Primordial Black Holes: Threshold prescription, QCD phase transition and cosmological impact

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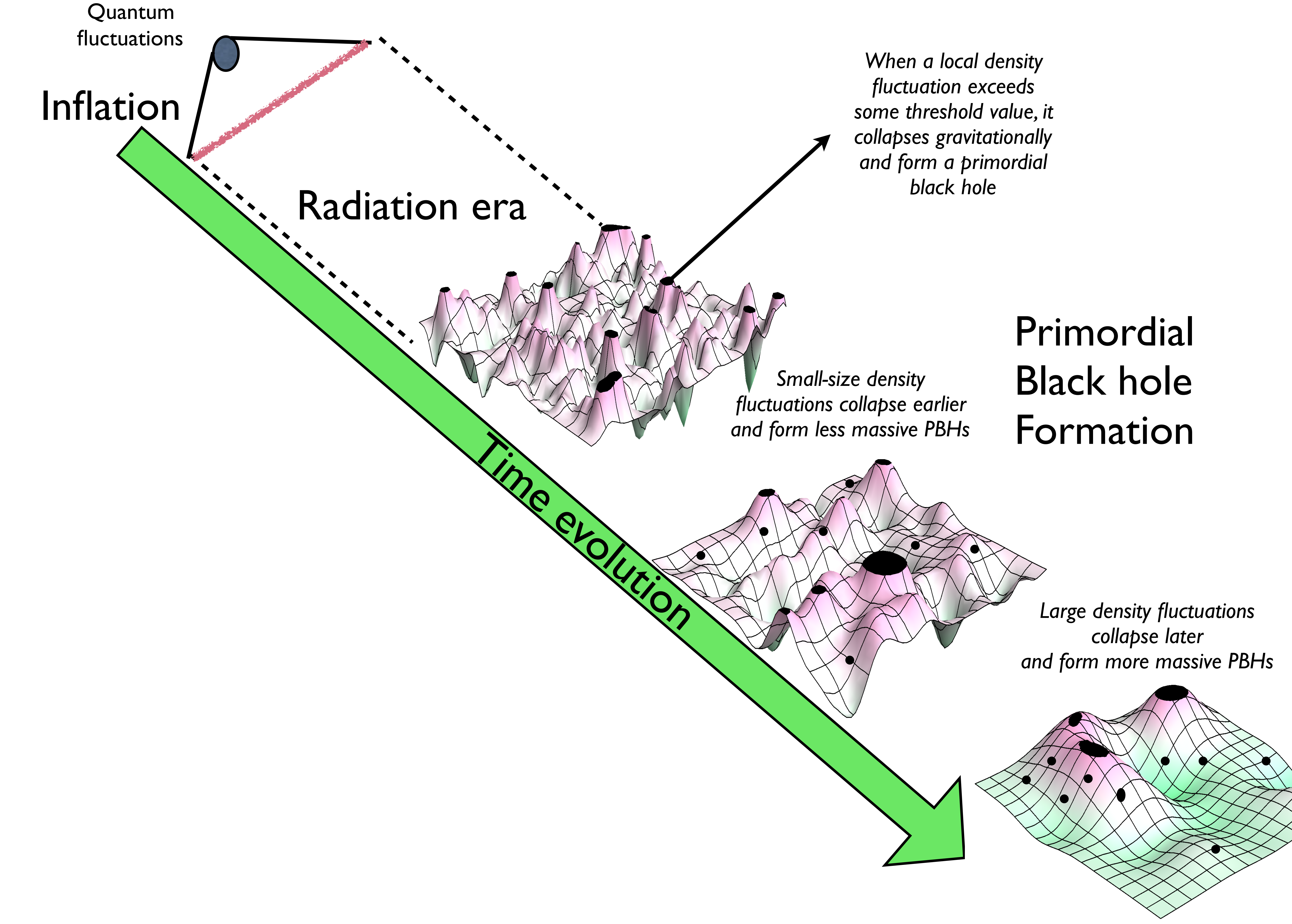
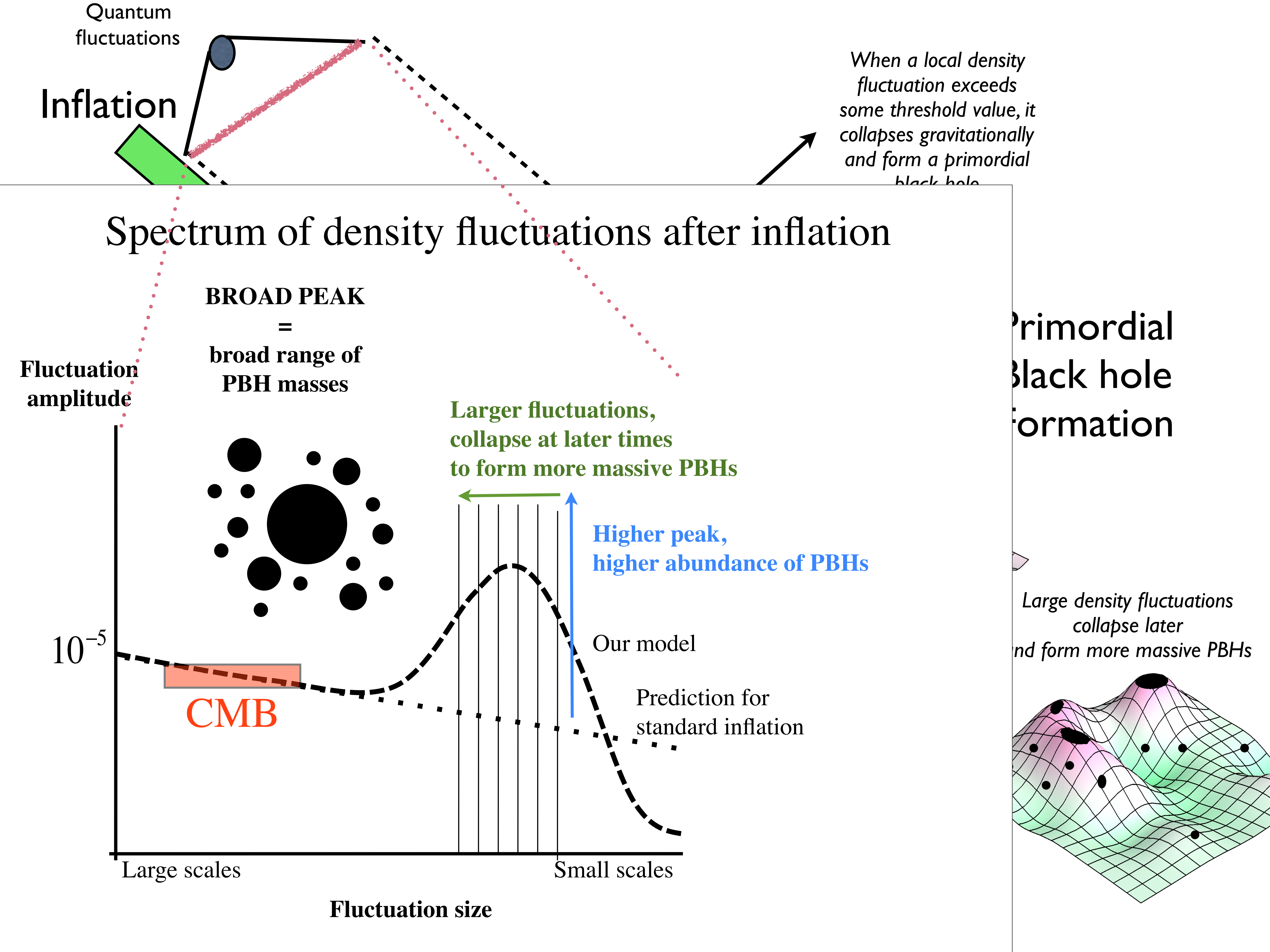
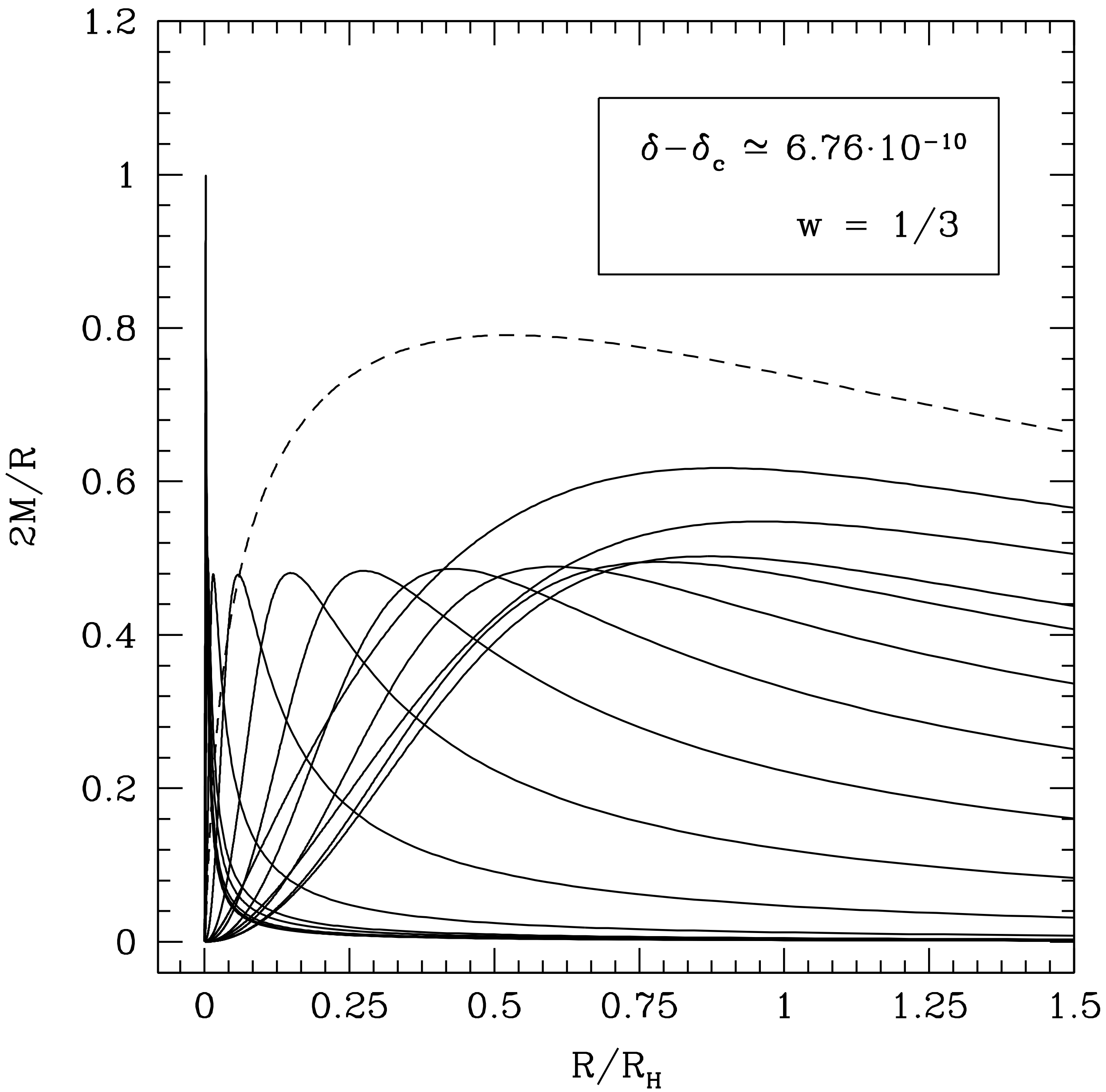
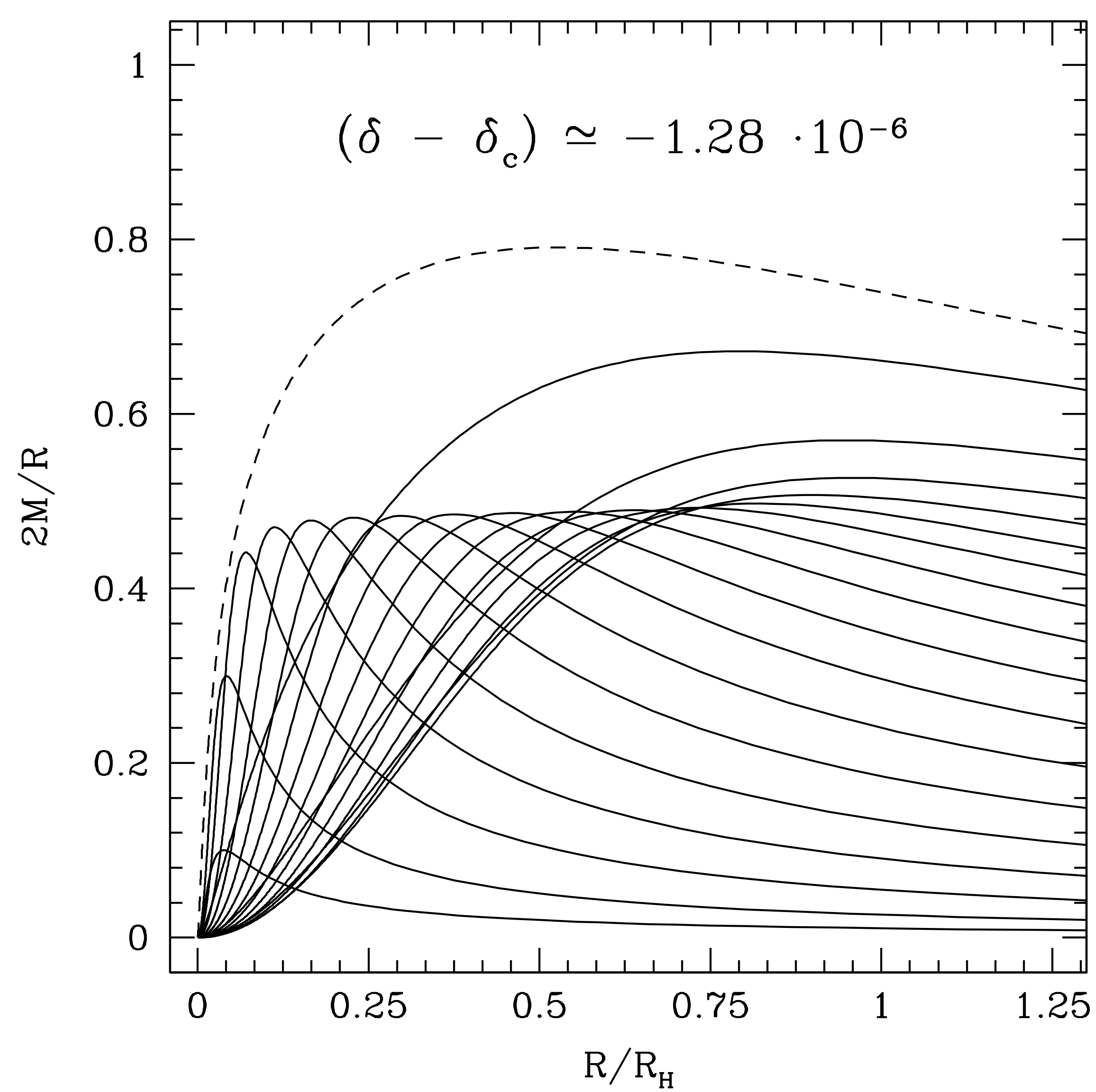

