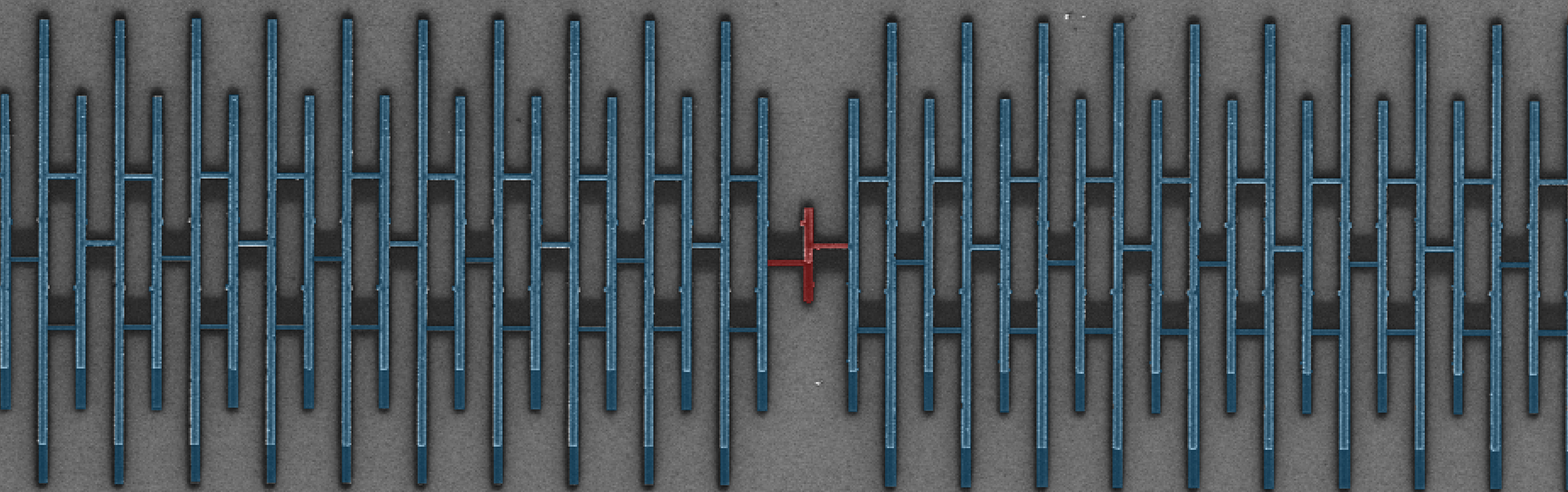
Circuit QED and analogue simulation of quantum impurities

Nicolas Roch,Neel Institute, Grenoble, France



cQED@Tn

3 October 2022

cnrs advancing the frontiers

AGENCE NATIONALE DE LA RECHERCHE ANR

Quantum Engineering
Univ. Grenoble Alpes

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European Research Council
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MARIE CURIE ACTIONS

lanef Laboratoire d'Alliances Nanosciences-Energies du Futur

Acknowledgments

Grenoble

Serge Florens

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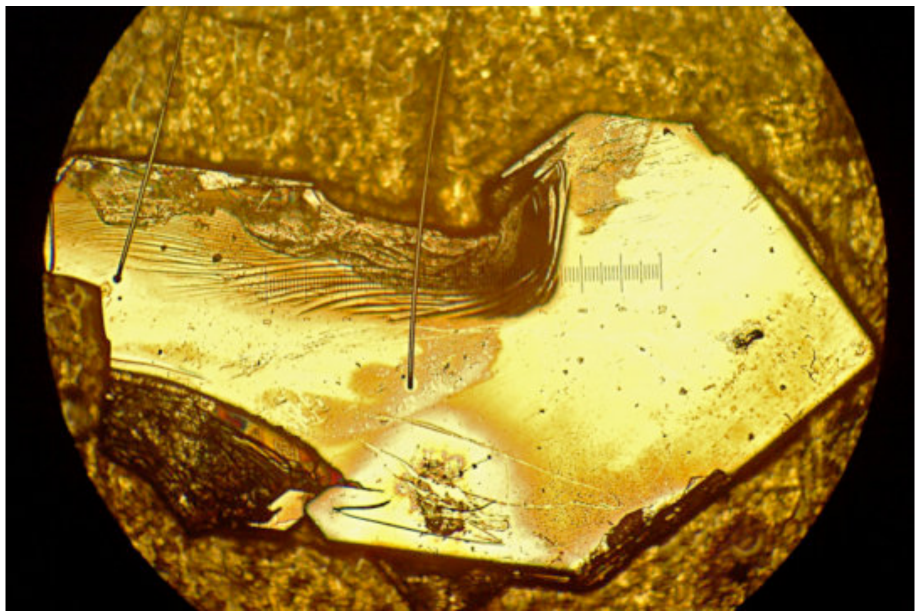
Denis Basko

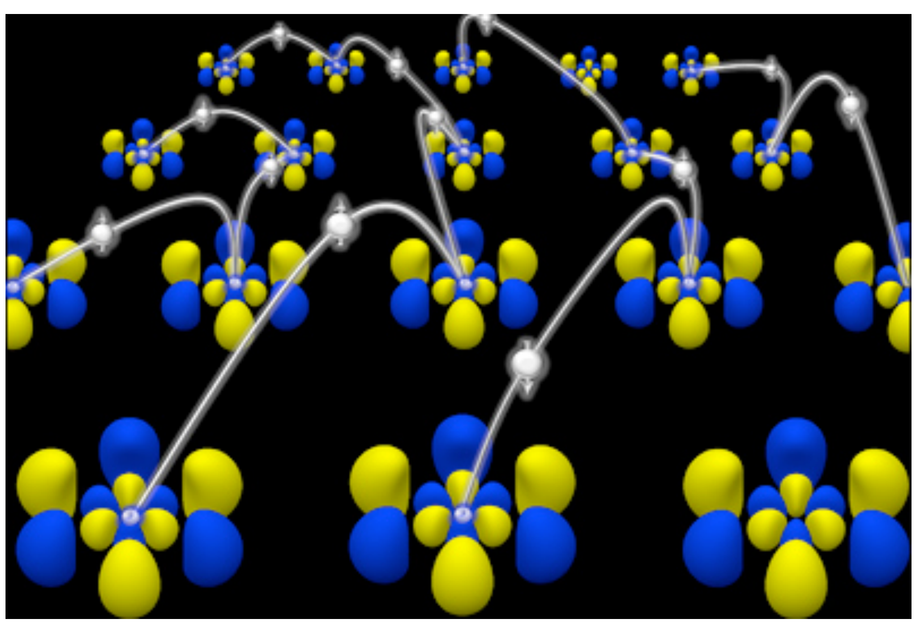
Grenoble

Motivation for designed superconducting circuits

Quantum matter



Credit: Marc Tippmann Munich

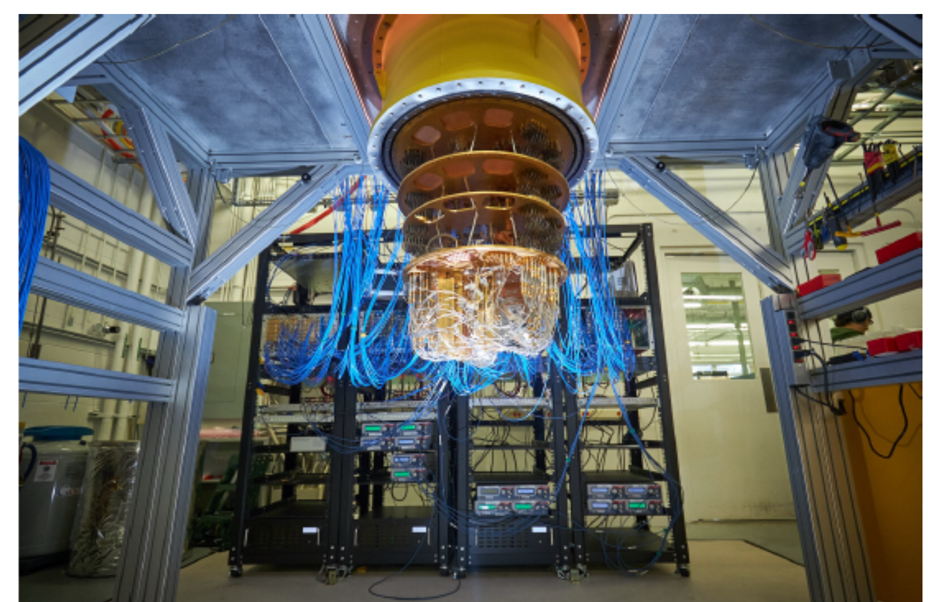


Credit: Mohammad Hamidian - Davis Lab

This research line

Quantum circuits

Credit: V. Milchakov/R. Dassonneville Grenoble

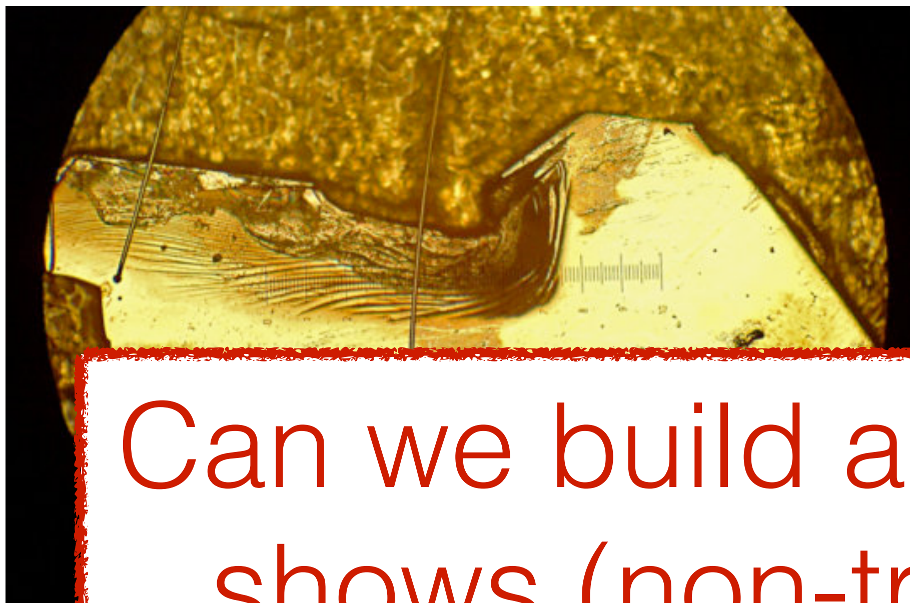


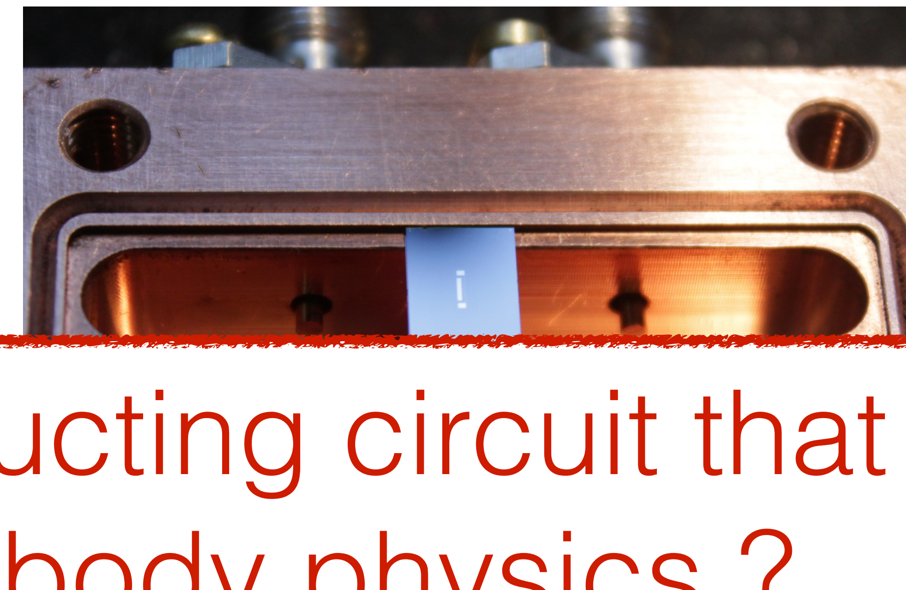
Credit: Google Quantum

Motivation for designed superconducting circuits

Quantum matter

Quantum circuits





Can we build a superconducting circuit that shows (non-trivial) many-body physics ?

Credit: Marc Tippmann Munich

Credit: V. Milchakov/R. Dassonneville Grenoble

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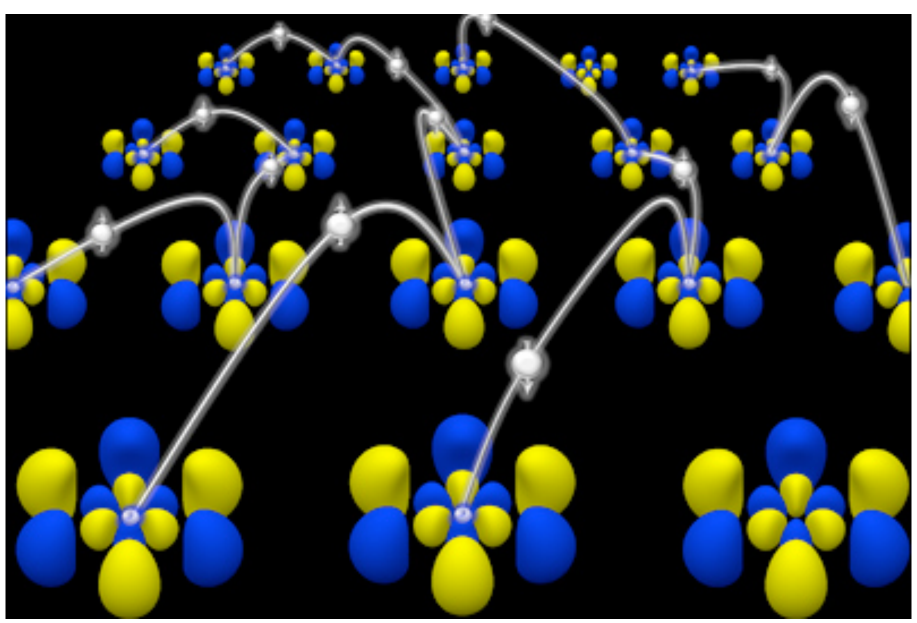

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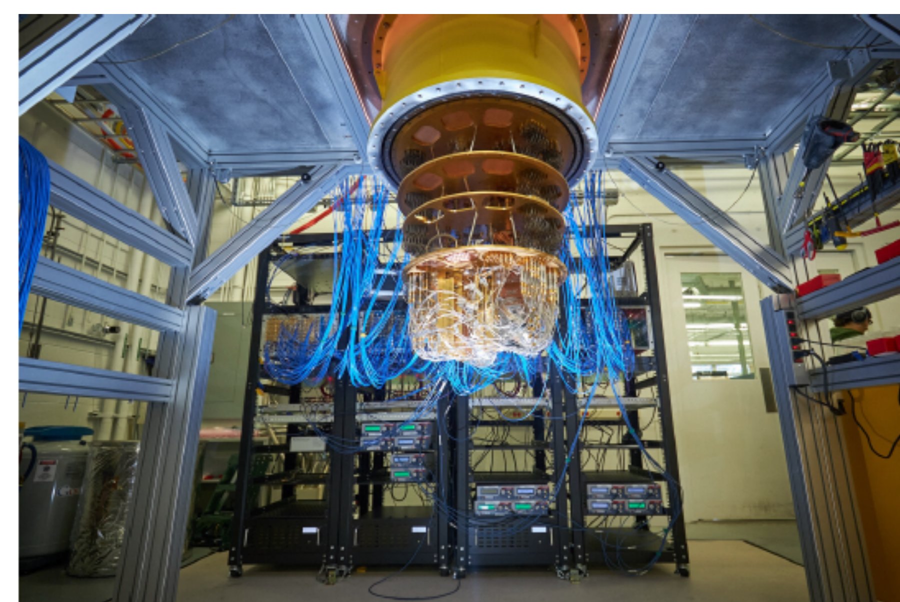
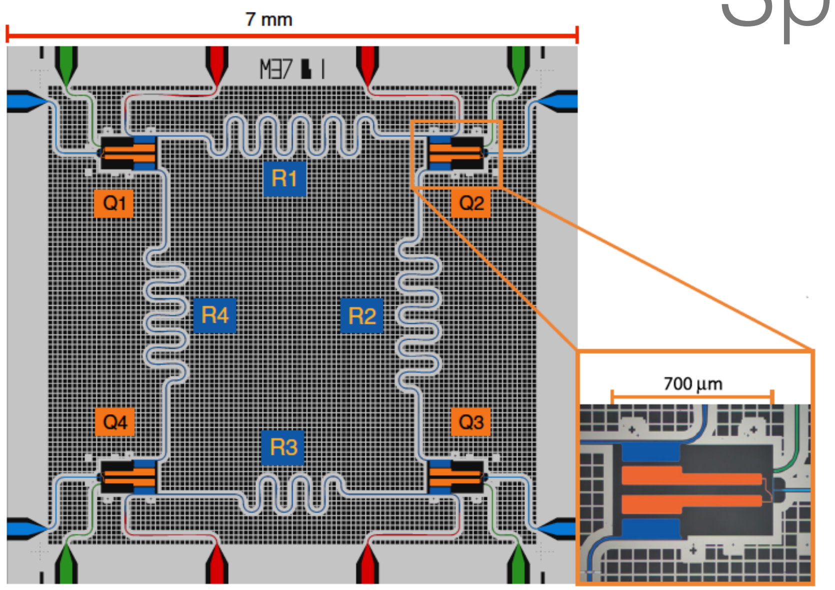


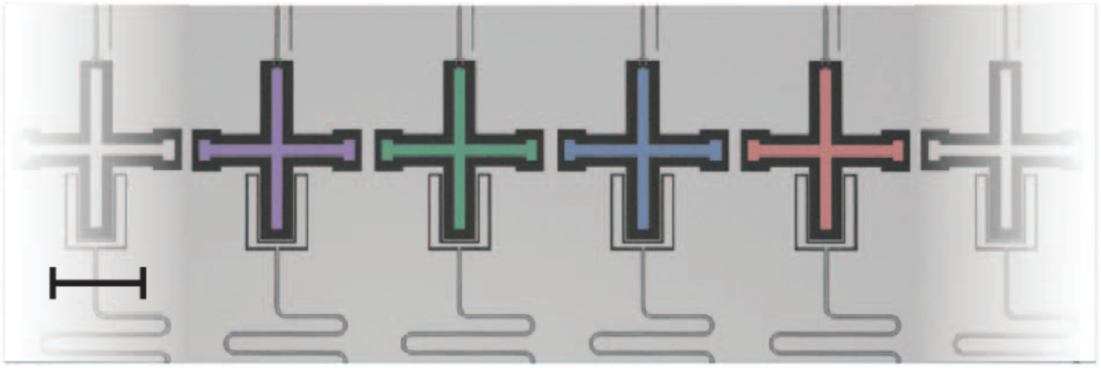
Credit: Google Quantum

What kind of many-body system?

Spin chains



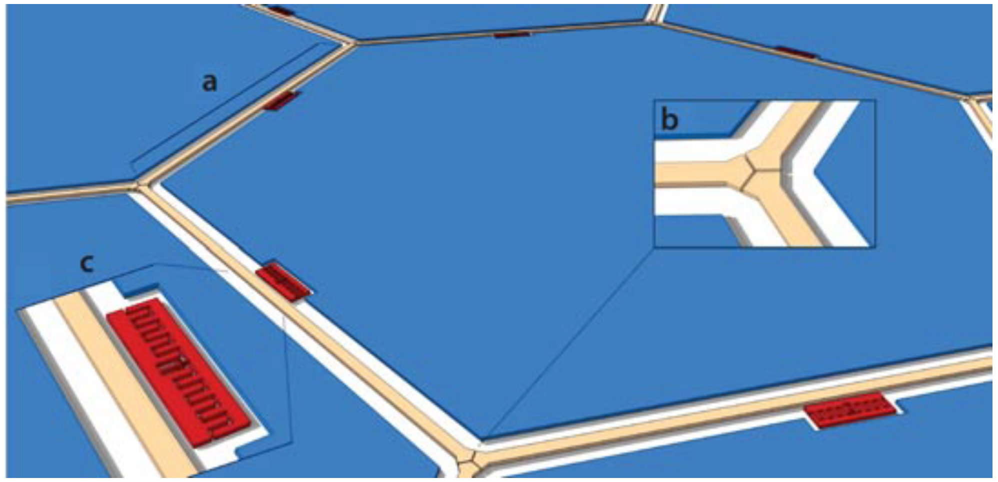
Salathé et al. PRX (2015)



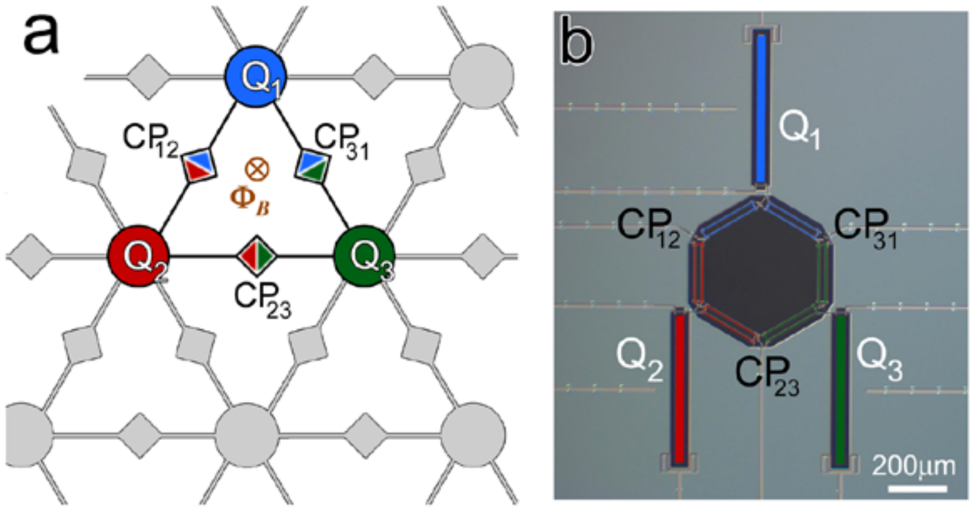
Barends et al. Nat. Com. (2015)

Quantum fluids of light

Review: I. Carusotto & C. Ciuti, *Rev. Mod. Phys. (2013)*



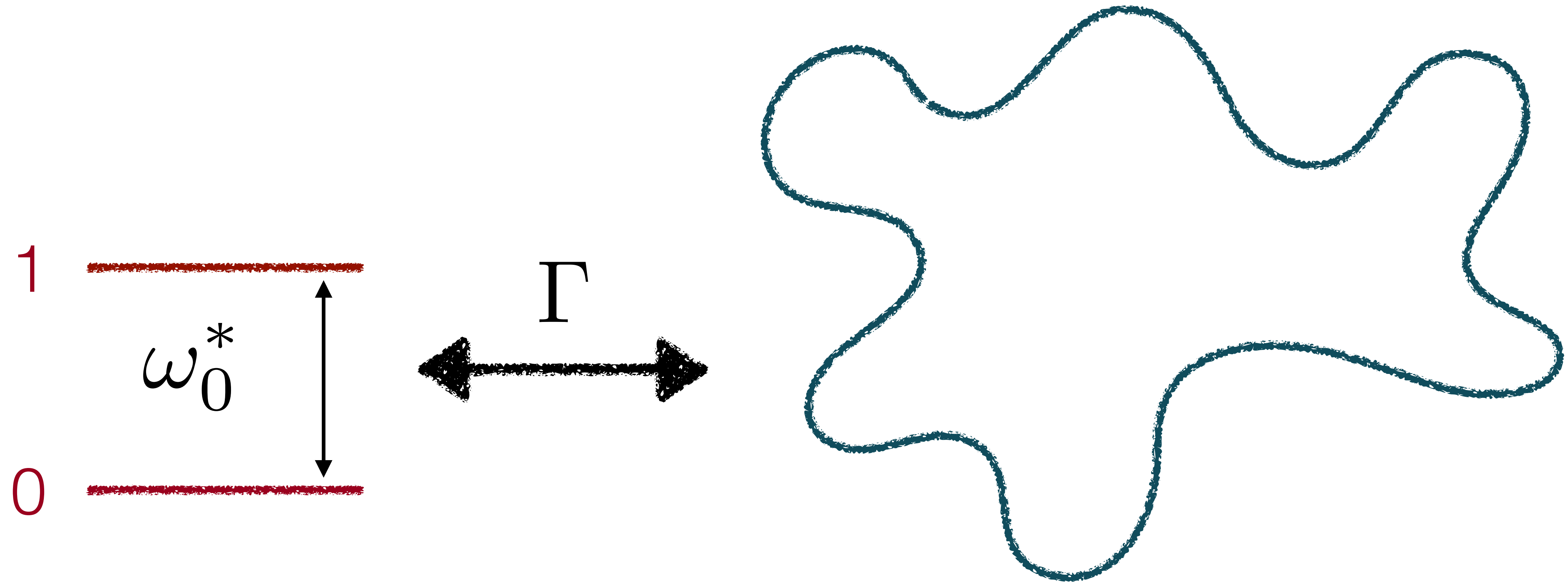
Houck et al. Nat. Phys. (2012)



Roushan et al. Nat. Phys. (2016)

What kind of many-body system?

Our choice: quantum impurities



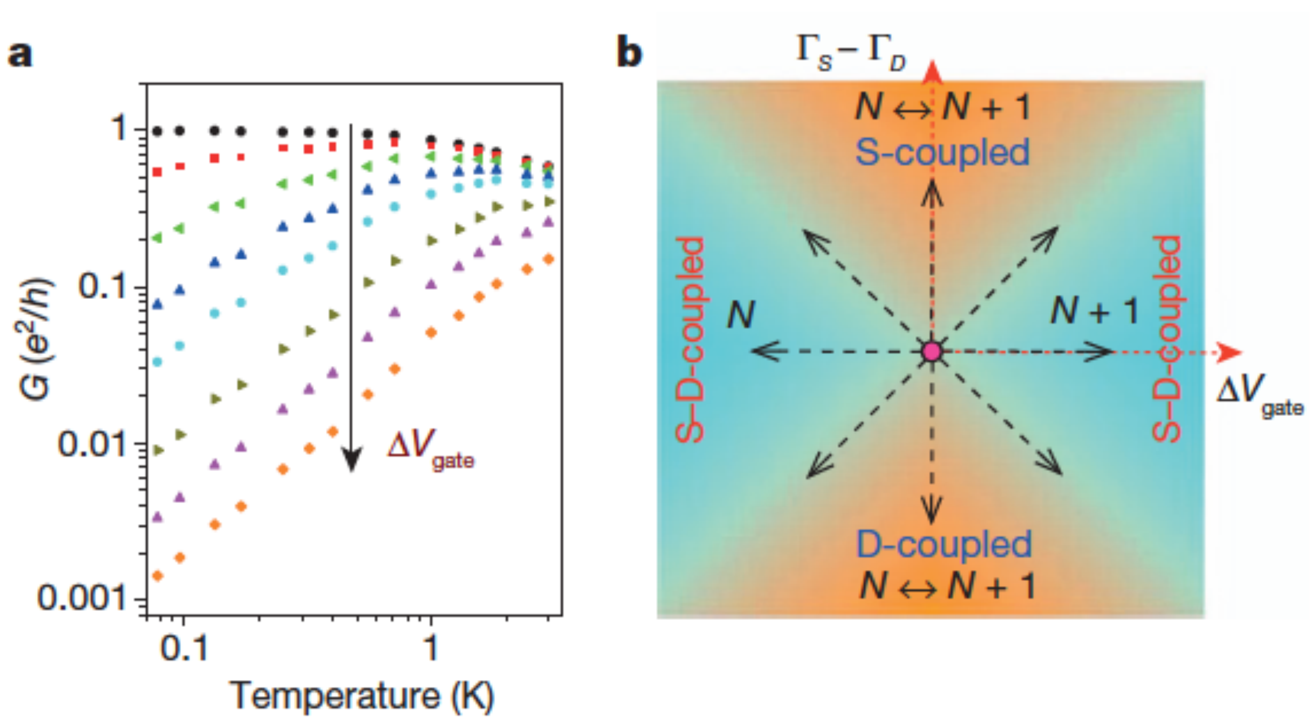
One quantum system coupled to a large bath:

The “hydrogen atom” of many-body physics

What kind of many-body system?

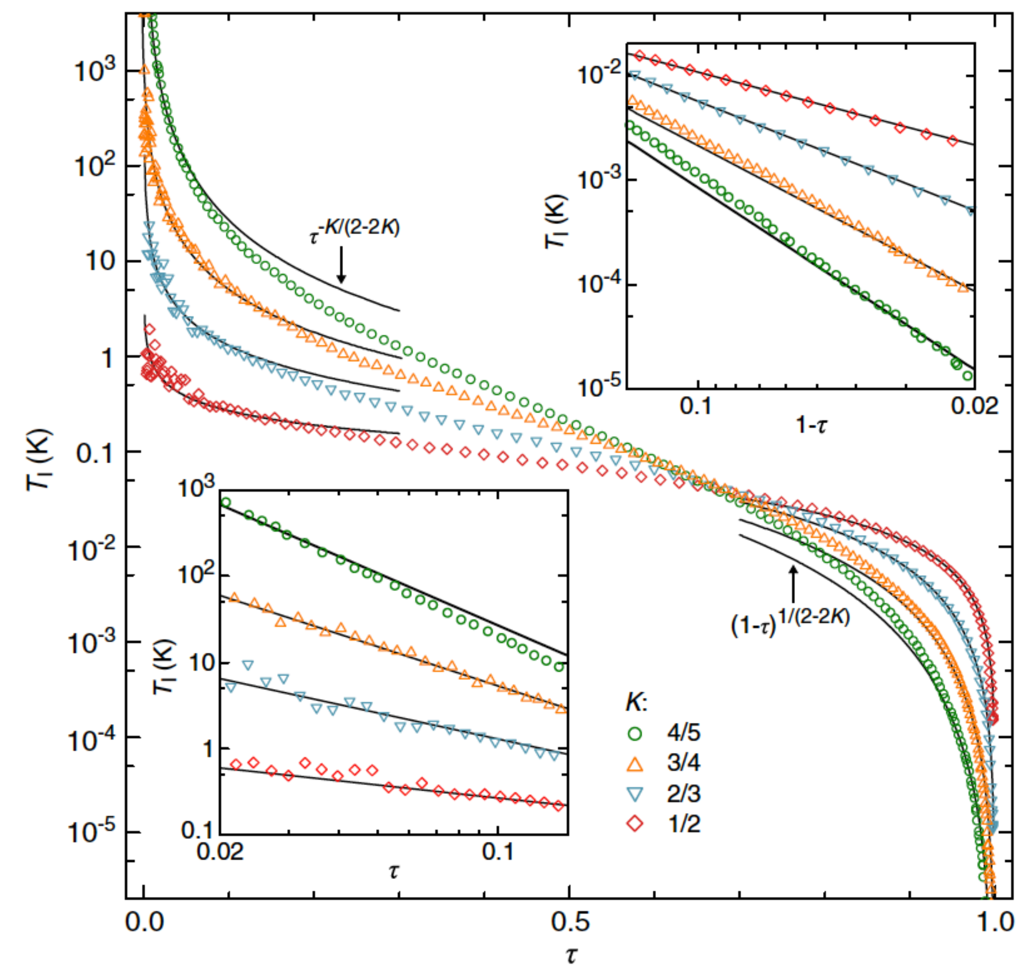
Our choice: quantum impurities

Quantum phase transition



H. T. Mebrahtu et al 12'

Scaling behaviour



A. Anthore et al 18'

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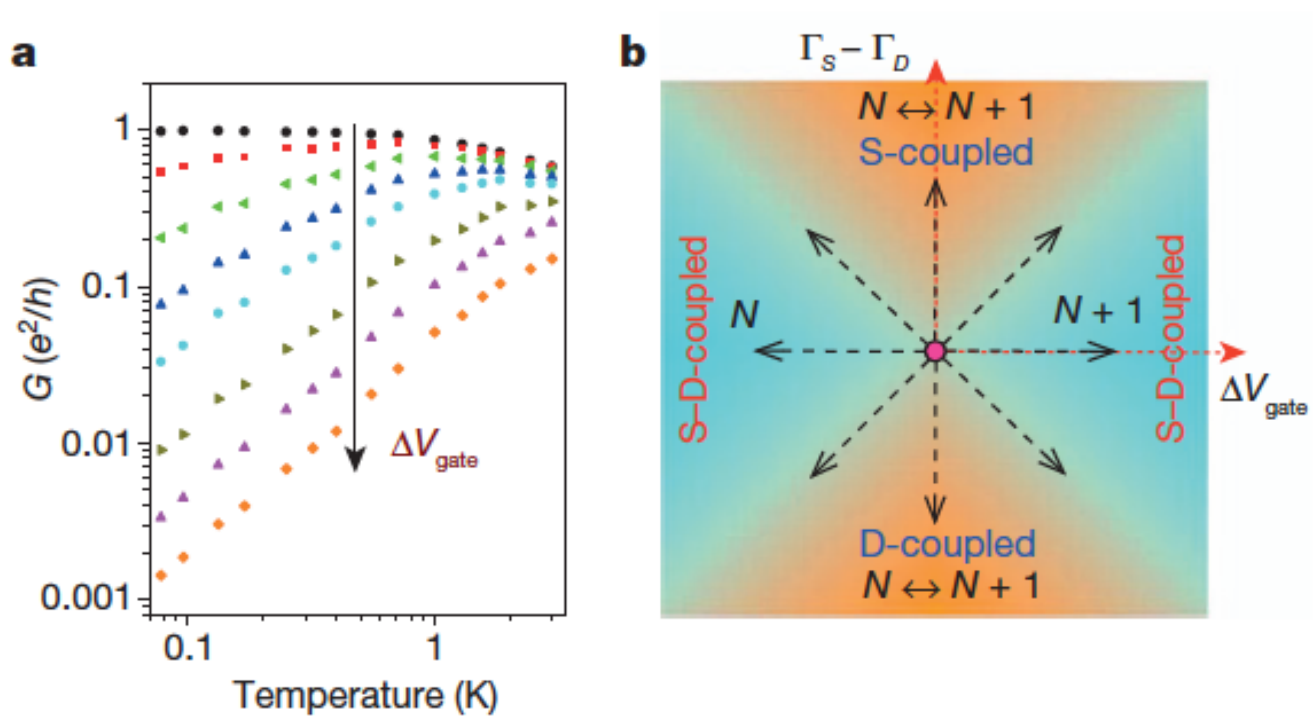

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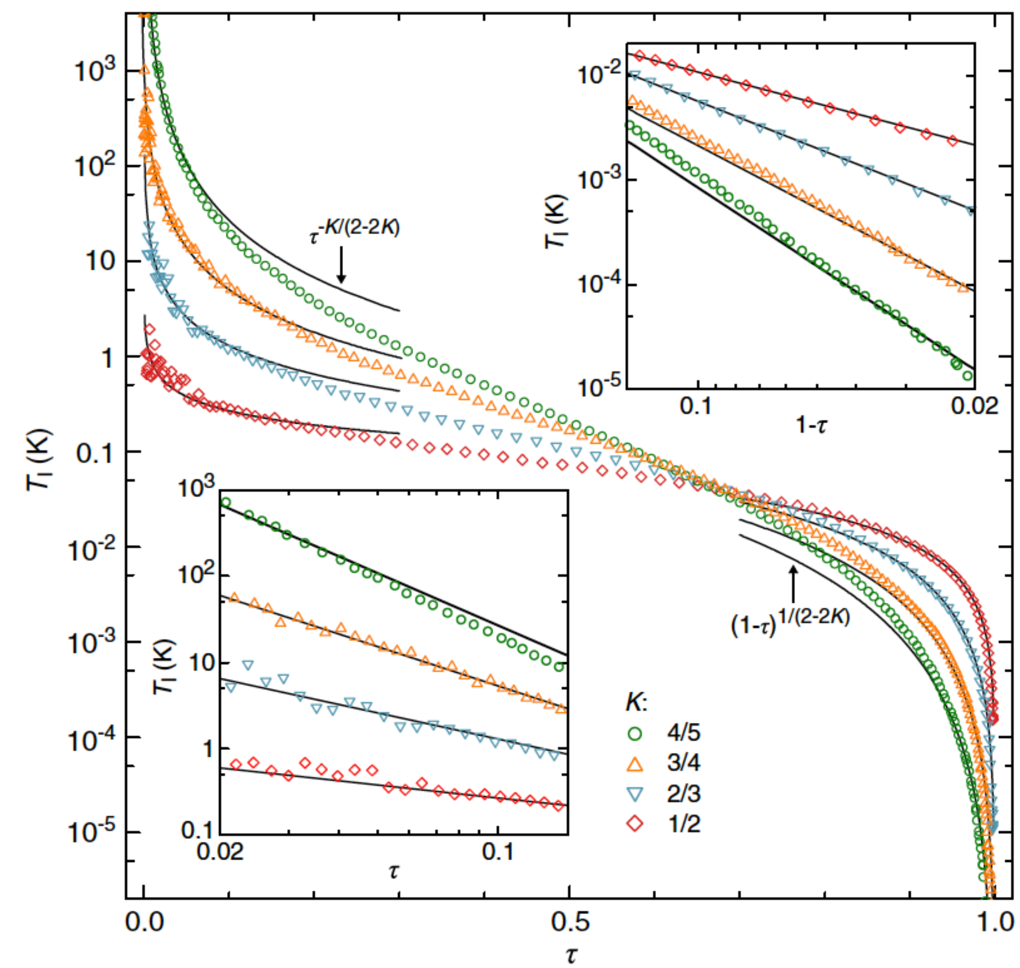

A. Anthore et al 18'

Mainly ground state properties

Can we probe the finite frequency response and the dynamics of quantum impurities ?

