THE PROBLEM OF COMPOSITION OF EXOTIC HADRONS

AD POLOSA

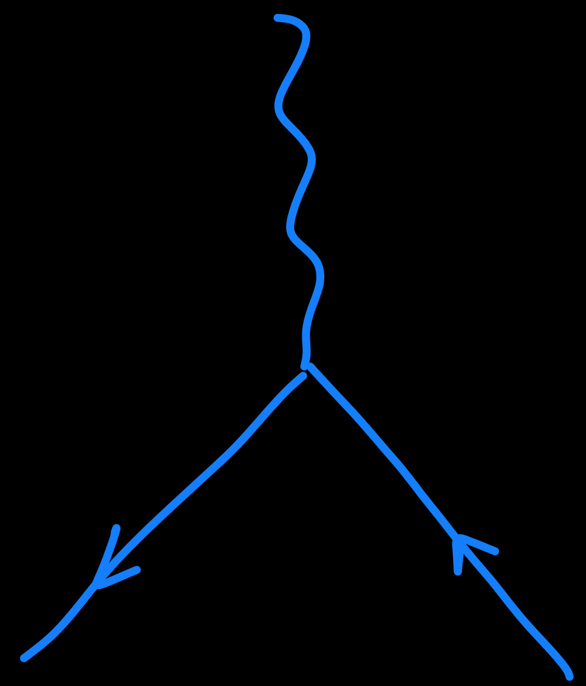
ADP, *Phys. Lett. B* 746 (2015) 248-250

Esposito, Maiani, Pilloni, ADP, Riquer *Phys. Rev. D* 105 (2022) 3, L031503

Esposito, Glioti, ADP, Rattazzi, Tarquini, in preparation.

STRUCTURELESS & POINTLIKE PARTICLES

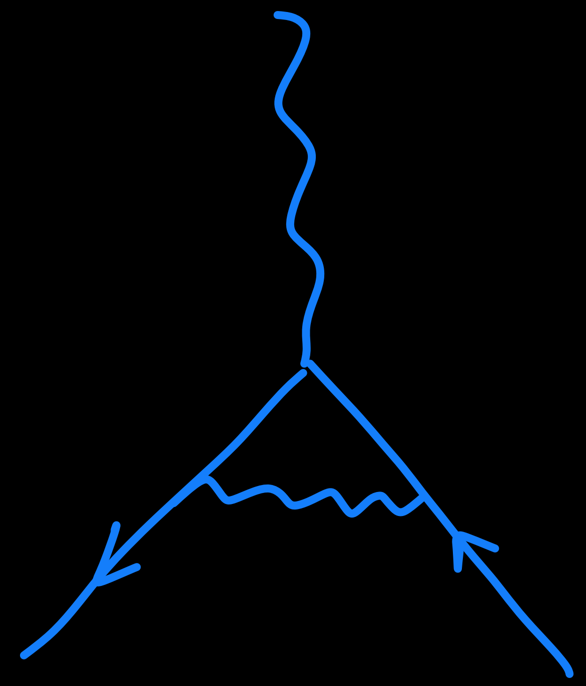
The interaction **strength** does not depend on Q



$$H_I = eA_\mu J_\mu$$

VIRTUAL STRUCTURE

The interaction strength depends on Q



E.g. the correction $1 + F_2(0)$ to $g = 2$

$$F_2(Q^2 \simeq 0) = \frac{\alpha}{2\pi}$$

FORM FACTORS

Form factors in nuclear physics are FT of the charge distribution due to the nucleons making the nucleus — they are partially virtual too.

Form factors in nuclei reflect the bound state structure

Form factors in elementary particles reflect the virtual structure

ELEMENTARY PARTICLES?

In this sense, elementary does not really mean structureless (the virtual cloud is always there)

Quarks are elementary in the sense that they are point-like (no size scale can be measured), and, at high-energy, we need only quark fields in the Lagrangian.

SCALING IN DIS

In DIS there are two main scales Q^2, $\nu = M(E - E')$.
If we select all events having fixed ratio Q^2/ν,
the a-dimensional quantity

$(\nu\sigma)$ should not depend on Q^2

unless Q^2 in σ (in FFs) comes in some other dimensionless combination Q^2/λ^2, where λ has to be related with the size of a (continuous) distribution of charge inside the proton.

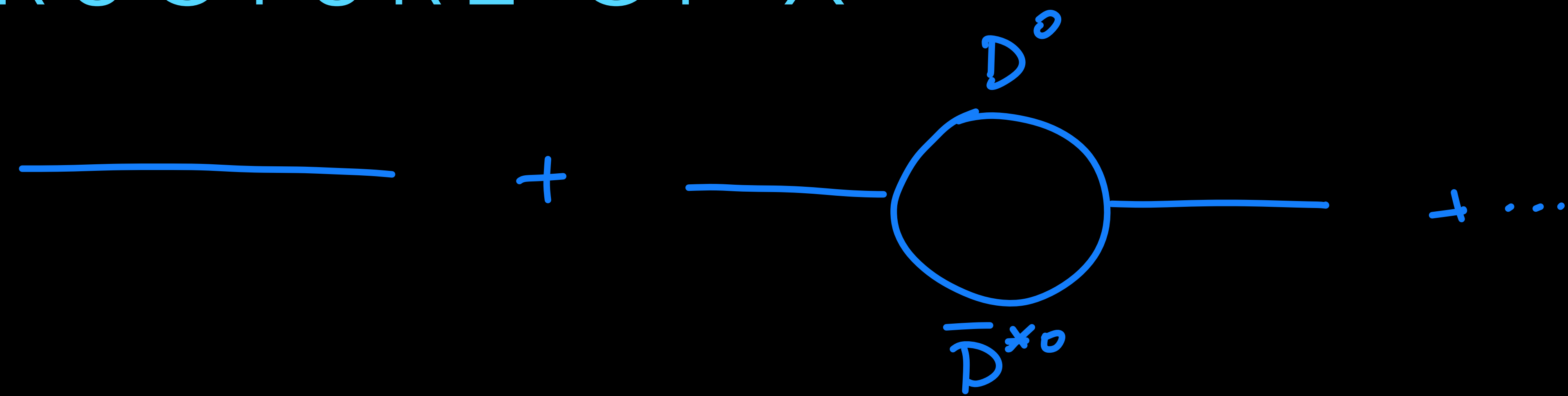
FEATURES OF $X(3872)$

1. $X(3872)$ has a mass precisely equal to $m_D + m_{D^*}$
2. It is an extremely narrow state $\lesssim 1$MeV
3. Its strong decays in $J/\psi\,\rho$ and $J/\psi\,\omega$ violate isospin
4. It is produced in prompt hadron collisions with very high cross section and hard p_T cuts
5. It has been found in the X^0 neutral charge state only, for the moment (?)

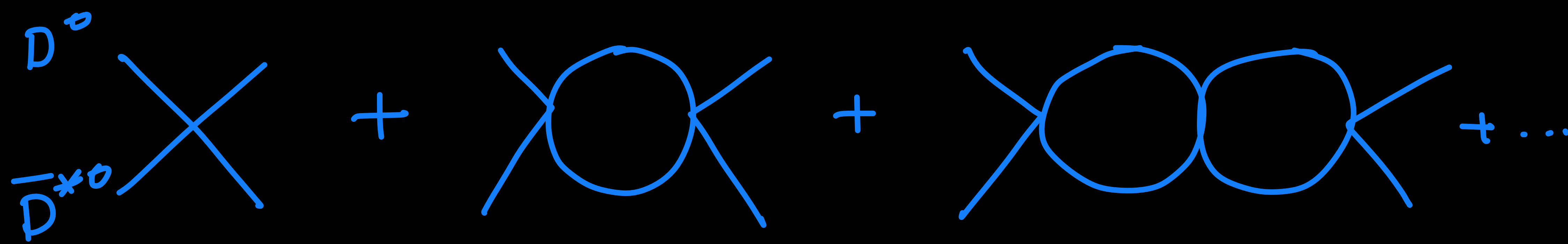
Some interpretations given over the years:
Compact tetraquark, DD^* **hadron molecule (deuson)**, kinematical effect, hadrocharmonium, standard cahrmonium, Georgi's unparticle!