Quantum fluctuations
Inflation
Radiation era
When a local density fluctuation exceeds some threshold value, it collapses gravitationally and form a primordial black hole
Primordial Black hole Formation
Small-size density fluctuations collapse earlier and form less massive PBHs
Time evolution
Large density fluctuations collapse later and form more massive PBHs


Quantum
fluctuations
Inflation
When a local density
fluctuation exceeds
some threshold value, it
collapses gravitationally
and form a primordial
black hole
Spectrum of density fluctuations after inflation
BROAD PEAK
=
broad range of
PBH masses
Fluctuation
amplitude
Larger fluctuations,
collapse at later times
to form more massive PBHs
Higher peak,
higher abundance of PBHs
Our model
Prediction for
standard inflation
10^{-5}
CMB
Large scales
Small scales
Fluctuation size
Primordial
Black hole
Formation
Large density fluctuations
collapse later
and form more massive PBHs

Numerical Results: PBH formation/bounce

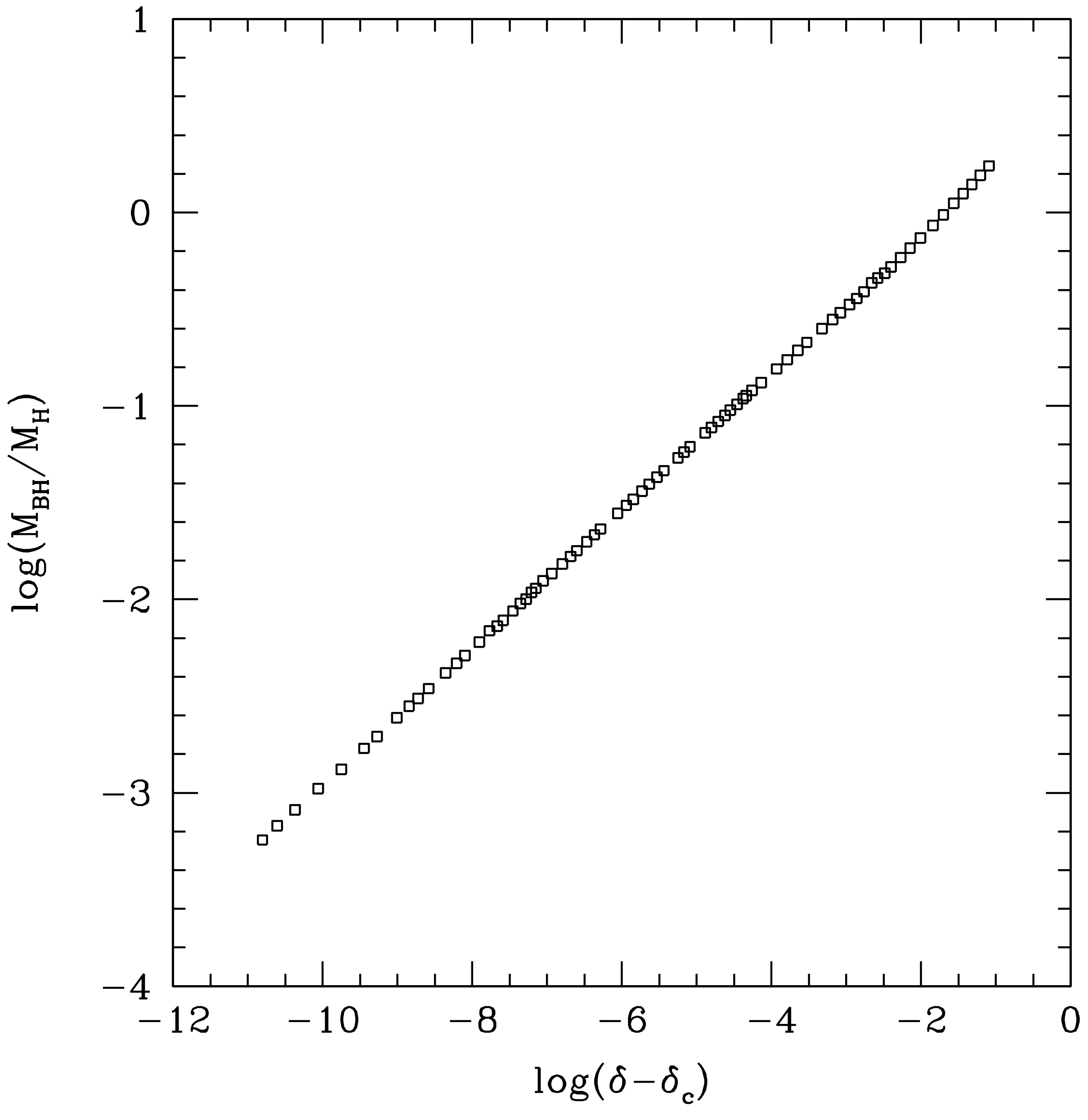
$$R(r,t) = 2M(r,t)$$





IM, J. Miller, Rezzolla - **CQG (2005)**

Numerical Results: Scaling Law / Critical collapse



$$\boxed{M_{PBH} = \mathcal{K}(\delta - \delta_c)^\gamma M_H}$$

M_H – cosmological horizon mass

$\mathcal{K}, \delta_c$ – shape dependent

$$\gamma \simeq 0.36$$

***IM, Miller, Polnarev* - CQG (2009, 2013)**

Curvature profile

- The asymptotic metric ($t \to 0$), describing super-horizon cosmological perturbations in the comoving synchronous gauge can be written as:

$$ds^2 \simeq -dt^2 + a^2(t)e^{2\zeta(r)}\left[dr^2 + r^2 d\Omega^2\right]$$

- In the "linear regime" of cosmological perturbations, adiabatic perturbations on super horizon scales can be described by a time independent curvature profile using the **quasi-homogeneous / gradient expansion approach**.

