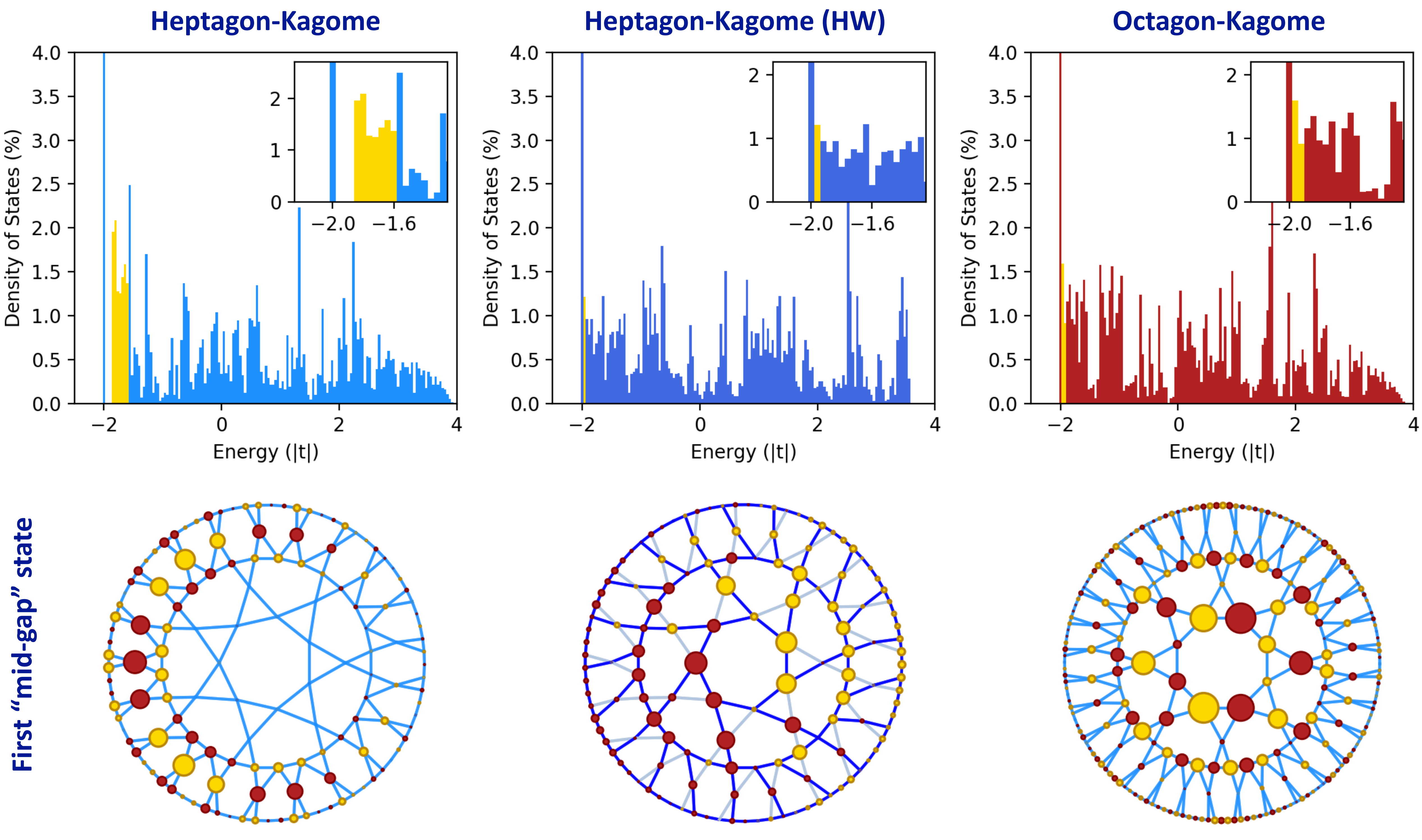
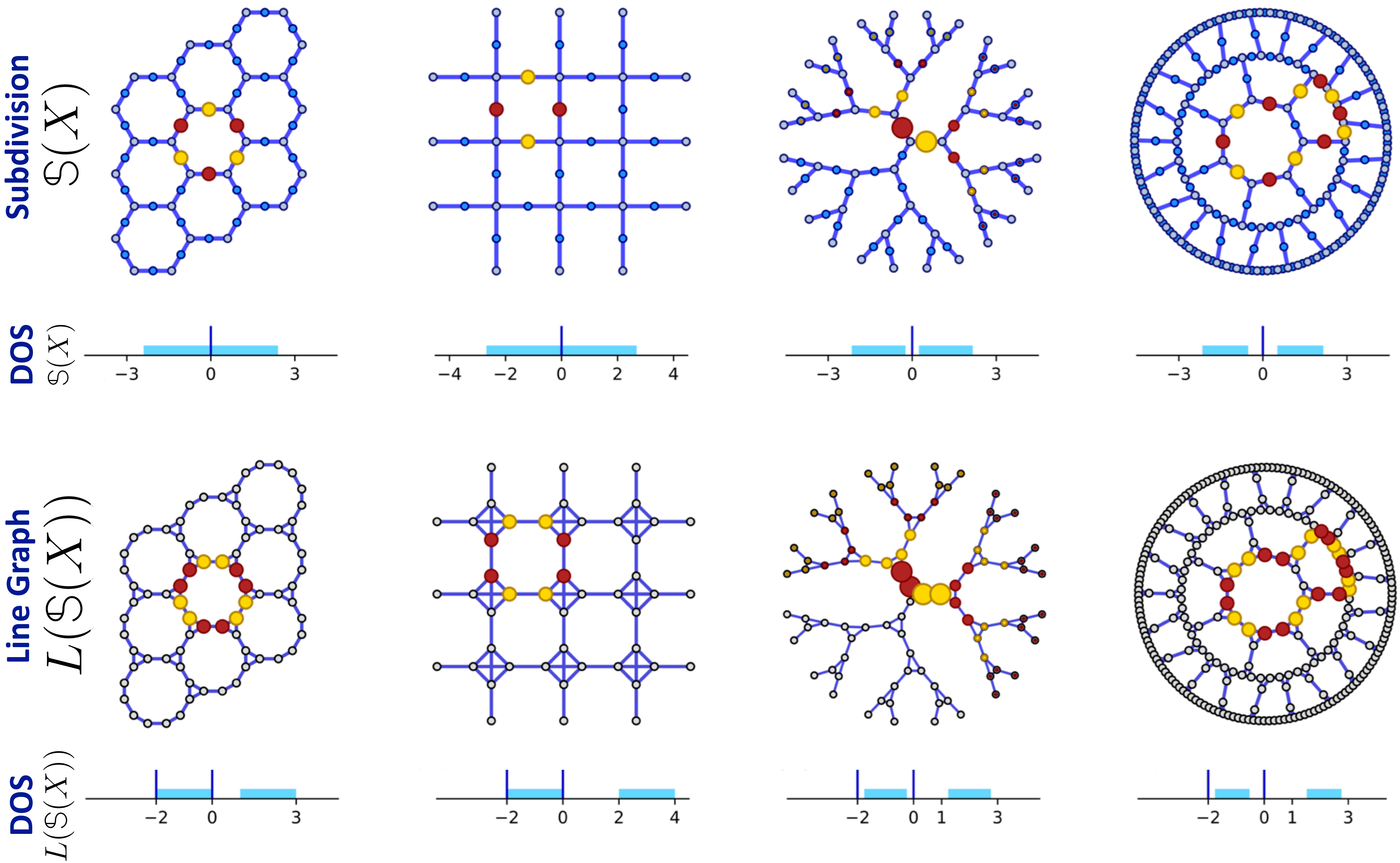
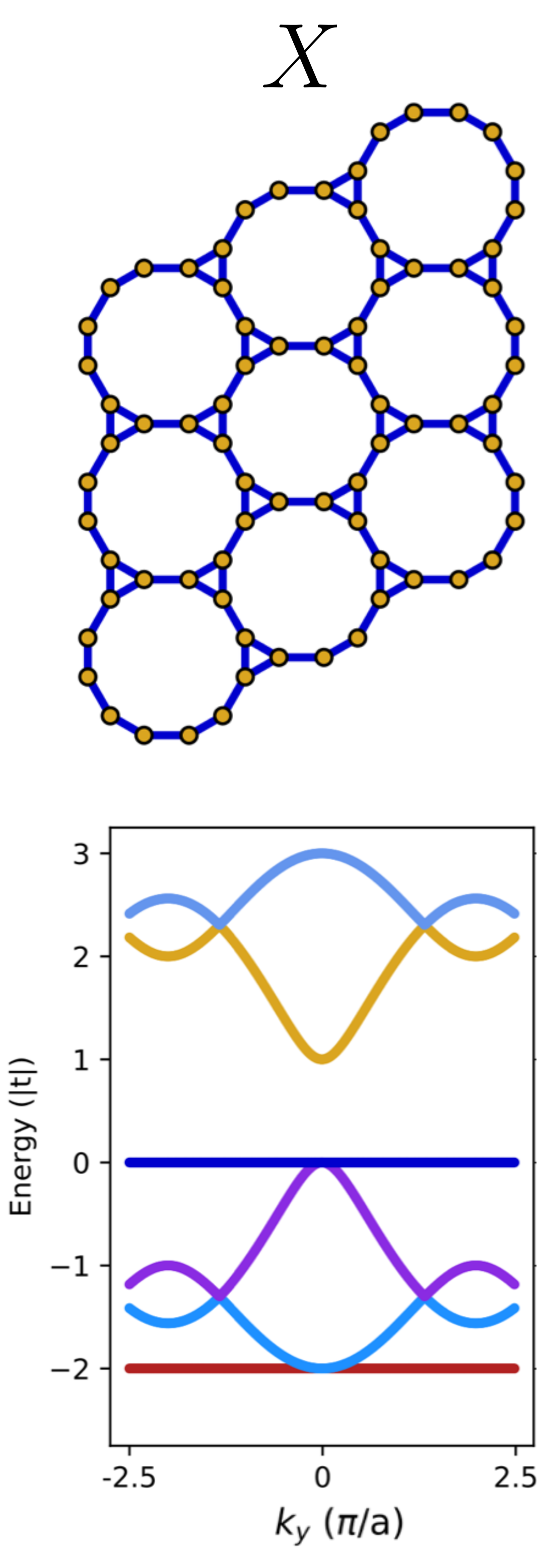


