Cusp effects in K→pai pi0 pi0

A case of constructive interaction
between experiment and theory

Nicola e la

FISICA DEI PIONI

Varenna 1958

B. Toushek

Determination of the $\pi\pi$ scattering length from $K^{\pm} \to \pi^{\pm} \pi^{0} \pi^{0}$ decays

Nicola Cabibbo [(1)] and Italo Mannelli [(2)]

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NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Why does NA48/2 collect so many $K^{\pm}$?

Direct CP -violation program

1997-2001 NA48/1 $Re(\varepsilon'/\varepsilon) = (14.7 \pm 2.2)\times 10^{-4}$ Neutral Kaon

2003-2004 NA48/2 $\Delta g = (0.5 \pm 3.8)\times 10^{-4}$ Charged Kaon

Future NA48/3 intent to measure $BR(K^{+} \to \pi^{+}\nu\nu)$

$K^{\pm} \to 3\pi$

$d\Gamma/du = 1 + gu$

$\Delta g = (g^{+} - g^{-})/(g^{+} + g^{-})$

looking for difference in the Dalitz plot distribution

Statistics Collected 2003-2004

$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ $\sim 2\cdot 10^{8}$

$K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ $\sim 4\cdot 10^{9}$

(loose selection cuts)

NA48 Detector

Main detector components:

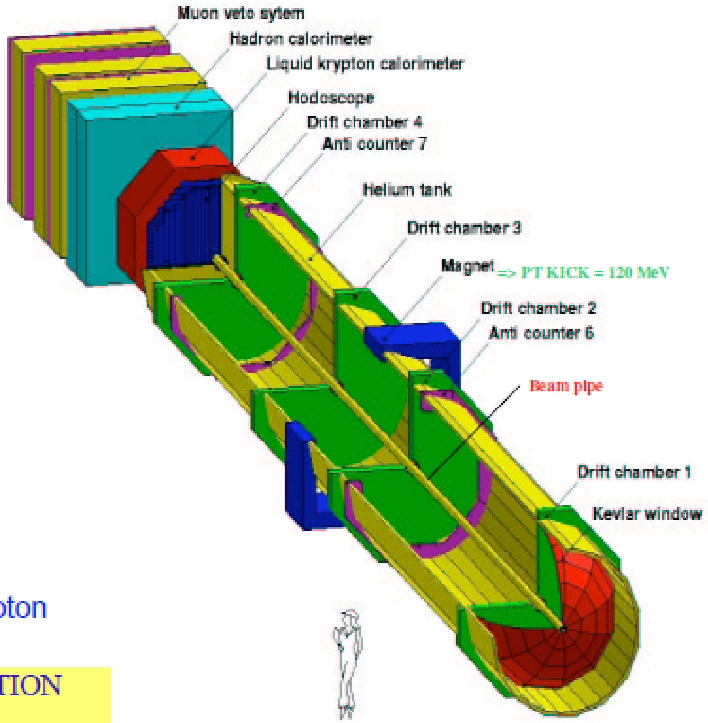
- Magnetic spectrometer (4 DCHs):
  redundancy ⇒ high efficiency;
  $\Delta p/p = 1.0\% + 0.044\% * p$ [GeV/c]

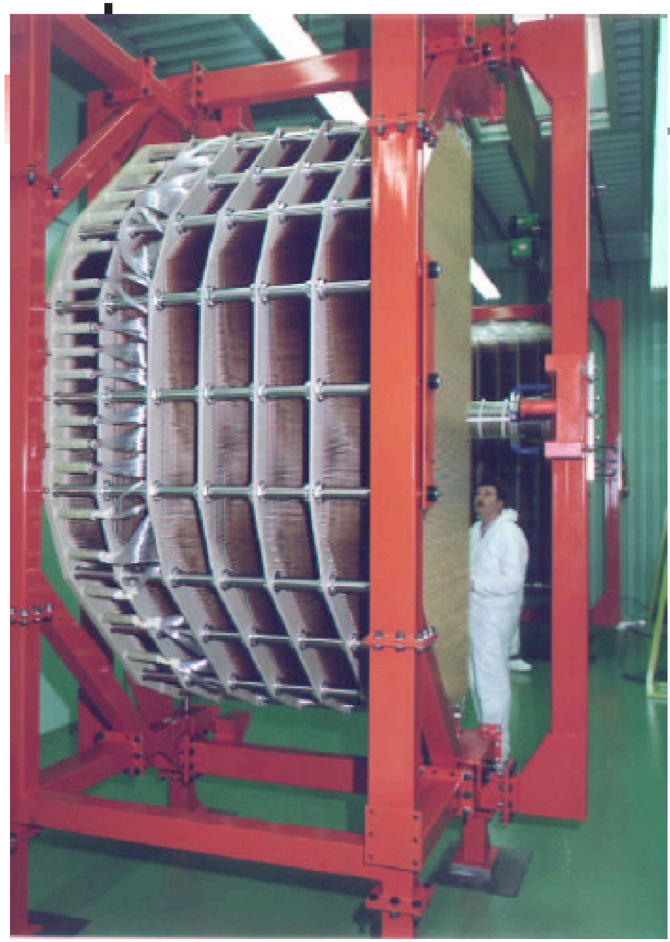
- Hodoscope
  fast trigger;
  precise time measurement (150ps).

- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogeneous;
  $\Delta\sigma(E)/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$ [GeV].

- Hadron calorimeter, muon veto counters, photon vetoes.

☺ HIGH RATE, HIGH RESOLUTION
☹ BEAM PIPE





NA48 set up: LKR Calorimeter

13248 projective cells, 2 x 2 cm²

$$\frac{\sigma(E)}{E} = \frac{0.032}{\sqrt{E}} \oplus \frac{0.09}{E} \oplus 0.0042$$

$$\sigma_x = \sigma_y = \frac{0.42}{\sqrt{E}} \oplus 0.06\,\text{cm}$$

(E in GeV)

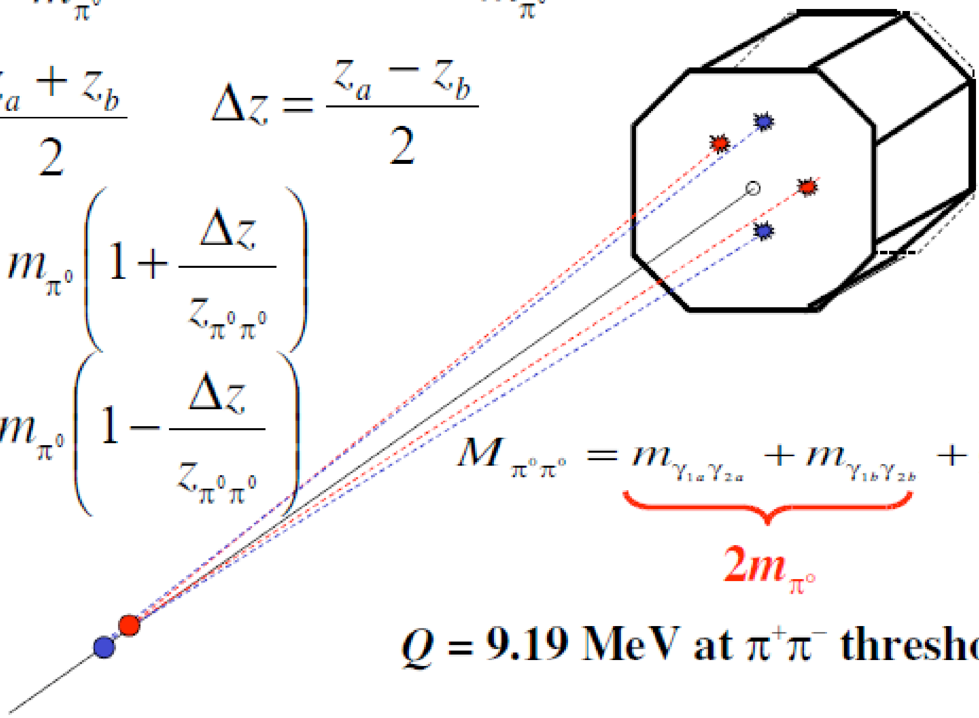
Multi π^0 detection capability

$$z_a = \frac{d_{12a}\sqrt{E_{1a}E_{2a}}}{m_{\pi^0}} \qquad z_b = \frac{d_{12b}\sqrt{E_{1b}E_{2b}}}{m_{\pi^0}}$$

$$z_{\pi^0\pi^0} = \frac{z_a + z_b}{2} \qquad \Delta z = \frac{z_a - z_b}{2}$$

$$m_{\gamma_{1a}\gamma_{2a}} = m_{\pi^0}\left(1 + \frac{\Delta z}{z_{\pi^0\pi^0}}\right)$$

$$m_{\gamma_{1b}\gamma_{2b}} = m_{\pi^0}\left(1 - \frac{\Delta z}{z_{\pi^0\pi^0}}\right)$$



$$M_{\pi^0\pi^0} = \underbrace{m_{\gamma_{1a}\gamma_{2a}} + m_{\gamma_{1b}\gamma_{2b}}}_{2m_{\pi^0}} + Q$$

Q = 9.19 MeV at $\pi^+\pi^-$ threshold

$\sigma(Q)$ ~ 0.42 MeV

Dirac experiment at Cern

The Dirac experiment is dedicated to measure the lifetime of Pionium

i.e. EM bound $\pi^+\pi^-$ state expected to decay almost exclusively into $\pi^0\pi^0$ with $\tau \approx 3\ 10^{-15}$ sec