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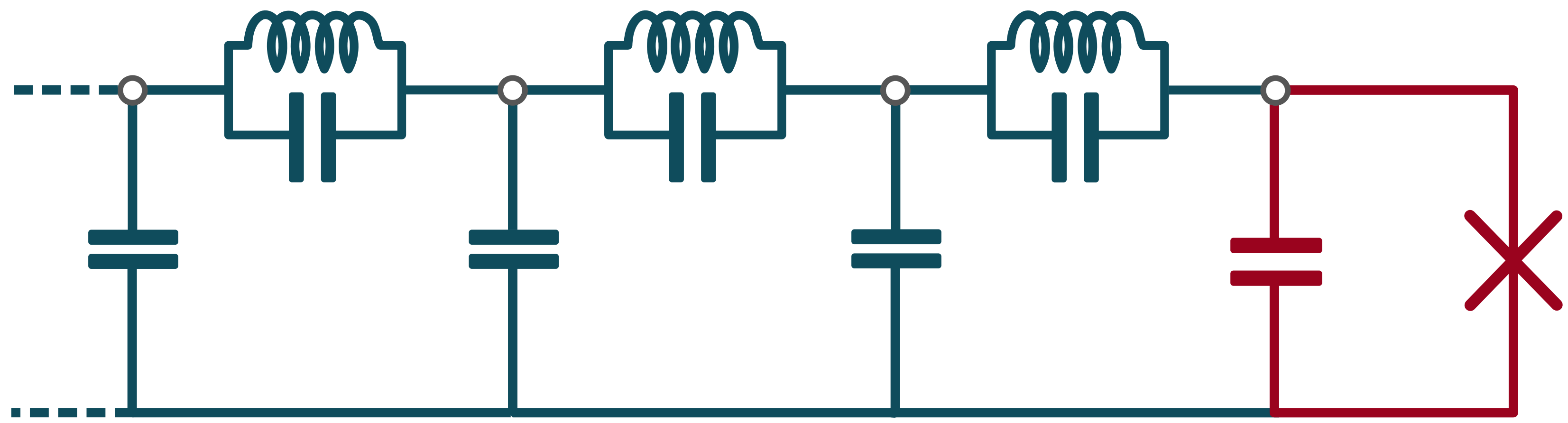
Bosonic quantum impurities

A Josephson junction coupled to many harmonic oscillators:



$$L = L_{\mathrm{env}} + \frac{\hbar^2}{4e^2}\frac{C_{\mathrm{J}}}{2}\left(\partial_t\varphi_0\right)^2 + E_{\mathrm{J}}\cos\varphi_0$$

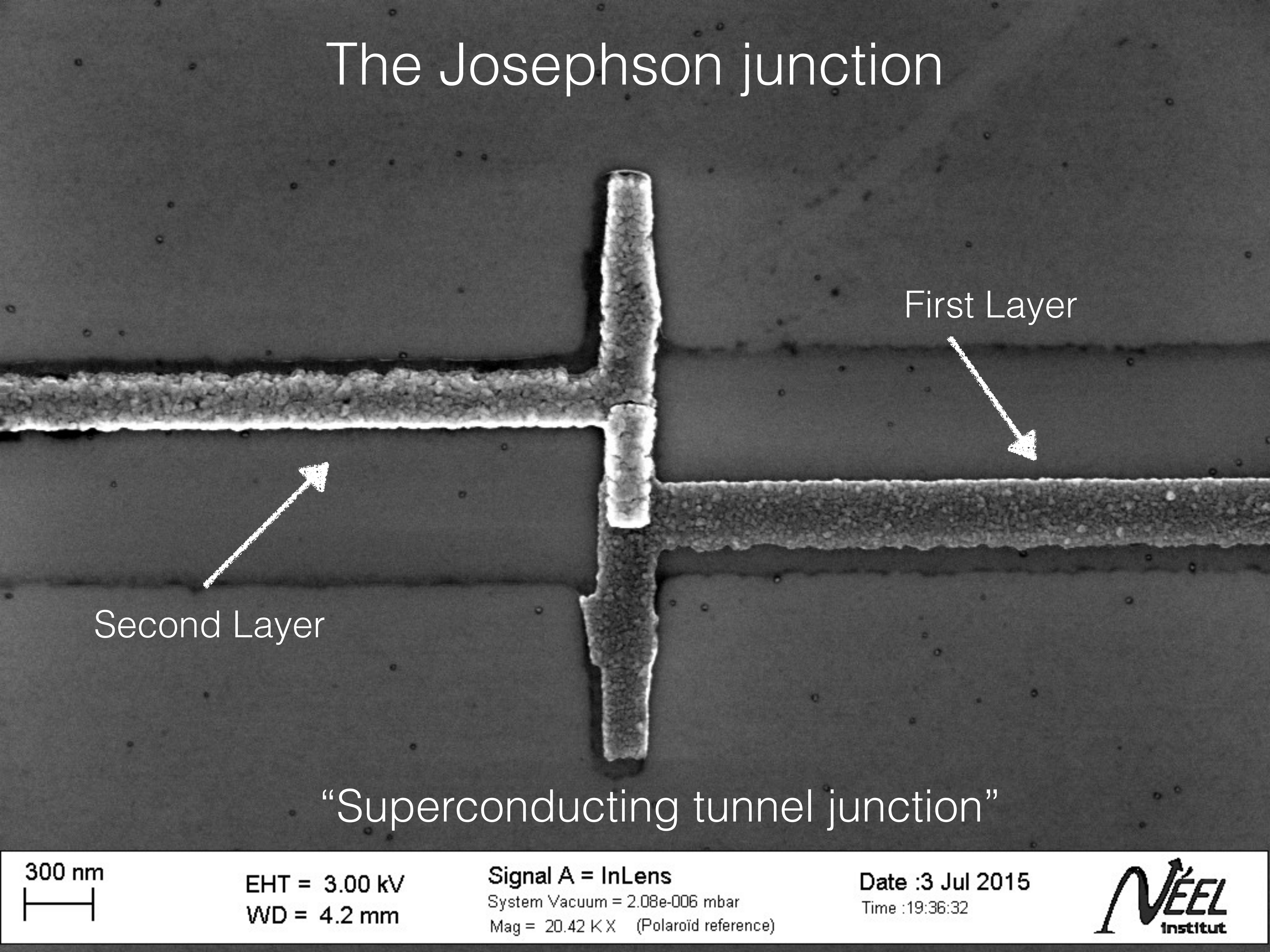
$$L_{\mathrm{env}} = \frac{\hbar Z_{\mathrm{q}}}{4\pi Z_{\mathrm{c}}}\int_0^\infty dx\left[1/c(\partial_t\varphi_x)^2 - c(\partial_x\varphi_x)^2\right]$$

the “Boundary Sine-Gordon” model

φ_0 boundary value of a continuous field φ_x

“localized/delocalized” quantum phase transition at $Z_{\mathrm{c}} = Z_{\mathrm{q}}$

The Josephson junction



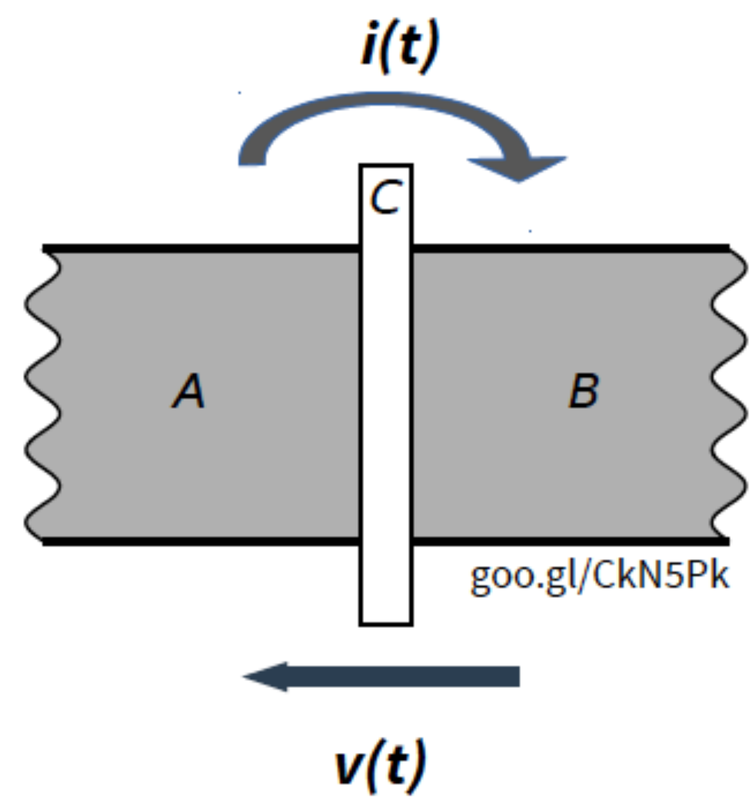
“Superconducting tunnel junction”

The Josephson junction

Josephson relations

$$i(t) = I_c \sin\varphi(t) \quad \text{with} \quad \varphi(t) = \frac{2e}{\hbar}\phi(t)$$

$$\phi(t) = \int_{-\infty}^{t} v(t')dt'$$



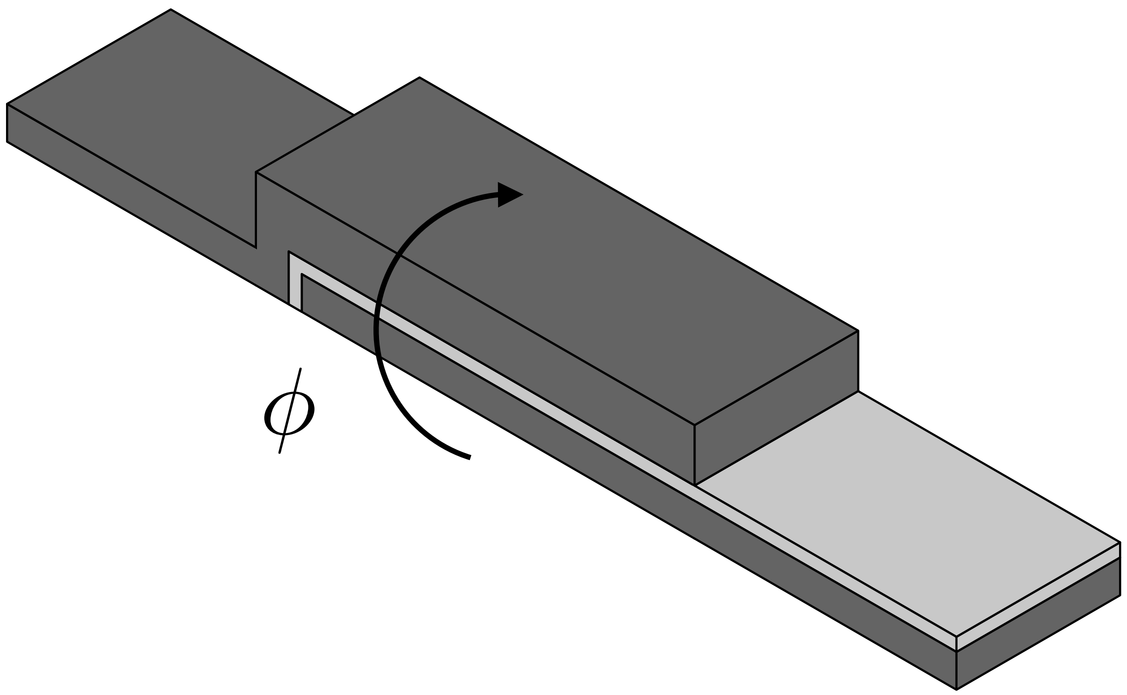
Josephson Hamiltonian

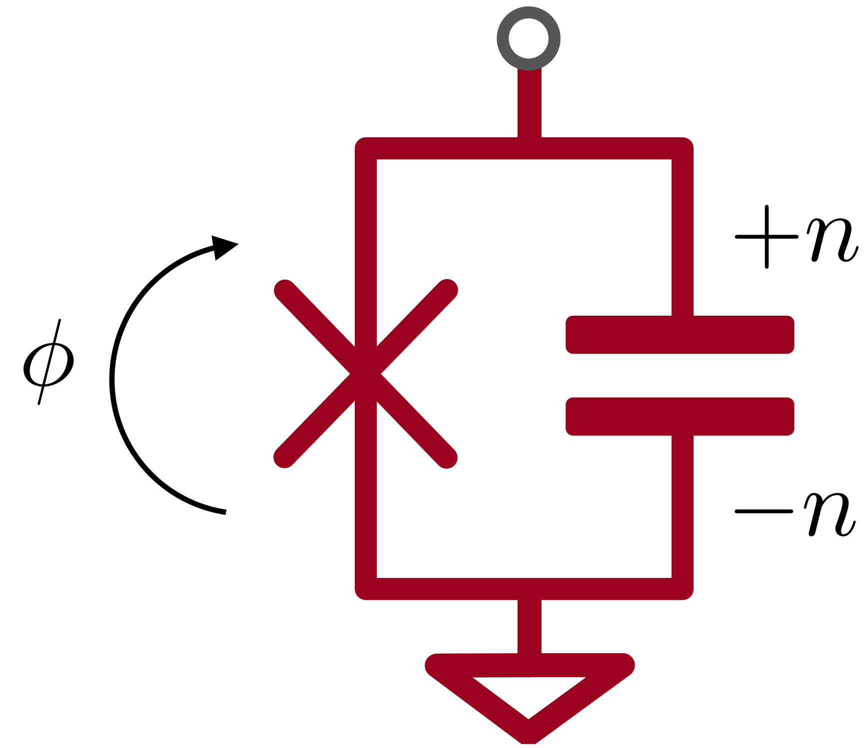
$$E_{\mathrm{J}}(\varphi) = \int i(t)v(t)dt = E_{\mathrm{J}}(1 - \cos\varphi)$$

Josephson inductance

$$L_{\mathrm{J}} = \left(\frac{\partial i}{\partial \phi}\right)^{-1} = \frac{L_{\mathrm{J},0}}{\cos\varphi}$$

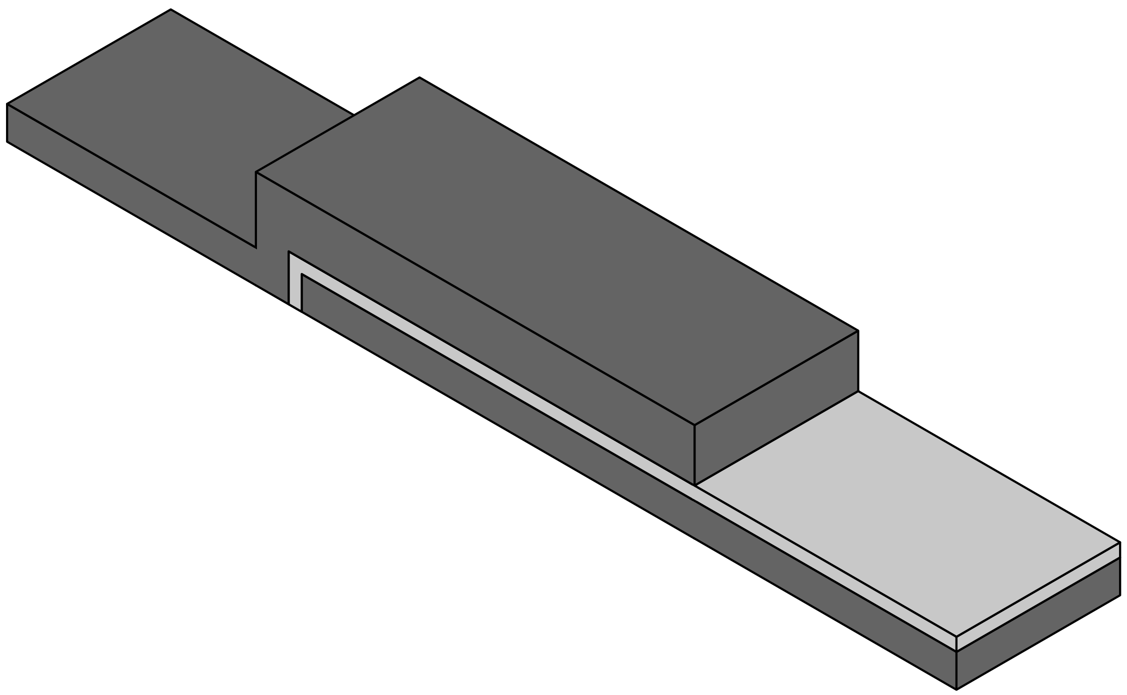
Josephson junction: basics

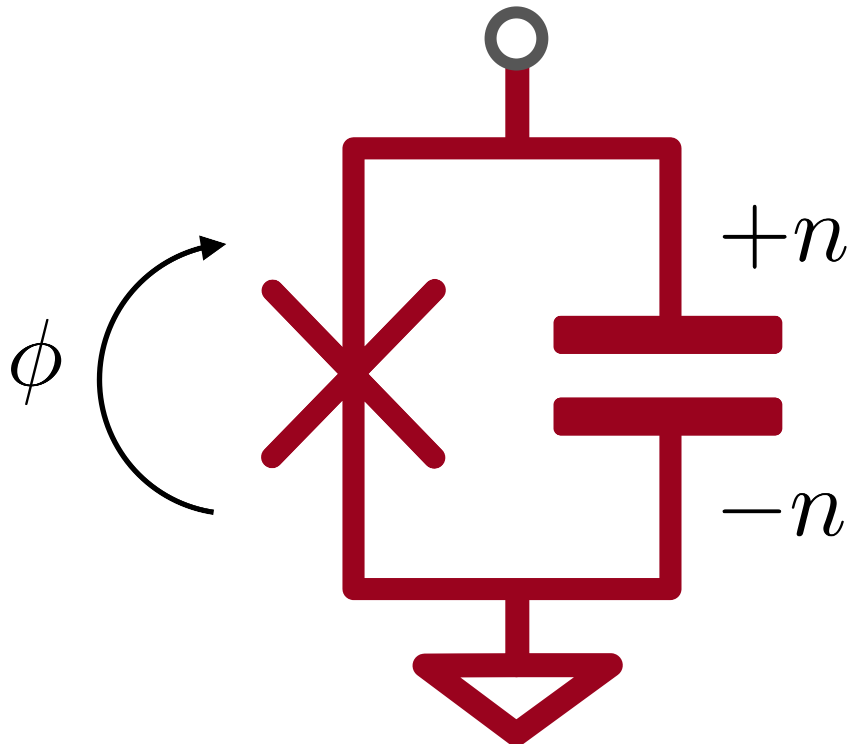




Circuit element	Associated energy	Associated variable
Capacitance C_J	$E_\mathrm{C} = \dfrac{(2e)^2}{2C_\mathrm{J}}$	Cooper pairs number $\hat{n}$
Junction L_J	$E_\mathrm{J} = \dfrac{\varphi_0^2}{L_\mathrm{J}}$	Macroscopic phase difference $\hat{\phi}$

Josephson junction: basics





Characteristic impedance

$$Z_{\mathrm{J}} = \frac{Z_{\mathrm{q}}}{2\pi}\sqrt{\frac{2E_{\mathrm{C}}}{E_{\mathrm{J}}}} \quad \text{with} \quad Z_{\mathrm{q}} = \frac{h}{(2e)^2} \simeq 6.5\mathrm{k}\Omega$$

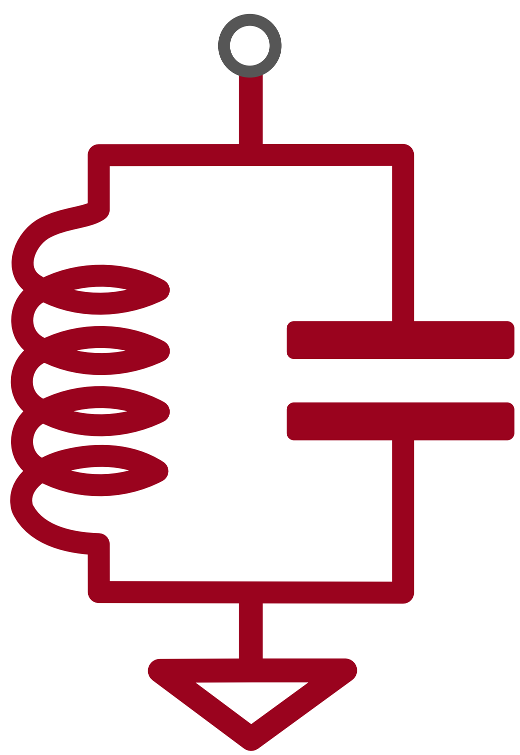
Resonant frequency

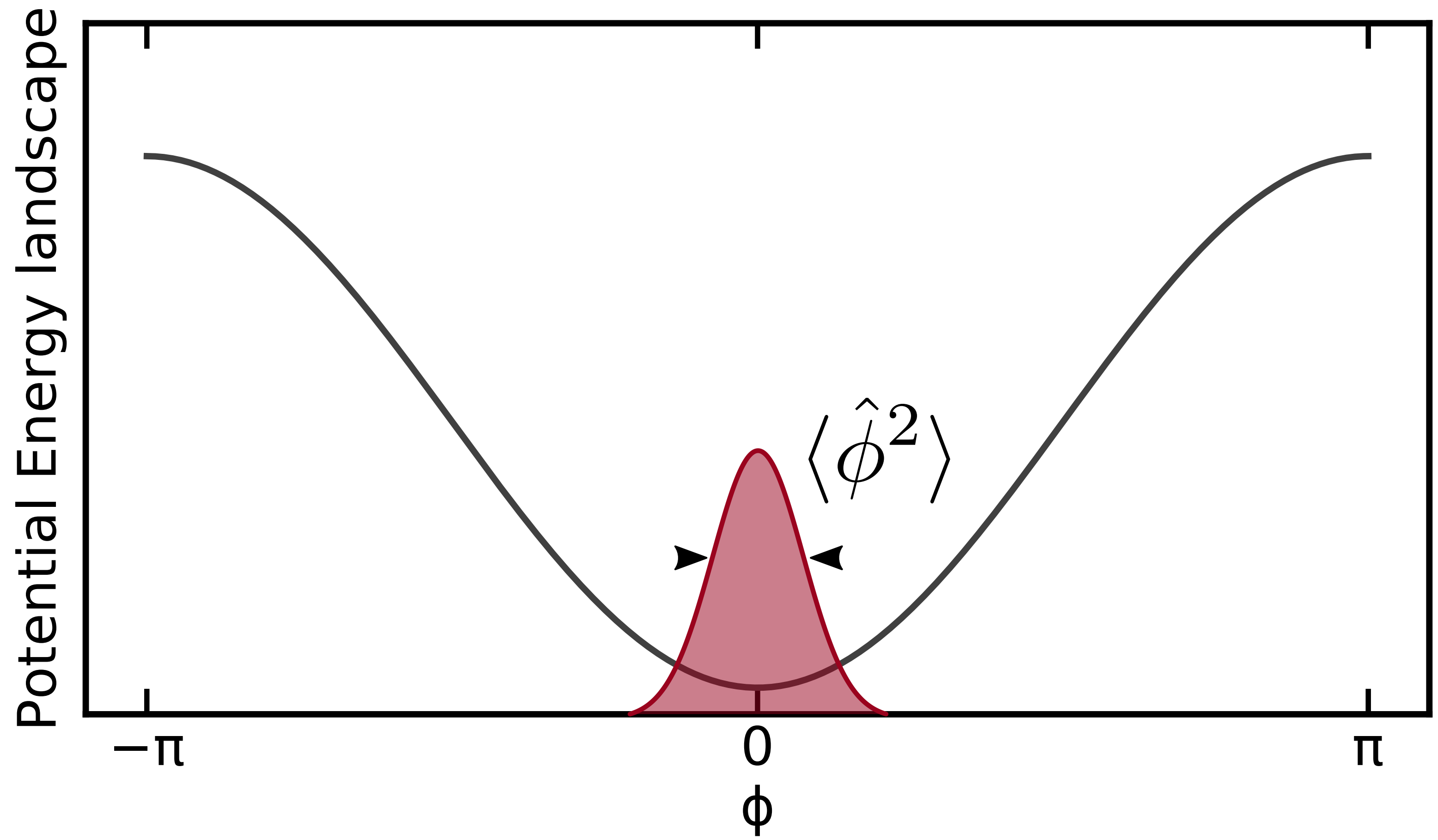
$$\omega_{\mathrm{J}} = \sqrt{2E_{\mathrm{J}}E_{\mathrm{c}}}$$

Josephson junction: linear regime

$$Z_\mathrm{J} \ll Z_\mathrm{q}$$

$$\hat{H} = E_\mathrm{C}\hat{n}^2 + \frac{E_\mathrm{J}}{2}\hat{\phi}^2$$



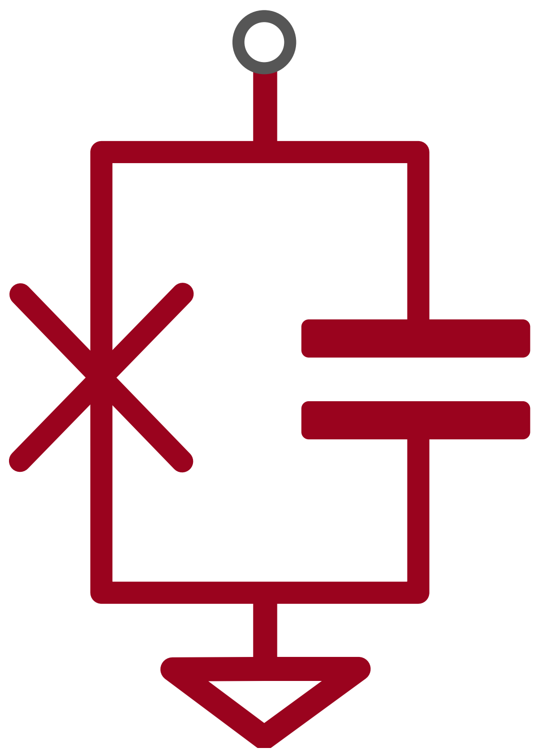


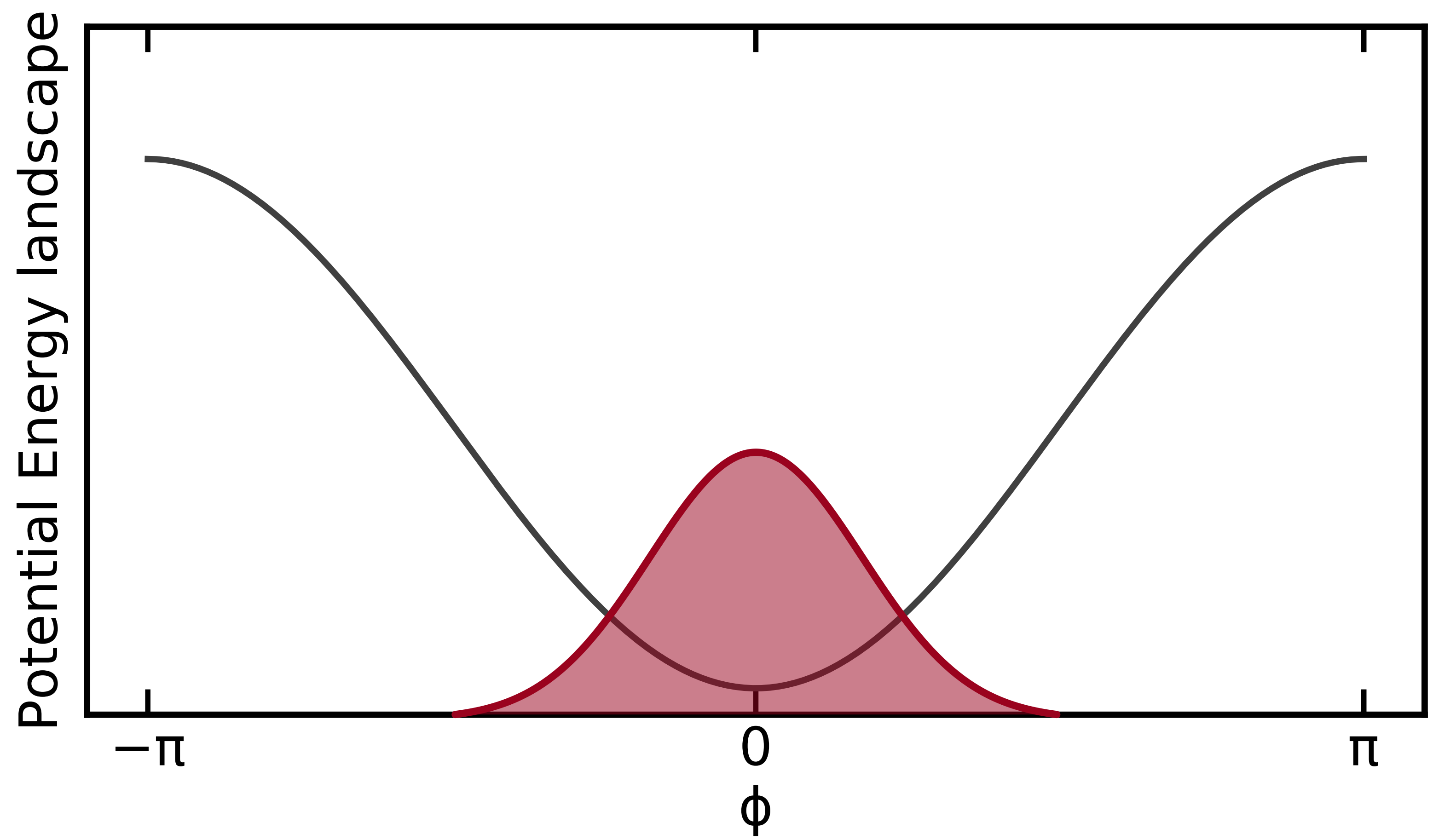
$$Z_\mathrm{J} = \sqrt{\frac{L}{C}} = \frac{Z_\mathrm{q}}{2\pi}\sqrt{\frac{2E_\mathrm{C}}{E_\mathrm{J}}} \text{ with } \quad Z_\mathrm{q} = \frac{h}{(2e)^2} \simeq 6.5\mathrm{k}\Omega$$

