

Searching for Gravitational Waves from Domain Walls in the Early Universe

Alessio Notari ¹

Universitat de Barcelona

Università di Firenze, May 2022

¹In collaboration with R.Z. Ferreira, F. Rompineve, O. Pujolas.
Based on: arXiv 2204.04228, Phys.Rev.Lett. 128 (2022) 14, 141101

Discrete symmetry breaking

GW from
Domain Walls

Domain Walls

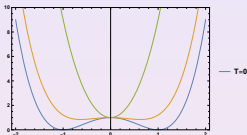
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple example: scalar field with Z_2 symmetry
$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$



- Symmetry broken **below** some Temperature T_{PT}

Discrete symmetry breaking

GW from
Domain Walls

Domain Walls

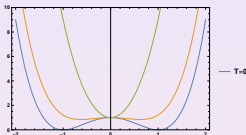
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple example: scalar field with Z_2 symmetry
$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$



- Symmetry broken **below** some Temperature T_{PT}
- ϕ takes different **(uncorrelated) values** ($\pm v$) in **different Hubble patches**

Discrete symmetry breaking

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

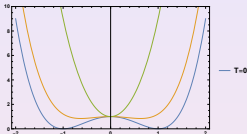
GW spectra
PTA

The QCD
Axion

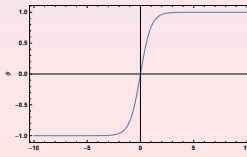
Heavy Axion

- Simple example: scalar field with Z_2 symmetry

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$



- Symmetry broken **below** some Temperature T_{PT}
- ϕ takes different (**uncorrelated**) values ($\pm v$) in **different Hubble patches**
- **Domain walls** are produced at T_{PT}
- $\phi(z) = v \tanh(\sqrt{\lambda/2} vz)$



Domain Walls

GW from
Domain Walls

Domain Walls

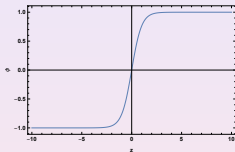
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\phi(z) = v \tanh(\sqrt{\lambda/2}vz)$.



- Thickness $\delta = (\sqrt{\lambda}v)^{-1}$
- Wall with energy per unit area (**tension**)

$$\sigma = 2 \int dz V(z) = \lambda v^3$$

Domain Walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

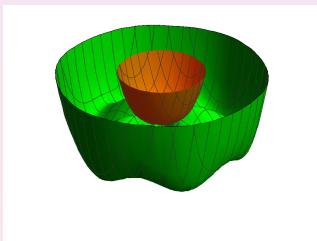
The QCD
Axion

Heavy Axion

- Another example: Complex field with $U(1)$ symmetry at high T , broken to Z_N at $T = 0$

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 + V_0 \cos\left(N\frac{a}{v}\right)$$

$$\Phi = |\Phi|e^{i\frac{a}{v}}$$



■ $T=0$
■ high T

- Symmetry broken below some T_{PT}
- Domain walls are produced at T_{PT}

Domain Walls Cosmology

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In expanding Universe with $H = \frac{\dot{a}}{a}$
- At T_{PT} (**uncorrelated**) values in different Hubble patches ($\mathcal{O}(H^{-1})$)

Domain Walls Cosmology

GW from
Domain Walls

Domain Walls

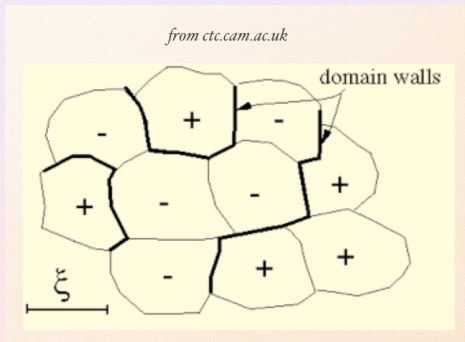
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In expanding Universe with $H = \frac{\dot{a}}{a}$
- At T_{PT} (**uncorrelated**) values in different Hubble patches ($\mathcal{O}(H^{-1})$)



Domain Walls Cosmology

GW from
Domain Walls

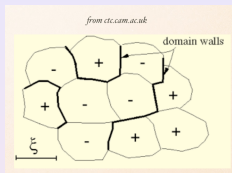
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



- Initial complicated dynamics (need simulations)
- Reach “Scaling regime”, $\mathcal{O}(1)$ walls per Hubble patch

Domain Walls Cosmology

GW from
Domain Walls

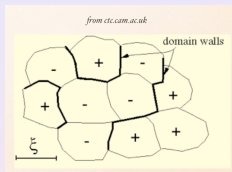
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



- Initial complicated dynamics (need simulations)
- Reach “Scaling regime”, $\mathcal{O}(1)$ walls per Hubble patch
- By dimensional analysis $\rho_{DW}|_{\text{scaling}} \approx \sigma H$

Domain Walls Cosmology

GW from
Domain Walls

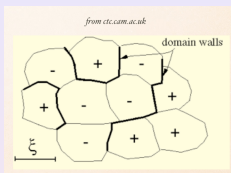
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



- Initial complicated dynamics (need simulations)
- Reach “Scaling regime”, $\mathcal{O}(1)$ walls per Hubble patch
- By dimensional analysis $\rho_{DW}|_{\text{scaling}} \approx \sigma H$
- For σ large enough they quickly dominate over radiation background, $\rho_{RAD} = 3H^2 M_{Pl}^2$
- \implies Domain wall problem!

Domain Walls Annihilation

GW from
Domain Walls

Domain Walls

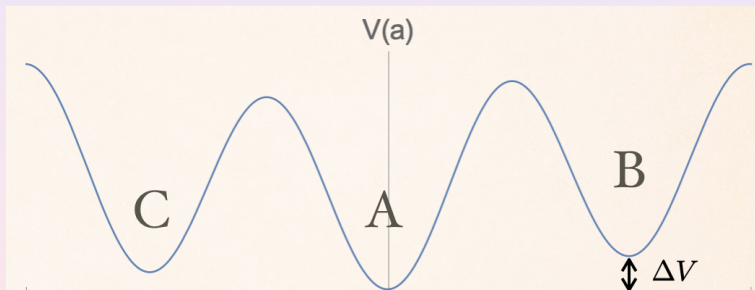
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible way out:
- Make them **unstable**, assuming a "bias" ΔV



Domain Walls Annihilation

GW from
Domain Walls

Domain Walls

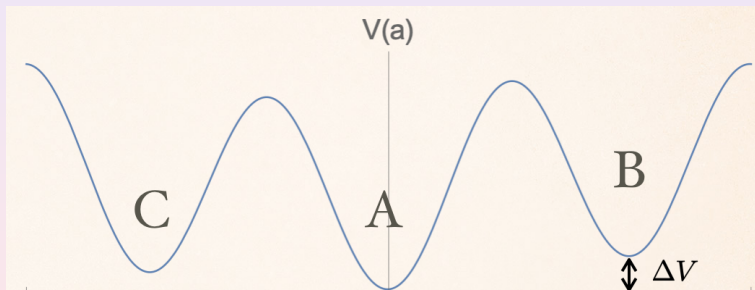
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible way out:
- Make them **unstable**, assuming a "bias" ΔV



- Annihilation happens when ΔV becomes $\simeq \rho_{DW}$

Domain Walls Annihilation

GW from
Domain Walls

Domain Walls

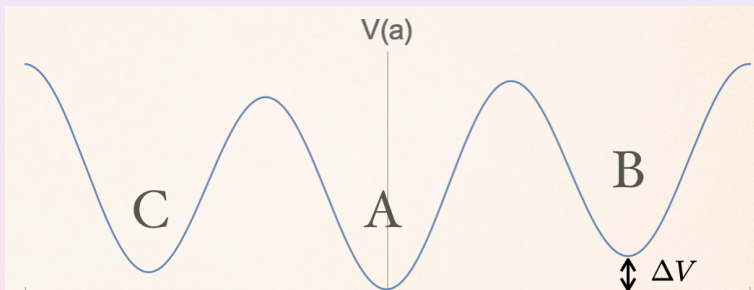
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible way out:
- Make them **unstable**, assuming a "bias" ΔV