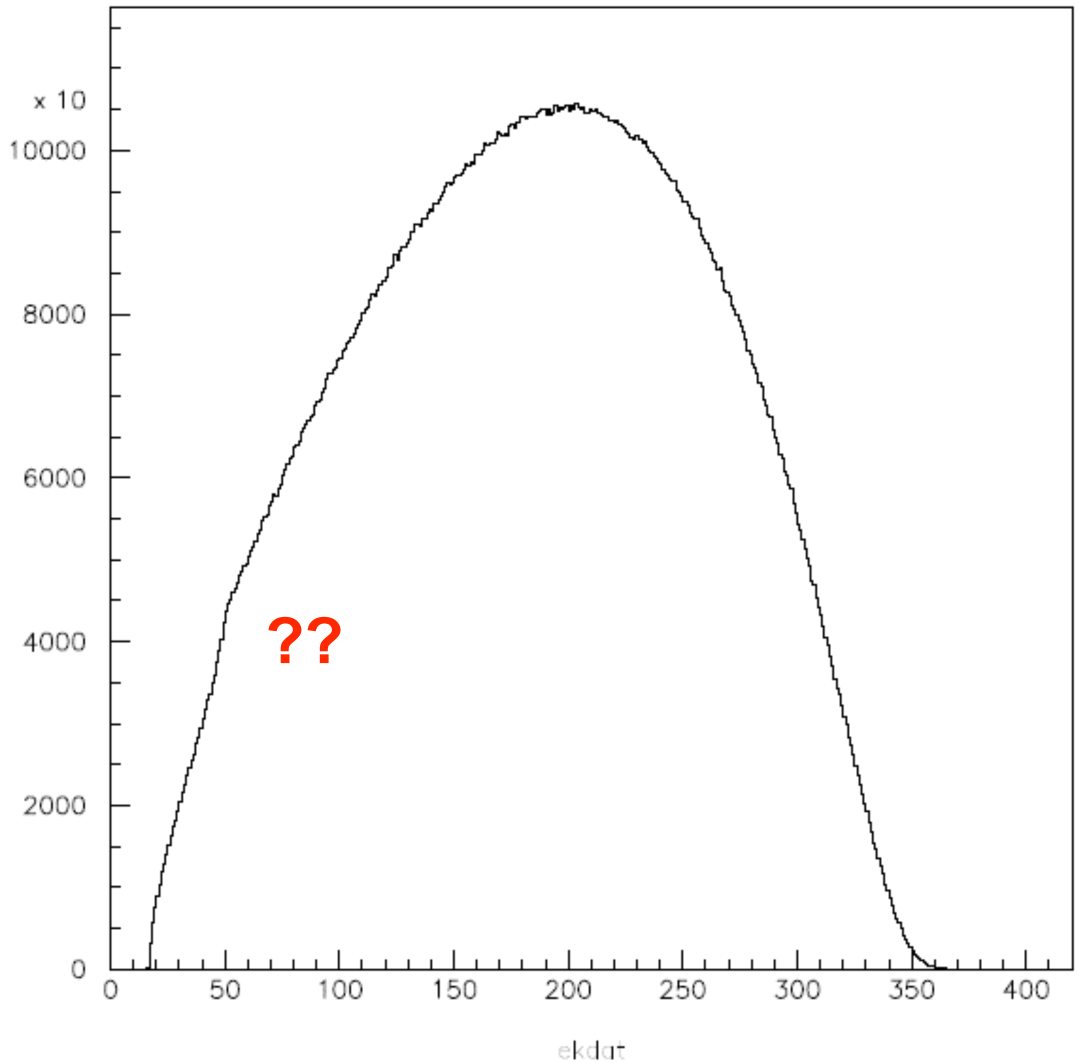
Collinear $\pi^+\pi^-$ of near equal momentum are inclusively produced in collisions onto nuclear targets of various atomic number and geometries proton

The rate of decay is directly connected to the difference $(a_0 - a_2)$ of the S-wave $\pi\pi$ scattering lengths

The pionium lifetime is deduced from the difference between the single and the multi-layer target distribution.

Only pionic atoms candidates which are "ionized" are counted:

No $\pi^0\pi^0$ are detected



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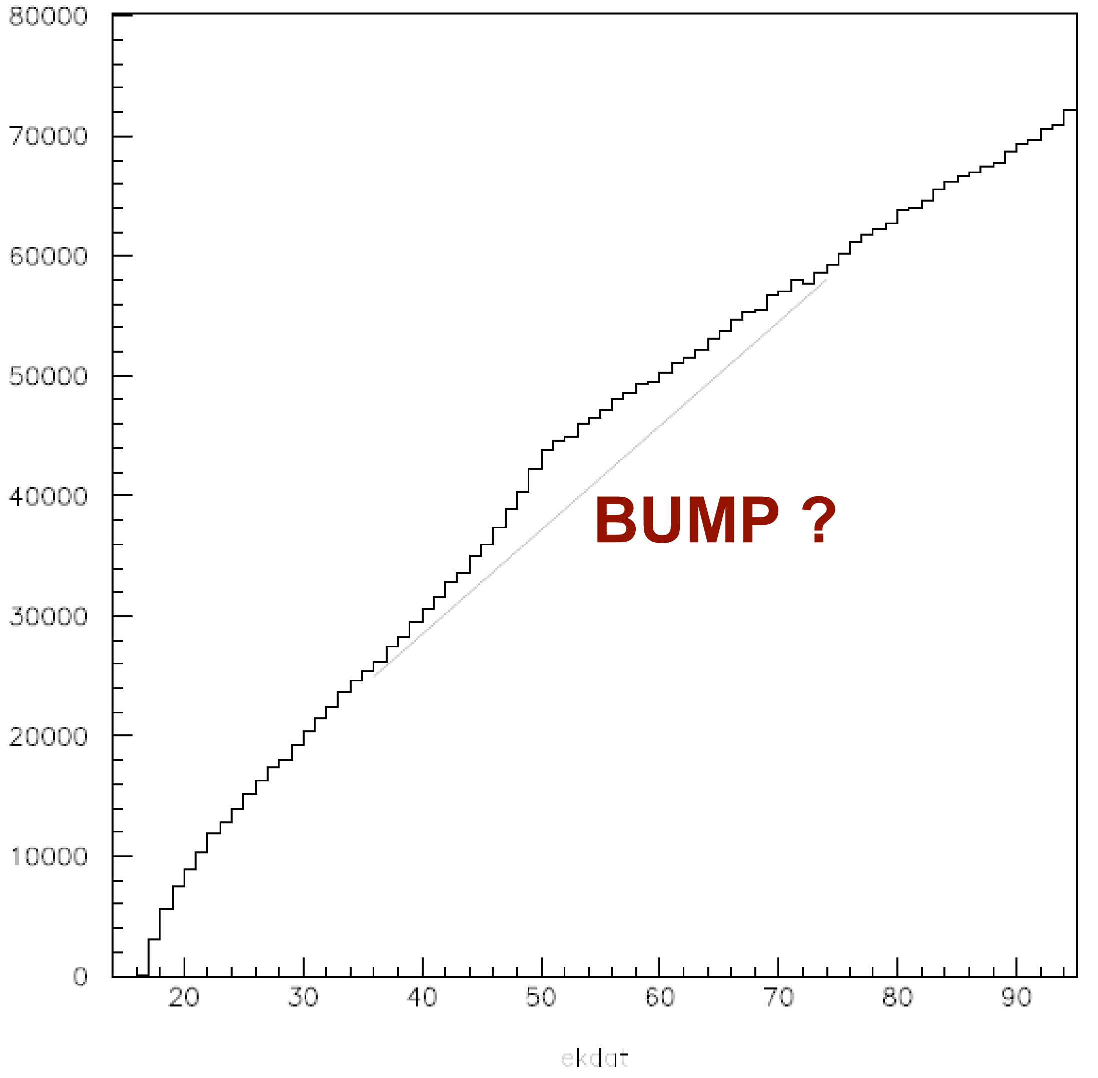
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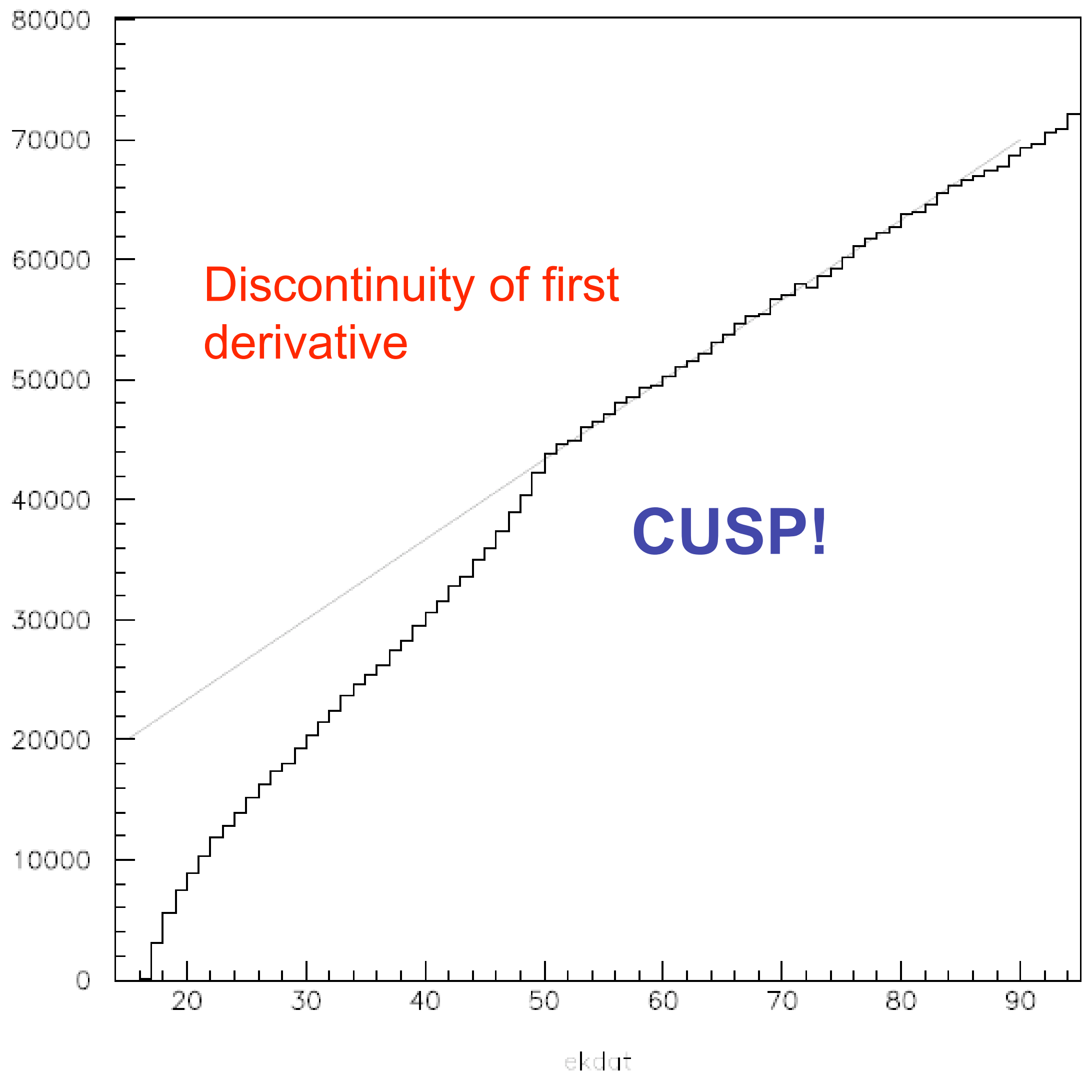
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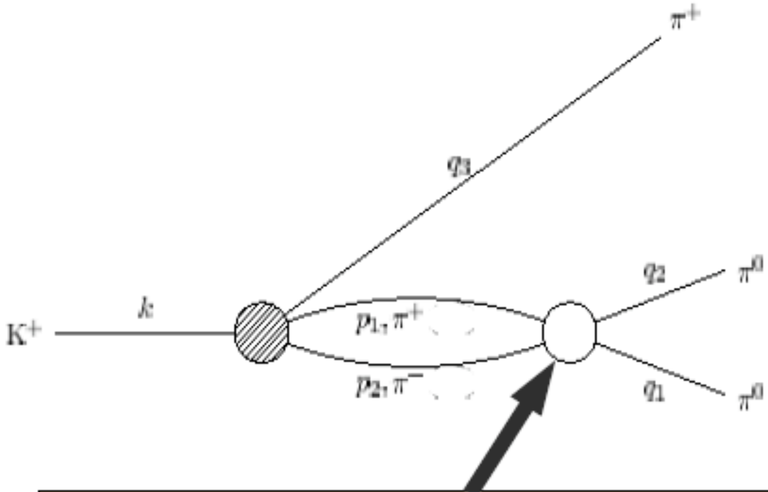
Discontinuity of first derivative
CUSP!