Josephson junction: strong nonlinearity

$$Z_{\mathrm{J}} \sim Z_{\mathrm{q}}$$

$$\hat{H} = E_{\mathrm{C}}\hat{n}^2 + E_{\mathrm{J}}(1 - \cos(\hat{\phi}))$$





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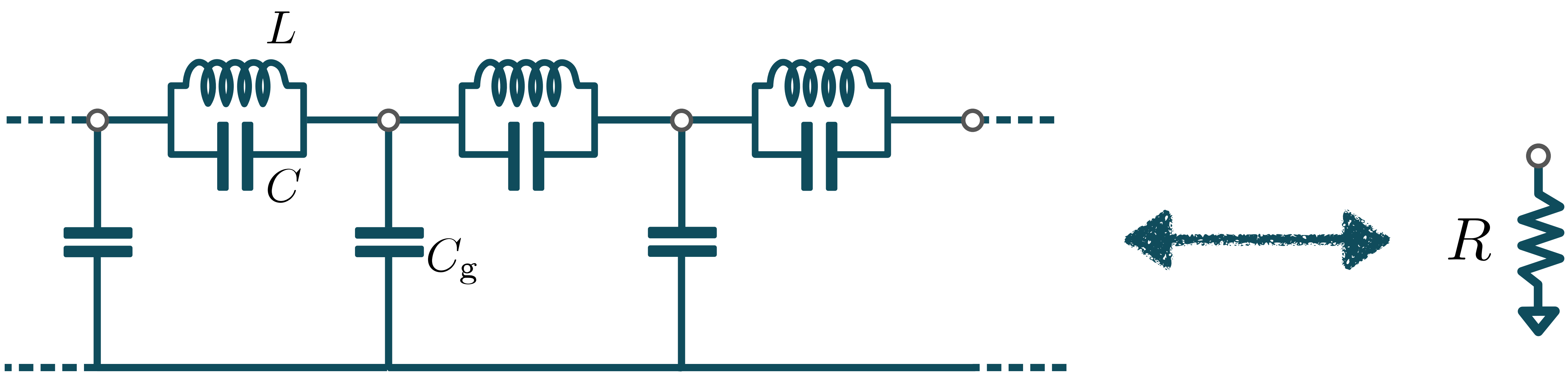
A Josephson junction coupled to a dissipative environment

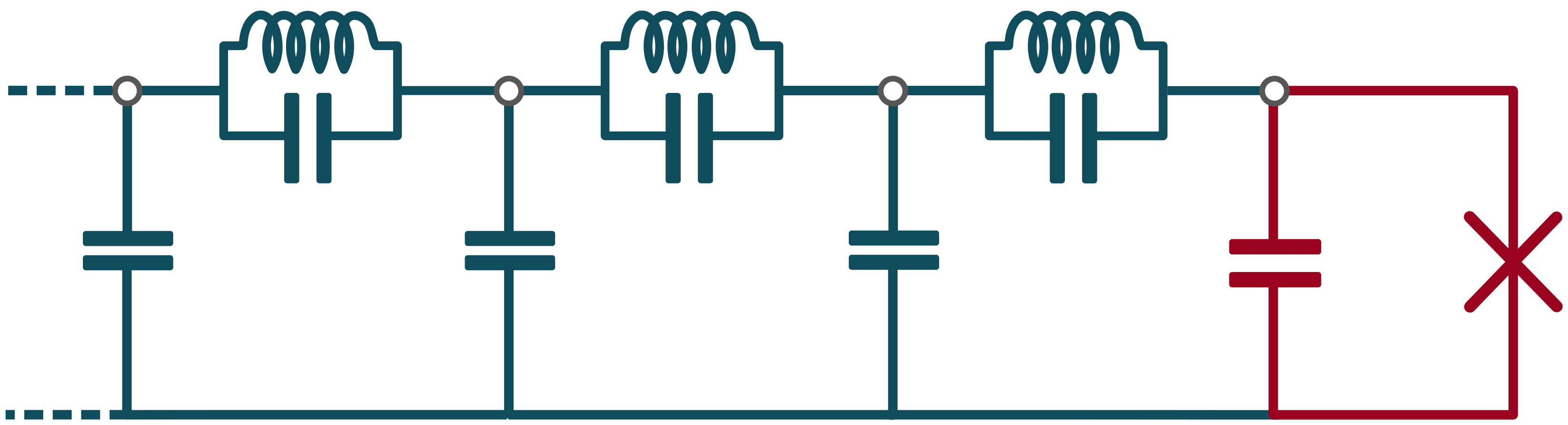


Infinite transmission line: $R = Z_c$

Dissipation in quantum mechanics

R. P. Feynman and F. L. Vernon Jr. (1963),
A. O. Caldeira and A. J. Leggett (1981)

A Josephson junction coupled to many harmonic oscillators:



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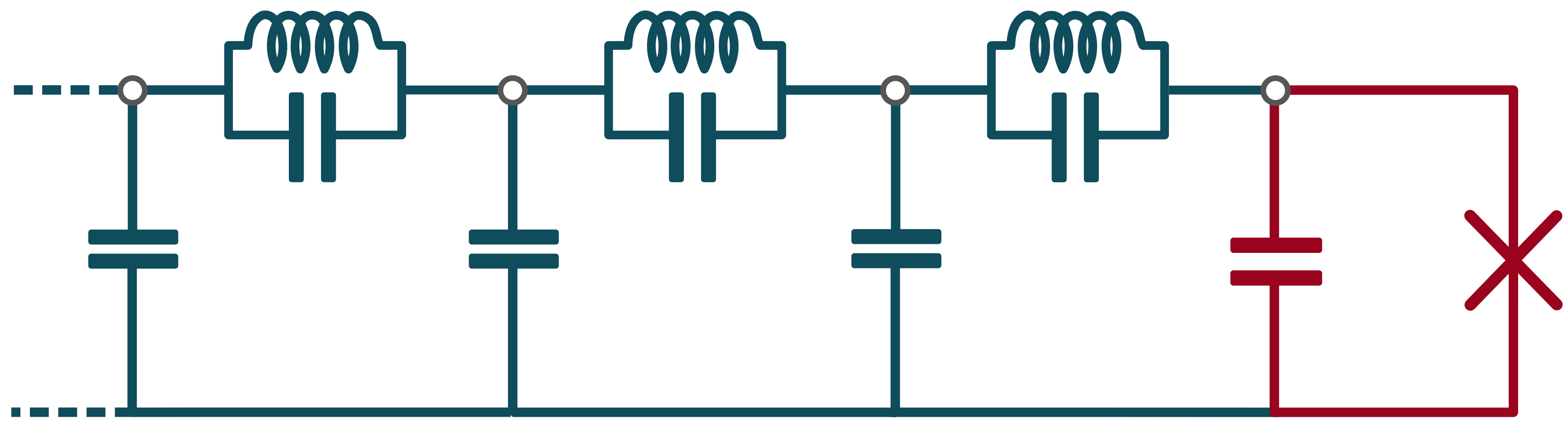
$$\langle\varphi_0^2\rangle \simeq 4Z_{\mathrm{c}}\ln(Z_{\mathrm{J}}/Z_{\mathrm{c}})/Z_{\mathrm{q}}$$

A Josephson junction coupled to many harmonic oscillators:



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Large Z_{J} ⟶ Small Josephson junction

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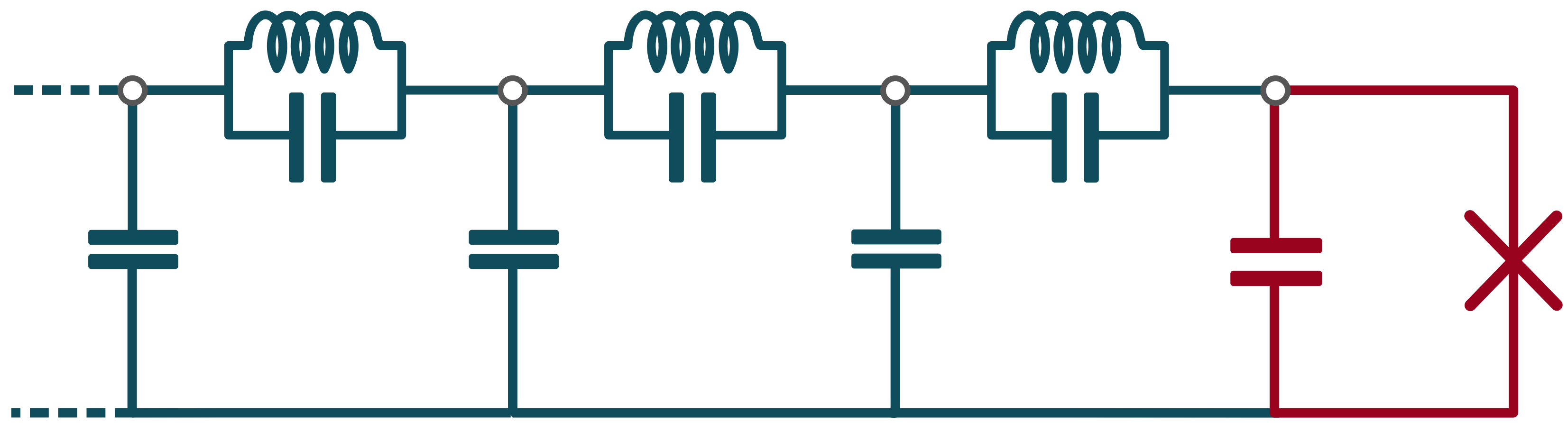

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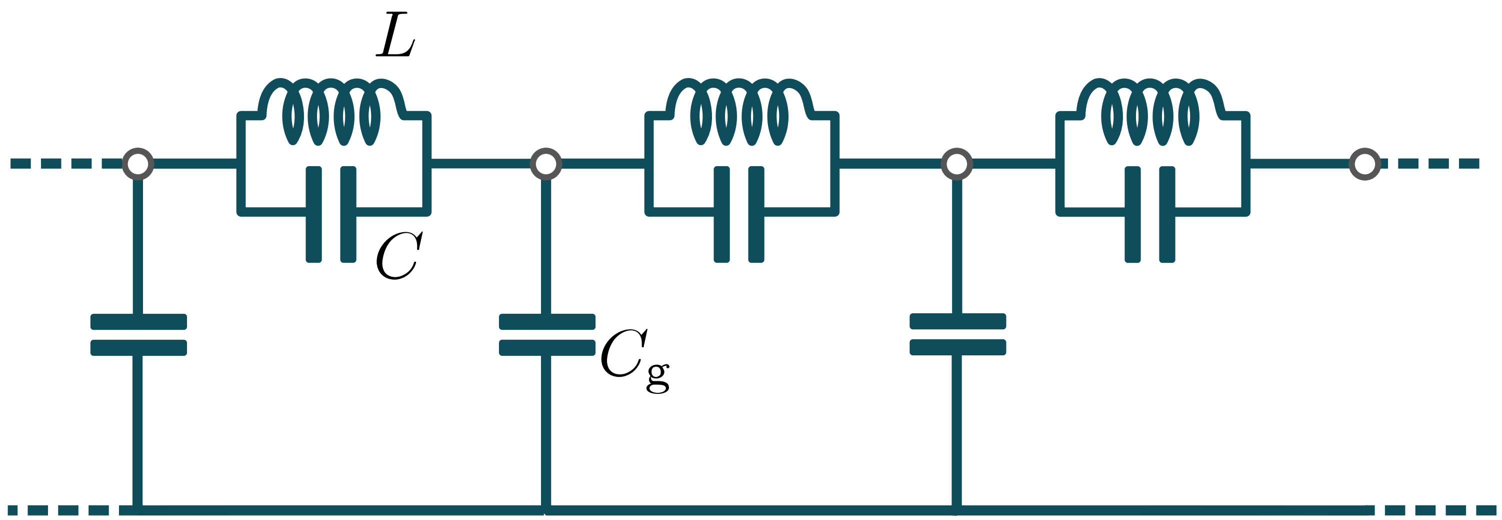
Large Z_{J} → Small Josephson junction

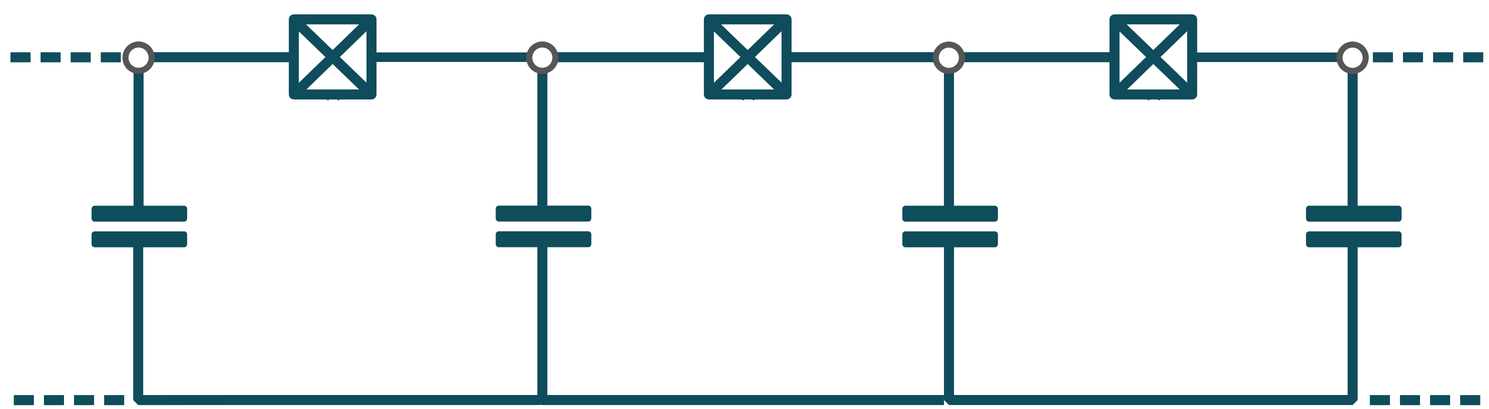
$Z_{\mathrm{c}} \gtrsim Z_{\mathrm{q}}$ Challenging since $Z_{\mathrm{q}} \sim 20Z_{\mathrm{vac}}$

Building a large impedance environment



$$Z_c = \sqrt{L/C_g}$$

Josephson junction meta-material



$$Z_c = \sqrt{L_J/C_g}$$

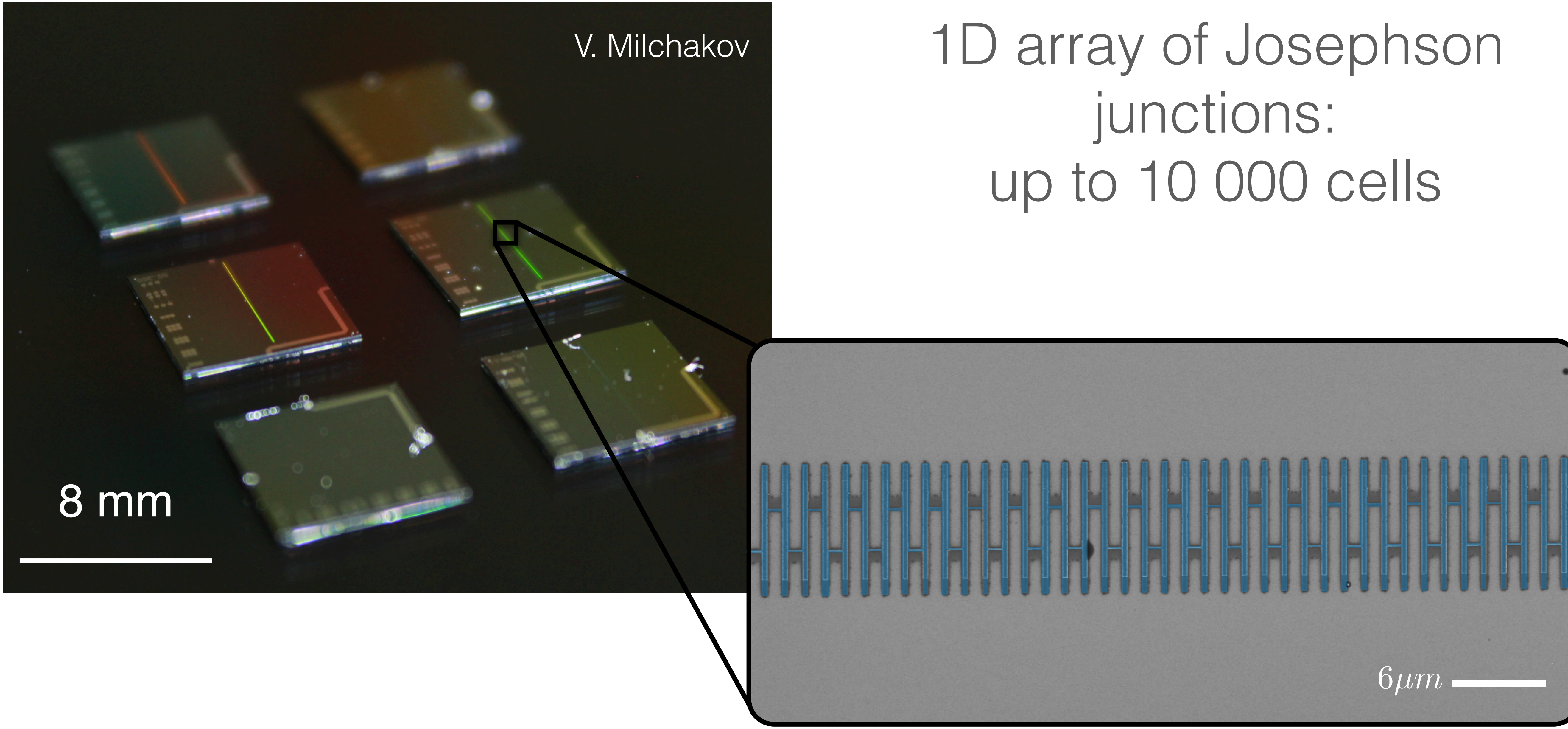
Seminal work:

S. Corlevi et al 06'
(Haviland's group)

See also:

N. Masluk et al 12', Bell et al 12', S. Butz et al. 13',
C. Altimiras et al. 13', R. Kuzmin et al 18'….

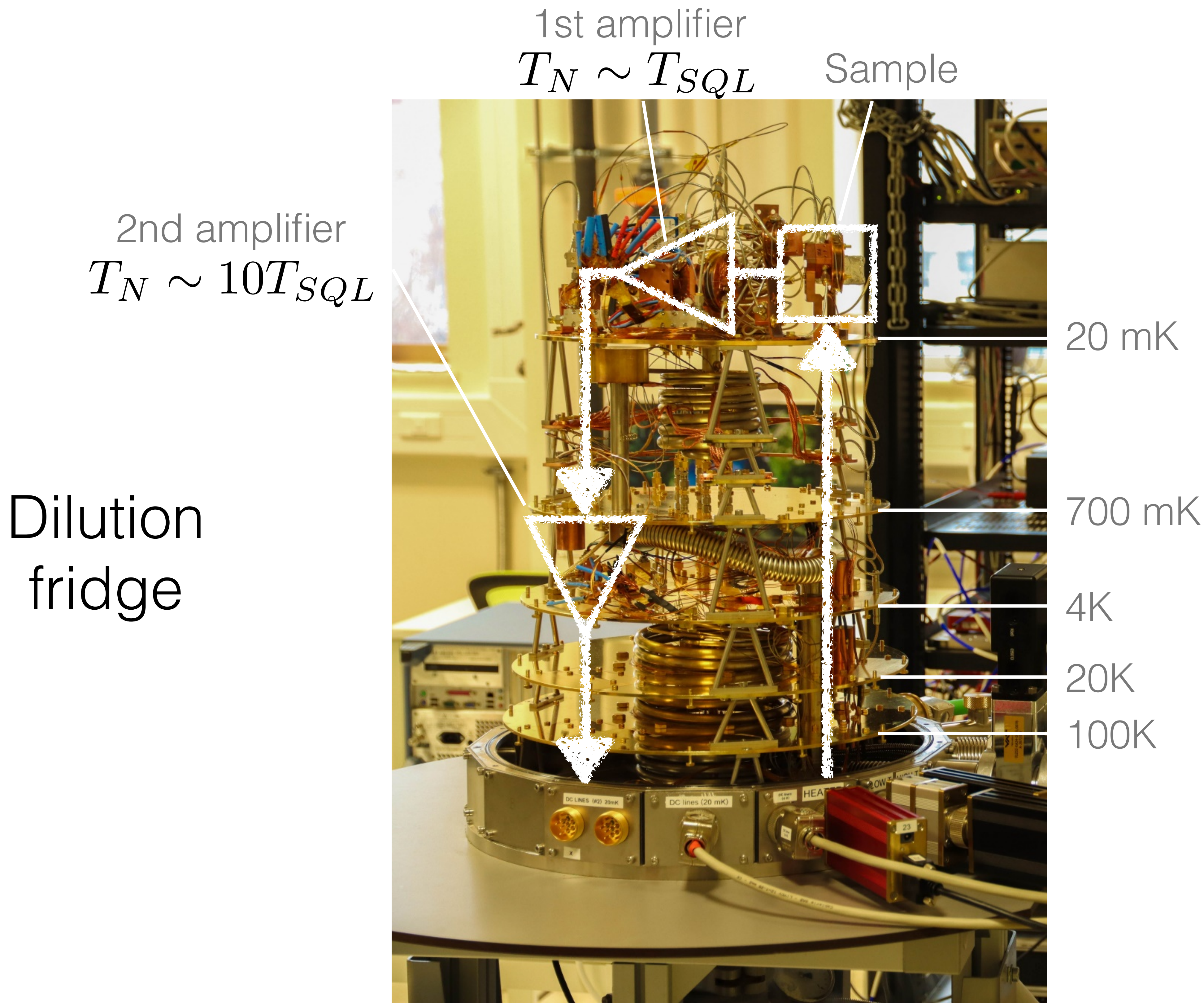
Josephson junction meta-material: Fabrication



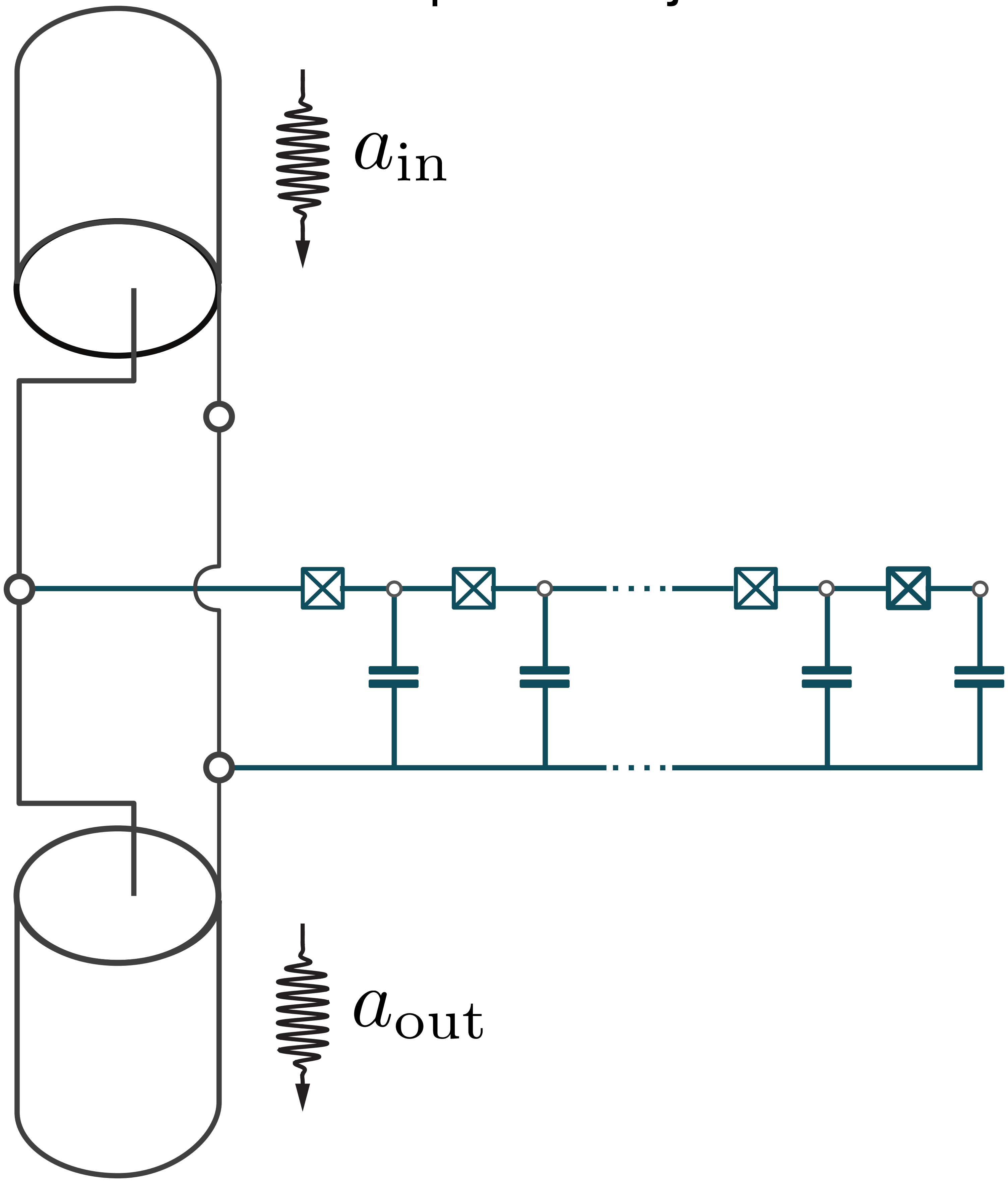
1D array of Josephson junctions:
up to 10 000 cells

Challenges faced: stitching errors, resist homogeneity, focus homogeneity, proximity effect….

Cooling down the circuit



Josephson junction meta-material

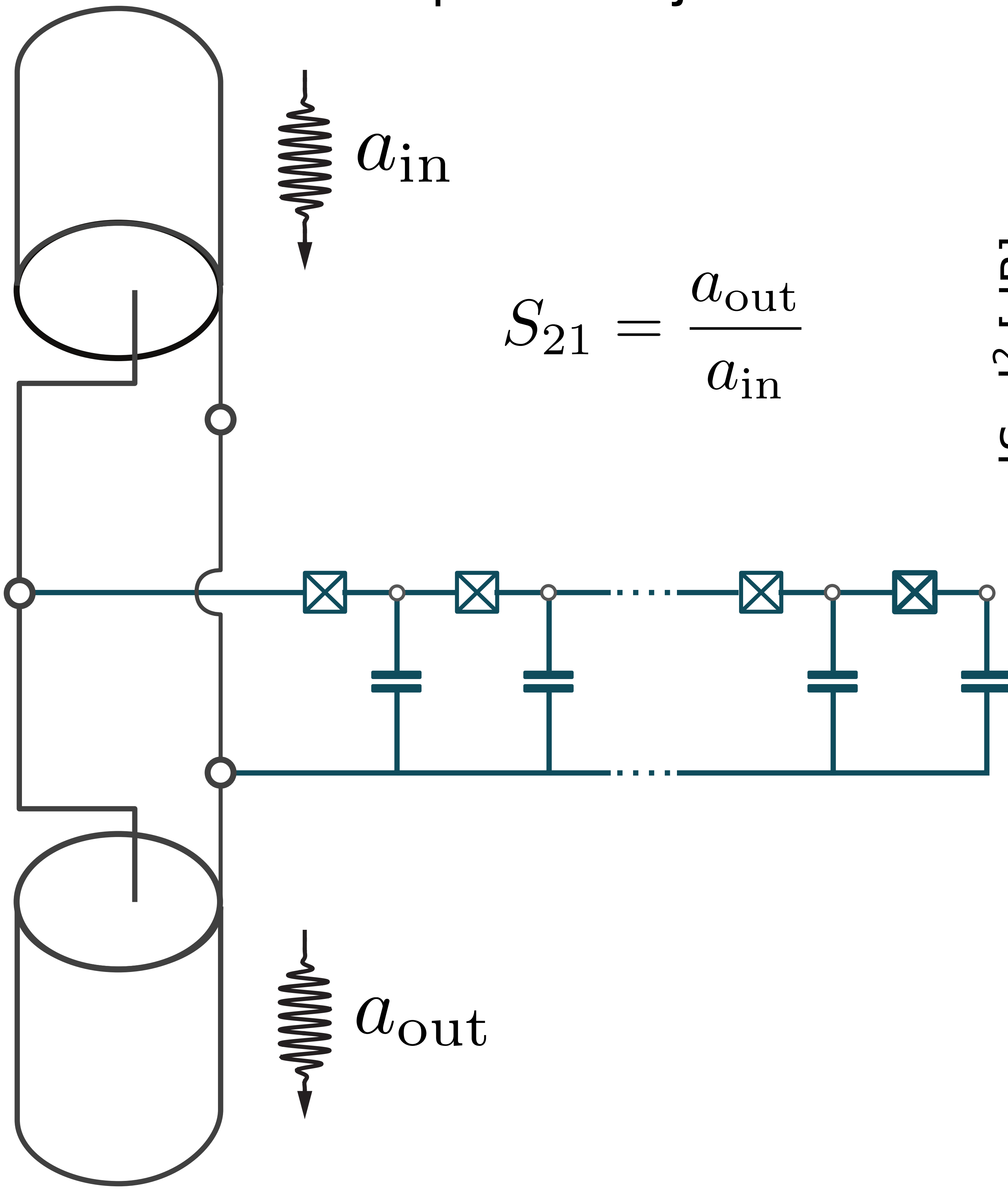


high frequency
& low temperature

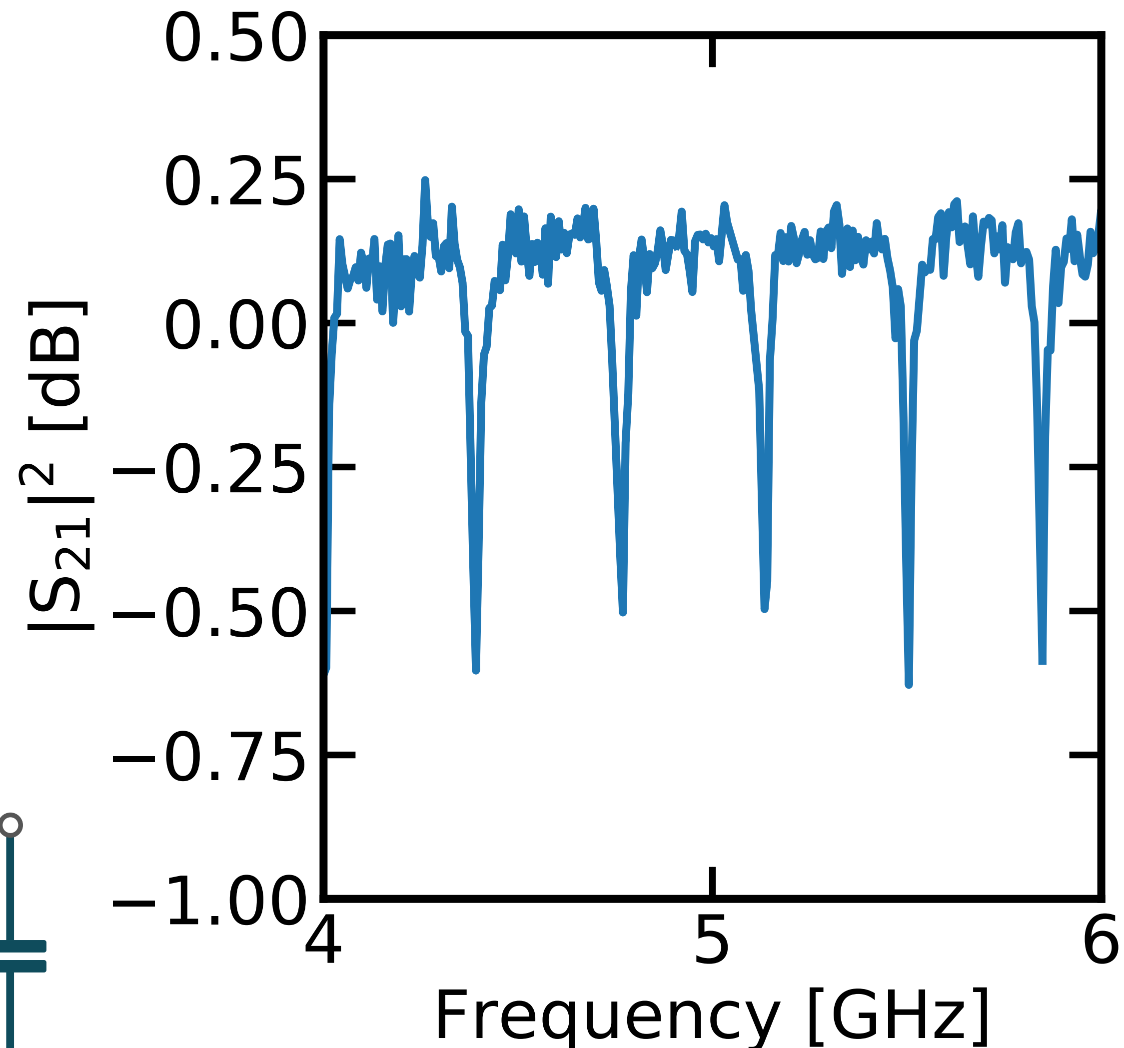
$$\hbar\omega \gg k_{\mathrm{B}}T$$

$$(T = 20\ \mathrm{mK})$$

Josephson junction meta-material



$$S_{21} = \frac{a_{\mathrm{out}}}{a_{\mathrm{in}}}$$



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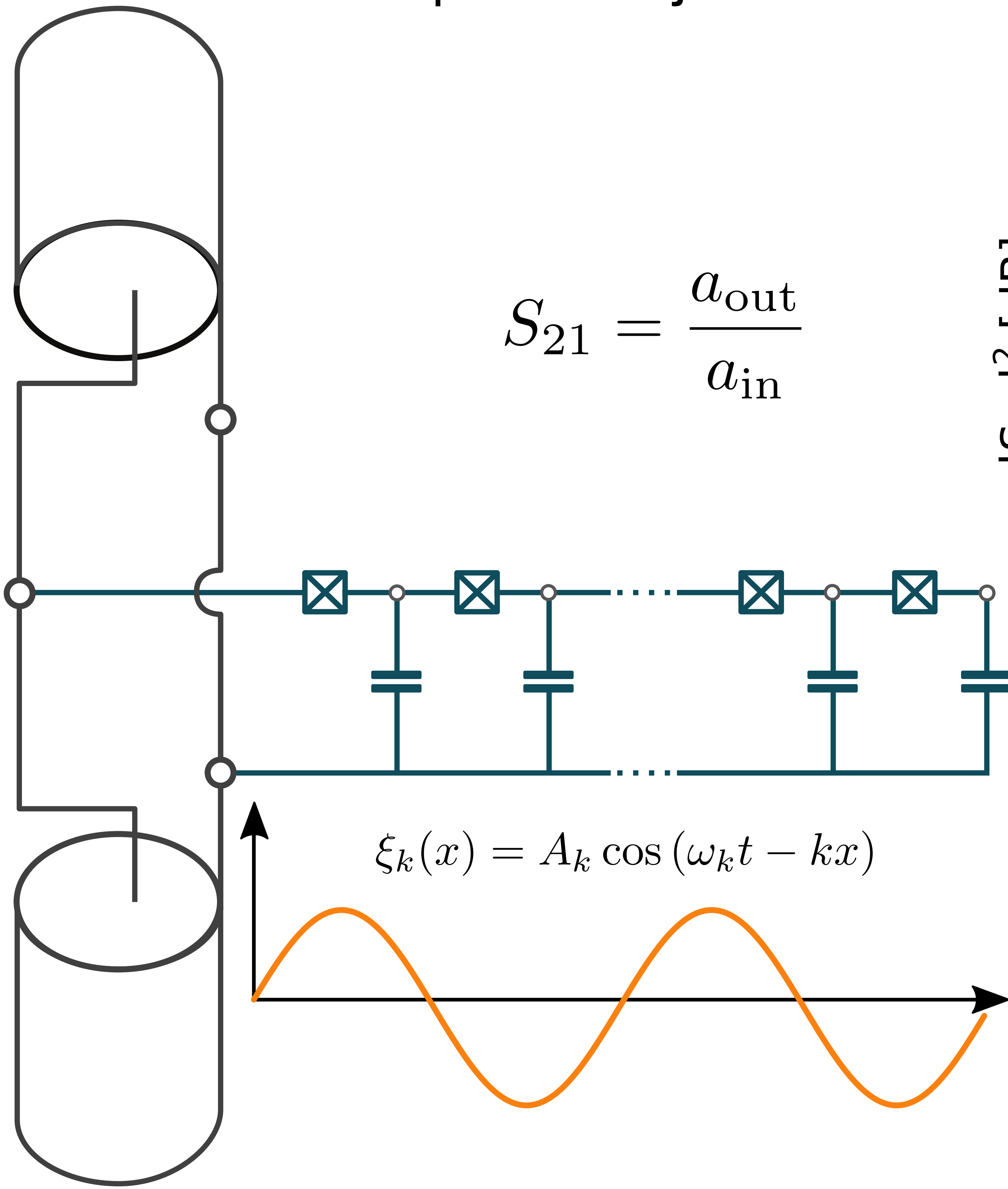