- Annihilation happens when ΔV becomes $\simeq \rho_{DW}$
- Alternative: ... maybe symmetry restoration at low- T ? "Inverse Phase Transition"

GW in a nutshell

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- The physical metric for a GW (traveling along the z-axis)

$$g_{ab} = \eta_{ab} + h_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 + h_+ & h_\times & 0 \\ 0 & h_\times & 1 - h_+ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $\eta_{ab} = \text{diag}\{-1, 1, 1, 1\}$ and

$$h_{+,\times} = h_{+,\times}(t - z) = \int_{-\infty}^{\infty} df e^{i2\pi f(t-z)} h_{+,\times}(f, \hat{z}).$$

GW in a nutshell

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- The physical metric for a GW (traveling along the z-axis)

$$g_{ab} = \eta_{ab} + h_{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 + h_+ & h_\times & 0 \\ 0 & h_\times & 1 - h_+ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where $\eta_{ab} = \text{diag}\{-1, 1, 1, 1\}$ and

$$h_{+, \times} = h_{+, \times}(t - z) = \int_{-\infty}^{\infty} df e^{i2\pi f(t-z)} h_{+, \times}(f, \hat{z}).$$

- GW are generated by any large inhomogeneous stress energy tensor T_{ab} (Traceless and Transverse)

$$\square h_{ab} = 2 \frac{T_{ab}^{TT}}{M_{Pl}^2}$$

- $\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}$

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(**constant in time**, as long as Domain walls exist)

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(**constant in time**, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(constant in time, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

- $\alpha|_* \equiv \frac{\rho_{GW}}{\rho_{RAD}}|_{ANN} \approx \frac{\frac{\sigma^2}{M_{Pl}^2}}{\rho_{RAD}}|_{ANN}$

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(constant in time, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

$$\alpha|_* \equiv \frac{\rho_{GW}}{\rho_{RAD}}|_{ANN} \approx \frac{\frac{\sigma^2}{M_{Pl}^2}}{\rho_{RAD}}|_{ANN} \times \frac{g_* T^4}{g_* T^4} = \left(\frac{\rho_{DW}}{\rho_{RAD}} \right)|_{ANN}^2$$

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(constant in time, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

$$\alpha|_* \equiv \frac{\rho_{GW}}{\rho_{RAD}}|_{ANN} \approx \frac{\frac{\sigma^2}{M_{Pl}^2}}{\rho_{RAD}}|_{ANN} \times \frac{g_* T^4}{g_* T^4} = \left(\frac{\rho_{DW}}{\rho_{RAD}} \right)|_{ANN}^2$$

- Today: $\Omega_{GW}^0 \approx \Omega_\gamma^0 \left(\frac{\rho_{DW}}{\rho_{RAD}} \right)|_{ANN}^2$

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(constant in time, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

$$\alpha|_* \equiv \frac{\rho_{GW}}{\rho_{RAD}}|_{ANN} \approx \frac{M_{Pl}^2}{\rho_{RAD}}|_{ANN} \times \frac{g_* T^4}{g_* T^4} = \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2$$

$$\bullet \text{ Today: } \Omega_{GW}^0 \approx \Omega_{\gamma}^0 \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2 \approx 10^{-5} \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2$$

Domain wall radiate GW

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Simple estimate,

$$\rho_{GW} = \frac{M_{Pl}^2}{4} \dot{h}_{ij} \dot{h}^{ij}, \implies \rho_{GW} \approx \frac{\sigma^2}{M_{Pl}^2}$$

(constant in time, as long as Domain walls exist)

- $\rho_{GW} \propto a^{-4}$ (like radiation) after Domain walls annihilate

- $\alpha|_* \equiv \frac{\rho_{GW}}{\rho_{RAD}}|_{ANN} \approx \frac{M_{Pl}^2}{\rho_{RAD}}|_{ANN} \times \frac{g_* T^4}{g_* T^4} = \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2$

- Today: $\Omega_{GW}^0 \approx \Omega_{\gamma}^0 \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2 \approx 10^{-5} \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)|_{ANN}^2$

- More precisely, simulations give

$$\Omega_{GW} h^2 \simeq 0.05 (\Omega_{\gamma}^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{DW}}{\rho_{RAD}}\right)_{T=T_{ann} \equiv T_*}^2,$$

- $\tilde{\epsilon} = 0.1 - 1$ is an efficiency parameter

Relic GW from Domain walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- $\Omega_{\text{GW}} h^2 \simeq 0.05 (\Omega_{\gamma}^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{\text{dw}}}{\rho_{\text{rad}}} \right)_{T=T_*}^2,$

Relic GW from Domain walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Omega_{\text{GW}} h^2 \simeq 0.05 (\Omega_{\gamma}^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{\text{dw}}}{\rho_{\text{rad}}} \right)_{T=T_*}^2,$

- Peak** at frequency $H|_{T=T_*}$ (DW annihilation), redshifted to today:

$$f_{\text{peak}}^0 \simeq H_* \left(\frac{T_0}{T_*} \right)$$

Relic GW from Domain walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Omega_{\text{GW}} h^2 \simeq 0.05 (\Omega_{\gamma}^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{\text{dw}}}{\rho_{\text{rad}}} \right)_{T=T_*}^2,$

- Peak** at frequency $H|_{T=T_*}$ (DW annihilation), redshifted to today:

$$f_{\text{peak}}^0 \simeq H_* \left(\frac{T_0}{T_*} \right) = \frac{T_*^2}{M_{\text{Pl}}} \left(\frac{T_0}{T_*} \right)$$

Relic GW from Domain walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Omega_{\text{GW}} h^2 \simeq 0.05 (\Omega_\gamma^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{\text{dw}}}{\rho_{\text{rad}}} \right)_{T=T_*}^2,$

- Peak** at frequency $H|_{T=T_*}$ (DW annihilation), redshifted to today:

$$f_{\text{peak}}^0 \simeq H_* \left(\frac{T_0}{T_*} \right) = \frac{T_*^2}{M_{\text{Pl}}} \left(\frac{T_0}{T_*} \right) \approx 10^{-9} \text{ Hz} \frac{g_*(T_*)^{1/6}}{10.75} \frac{T_*}{10 \text{ MeV}}$$

Relic GW from Domain walls

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Omega_{\text{GW}} h^2 \simeq 0.05 (\Omega_\gamma^0 h^2) \tilde{\epsilon} \left(\frac{\rho_{\text{dw}}}{\rho_{\text{rad}}} \right)_{T=T_*}^2,$

- Peak** at frequency $H|_{T=T_*}$ (DW annihilation), redshifted to today:

$$f_{\text{peak}}^0 \simeq H_* \left(\frac{T_0}{T_*} \right) = \frac{T_*^2}{M_{\text{Pl}}} \left(\frac{T_0}{T_*} \right) \approx 10^{-9} \text{ Hz} \frac{g_*(T_*)^{1/6}}{10.75} \frac{T_*}{10 \text{ MeV}}$$

- Two free parameters σ (or α_*) and T_*

GW spectra

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

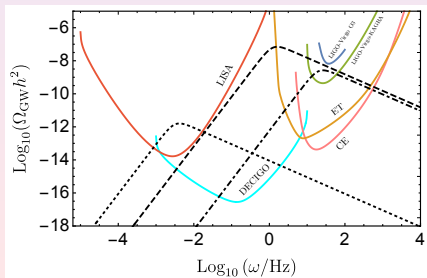
The QCD
Axion

Heavy Axion

- GW spectrum $\rho_{\text{GW}} \equiv \int \frac{d\rho_{\text{GW}}}{d \log k} \frac{dk}{k}$:

$$\frac{d\rho_{\text{GW}}}{d \log k} = \begin{cases} f^3 & \text{for } f < f^0_{\text{peak}}, \text{ (causality)} \\ f^{-1} & \text{for } f > f^0_{\text{peak}}, \text{ (until cutoff given by DW width)}. \end{cases}$$

(e.g. simulations, Hiramatsu, Kawasaki, Saikawa, 2014)



Pulsar Timing redshift

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Consider a **pulsar** emitting in the \hat{p} direction with frequency ν_0
- And a **GW** traveling in the direction $\hat{\Omega}$

²see e.g. Anholm et al. PRD (2009)

Pulsar Timing redshift

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Consider a **pulsar** emitting in the \hat{p} direction with **frequency** ν_0
- And a **GW** traveling in the **direction** $\hat{\Omega}$
- The pulsar is redshifted as ²

$$z(t, \hat{\Omega}) \equiv \frac{\nu_0 - \nu(t)}{\nu_0} = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} \Delta h_{ij},$$

where

$$\Delta h_{ij} \equiv h_{ij}(t_P, \hat{\Omega}) - h_{ij}(t, \hat{\Omega}),$$

difference at the pulsar (t_P) and at the center of the solar system (t).