STRUCTURE OF X

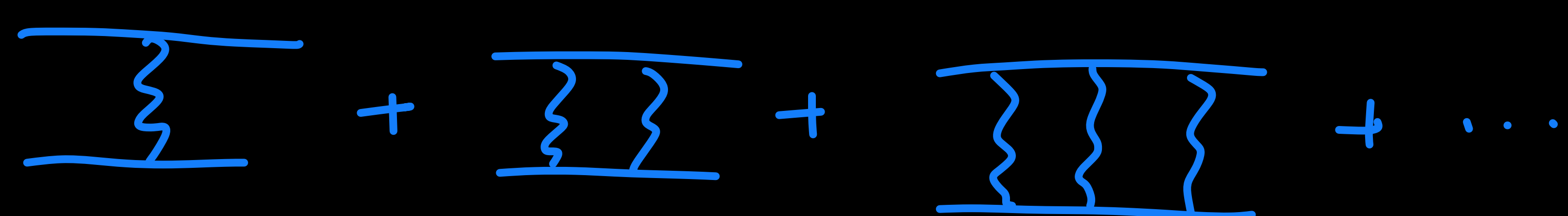
I)



II)

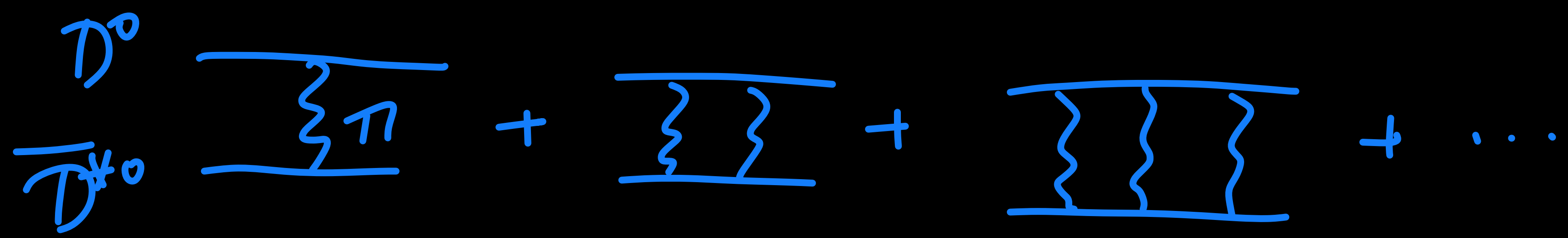


the X might result as a bound state from this ladder of bubbles, as Hydrogen results from



STRUCTURE OF X

It is very (!) unlikely that the X might be a $D^0\bar{D}^{*0}$ bound state due to pion exchange



$1/r^3$ potentials in QM have no bound states!

THE COMPACT `COMPONENT`

The X made of quarks, as in (I), is compact.
The X as in (II), is a sort of deuson.

How do we know if we are in case (I) or case (II)?

Are you able to tell from data that deuteron is a pn nucleus rather than a compact state?

DEUTERONS & `DEUSONS`

The effective range deduced from the np scattering amplitude is the discriminating observable [Weinberg '65].

For the $X(3872)$ mesonic deuteron there is no way of performing $D\bar{D}^*$ scatterings, but the resonance lineshape is well studied experimentally, and it encodes r_0.

DEUTERONS & `DEUSONS`

The deuteron state in a NR description

$$|d\rangle = \sqrt{Z}\,|\mathfrak{d}\rangle + \int d\beta\, C_\beta\,|\beta\rangle$$

Compact

$$\delta \longrightarrow \left.\begin{matrix} n \\ p \end{matrix}\right\} \vec{k}_{rel}$$

WEINBERG & DEUTERON (1965)

Weinberg finds, for *shallow bound states*, a relation between Z and a quantity known in low energy scattering theory as the *effective range* r_0 (Schwinger)

$$r_0 = -\frac{Z}{1-Z}R + O(\frac{1}{m_\pi})$$

$$R = \frac{1}{\sqrt{2mB}}$$

The "molecule" has $Z = 0$ thus $r_0 = O(1/m_\pi)$.

What is the sign of the unknown corrections?

See Weinberg Phys. Rev. 137, B672 (1965)

A THEOREM ON SHALLOW BOUND STATES IN QM

BETHE ('49), LANDAU-SMORODINSKY ('48)

$$\mathbf{r_0 > 0}$$

(indeed $r_0 = +1.74$ fm for deuteron)

This is a general theorem, together with case by case analyses (see e.g. Blatt & Weisskopf).

This agrees with original Weinberg's $Z = 0$ molecule:

"…an elementary deuteron would have $0 < Z < 1$"

"The true token that the deuteron is composite is an r_0 small and positive rather than large and negative "

"…an elementary deuteron would entail a large and negative r_0"

Esposito et al. 2108.11413

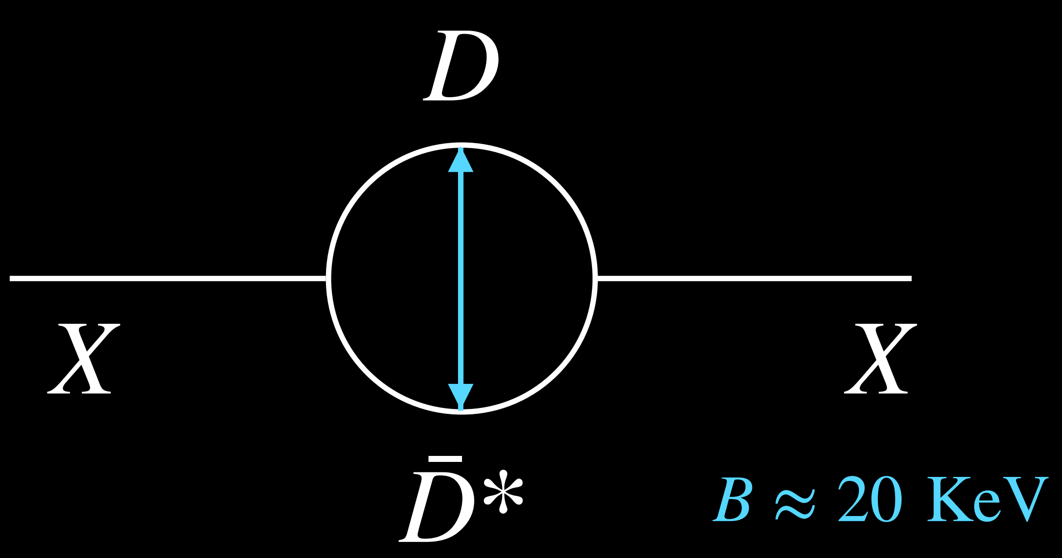
LHCB (2020)

arXiv:2005.13419

Allows to compute the effective range r_0 for the $X(3872)$. This was dubbed as the "deuson", a $D\bar{D}^*$ mesonic molecule analogue of deuteron: a viable option **iff** $Z = 0$ or $r_0 > 0$ and $O(1/m_\pi)$.

