Computers and the Weak Interactions

Nicola Cabibbo Memorial Symposium

November 12, 2010

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RBC/UKQCD Collaboration

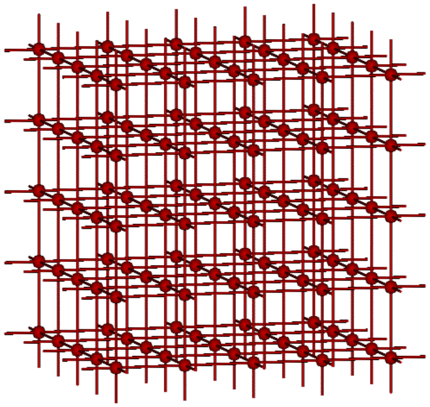
Outline

- Computers, Weak Interactions and Cabibbo.
- Technique
  - Lattice regularization
  - Domain wall fermions
  - Ensembles
- Cabibbo Universality
- $\overline{K^0} - K^0$ mixing
- $K \rightarrow \pi\,\pi$ decay
- Conclusion

Technique

Lattice QCD

- The **<u>only</u>** first-principles approach to the theoretical study of low energy QCD.
- Lattice calculation now accurate at the few percent level.
- No known barrier to arbitrarily increased accuracy.



- Present success driven by algorithm + computer technology.

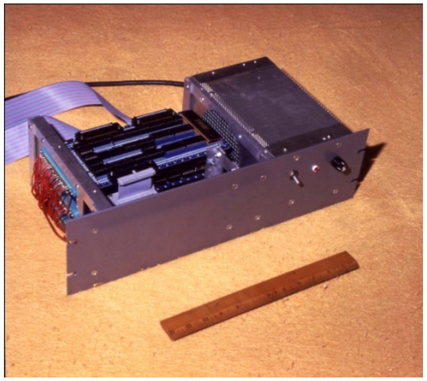
10^7 x

VAX 780 (1984)
1 Mflops (10^6)

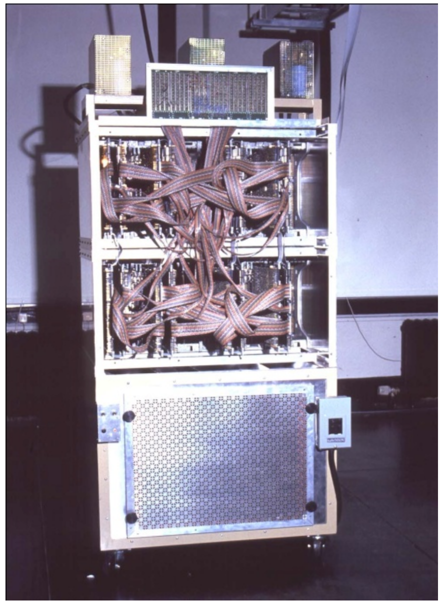
BG/P (2008)
10 Tflops (10^{13})

“Columbia” machines

1981



1985



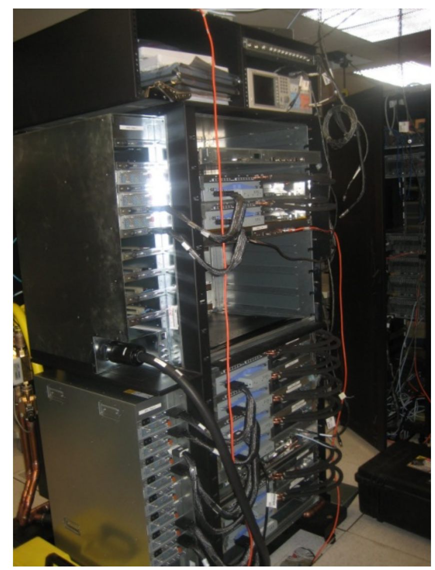
1987

1989

1998

2005

2010



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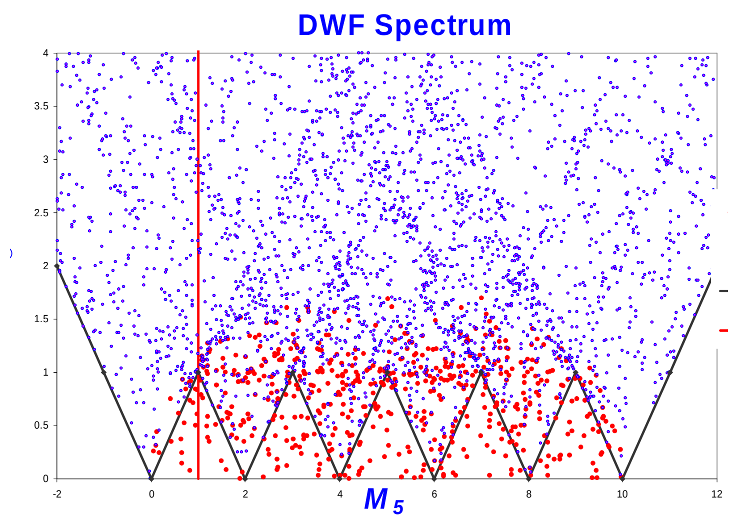
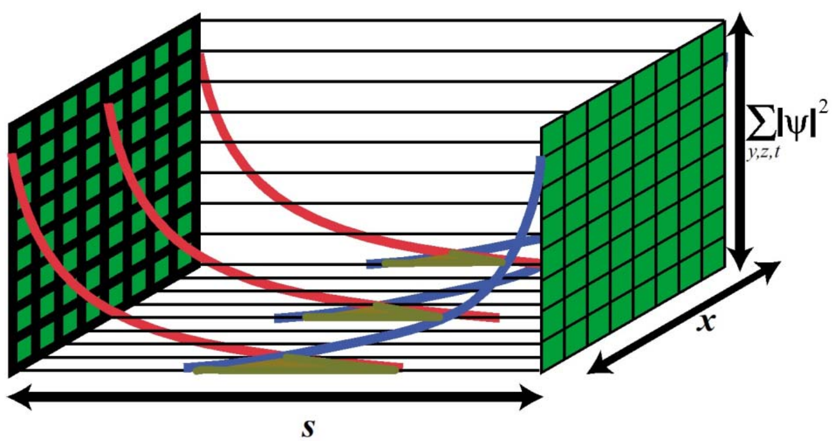
Domain Wall Fermions

- Invented by Kaplan, 1993.
- 5-D theory with 4-D, chiral surface states.
- Typical 5-D extent of 16.
- $L_s \rightarrow \infty$ gives the overlap operator of Neuberger.



