ALL ORDERS IN GAUGE THEORIES

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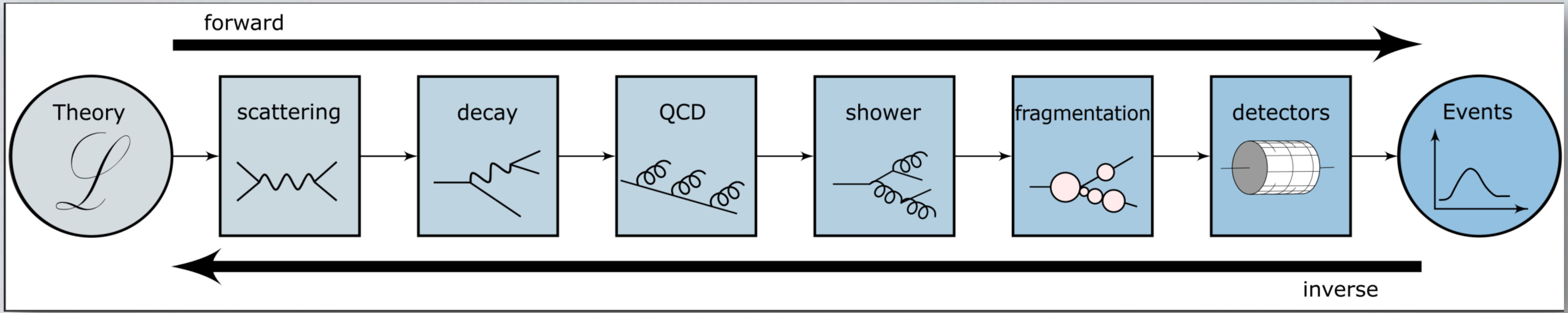
PRECISION FOR COLLIDER PHENOMENOLOGY

- Precision as a tool for the discovery of new physics as small deviations from the SM.



(Figure from 2203.07460)

- Focus on the first step, the perturbative calculation of hard scattering kernels:

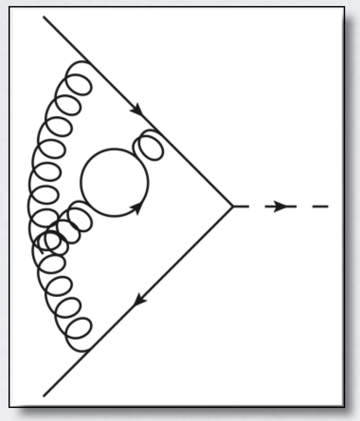
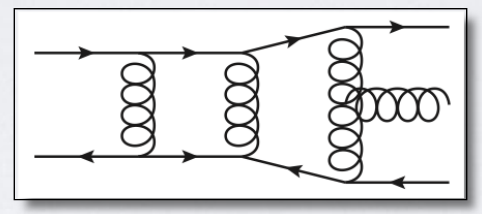
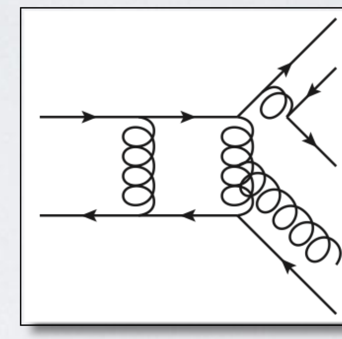
$$\sigma_X = \sum_{a,b} \int_0^1 dx_1\, dx_2\, f_a(x_1, \mu_F^2)\, f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab\to X}\left(x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2}\right),$$

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_s\, \sigma_1 + \alpha_s^2\, \sigma_2 + \dots$$

PRECISION FOR COLLIDER PHENOMENOLOGY

$$\hat{\sigma}_{ab\to X} = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2 + \ldots$$

- Hard scattering processes are calculated in perturbation theory.

- Going beyond NNLO and N3LO turns out to be incredibly difficult, yet necessary to match the precision of current and forthcoming experiments!
- Loop and phase space integrals:
    - Analytic vs numerical evaluation
    - Space of functions
    - Infrared divergences
    - Large logarithms

Express Feynman integrals in terms of known functions: Log(x), Li2(x), H(a1,...an;x), ...

KLN theorem: IR divergences cancel among virtual and real diagrams, yet structure of IR divergences needed for analytical and numerical evaluation of scattering amplitudes.

Many Scales;

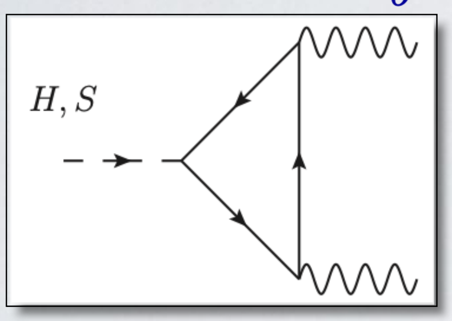
Dynamical enhancement of soft and collinear radiation;

Spoil the convergence of the perturbative series: need resummation.