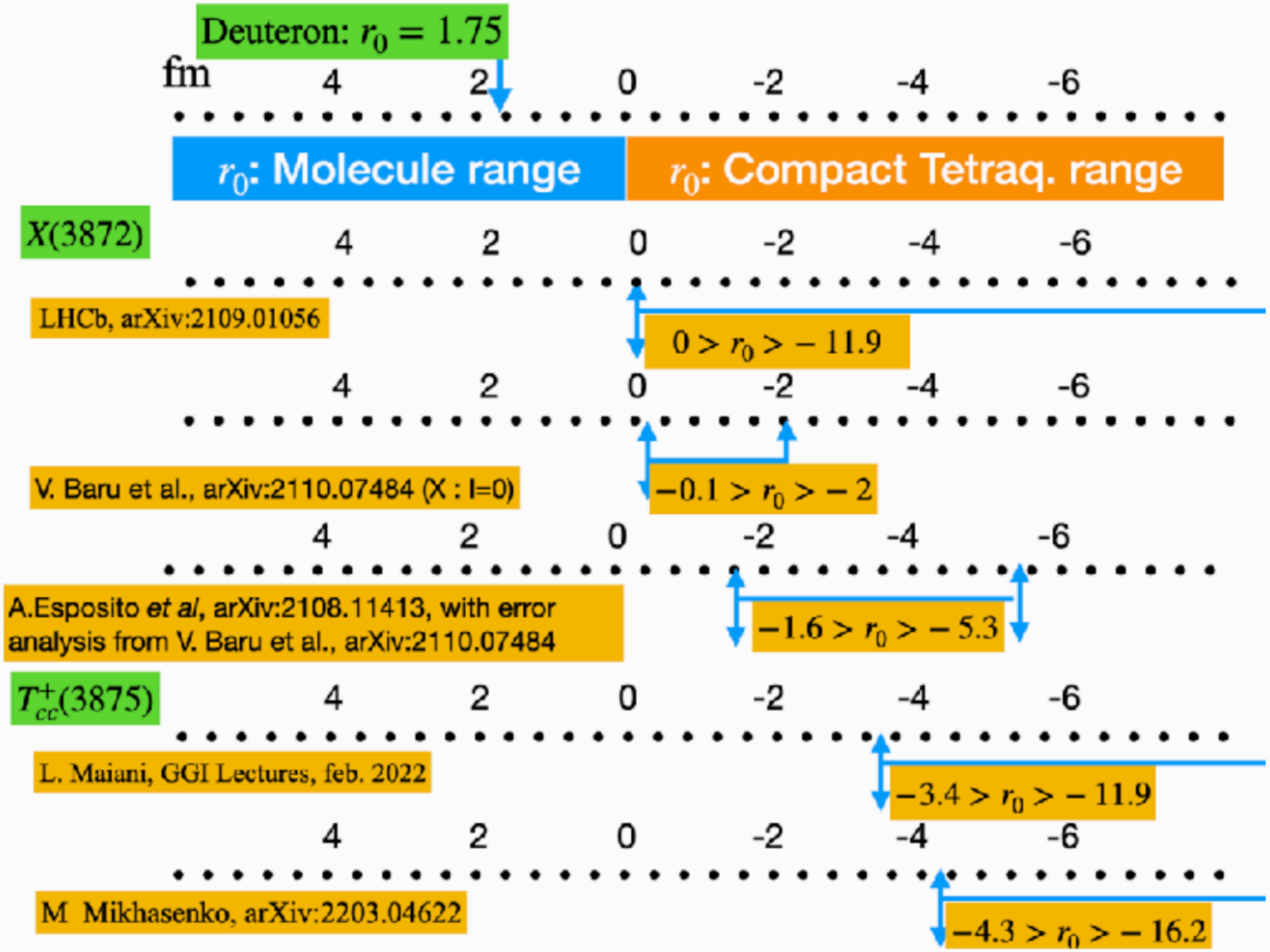
However we find $r_0 = -5.43$ fm and $|r_0| > 1/m_\pi$!

"…an elementary deuteron would entail a large and negative r_0"



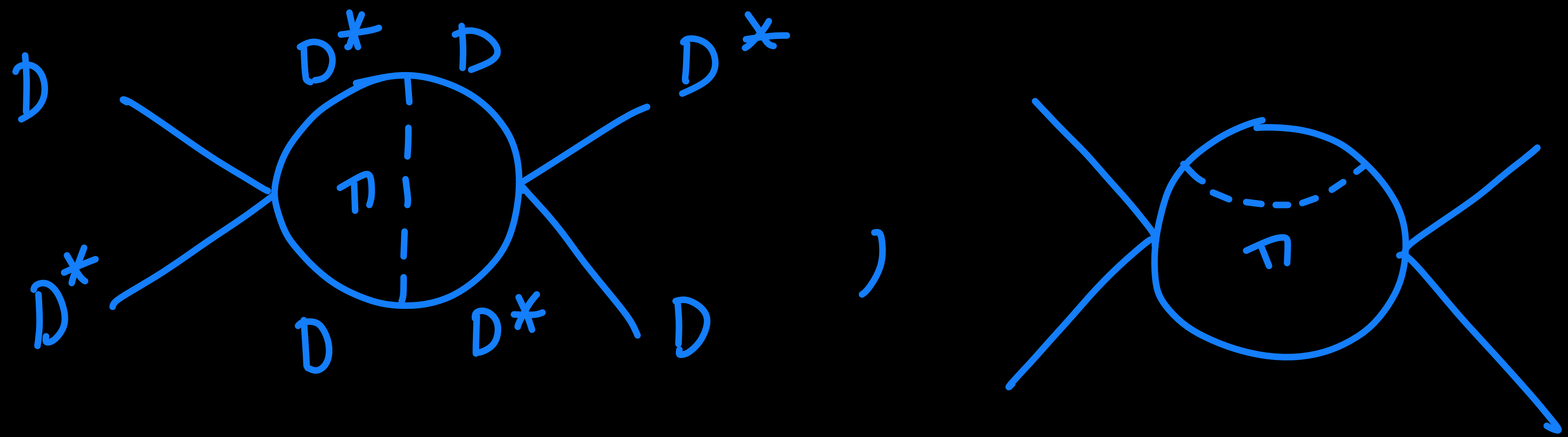
Esposito et al. 2108.11413

My Summary about r_0



No consensus yet, but we are on a very promising road.
Stay tuned!!

CASE (II) AND THE PION ROLE



2-loop diagrams of this sort have been computed in papers with NR-EFT. They address the potentially dangerous question: are Weinberg's corrections $O(1/m_\pi)$ or $O(1/\mu)$ with $\mu \ll m_\pi$?

[Esposito, Glioti, ADP, Rattazzi, in preparation]

WHAT IS μ?

The potential is the FT of the propagator in the no-recoil approximation

$$\int \frac{q_i q_j \, e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2 - \mu^2 - i\epsilon} \, d^3 q$$

where

$$\mu^2 = (m_{D*} - m_D)^2 - m_\pi^2 \approx 40 \text{ MeV}$$

[Esposito, Glioti, ADP, Rattazzi, in preparation]

A COMPLEX POTENTIAL

The previous integral gives a complex potential of the form

$$V(r) = A\delta^3(r) + B\mu^2 \frac{e^{i\mu r}}{r}$$

We reproduced the results obtained with NREFT using only low-energy scattering theory (no diagrams) and found a very small contribution to $|r_0|$ and, an imaginary part $\Im(r_0)$!

[Esposito, Glioti, ADP, Rattazzi, in preparation]

ADDITIONAL

THIS CONCLUSIONS IS DEBATED

This conclusion has been disputed in a recent paper by C. Hanhart and collaborators 2110.07484.

Their approach is: we measure a `small` Z (even if for X and the new tetraquark this means $14\% \div 30\,\%$). That means that the state is essentially a molecule and marginally a compact quark state.

SCATTERING AMPLITUDE

$$f = \frac{1}{k\cot\delta(k) - ik} = \frac{1}{-1/a + \frac{1}{2}r_0k^2 - ik + \dots}$$

Compares with NR-BW formula

$$f = -\frac{\frac{1}{2}g_{\mathrm{BW}}^2}{E - m_{\mathrm{BW}} + \frac{i}{2}g_{\mathrm{BW}}^2 k}$$

$$g_{\mathrm{BW}}^2 = -\frac{2}{\mu r_0} \quad m_{\mathrm{BW}} = \frac{1}{a\mu r_0} \quad E = \frac{k^2}{2\mu}$$

Esposito et al. [2108.11413]

A FORMULA FOR r_0

We find

$$r_0 = -\frac{2}{\mu g_{\text{Flatte}}} - \sqrt{\frac{\mu'}{2\mu^2\delta}} = -5.34 \text{ fm}$$

Where μ' is the reduced mass of the charged open charm pair, μ of the neutral and δ is

$$\delta = m_{D^+} + m_{D^{*-}} - m_{D^0} - m_{\bar{D}^{*0}}$$

Esposito et al. 2108.11413

FIELD THEORY DESCRIPTION

In the field theory description the $\delta^3(r)$ potential (which might have bound states) corresponds to

$$\lambda(\phi^\dagger\phi)^2$$

where ϕ are the fields for $D^0, \bar{D}^{*0}$ particles, whereas H_{PQ} corresponds to

$$g(\psi^\dagger\phi^2 + (\phi^\dagger)^2\psi)$$

coupling to the elementary field ψ of the X.