5-D mass

Simulations run at $M_5 = 1.8$



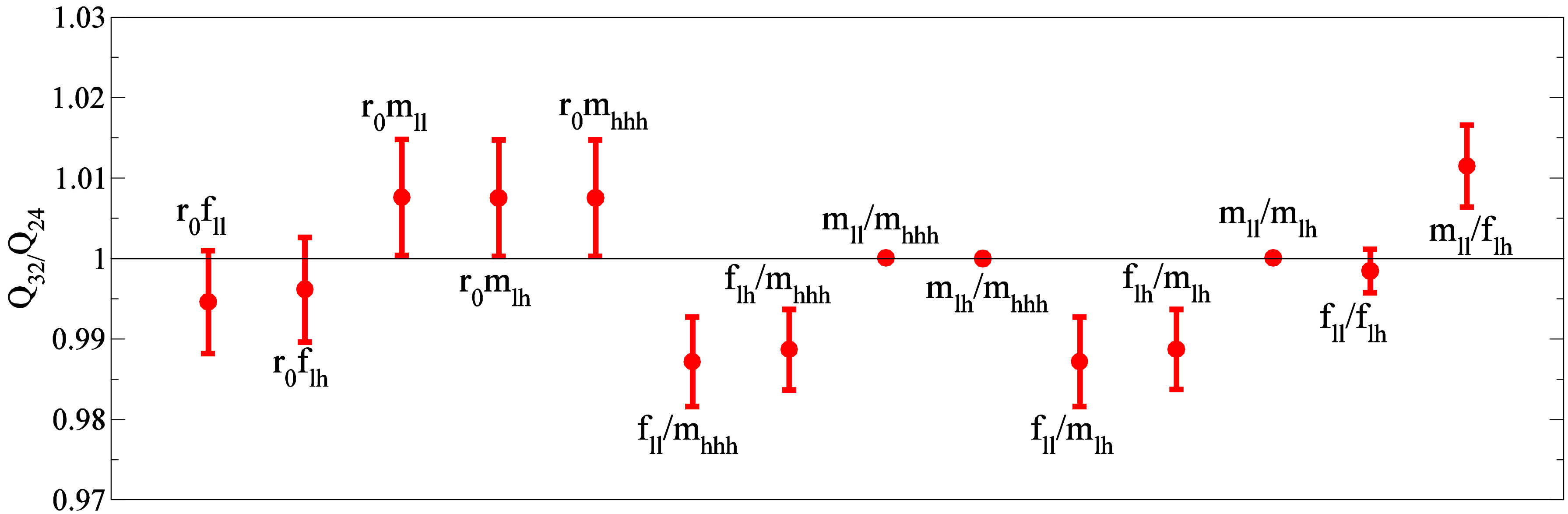
Monte Carlo Ensembles

- RBC/UKQCD gauge ensembles:

Volume	*1/a*	*L*	m_π	Time units	$m_{\text{quark}}a$	Gauge Action
24^3 x 64	1.73 GeV	2.7 fm	315 MeV	9000	0.005+0.0032	Iwasaki
			402 MeV	9000	0.01+0.0032	
32^3 x 64	2.28 GeV	2.7 fm	290 MeV	7000	0.004+0.0006	
			350 MeV	8000	0.006+0.0006	
			410 MeV	6000	0.008+0.0006	
32^3 x 64	1.4 GeV	4.5 fm	180 MeV	1000	0.001+0.0018	Iwasaki + DSDR
			250 MeV	1800	0.004+0.0018	

Scaling: 1.73 GeV (24^3) – 2.28 GeV (32^3)

(Chris Kelly)

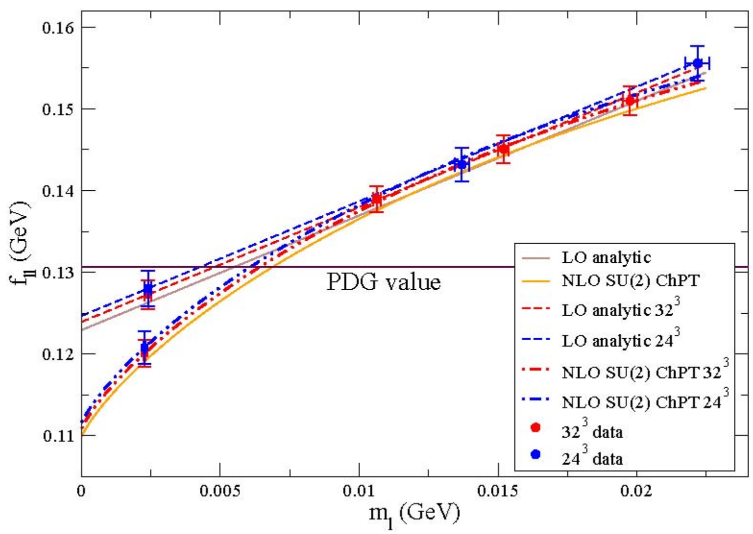


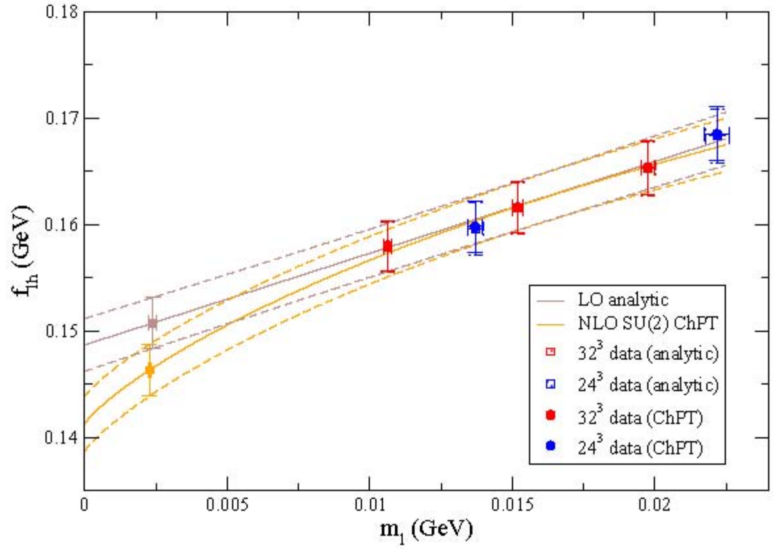
Ratios of dimensionless combinations of physical quantities computed using $1/a$ = 1.73 and 2.28 GeV.