²see e.g. Anholm et al. PRD (2009)

Pulsar Timing redshift

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Consider a **pulsar** emitting in the \hat{p} direction with **frequency** ν_0
- And a **GW** traveling in the **direction** $\hat{\Omega}$
- The pulsar is redshifted as ²

$$z(t, \hat{\Omega}) \equiv \frac{\nu_0 - \nu(t)}{\nu_0} = \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}} \Delta h_{ij},$$

where

$$\Delta h_{ij} \equiv h_{ij}(t_P, \hat{\Omega}) - h_{ij}(t, \hat{\Omega}),$$

difference at the pulsar (t_P) and at the center of the solar system (t).

- Common assumption: **Neglect** the pulsar (t_P) term

²see e.g. Anholm et al. PRD (2009)

Pulsar Timing Arrays

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- Fourier transform and consider $\langle z_1^*(f, \hat{\Omega}) z_2(f', \hat{\Omega}) \rangle$ from two Pulsars (1 and 2)
- Integrate over all possible $\hat{\Omega}$:

Pulsar Timing Arrays

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- Fourier transform and consider $\langle z_1^*(f, \hat{\Omega}) z_2(f', \hat{\Omega}) \rangle$ from two Pulsars (1 and 2)
- Integrate over all possible $\hat{\Omega}$:

$$\langle \tilde{z}_1^*(f) \tilde{z}_2(f') \rangle = \frac{H_0^2}{8\pi^2} \delta(f - f') |f|^{-3} \Omega_{\text{GW}}(|f|) \Gamma_{12},$$

Pulsar Timing Arrays

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Fourier transform and consider $\langle z_1^*(f, \hat{\Omega}) z_2(f', \hat{\Omega}) \rangle$ from two Pulsars (1 and 2)
- Integrate over all possible $\hat{\Omega}$:

$$\langle \tilde{z}_1^*(f) \tilde{z}_2(f') \rangle = \frac{H_0^2}{8\pi^2} \delta(f - f') |f|^{-3} \Omega_{\text{GW}}(|f|) \Gamma_{12},$$

where

$$\begin{aligned} \Gamma_{12} &= \frac{3}{4\pi} \sum_A \int_{\mathcal{S}^2} d\hat{\Omega} F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}) \\ &= 3 \left\{ \frac{1}{3} + \frac{1 - \cos \xi}{2} \left[\ln \left(\frac{1 - \cos \xi}{2} \right) - \frac{1}{6} \right] \right\}, \end{aligned}$$

$$\xi \equiv \arccos(\hat{p}_1 \cdot \hat{p}_2), \text{ and } F^A(\hat{\Omega}) \equiv e_{ij}^A(\hat{\Omega}) \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}}.$$

Pulsar Timing Arrays

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Fourier transform and consider $\langle z_1^*(f, \hat{\Omega}) z_2(f', \hat{\Omega}) \rangle$ from two Pulsars (1 and 2)
- Integrate over all possible $\hat{\Omega}$:

$$\langle \tilde{z}_1^*(f) \tilde{z}_2(f') \rangle = \frac{H_0^2}{8\pi^2} \delta(f - f') |f|^{-3} \Omega_{\text{GW}}(|f|) \Gamma_{12},$$

where

$$\begin{aligned} \Gamma_{12} &= \frac{3}{4\pi} \sum_A \int_{\mathcal{S}^2} d\hat{\Omega} F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}) \\ &= 3 \left\{ \frac{1}{3} + \frac{1 - \cos \xi}{2} \left[\ln \left(\frac{1 - \cos \xi}{2} \right) - \frac{1}{6} \right] \right\}, \end{aligned}$$

$\xi \equiv \arccos(\hat{p}_1 \cdot \hat{p}_2)$, and $F^A(\hat{\Omega}) \equiv e_{ij}^A(\hat{\Omega}) \frac{1}{2} \frac{\hat{p}^i \hat{p}^j}{1 + \hat{\Omega} \cdot \hat{p}}$.

- Common spectrum $|f|^{-3} \Omega_{\text{GW}}(|f|)$
- **Angular "Hellings-Downs" (HD) correlation** Γ_{12} between two pulsars, 1 and 2

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- *North American Nanohertz Observatory for Gravitational Waves*
- **45** analyzed **pulsars** (Arzoumanian et al. Ap.J. Lett. (2020)) with at least 3 years data
- **Strong evidence for common-spectrum** stochastic process

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

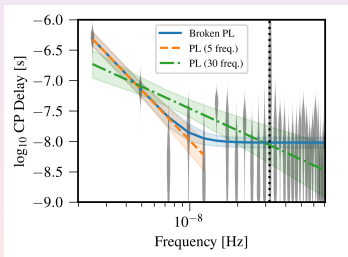
GW spectra

PTA

The QCD
Axion

Heavy Axion

- *North American Nanohertz Observatory for Gravitational Waves*
- **45 analyzed pulsars** (Arzoumanian et al. Ap.J. Lett. (2020)) with at least 3 years data
- **Strong evidence for common-spectrum** stochastic process



NANOGrAV 12.5 year

GW from
Domain Walls

Domain Walls

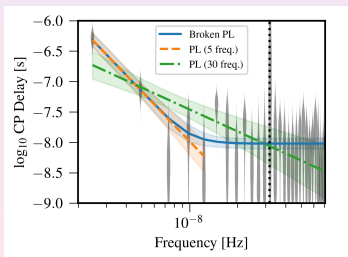
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

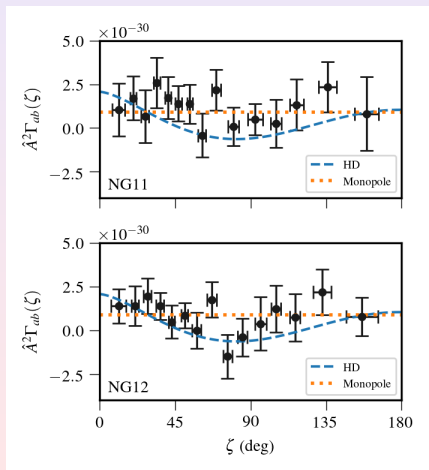
- *North American Nanohertz Observatory for Gravitational Waves*
- **45 analyzed pulsars** (Arzoumanian et al. Ap.J. Lett. (2020)) with at least 3 years data
- **Strong evidence for common-spectrum** stochastic process



- Pulsar-intrinsic **noise** at **high frequencies**
- NANOGrav Collaboration simple solution: consider **only 5 lowest frequencies**.