$$\frac{\delta\rho}{\rho_b} = -\left(\frac{1}{aH}\right)^2 \frac{4}{9}\left[\nabla^2\zeta(r) + \frac{1}{2}\left(\nabla\zeta(r)\right)^2\right] e^{-2\zeta(r)}$$

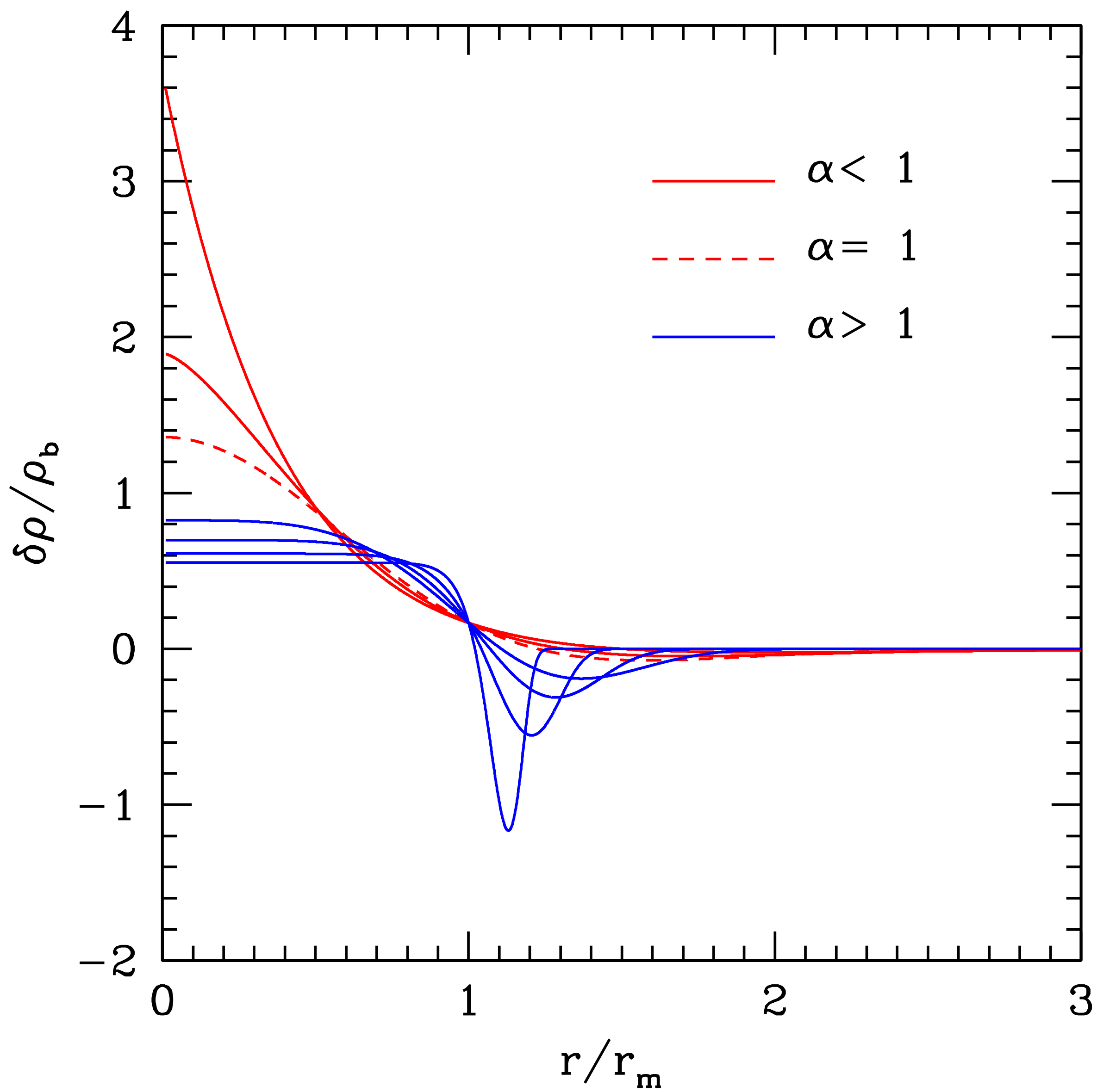
- The perturbation amplitude δ is measured by the peak of the compaction function, measuring the excess of mass of the over density.

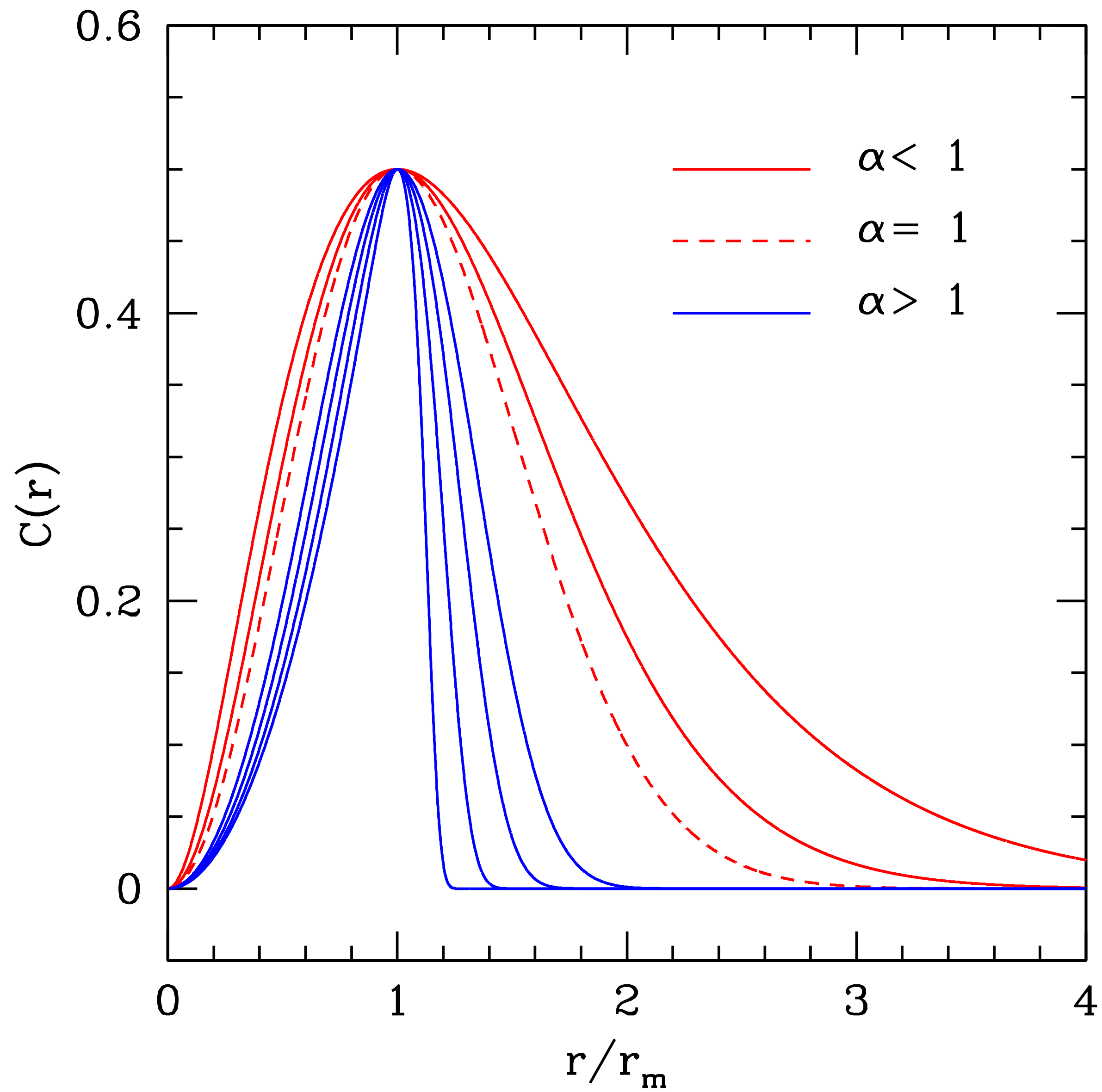
$$\mathcal{C}(r) := \frac{2[M(r,t) - M_b(r,t)]}{R(r,t)} = -\frac{4}{3}\tilde{r}\zeta'(r)\left[1 + \frac{1}{2}\tilde{r}\zeta'(r)\right]$$