Subdivision Graphs: Flat Bands at 0

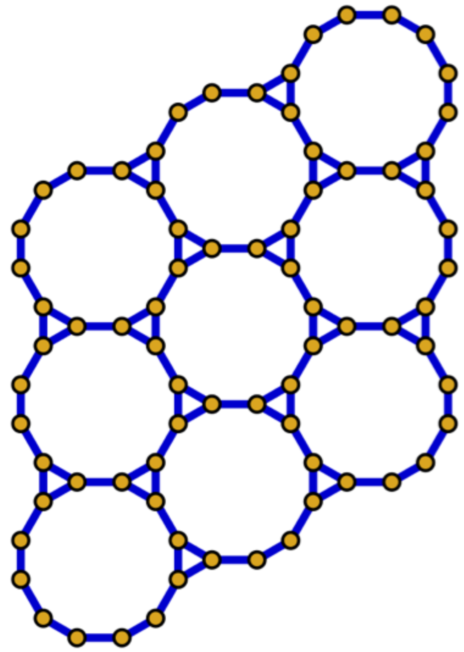


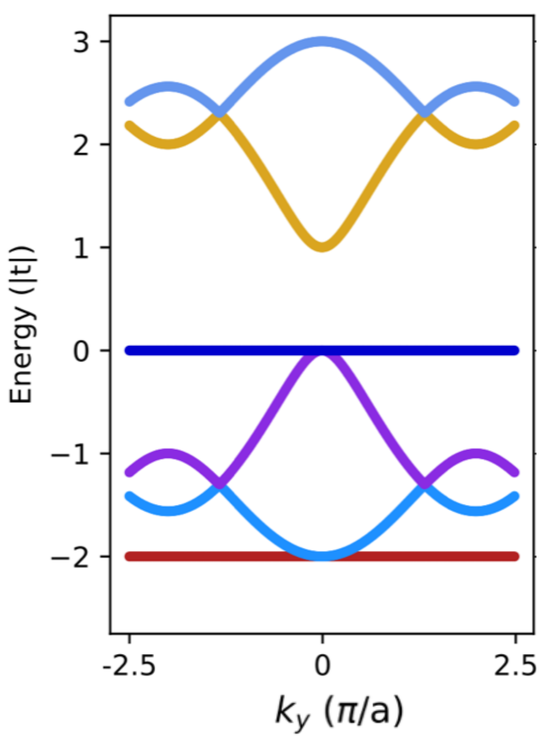
Subdivision Graphs and Optimally Gapped Flat Bands



Subdivision Graphs and Optimally Gapped Flat Bands

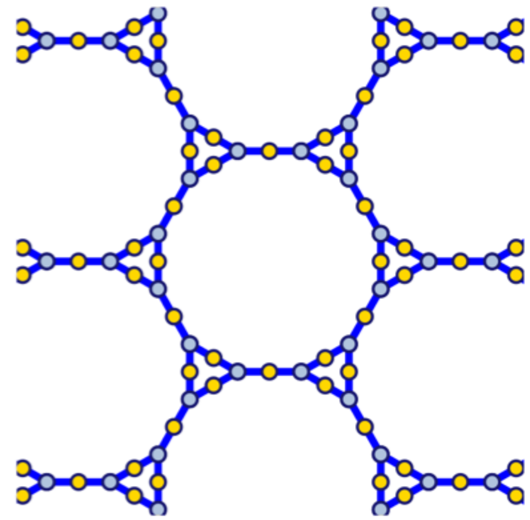
X

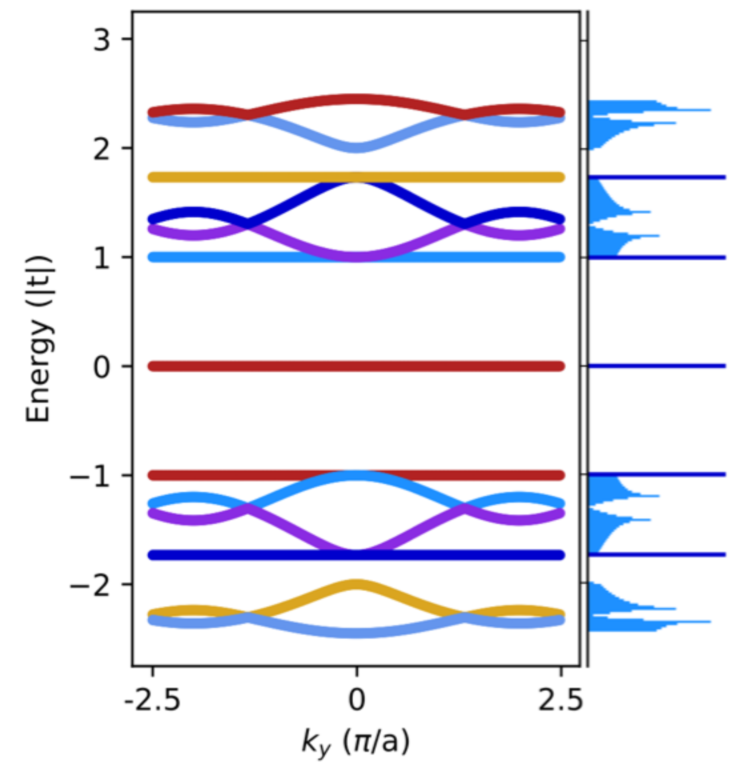




$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

$\mathbb{S}(X)$





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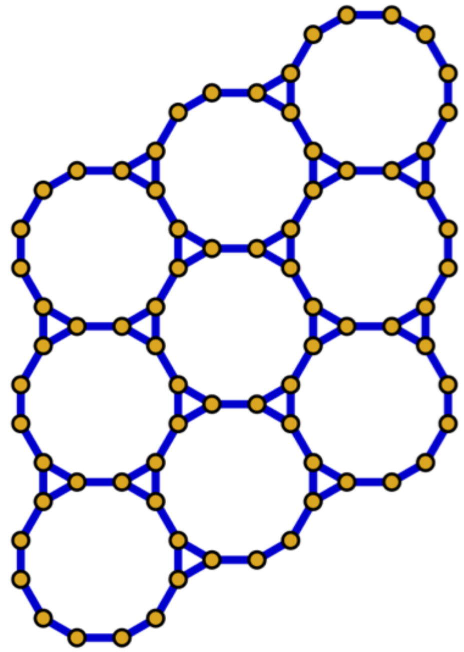