See T. Kinugawa and T. Hyodo 2112.00249

The negative r_0 originates in the coupling to the (bare) field ψ
What is the role of the complex potential in $D\bar{D}^*$?

FIELD THEORY DESCRIPTION

There could also be another 4-linear term contributing to r_0

$$\rho \nabla(\phi^\dagger\phi) \cdot \nabla(\phi^\dagger\phi)$$

however in the NR limit, dimensional analysis tells

$$g \sim \frac{1}{\sqrt{v}} \quad \lambda \sim v \quad \rho \sim v^3$$

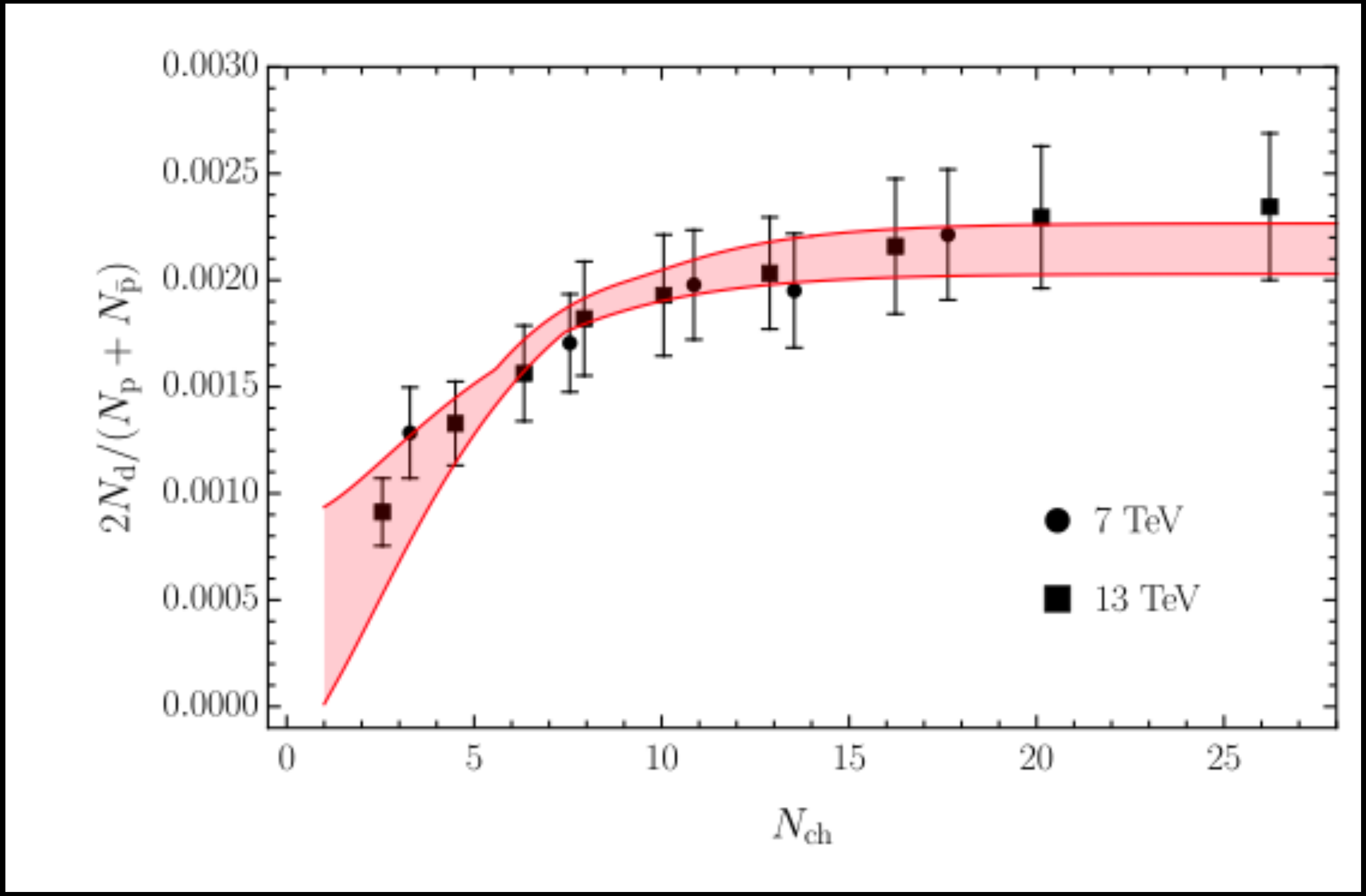
with $v \to 0$

See T. Kinugawa and T. Hyodo 2112.00249

DOES THE X BEHAVE LIKE A DEUTERON?