Pseudo-scalar decay constants

Cabibbo Universality: f_π & f_K

$$225\ \text{MeV} \leq m_\pi \leq 420\ \text{MeV}$$

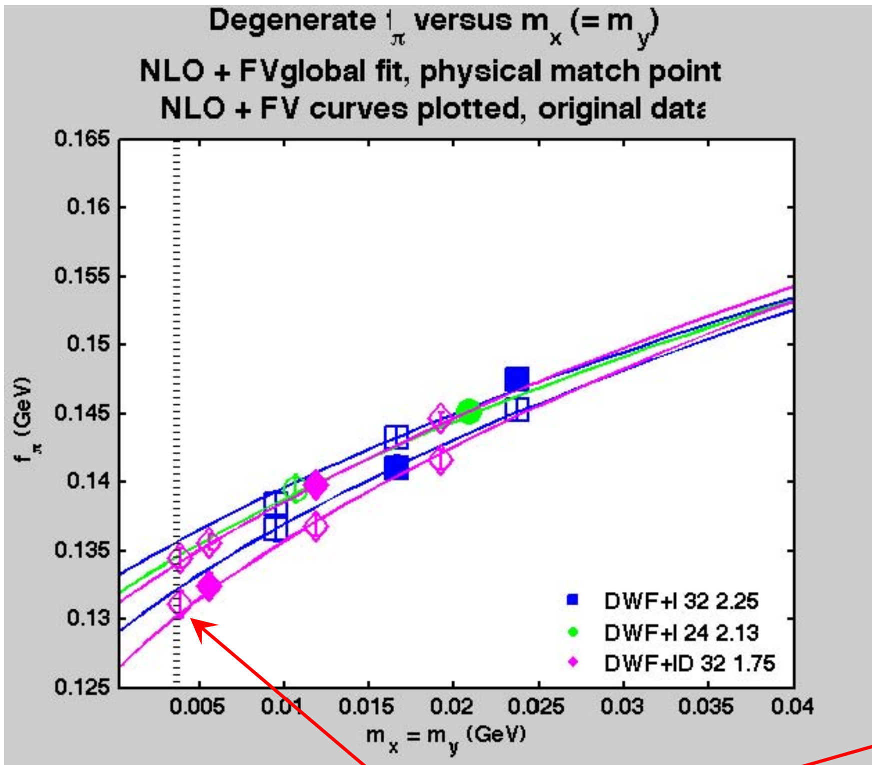


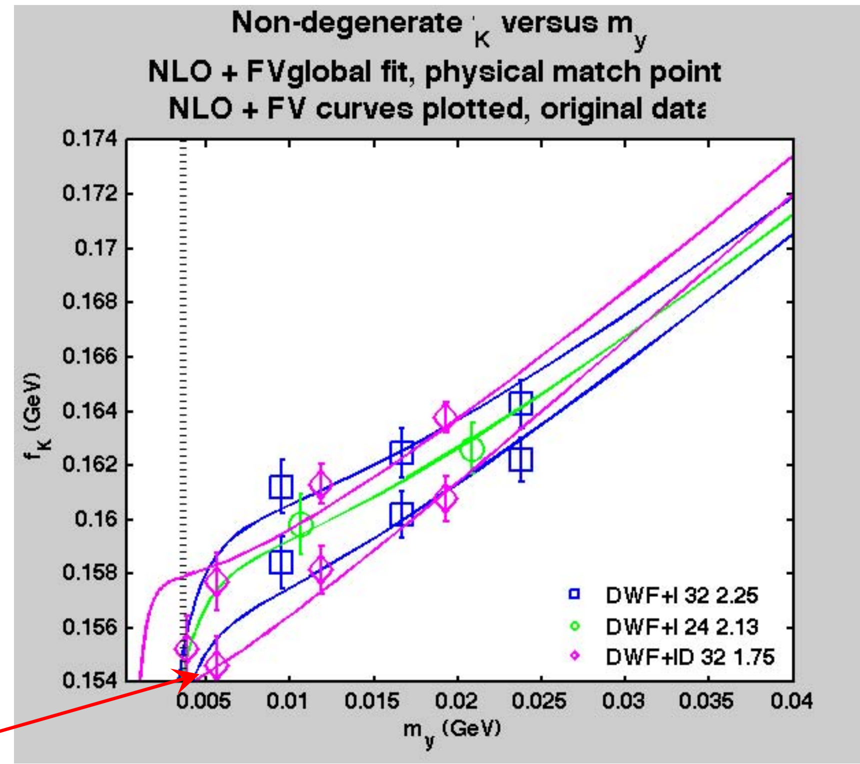


f_π (MeV)	$124\ (2)_{stat}(5)_{sys}$	130.4(0.2) [expt]
f_K (MeV)	$149(2)_{stat}(3)_{sys}$	156.1(0.9) [expt]
f_K / f_π	$1.204(7)_{stat}(25)_{sys}$	1.197(6) [expt]

Cabibbo Universality: f_π & f_K

145 MeV ≤ m_π ≤ 370 MeV





m_π = 145 MeV close to expt.

Indirect CP violation
$K^0 - \bar{K}^0$ mixing

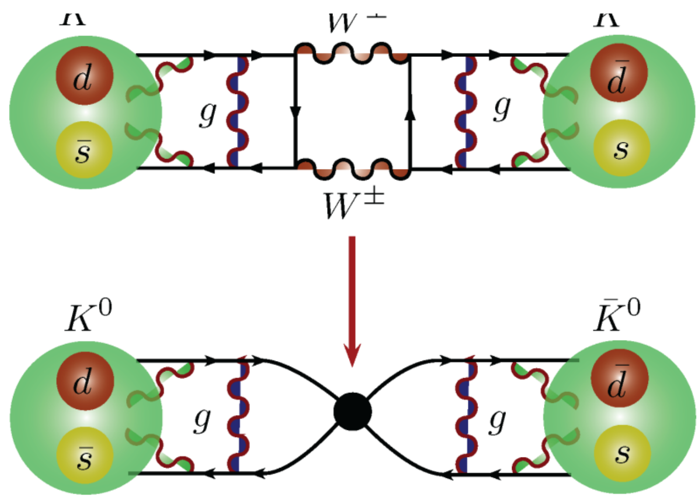
Indirect CP Violation

- CP violating phase of K^0 - $\overline{K^0}$ mixing amplitude specified by the CP odd parameter ε

$$\epsilon = \hat{B}_K \, \mathrm{Im}\lambda_t \, \frac{G_F^2 f_K^2 m_K M_W^2}{12\sqrt{2}\pi^2 \Delta M_K} \left\{ \mathrm{Re}\lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t) \right] - \mathrm{Re}\lambda_t \, \eta_2 S_0(x_t) \right\} \exp(i\pi/4)$$

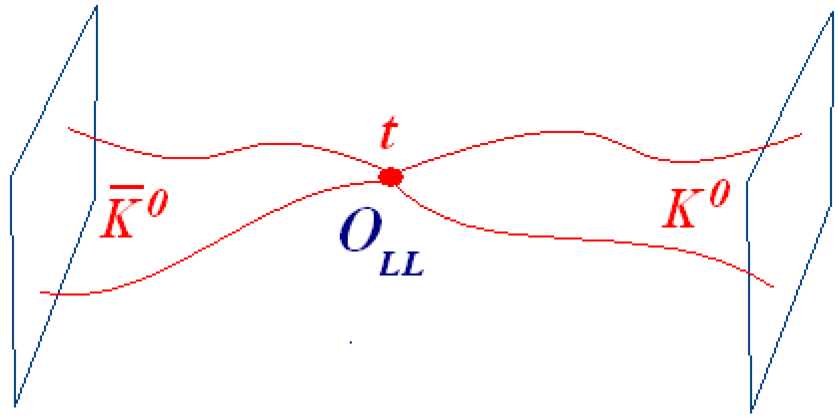
$$\langle \overline{K}^0 | Q^{(\Delta S=2)}(\mu) | K^0 | \rangle \equiv \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$

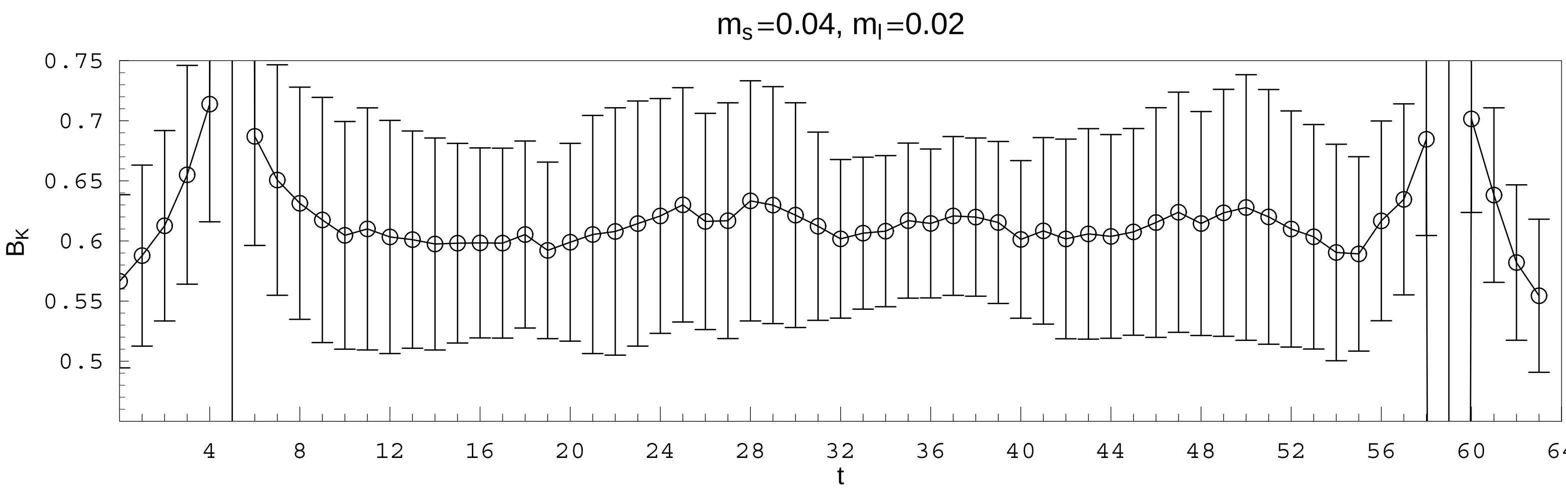
- $\lambda_k = V_{kd} V_{ks}^*$, $k = u, c, t$
- $x_k = m_k^2 / m_W^2$
- The matrix element B_K which can be only computed from lattice QCD.