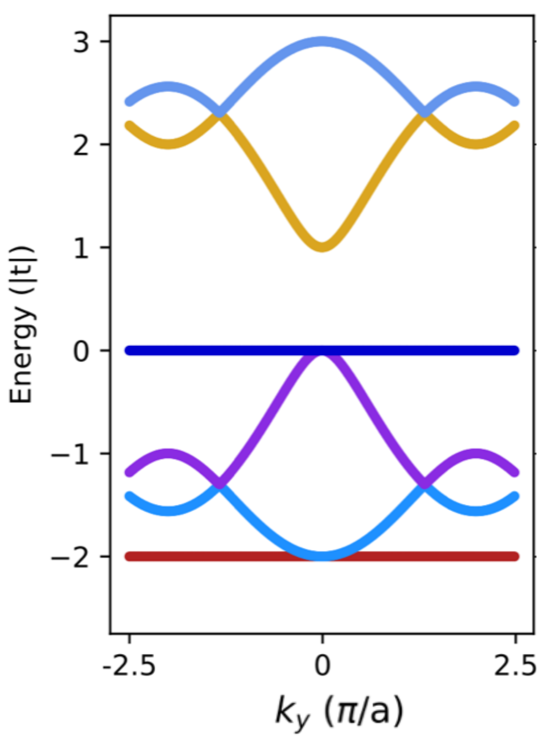

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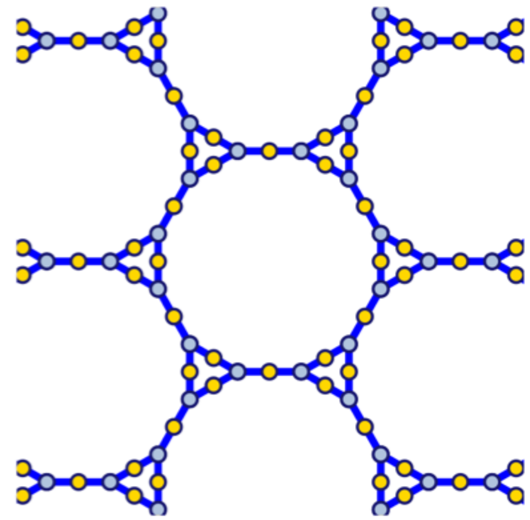
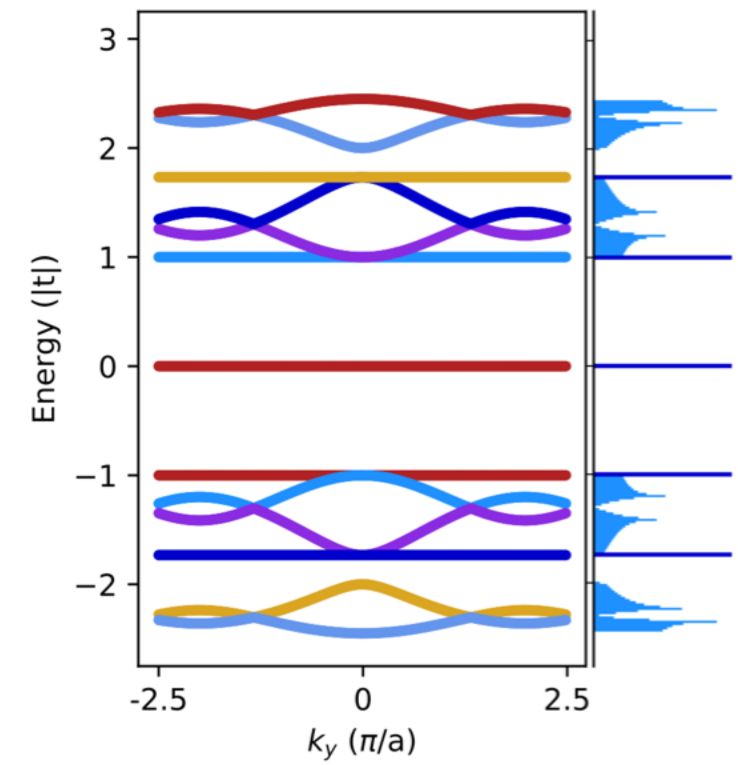
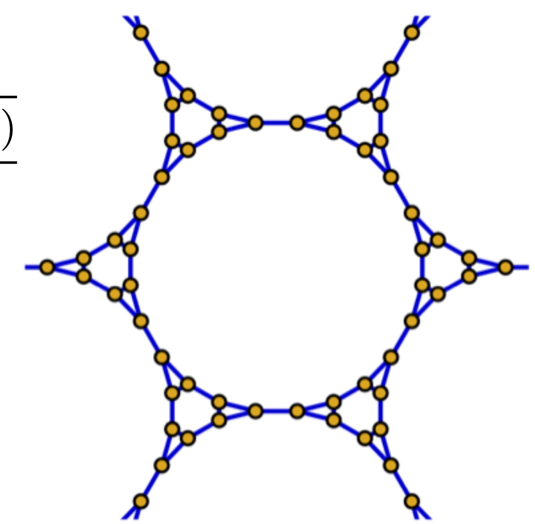
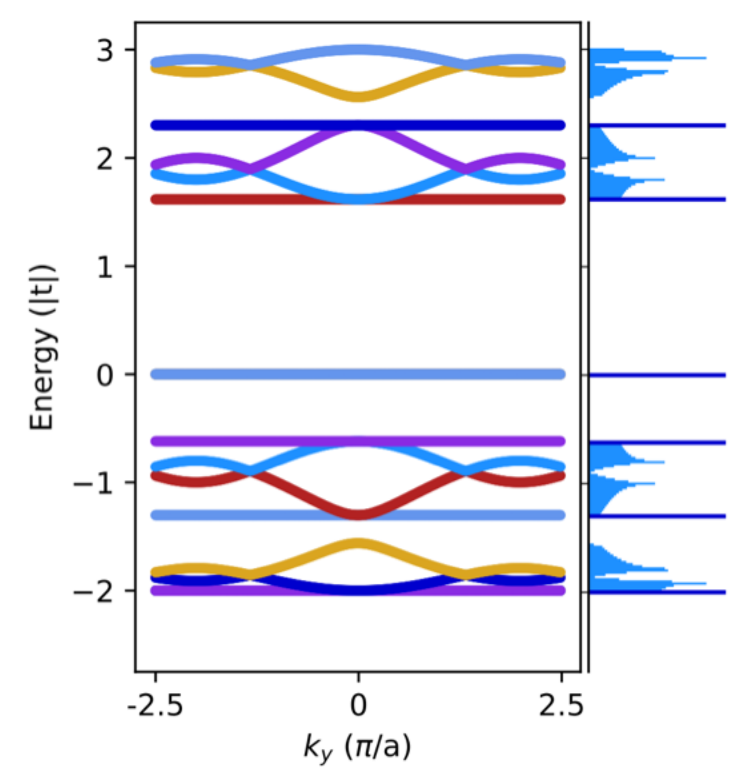




$L(\mathbb{S}(X))$

$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1 \pm \sqrt{1 + 4(E_X + 3)}}{2} \\ 0 \\ -2 \end{cases}$$