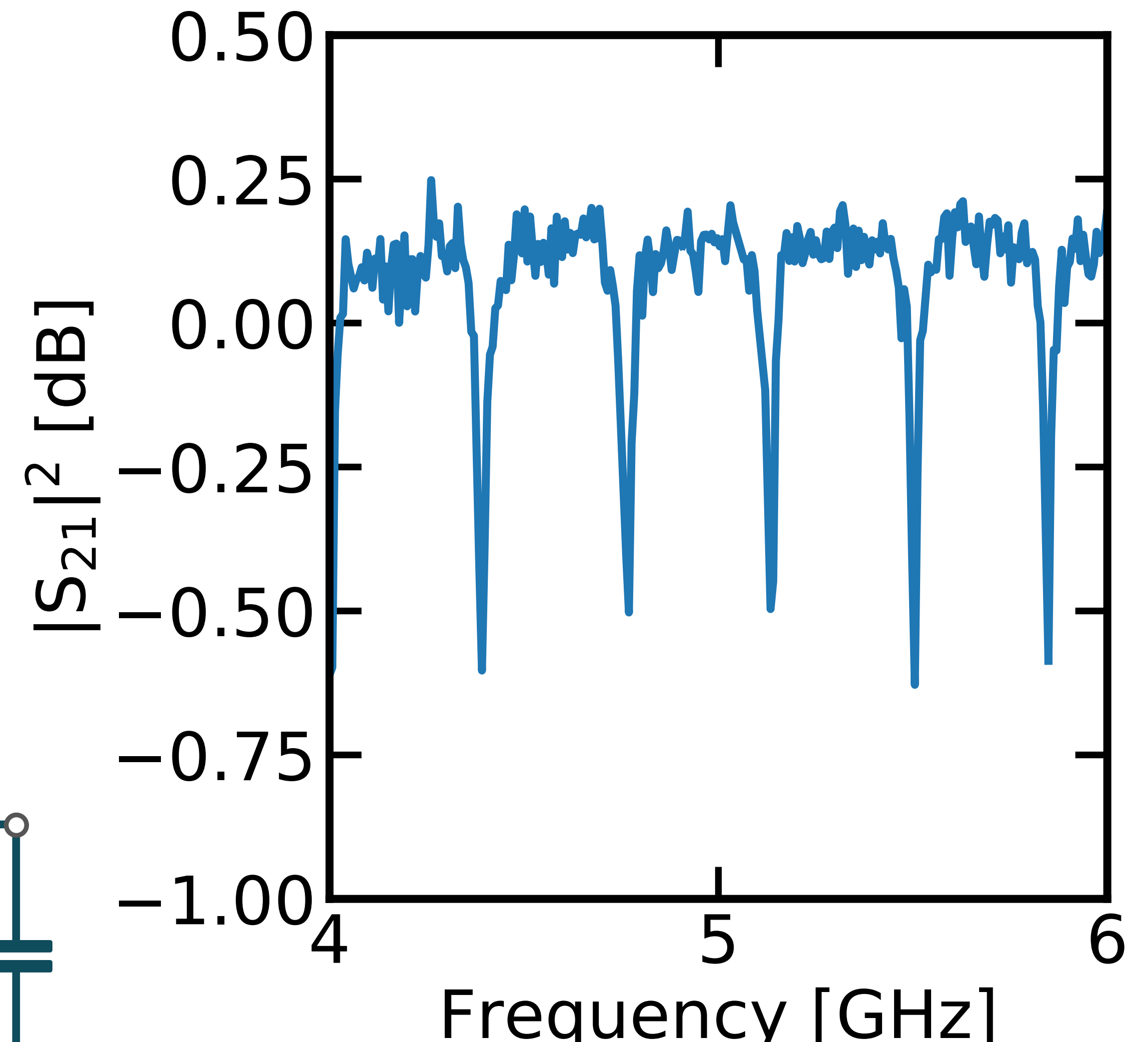

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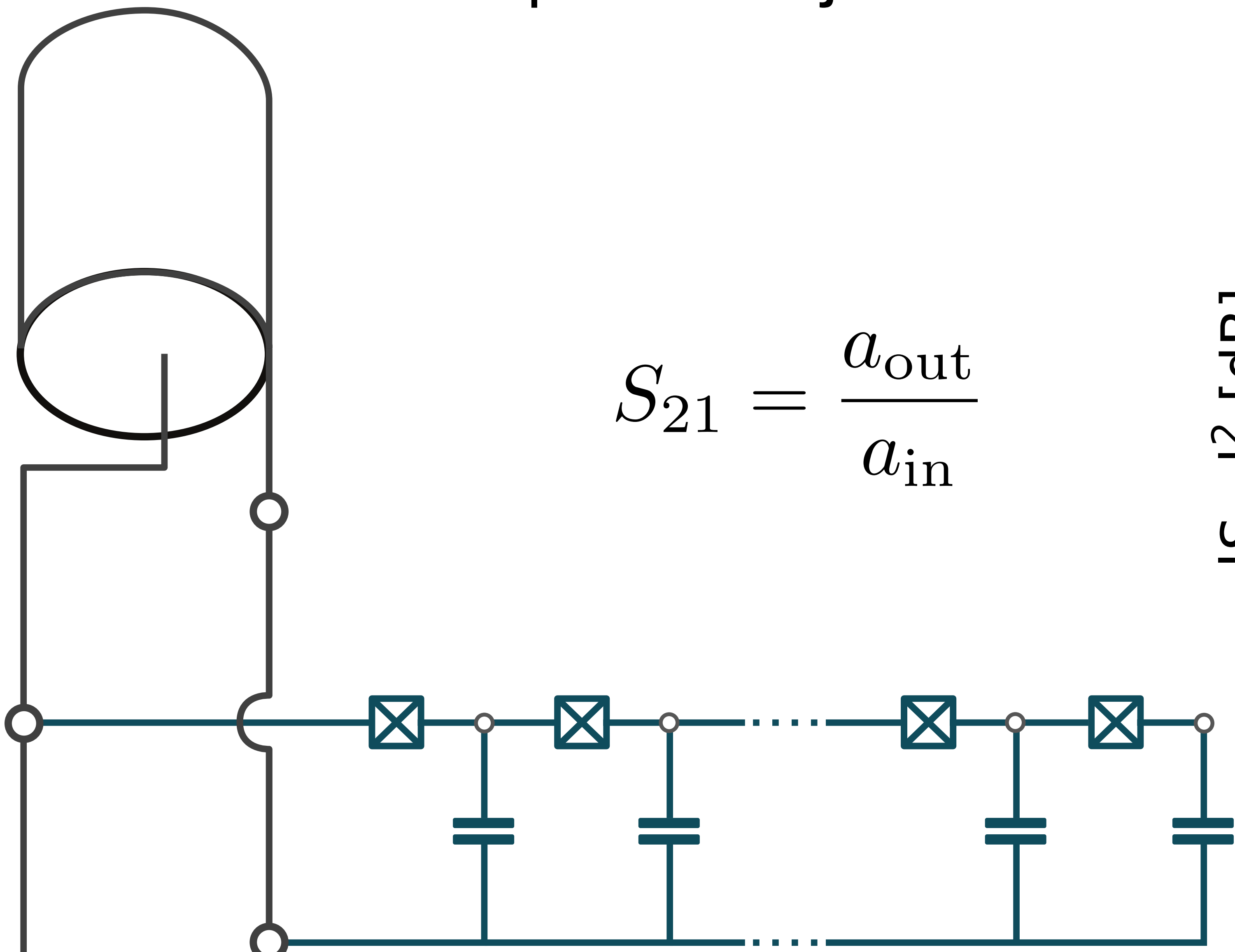
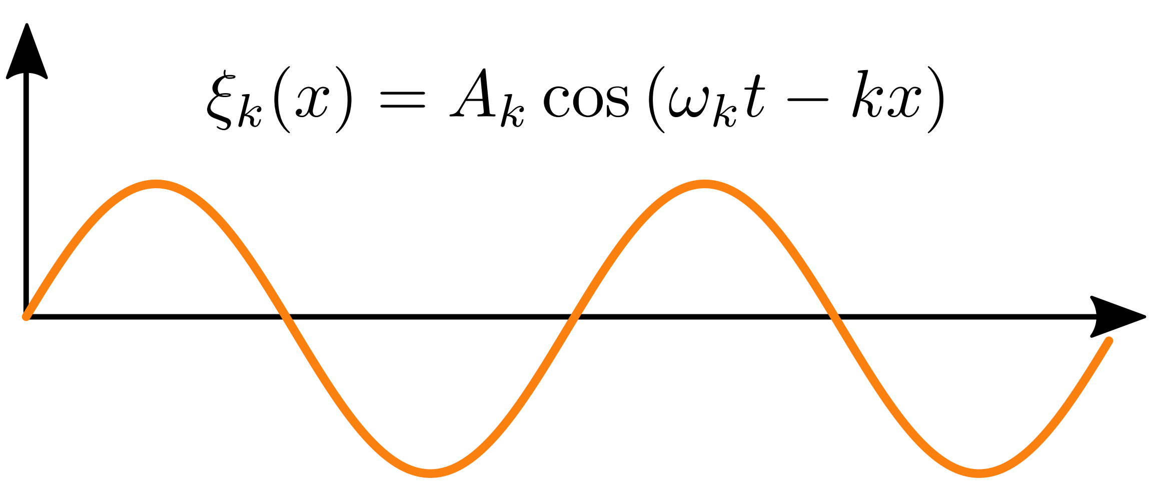
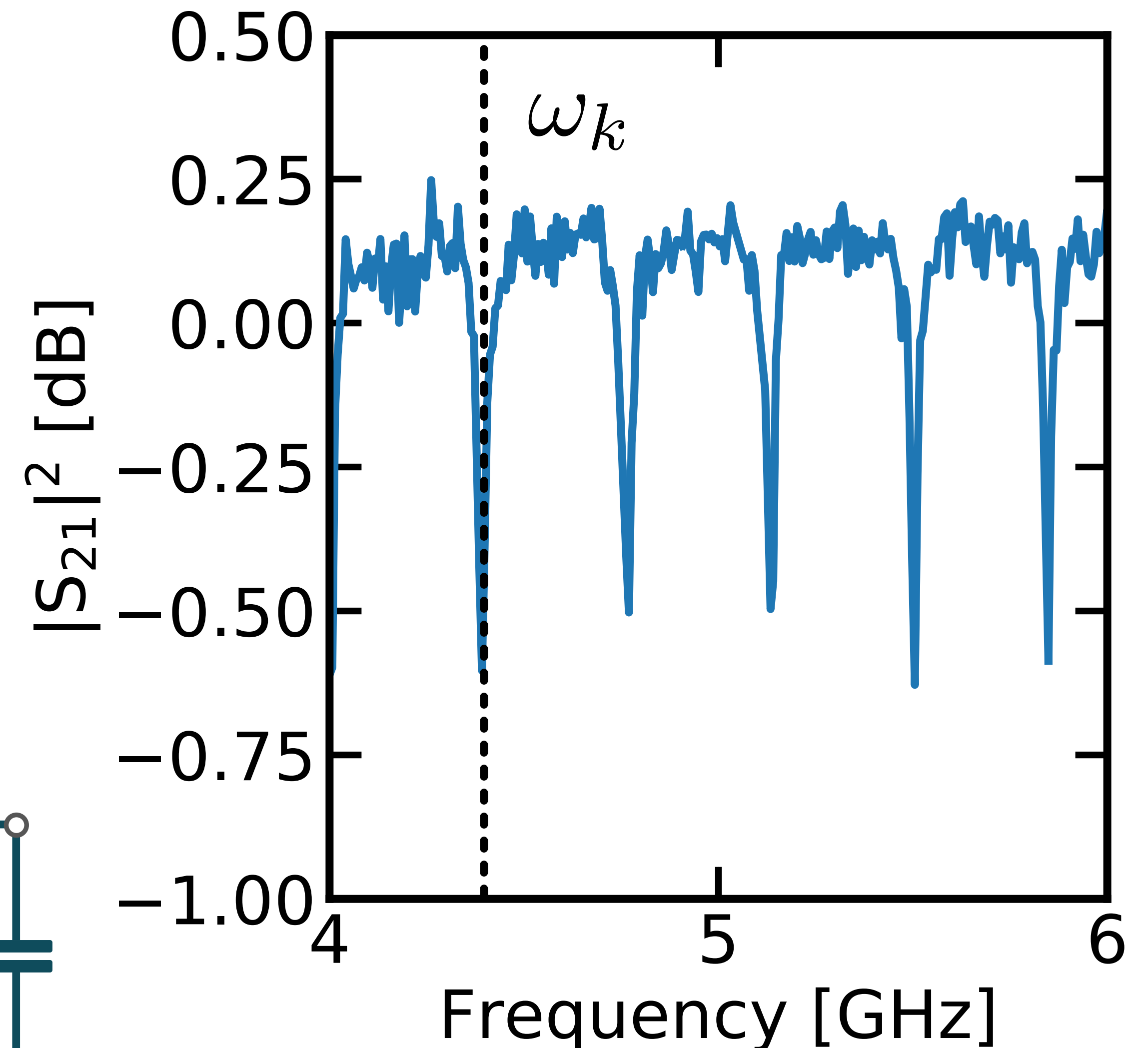


$$S_{21} = \frac{a_{\text{out}}}{a_{\text{in}}}$$

$$\xi_k(x) = A_k \cos\left(\omega_k t - kx\right)$$





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$$\hbar\omega \gg k_{\text{B}}T$$

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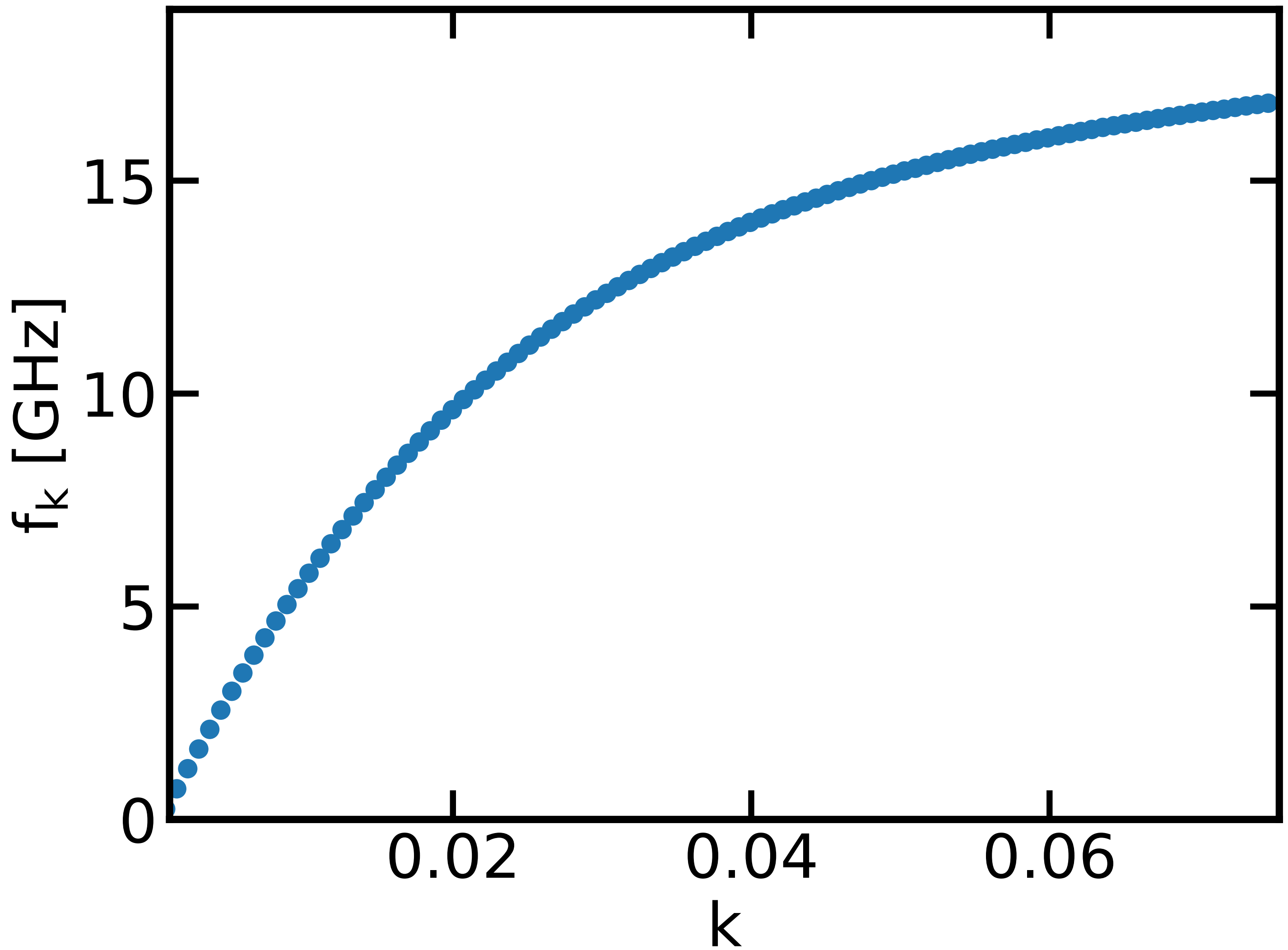
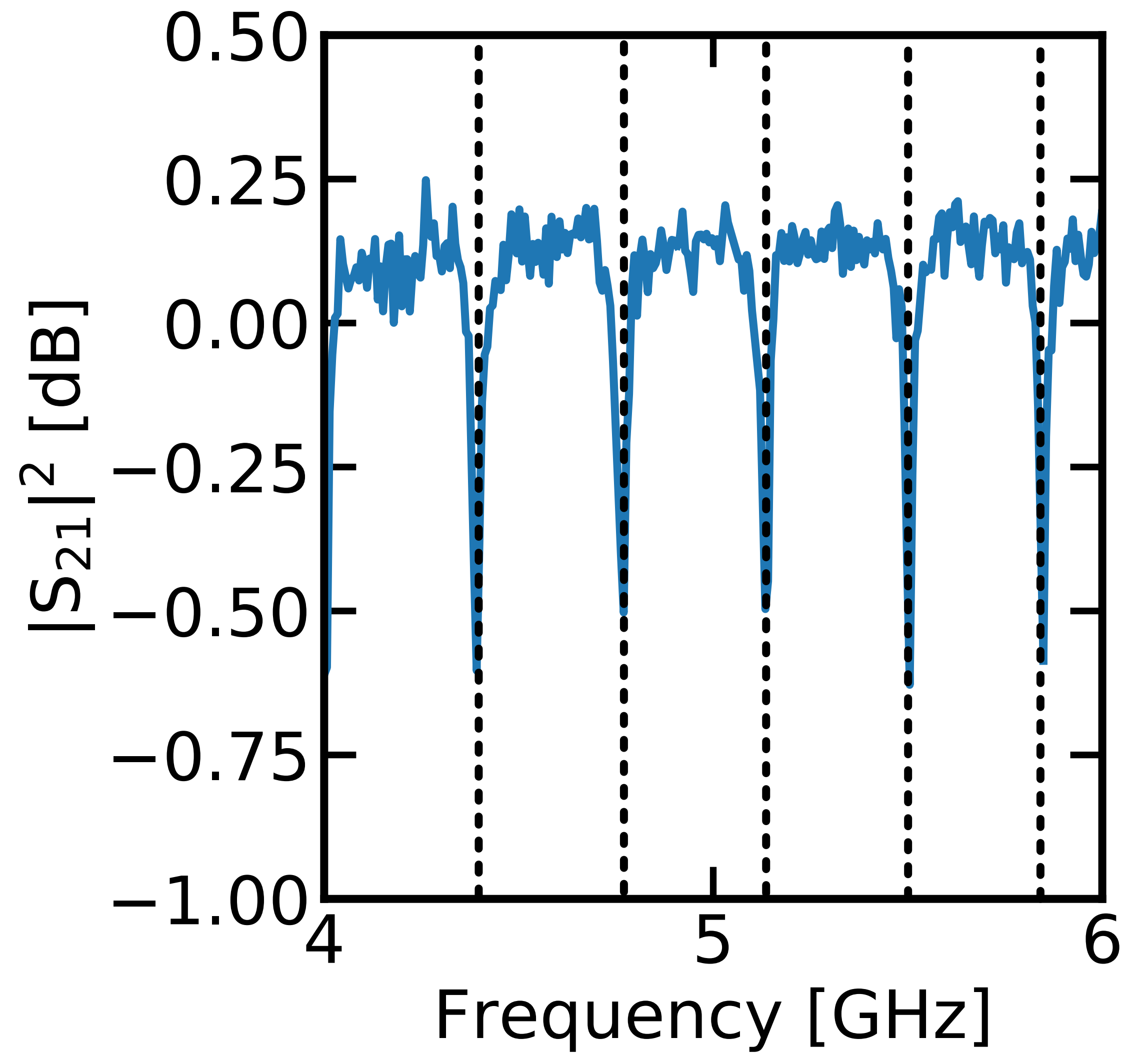
JJ meta-material: dispersion relation





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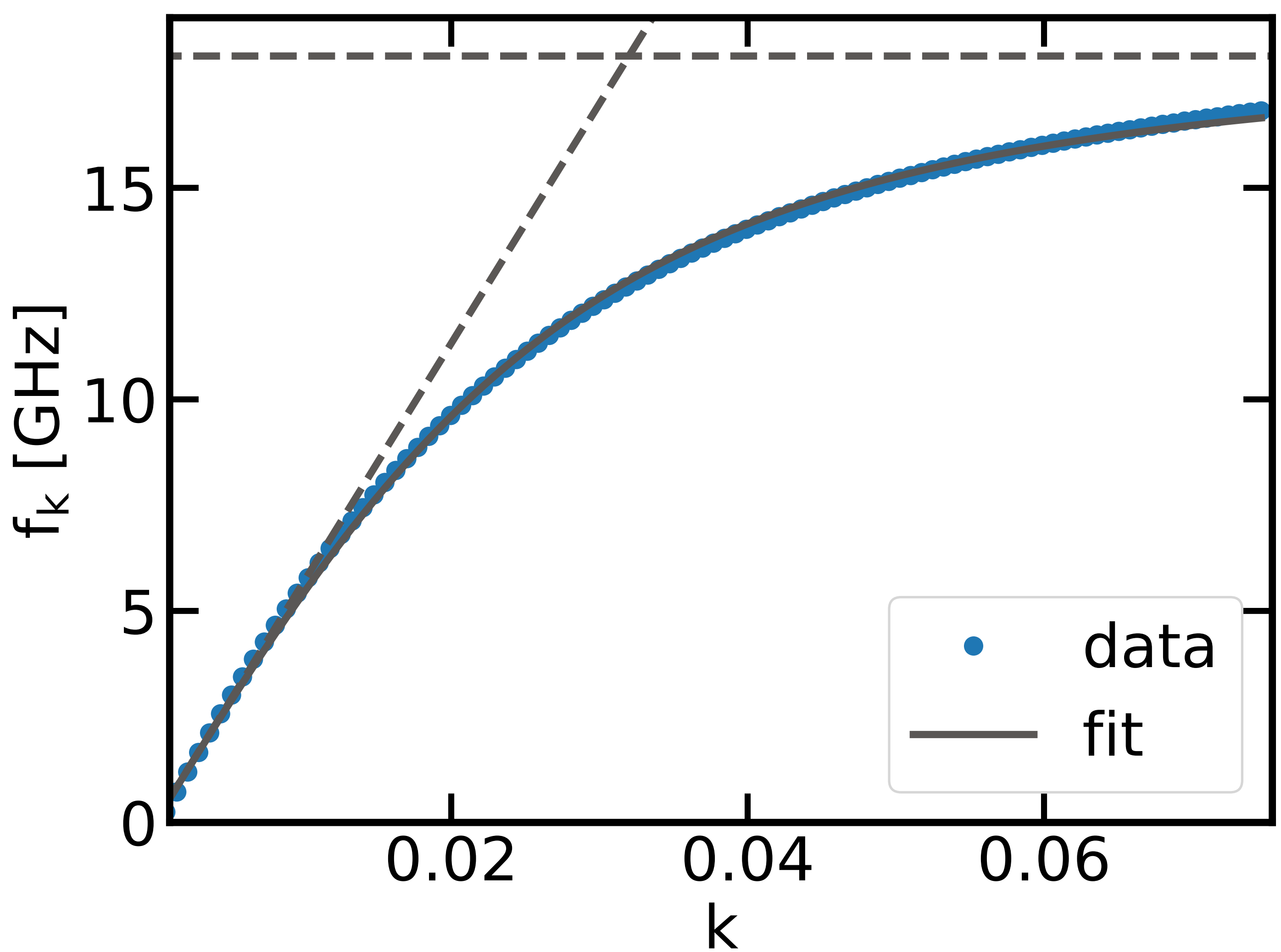
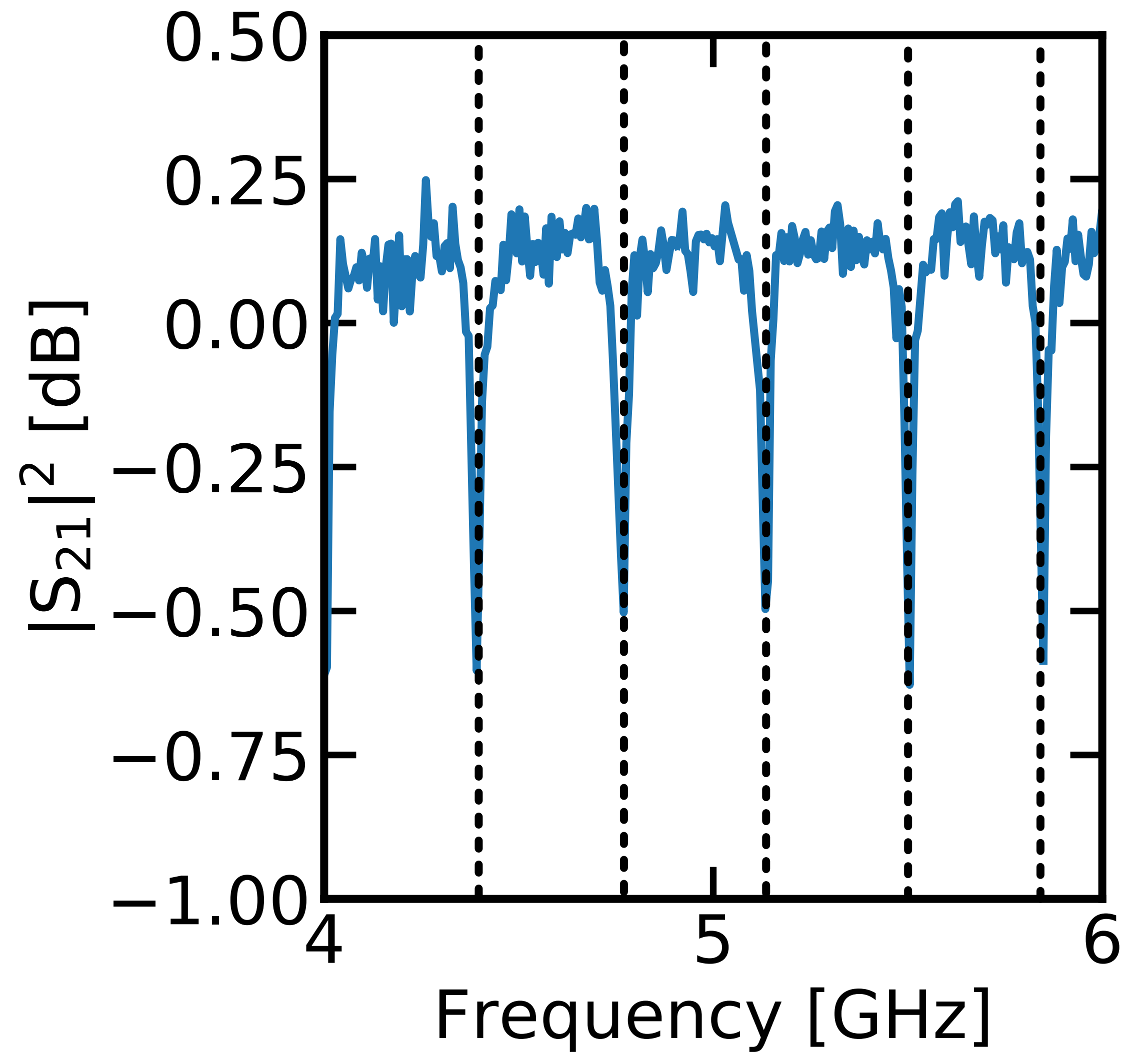
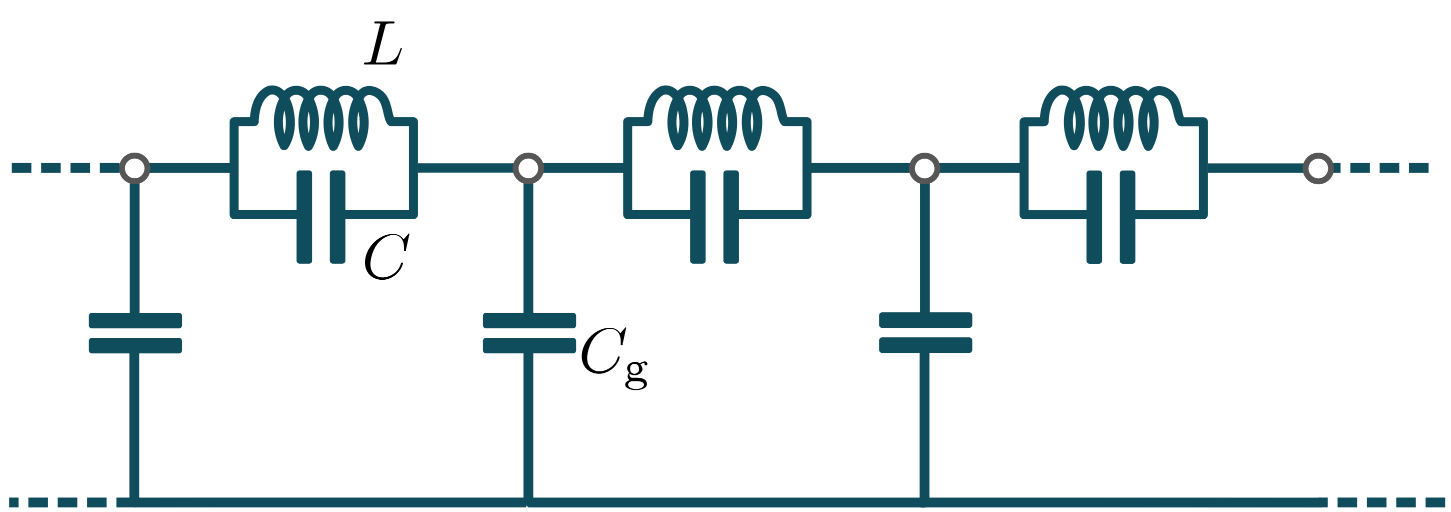