NANOGRAV 12.5 year

- No evidence yet for HD angular correlation from GW



GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Power-law fit, exponent γ_{CP}

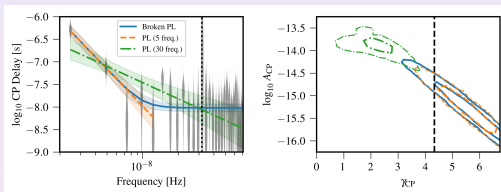


Figure: Arzoumanian et al. Ap.J. Lett. (2020)

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Power-law fit, exponent γ_{CP}

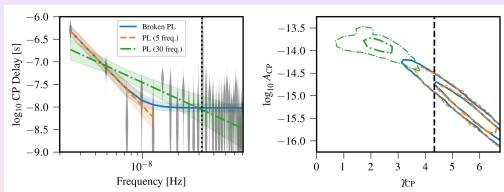


Figure: Arzoumanian et al. Ap.J. Lett. (2020)

- Most “conservative” interpretation: GW from **SuperMassive Black Hole Binaries (SMBHB)**

$$h(f) = A_{GWB} \left(\frac{f}{f_{yr}} \right)^{-\frac{2}{3}} =$$

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Power-law fit, exponent γ_{CP}

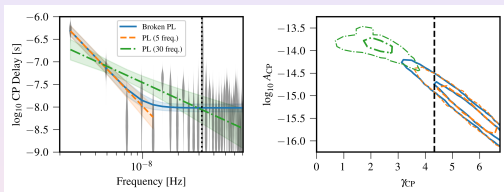


Figure: Arzoumanian et al. Ap.J. Lett. (2020)

- Most “conservative” interpretation: GW from **SuperMassive Black Hole Binaries (SMBHB)**

$$h(f) = A_{GWB} \left(\frac{f}{f_{yr}} \right)^{-\frac{2}{3}} = A_{GWB} \left(\frac{f}{f_{yr}} \right)^{\frac{3-\gamma_{CP}}{2}}, \quad (\gamma_{CP} = 4.33)$$

NANOGRAV 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Power-law fit, exponent γ_{CP}

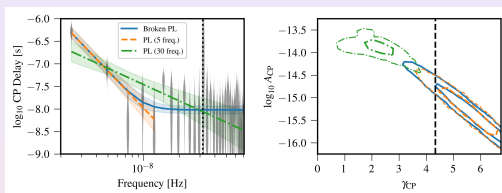


Figure: Arzoumanian et al. Ap.J. Lett. (2020)

- Most “conservative” interpretation: GW from **SuperMassive Black Hole Binaries (SMBHB)**

$$h(f) = A_{GWB} \left(\frac{f}{f_{yr}} \right)^{-\frac{2}{3}} = A_{GWB} \left(\frac{f}{f_{yr}} \right)^{\frac{3-\gamma_{CP}}{2}}, \quad (\gamma_{CP} = 4.33)$$

- **Alternative:** GWB from **Early Universe**

NANOGrav 12.5 year

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Example: **NANOGrav search** for **GWB** from **Phase Transitions** (bubble collisions)

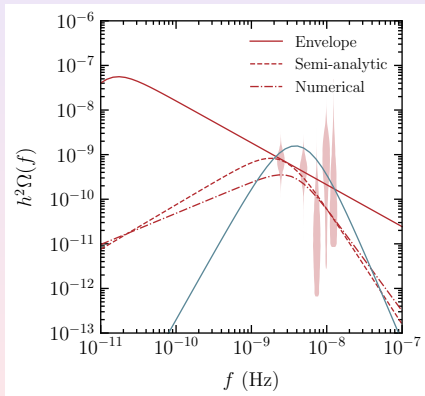


Figure: Arzoumanian et al. Phys.Rev.Lett. 127 (2021)

IPTA DR2 Dataset

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- International Collaboration (North America, Europe, Australia) (J. Antoniadis et al. MNRAS (2022))
- **Combination** of European Pulsar Timing Array (EPTA), NANOGrav, and the Parkes Pulsar Timing array (PPTA)
- 53 pulsars

IPTA DR2 Dataset

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion

- International Collaboration (North America, Europe, Australia) (J. Antoniadis et al. MNRAS (2022))
- **Combination** of European Pulsar Timing Array (EPTA), NANOGrav, and the Parkes Pulsar Timing array (PPTA)
- 53 pulsars
- Use only **first 13 datapoints**

IPTA DR2 Dataset

GW from
Domain Walls

Domain Walls

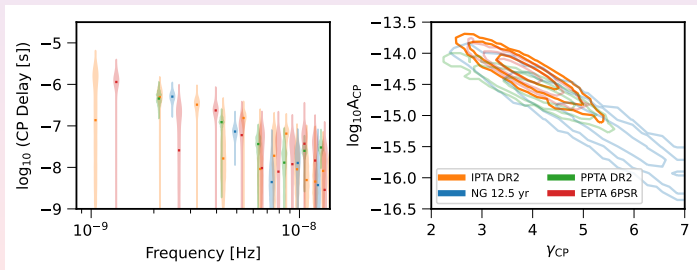
Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- International Collaboration (North America, Europe, Australia) (J. Antoniadis et al. MNRAS (2022))
- **Combination** of European Pulsar Timing Array (EPTA), NANOGrav, and the Parkes Pulsar Timing array (PPTA)
- 53 pulsars
- Use only **first 13 datapoints**



- Similar results (slightly smaller γ_{CP})

GW Search from Domain Walls in NANOGRAV and IPTA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Search for GW from Domain Walls³:

$$\Omega_{\text{GW,DW}}(f)h^2 \simeq 10^{-10} \tilde{\epsilon} \left(\frac{10.75}{g_*(T_*)} \right)^{\frac{1}{3}} \left(\frac{\alpha_*}{0.01} \right)^2 S \left(\frac{f}{f_p^0} \right),$$

where $\tilde{\epsilon} \simeq 0.1 - 1$ (efficiency parameter)

- $S(x)$ models the shape:

³R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, ePrint: 2204.04228

GW Search from Domain Walls in NANOGRAV and IPTA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Search for GW from Domain Walls³:

$$\Omega_{\text{GW,DW}}(f)h^2 \simeq 10^{-10} \tilde{\epsilon} \left(\frac{10.75}{g_*(T_*)} \right)^{\frac{1}{3}} \left(\frac{\alpha_*}{0.01} \right)^2 S \left(\frac{f}{f_p^0} \right),$$

where $\tilde{\epsilon} \simeq 0.1 - 1$ (efficiency parameter)

- $S(x)$ models the shape:

$$S(x) = \frac{(\gamma + \beta)^\delta}{(\beta x^{-\frac{\gamma}{\delta}} + \gamma x^{\frac{\beta}{\delta}})^\delta},$$

$$\begin{cases} \text{At low frequency } S \propto f^3 \\ \text{At high } f, \text{ simulations suggest } \delta \approx \beta \approx 1 \implies S \propto f^{-1} \end{cases}$$

³R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, ePrint: 2204.04228

Decay of the network

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Assume DW decay into ϕ quanta and subsequently:

- Two scenarios

$\left\{ \begin{array}{l} \phi \text{ Decay to Dark Radiation} \text{ problem if too much} \\ \phi \text{ Decay to Standard Model} \text{ Before BBN } T_* \gtrsim 3\text{MeV} \end{array} \right.$

Decay of the network

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Assume DW decay into ϕ quanta and subsequently:

- Two scenarios

$\left\{ \begin{array}{l} \phi \text{ Decay to Dark Radiation} \text{ problem if too much} \\ \phi \text{ Decay to Standard Model} \text{ Before BBN } T_* \gtrsim 3\text{MeV} \end{array} \right.$

- **CASE I: Decay into DR**
- Abundance of DR, standard parameterization

$$\Delta N_{\text{eff}} = \frac{\rho_{\text{DR}}}{\rho_{\nu}}$$

Decay of the network

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Assume DW decay into ϕ quanta and subsequently:

- Two scenarios

$\left\{ \begin{array}{l} \phi \text{ Decay to Dark Radiation} \text{ problem if too much} \\ \phi \text{ Decay to Standard Model} \text{ Before BBN } T_* \gtrsim 3\text{MeV} \end{array} \right.$

- **CASE I: Decay into DR**
- Abundance of DR, standard parameterization

$$\Delta N_{\text{eff}} = \frac{\rho_{\text{DR}}}{\rho_{\nu}} \approx \frac{\rho_{\text{DW}}}{\rho_{\nu}}$$

Decay of the network

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Assume DW decay into ϕ quanta and subsequently:

- Two scenarios

$$\left\{ \begin{array}{l} \phi \text{ Decay to Dark Radiation } \text{problem if too much} \\ \phi \text{ Decay to Standard Model } \text{Before BBN } T_* \gtrsim 3\text{MeV} \end{array} \right.$$