Musco - **PRD (2019)**

$$K(r) = \mathcal{A} \exp\left[-\frac{1}{\alpha}\left(\frac{r}{r_m}\right)^{2\alpha}\right] \Rightarrow \frac{\delta\rho}{\rho_b} = \frac{\delta\rho_0}{\rho_b}\left[1 - \frac{2}{3}\left(\frac{r}{r_m}\right)^{2\alpha}\right]\left[-\frac{1}{\alpha}\left(\frac{r}{r_m}\right)^{2\alpha}\right]$$

Shape parameter: $$\alpha = -\frac{\mathcal{C}''(r_m) r_m^2}{4\mathcal{C}(r_m)} \Rightarrow 0.4 \leq \delta_c \leq \frac{2}{3}$$





PBH threshold prescription

Curvature power spectrum $\mathcal{P}_\zeta$

$\Downarrow$

Characteristic overdensity scale $k_* \hat{r}_m$

$\Downarrow$

Characteristic shape parameter α

$\Downarrow$

Threshold δ_c

IM, De Luca, Franciolini, Riotto - PRD (2021)

1. **The power spectrum of the curvature perturbation**: take the primordial power spectrum $\mathcal{P}_\zeta$ of the Gaussian curvature perturbation and compute, on superhorizon scales, its convolution with the transfer function $T(k,\eta)$

$$P_\zeta(k,\eta) = \frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k) T^2(k,\eta).$$

2. **The comoving length scale** $\hat{r}_m$ of the perturbation is related to the characteristic scale k_* of the power spectrum P_ζ. Compute the value of $k_* \hat{r}_m$ by solving the following integral equation

$$\int dk k^2 \left[(k^2 \hat{r}_m^2 - 1)\frac{\sin(k\hat{r}_m)}{k\hat{r}_m} + \cos(k\hat{r}_m) \right] P_\zeta(k,\eta) = 0\,.$$

3. **The shape parameter:** compute the corresponding shape parameter α of the collapsing perturbation, including the correction from the non linear effects, by solving the following equation

$$F(\alpha)\left[1+F(\alpha)\right]\alpha = -\frac{1}{2}\left[1+\hat{r}_m \frac{\int dk k^4 \cos(k\hat{r}_m) P_\zeta(k,\eta)}{\int dk k^3 \sin(k\hat{r}_m) P_\zeta(k,\eta)}\right]$$

$$F(\alpha) = \sqrt{1 - \frac{2}{5} e^{-1/\alpha} \frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}}\,.$$

4. **The threshold δ_c:** compute the threshold as function of α, fitting the numerical simulations.

- At *superhorizon scales* making a linear extrapolation at horizon crossing ($aHr_m = 1$).

$$\delta_c \simeq \begin{cases} \alpha^{0.047} - 0.50 & 0.1 \lesssim \alpha \lesssim 7 \\ \alpha^{0.035} - 0.475 & 7 \lesssim \alpha \lesssim 13 \\ \alpha^{0.026} - 0.45 & 13 \lesssim \alpha \lesssim 30 \end{cases}$$

- At *horizon crossing* taking into account also the non linear effects.

$$\delta_c \simeq \begin{cases} \alpha^{0.125} - 0.05 & 0.1 \lesssim \alpha \lesssim 3 \\ \alpha^{0.06} + 0.025 & 3 \lesssim \alpha \lesssim 8 \\ 1.15 & \alpha \gtrsim 8 \end{cases}$$

The difference between these two values of the threshold δ_c is discussed later in section V.

PBH Abundance (Peak Theory)

- **PDF of δ follow a Gaussian distribution:** $P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\delta^2}{2\sigma^2}\right)$

$$\sigma^2 = \langle \delta^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\delta(k, r_m) = \left(\frac{2}{3}\right)^4 \int_0^\infty \frac{dk}{k} (k r_m)^4 T^2(k, r_m) W^2(k, r_m) \mathcal{P}_\zeta(k)$$

$$\beta_f \simeq \sqrt{\frac{2}{\pi}} \mathcal{K} \left(\frac{k_*}{a_m H_m}\right)^3 \sigma^\gamma \nu_c^{1-\gamma} \gamma^{\gamma+1/2} e^{-\frac{\nu_c^2}{2}} \qquad \nu_c \equiv \frac{\delta_c}{\sigma}$$

- **If** $M_{PBH} \sim 10^{16} g$ are Dark Matter $\Rightarrow \beta_f \simeq 10^{-8} \sqrt{\frac{M_{PBH}}{M_\odot}} \simeq 10^{-16}$

- **Narrow peak:** $\frac{k_*}{\sigma} \gg 1 \;\Rightarrow\; \nu_c \simeq 0.22 \sqrt{\frac{k_*}{\sigma \mathcal{P}_0}} \;\Rightarrow\; \mathcal{P}_0 \sim 7 \times 10^{-4} \frac{k_*}{\sigma} \gg 10^{-3}$

- **Broad peak:** $\frac{k_*}{\sigma} \ll 1 \;\Rightarrow\; \nu_c \simeq 0.46 (\mathcal{P}_0)^{-1/2} \;\Rightarrow\; \mathcal{P}_0 \sim 3 \times 10^{-3}$