Indirect CP Violation

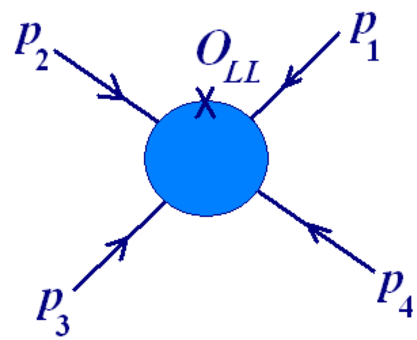
- O_{LL} matrix element:
  - K^0 on right
  - $\bar{K}^0$ on left
  - Operator at t

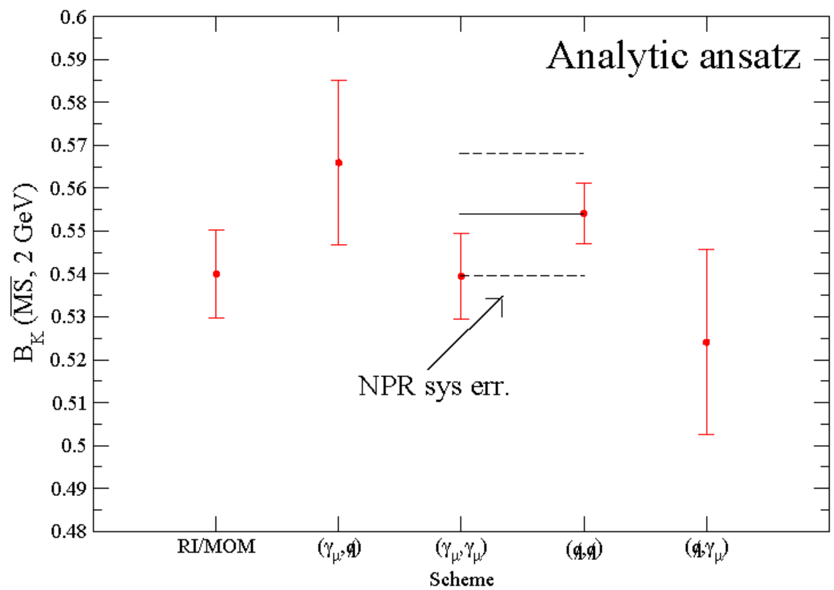




Operator normalization: Z_{BK}

- Use RI/MOM renormalization scheme (Rome/Southampton)
  - Fix to Landau gauge
  - Evaluate off-shell Green's functions
  - Impose momentum-space normalization condition: $\Gamma_{abcd}\,\Lambda_{abcd}(p_1, p_2, p_3, p_4)\,|_{\mu^2} = 1$
  - Use non-exceptional momenta
- Try 4 choices for Γ_{abcd} and chose the two that agree best with perturbative running:

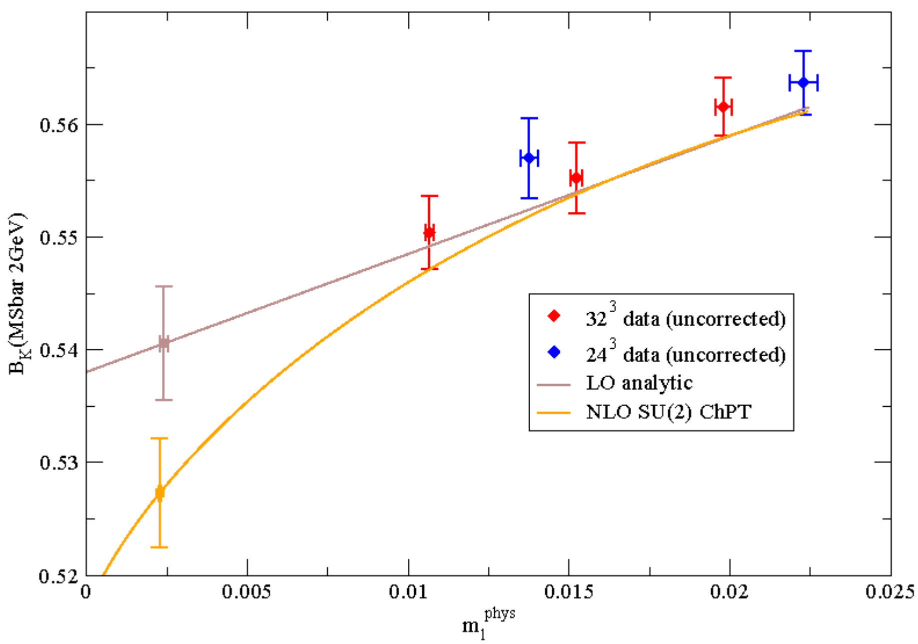




Continuum results

- Results from two lattice spacings
  - Small, 1-2% , $O(a^2)$ errors
  - $B_K = 0.524(10)_{stat}(28)_{sys}$ [PRL, 2008]

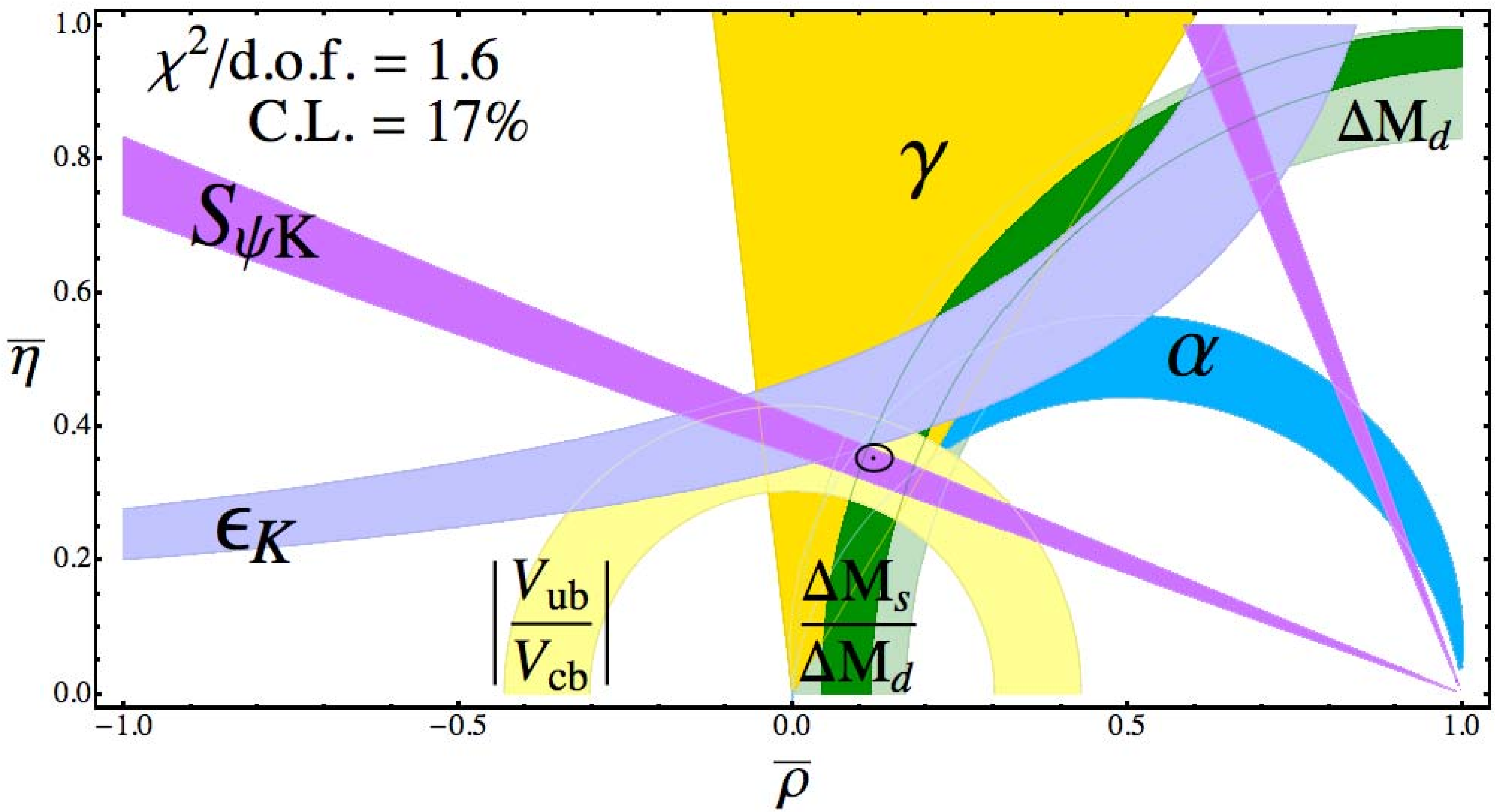
    → $B_K = 0.546(7)_{stat}(16)_{\chi}\ (3)_{FV}\ (14)_{ren}$ [preliminary]



Comparison with Expt.