$\pi^- \pi^+ \longrightarrow \pi^0 \pi^0$ charge exchange scattering

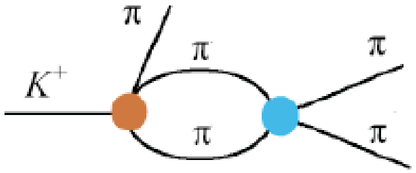
A diagram contributing to $K^+ \dashrightarrow \pi^+ \pi^0 \pi^0$



- Strong rescattering + charge exchange with effective coupling constant $\propto (a_0 - a_2)$
- Cusp effect induced

Theoretical Approach (CI)

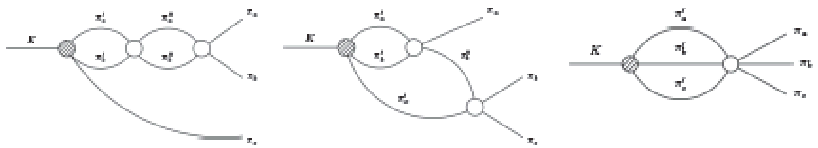
Explanation: (Cabibbo, PRL 93 (2004) 121801)

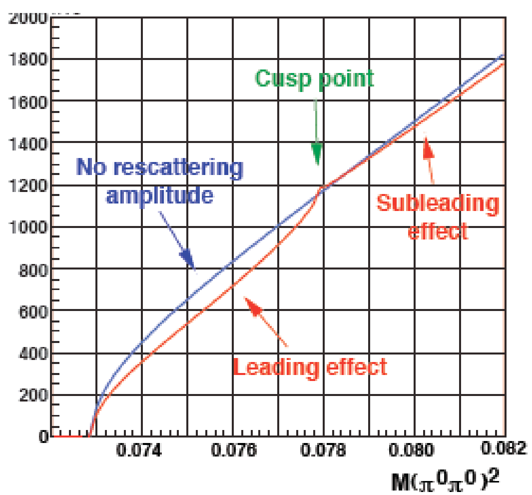


- $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ rescattering amplitude
- depends on **a_0 - a_2**

Full computation: (Cabibbo, Isidori, JHEP03 (2005) 21)

- rescattering corrections from
  $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$, $\pi^+ \pi^0 \rightarrow \pi^+ \pi^0$, ...
- two-loop level $O(a_i^2)$ corrections





⇒ sensitivity to a_2 alone

- no $O(a_i^3)$, no radiative corrections

Theoretical predictions

Weinberg (1966)
Effective field theory for
strong interaction at low E

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Most recently
Colangelo et al. (2001)
CH-pt -theory two loops
Ref: hep- ph/0103088