C.Germani, IM - PRL (2019)

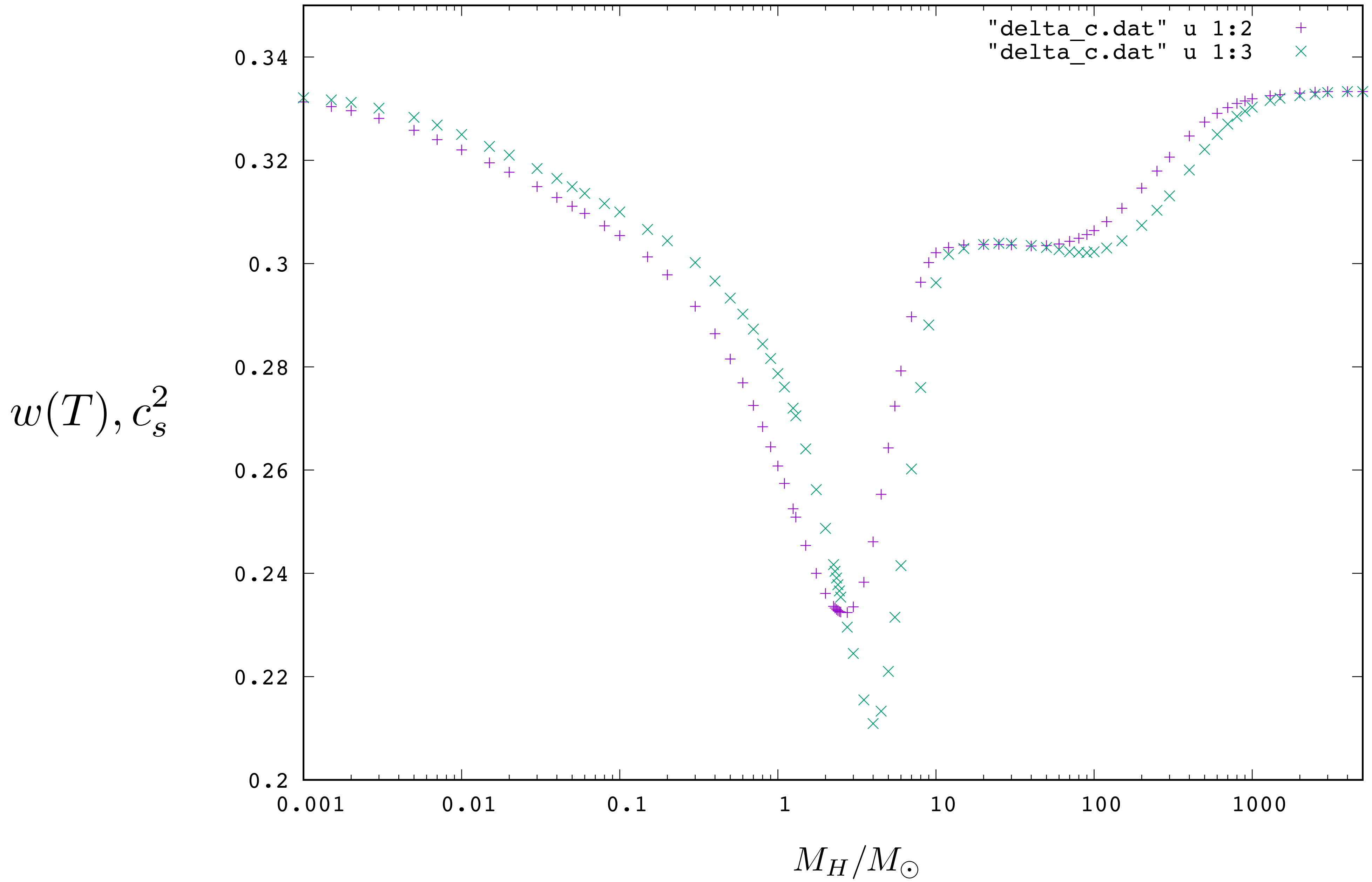
See also *Chul-Moon Yoo, Harada, Garriga, Kohri* (2019)

PBHs during QCD phase transition

$$\rho = \frac{\pi^2}{30} g_{\rm eff}(T) T^4 \qquad s = \frac{2\pi^2}{45} h_{\rm eff}(T) T^3 \qquad c_s^2(T) = \frac{\partial p}{\partial \rho} = \frac{4(4h_{\rm eff} + T h'_{\rm eff})}{3(4g_{\rm eff} + T g'_{\rm eff})} - 1$$

$$p = sT - \rho = w(T)\rho \qquad w(T) = \frac{4h_{\rm eff}(T)}{3g_{\rm eff}(T)} - 1 \qquad g_{\rm eff}(T)\,,\ h_{\rm eff}(T)$$

particle degrees of freedom (lattice QCD simulations)



$$P_\zeta(k) = A\,(k/k_{\min})^{n_s-1}\,\Theta(k-k_{\min})\Theta(k_{\max}-k)$$

IM, K. Jedamzik, Sam Young - in progress...