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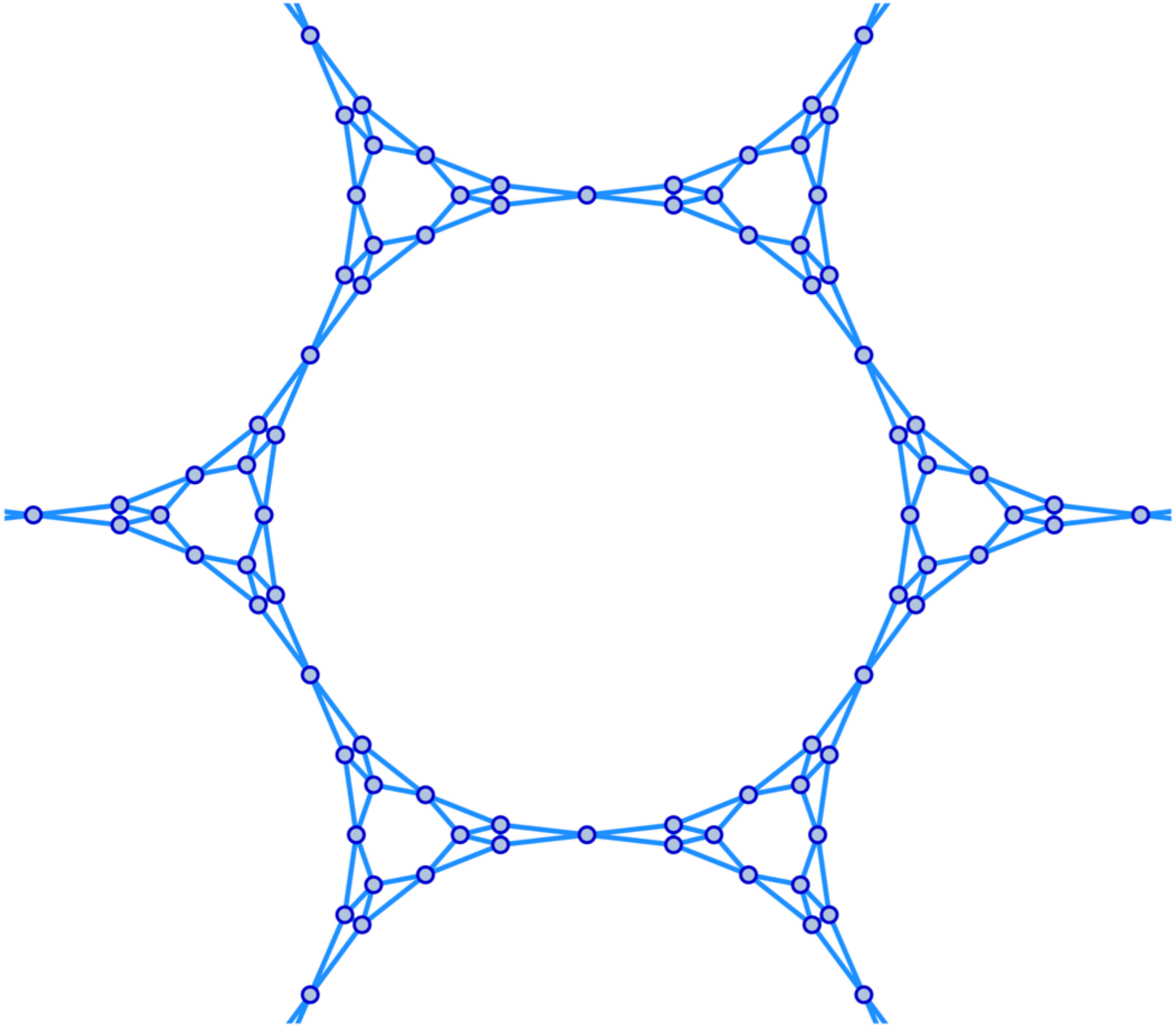
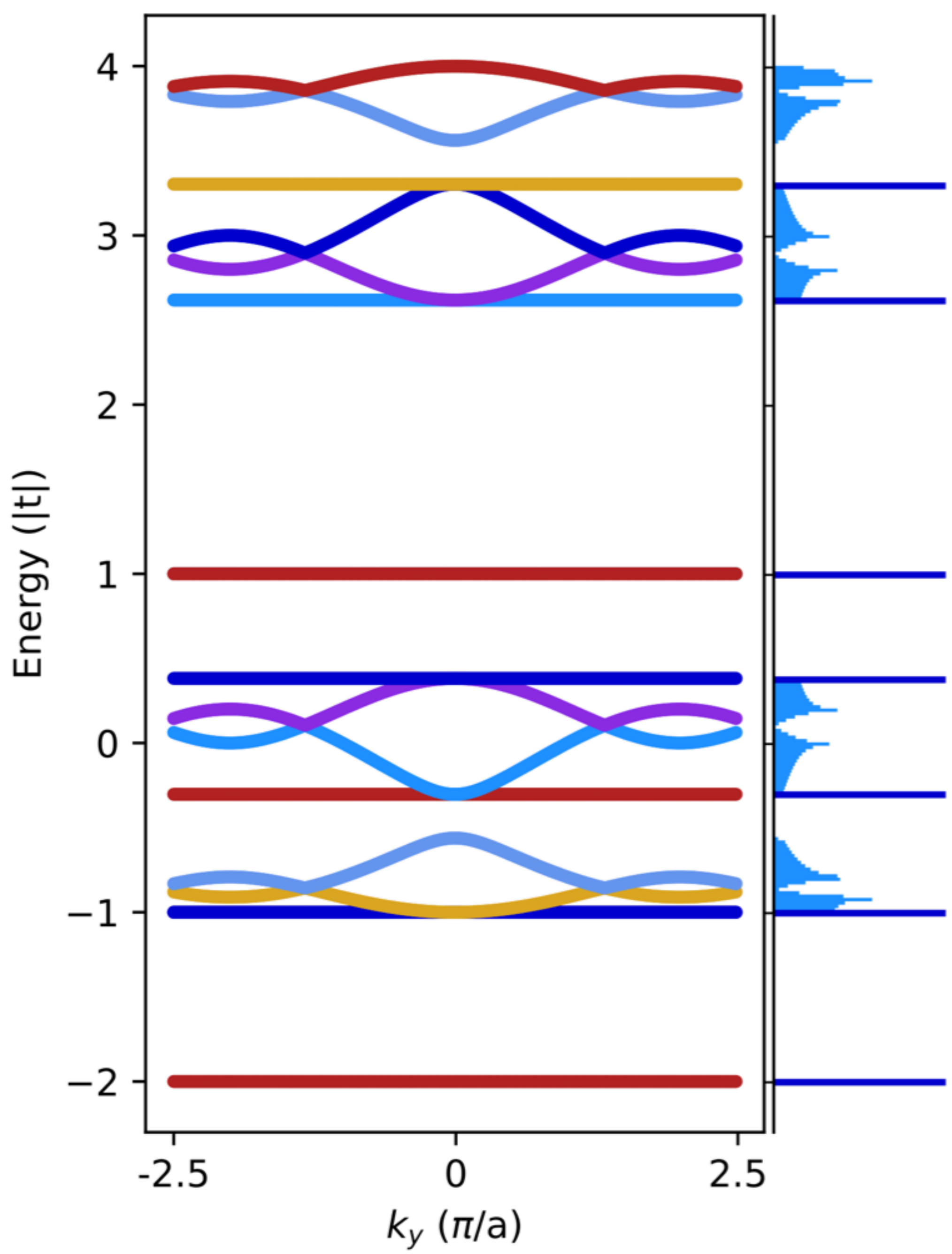
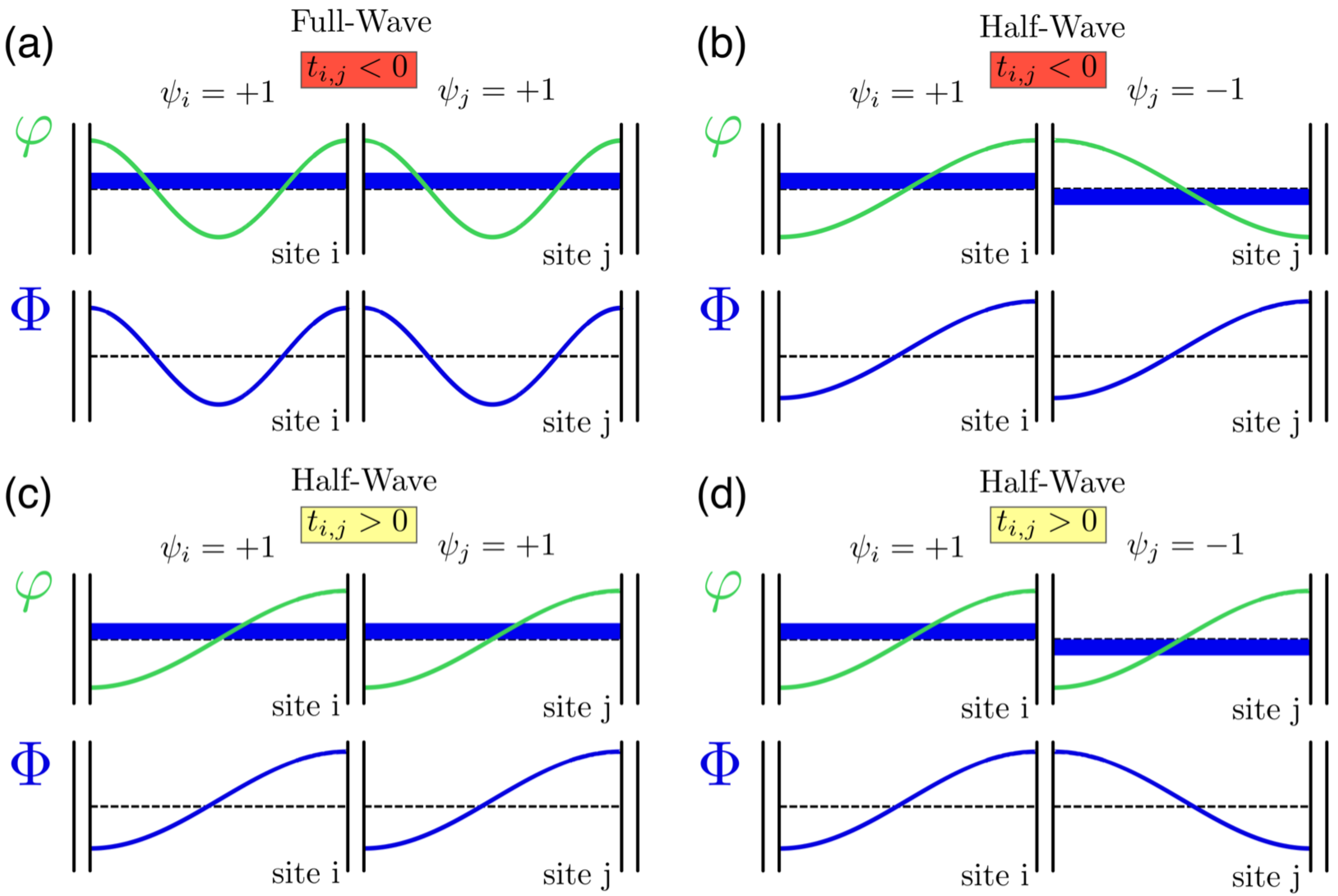




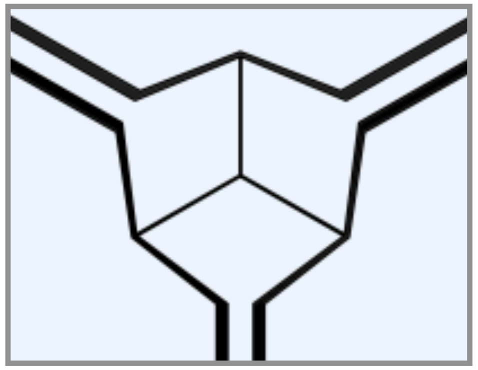
Kollár *et al*. arXiv:1902.02794 (2019)

Tight Binding



S-Wave and P-Wave On-Site Wave Functions

$$\mathcal{H} = \sum_{\substack{coupling \\ capacitors}} \omega C_c \Phi^+ \Phi^-$$



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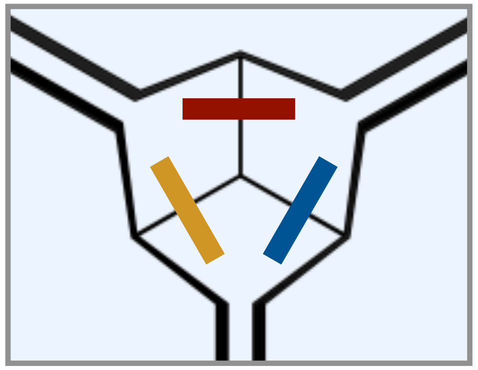

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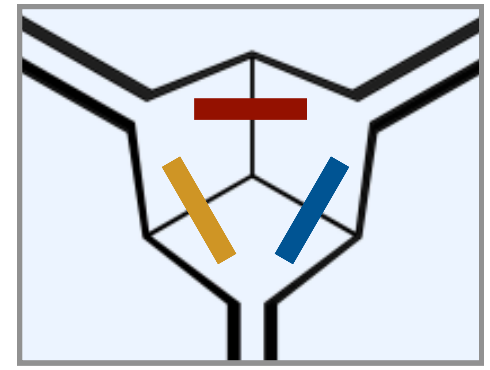
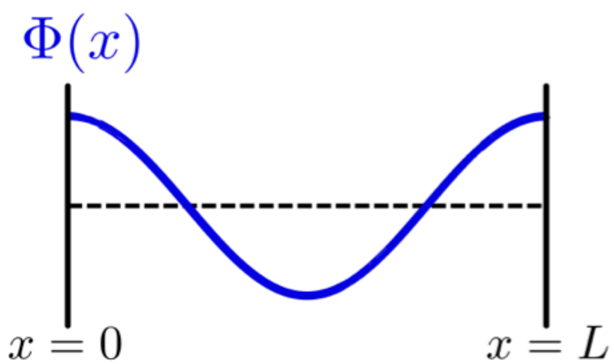
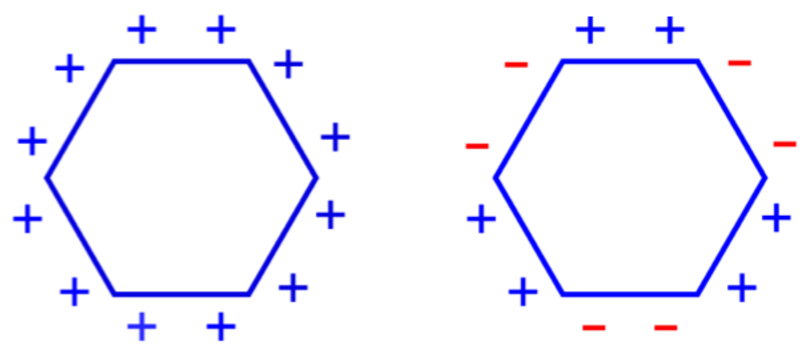