- **CASE I: Decay into DR**
- Abundance of DR, standard parameterization

$$\Delta N_{\text{eff}} = \frac{\rho_{\text{DR}}}{\rho_\nu} \approx \frac{\rho_{\text{DW}}}{\rho_\nu} = 13.6 g_* |T_*|^{-1/3} \alpha_*$$

- Current limits $\Delta N_{\text{eff}} \lesssim 0.3$ (*Planck 2018 + BAO*)

Results (CASE I): Decay into Dark Radiation

GW from
Domain Walls

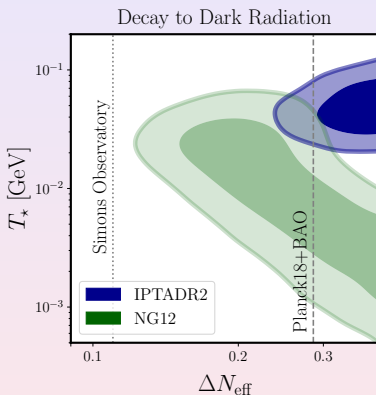
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



Results (CASE I): Decay into Dark Radiation

GW from
Domain Walls

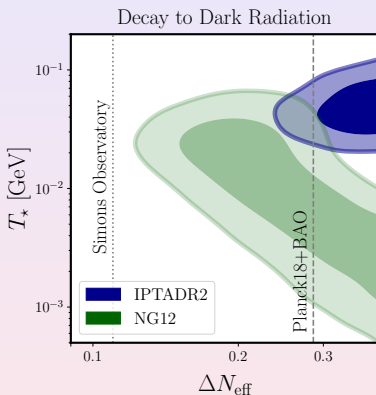
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



- **Currently constrained** (Planck+BBN)
- Future Forecast: **visible** by CMB experiments

Results (CASE II): Decay into Standard Model

GW from
Domain Walls

Domain Walls

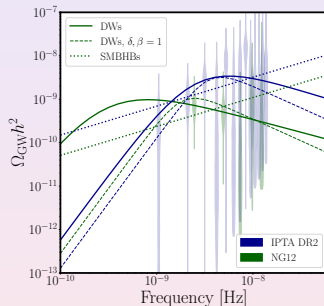
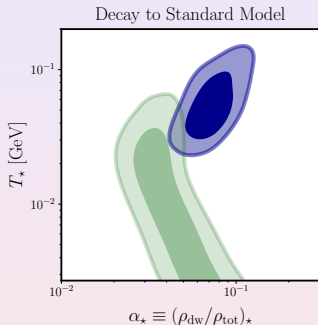
Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

Heavy Axion



Results (CASE II): Decay into Standard Model

GW from
Domain Walls

Domain Walls

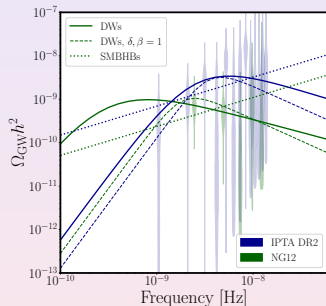
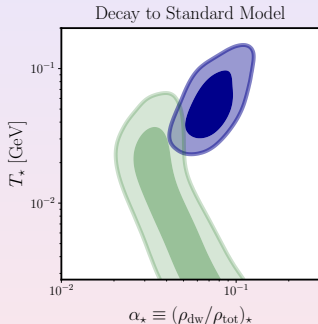
Gravitational
Waves from
DWs

GW spectra

PTA

The QCD
Axion

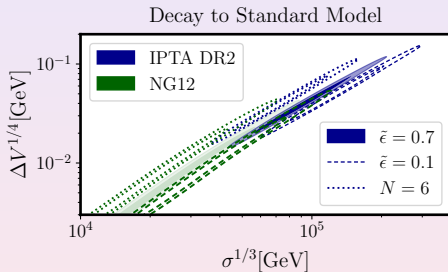
Heavy Axion



- **IPTA** prefers a **peak**
- **NANOGrav** ok with a power-law

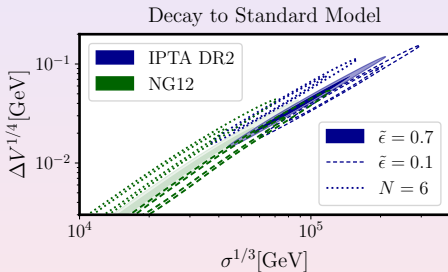
Results: Decay into Standard Model

- Decay Temperature T_* and fraction α_* could be traded for bias (ΔV) and tension (σ),



Results: Decay into Standard Model

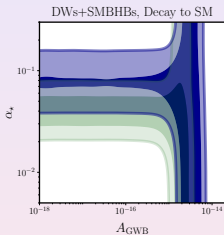
- Decay Temperature T_* and fraction α_* could be traded for bias (ΔV) and tension (σ),



- In a \mathbb{Z}_2 model with $V(\phi) = \lambda(\phi^2 - v^2)^2$, \implies
 $v \approx (10 - 100 \text{ TeV})/\lambda^{1/3}$
- Bias points to a scale of $\Delta V^{1/4} = 10 - 100 \text{ MeV}$, close to QCD scale

Results: Combine with SMBHM

- We also combined with "standard" expected signal from Supermassive Black Holes Mergers (SMBHM)



Results: Combine with SMBHM

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

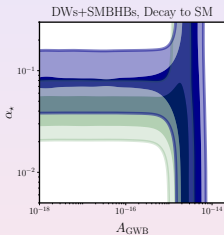
GW spectra

PTA

The QCD
Axion

Heavy Axion

- We also combined with "standard" expected signal from Supermassive Black Holes Mergers (SMBHM)



- We also compared models via Bayes factors $\log_{10} B_{i,j}$

Results: Combine with SMBHM

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

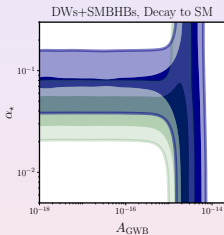
GW spectra

PTA

The QCD
Axion

Heavy Axion

- We also combined with "standard" expected signal from Supermassive Black Holes Mergers (SMBHM)



- We also compared models via Bayes factors $\log_{10} B_{i,j}$
- For NG12, we find: $\log_{10} B_{\text{SMBHBs, DW}} \simeq 0.16$,
 $\log_{10} B_{\text{DW, DW+SMBHBs}} \simeq 0.07$.
- For IPTADR2, we find: $\log_{10} B_{\text{DW, SMBHBs}} \simeq 0.48$,
 $\log_{10} B_{\text{DW, DW+SMBHBs}} \simeq 0.38$.
- \implies **no substantial evidence for one model against any other one.**

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- I discuss now realizations of **Decaying DW** with **Axions**
- Many axion models have a **Z_N symmetry at low T** .

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- I discuss now realizations of **Decaying DW** with **Axions**
- Many axion models have a **Z_N symmetry at low T** .
- Decaying DW in **“Heavy QCD Axion”** scenario ⁴

⁴R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, PRL 2022 

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- I discuss now realizations of **Decaying DW** with **Axions**
- Many axion models have a **Z_N symmetry at low T** .
- Decaying DW in **"Heavy QCD Axion"** scenario ⁴
- Introduce first the "Standard QCD Axion"

⁴R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, PRL 2022 

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect
- In non-abelian theory: Boundary term sensitive to Instantons \implies **has physical effects**⁵

⁵Unless a quark is massless

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect
- In non-abelian theory: Boundary term sensitive to Instantons \implies **has physical effects**⁵
 - Violates **P** and **T** (or equivalently, P and CP)
 - **Periodic**: $\theta = \theta + 2\pi$.