- Some tension between ε_K and other constraints:

Laiho, Lunghi, Van de Water, arXiv:0910.2928



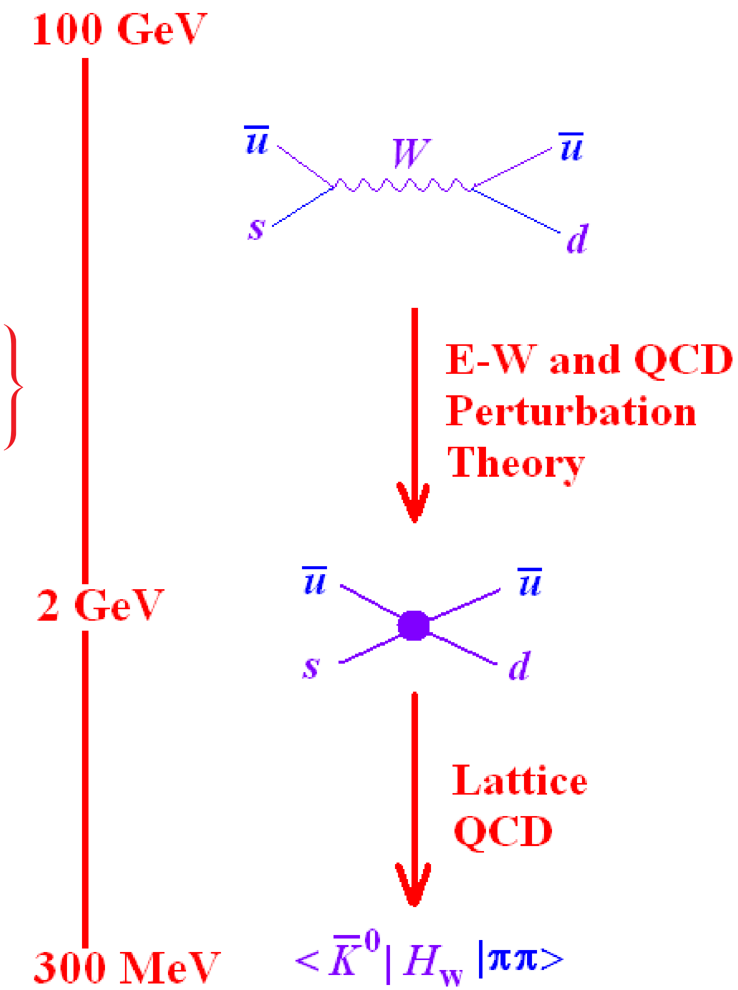
$K \rightarrow \pi\pi$ Decays

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

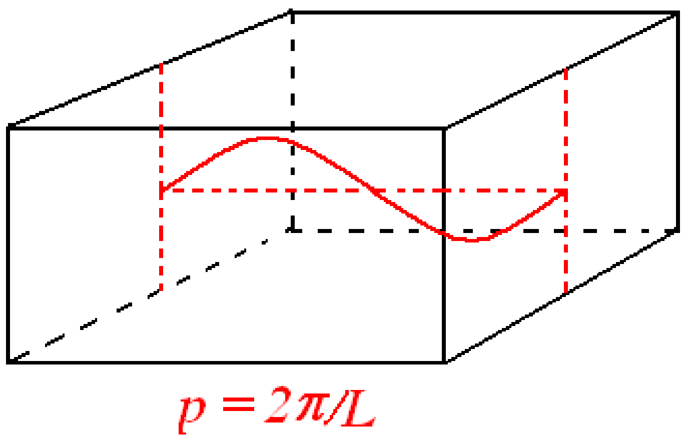
$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{us}^*}\frac{V_{ts}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Calculate π - π final state directly

- SU(3) ChPT failure:
  Abandon $<K|H_W|\pi>$ & $<K|H_W|0>$ → $<K|H_W|\pi\pi>$
- Maiani-Testa theorem (1990):
  - Euclidean space methods use e^{-Ht} to project onto lowest energy state
  - For π - π state this state will have zero relative momentum
- Lellouch-Luscher method (2000):
  - Use finite-volume quantization
  - Adjust volume so 1st or 2nd excited state has correct p
  - Correctly include π - π interactions
  - Extra finite-volume normalization factor.

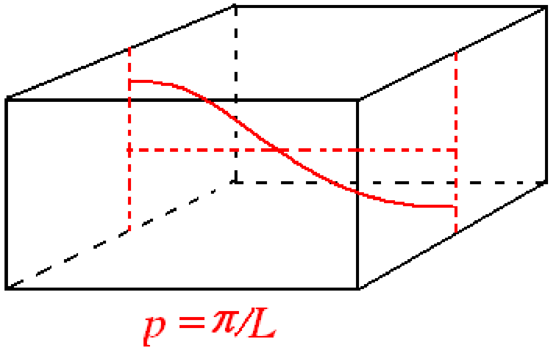


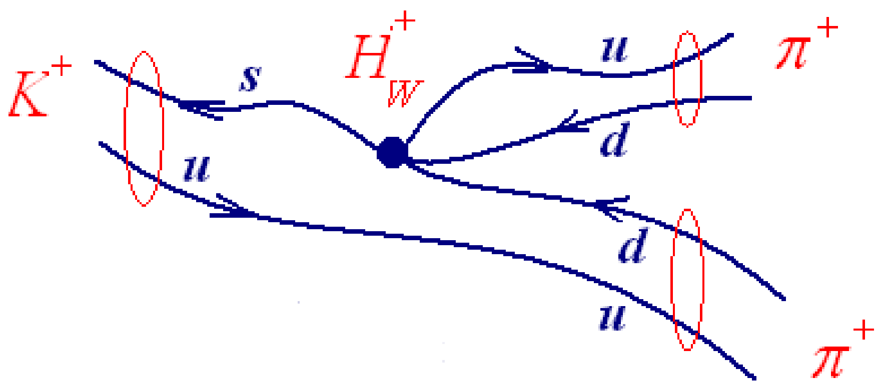
Use Iwasaki + DSDR Action

- Work at larger lattice spacing: 1/a = 1.4 GeV
- Use $32^3 \approx (4.5\ \mathrm{fm})^3$ volume.
- Exploit DSDR action to reduce residual chiral symmetry breaking
  - Iwasaki: $1/a = 1.73$ GeV $m_{\mathrm{res}} = 0.0030$
  - I + DSDR: $1/a = 1.40$ GeV $m_{\mathrm{res}} = 0.0018$
- Unitary $m_\pi = 180$ MeV, valence $m_\pi = 145$ MeV

$\Delta I = 3/2\ \ K \rightarrow \pi\pi$

- I = 2 final state has no vacuum overlap.
- Use twisted boundary conditions (Changhoan Kim, hep-lat/0210003).
- I = 2 quantum number must be carried by four I=1/2 valence quarks.
  - Twist only valence quarks Sachrajda and Villadoro (hep-lat/0411033).
  - Safe to use slightly different valence and sea quark masses.