G. Franciolini, IM, P.Pani, A.Urbano - in progress...

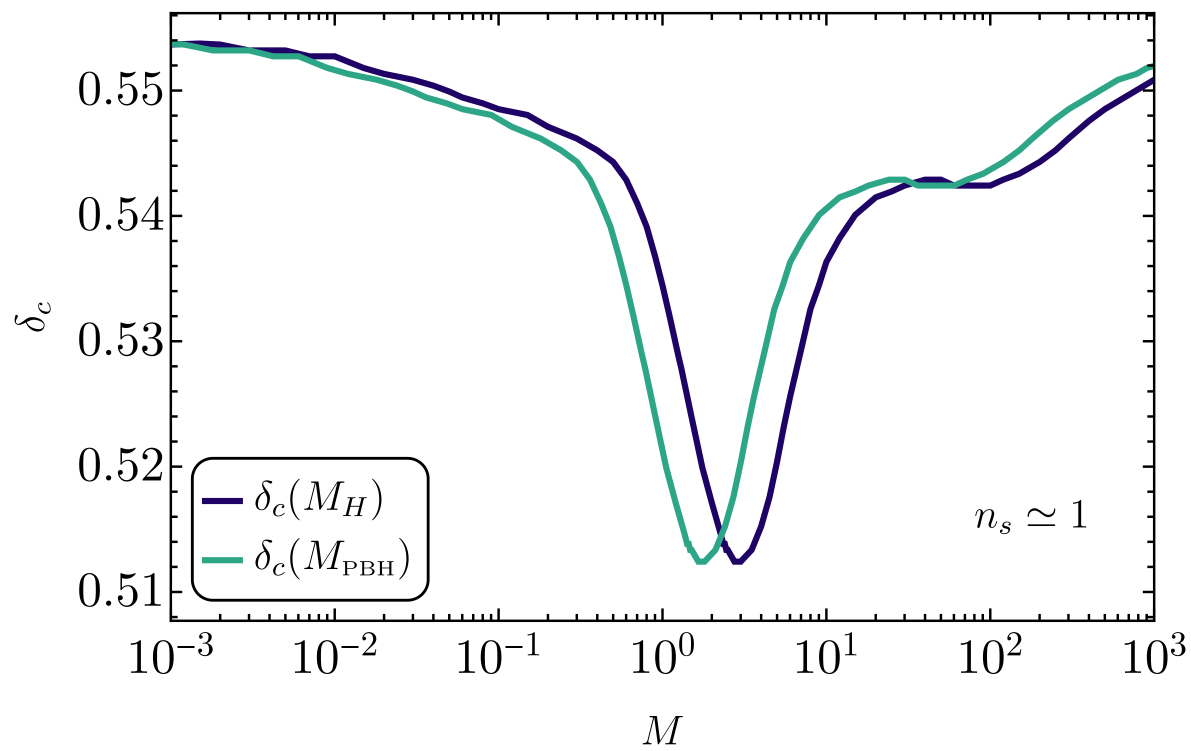


FIG. 1: Threshold as a function of the horizon mass M_H or the corresponding effective PBH mass $M_{\rm PBH} \simeq 0.602 \times M_H$

$$f_{\rm PBH} \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}$$

$$\psi(M_{\rm PBH}) = \frac{1}{\Omega_{\rm PBH}}\frac{{\rm d}\Omega_{\rm PBH}}{{\rm d}M_{\rm PBH}}$$

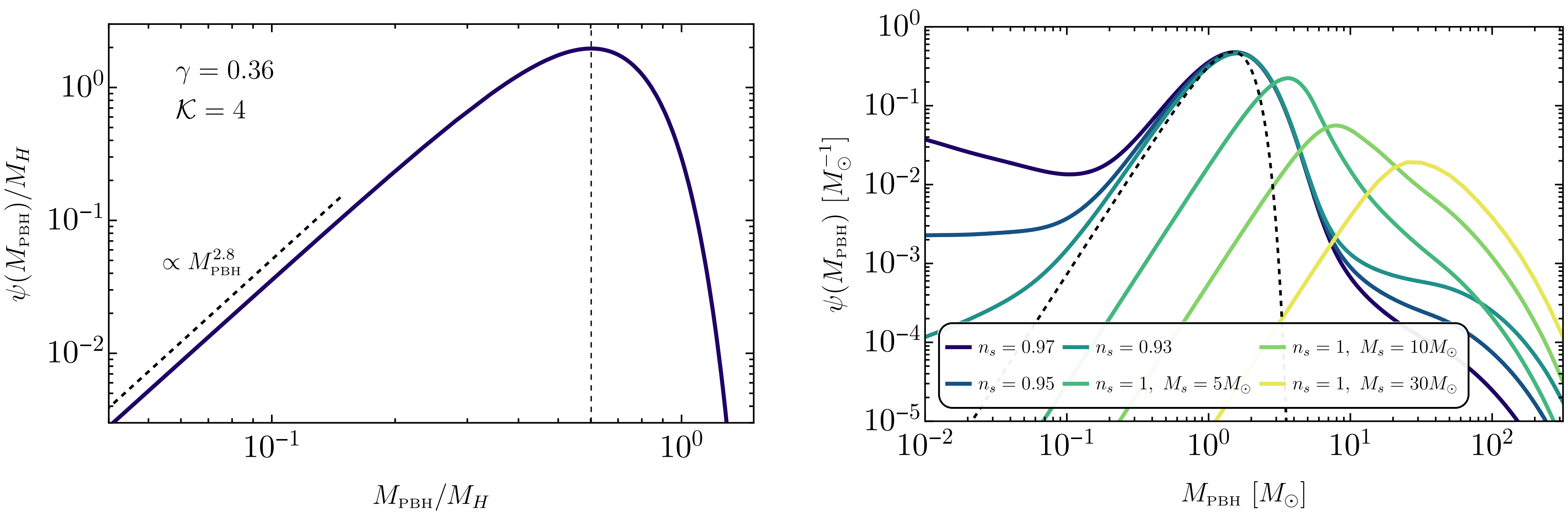


FIG. 2: **Left:** Mass distribution resulting from the collapse of a single scale crossing the horizon at M_H. The low mass tail is fixed by the critical collapse scaling γ. The vertical gridline indicates the peak of the contribution sitting at $M_{\rm PBH}/M_H = 0.602$. **Right:** Mass function obtained with few nearly scale invariant power spectrum. Where not specified, the minimum horizon mass is assumed to be smaller than $M_{\rm s} \lesssim 10^{-2} M_\odot$. We also show the critical mass function produced by the collapse of a single spectrum scale at the peak of the QCD, which has a key role in shaping the peak of the mass distribution around the solar mass.

Conclusions

- The curvature profile (pressure gradients) play a key role determining the particular value of the threshold. **This can be related to the morphology of the power spectrum of cosmological perturbations.**

- **PBH formation is characterised by non liner curvature profile.** The linear approximation does not gives accurate results.

- **The threshold, including also the non linear effects of the horizon crossing, could be fully computed using Gaussian statistics plus an algebraic correction accounting for the non linear effects. -** *IM, De Luca, Franciolini, Riotto* - PRD (2021)

- **The abundance of PBHs is exponentially sensitive to the threshold.** The shape of the peak of the power spectrum is very important.

- A large enough feature of the power spectrum on small scales (large k) could account for an important component of dark matter in PBHs.