⁵Unless a quark is massless

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect
- In non-abelian theory: Boundary term sensitive to Instantons \implies **has physical effects**⁵
 - Violates **P** and **T** (or equivalently, P and CP)
 - **Periodic**: $\theta = \theta + 2\pi$.
 - One effect: Neutron Electric Dipole Moment (**nEDM**)
 $d_n = 5 \times 10^{-16} \theta$ e cm

⁵Unless a quark is massless

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect
- In non-abelian theory: Boundary term sensitive to Instantons \implies **has physical effects**⁵
 - Violates **P** and **T** (or equivalently, P and CP)
 - **Periodic**: $\theta = \theta + 2\pi$.
 - One effect: Neutron Electric Dipole Moment (**nEDM**)
 $d_n = 5 \times 10^{-16} \theta$ e cm
 - Measurement $d_n < \mathcal{O}(10^{-26})$ e cm $\implies |\theta| \lesssim 10^{-10}$

⁵Unless a quark is massless

Strong CP problem in QCD

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- In QCD lagrangian a term is allowed:

$$\mathcal{L}_\theta = \frac{\alpha_s}{8\pi} \theta G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- $G_{\mu\nu} \tilde{G}^{\mu\nu} = \partial_\mu K^\mu$: total derivative \implies no classical effect
- In non-abelian theory: Boundary term sensitive to Instantons \implies **has physical effects**⁵
 - Violates **P** and **T** (or equivalently, P and CP)
 - **Periodic**: $\theta = \theta + 2\pi$.
 - One effect: Neutron Electric Dipole Moment (**nEDM**)
 $d_n = 5 \times 10^{-16} \theta$ e cm
 - Measurement $d_n < \mathcal{O}(10^{-26})$ e cm $\implies |\theta| \lesssim 10^{-10}$
- **Why so small?**

⁵Unless a quark is massless

Solving the Strong CP problem: QCD Axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Promote θ to a new scalar field, **QCD Axion** ($\theta \rightarrow \frac{a}{f}$):

Solving the Strong CP problem: QCD Axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Promote θ to a new scalar field, **QCD Axion** ($\theta \rightarrow \frac{a}{f}$):

- Solves the **“Strong CP problem”**

$$\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{a}{f} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Solving the Strong CP problem: QCD Axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Promote θ to a new scalar field, **QCD Axion** ($\theta \rightarrow \frac{a}{f}$):

- Solves the “**Strong CP problem**”

$$\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{a}{f} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Integrating by parts: $\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{\partial_\mu a}{f} K^\mu$,
 \implies continuous shift symmetry $a \rightarrow a + c$
(No potential)

Solving the Strong CP problem: QCD Axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Promote θ to a new scalar field, **QCD Axion** ($\theta \rightarrow \frac{a}{f}$):

- Solves the “**Strong CP problem**”

$$\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{a}{f} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Integrating by parts: $\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{\partial_\mu a}{f} K^\mu$,
 \implies continuous shift symmetry $a \rightarrow a + c$
(No potential)

- But boundary term sensitive to QCD Instantons,

- 1 Induces a potential $V(a) \propto -\cos(a/f)$;
- 2 $a \rightarrow 0 \implies$ Drives ~~CP~~ to zero
- 3 \implies Axion mass $m_a \approx \sqrt{V''}|_{a=0} = 0.57 \left(\frac{10^7 \text{ GeV}}{f} \right) \text{ eV}$

Solving the Strong CP problem: QCD Axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Promote θ to a new scalar field, **QCD Axion** ($\theta \rightarrow \frac{a}{f}$):

- Solves the “**Strong CP problem**”

$$\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{a}{f} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Integrating by parts: $\mathcal{L}_a = \frac{\alpha_s}{8\pi} \frac{\partial_\mu a}{f} K^\mu$,
 \implies continuous shift symmetry $a \rightarrow a + c$
(No potential)
- But boundary term sensitive to QCD Instantons,
 - 1 Induces a potential $V(a) \propto -\cos(a/f)$;
 - 2 $a \rightarrow 0 \implies$ Drives ~~CP~~ to zero
 - 3 \implies Axion mass $m_a \approx \sqrt{V''}|_{a=0} = 0.57 \left(\frac{10^7 \text{ GeV}}{f} \right) \text{ eV}$
- f (Axion “decay constant”) $\Leftrightarrow m_a$

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Coupling to gluons is an **effective** dim.5 operator
- Needs a **UV** complete model above the scale f

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

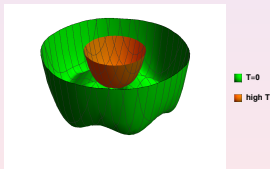
Heavy Axion

- Coupling to gluons is an **effective** dim.5 operator
- Needs a **UV** complete model above the scale f
- Axions arise from a **global $U(1)$ (Peccei-Quinn)**

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 + V_0 \cos\left(N_{DW} \frac{a}{v}\right)$$

$$\Phi = |\Phi| e^{i \frac{a}{v}}, \quad v = f N_{DW}$$

- **If $N_{DW} > 1$ (integer) \implies $V(a)$ has N_{DW} minima**



Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

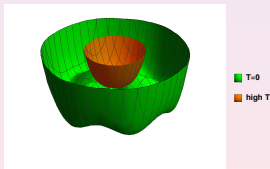
Heavy Axion

- Coupling to gluons is an **effective** dim.5 operator
- Needs a **UV** complete model above the scale f
- Axions arise from a **global $U(1)$ (Peccei-Quinn)**

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 + V_0 \cos\left(N_{DW} \frac{a}{v}\right)$$

$$\Phi = |\Phi| e^{i \frac{a}{v}}, \quad v = f N_{DW}$$

- If $N_{DW} > 1$ (integer) $\implies V(a)$ has N_{DW} minima



- \implies **Domain Walls**

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

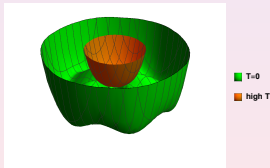
Heavy Axion

- Coupling to gluons is an **effective** dim.5 operator
- Needs a **UV** complete model above the scale f
- Axions arise from a **global $U(1)$ (Peccei-Quinn)**

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 + V_0 \cos\left(N_{DW} \frac{a}{v}\right)$$

$$\Phi = |\Phi| e^{i \frac{a}{v}}, \quad v = f N_{DW}$$

- If $N_{DW} > 1$ (integer) $\implies V(a)$ has N_{DW} minima



- \implies **Domain Walls**
- Needs a bias term ΔV
- In QCD $V_0 \approx \Lambda_{QCD}^4 \implies$ **too small Tension** σ for observable GW

Axion realization

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

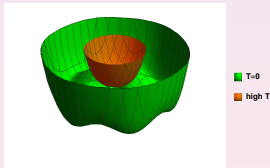
Heavy Axion

- Coupling to gluons is an **effective** dim.5 operator
- Needs a **UV** complete model above the scale f
- Axions arise from a **global $U(1)$ (Peccei-Quinn)**

$$V(\Phi) = \lambda(|\Phi|^2 - v^2)^2 + V_0 \cos\left(N_{DW} \frac{a}{v}\right)$$

$$\Phi = |\Phi| e^{i \frac{a}{v}}, \quad v = f N_{DW}$$

- If $N_{DW} > 1$ (integer) $\implies V(a)$ has N_{DW} minima



- \implies **Domain Walls**
- Needs a bias term ΔV
- In QCD $V_0 \approx \Lambda_{QCD}^4 \implies$ **too small Tension** σ for observable GW \implies **"Heavy axion scenario"**

Axion “Quality” Problem

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Phi = |\Phi| e^{i \frac{a}{v}}$, with

$$V_a \propto \Lambda_{\text{QCD}}^4 \left(1 - \cos \frac{a}{f} \right)$$

Axion “Quality” Problem

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Phi = |\Phi| e^{i \frac{a}{v}}$, with

$$V_a \propto \Lambda_{\text{QCD}}^4 \left(1 - \cos \frac{a}{f} \right)$$

- **Danger:** Global $U(1)$ may be broken by unknown high-energy physics (Quantum gravity? Axion couples to another gauge group?) \implies **possible extra term:**

$$V_b \simeq -\mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

Axion “Quality” Problem

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- $\Phi = |\Phi| e^{i \frac{a}{v}}$, with

$$V_a \propto \Lambda_{\text{QCD}}^4 \left(1 - \cos \frac{a}{f} \right)$$

- **Danger:** Global $U(1)$ may be broken by unknown high-energy physics (Quantum gravity? Axion couples to another gauge group?) \implies **possible extra term:**

$$V_b \simeq -\mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

- Generically $\delta_0 \neq 0 \implies$ Minimum is **NOT** at $\theta = \frac{a}{f} = 0 \implies$ Strong CP problem **NOT** solved