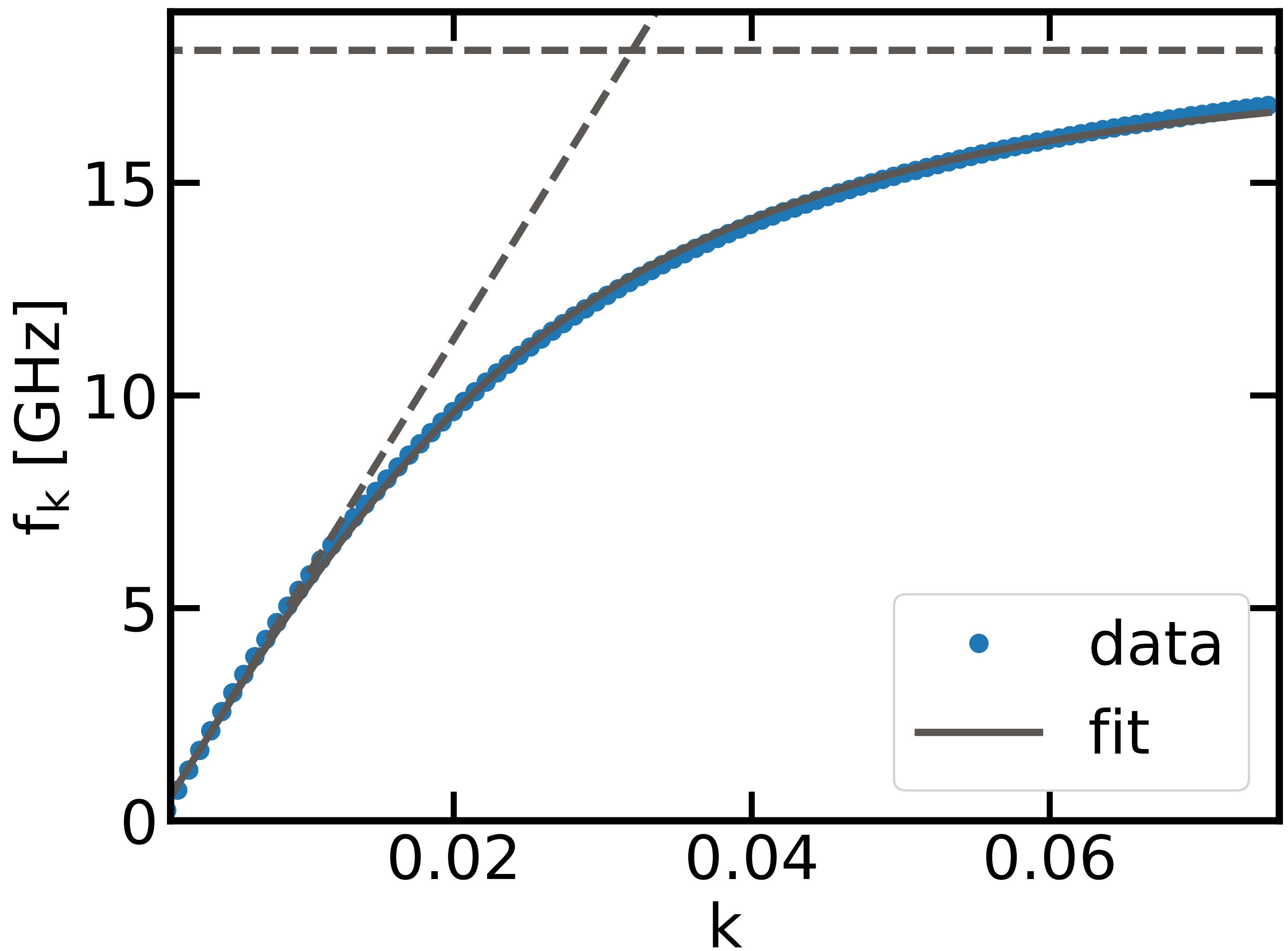
$L = 0.53$ nH $\quad C = 144$ fF $\quad C_{\mathrm{g}} = 0.147$ fF

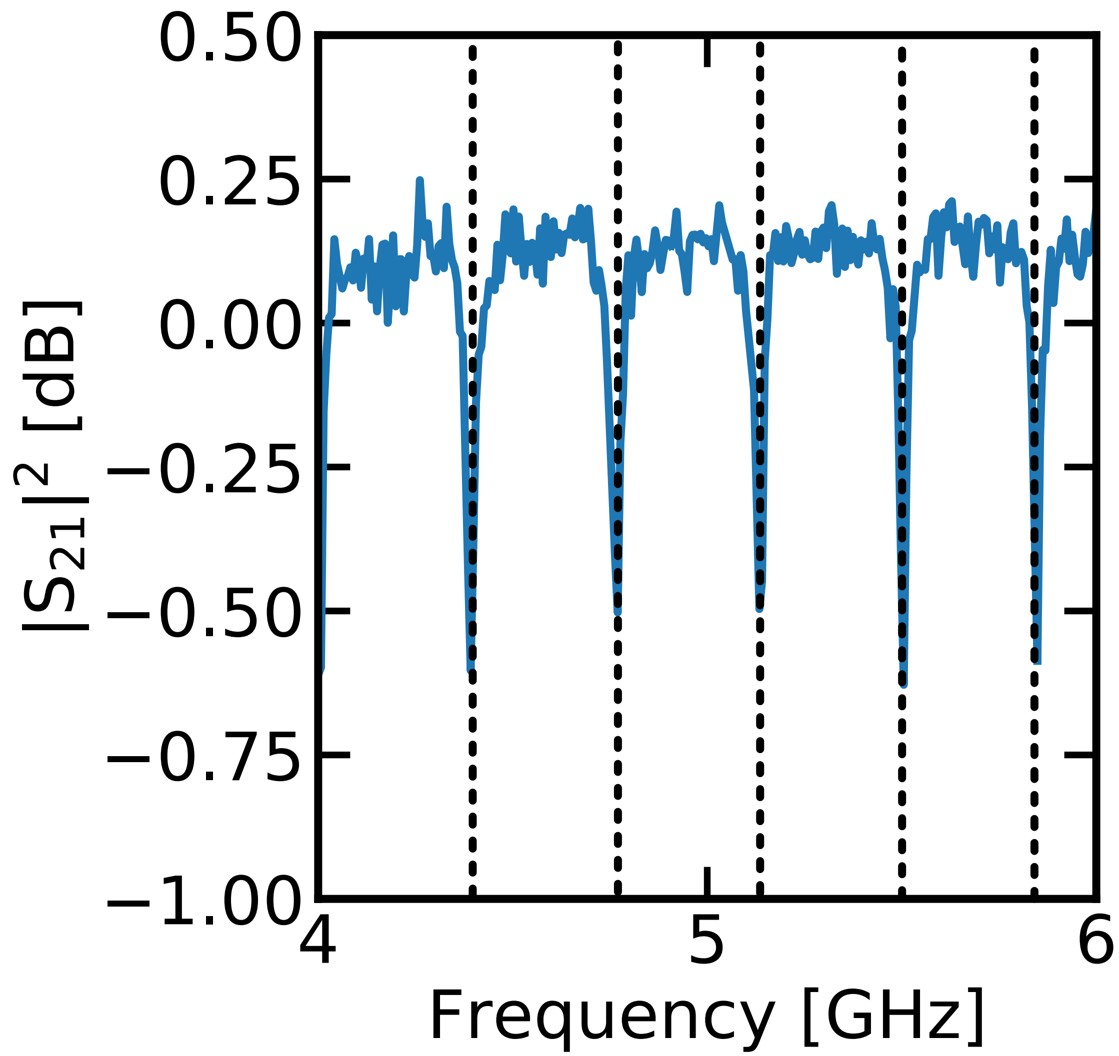
Two scales

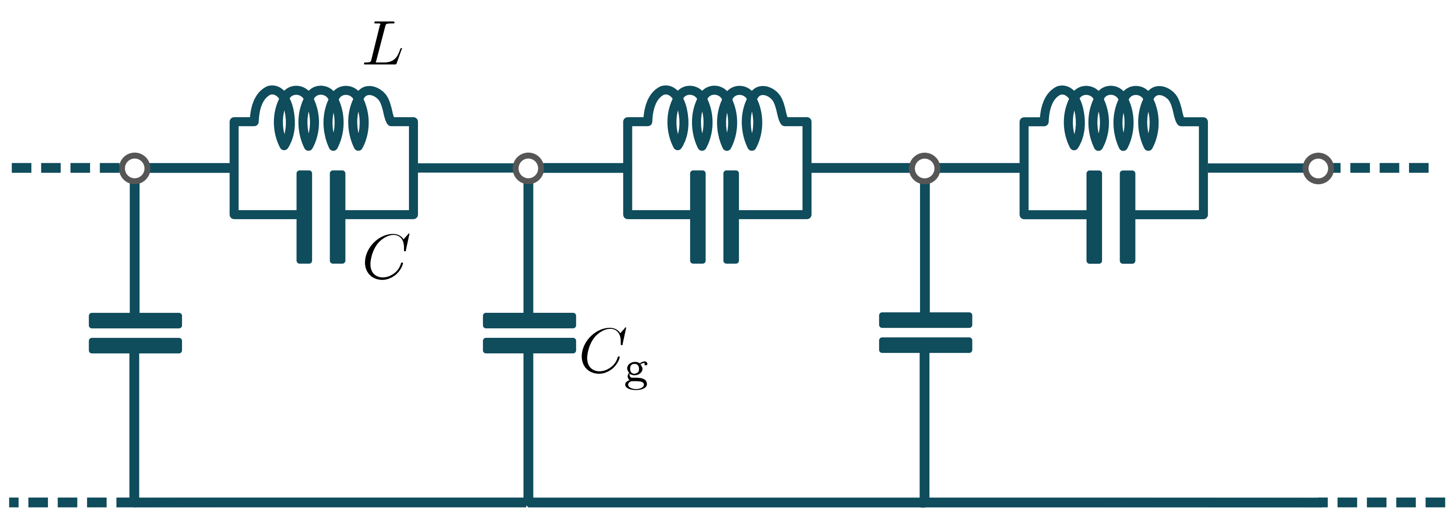
$$Z_{\mathrm{J}} = \sqrt{\frac{L}{C}} \sim 60\Omega$$

$$Z_c = \sqrt{\frac{L}{C_g}} \sim 1.9\mathrm{k}\Omega$$

JJ meta-material: dispersion relation







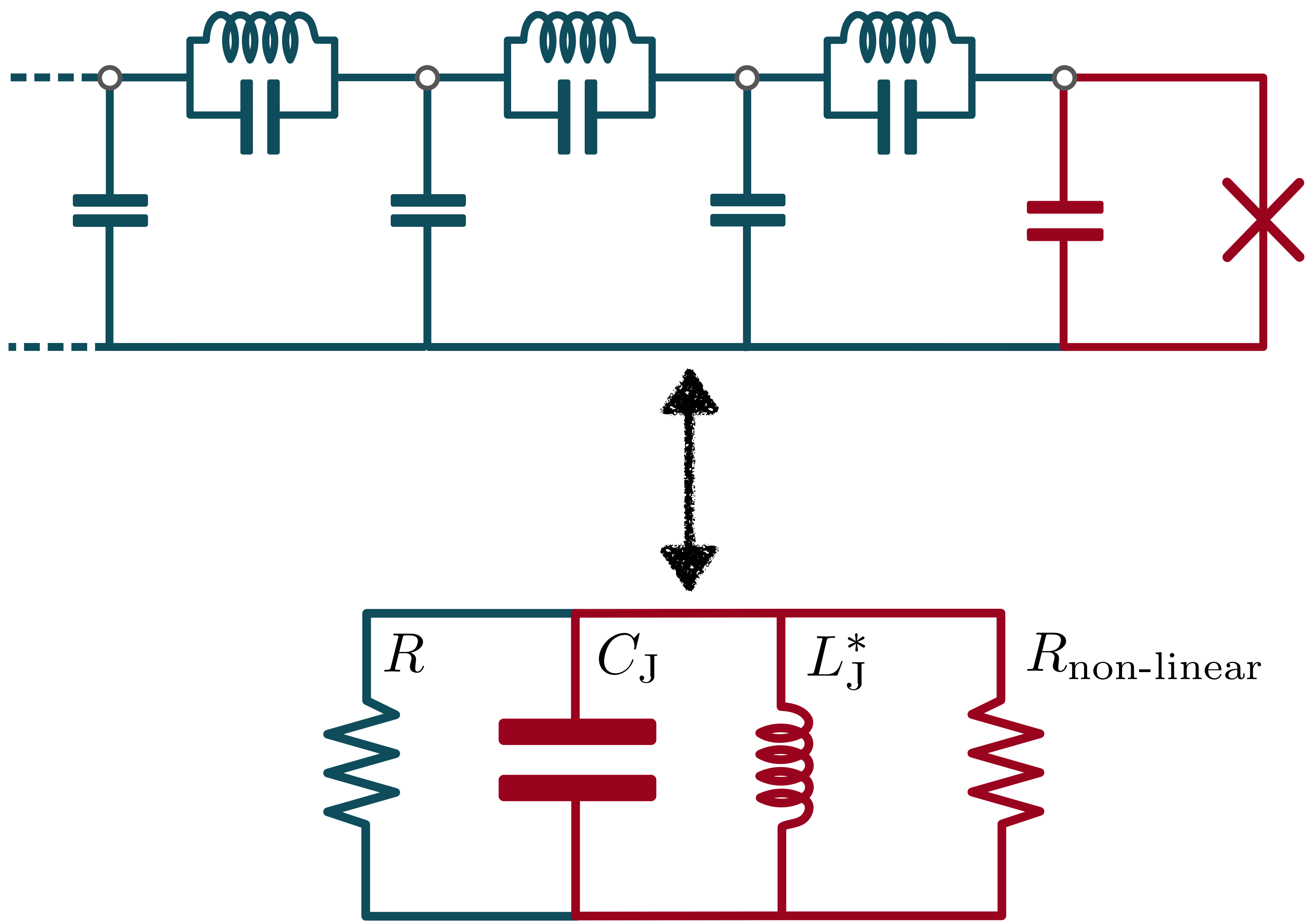
$L = 0.53$ nH $\quad C = 144$ fF $\quad C_\mathrm{g} = 0.147$ fF

Two scales

$$Z_\mathrm{J} = \sqrt{\frac{L}{C}} \sim 60\Omega$$

$$Z_c = \sqrt{\frac{L}{C_g}} \sim 1.9\mathrm{k}\Omega$$

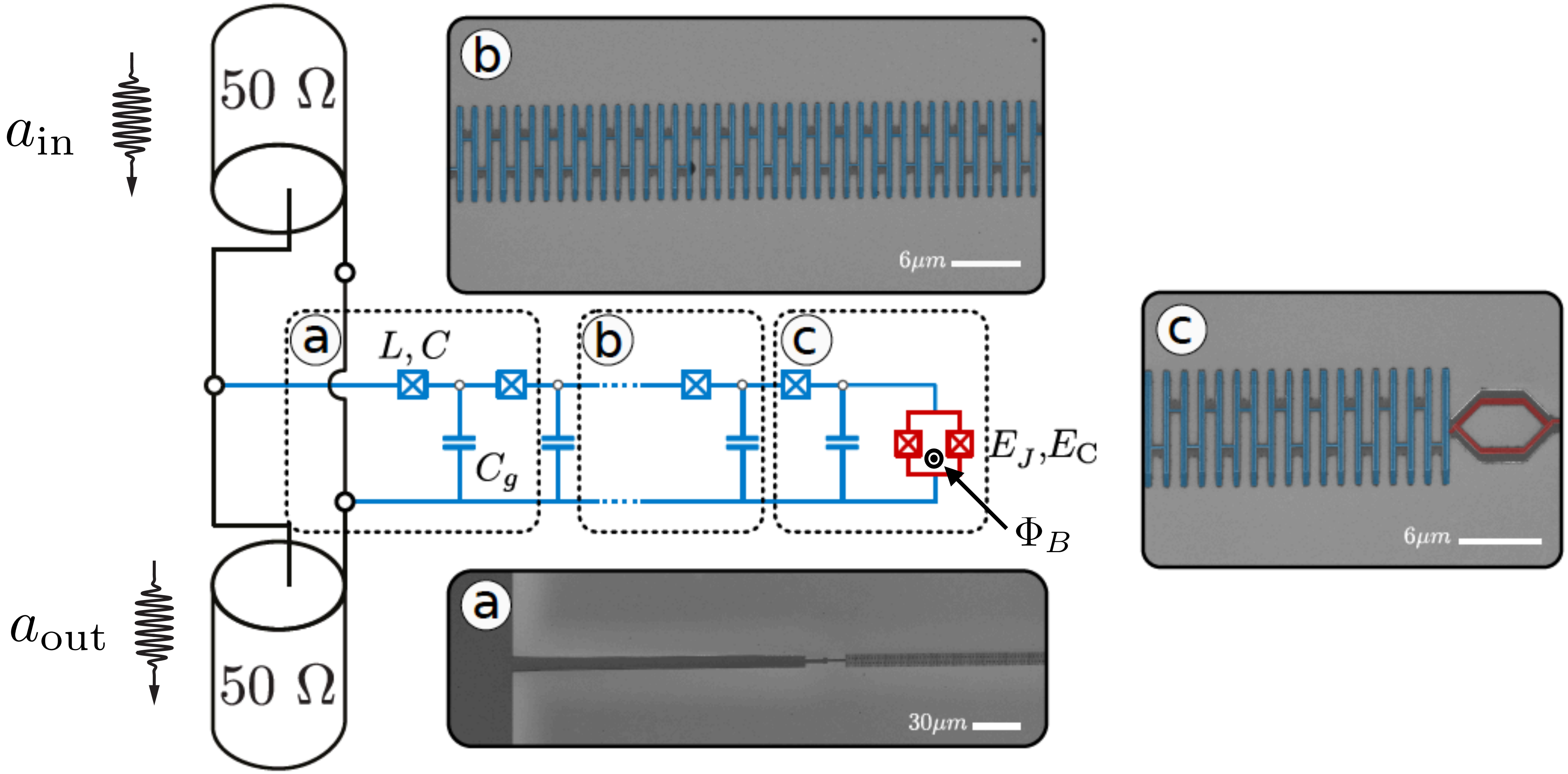
Outline: Finite-frequency properties of the Boundary Sine-Gordon model



L_{J}^* : Renormalisation of the Josephson energy from zero-point fluctuations

$R_{\text{non-linear}}$: Losses from Many-body effects

A small Josephson junction in a high impedance environment



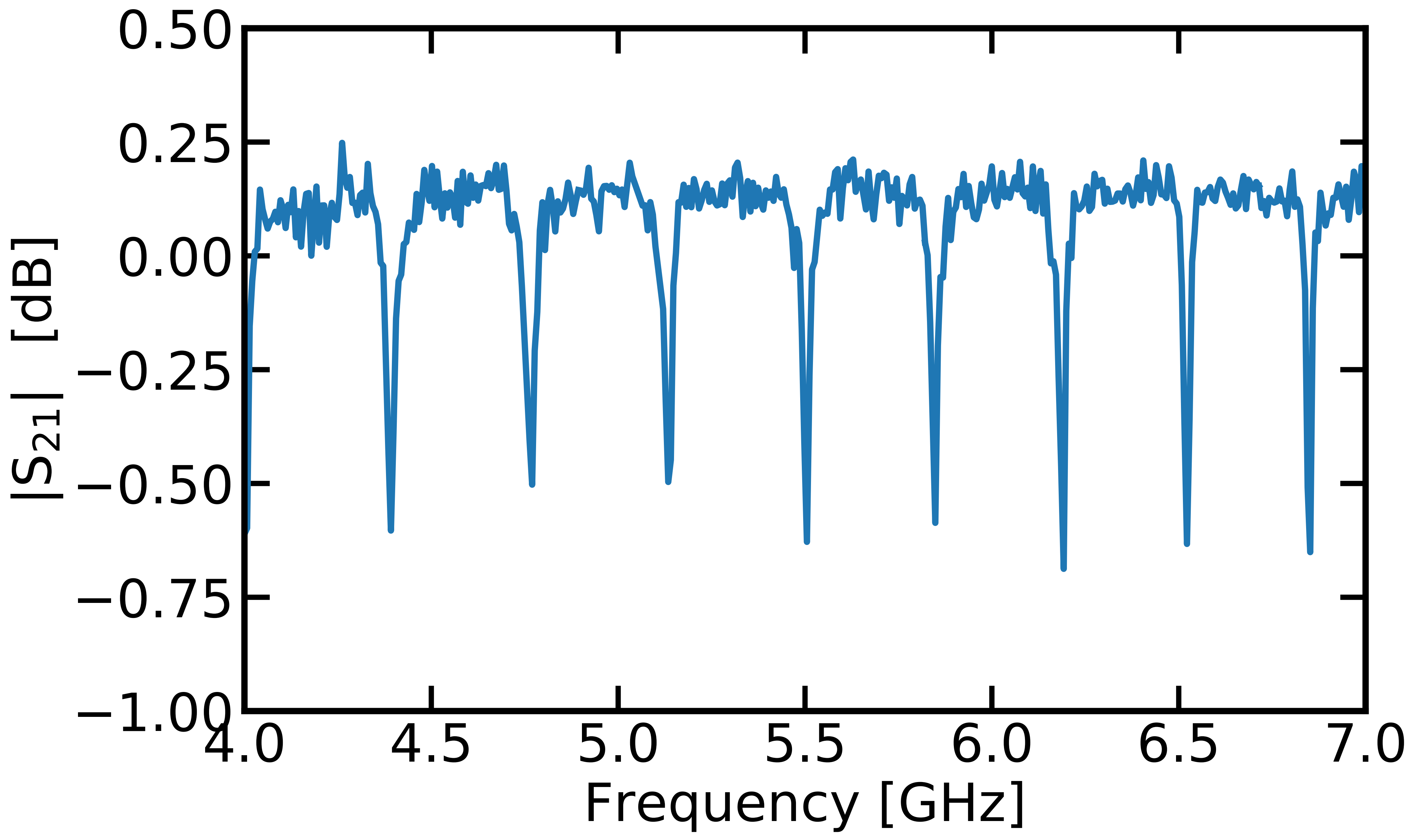
small junction (non-linear): $Z_{\rm J} \simeq 2k\Omega$

$$E_{\rm J}(\Phi_B) = E_{\rm J}(0)\,|\cos(\pi\Phi_B/\Phi_q)|$$

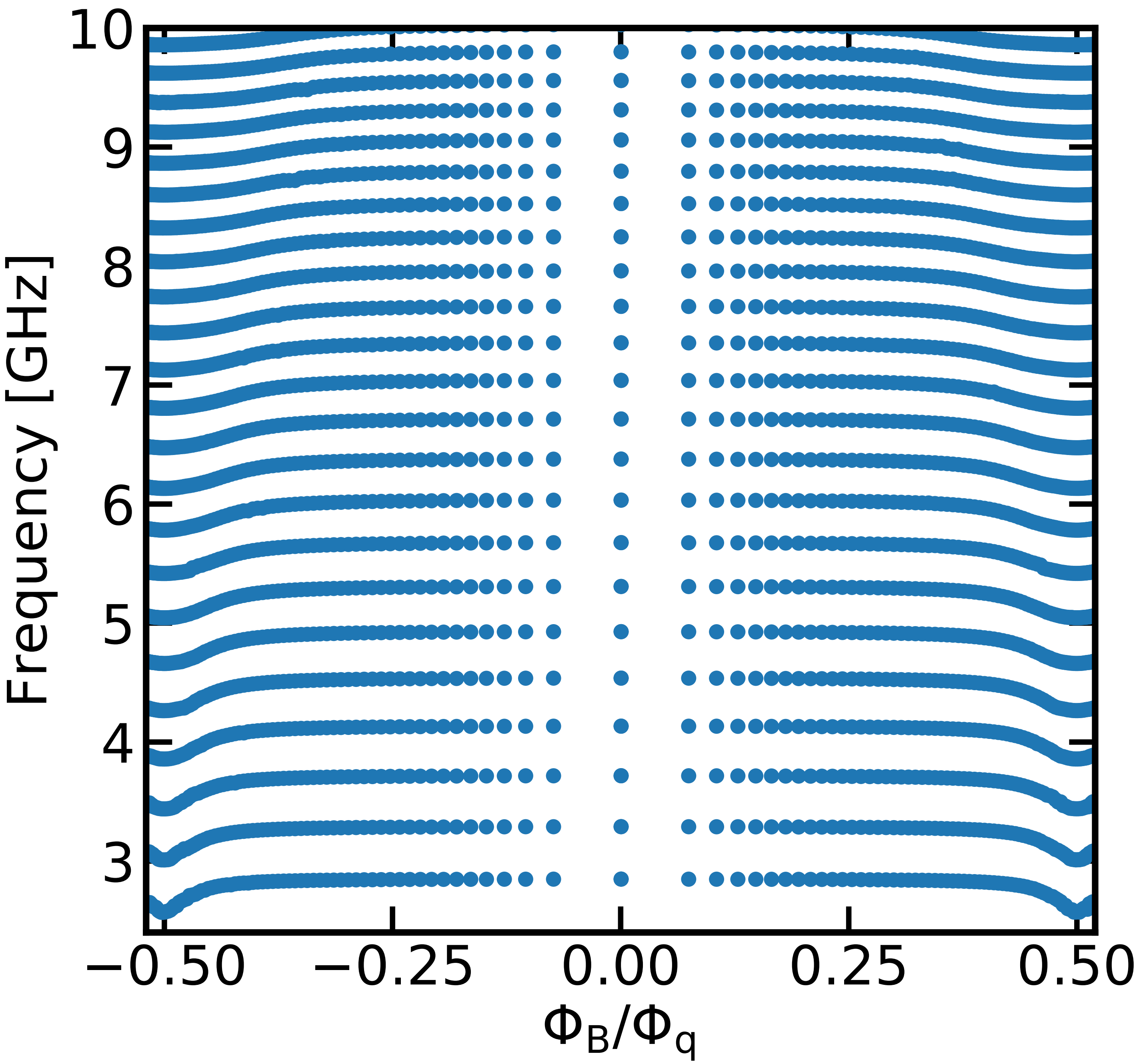
chain junctions (linear): $Z_{\rm J} \simeq 10\Omega$

$$Z_c = 1.9\ \mathrm{k}\Omega$$

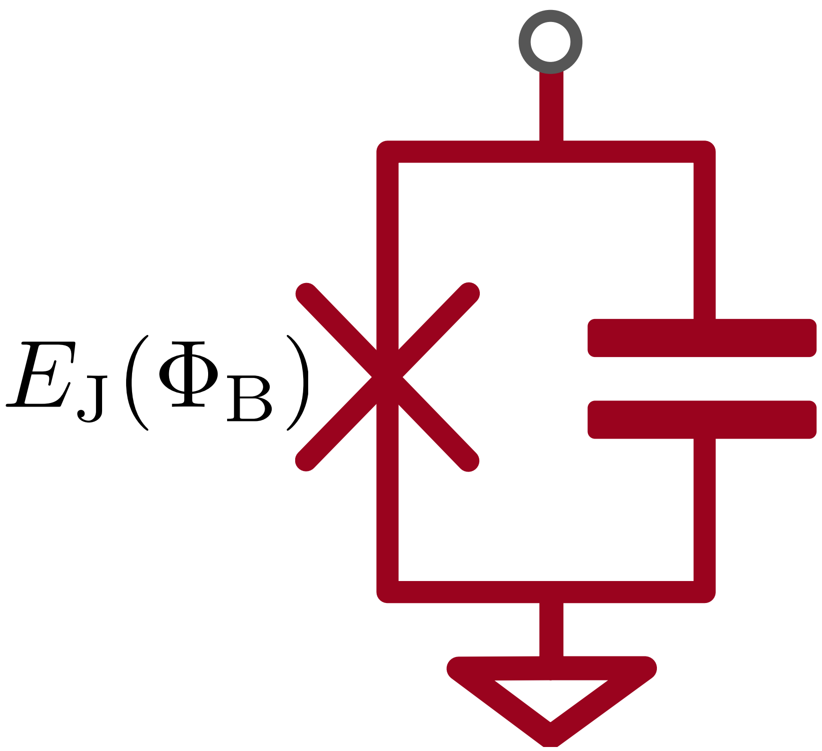
Transmission measurement



Transmission measurement: flux dependence

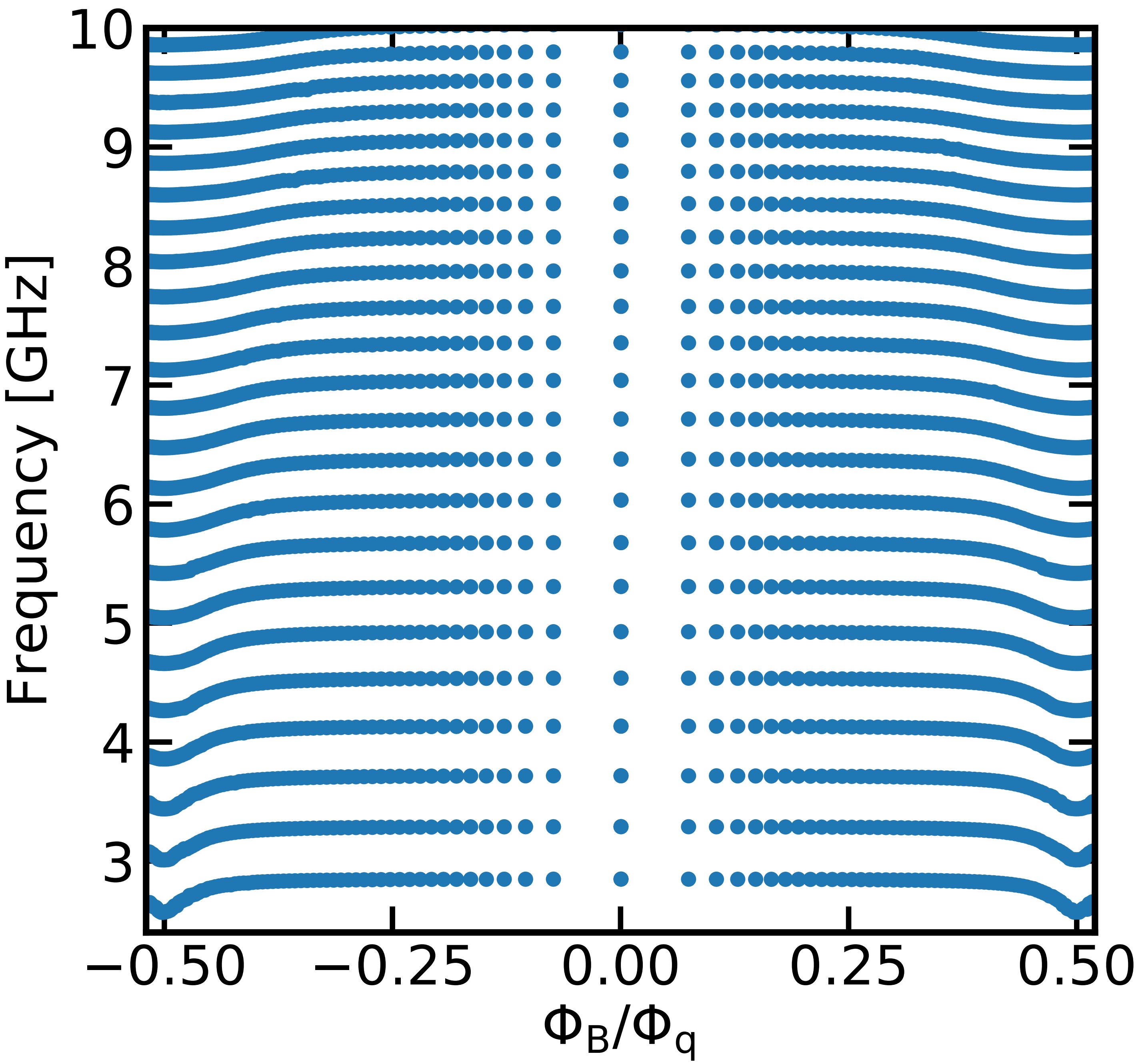


Tunable nonlinearity

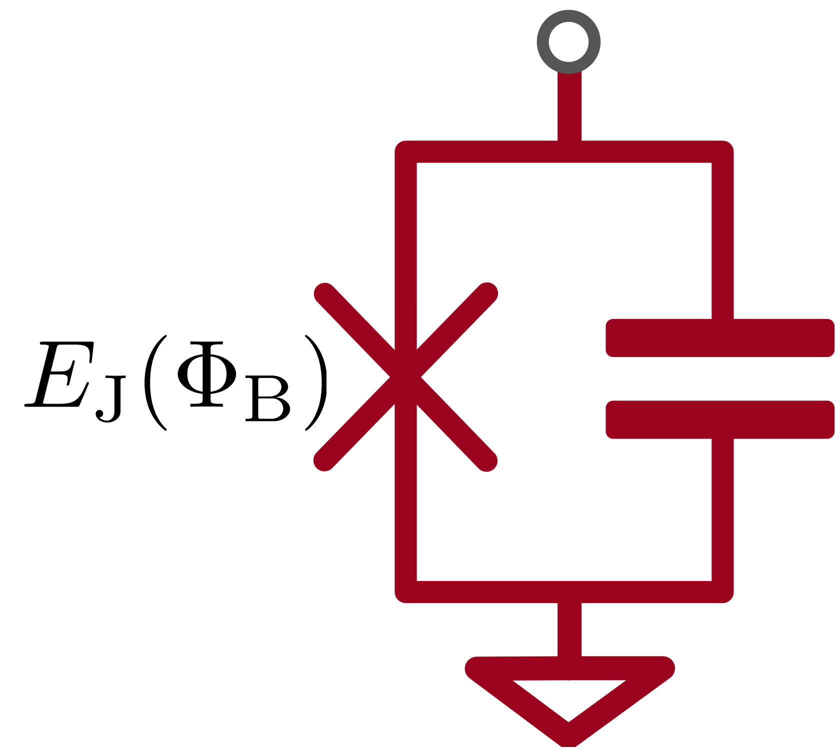


$$E_J(\Phi_B) = E_J(0)\,|\cos(\pi\Phi_B/\Phi_q)|$$

Transmission measurement: flux dependence



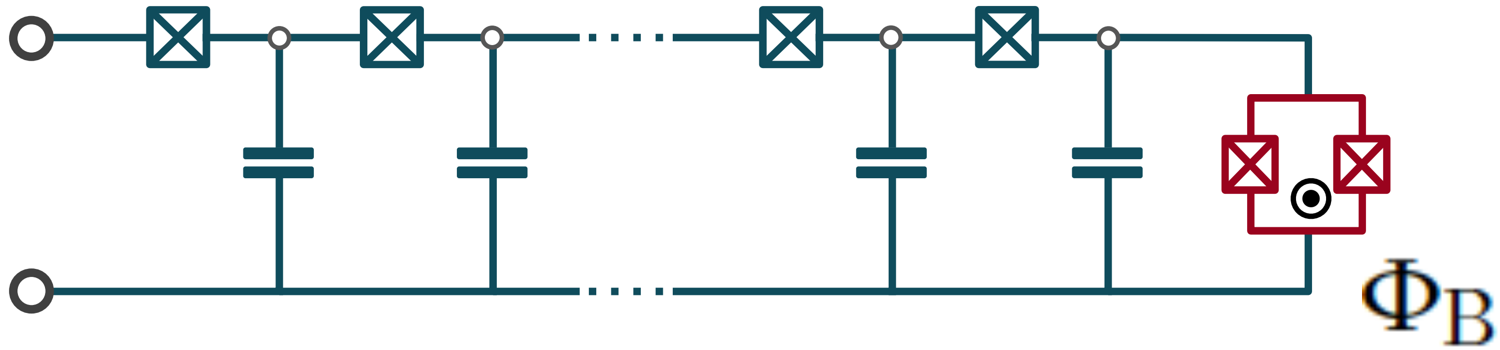
Tunable nonlinearity

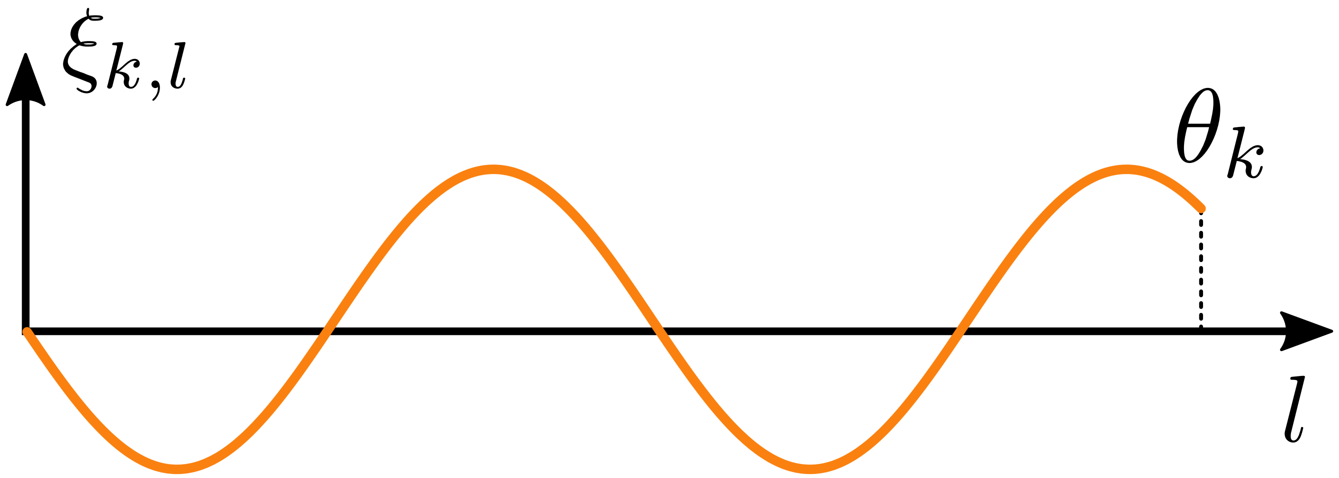


$$E_J(\Phi_B) = E_J(0)\,|\cos(\pi\Phi_B/\Phi_q)|$$

How to infer the small junction properties
(frequency, line-width..) ?