$\Delta I = 3/2\ K \rightarrow \pi\pi$

(Matthew Lightman and Elaine Goode)

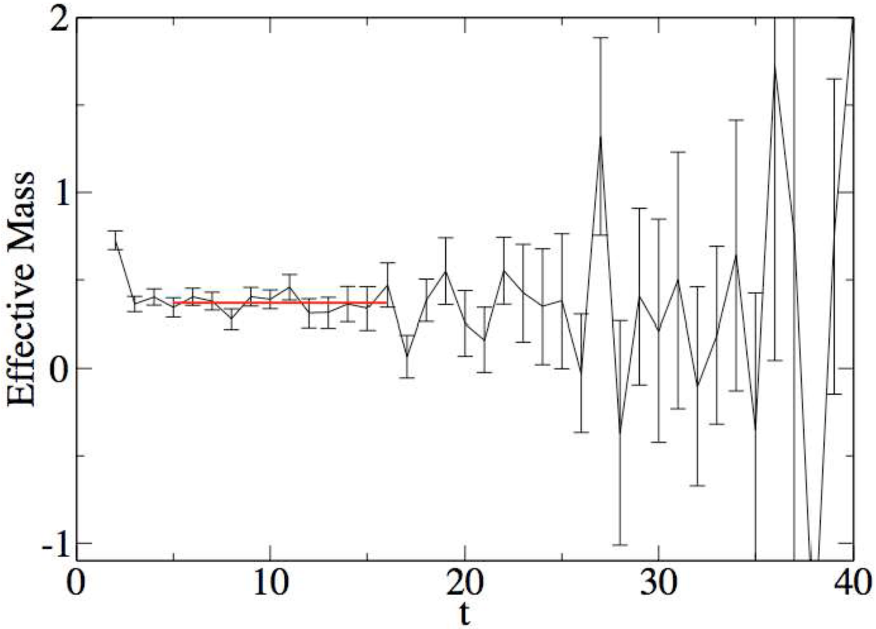
- Use 4.5 fm DSDR DWF ensembles.
  - $m_\pi = 250$ and 180 MeV
  - $1/a = 1.4$ GeV
  - Finite a errors $\leq 5\%$.
- Use physical valence light quark mass.
  - Sea quark mass dependence of I=2, $K \rightarrow \pi\pi$ expected to be very small
  - $m_{sea} = 0.008 \rightarrow 0.004$, < 3% (Lightman, arXiv:0906.1847 [hep-lat])
- Use anti-periodic boundary condition in two space directions (47 *configurations – preliminary*!)
  - $m_\pi = 145.6(5)$ MeV
  - $m_K = 519(2)$
  - $E_{\pi\pi} = 516(9)$ MeV

$\Delta I = 3/2\ \ K \rightarrow \pi\pi$

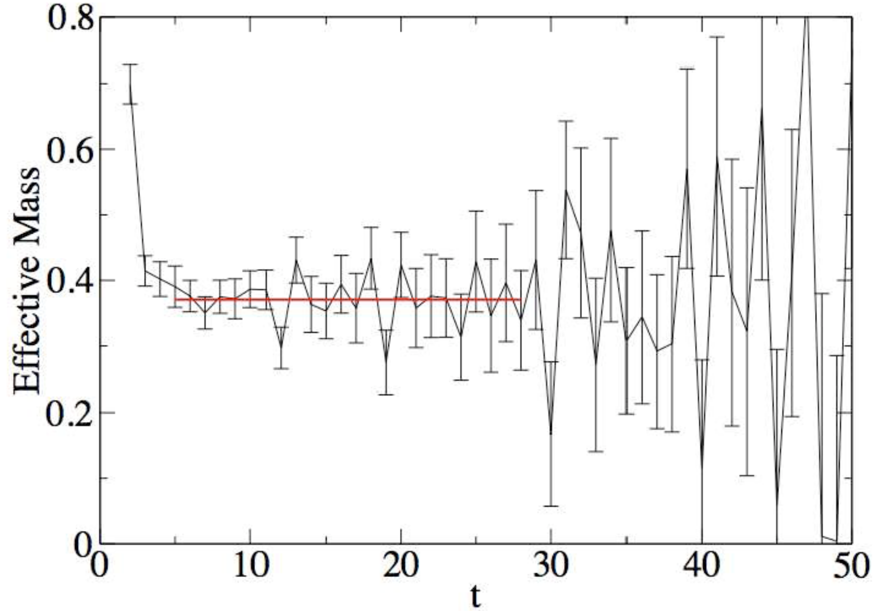
(Matthew Lightman and Elaine Goode)

$\pi\pi$ and K effective mass: $m_{\text{eff}}(t) = \ln\big(C(t) / C(t+1) \big)$

$\pi\pi\ (\text{p} = \sqrt{2}\pi/L)$



K



$\boldsymbol{E_{\pi\pi}}$ = 516(9) MeV

$\boldsymbol{m_K}$ = 519(2) MeV

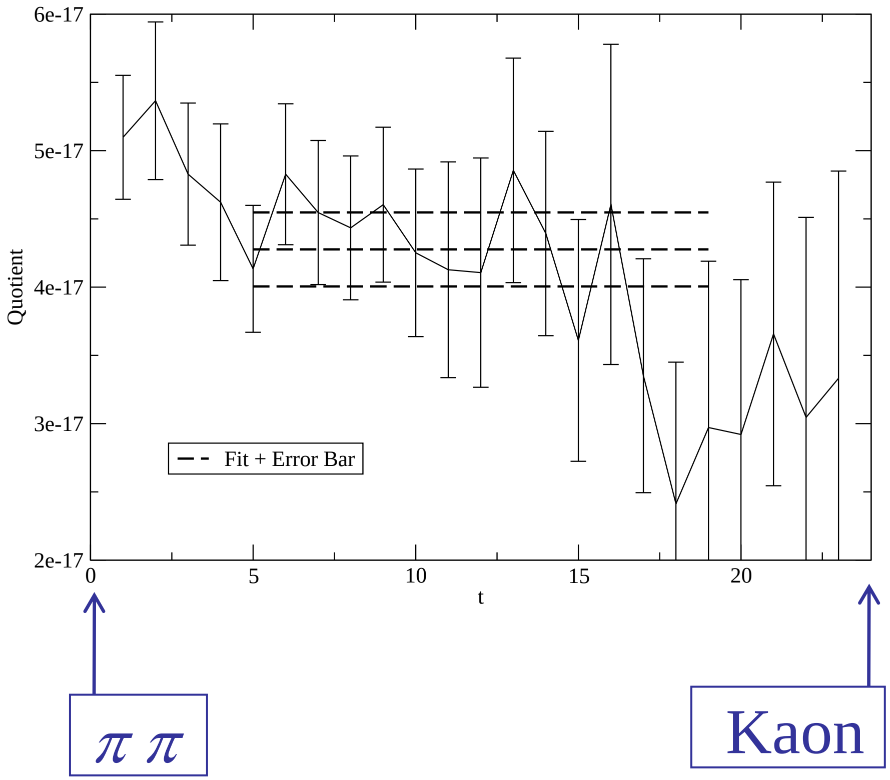
$< \pi\,\pi \,|\, O^{(27,1)} \,|\, K >$ from 47 configurations

(Matthew Lightman and Elaine Goode)

Quantity	This Calculation	Physical
m_π	145.6(5) MeV	139.6 MeV
m_K	519(2) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx 0)$	294(1) MeV	-
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L)$	516(9) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L) - m_K$	-2.7(8.3) MeV	0 MeV

$O^{(27,1)}$	0.000945(56)
$O^{(8,8)}$	0.0192(11)
$O^{(8,8)\text{m}}$	0.0641(38)

$O^{(27,1)}$ Amplitude



Determine physical A_2

(Matthew Lightman and Elaine Goode)

- Recall $\langle \pi\pi(I=2)|\mathcal{L}_W(0)|K\rangle = A_2 e^{i\delta_2}$

$$A_2 = \frac{\sqrt{3}}{2\sqrt{2}} \frac{1}{\pi q_\pi} \sqrt{\frac{\partial \phi}{\partial q_\pi} + \frac{\partial \delta}{\partial q_\pi}} L^{3/2} a^{-3} G_F V_{ud} V_{us} \sqrt{m_K} E_{\pi\pi} \times \sum_{i,j} C_i(\mu) Z_{ij}(\mu) \langle \pi\pi | Q_j | K\rangle$$

- Re(A_2) dominated by single operator $O^{(27,1)}$.
- Determine Lellouch-Luscher factor.