Full-wave

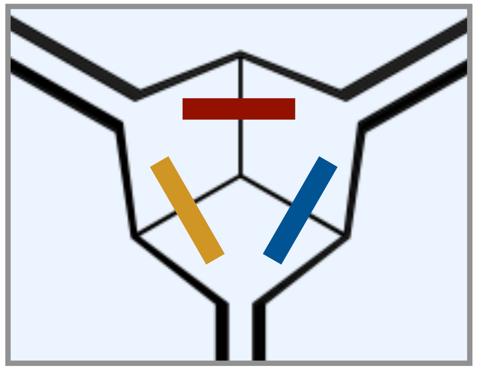




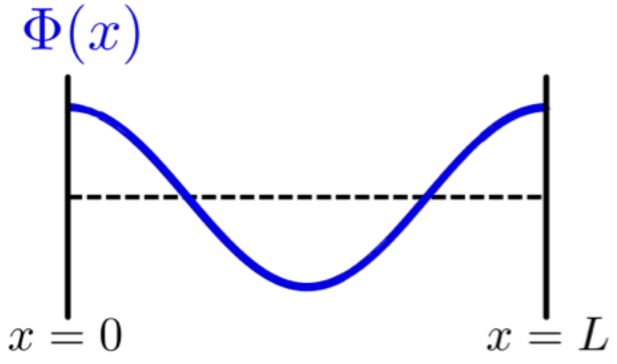
$E = +2$ $E = -2$

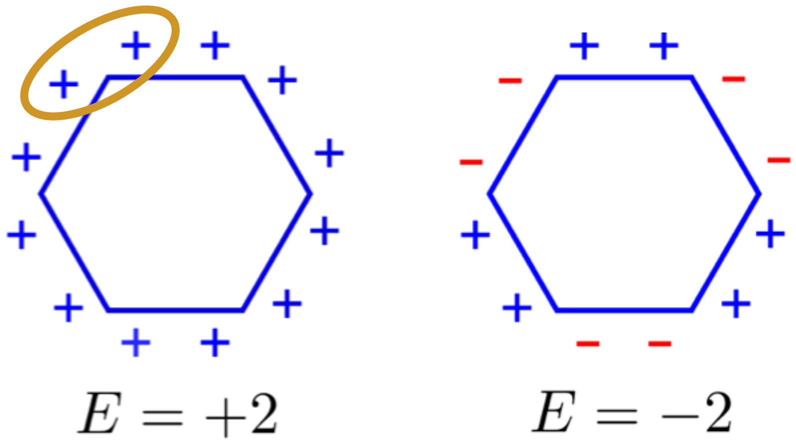
S-Wave and P-Wave On-Site Wave Functions

$$\mathcal{H} = \sum_{\substack{coupling \\ capacitors}} \omega C_c \Phi^+ \Phi^-$$



Full-wave





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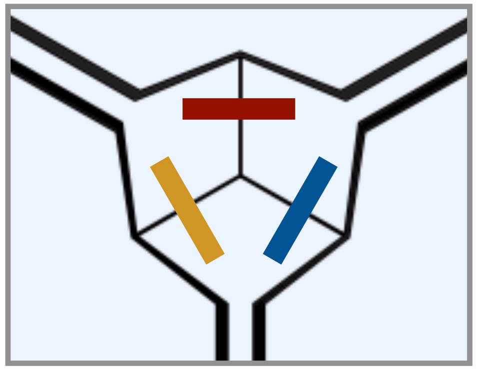

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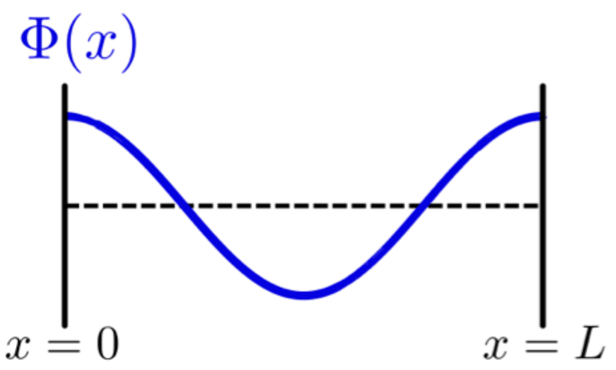

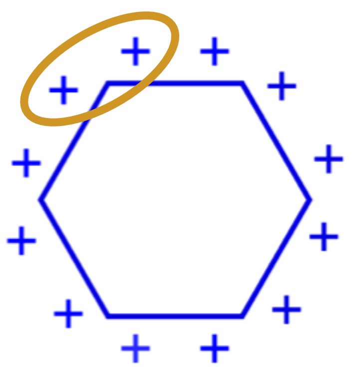

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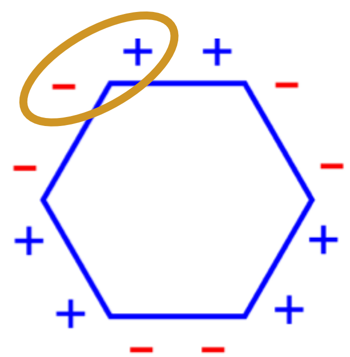

$E = -2$

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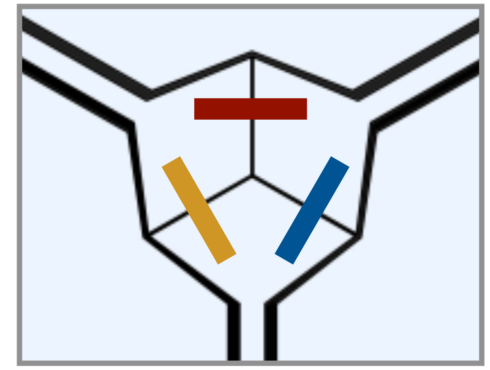
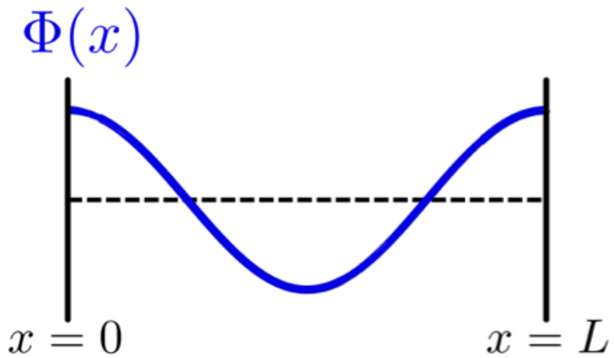
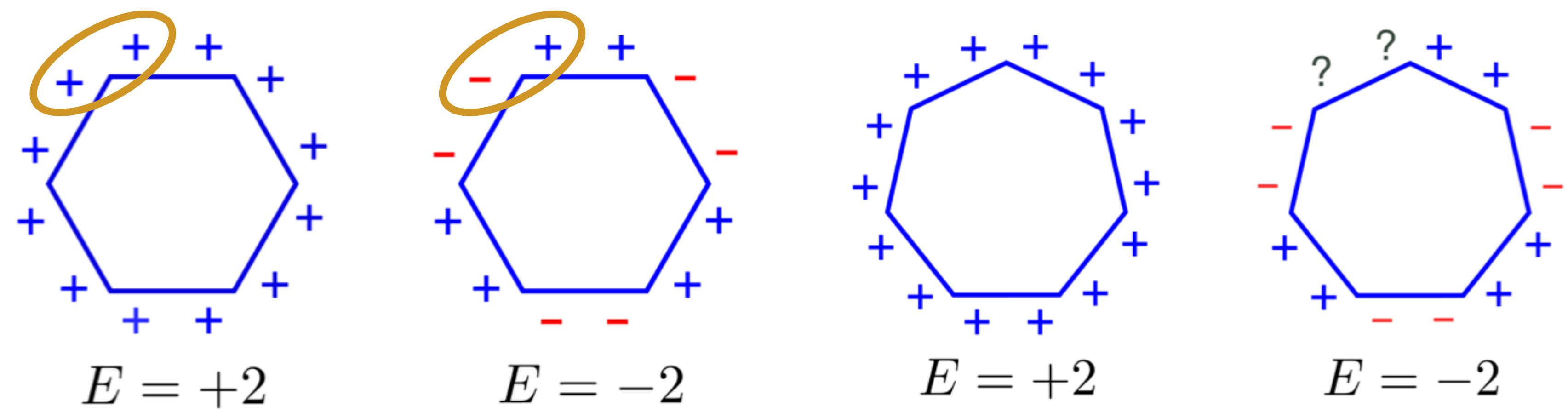