Idea: relative phase shift

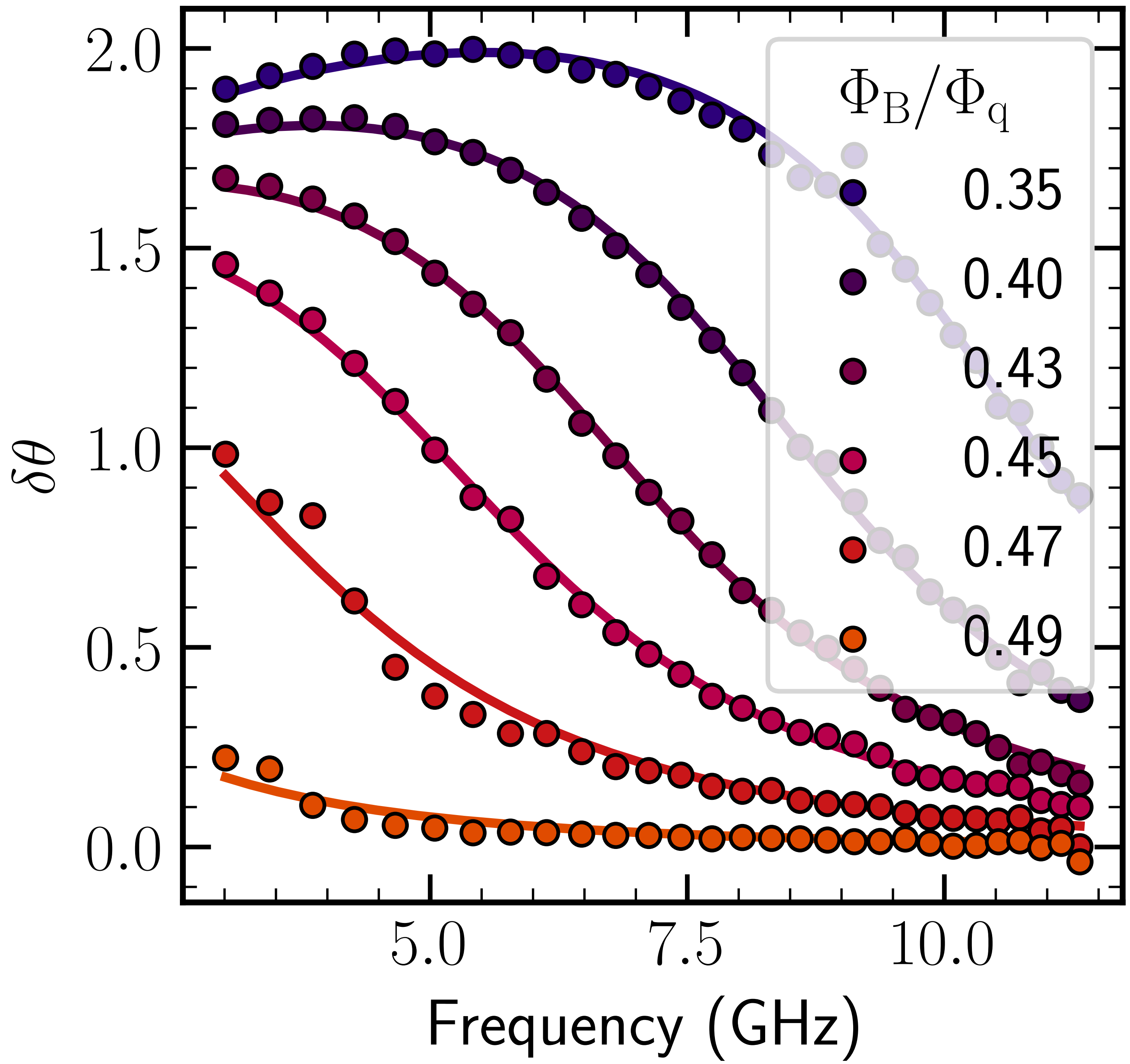




$$\xi_{k,l} = A_k \cos(kl + \boxed{\theta_k})$$

DeWitt (1956)

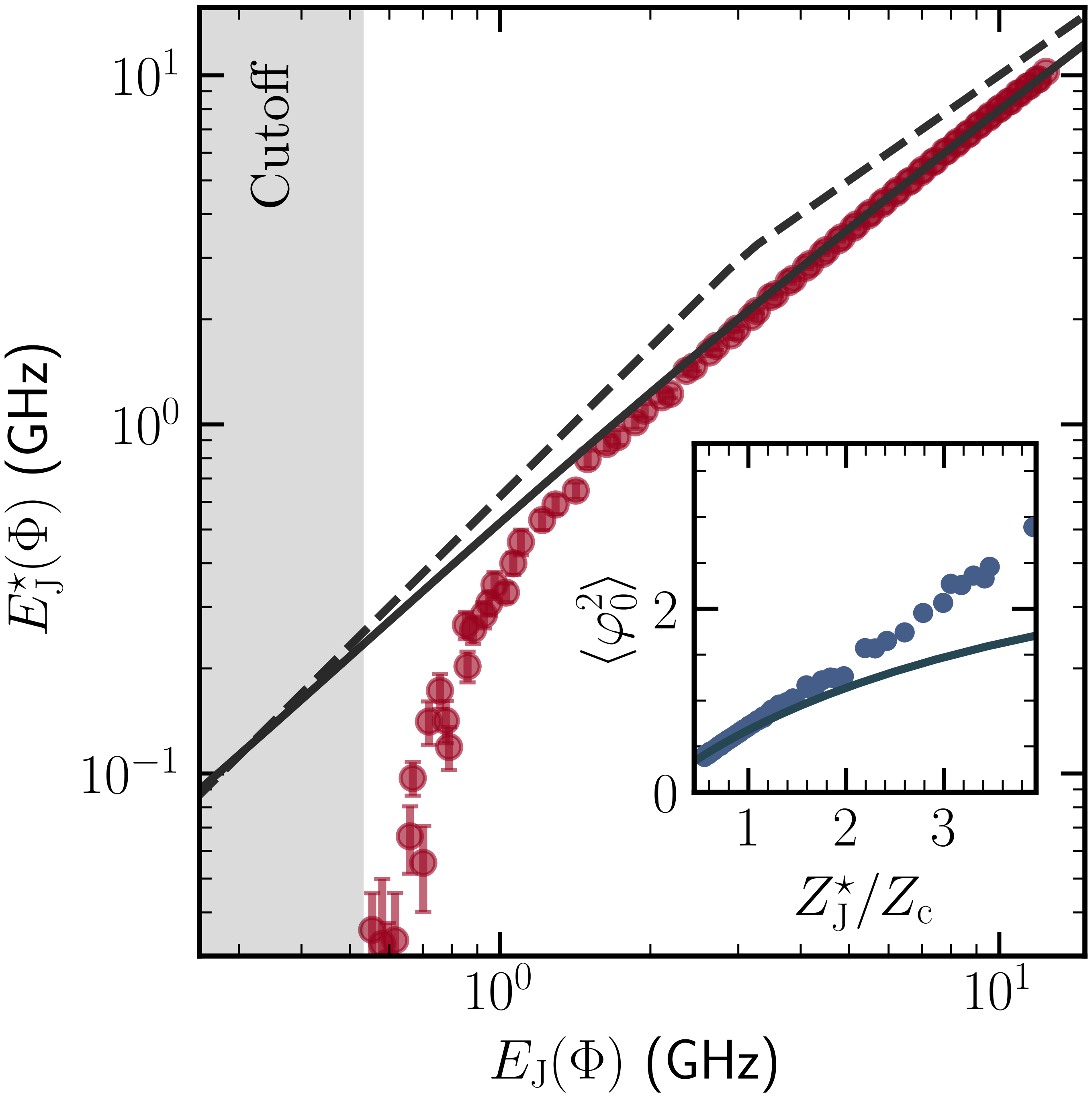
Phase Shift



$$\delta\theta(\omega_k) = \theta_k(E_J^\star) - \theta_k(E_J^\star \simeq 0)$$

→ We can infer $E_J^\star$ for each Φ_B

Renormalisation of the Josephson energy from zero-point fluctuations



Self-consistent harmonic approximation

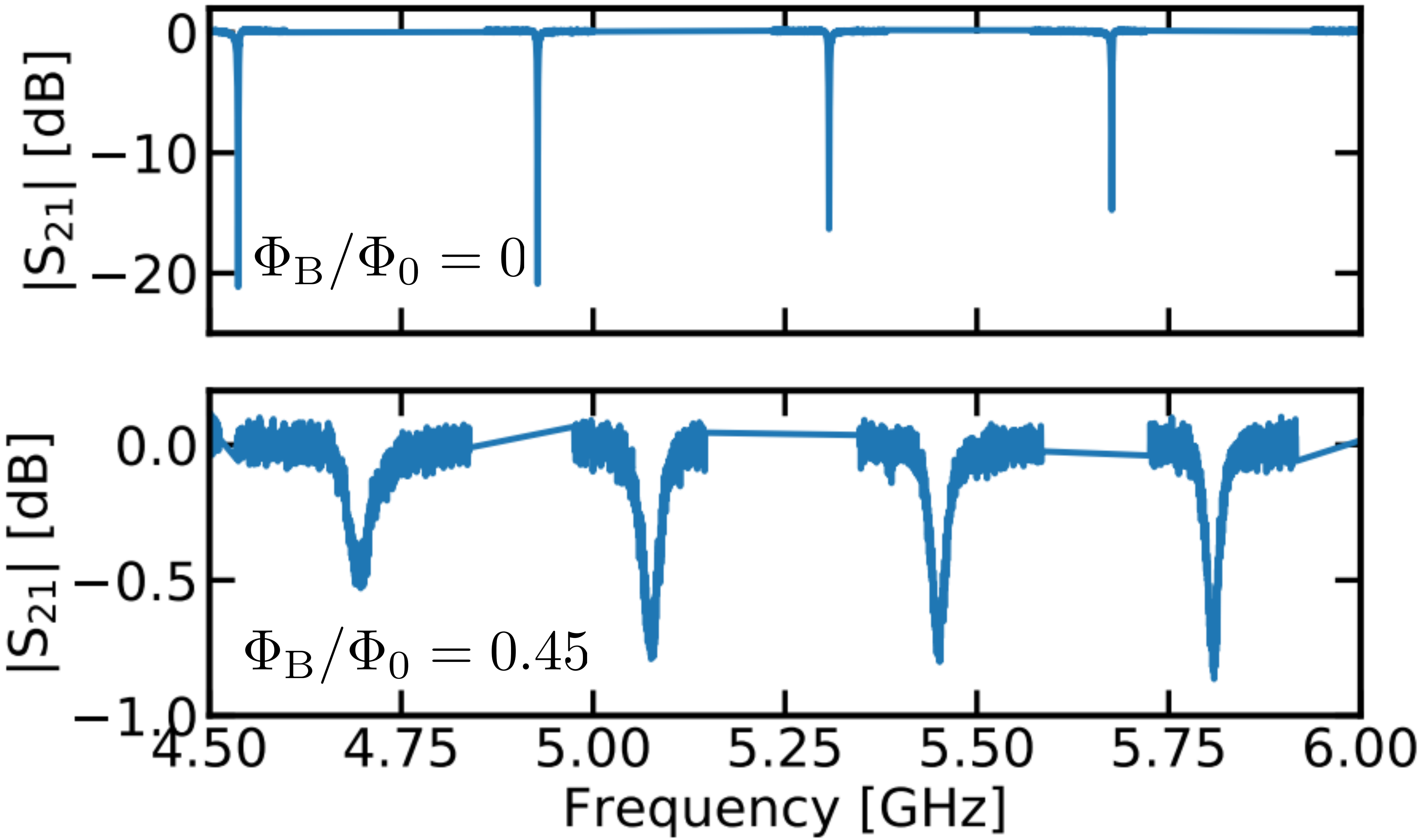
$$E_J^\star = E_J e^{-\langle \phi(E_J\star)^2 \rangle/2}$$

Scaling limit

$$E_J^\star = \mathrm{Min}\left(E_J, E_J\left[\frac{(2\pi\alpha)^2 E_J}{2E_c}\right]^{\frac{\alpha}{1-\alpha}}\right)$$

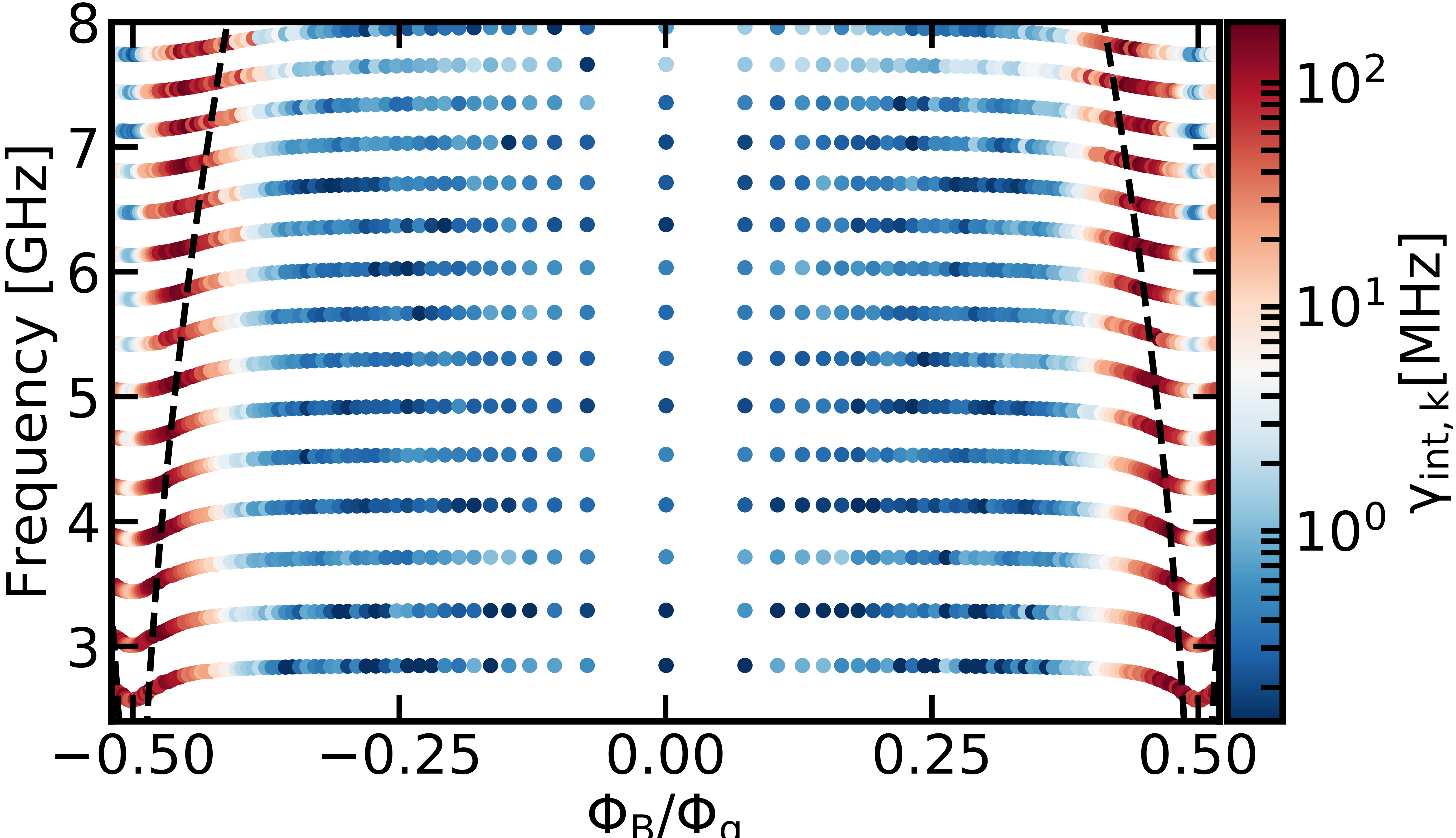
with $\alpha = Z_c/R_q$

Experimental Observation



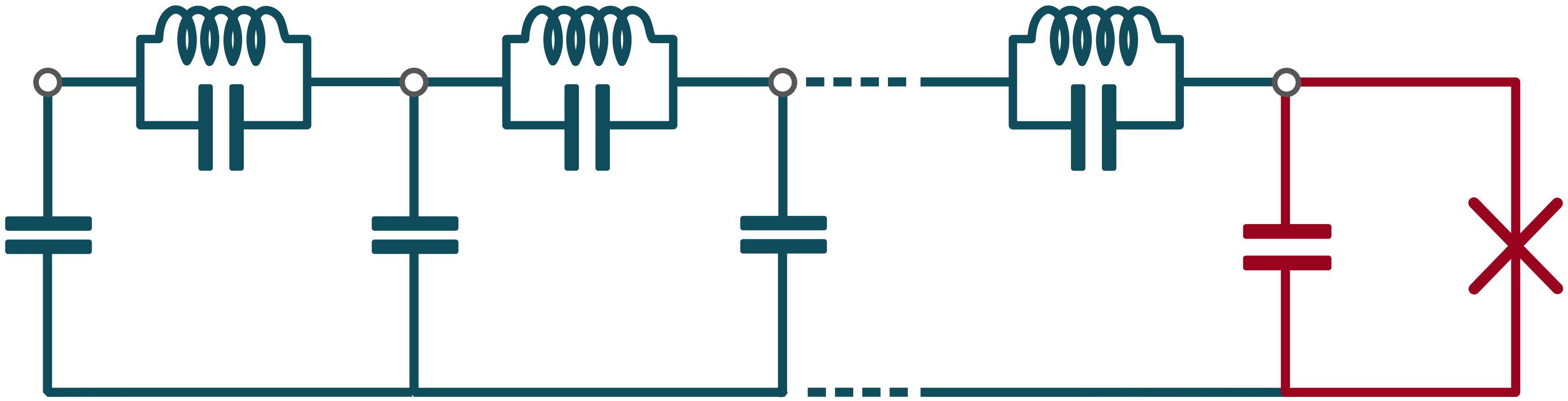
Modes quality factors depend strongly on the magnetic flux threading the small junction

Experimental Observation



f_J^* --------

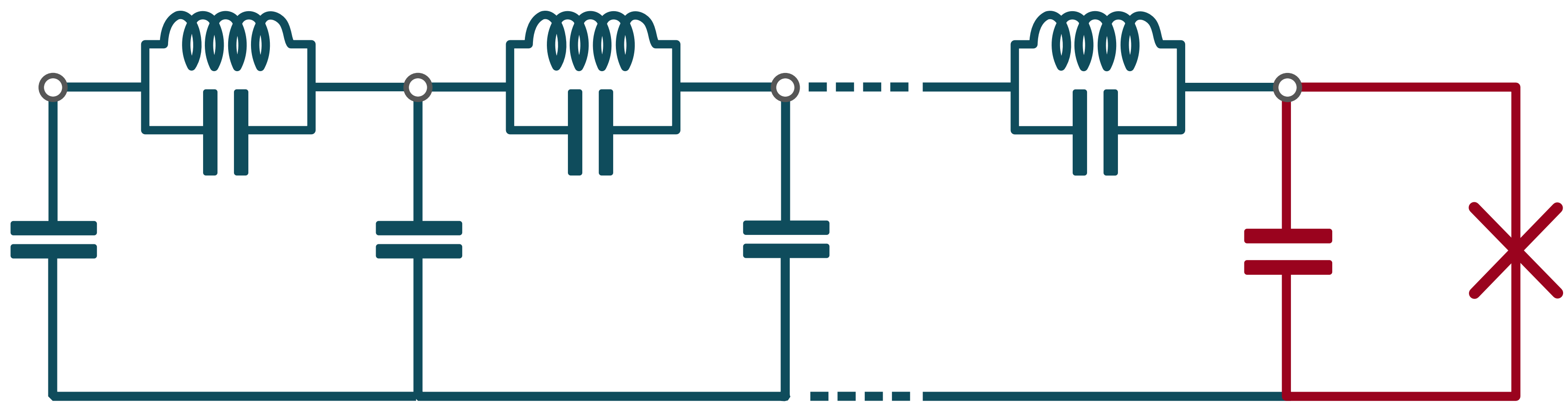
Losses from Many-body effect



$$\hat{H} = \sum_{k=0}^{N} \hbar\omega_k \hat{a}_k^\dagger \hat{a} - \left(E_\mathrm{J} \cos(\hat{\varphi}_0) + \frac{E_\mathrm{J}}{2} \hat{\varphi}_0^2 \right)$$

$$\hat{\varphi}_0 = \sum_k \xi_k (\hat{a}_k^\dagger + \hat{a}_k)$$

Losses from Many-body effect

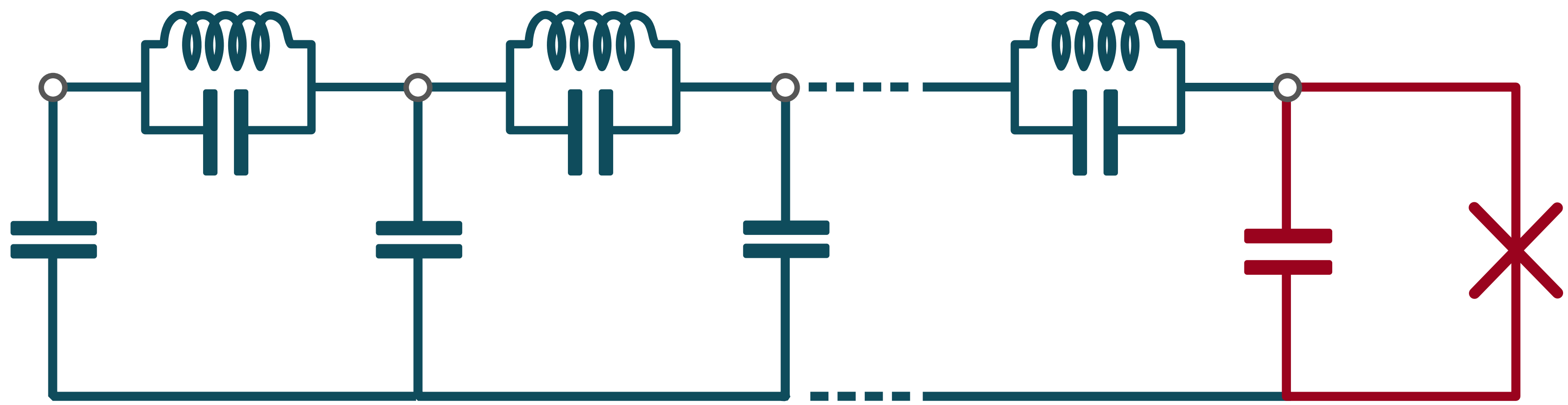


$$\hat{H} = \sum_{k=0}^{N} \hbar\omega_k \hat{a}_k^\dagger \hat{a} - \left(E_\mathrm{J} \cos(\hat{\varphi}_0) + \frac{E_\mathrm{J}}{2}\hat{\varphi}_0^2 \right)$$

$$\hat{\varphi}_0 = \sum_k \xi_k (\hat{a}_k^\dagger + \hat{a}_k)$$

$$\longrightarrow \hat{V} = \cos\left(\sum_k \xi_k (\hat{a}_k^\dagger + \hat{a}_k) \right)$$

Losses from Many-body effect



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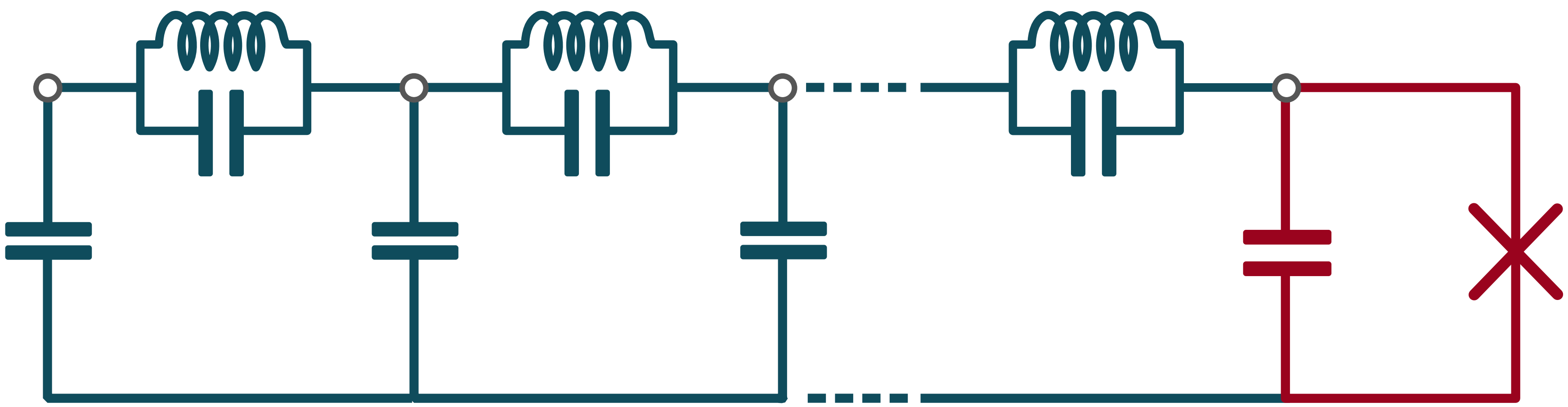
$$\longrightarrow \hat{V} = \cos\left(\sum_k \xi_k (\hat{a}_k^\dagger + \hat{a}_k) \right)$$

$$\longrightarrow \hat{V}^{1\to 3} \propto \hat{a}_\mathrm{k,in} \hat{a}_\mathrm{k1,out}^\dagger \hat{a}_\mathrm{k2,out}^\dagger \hat{a}_\mathrm{k3,out}^\dagger$$

Losses from Many-body effect



$$\hat{H} = \sum_{k=0}^{N} \hbar\omega_k \hat{a}_k^\dagger \hat{a} - \left(E_\mathrm{J} \cos(\hat{\varphi}_0) + \frac{E_\mathrm{J}}{2}\hat{\varphi}_0^2 \right)$$

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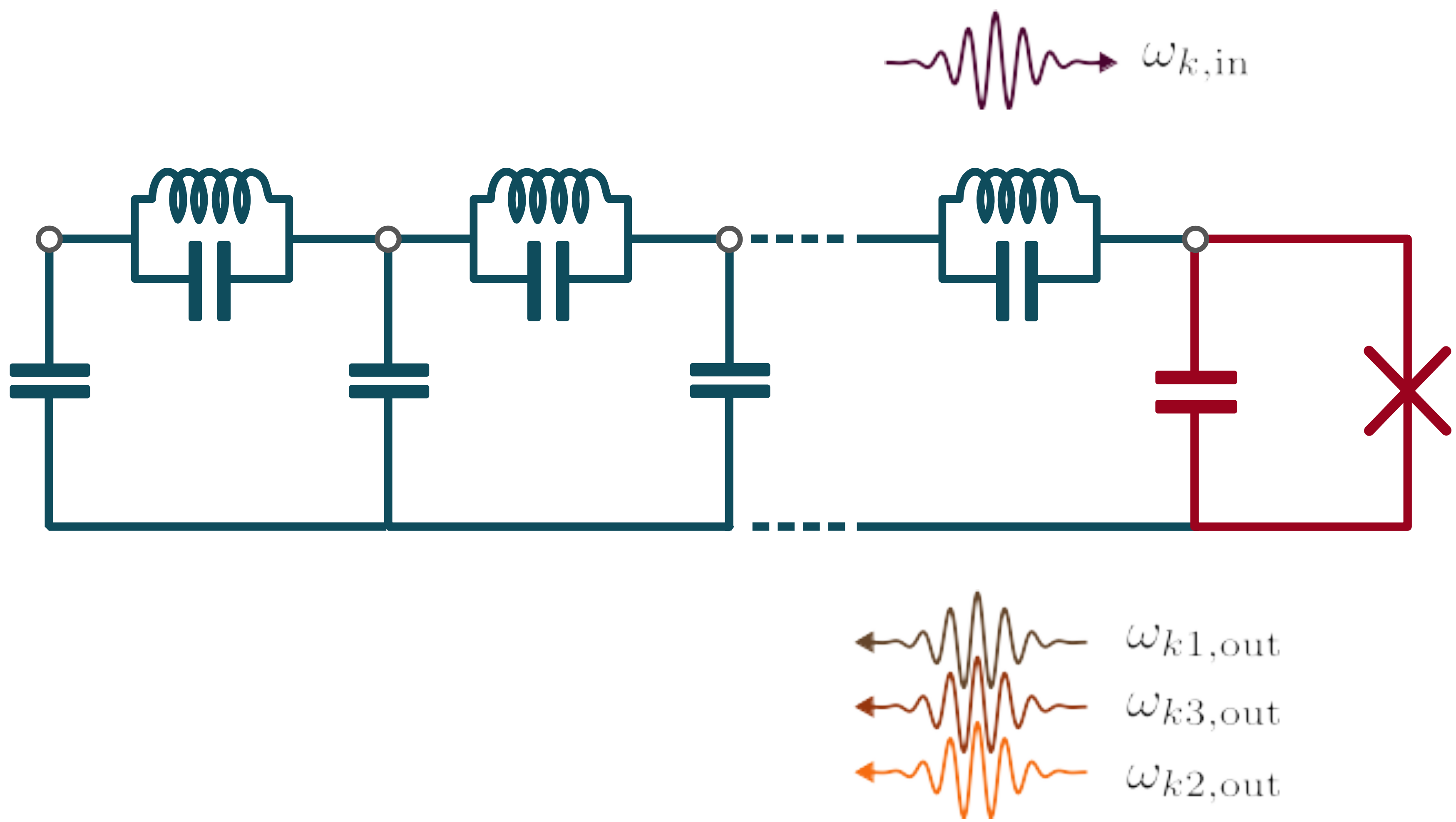
$$\longrightarrow \hat{V}^{1\to 3} \propto \hat{a}_{\mathrm{k,in}} \hat{a}_{\mathrm{k1,out}}^\dagger \hat{a}_{\mathrm{k2,out}}^\dagger \hat{a}_{\mathrm{k3,out}}^\dagger$$