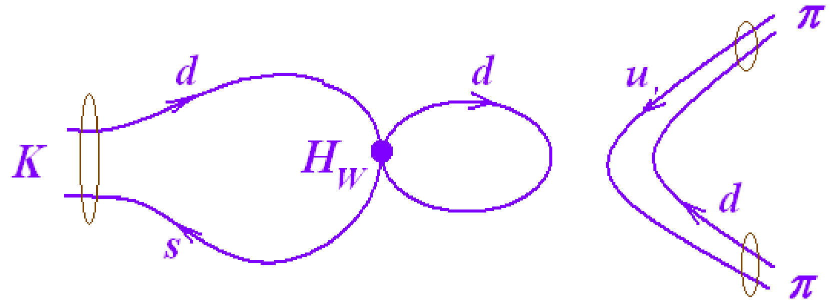
$$\frac{\partial \phi}{\partial q_\pi} = 5.141 \qquad \frac{\partial \delta}{\partial q_\pi} = 0.305$$

- **Re(A_2) = 1.40(7)$_{\text{stat}}$(11)$_{\text{sys}}$ 10^{-8} GeV** [Expt: 1.5 10^{-8} GeV]
- Im(A_2) equally easy, awaits NPR Z factors.

$\Delta I = 1/2$

$\Delta I = 1/2 \; K \rightarrow \pi\pi$
(Qi Liu)

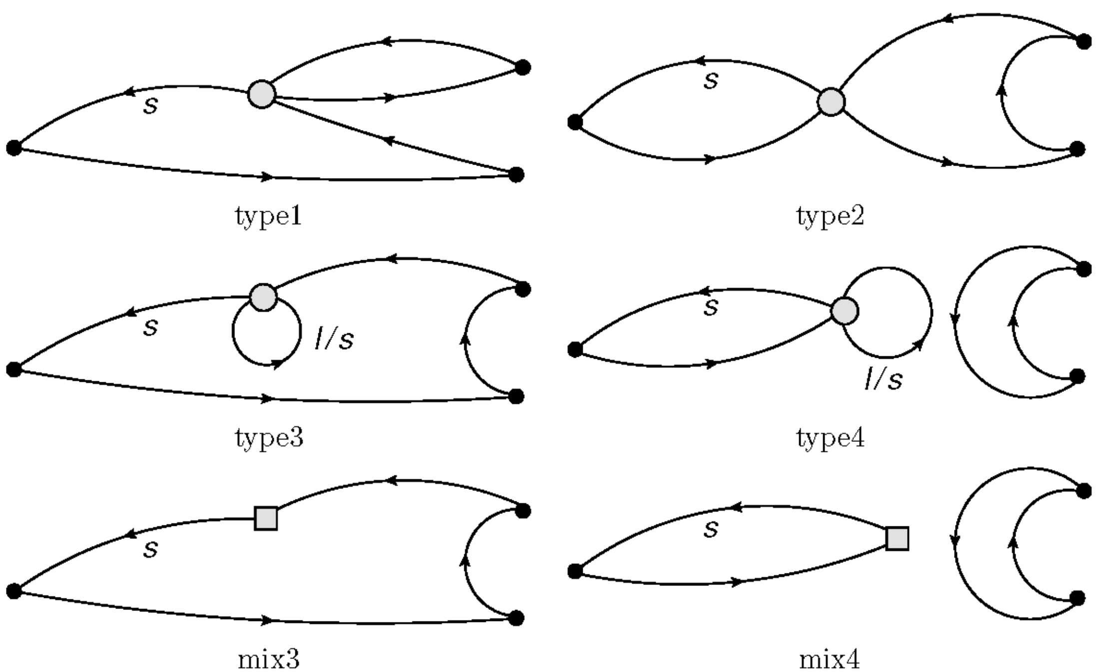
- Made much more difficult by disconnected diagrams:



- Experiment on 16^3 x 32 ensembles.
- $1/a$ = 1.73 GeV, m_π = 420 MeV, L = 1.8 fm
- Start with 4000 time units, measure on every 10.
- Adjust valence strange mass for on-shell, threshold kinematics ($\pi\pi$ state is unitary)

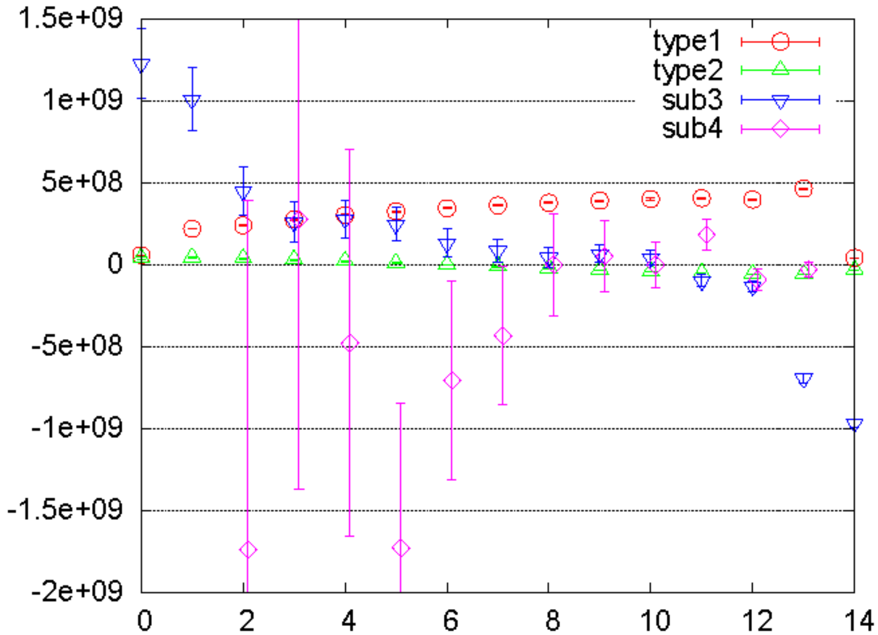
$\Delta I = 1/2 \; K \rightarrow \pi\pi$
(Qi Liu)

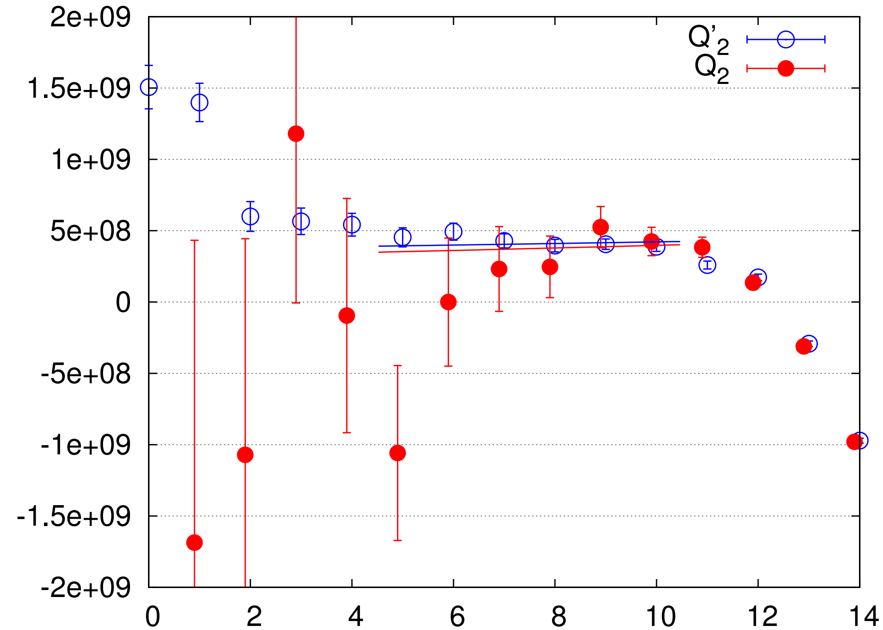
- Code 48 different contractions
- For each of 400 configurations invert with source at each of 32 times.
- Use Ran Zhou's deflation code