DEUTERON FROM ALICE

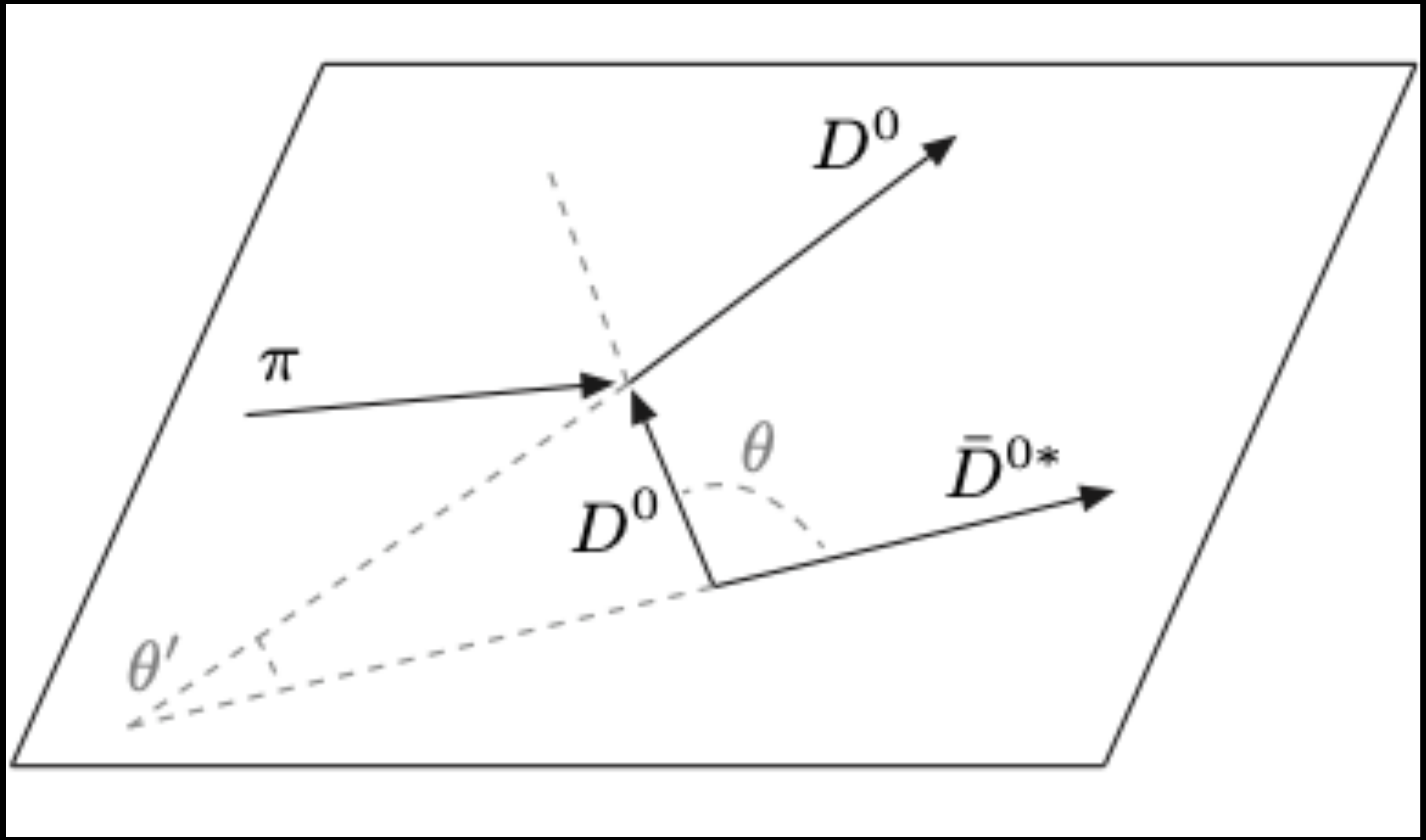
ALICE: 1902.09290; 2003.03184



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

Numbers of molecules as a function of multiplicity, computed with Boltzmann eq. in a coalescence model.

COALESCENCE MODEL

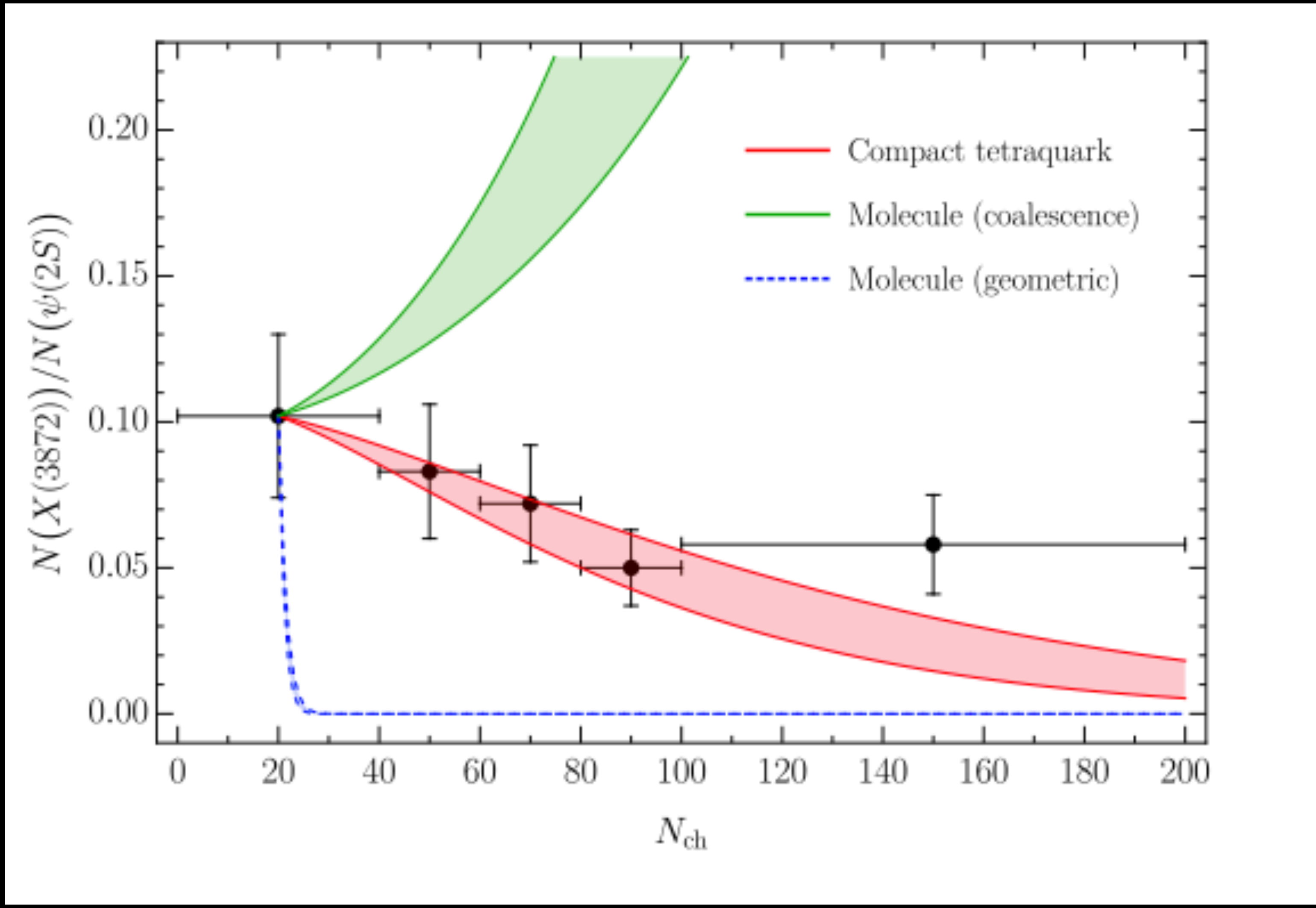


Esposito, Piccinini, Pilloni, ADP, *J. Mod. Phys.* 4 (2013) 1569-1573
Guerrieri, Piccinini, Pilloni, ADP, *Phys. Rev. D* 90 (2014) 3, 034003

In final states of hadron collisions, the would-be molecule constituents have large $(k > \Lambda)$ relative momenta and, after an interaction with a GeV comovers, the prob. of falling within $k < \Lambda$ is small. On the other hand a bound pair is most likely broken.