Large if $\langle \phi^2 \rangle$ is large

Losses from Many-body effect



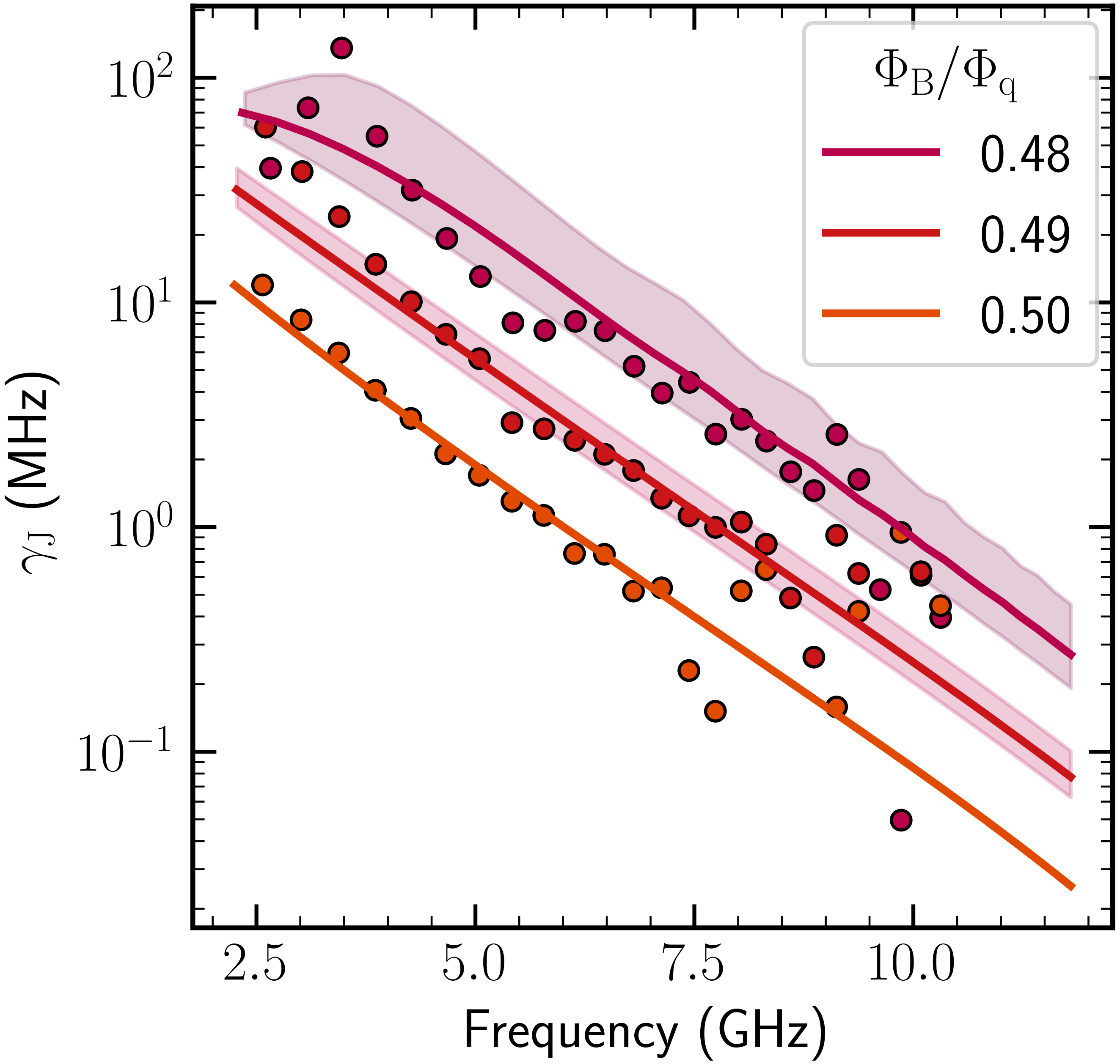
$$\omega_{k,in} = \omega_{k_1,out} + \omega_{k_2,out} + \omega_{k_3,out}$$

Coupling between single-photon states and multi-photons states

→ Interactions induced losses

Losses from Many-body effect

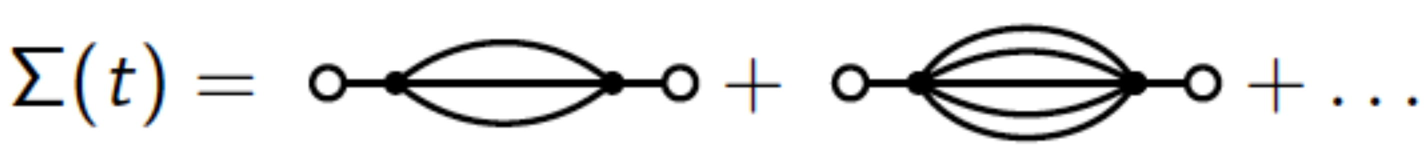
Agreement with theory at small flux (no fit)



Capacitive loss

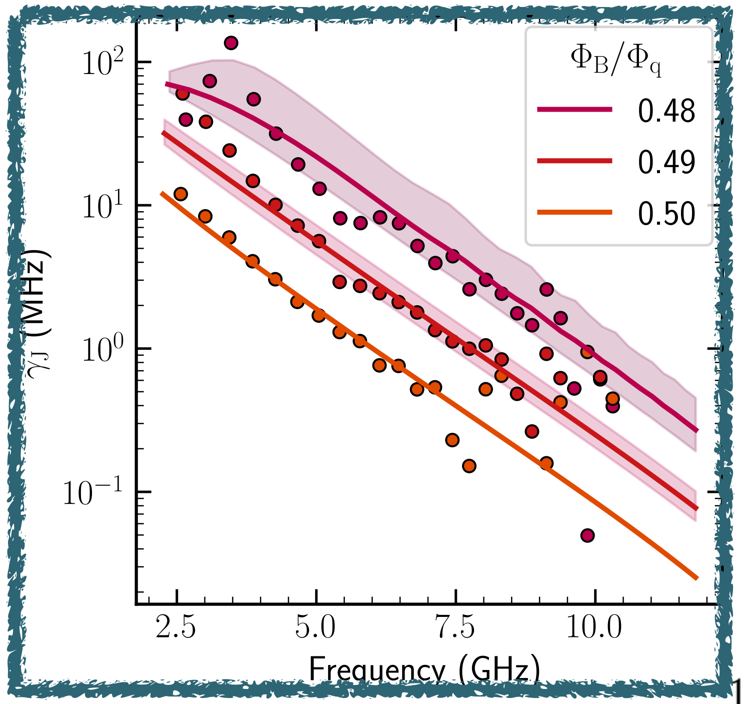
$$\gamma_J = \gamma_{int} - \gamma_C$$

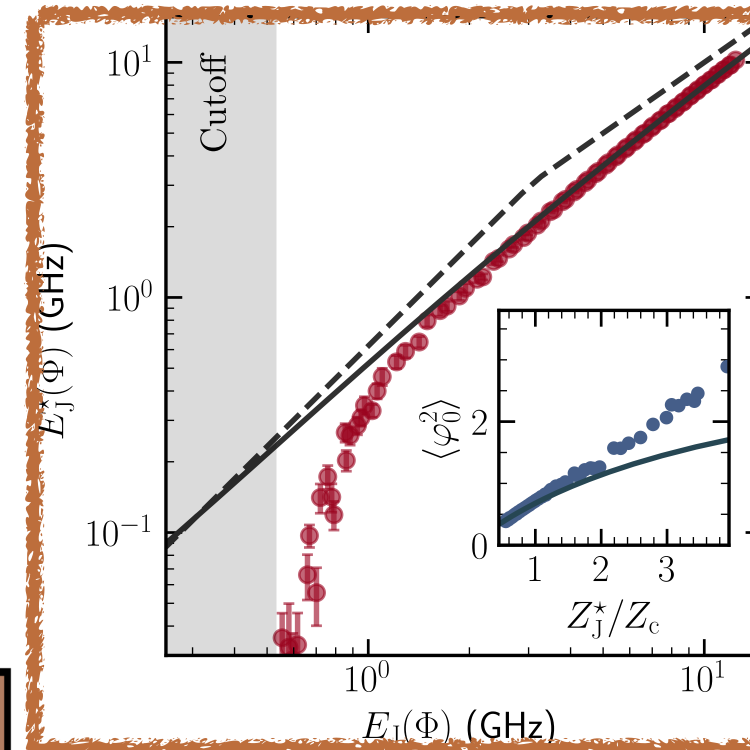
Theory: resummation of Josephson-Feynman diagrams

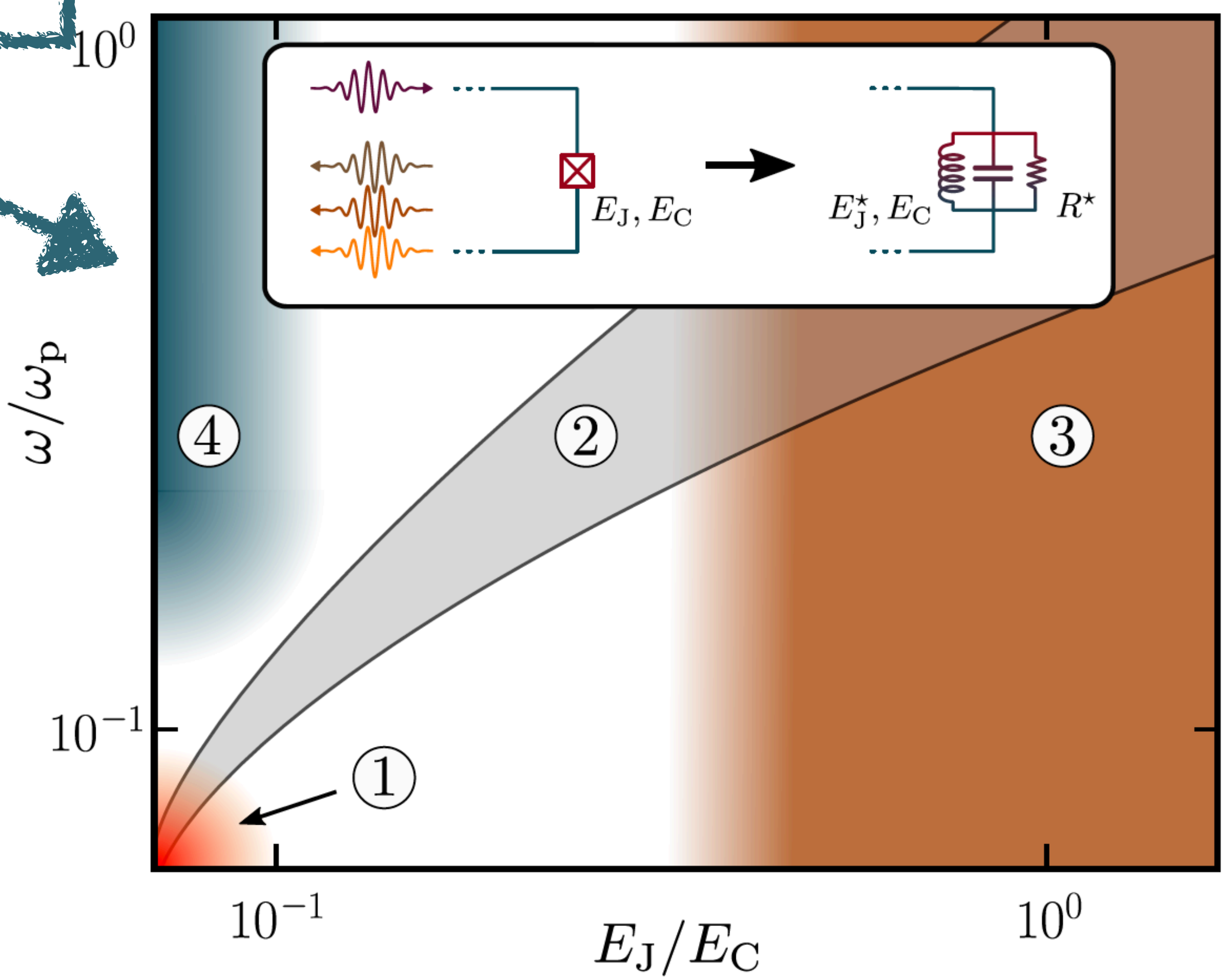


$$= E_J^2 [\sin(G(t)) - G(t)]$$

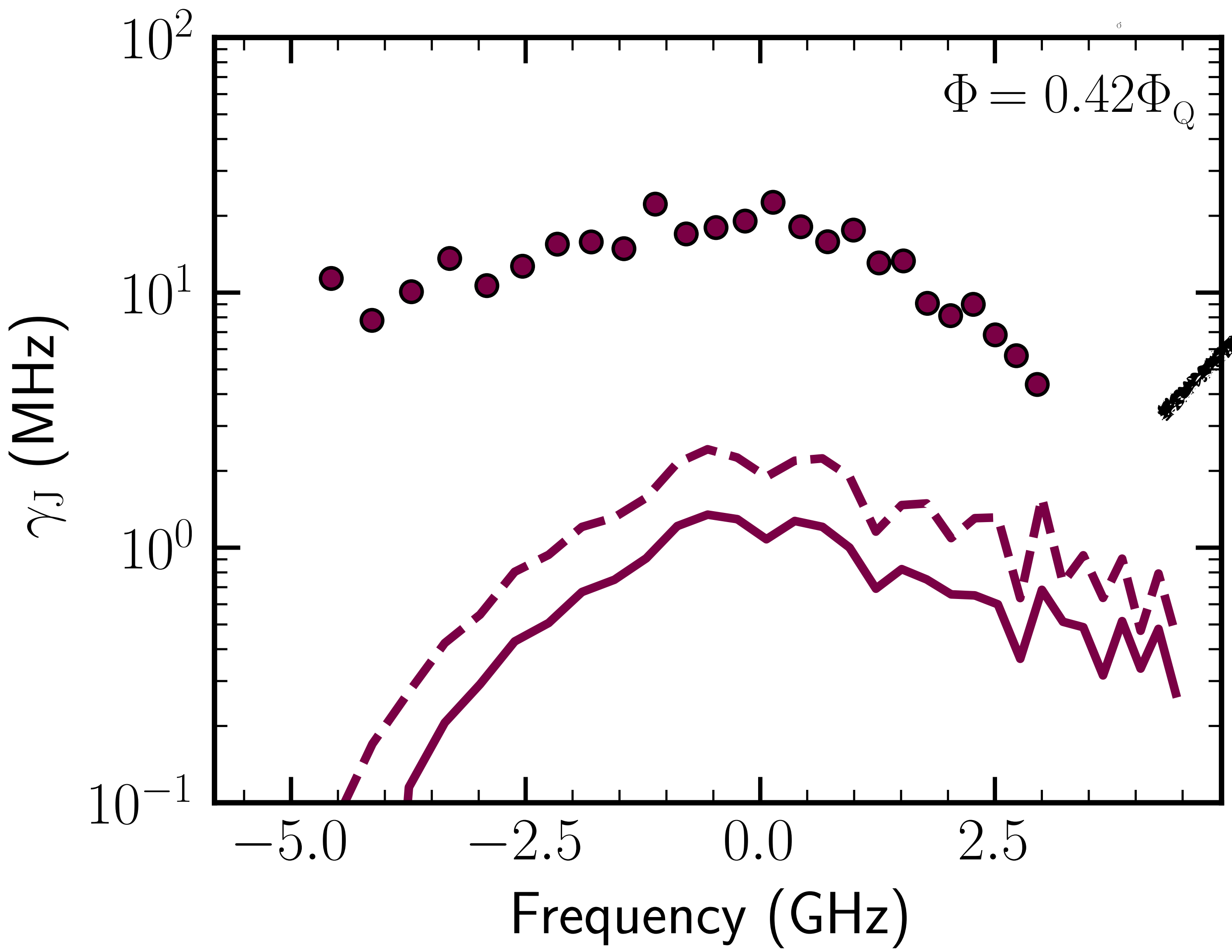
Finite-frequency properties of the Boundary Sine-Gordon model

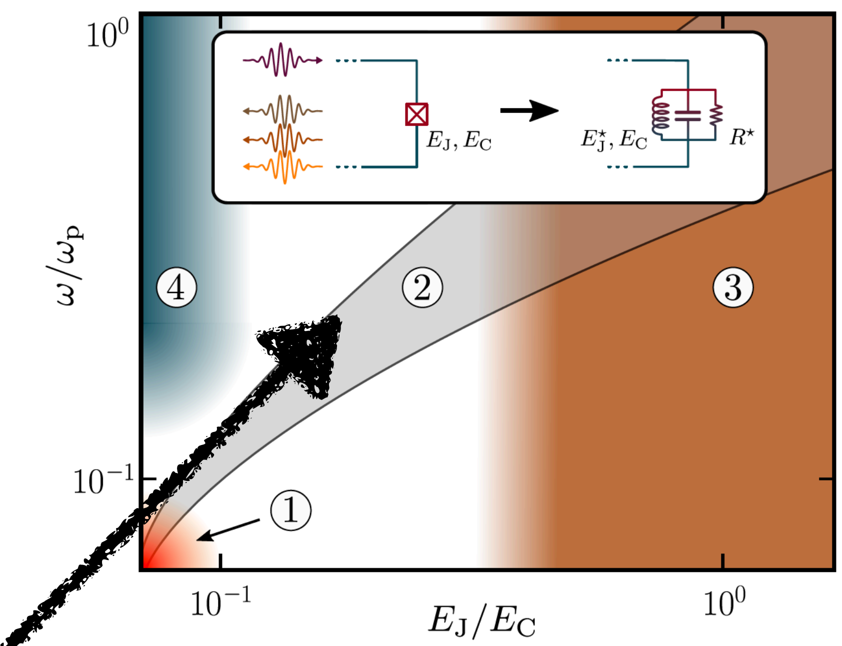






Quantum simulation of the Boundary Sine-Gordon model





Exp data
Third order
Second order

BSG: Conclusion and Perspectives

Dissipation from a lossless environment

Y. Krupko et al.,
Phys. Rev. B (2018)

J. Puertas-Martinez et al., npjQI (2019)
(See also R. Kuzmin et al., npjQI (2019))

Many-body renormalisation and large phase fluctuations

S. Leger et al., Nat. Commun. (2019)

Quantitative understanding using a variational ansatz

K. Kaur, T. Sepulcre et al., Phys. Rev. Lett. (2021)

Circuit QED implementation of the non-perturbative boundary sine-Gordon model

S. Leger, T. Sepulcre et al., arxiv:2208.03053

What about Quantum Phase Slips?

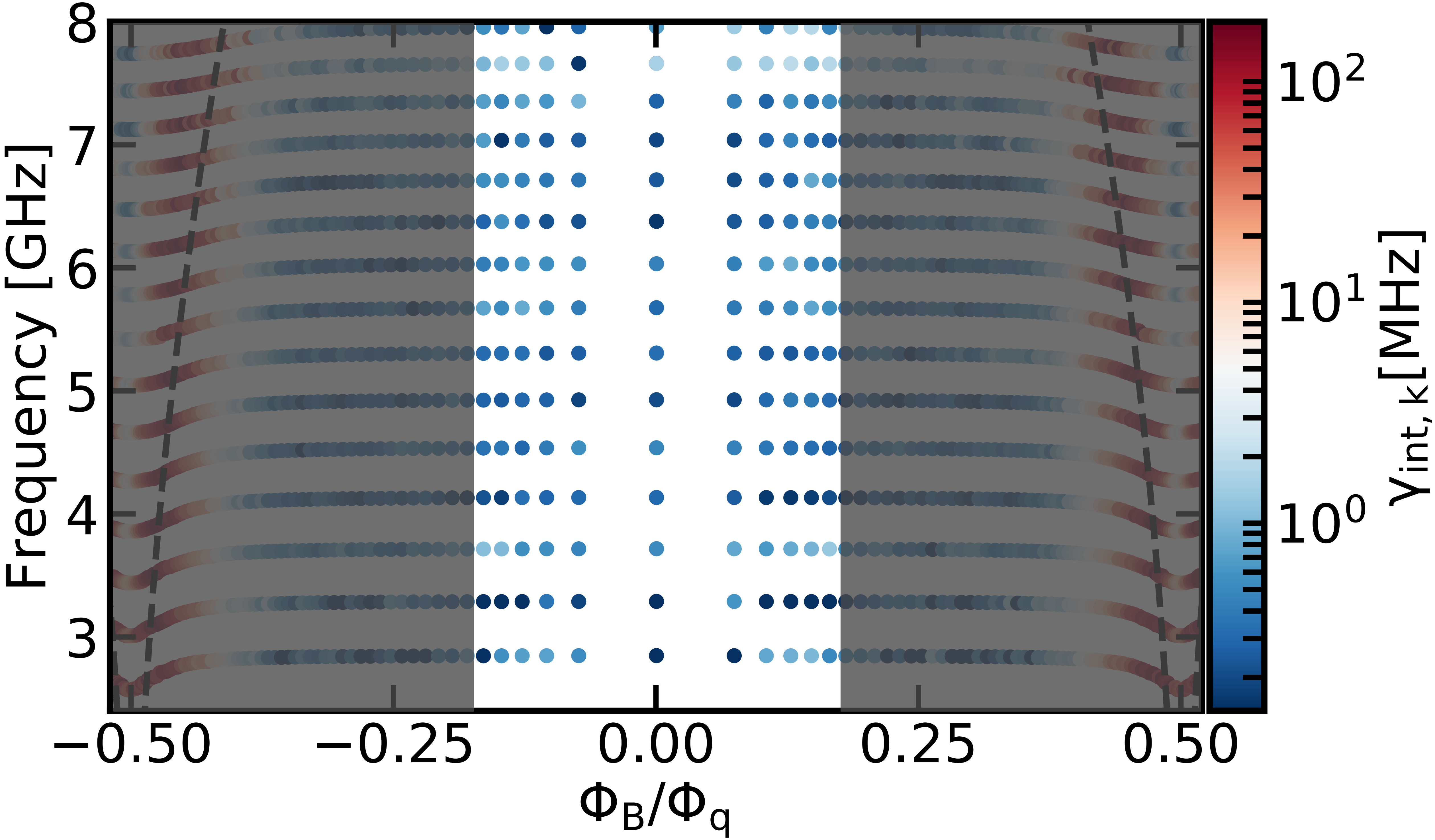
Houzet & Glazman, PRL (2020), Burshtein et al., PRL (2021) Kuzmin et al., PRL (2021)

Signature of quantum criticality?

Quantum metrology

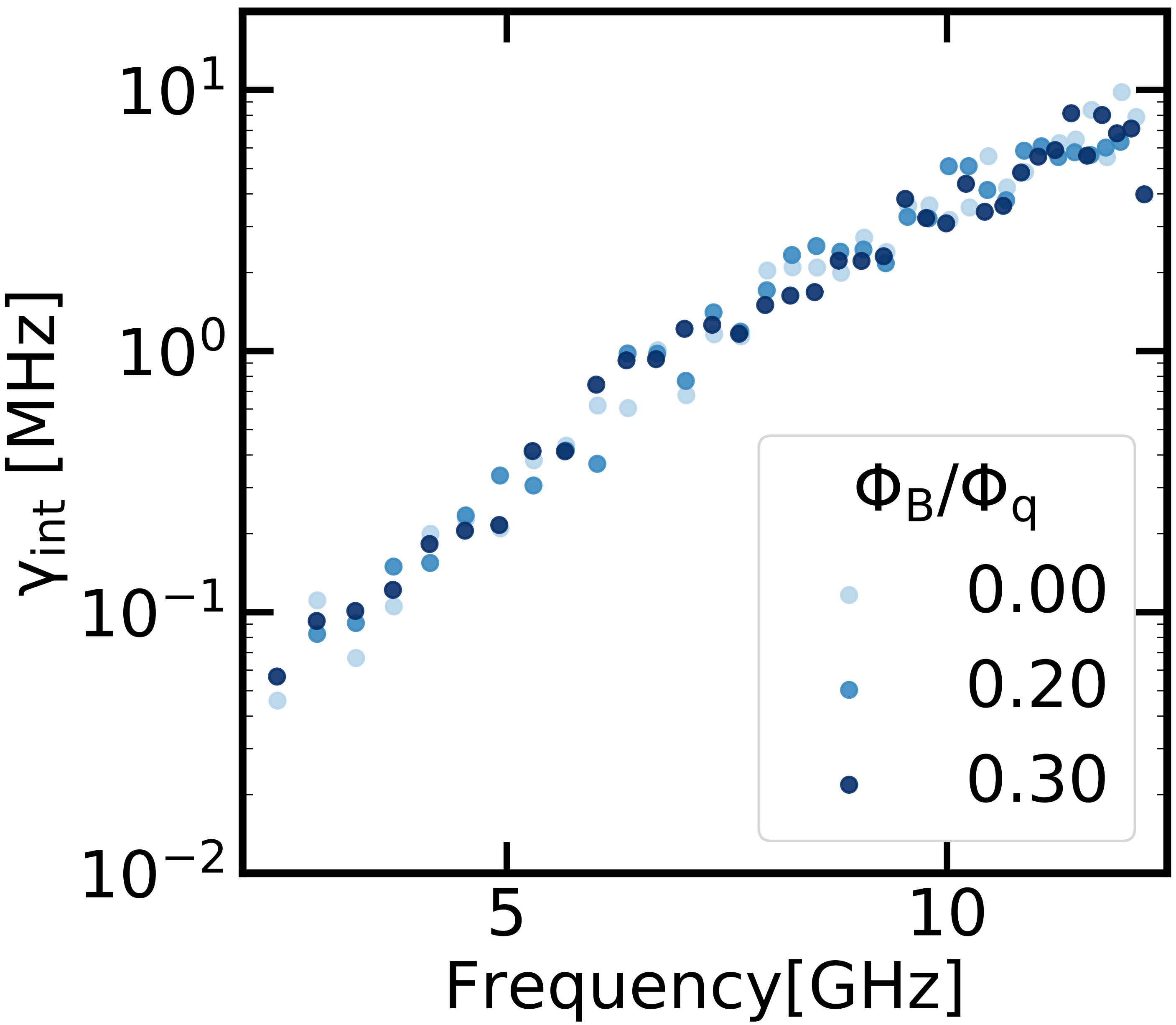

D. Fraudet
S. Leger

Experimental Observation

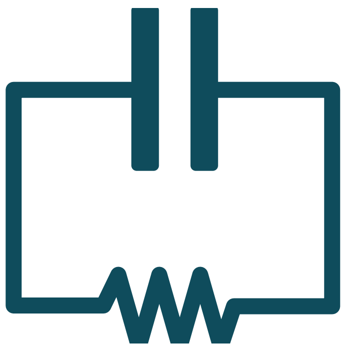


f_J^* --------

Dielectric loss



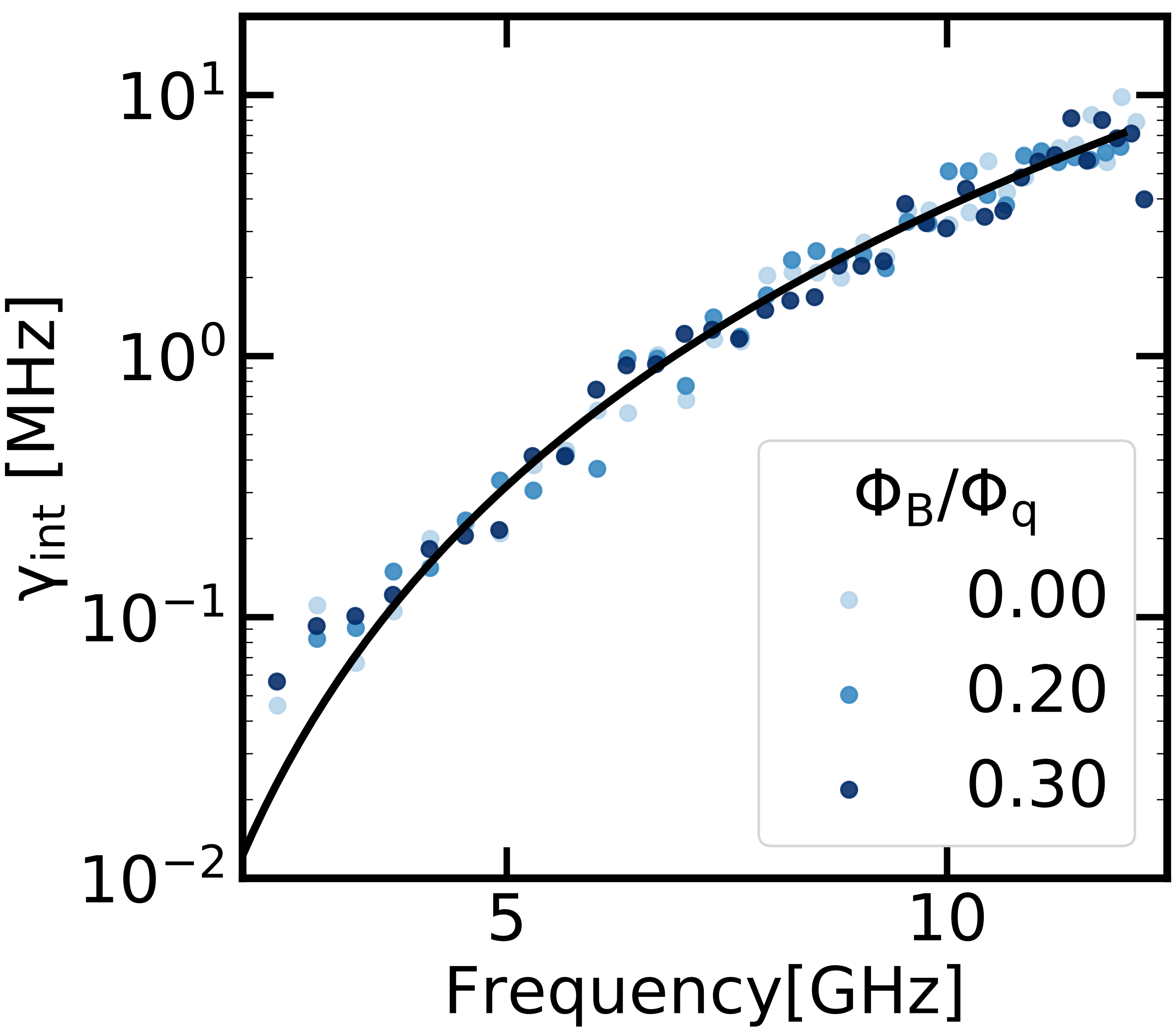
Capacitive loss in the chain



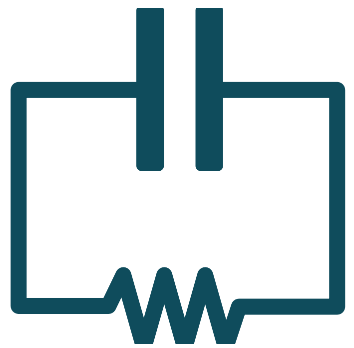
$$Y(\omega) = i\omega \mathrm{Re}(C) + \omega \mathrm{Im}(C)$$

$$\tan\delta = \frac{\mathrm{Im}(C)}{\mathrm{Re}(C)}$$

Dielectric loss



Capacitive loss in the chain



$$Y(\omega) = i\omega \mathrm{Re}(C) + \omega \mathrm{Im}(C)$$

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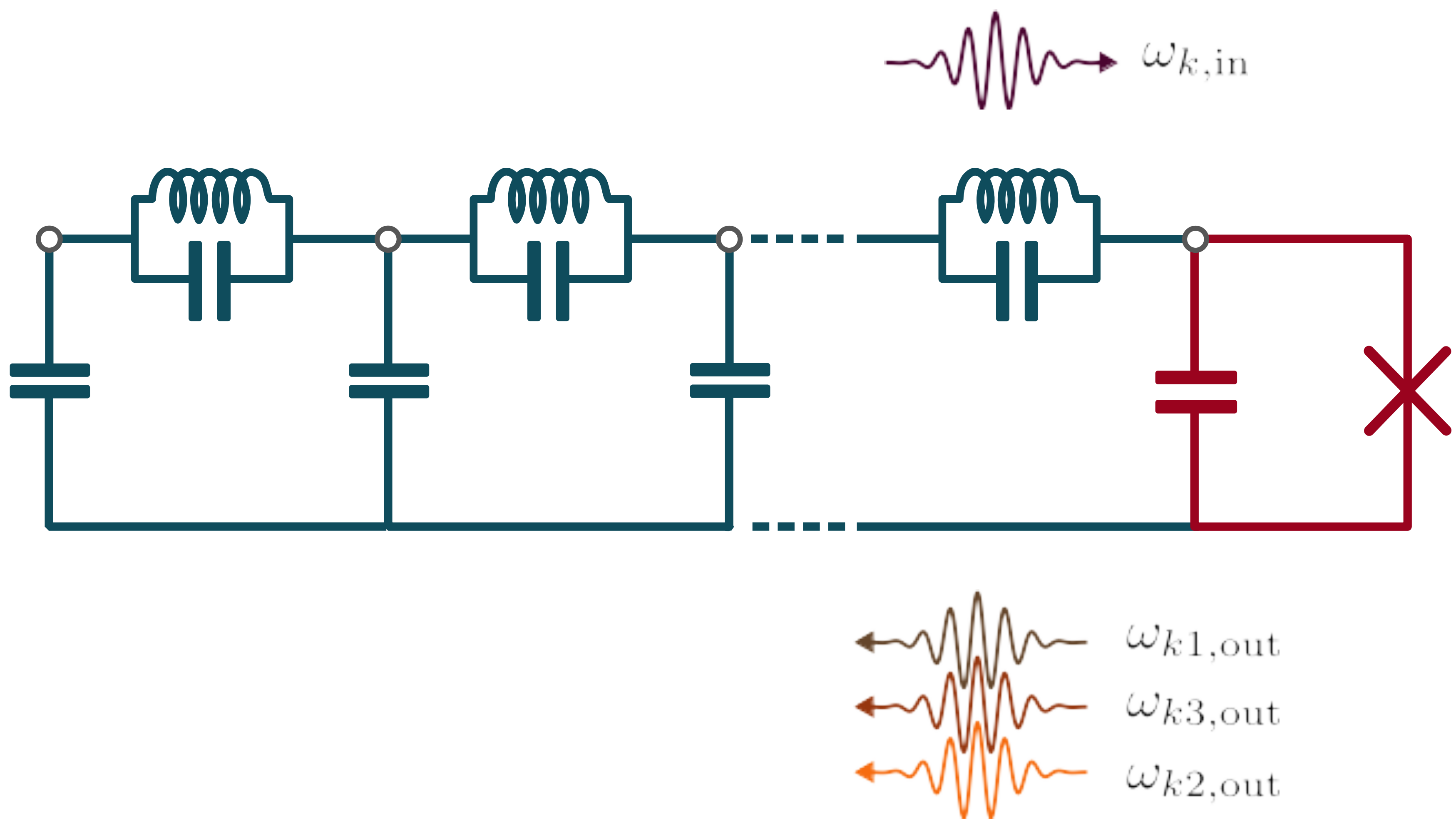
Losses from Many-body effect