Full-wave





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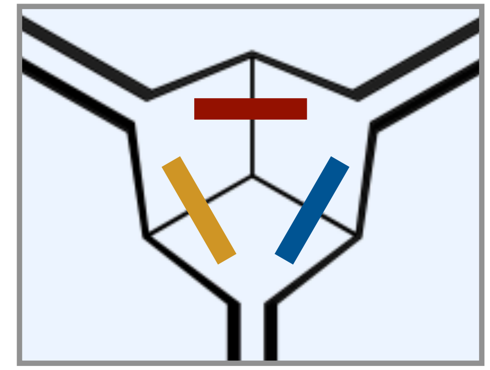
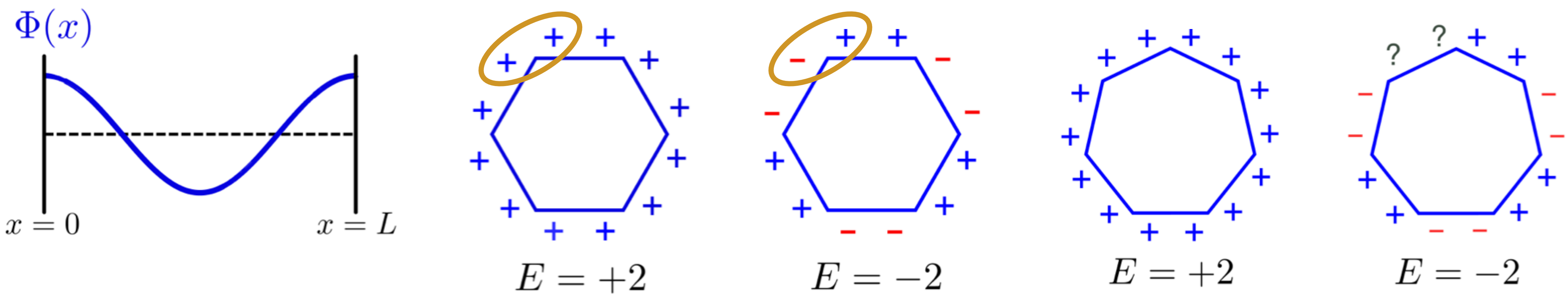
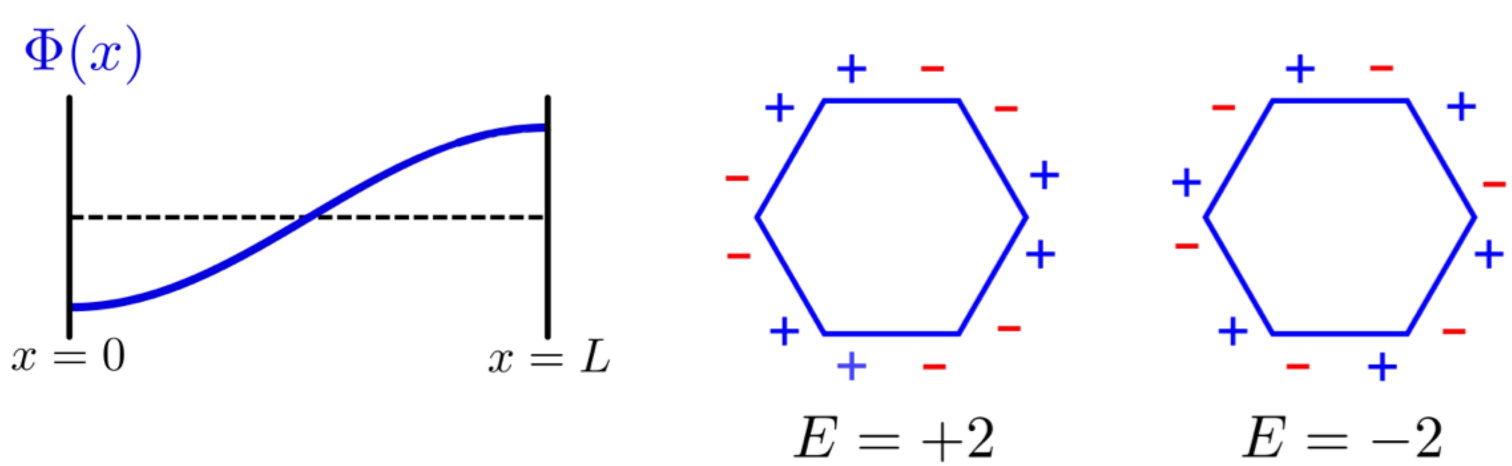


Full-wave



Half-wave



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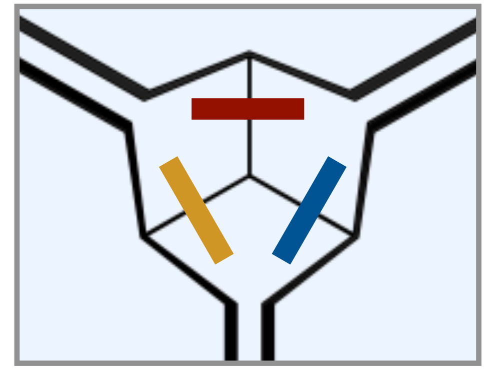

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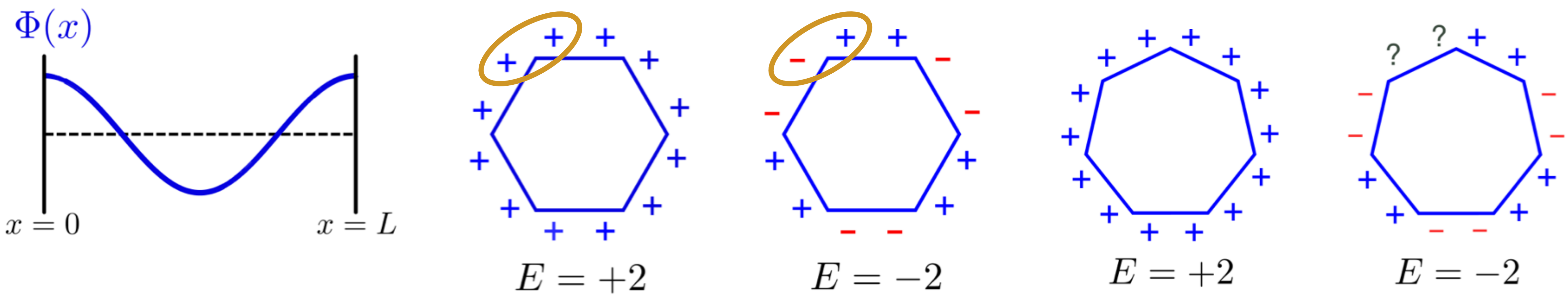
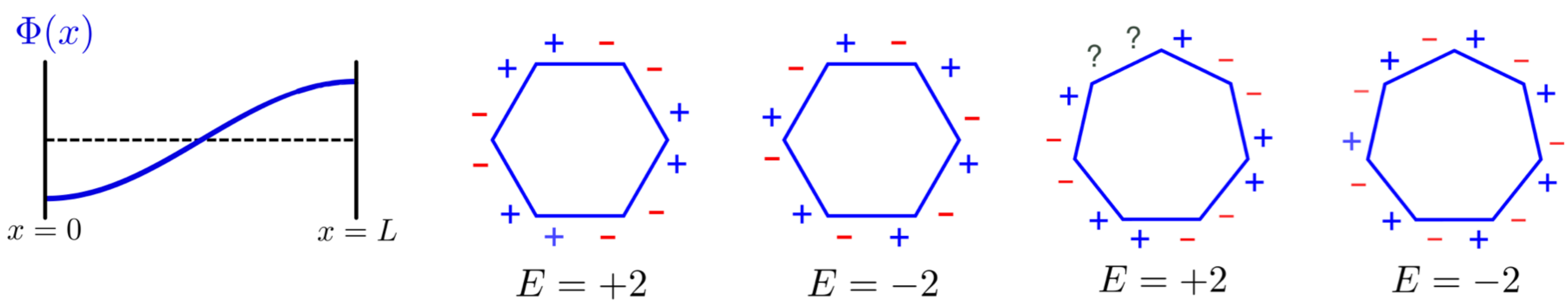


Half-wave



Half-Wave Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Kollár *et al*. in preparation

Half-Wave Band Structure Correspondence

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Effective Hamiltonian

- Bounded self-adjoint operator on L(X)
- Mixed positive and negative hopping

$$\bar{H}_a(X) \neq H_{L(X)}$$

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Half-Wave Band Structure Correspondence

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Incidence Operator

- From X to L(X)

$$N : \ell^2(X) \to \ell^2(L(X))$$

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Kollár *et al*. in preparation

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- Identical on bipartite graphs

Kollár *et al*. in preparation

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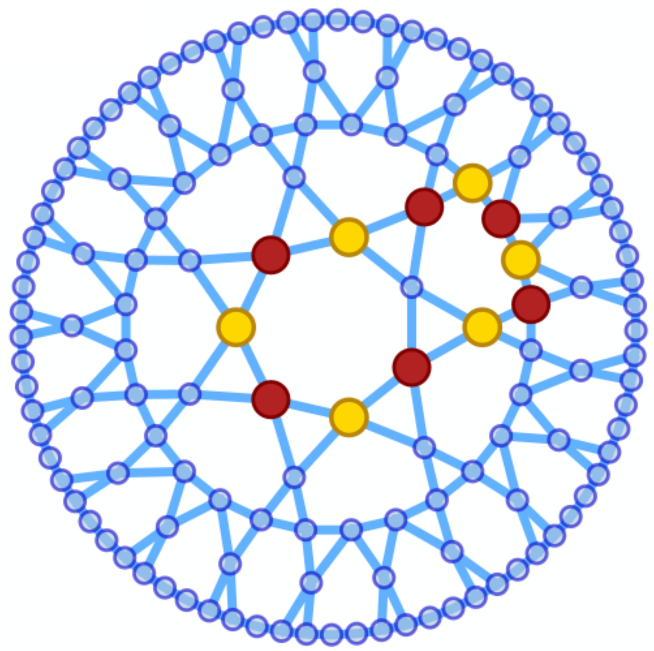
$$E_{\bar{H}_a} = \begin{cases} d - 2 - E_{H_X} \\ -2 \end{cases}$$