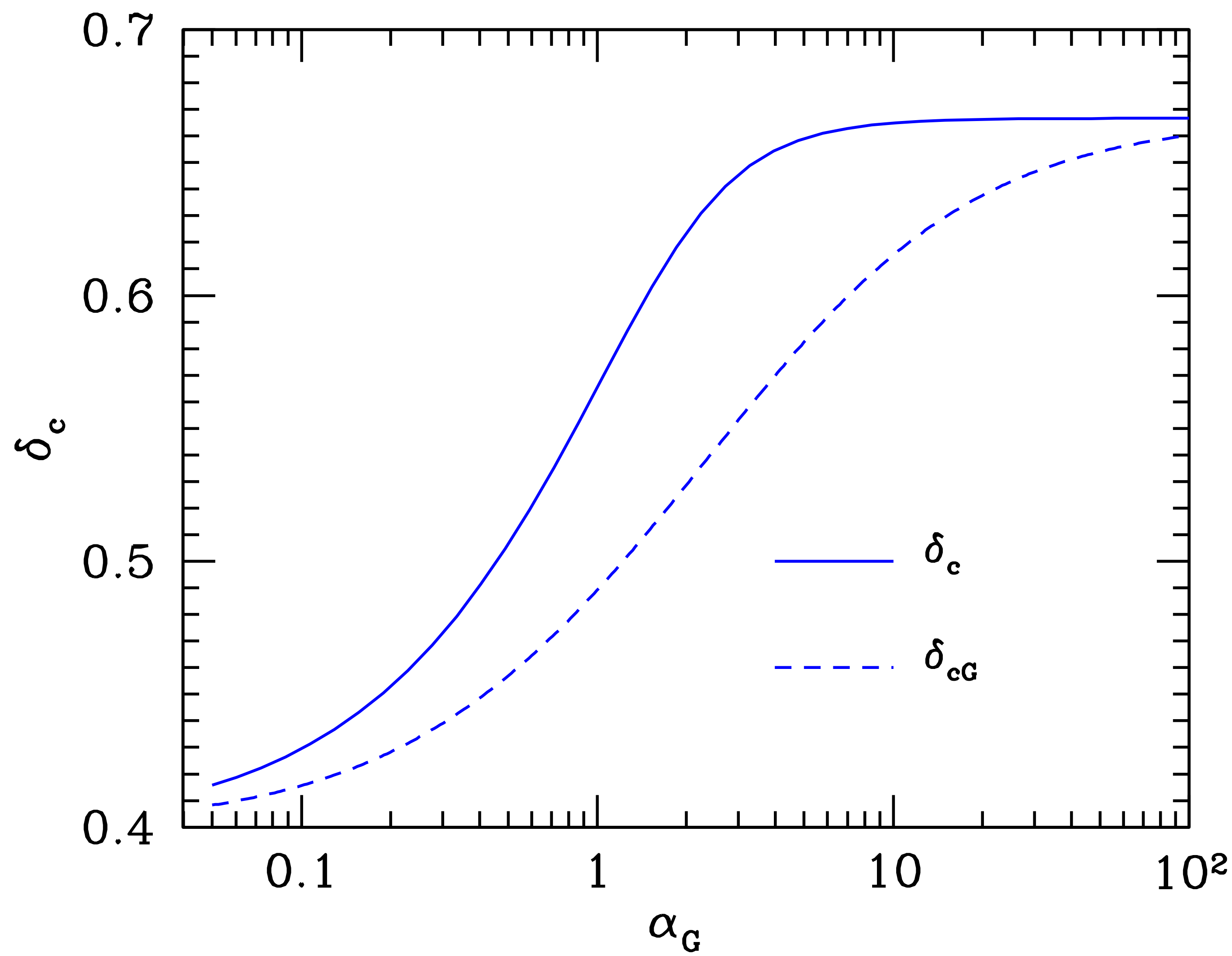
- A **softening of the equation of state (QCD)** significantly **enhances the formation of PBHs:** at the minimum of the QCD the threshold δ_c is about 10% smaller, increasing the abundance of PBHs, **typically between 1 and 2 solar masses**, of about 100 times - **in progress…**

PBH threshold

- *Escrivá, Germani, Sheth* - PRD (2020)

$$\bar{\mathcal{C}}(r_m) = 0.4 \quad \text{(shape independent)}$$

$$\delta_c \simeq \frac{4}{15} e^{-\frac{1}{\alpha}} \frac{\alpha^{1-5/2\alpha}}{\Gamma\left(\frac{5}{2\alpha}\right) - \Gamma\left(\frac{5}{2\alpha}, \frac{1}{\alpha}\right)}$$



- *IM, De Luca, Franciolini, Riotto* - PRD (2021)

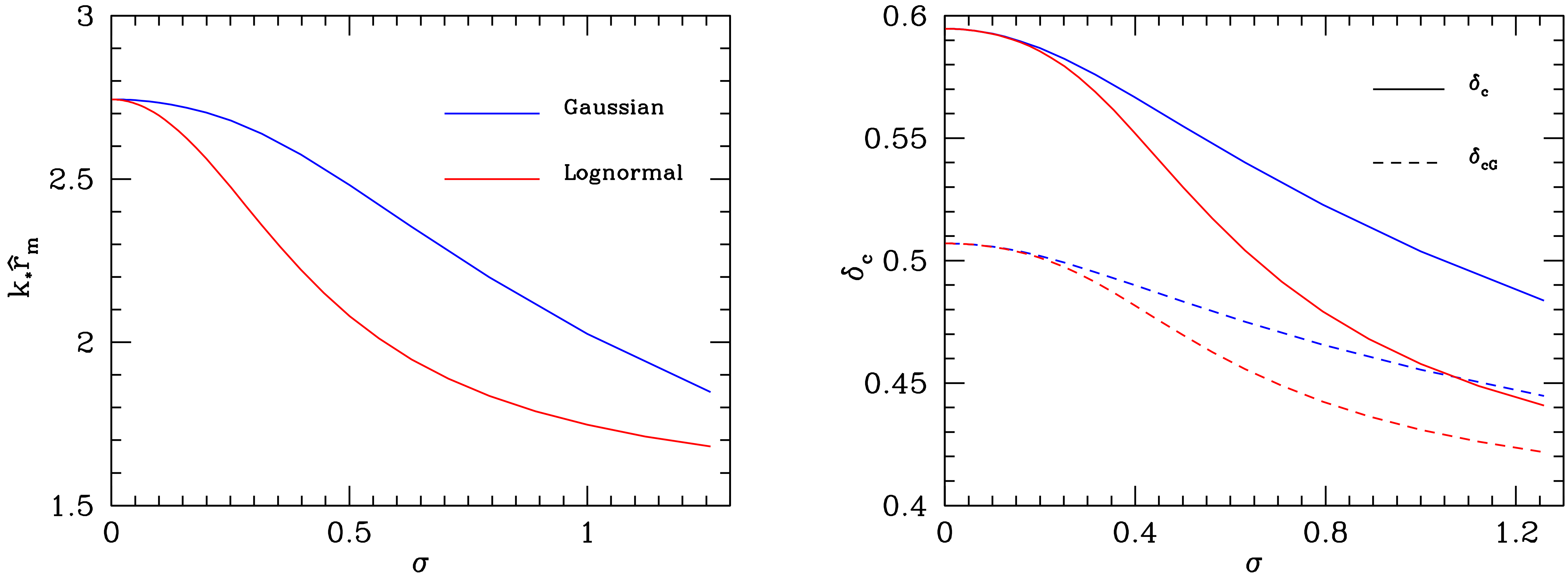
$$\Phi_m = -\tilde{r}_m \zeta'(\tilde{r}_m)$$

$$\delta_m = \frac{4}{3}\Phi_m \left(1 - \frac{1}{2}\Phi_m\right) = \delta_G \left(1 - \frac{3}{8}\delta_G\right)$$

$$\alpha = -\frac{\Phi''_m \, \tilde{r}_m^2}{4\Phi_m \left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$

$$\alpha = -\frac{\alpha_G}{\left(1 - \frac{1}{2}\Phi_m\right)\left(1 - \Phi_m\right)}$$

Power Spectrum



$$\text{Gaussian: } \mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-(k-k_*)^2/2\sigma^2\right]$$

$$\text{Lognormal: } \mathcal{P}_\zeta(k) = \mathcal{P}_0 \exp\left[-\ln^2(k/k_*)/2\sigma^2\right]$$