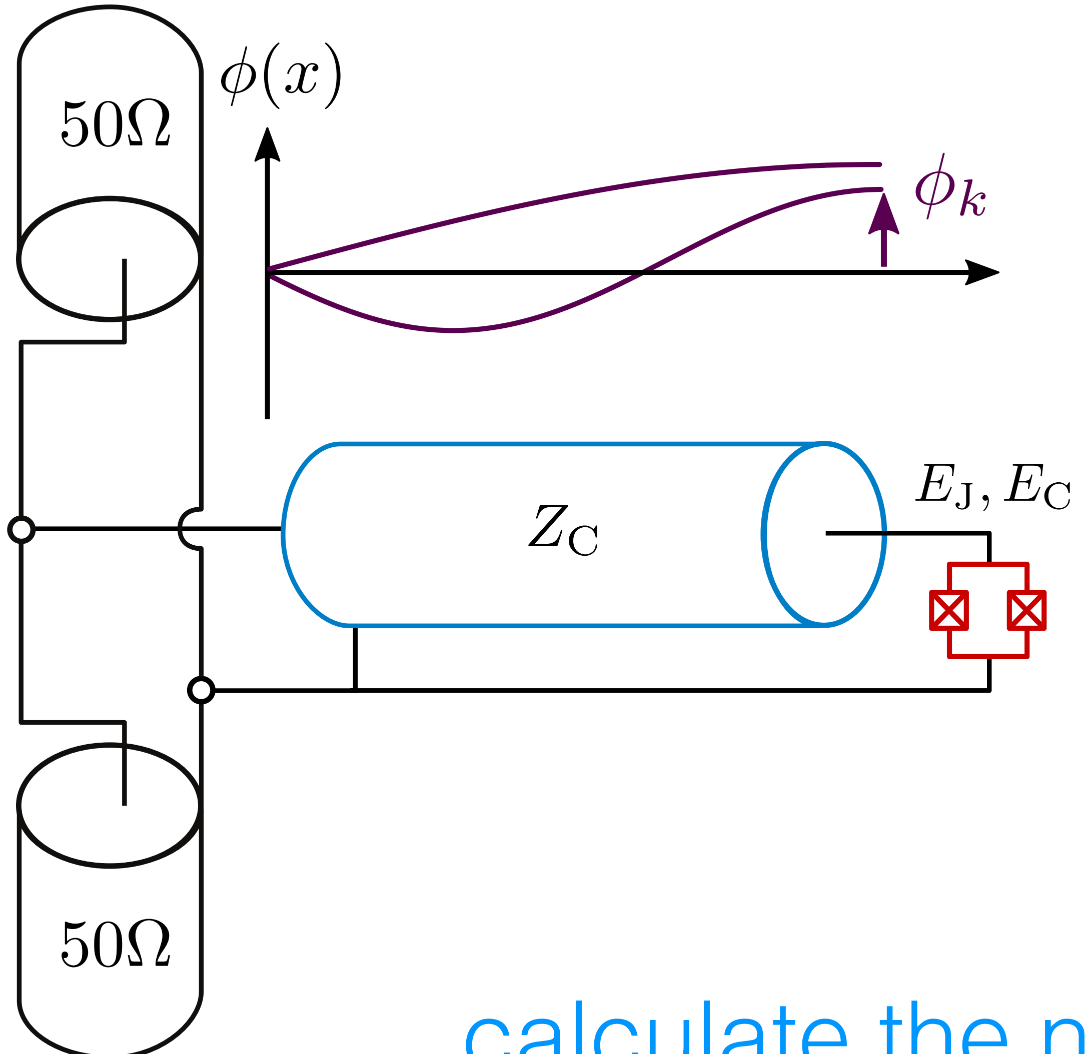
Linear response theory

$$\frac{\gamma_{\mathrm{J}}}{\Delta\omega_{\mathrm{FSR}}} = \frac{2\hbar}{\pi}\Sigma''(\omega_k)\chi''_{\phi_0}(\omega_k)$$

Self-energy

Response function

Modelling: Self-Consistent Harmonic Approximation



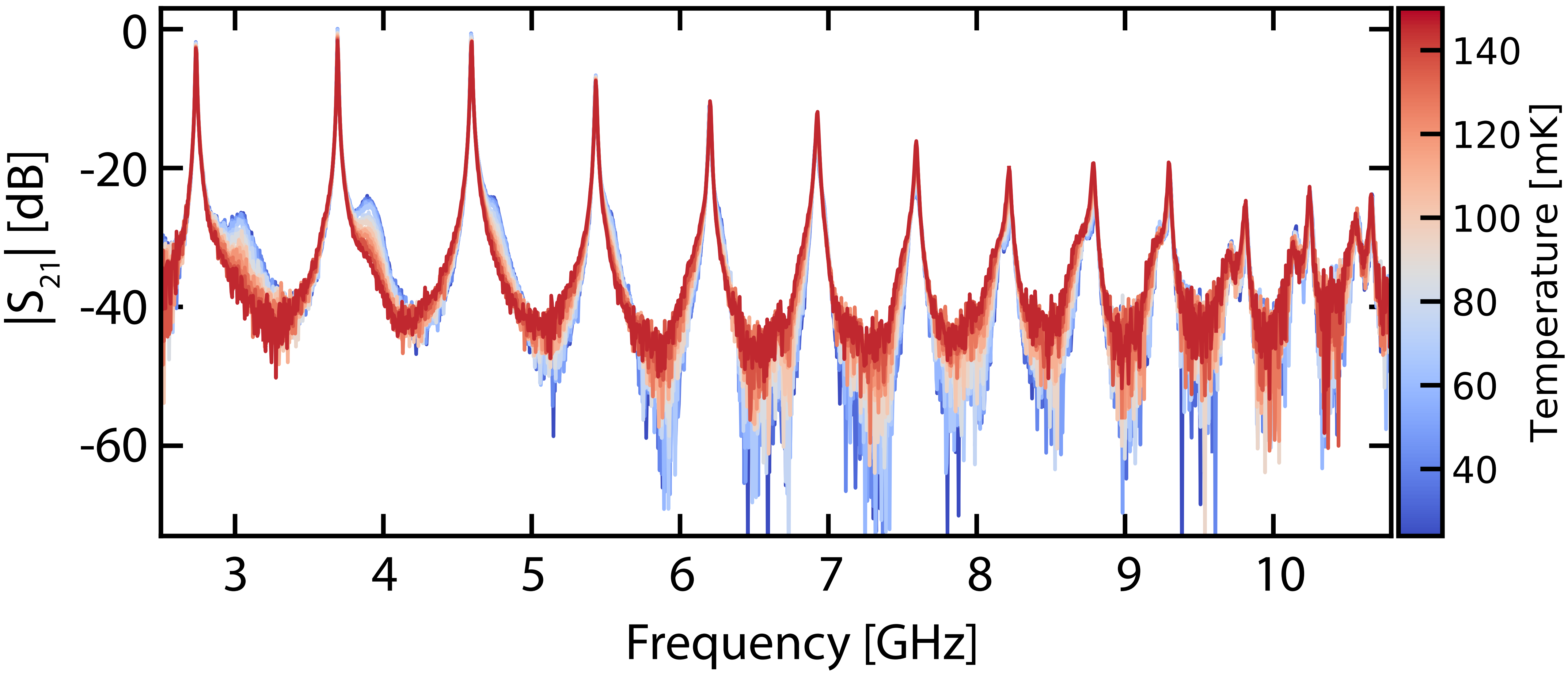
calculate the normal modes

Implementation

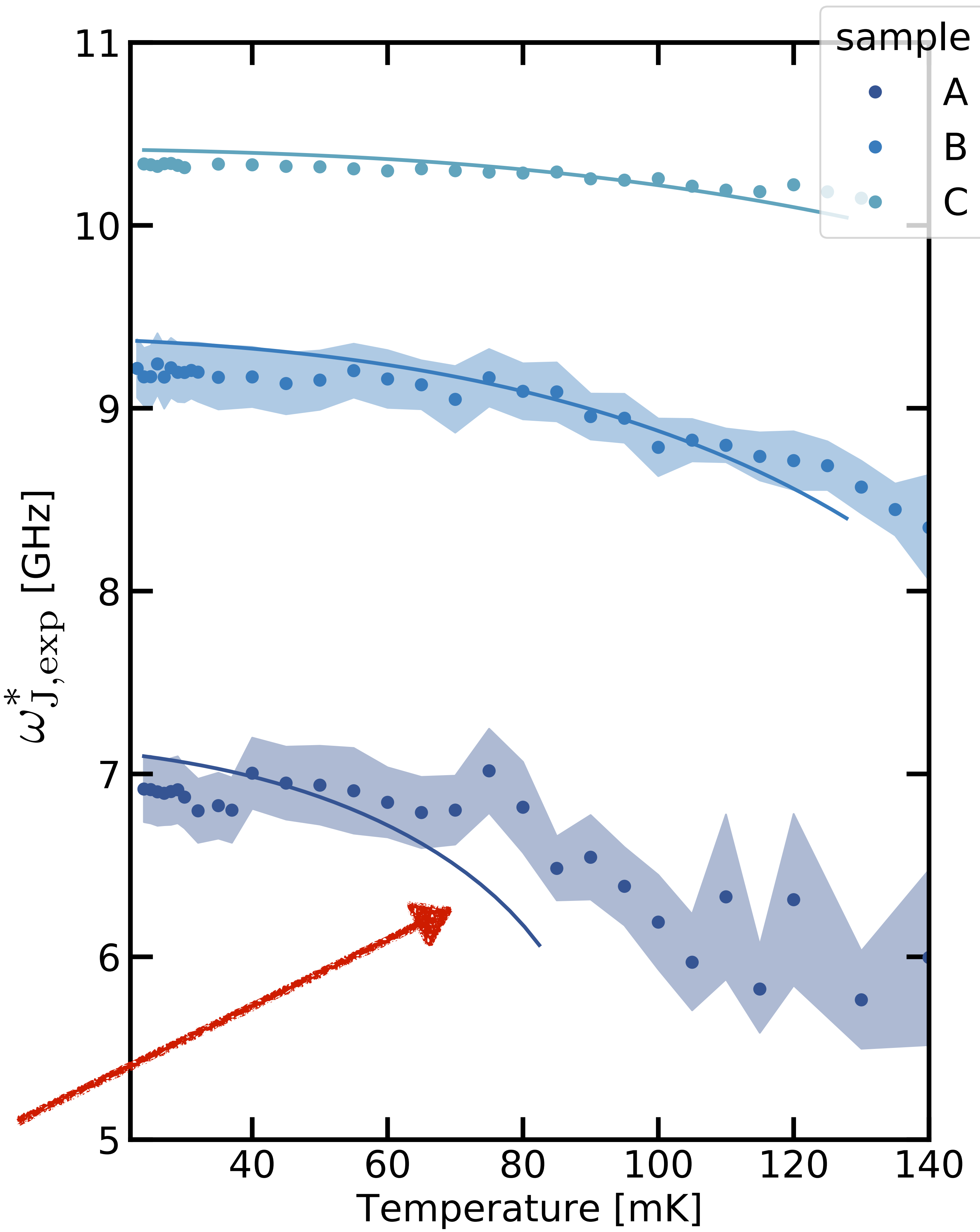
$$E_{\rm J}^* = E_{\rm J} e^{-\langle \phi(E_{\rm J}*)^2 \rangle /2} \text{ with } \langle \phi^2 \rangle = \sum_{\rm k}^{\rm M} \phi_{\rm k}^2$$

$$\omega_{\rm J,th}^* = \sqrt{2 E_{\rm J}^* E_{\rm c}}$$

ZPF versus temperature



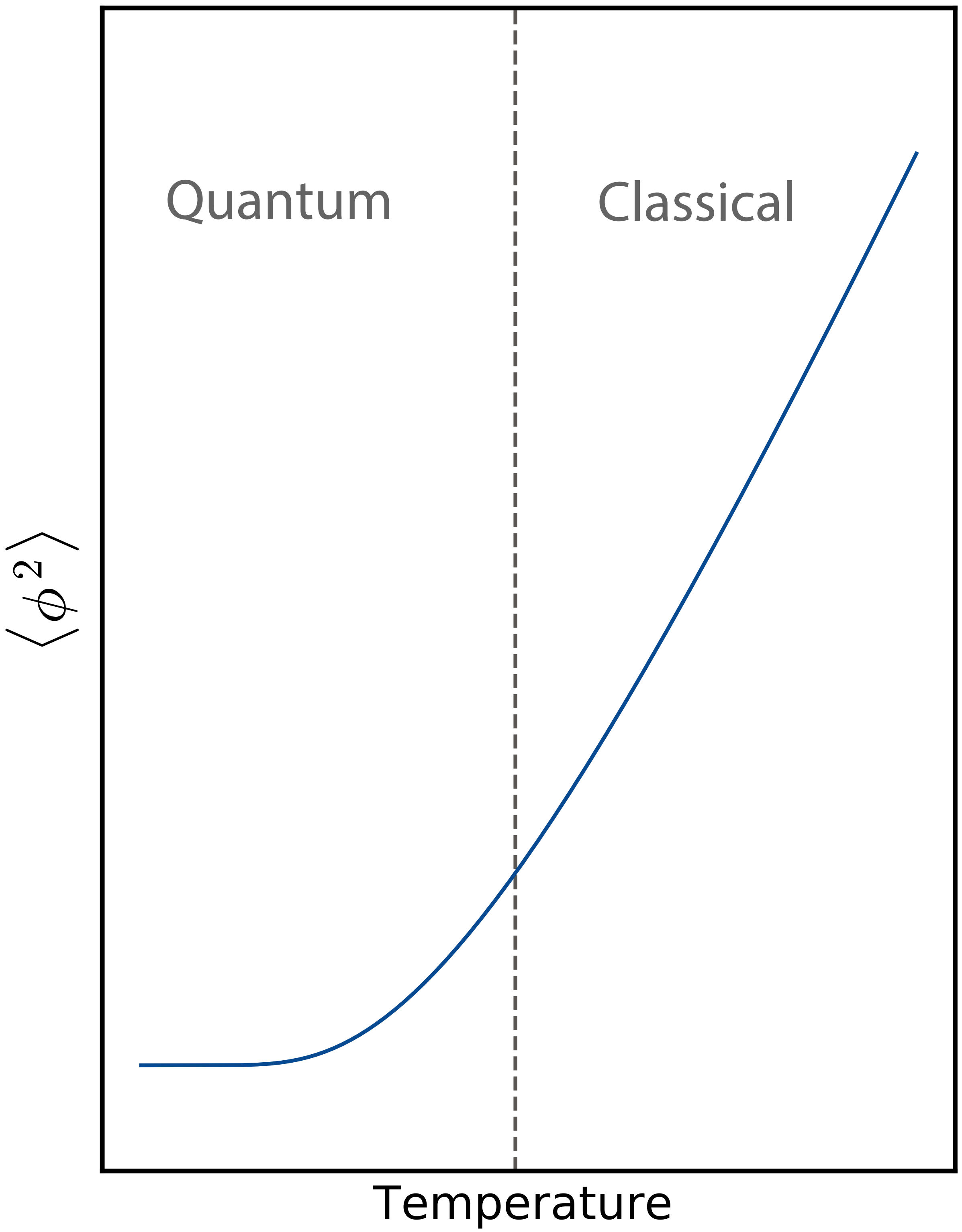
ZPF versus temperature



Deviation: SCHA breakdown

$\langle \phi^2 \rangle \simeq \pi^2$

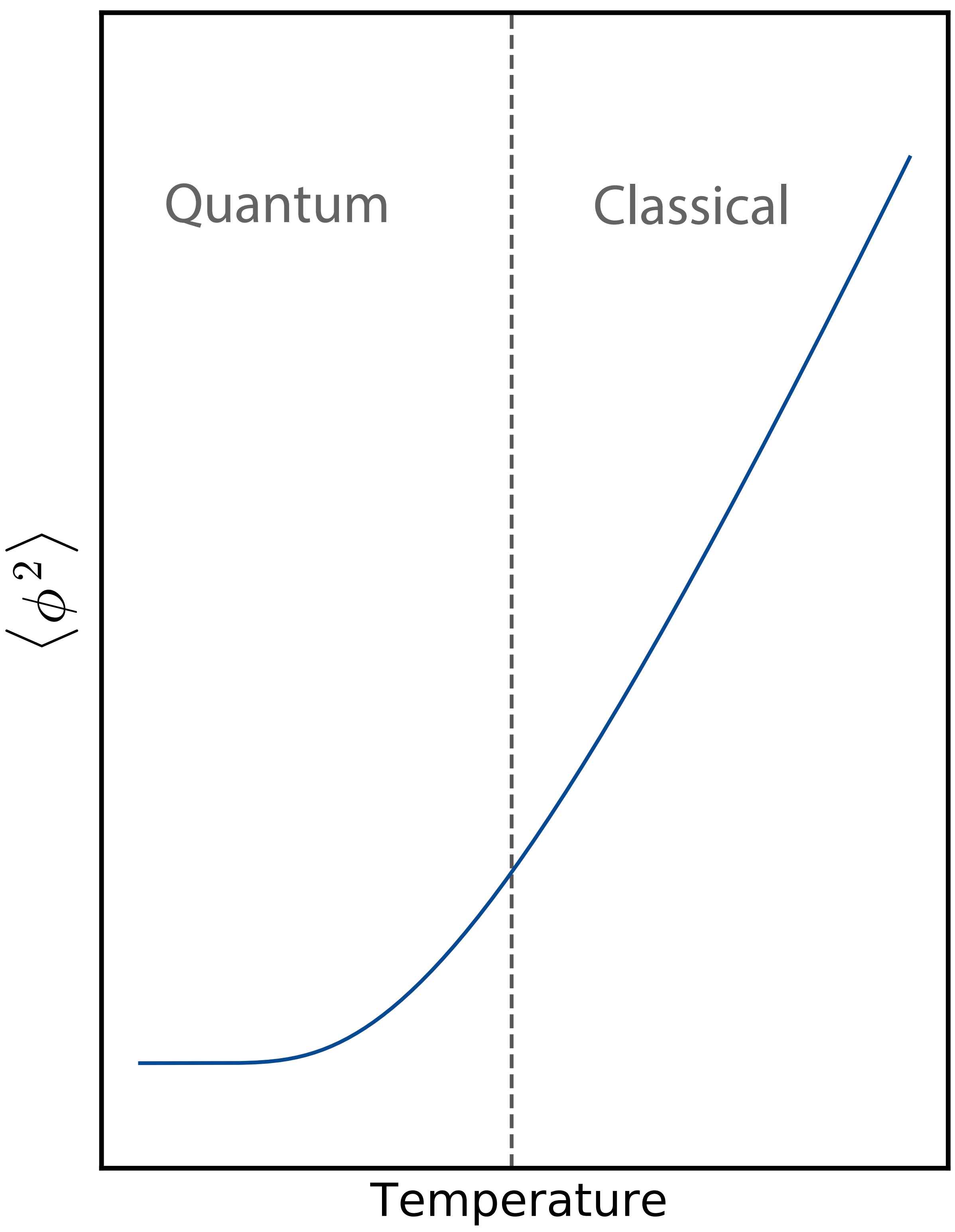
ZPF versus temperature

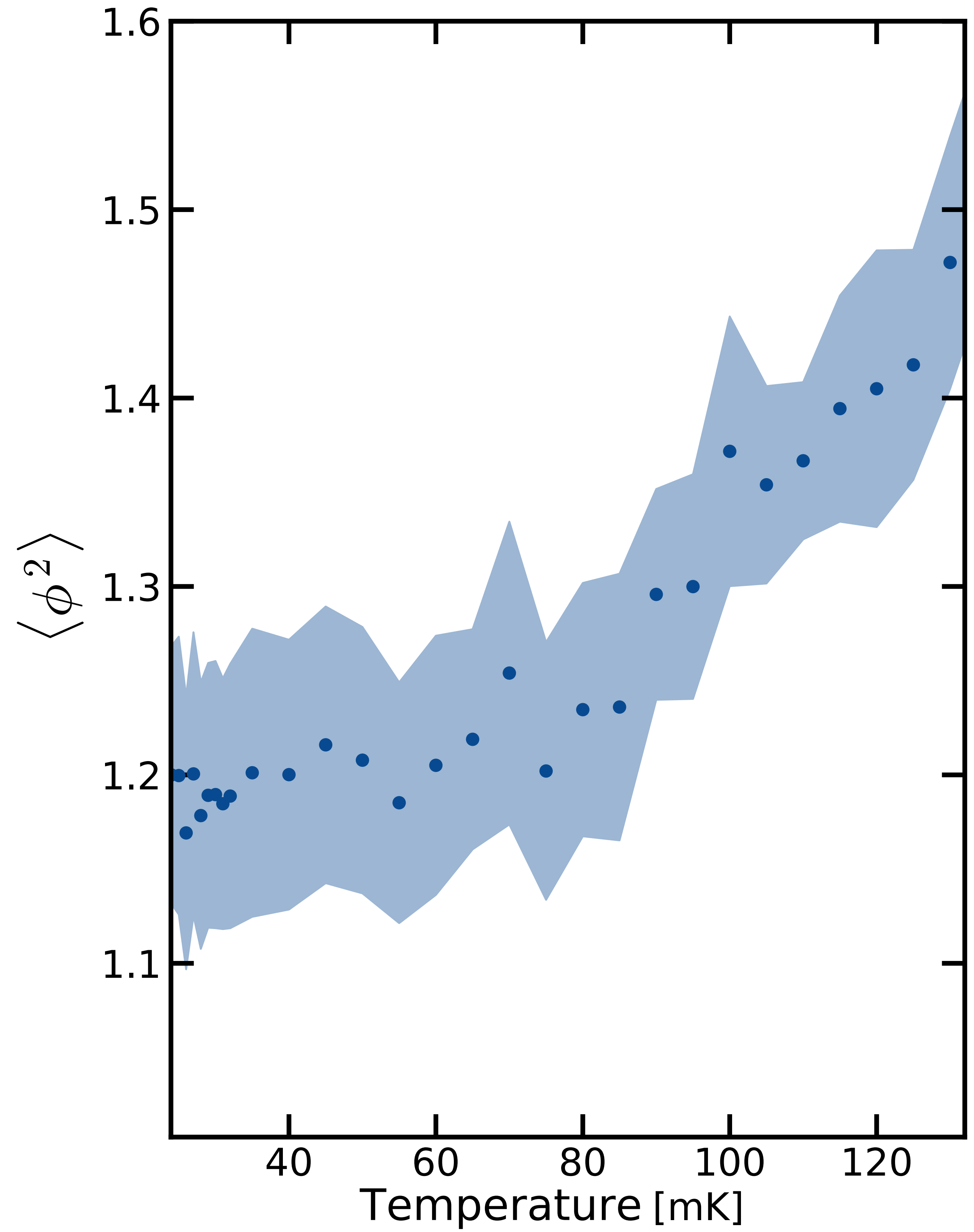


$$\omega^*_{\mathrm{J,th}} = \sqrt{2E^*_{\mathrm{J}} E_{\mathrm{c}}}$$

$$\langle \phi^2 \rangle = 4 \ln \left(\frac{\omega_{\mathrm{J,bare}}}{\omega^*_{\mathrm{J}}} \right)$$

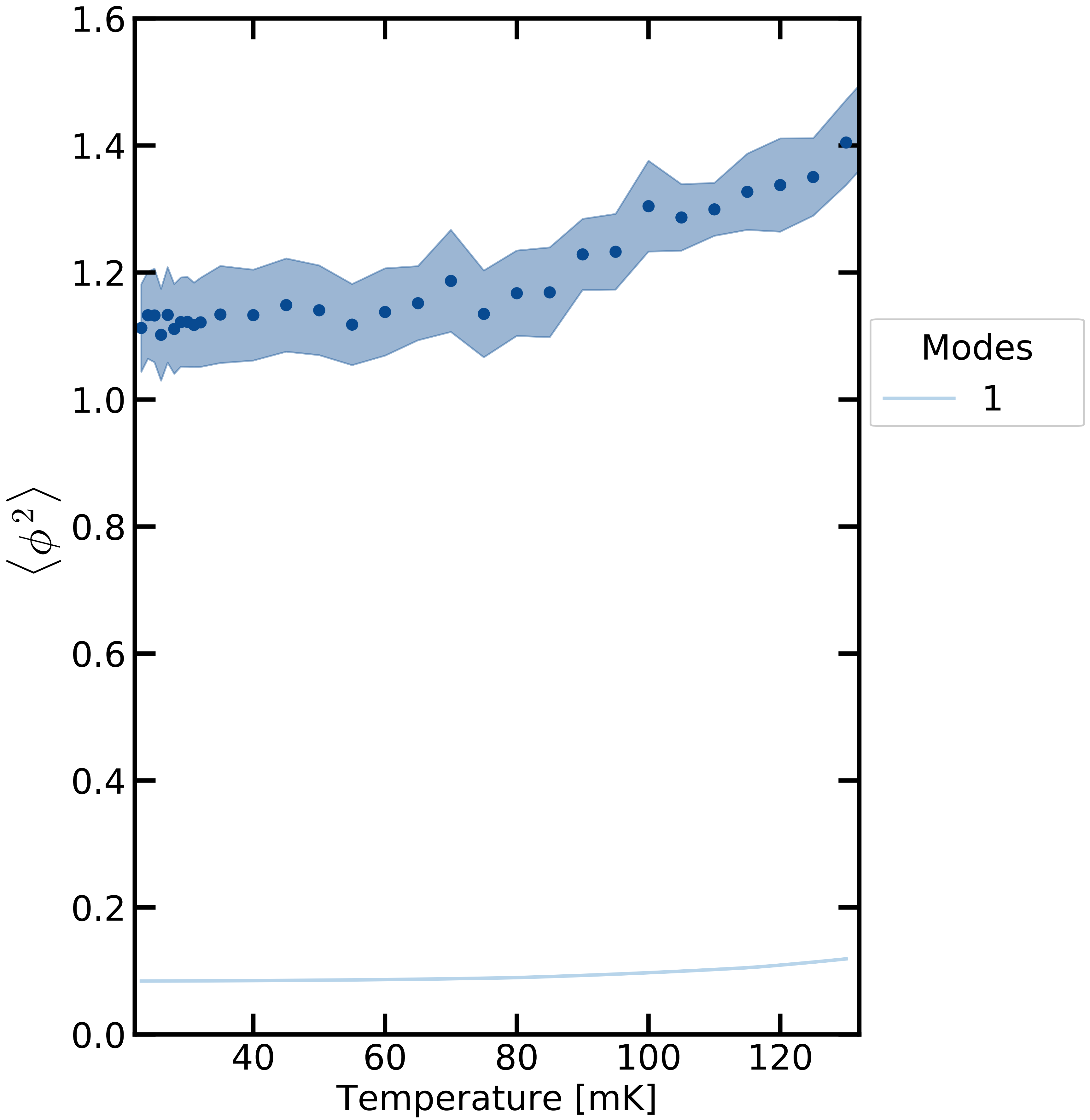
ZPF versus temperature



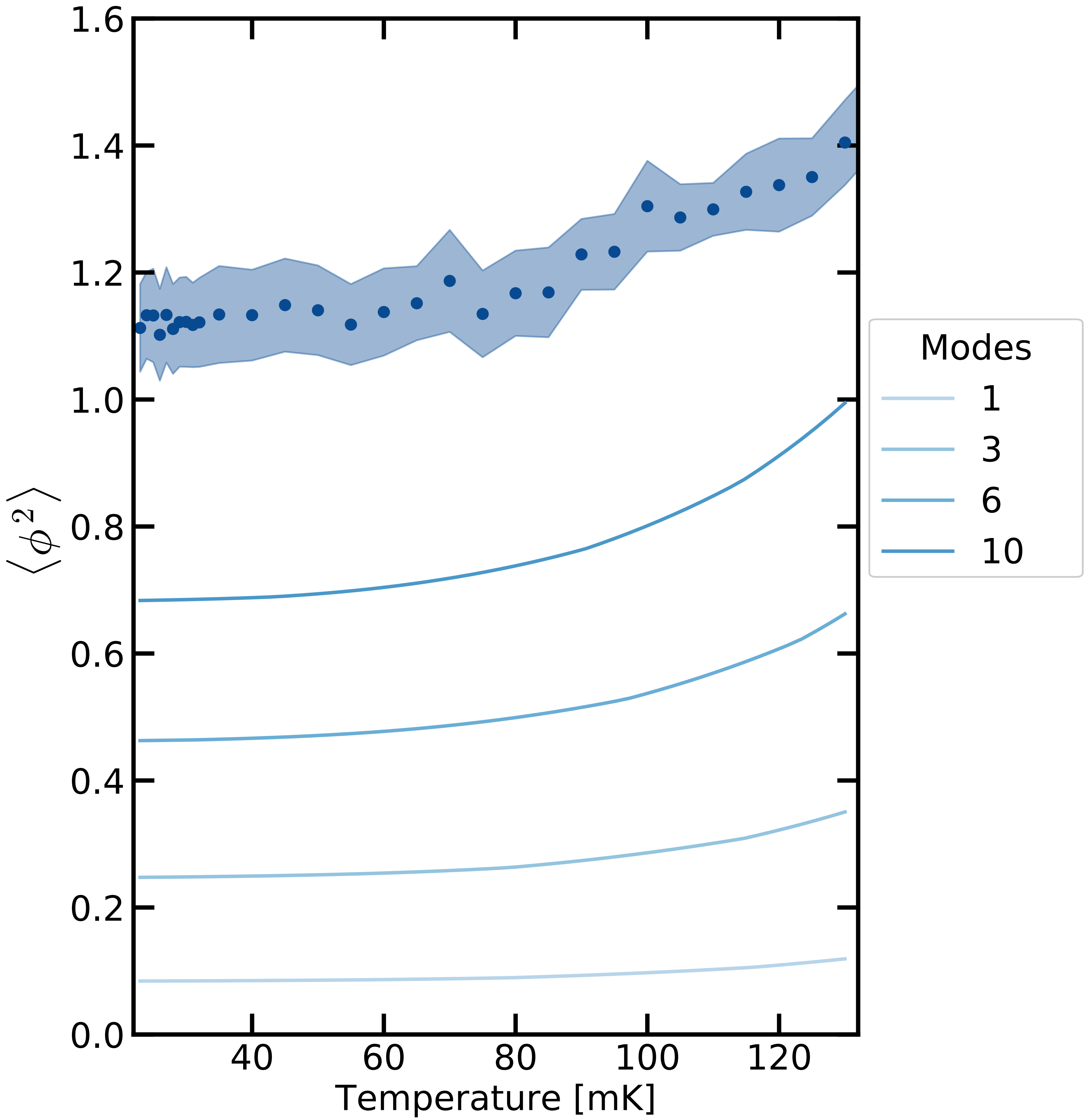


What about the many-body nature ?

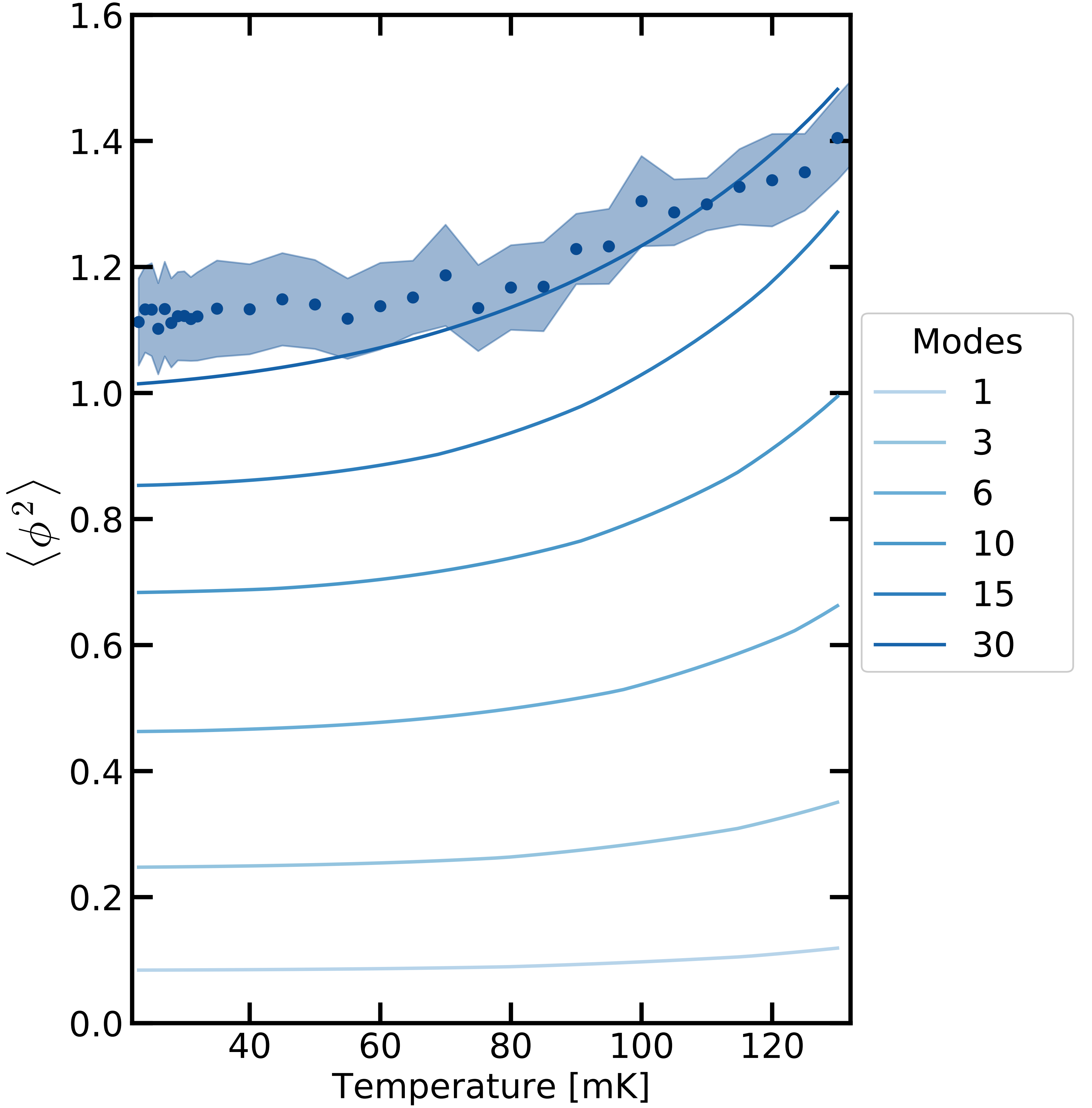
Many-body problem



Many-body problem



Many-body problem



Many-body problem