$\Delta I = 1/2$ $K \rightarrow \pi\pi$
(Qi Liu)

- Results for Q2, largest part of Re (A_0):



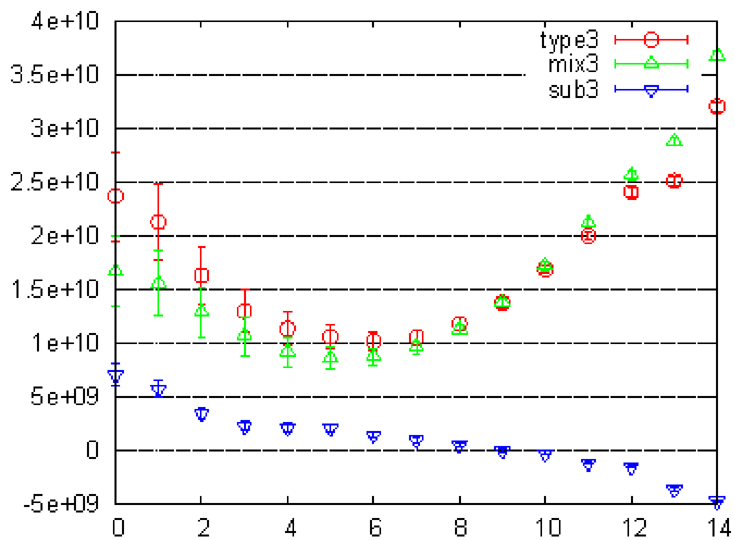


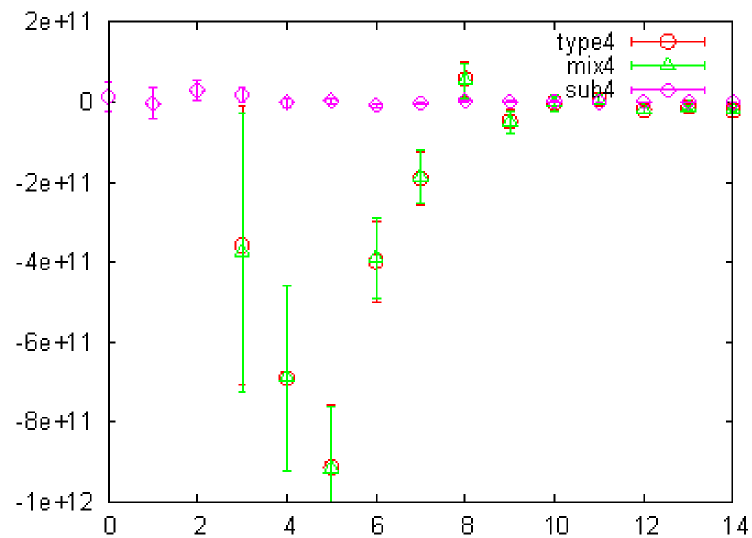
- Re (A_0) = 43(7) 10^{-8}
- Recall, p = 0, m_π = 420 MeV!

$\Delta I = 1/2 \;\; K \rightarrow \pi\pi$

(Qi Liu)

- Removing quadratic divergence from Q_6 :

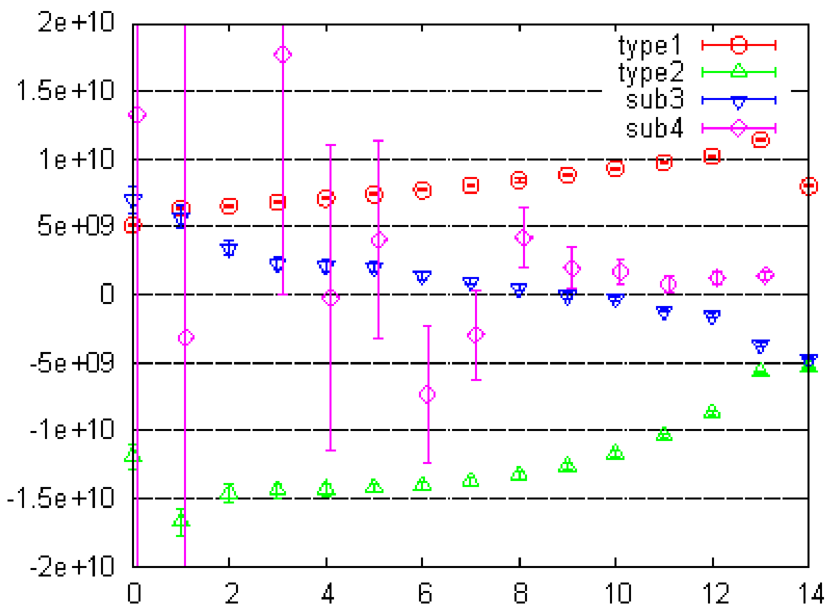


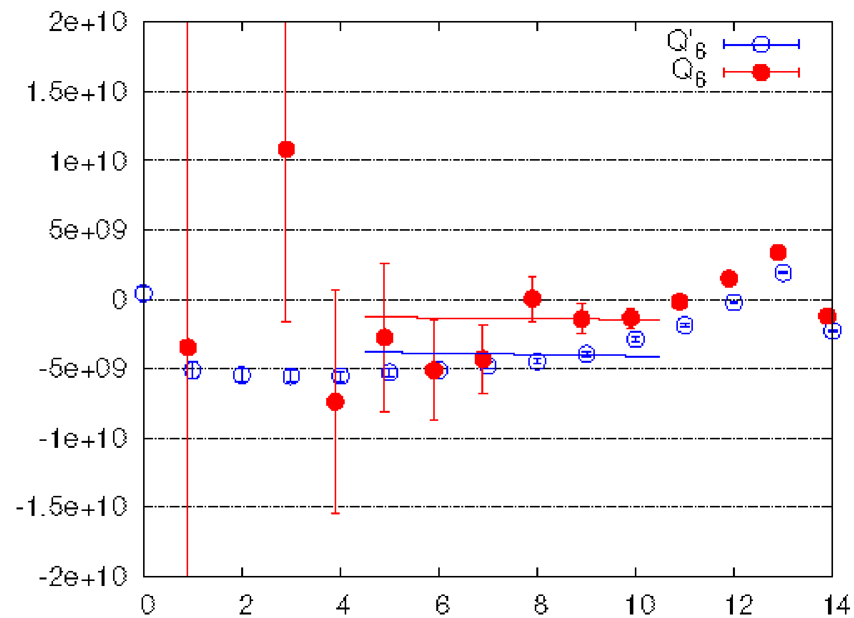


- Subtraction is needed even though it vanishes on-shell
- Off-shell excited states contributions are large.

$\Delta I = 1/2$ $K \rightarrow \pi\pi$
(Qi Liu)

- Results for Q_6, largest part of im(A_0):





- No result – errors are too large.
- If $Q_6{}'$(without disconnected part) is physical then need 4 x statistics?

Future prospects

- Re (A_2) and Im (A_2) known soon to 10%
  - 48^3 x 64 will allow unitary pions
  - Second lattice spacing → 2-3% error

  } 1 Tf yr → 10 Tf yr
- Re (A_0) and Im (A_0) with physical kinematics

ΔI=½ rule: Re(A_0)/Re(A_2)

$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

	Factor	Pflops yr
2000 configurations	1	1.40
$5^2 \cdot 3$ statistics for p != 0	75	105
Benefit of 2x time slices	0.5	52.50
Benefit of split sources	0.25	13.13
Gain from lighter kaon mass	0.53	3.69
Reduced precision	0.75	2.77
Benefit of large volume 16^3 → 32^3	0.13	0.35
Deflation	0.3	**0.10**

Conclusion

- QCD machines begun 30 years ago have blossomed into commercial tools providing 100’s of Teraflops
- Methods pioneered by the Italian school now enable weak interaction studies with few % precision.

- Expect continued important progress:
  - Compute long-distance 2nd order weak amplitudes
  - Incorporate E&M