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

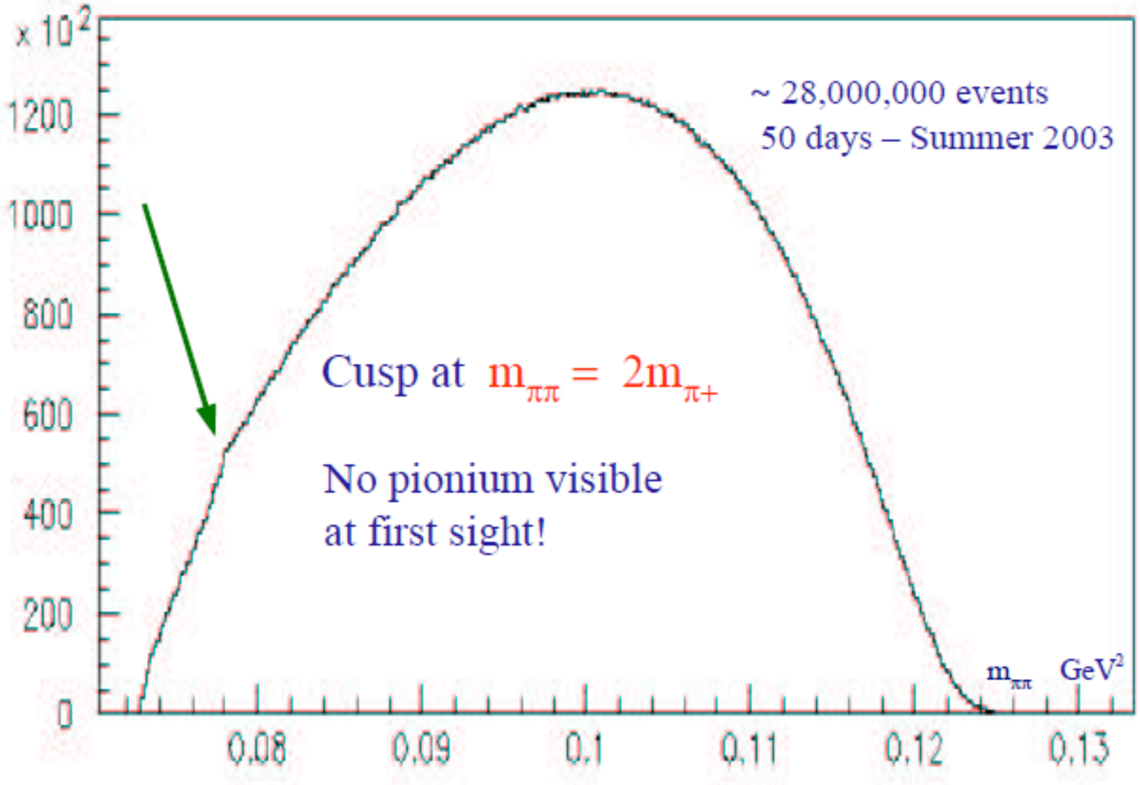
$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

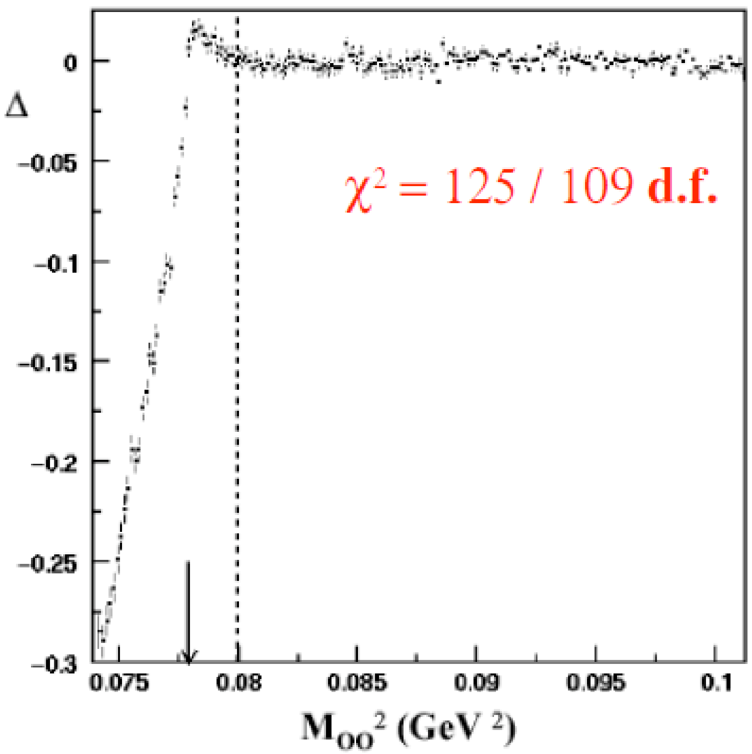
- 2% level of accuracy is quite unusual for hadronic physics