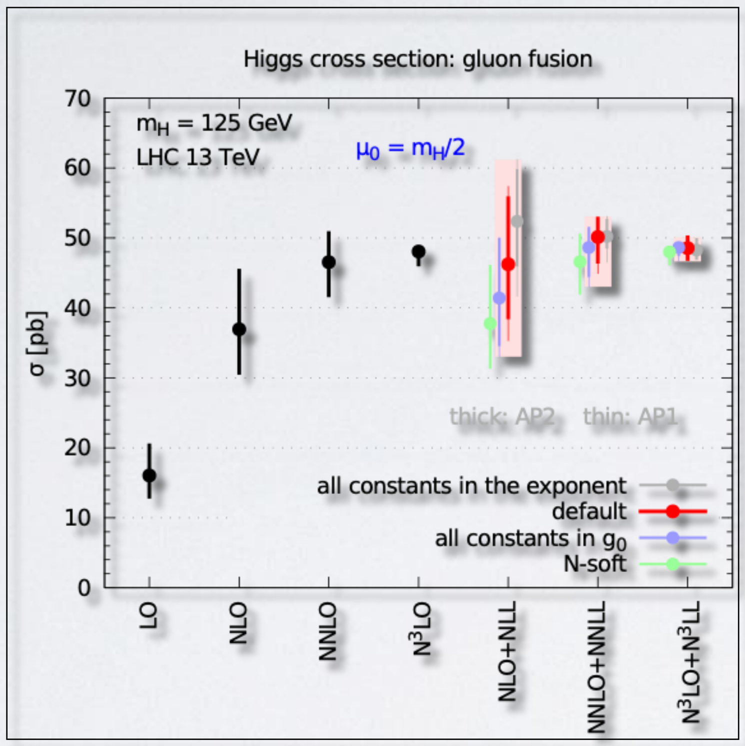
ANALYTIC TOOLS FOR MULTI-SCALE SCATTERING

- Multiple scales gives rise to large logarithms:

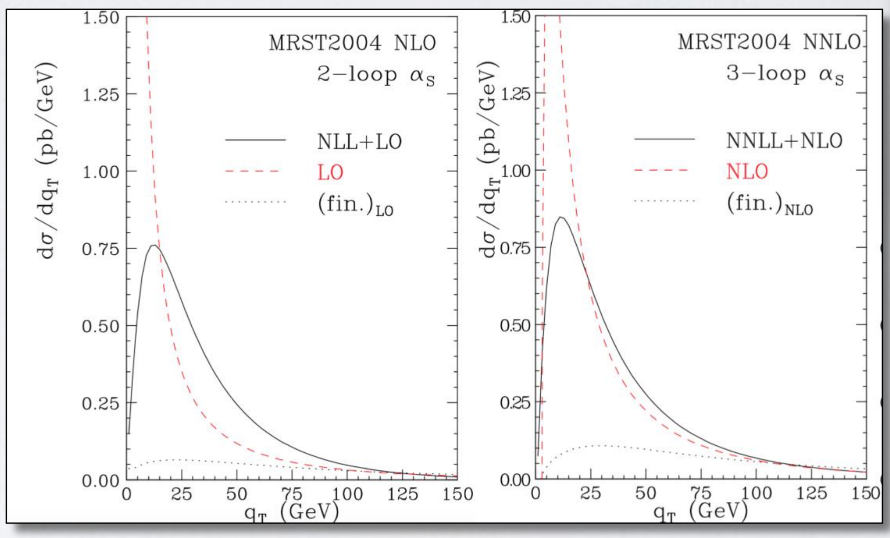
$$\sim \log^2 \frac{m_H^2}{m_b^2}$$



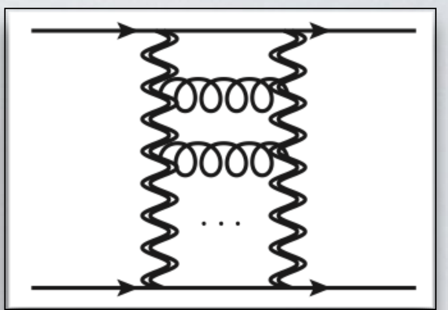
$$\sim \log^2(1-z)$$



$$\sim \log \frac{m_H^2}{p_T^2}$$



$$\sim \log \frac{s}{-t}$$



$$d\sigma \sim 1 + \alpha_s(L+1) + \alpha_s^2(L^3+L^2+L+1) + \ldots$$

- Resummation is necessary to restore the convergence of the perturbative series.
- The development of resummation is necessary for phenomenology, and feeds into formal aspects of quantum field theory.

ANALYTIC TOOLS FOR MULTI-SCALE SCATTERING

- Resummation requires to understand all order properties of gauge theories.
- As such, it feeds into several aspects of quantum field theory, providing also important results for fixed order perturbation theory and effective field theories.
- I will illustrate these aspects focusing on two cases:

Scattering in the high-energy limit

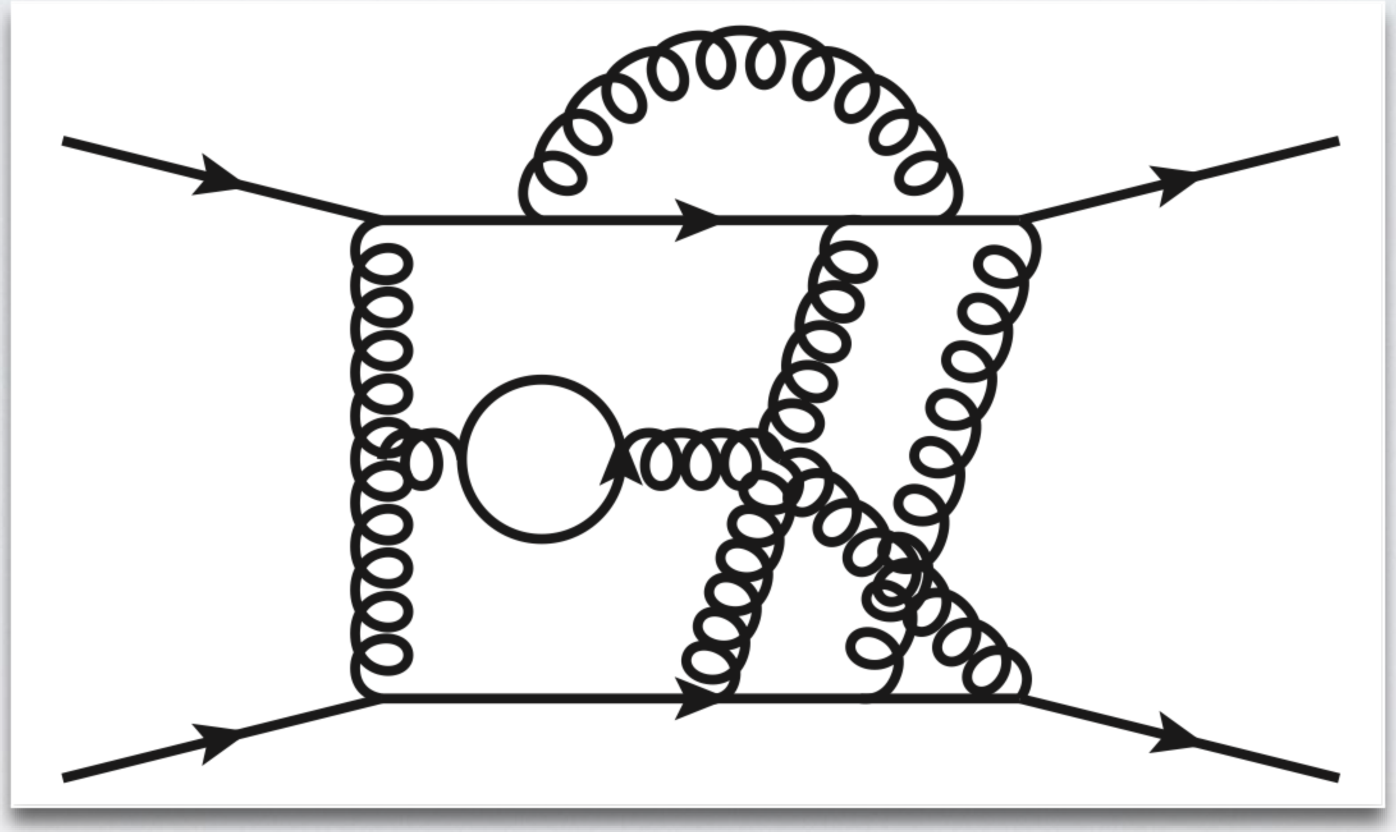
implications for fixed order PT:
→ Infrared divergences
→ Analytic structure

Scattering near threshold

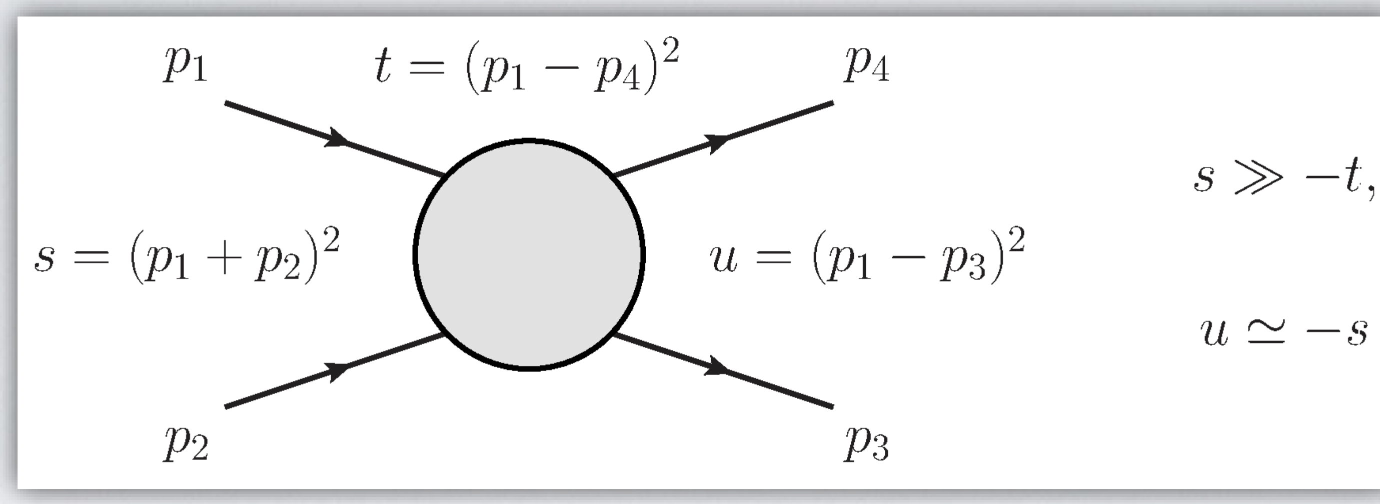
implications for phenomenology and EFTs

- I have developed new frameworks which allows to calculate large logarithms;
- In turn, it allows us to clarify/solve long standing problems.

SCATTERING
IN THE HIGH-ENERGY LIMIT



THE HIGH-ENERGY LIMIT



- Expansion in the strong coupling and in towers of (large) logarithms:

$$\mathcal{M}_{ij\to ij} = \mathcal{M}^{(0)} + \frac{\alpha_s}{\pi}\log\frac{s}{-t}\,\mathcal{M}^{(1,1)} + \frac{\alpha_s}{\pi}\mathcal{M}^{(1,0)} + \left(\frac{\alpha_s}{\pi}\right)^2\log^2\frac{s}{-t}\,\mathcal{M}^{(2,2)} + \left(\frac{\alpha_s}{\pi}\right)^2\log\frac{s}{-t}\,\mathcal{M}^{(2,1)} + \left(\frac{\alpha_s}{\pi}\right)^2\mathcal{M}^{(2,0)} + \dots$$

LL | NLL | NNLL

THE HIGH-ENERGY LIMIT

- Very interesting theoretical problem:
  - toy model for full amplitude, yet
    → retain rich dynamic in the 2D transverse plane,
    → non-trivial function spaces;
  - Understand the high-energy QCD asymptotic in terms of Regge poles and cuts;
  - predict amplitudes and other observables in overlapping limits:
    → soft limit, infrared divergences.