RECENT IMPLICATIONS

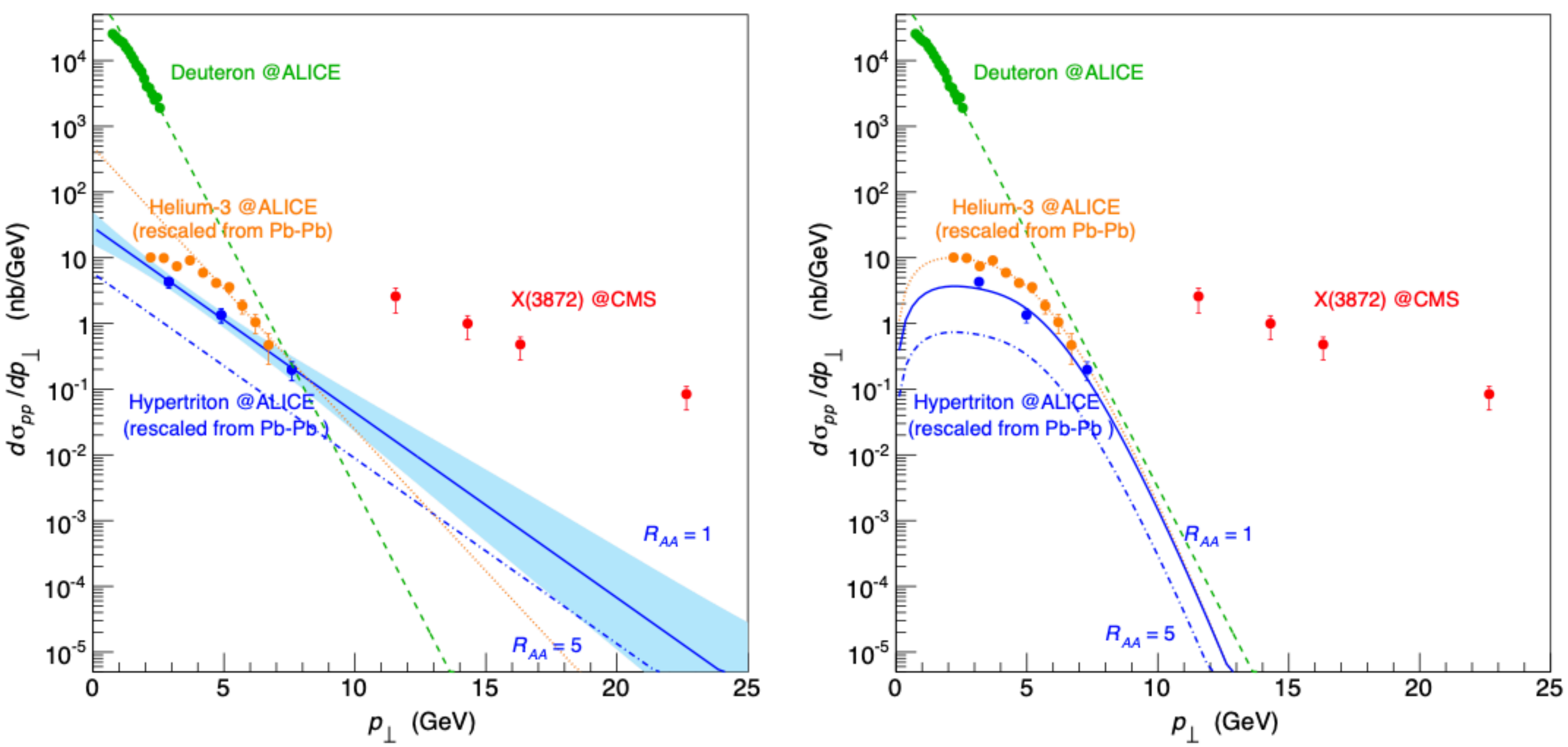
LHCb:2009.06619



Esposito, Ferreiro, Pilloni, ADP, Salgado *Eur. Phys. J. C* 81 (2021) 669

The coalescence picture predicts a behavior (green band) qualitatively different from data.

FROM MULTIPLICITY TO PT



Esposito, Guerrieri, Maiani, Piccinini, Pilloni, ADP, Riquer, *Phys. Rev. D* 92 (2015) 3, 034028

THE MOST RECENT TETRAQUARKS

DIQUARK-ANTIDIQUARK

$$X(1^{++}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 + |0,1\rangle_1) \qquad X(3872)$$

$$X(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{\sqrt{2}}(|1,0\rangle_1 - |0,1\rangle_1) \qquad Z(3900)$$

$$X'(1^{+-}) = [cq][\bar{c}\bar{q}] = \frac{1}{2\sqrt{2}}(|1,1\rangle_1) \qquad Z(4020)$$

Here $q = u, d$. What about strange quarks?

Maiani, Piccinini, ADP, Riquer, Phys. Rev. D71, 014028 (2005); D89 114010 (2014); PLB778, 247 (2018)

THE EQUAL SPACING RULE

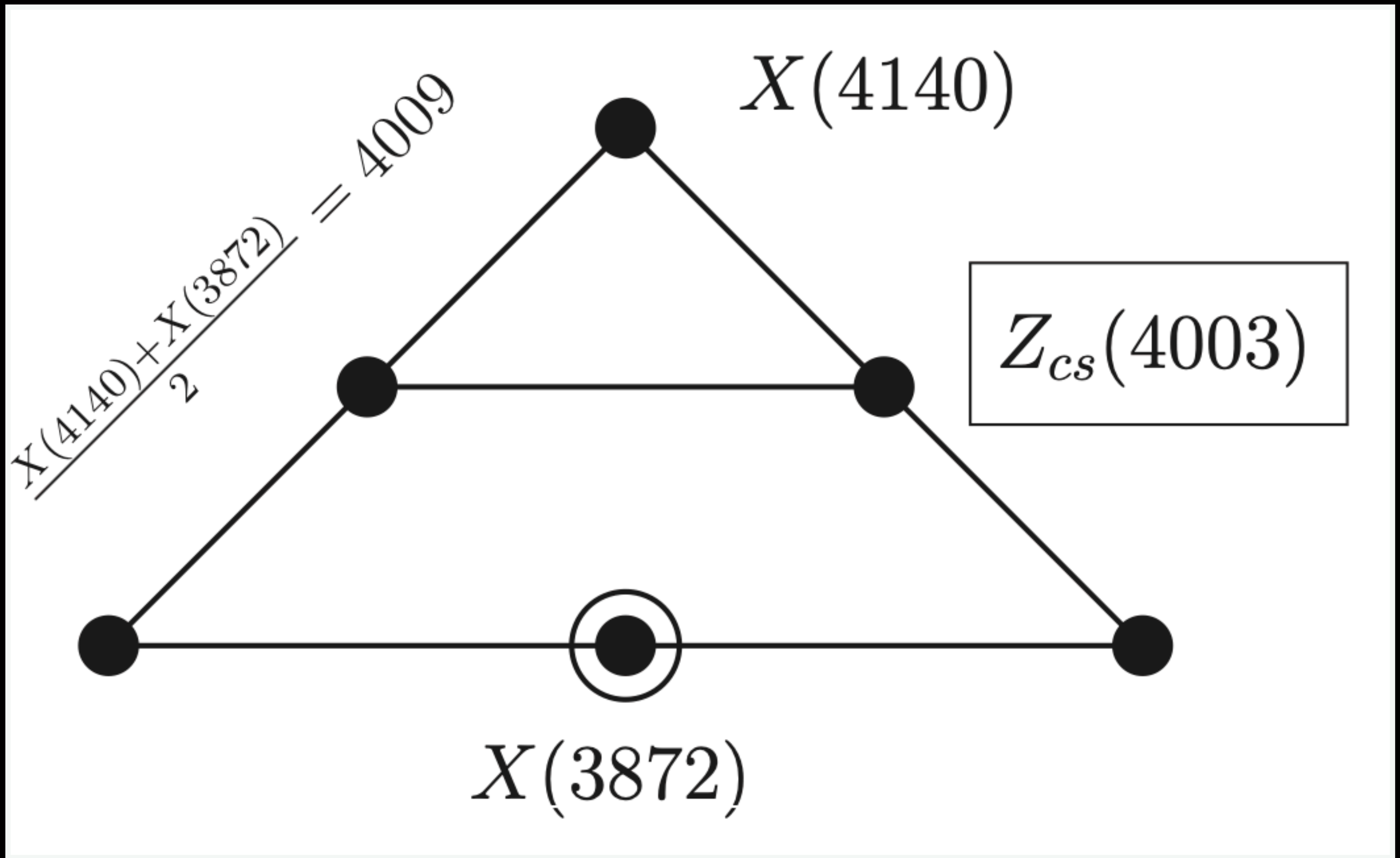
$$K^* \approx (\phi + \rho)/2$$

$$X(1^{++}) = [cs][\bar{c}\bar{s}] \qquad X(4140)$$

To first order of SU(3) flavor symmetry breaking we predict

$$Z_{cs} \stackrel{!}{=} (X(4140) + X(3872))/2 = 4009 \text{ MeV}$$

Spacing = 275 MeV
wrt 244 MeV for $\phi - \rho$



$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)

LHCB (2021)