- Identical on bipartite graphs
- Inverted otherwise

Kollár *et al*. in preparation

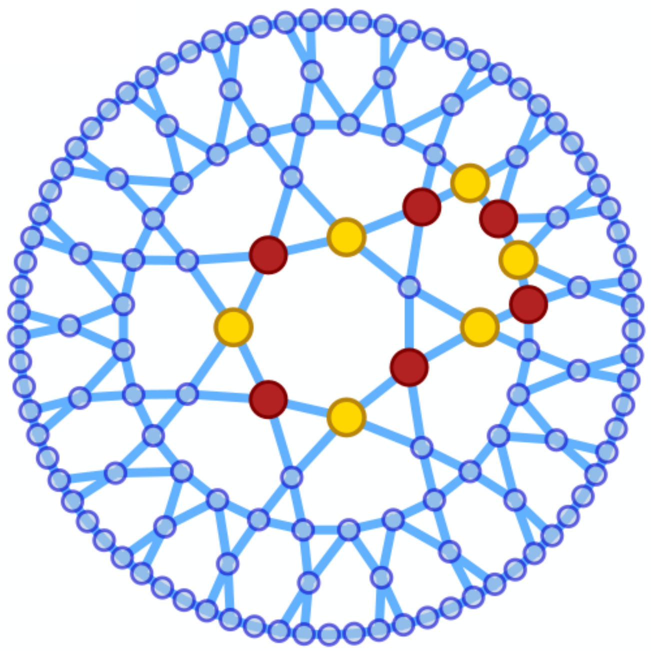
Full-Wave v Half-Wave Flat Band States

FW

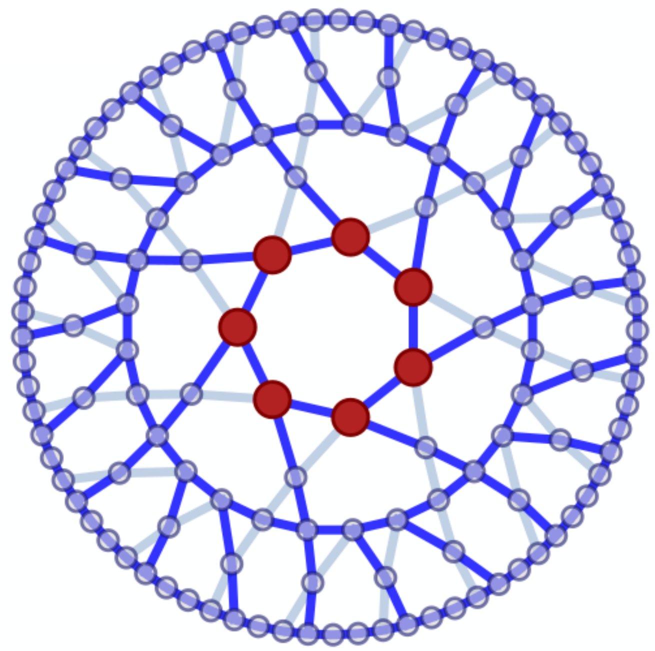


Full-Wave v Half-Wave Flat Band States

FW

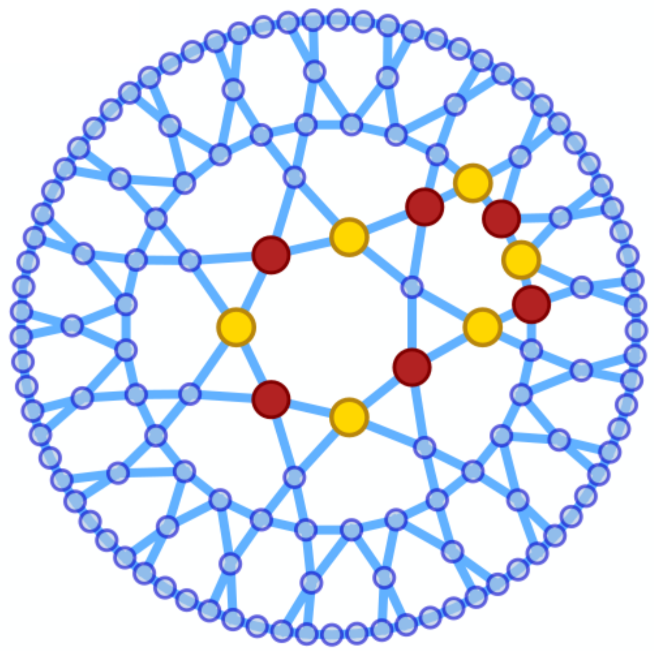


HW

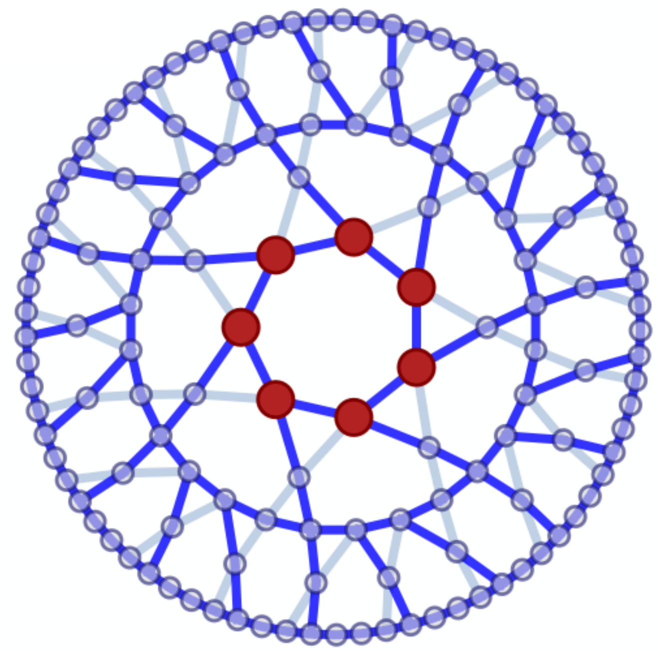


Full-Wave v Half-Wave Flat Band States

FW



HW



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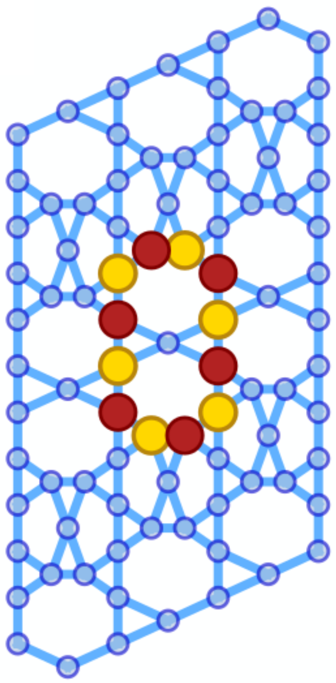

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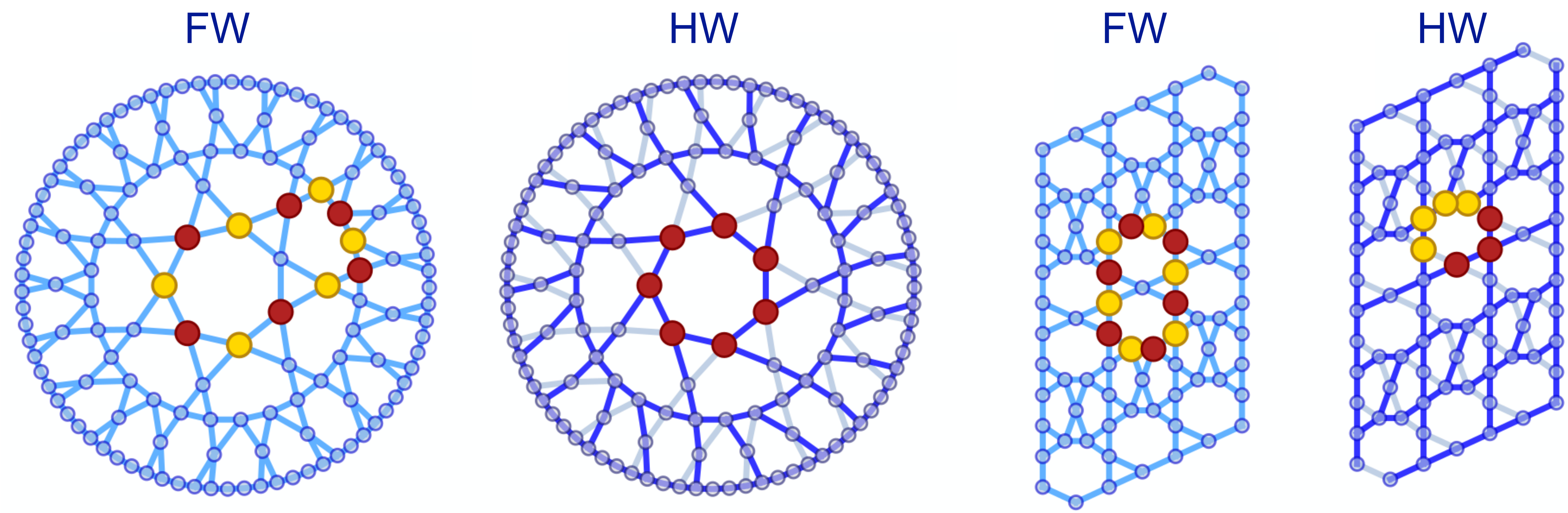