MRK in N=4 SYM:
Dixon, Pennington, Duhr, 2012;
Del Duca, Dixon, Pennington, Duhr, 2013;
Del Duca, Druc, Drummond, Duhr, Dulat, Marzucca, Papathanasiou, Verbeek 2019

- Relevant for phenomenology at the LHC and future colliders:
  - perturbative phenomenology of forward scattering, e.g.
    → Deep inelastic scattering/saturation (small x = Regge, large Q^2 = perturbative),
    → Mueller-Navelet: pp → X+2jets, forward and backward.

See e.g. Andersen, Smillie, 2011; Andersen, Medley Smillie, 2016; Andersen, Hapola, Maier, Smillie, 2017; …

THE HIGH-ENERGY LIMIT

- I have developed a formalism that allows us to evaluate scattering amplitudes in the high-energy limit as a vacuum expectation value of Wilson lines:

$$\mathcal{M} \sim \langle \psi_j | e^{-HL} | \psi_i \rangle,$$

Korchemskaya, Korchemsky, 1994,1996; Balitsky 1995; Babansky, Balitsky 2002.

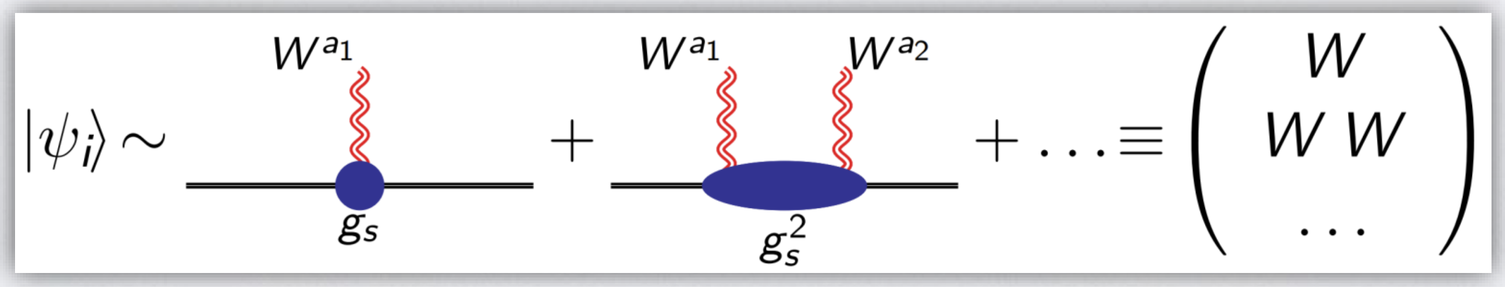
where $\psi_{i,j}$ are states made out of Wilson lines U:

$$U(z_\perp) = \mathcal{P} \exp \left[i g_s \int_{-\infty}^{+\infty} A_+^a(x^+, x^- = 0, z_\perp) T^a \, dx^+ \right],$$

which obeys the (non linear!) Balitsky-JIMWLK evolution equation:

$$\frac{d}{d\eta} UU \sim g_s^2 \int d^2 z_0 K(z_0, z_1, z_2) \Big[U(z_0) UU - UU \Big], \qquad \eta = L \equiv \log \frac{s}{-t} - i\frac{\pi}{2}.$$

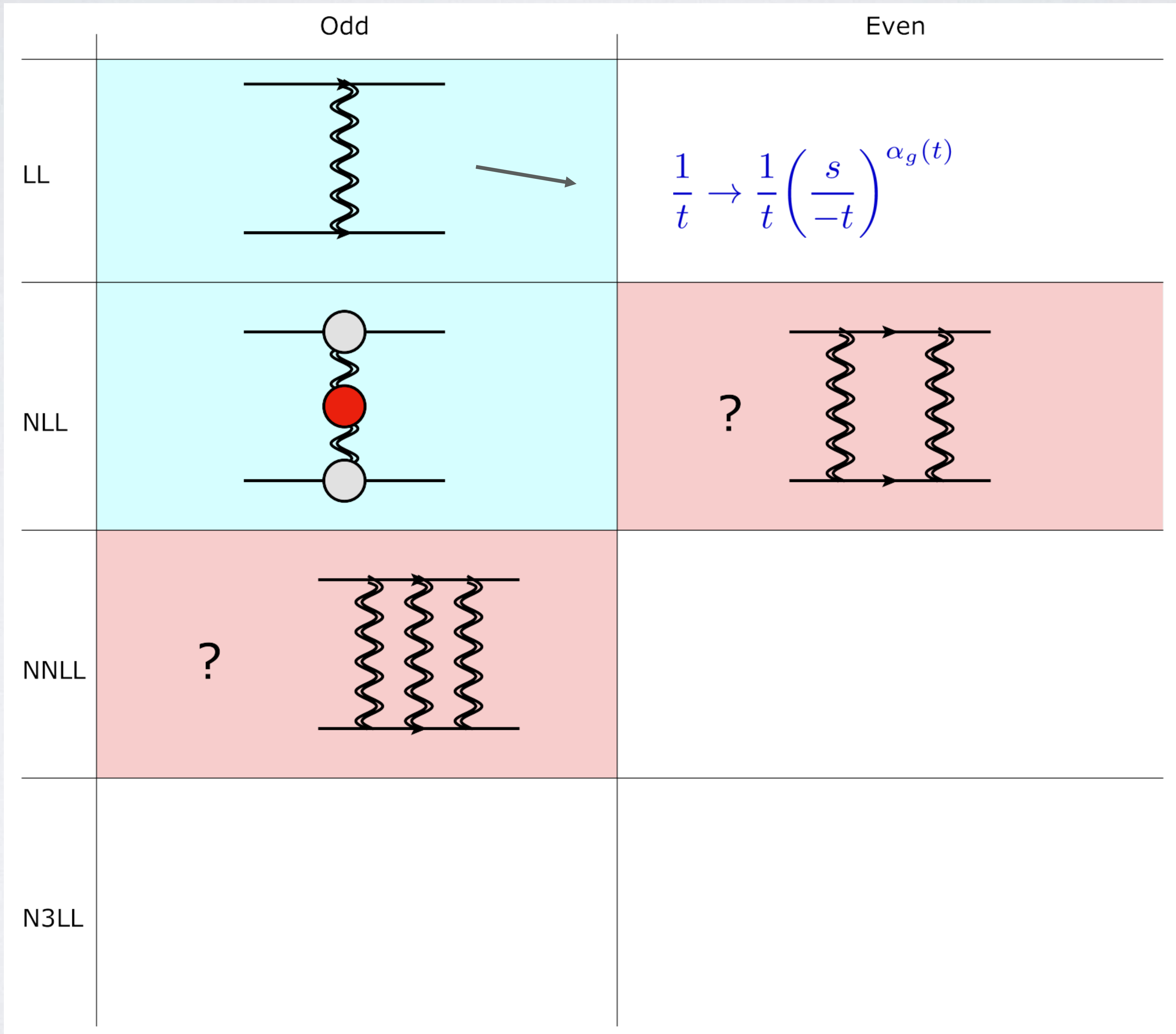
- A fundamental step is the identification of an effective degree of freedom, the so-called *Reggeon*, $U(z) = e^{i g_s T^a W^a(z)}$, in which the states are expanded:



Caron-Huot, 2013; Caron-Huot, Gardi, LV, 2017.

TWO PARTON SCATTERING AMPLITUDES

- Status pre ~ 2014:



TWO PARTON SCATTERING AMPLITUDES

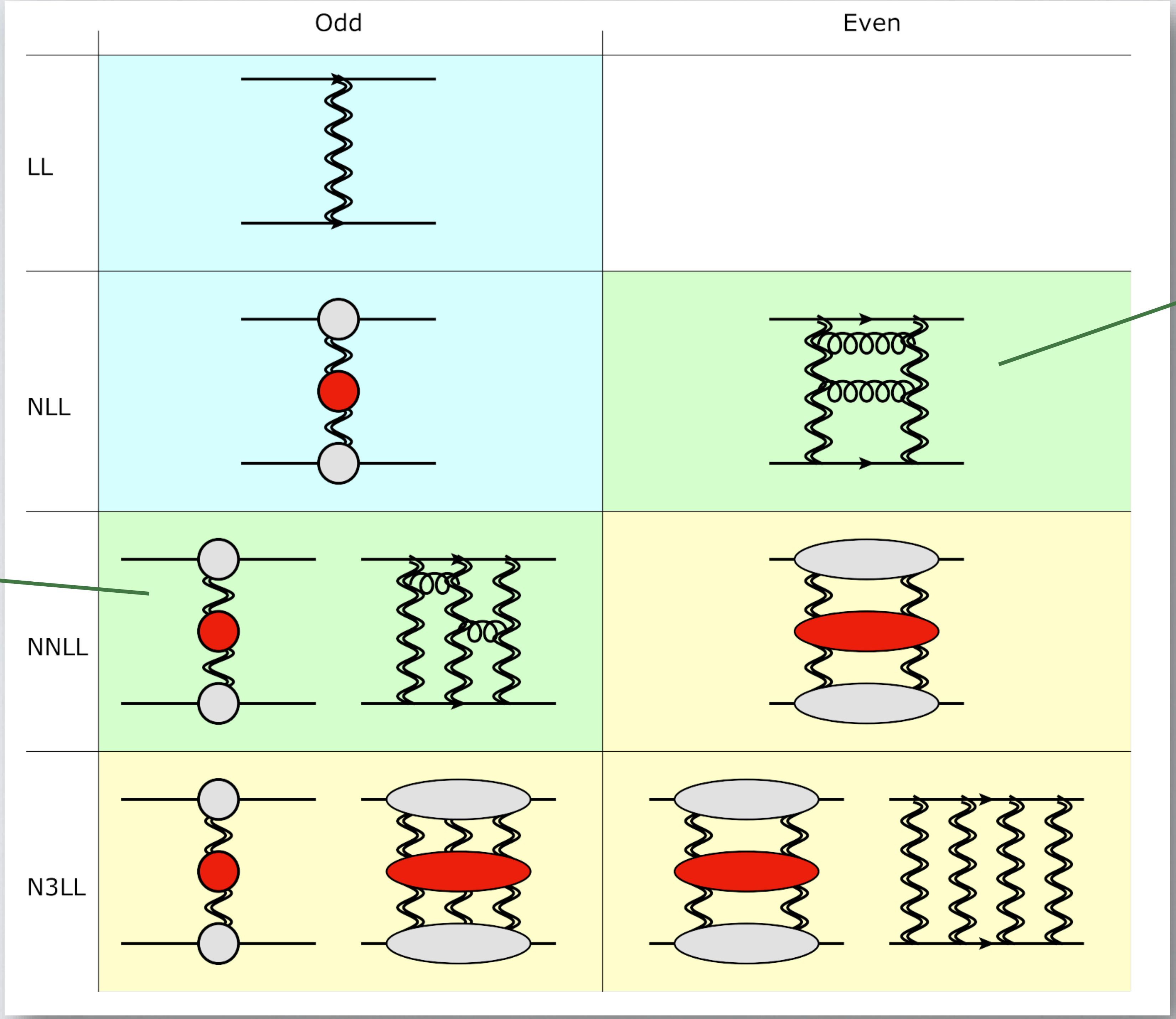
- Developed a framework for the calculation of amplitudes in the high-energy limit;
- Systematic relation between logarithmic accuracy and number of Reggeons.

Analysed to 2 loops in Del Duca, Falcioni, Magnea, LV 2014;

Calculated to 3 loops in Caron-Huot, Gardi, LV, 2017;

Calculated to 4 loops in Falcioni, Gardi, Milloy, LV, 2020; Falcioni, Gardi, Maher, Milloy, LV, 2021;

Falcioni, Gardi, Maher, Milloy, LV, PRL 128, 2022.