The $Z_{cs}(4003)$ was observed by LHCb in the decay

$$B^+ \to \phi + Z_{cs}^+(4003) \to \phi + K^+ + J/\psi$$

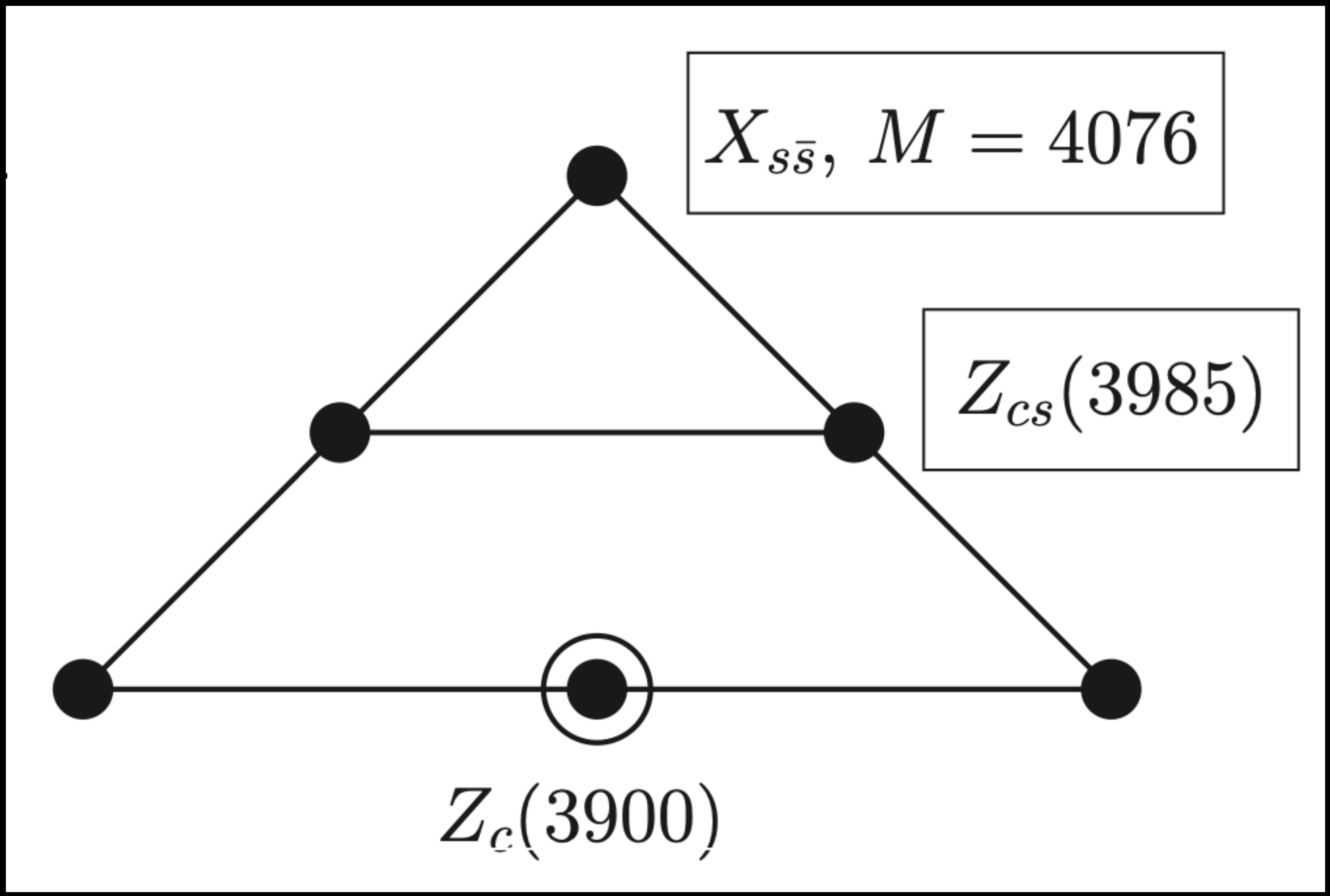
At a mass very close to that given by the equal spacing rule

NEGATIVE CHARGE CONJUGATION

The diquark-antidiquark model requires the quasi-degeneracy $M(X(1^{++})) = M(Z(1^{+-}))$ and we expect a similar multiplet for 1^{+-}

Spacing = 188 MeV
wrt 200 MeV for $f_2' - a_2$

Maiani, ADP, Riquer
arXiv:2103.08331
Sci. Bulletin 66, 1616 (2021)



Prediction

$[cs][\bar{c}\bar{q}] \vee [cq][\bar{c}\bar{s}]$

BESIII recently observed $e^+e^- \to K^+ Z_{cs}^-(3985) \to K^+(D_s^{*-}D^0 + D_s^- D^{*0})$

CONCLUSIONS

- It is always mentioned that symmetry arguments predict ‘too many’ states. However a particle zoo was identified — not expected in molecular models. Recently a new kind of tetraquark $[cc][qq']$ has been reported.

- In quark models the vicinity to threshold, from below and from above, is natural — not for molecules.

- Vicinity to threshold does not necessarily mean loosely bound state of hadrons.

- **Independent measurements of r_0 in $X(3872)$ would be crucial to address the compositeness `vexata questio`.**

CONCLUSIONS

$X(3872)$	$Z_c^{0\pm}(3900)$	$Z_c^{0\pm}(4020)$	$Z_b^{0\pm}(10610)$	$Z_b^{0\pm}(10650)$
$D^0\bar{D}^{*0}$	$D^0\bar{D}^{*0\pm}$	$D^{*0}\bar{D}^{*0\pm}$	$B^0\bar{B}^{*0\pm}$	$B^{*0}\bar{B}^{*0\pm}$
$\delta \approx 0$	$+7.8$	$+6.7$	$+2.7$	$+1.8$

(MeV)

LINESHAPE OF THE $X(3872)$

The LHCb data

- To apply the composite criterion we must set $\Gamma_\rho^0 = \Gamma_\omega^0 = \Gamma_0^0 = 0$:

$$f\left(X \to J/\psi\pi^+\pi^-\right) \simeq -\frac{N\frac{2}{g_{LHCb}}}{\frac{2}{g_{LHCb}}\left(E - m_X^0\right) - \sqrt{2\mu_+\delta} + E\sqrt{\frac{\mu_+}{2\delta}} + ik}$$

- From here we extract

$$\boxed{r_0 = -\frac{2}{\mu g_{LHCb}} - \sqrt{\frac{\mu_+}{2\mu^2\delta}} \simeq -5.34 \text{ fm}}$$

[Esposito, Maiani, Pilloni, Polosa, Riquer — 2108.11413]

- The effective range is negative and well beyond m_π^{-1} $\longrightarrow$ the dynamics can only be explained if the X is an elementary, interacting tetraquark

POINTLIKE/COMPOSITE PARTICLES

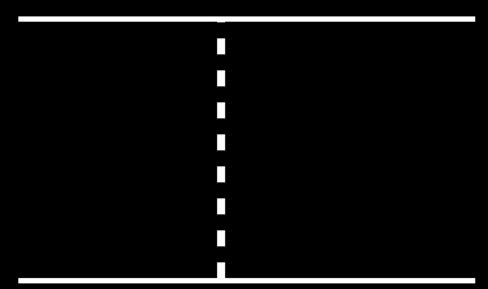
If a pointlike particle is hit by another pointlike particle, the only effect is that its momentum will change and the strength of the interaction is insensitive to the exchanged momentum.

However if the target is composite, a charge distribution in space may result, and the coupling is $\mathbf{k}$ dependent differently from the constant e coupling

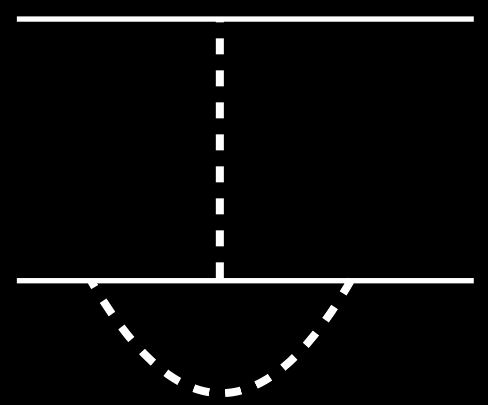
$$\delta\phi(P) = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

with $F(\mathbf{k})$ in place of e (which is obtained if $\rho(\mathbf{r}) = e\delta^3(\mathbf{r})$)

QUANTUM LOOPS FORM FACTORS



The interaction strength depends only on g (cubic coupling)



The interaction strength is a function of the exchanged momentum as a result of the loop. A *composite system of virtual* particles.

Particles are composite if their interaction with probes depends on momentum.

Then, due to quantum fluctuations, all particles are composite!