Solution: "Heavy" axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Suppose one engineers a high energy contribution **aligned** with QCD **at high scale** Λ_H :

$$V_a = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos N_{DW} \frac{a}{v} \right),$$

Solution: "Heavy" axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Suppose one engineers a high energy contribution **aligned** with QCD **at high scale Λ_H** :

$$V_a = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos N_{DW} \frac{a}{v} \right),$$

- \implies non-aligned contributions become **less dangerous**, **if $\Lambda_H \gg \mu_b$** (only a small perturbation)

$$V_b \simeq -\mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

Solution: "Heavy" axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Suppose one engineers a high energy contribution **aligned** with QCD **at high scale** Λ_H :

$$V_a = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos N_{DW} \frac{a}{v} \right),$$

- \implies non-aligned contributions become **less dangerous**, if $\Lambda_H \gg \mu_b$ (only a small perturbation)

$$V_b \simeq -\mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

- Small, potentially observable, CP violation:

$$\Delta\theta \simeq \left(\frac{\mu_b^4}{\Lambda_H^4} \right) \sin \delta_0 \ll 1$$

- nEDM measurements require $\Delta\theta \lesssim 10^{-10}$

Solution to “Axion Quality Problem”: Heavy QCD axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible high energy aligned contributions:
 - Additional gauge group at scale Λ_H unified with QCD at high energy:
 - V. A. Rubakov, Grand unification and heavy axion, JETP Lett. 65 (1997) 621D624.*
 - T. Gherghetta, N. Nagata, and M. Shifman, A Visible QCD Axion from an Enlarged Color Group, Phys. Rev. D 93 (2016), no. 11 115010.*
 - T. Gherghetta and M. D. Nguyen, A Composite Higgs with a Heavy Composite Axion, JHEP 12 (2020) 094.*
 - QCD strong again at high energies Λ_H :
 - B. Holdom and M. E. Peskin, Raising the Axion Mass, Nucl. Phys. B 208 (1982) 397D412,*
 - B. Holdom, Strong QCD at High-energies and a Heavy Axion, Phys. Lett. B 154 (1985) 316.*
 - T. Gherghetta, V. V. Khoze, A. Pomarol, and Y. Shirman, The Axion Mass from 5D Small Instantons, JHEP 03 (2020) 063.*

Solution: “Heavy” axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible high energy aligned contributions:

- \mathbf{Z}_2 symmetry (copy of SM, but at higher energy):

Z. Berezhiani, L. Gianfagna, and M. Giannotti, Strong CP problem and mirror world: The Weinberg-Wilczek axion revisited, Phys. Lett. B 500 (2001) 286D296.

Solution: "Heavy" axion?

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Possible high energy aligned contributions:

- \mathbf{Z}_2 symmetry (copy of SM, but at higher energy):

Z. Berezhiani, L. Gianfagna, and M. Giannotti, Strong CP problem and mirror world: The Weinberg-Wilczek axion revisited, Phys. Lett. B 500 (2001) 286D296.

- Observability at colliders:

S. Dimopoulos, A. Hook, J. Huang, and G. Marques-Tavares, A collider observable QCD axion, JHEP 11 (2016) 052,
A. Hook, S. Kumar, Z. Liu, and R. Sundrum, High Quality QCD Axion and the LHC, Phys. Rev. Lett. 124 (2020), no. 22 221801,
M. Bauer, M. Heiles, M. Neubert, and A. Thamm, Axion-Like Particles at Future Colliders, Eur. Phys. J. C 79 (2019), no. 1 74,
S. Chakraborty, M. Kraus, V. Loladze, T. Okui, and K. Tobioka, Heavy QCD Axion in $b \rightarrow s$ transition: Enhanced Limits and Projections, arXiv:2102.04474.

Cosmology of "Heavy" axion

- Summary:

$$V_{TOT} = \left(\Lambda_{QCD}^4 + \Lambda_H^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

with $\Lambda_H \gg \mu_b$ (and Λ_{QCD} negligible)

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Cosmology of "Heavy" axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Summary:

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

with $\Lambda_H \gg \mu_b$ (and Λ_{QCD} negligible)

- When $U(1)$ symmetry of $\Phi = |\Phi|e^{j\frac{a}{v}}$ is broken at scale f (V_{TOT} is negligible)
- a takes random values in different Hubble patches

Cosmology of "Heavy" axion

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Summary:

$$V_{TOT} = \left(\Lambda_{QCD}^4 + \Lambda_H^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

with $\Lambda_H \gg \mu_b$ (and Λ_{QCD} negligible)

- When $U(1)$ symmetry of $\Phi = |\Phi| e^{i\frac{a}{v}}$ is broken at scale f (V_{TOT} is negligible)
- a takes random values in different Hubble patches
- Cosmic strings formation (where a goes from 0 to 2π)
- Strings radiate axion quanta, reach scaling regime
 $\rho_S \approx f^2 H^2$

Cosmology of "Heavy" axion

GW from
Domain Walls

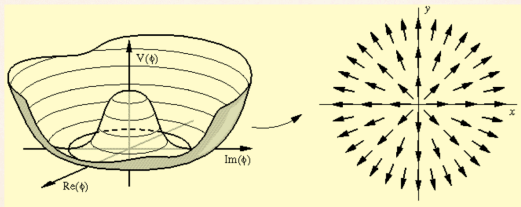
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

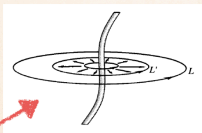
The QCD
Axion

Heavy Axion



From [Vilenkin](#), *Cosmic Strings and other topological defects*

$$\Delta a = 2\pi v$$



Cosmology of “Heavy” axion, with $N_{DW} > 1$

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_{\text{H}}^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

$$\implies m_a^2 \approx \frac{\Lambda_{\text{H}}^4}{f^2}$$

Cosmology of “Heavy” axion, with $N_{DW} > 1$

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_{\text{H}}^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

$$\implies m_a^2 \approx \frac{\Lambda_{\text{H}}^4}{f^2}$$

- When $m_a \approx 3H$, potential becomes important,
- A homogeneous field would simply oscillate

Cosmology of “Heavy” axion, with $N_{DW} > 1$

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_{\text{H}}^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

$$\implies m_a^2 \approx \frac{\Lambda_{\text{H}}^4}{f^2}$$

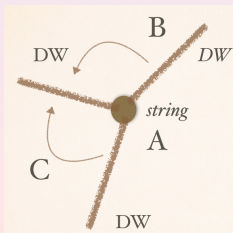
- When $m_a \approx 3H$, potential becomes important,
- A homogeneous field would simply oscillate
- **Inhomogeneous** field \implies large energy density in **domain walls** (where $\frac{a}{f} \approx \pi$)
- Domain walls attached to strings

Cosmology of “Heavy” axion, with $N_{DW} > 1$

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

$$\implies m_a^2 \approx \frac{\Lambda_H^4}{f^2}$$

- When $m_a \approx 3H$, potential becomes important,
- A homogeneous field would simply oscillate
- **Inhomogeneous** field \implies large energy density in **domain walls** (where $\frac{a}{f} \approx \pi$)
- Domain walls attached to strings

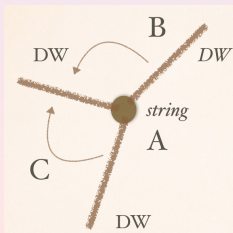


Cosmology of “Heavy” axion, with $N_{DW} > 1$

$$V_{TOT} = \left(\Lambda_{\text{QCD}}^4 + \Lambda_H^4 \right) \left(1 - \cos \frac{a}{f} \right) - \mu_b^4 \cos \left(\frac{a}{v} - \delta_0 \right),$$

$$\implies m_a^2 \approx \frac{\Lambda_H^4}{f^2}$$

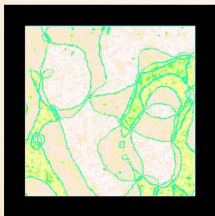
- When $m_a \approx 3H$, potential becomes important,
- A homogeneous field would simply oscillate
- **Inhomogeneous** field \implies large energy density in **domain walls** (where $\frac{a}{f} \approx \pi$)
- Domain walls attached to strings