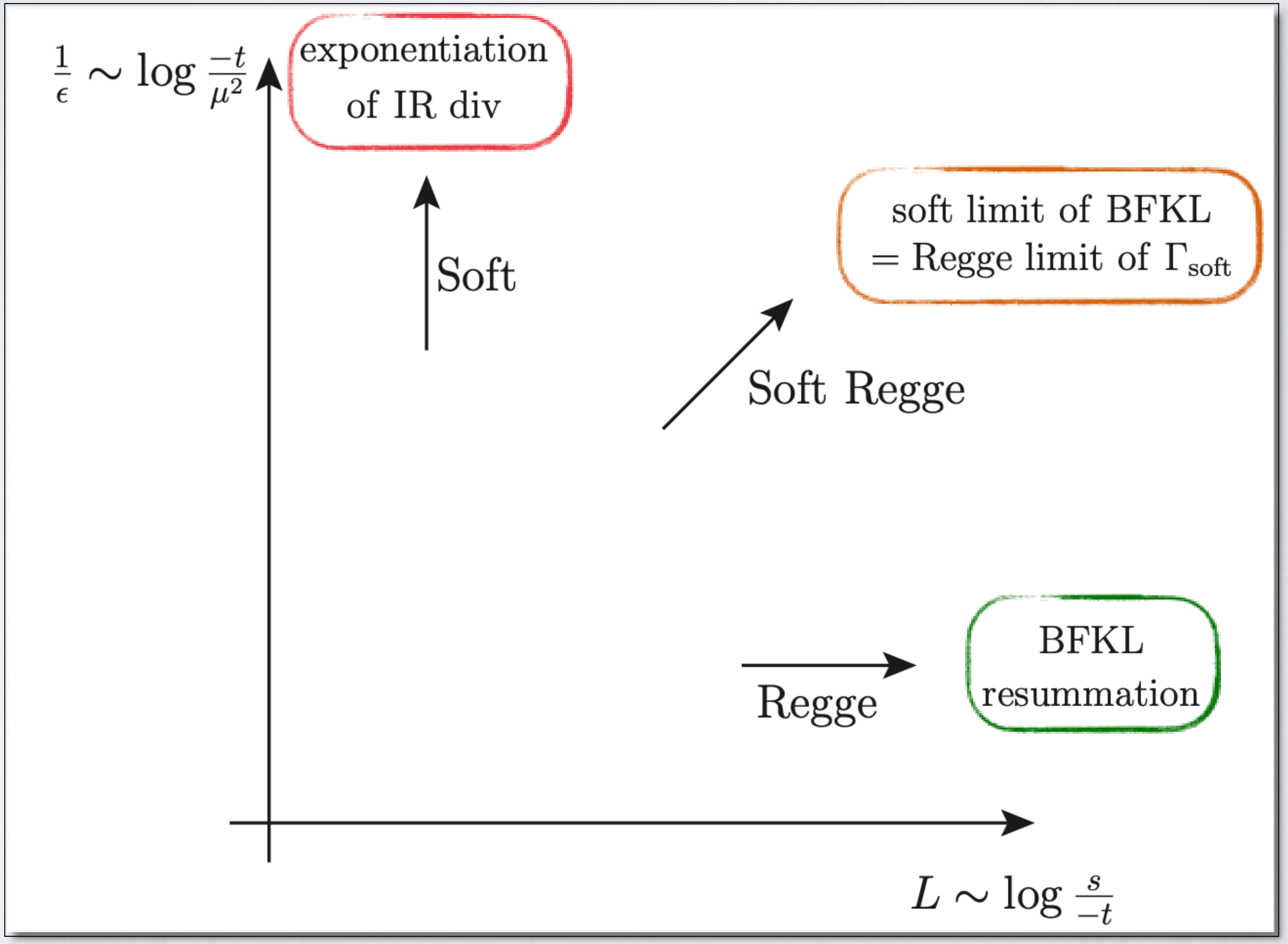


IR rivergences calculated to all orders in Caron-Huot, Gardi, Reichel, LV, 2017;

Finite terms calculated to 13 loops in Caron-Huot, Gardi, Reichel, LV, 2020.

REGGE VS INFRARED FACTORISATION

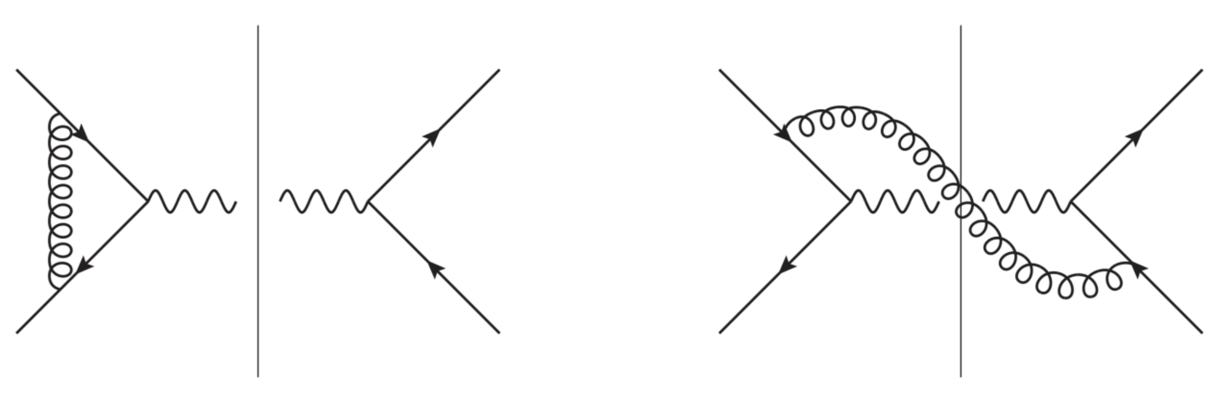
- One application: test (and predict) the analytic structure of infrared divergences.



