g-mode Oscillations in Neutron Stars

C. Constantinou

Ferrara, 31 May 2022

- Global, long-lived, nonradial fluid oscillations resulting from fluid-element perturbations in a stratified environment.
- Slow chemical equilibration generates buoyancy forces to oppose dispacement.
- In stably-stratified systems the opposing force sets up oscillations with a characteristic frequency (Brunt-Vaisala) which depends on both the equilibrium and the adiabatic sound speeds.
- g-mode oscillations couple to tidal forces; they can be excited in a NS merger and provide information on the interior composition.
- Detection remains a challenge; but within sensitivity of 3rd generation detectors.

- In linearized GR, the calculation of g-mode frequencies, damping times, and amplitudes requires the solution of 4 coupled ODEs.
- The relativistic Cowling approximation neglects metric perturbations that must accompany matter perturbations in a GR treatment reducing complexity:

$$\begin{aligned}\frac{dU}{dr} &= \frac{g}{c_{\rm ad}^2}U + \mathrm{e}^{\lambda/2}\left[\frac{l(l+1)\mathrm{e}^{\nu}}{\omega^2} - \frac{r^2}{c_{\rm ad}^2}\right]V \\ \frac{dV}{dr} &= \mathrm{e}^{\lambda/2-\nu}\,\frac{\omega^2 - N^2}{r^2}U + g\Delta(c^{-2})V\end{aligned}$$

where $U = r^2\mathrm{e}^{\lambda/2}\,\xi_r$, $V = \omega^2 r\,\xi_h$, $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$,
$N^2 = g^2\Delta(c^{-2})\mathrm{e}^{\nu-\lambda}$, $g = -\nabla P/(\varepsilon + P)$,
and λ, ν are Schwarzchild metric functions.

- Accurate to a few % compared to GR.
- Cannot compute imaginary part of eigenfrequeny (damping time).

Sound Speeds

- The difference $\Delta(c^{-2}) = c_{\rm eq}^{-2} - c_{\rm ad}^{-2}$ drives the restoring force for g-mode oscillations. For example, in *npe* matter

$$
\begin{aligned}
c_{\rm ad}^2 &= c_{\rm eq}^2 + \frac{\left[n_B \left(\frac{\partial\tilde{\mu}}{\partial n_B}\right)_x\right]^2}{\mu_n \left(\frac{\partial\tilde{\mu}}{\partial x}\right)_{n_B}} \\
\tilde{\mu} &= \mu_e + \mu_p - \mu_n \stackrel{\beta-{\rm eq.}}{\longrightarrow} 0
\end{aligned}
$$

- $c_{\rm eq}^2(n_B) = \frac{dp}{d\epsilon}$; β-eq. restored instantaneously.
- $c_{\rm ad}^2 = \left(\frac{\partial p}{\partial\epsilon}\right)_x$; $\tau_\beta \gg \tau_{\rm oscillation}$.

Equation of State

- Nucleons: Zhao - Lattimer

$$\begin{aligned}\epsilon_B &= \sum_{h=n,p} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{M_B^2 + k^2}\, dk + n_B V(u,x) \\ V(u,x) &= 4x(1-x)(a_0 u + b_0 u^\gamma) + (1-2x)^2 (a_1 u + b_1 u^{\gamma_1})\end{aligned}$$

- Quarks: vMIT

$$\begin{aligned}\mathcal{L} &= \sum_{q=u,d,s} \left[\bar{\psi}_q \left(i\partial\!\!\!/ - m_q - B\right) \psi_i + \mathcal{L}_{\rm int}\right] \Theta \\ \mathcal{L}_{\rm int} &= -G_v \sum_q \bar{\psi}\gamma_\mu V^\mu \psi + \left(m_V^2/2\right) V_\mu V^\mu \\ \epsilon_Q &= \sum_q \epsilon_{\rm FG,q} + \frac{1}{2}\left(\frac{G_v}{m_V}\right)^2 n_Q^2 + B\end{aligned}$$

- Leptons: noninteracting, relativistic fermions

$$\epsilon_L = \sum_{l=e,\mu} \frac{1}{\pi^2} \int_0^{k_{Fh}} k^2 \sqrt{m_L^2 + k^2}\, dk$$

- Gibbs

$$\begin{aligned} \varepsilon^* &= (1-\chi)\varepsilon_H + \chi\varepsilon_Q \quad ; \quad 0 \leq \chi \leq 1 \\ P_Q &= P_H \end{aligned}$$