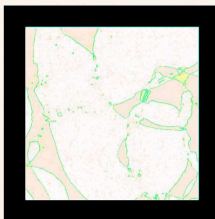
Cosmology of “Heavy” axion, with $N_{DW} > 1$

Simulations from Kawasaki, Saikawa, Sekiguchi 14, PRD 91

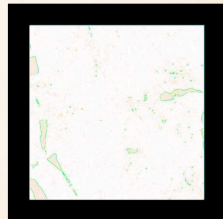
$N_{DW} = 6$



(b) $\Xi = 0.00006$, $\tau = 42$



(d) $\Xi = 0.00006$, $\tau = 62$



(f) $\Xi = 0.00006$, $\tau = 82$

← T

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

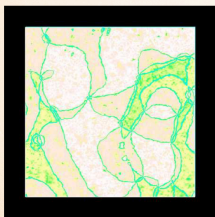
The QCD
Axion

Heavy Axion

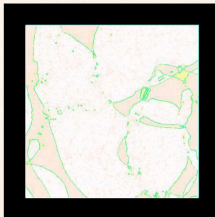
Cosmology of “Heavy” axion, with $N_{DW} > 1$

Simulations from Kawasaki, Saikawa, Sekiguchi 14, PRD 91

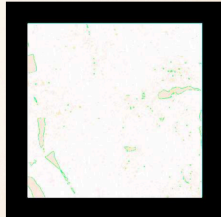
$N_{DW} = 6$



(b) $\Xi = 0.00006$, $\tau = 42$



(d) $\Xi = 0.00006$, $\tau = 62$



(f) $\Xi = 0.00006$, $\tau = 82$

← T

- Later μ_b breaks degeneracy among vacua
⇒ DW decay ⇒ a sits in true vacuum

Small CP violation at the minimum

GW from
Domain Walls

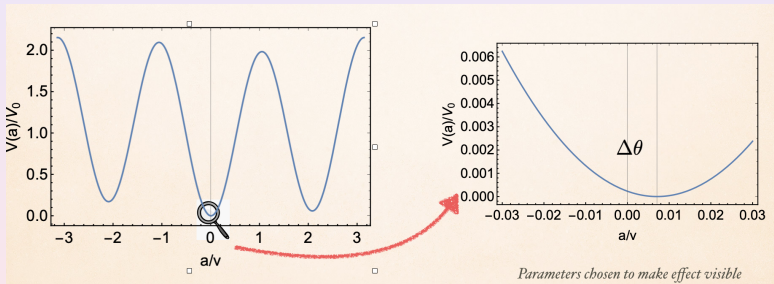
Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion



Heavy Axion at NANOGrav - IPTA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Tension $\sigma = m_a f^2$
(much larger than for “Standard” QCD Axion)

Heavy Axion at NANOGrav - IPTA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Tension $\sigma = m_a f^2$
(much larger than for “Standard” QCD Axion)

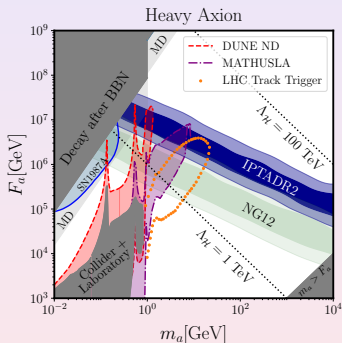


Figure: Marginalized over bias μ_b

(R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, arXiv: 2204.04228 (2022))

- Decay rate into gluons/photons $\Gamma \approx m_a^3 / f^2$

Heavy Axion at LIGO/Virgo/KAGRA and LISA

- Heavy axion with **High scale** $\Lambda_H \implies$ signals at **Interferometers** (R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, PRL 2022)

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

Heavy Axion at LIGO/Virgo/KAGRA and LISA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Heavy axion with **High scale** $\Lambda_H \implies$ signals at **Interferometers** (R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, PRL 2022)
- Correlated with nEDM signal:

$$\Delta\theta \simeq \left(\frac{\mu_b^4}{\Lambda_H^4} \right) \sin \delta_0 \ll 1$$

Heavy Axion at LIGO/Virgo/KAGRA and LISA

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Heavy axion with **High scale** $\Lambda_H \implies$ signals at **Interferometers** (R. Z. Ferreira, A.N., O. Pujolas, F. Rompineve, PRL 2022)
- Correlated with nEDM signal:

$$\Delta\theta \simeq \left(\frac{\mu_b^4}{\Lambda_H^4} \right) \sin \delta_0 \ll 1$$

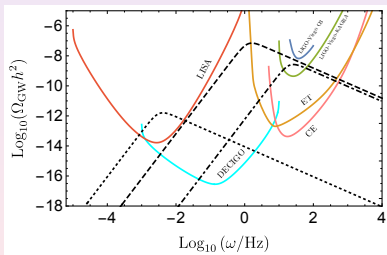


Figure: GW spectra ($N_b = 1$, $N_{DW} = 6$, $\delta_0 = 0.3$).

Dashed: $\Lambda_H = 10^{10}$ GeV, $f = 10^{11}$ GeV and $\Delta\theta \simeq 8 \cdot 10^{-13}$.

Dotted: $\Lambda_H = 10^7$ GeV, $f = 2.5 \cdot 10^{10}$ GeV $\Delta\theta \simeq 8 \cdot 10^{-13}$.

Dot-dashed: $\Lambda_H = 10^{11}$ GeV, $f = 1.6 \cdot 10^{11}$ GeV and $\Delta\theta \simeq 1.5 \cdot 10^{-11}$.

Heavy Axion : GW and nEDM experiments

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

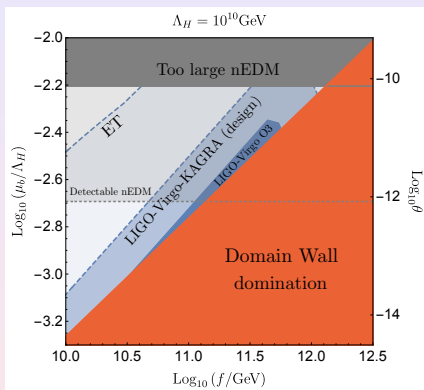


Figure: Here $\Lambda_H = 10^{10} \text{ GeV}$.

- GW from decaying DWs correlated to Neutron Electric Dipole (nEDM)

Conclusions

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Did **NANOGrav/IPTA** see **GWs**?
- Wait for **Hellings-Downs** angular **correlations**

Conclusions

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Did **NANOGrav/IPTA** see **GWs**?
- Wait for **Hellings-Downs** angular **correlations**
- If yes, **decaying DWs** fit well the data
- Interesting scales: $\sigma^{1/3} \approx 10 - 100 \text{ TeV}$ and $\Delta V \approx 10 - 100 \text{ MeV}$ (close to **QCD PT**)

Conclusions

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Did **NANOGrav/IPTA** see **GWs**?
- Wait for **Hellings-Downs** angular **correlations**
- If yes, **decaying DWs** fit well the data
- Interesting scales: $\sigma^{1/3} \approx 10 - 100 \text{ TeV}$ and $\Delta V \approx 10 - 100 \text{ MeV}$ (close to **QCD PT**)
- Could be related to **heavy axions** (with misaligned terms in the potential, with different N_{DW})

Conclusions

GW from
Domain Walls

Domain Walls

Gravitational
Waves from
DWs

GW spectra
PTA

The QCD
Axion

Heavy Axion

- Did **NANOGrav/IPTA** see **GWs**?
- Wait for **Hellings-Downs** angular **correlations**
- If yes, **decaying DWs** fit well the data
- Interesting scales: $\sigma^{1/3} \approx 10 - 100 \text{ TeV}$ and $\Delta V \approx 10 - 100 \text{ MeV}$ (close to **QCD PT**)
- Could be related to **heavy axions** (with misaligned terms in the potential, with different N_{DW})
- Heavy Axion models could also give a **signal at LISA, LIGO/Virgo/KAGRA**, correlated with **nEDM**)