REGGE VS INFRARED FACTORISATION

- Individual terms of matrix element squared are infrared divergent;
- Infrared divergences cancel in the sum over equivalent final (and initial) states.

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \, V \, \delta_n(X) + \int d\Phi_{n+1} \, R \, \delta_{n+1}(X).$$



See for instance Agarwal, Magnea, Signorile-Signorile, Tripathi, 2021.

- In practice, need to construct counterterms for both terms.

$$\frac{d\sigma_{\rm NLO}}{dX} = \int d\Phi_n \Big(V + I \Big) \delta_n(X) + \int \Big(d\Phi_{n+1} \, R \, \delta_{n+1}(X) - d\widehat{\Phi}_{n+1} \, \overline{K} \, \delta_n(X) \Big), \qquad I = \int d\widehat{\Phi}_{\rm rad} \, \overline{K}.$$

- Structure of infrared divergences is universal: depends on features of soft and collinear radiation in a gauge theory. A lot of work has been devoted to constraint it.

REGGE VS INFRARED FACTORISATION

- Infrared divergences are calculated in terms of the so-called soft anomalous dimension:

$$\mathcal{M}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right)\mathcal{H}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right),$$

$$\mathbf{Z}_n\left(\{p_i\},\mu,\alpha_s(\mu^2)\right) = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu^2}\frac{d\lambda^2}{\lambda^2}\,\mathbf{\Gamma}_n\left(\{p_i\},\lambda,\alpha_s(\lambda^2)\right)\right\}.$$

Re	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{4}\widehat{\gamma}_K^{(1)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(1)}$	$\frac{1}{2}\widehat{\gamma}_K^{(1)}\mathbf{T}_t^2$					
α_s^2	$\frac{1}{4}\widehat{\gamma}_K^{(2)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(2)}$	$\frac{1}{2}\widehat{\gamma}_K^{(2)}\mathbf{T}_t^2$	0				
α_s^3	$\frac{1}{4}\widehat{\gamma}_K^{(3)}\ln\frac{-t}{\lambda^2}\sum_{i=1}^4 C_i+\sum_{i=1}^4\gamma_i^{(3)}+\Delta^{(+,3,0)}$	$\frac{1}{2}\widehat{\gamma}_K^{(3)}\mathbf{T}_t^2$	0	0			
α_s^4				0	0		
α_s^5					0	0	
α_s^6						0	0

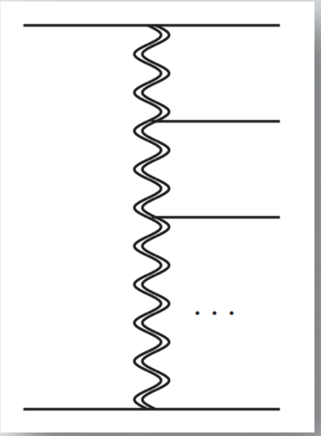
Caron-Huot, Gardi, LV, 2017, 2021

Im	L^0	L^1	L^2	L^3	L^4	L^5	L^6
α_s^1	$\frac{1}{2}\widehat{\gamma}_K^{(1)}i\pi\mathbf{T}_{s-u}^2$	0					
α_s^2	$\frac{1}{2}\widehat{\gamma}_K^{(2)}i\pi\mathbf{T}_{s-u}^2$	0	0				
α_s^3	$\frac{1}{2}\widehat{\gamma}_K^{(3)}i\pi\mathbf{T}_{s-u}^2+\Delta^{(-,3,0)}$	$\Delta^{(-,3,1)}$	0	0			
α_s^4				$\Gamma_{\text{NLL}}^{(-,4)}$	0		
α_s^5					$\Gamma_{\text{NLL}}^{(-,5)}$	0	
α_s^6						$\Gamma_{\text{NLL}}^{(-,6)}$	0