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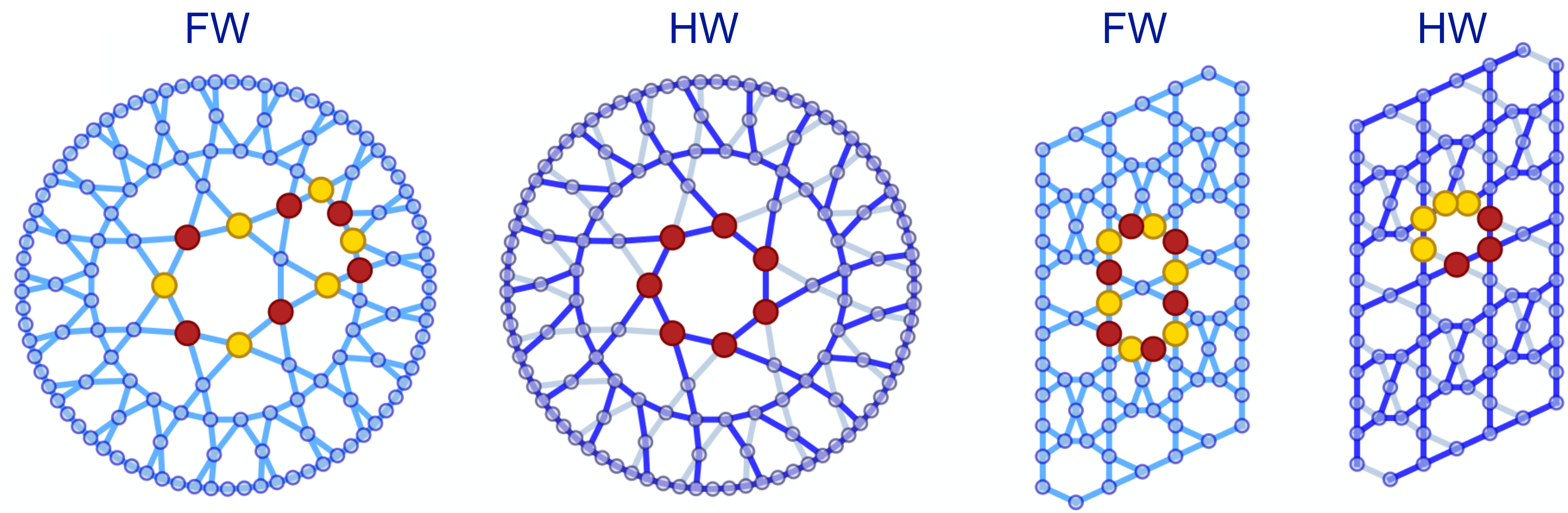
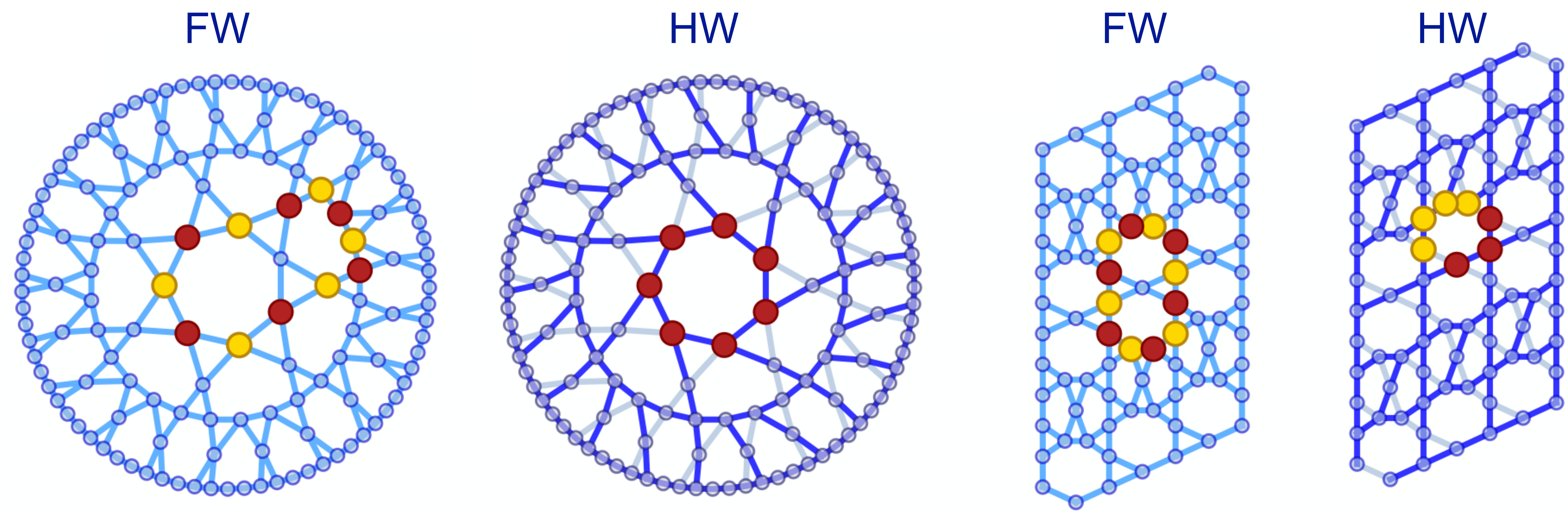


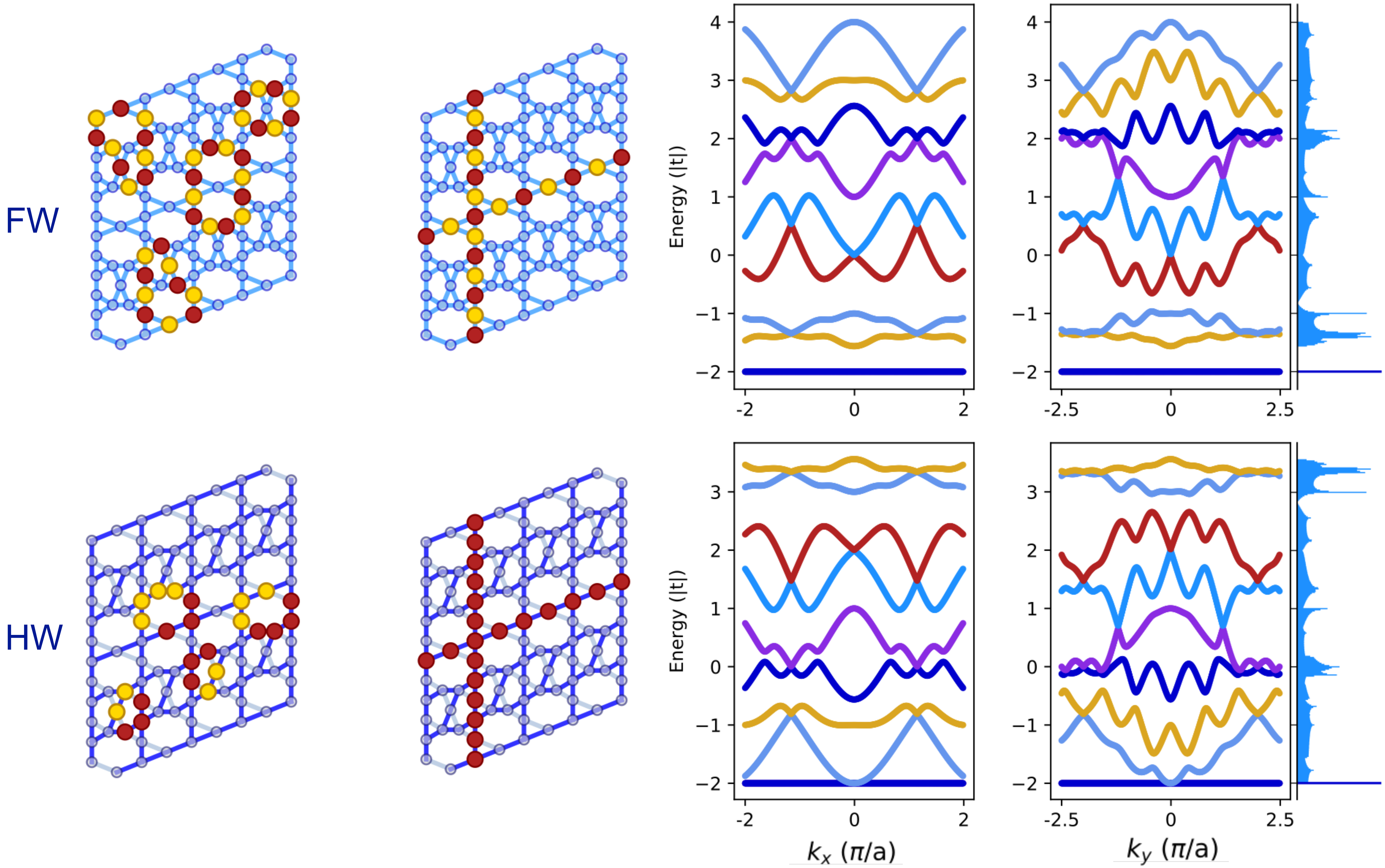
- Full-wave has localized states on only even cycles of the layout.

Full-Wave v Half-Wave Flat Band States



- Full-wave has localized states on only even cycles of the layout.
- Half-wave has localized states on any cycle of the layout.

Full-Wave Half-Wave Correspondence



Real-Space Topology