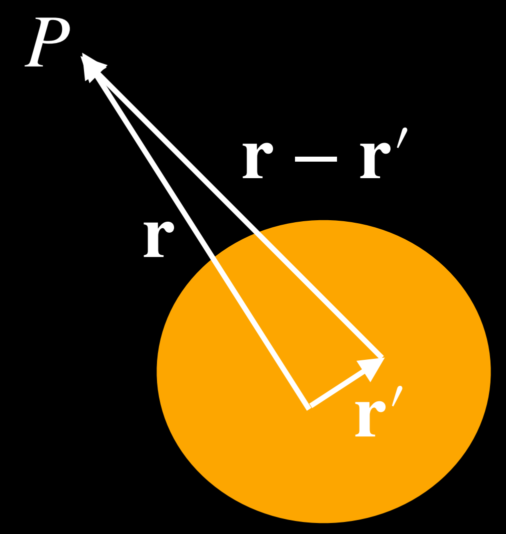
What is the meaning of elementary?

FORM FACTORS & COMPOSITENESS

$$\delta\phi(P) = \int d^3r' \frac{\rho(\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3r' \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|}\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}'} F(\mathbf{k})$$

$$= \int d^3k\, F(\mathbf{k}) \int d^3\mathbf{r}' \frac{e^{i\mathbf{k}\cdot\mathbf{r}'}}{4\pi|\mathbf{r}-\mathbf{r}'|} = \int d^3k\, e^{i\mathbf{k}\cdot\mathbf{r}} \frac{F(\mathbf{k})}{\mathbf{k}^2}$$

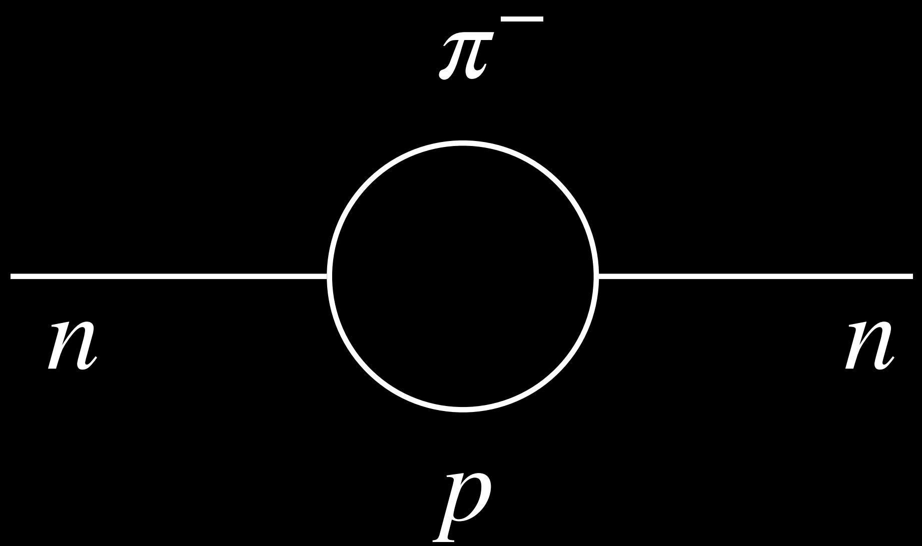
$$= \int d^4x'\, i\Delta(x-x')\, J(x') \text{ with } J(x') \equiv \rho(\mathbf{r}')$$



$F(\mathbf{k})$ in place of $e \Rightarrow$

the strength of the inter. depends on $\mathbf{k}$

THE $\pi^- p$ MOLECULE EXAMPLE



$$|n, \mathrm{in}\rangle = \sqrt{Z}\,|n, \mathrm{bare}\rangle + \int_{\mathbf{k}} \Psi_\pi(\mathbf{k})\,|p\,\pi^-(\mathbf{k}), \mathrm{bare}\rangle$$

$$\int_{\mathbf{k}} |\Psi_\pi(\mathbf{k})|^2 = 1 - Z$$

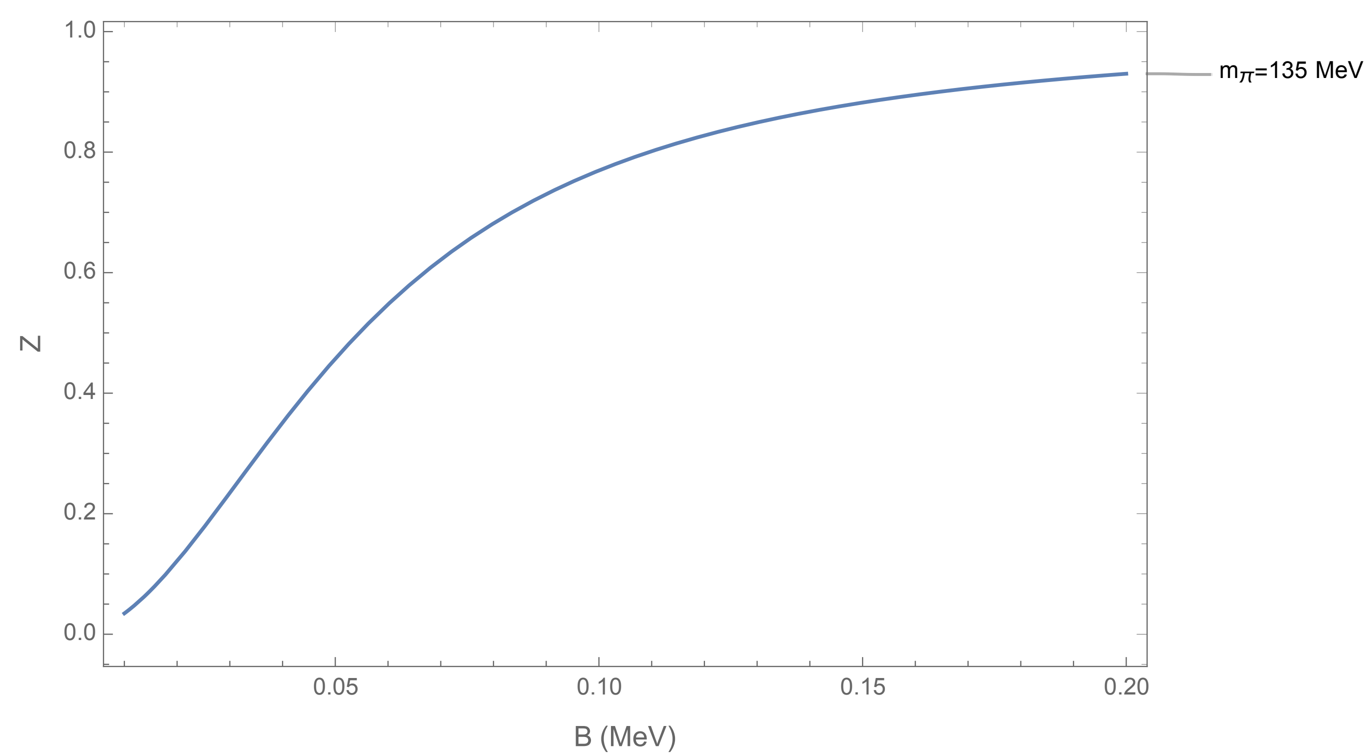
Same equations as Weinberg's

See the "Lee-model" ('54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

THE $\pi^- p$ MOLECULE EXAMPLE

$$Z^{-1} = 1 + \left(\frac{M_p}{F_\pi} g_A\right)^2 \frac{1}{2\pi^2} \int_0^\infty \frac{k^2}{(\sqrt{2m_\pi})^2 (B + \frac{k^2}{2(M_p + m_\pi)})^2 (1 + \frac{k^2}{M_A^2})^4} dk$$

Tune the mass of the neutron, i.e. tune B



See the “Lee-model” (’54) in Henley & Thirring, Elementary Quantum Field Theory, McGraw-Hill
T.D. Lee, Phys. Rev. 95, 1329 (1954)

THE LANDAU COUPLING & $X \to DD\pi$

$$g^2 = 16\pi\sqrt{\frac{2B}{m}}\frac{(m_a + m_b)^2}{1 - r_0\sqrt{2mB}}$$

To compute $\Gamma(X \to DD\pi)$

ADP, 1505.03083