→ experiments have not yet reached the same level of accuracy

NA48/2 data $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$



Fit above 0.08 Gev^2: PDG prescription



Δ = (data-fit)/data

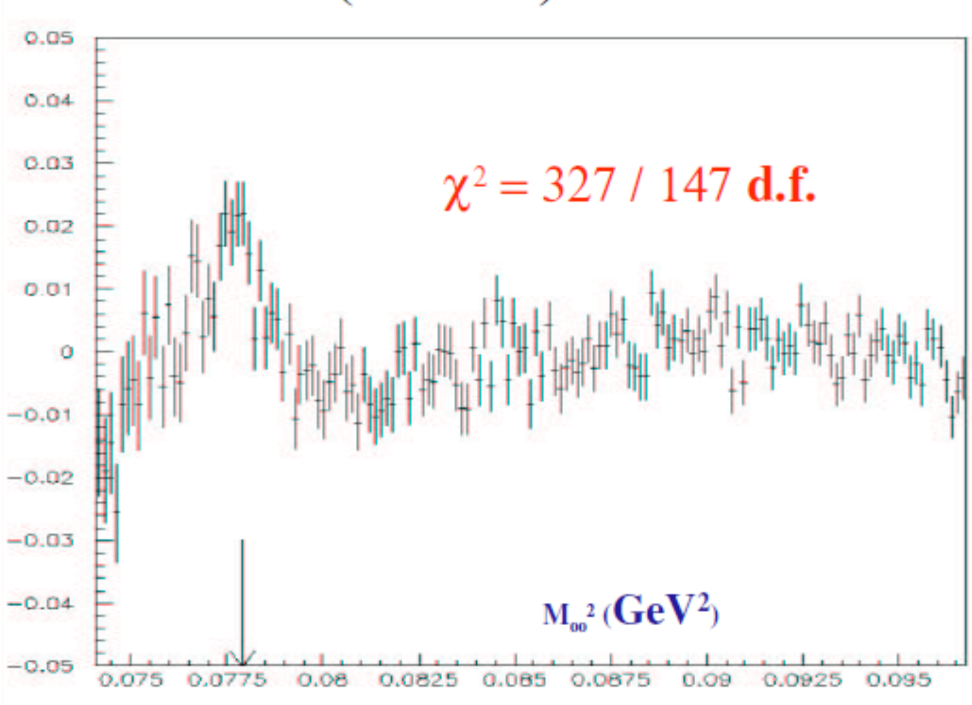
$$u = \frac{M_{00}^2 - s_0}{m_+^2}$$

$$s_0 = \frac{1}{3}\left(m_K^2 + m_+^2 + 2m_0^2\right)$$

$$\mathcal{M}_0 = 1 + \frac{1}{2} g_0 u$$

One – loop charge – exchange $\pi^+\pi^- \rightarrow \pi^\circ\pi^\circ$

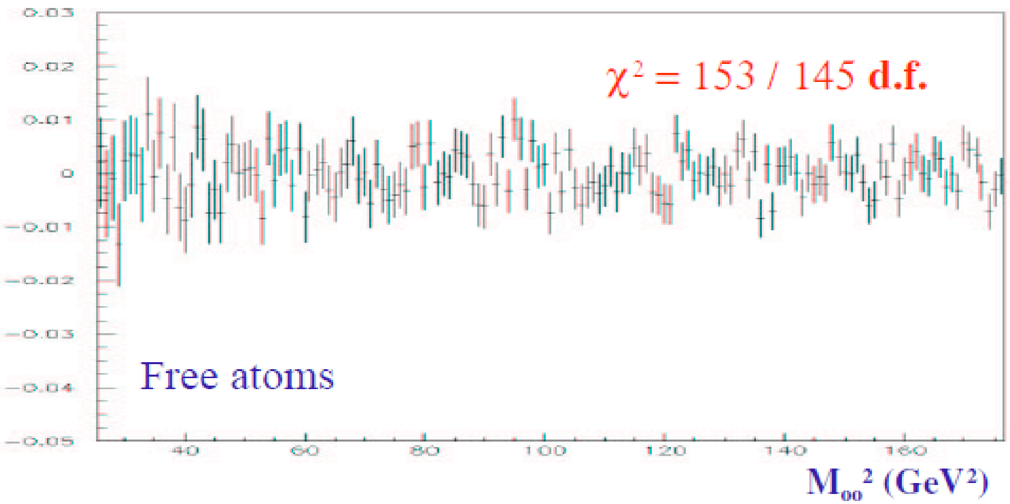
Δ = (data-fit)/data



N.Cabibbo,G.Isidori JHEP03 (2005) 21

Fit including $\pi^+\pi^-$ atoms

Δ = (data-fit)/data



Fit result

$$\frac{K^\pm \to \pi^\pm + \text{atom}}{K^\pm \to \pi^\pm\pi^+\pi^-} \approx 1.4 \cdot 10^{-5}$$

$$\frac{K^\pm \to \pi^\pm + \text{atom}}{K^\pm \to \pi^\pm\pi^+\pi^-} \approx 0.8\times10^{-5}$$