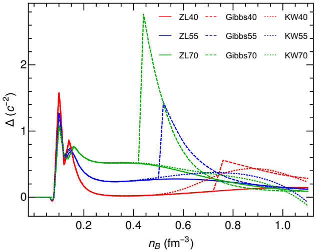
- Crossover (Kapusta-Welle)

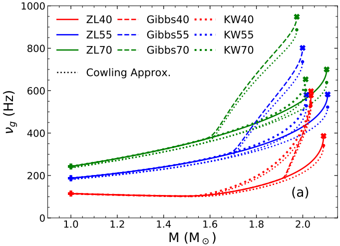
$$\begin{aligned} P_B &= (1-S)P_H + S\ P_Q \\ S &= \exp\left[-\left(\frac{\mu_0}{\mu}\right)^4\right] \\ \mu_0 &\sim 2\ \mathrm{GeV} \end{aligned}$$

- Neutron-star matter
  - Strong equilibrium: $\mu_n = 2\mu_d + \mu_u \quad ; \quad \mu_p = 2\mu_u + \mu_d$
  - Weak equilibrium: $\mu_n = \mu_p + \mu_e \quad ; \quad \mu_e = \mu_\mu \quad ; \quad \mu_d = \mu_s$
  - Charge neutrality: $n_p^* + (2n_u^* - n_d^* - n_s^*)/3 - (n_e + n_\mu) = 0$
  - Baryon number cons: $n_n^* + n_p^* + (n_u^* + n_d^* + n_s^*)/3 - n_B = 0$

Sound-speed difference vs. g-mode frequency

- g-modes in Gibbs hybrid matter have a larger frequency range compared to the pure-nucleon and crossover cases corresponding to the behavior of $\Delta(c^{-2})$ in the mixed phase.
- Dramatic changes in ν_g require the appearance of new particle species not merely a smooth change in composition.
- The Cowling approximation is qualitatively similar to GR but underestimates ν_g by up to 10%; does better for low-mass stars.



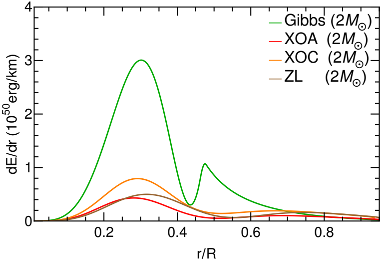


Energy per unit distance

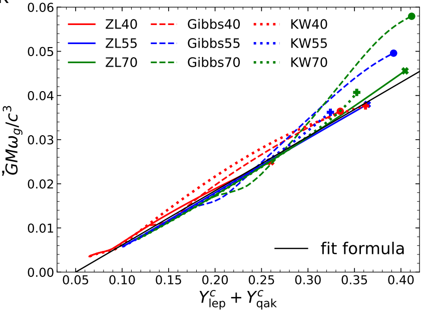
- Energy per unit radial distance in oscillatory motion:
$\frac{dE_T}{dr} = \frac{\omega^2 r^2}{2}(\varepsilon + P)\mathrm{e}^{(\lambda-\nu)/2}\left[\xi_r^2 \mathrm{e}^{\lambda} + l(l+1)\xi_h^2\right]$

- ZL, XO $\times$ 10

- The Gibbs energy scale is one order of magnitude larger than ZL and KW once quark matter appears ($\sim 10^51$ ergs/km vs. $\sim 10^50$ ergs/km).



- Universal relations depend weakly on the EOS and can be used to break degeneracies and otherwise constrain difficult-to-access observables.
- Given the sensitivity of g-modes to departures from chemical equilibrium, it is likely that N and ν_g depend strongly on composition
- $\Omega_g = GM\omega_g/c^3 = 1.228(Y^c - 0.05)$



Summary

- First calculation of g-mode properties under Gibbs phase rules and for the KW model (both with the Cowling approximation as well as linearized GR).
- g-modes can detect nonnucleonic matter in the cores of NS; assuming quark matter (by some other means), g-modes can distinguish between a first-order phase transition and a crossover.
- Universal relation between Ω_g and Y^c.
- (Near) Future:
  - Extend KW to finite T.
  - Applications to protoneutron stars (cooling, superfluidity)
  - Other signals?
  - Construct EOS that uses the same underlying description for quarks and hadrons; explore hybrid matter microscopically.