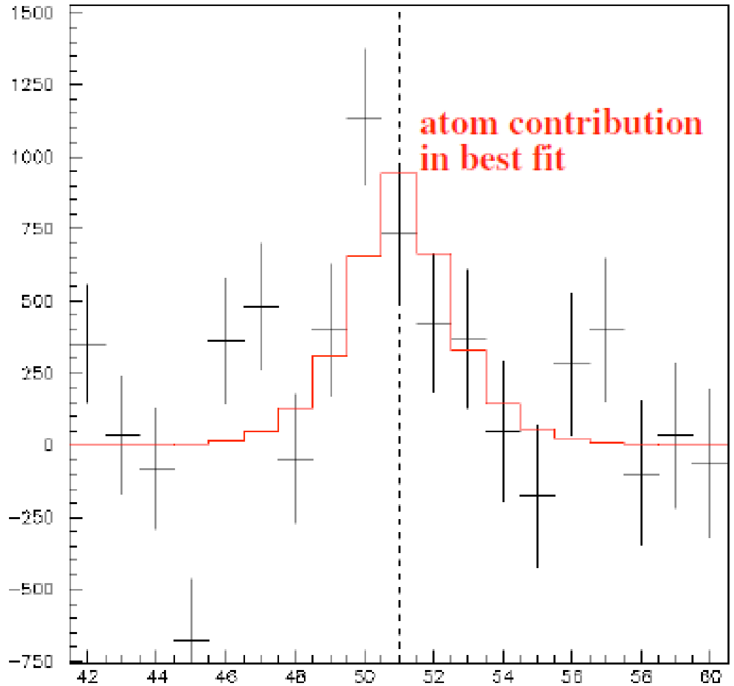
Silagadze 1994

$\chi^2 = 154 / 146$ **d.f.**

Forcing 0-atoms

$\chi^2 = 160 / 146$ **d.f.**

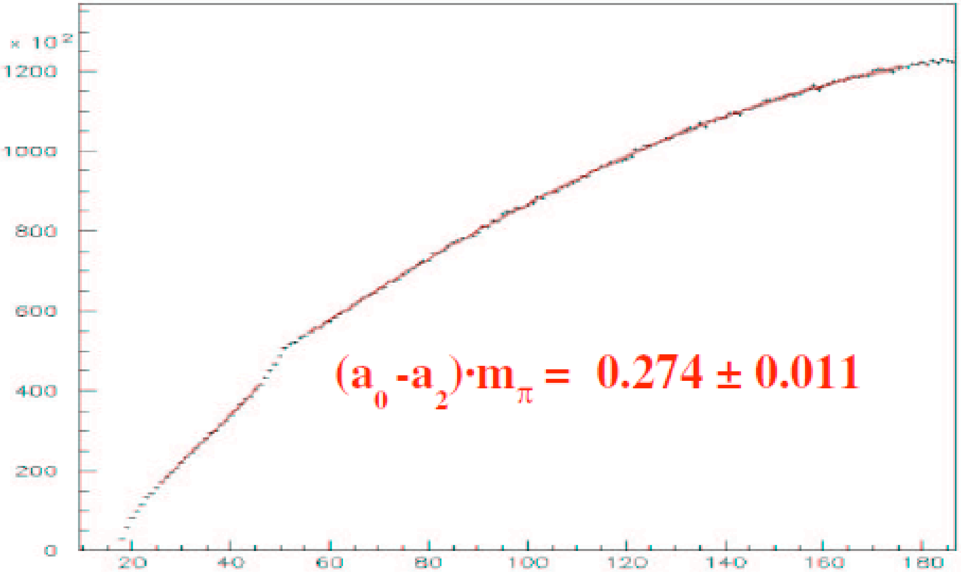
Data – Fit (excluding atoms)



1 **bin** = 0.00015 **GeV**^{2}

Systematics Check: fitting region

Fit excluding 10 bins around the cusp



Choice of the fitting region systematic on $(a_0 - a_2) \cdot m_\pi$ ± 0.008

Conclusions

By including the full 2003-2004 data sample we expect an increase in statistics from the 30M events used in the present analysis by a factor of at least 4

The study of the systematic uncertainties is well under way, (detailed MonteCarlo, additional systematics effects)

The data quality calls for additional theoretical effort in order to extract precise values of the pion-pion scattering parameters

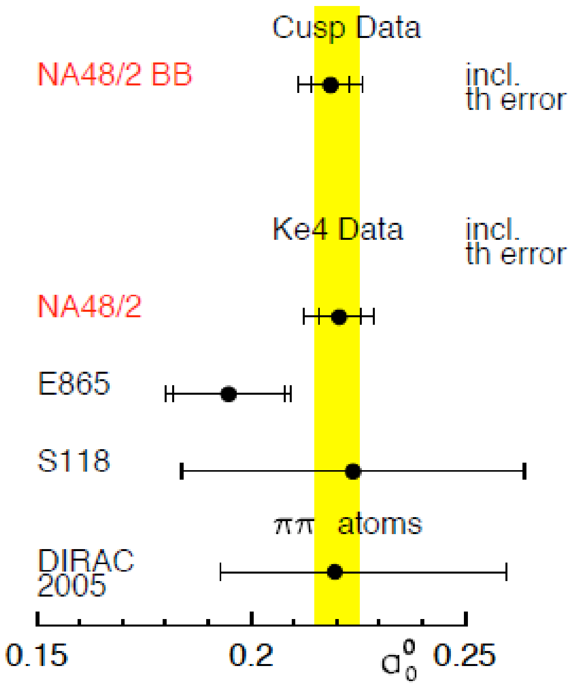
A study of the corresponding effects in $K_L \rightarrow 3\pi^0$ from the 2002 NA48 has been started

...the PDG pages related to the K Dalitz plot description will be deeply modified ......

Data have also been collected by NA48/2 on Ke4 decays, which will provide independent information on pion-pion scattering

Combination with other Measurements

All experiments:



Experimental data has reached theoretical precision

Perfect agreement with ChPT prediction

Yellow line: Theory prediction
(*not* combination of measurements)

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