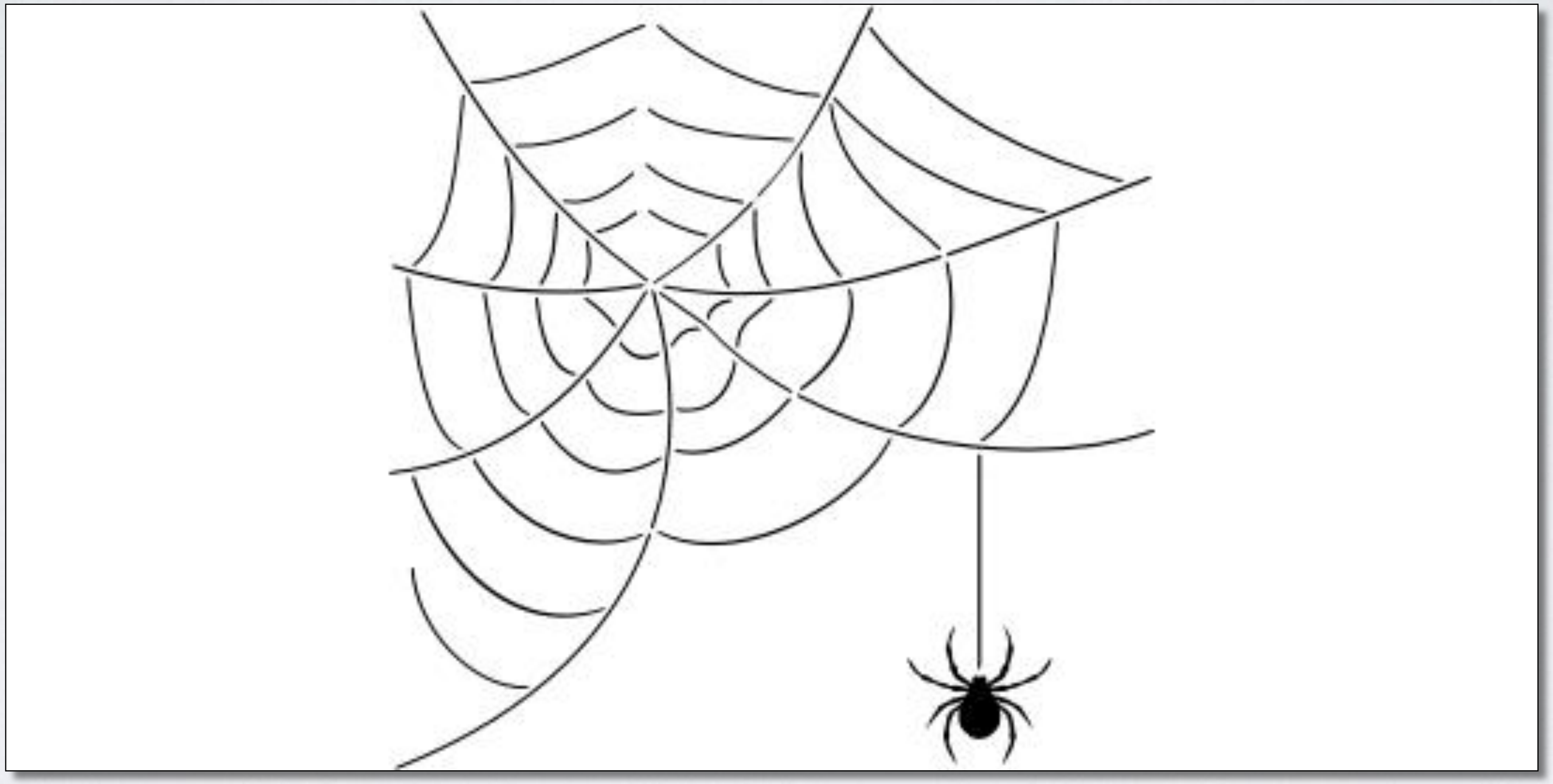
Caron-Huot, Gardi, Reichel, LV, 2017

THE HIGH-ENERGY LIMIT

- Extend the formalism to multi-Regge kinematic;
- Boundary conditions for scattering amplitudes in general kinematic;
- Bootstrap approach to infrared divergences;
- Convergence of the perturbative expansion;
- Gauge-gravity duality …

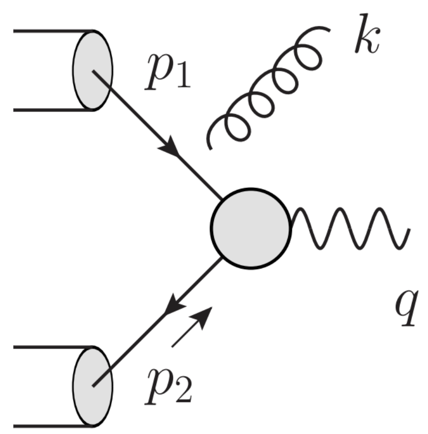


PARTICLE SCATTERING NEAR THRESHOLD

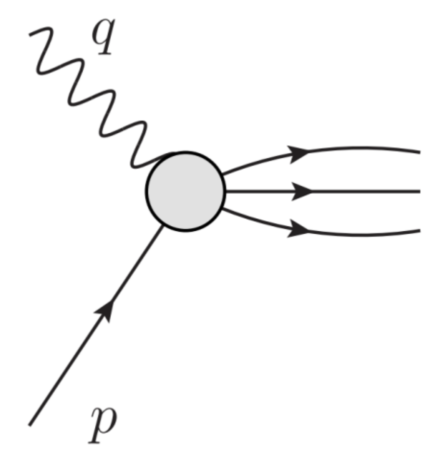


PARTICLE SCATTERING NEAR THRESHOLD

- Consider Drell-Yan and DIS near partonic threshold:



$(p_1 + p_2)^2 \equiv \hat{s}, \quad z \equiv \frac{q^2}{\hat{s}} \to 1$



$Q^2 = -q^2, \quad x \equiv \frac{Q^2}{2p \cdot q} \to 1$

- The partonic cross section has singular expansion

$$\Delta_{ab}(\xi) \sim \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \left[c_n \delta(1-\xi) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-\xi)}{1-\xi}\right]_+ + d_{nm} \ln^m(1-\xi)\right) + \ldots\right],$$

LP **NLP**

with $\xi = z$ for DY or x for DIS.

- Resummation of large logarithms at next-to-leading power (NLP):

→ interesting theoretical challenge, relevant for precision phenomenology!

FACTORIZATION AND RESUMMATION AT NLP

- Lot of work in the past few years!
- Drell-Yan, Higgs and DIS near threshold

  Del Duca, 1990; Bonocore, Laenen, Magnea, LV, White, 2014, 2015, 2016;
  Bahjat-Abbas, Bonocore, Sinninghe Damsté, Laenen, Magnea, LV, White, 2019;
  van Beekveld, Beenakker, Laenen, White, 2019; van Beekveld, Laenen, Sinninghe Damsté, LV, 2021;
  Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
  Beneke, Broggio, Jaskiewicz, LV, 2019;
  Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019, 2020.

- Operators and Anomalous dimensions

  Larkoski, Neill, Stewart 2014;
  Moult, Stewart, Vita 2017; Feige, Kolodrubetz, Moult, Stewart 2017;
  Beneke, Garny, Szafron, Wang, 2017, 2018, 2019.

- Thrust

  Moult, Stewart, Vita, Zhu 2018, 2019.

- pT and Rapidity logarithms

  Ebert, Moult, Stewart, Tackmann, Vita, 2018,
  Moult, Vita Yan 2019;
  Cieri, Oleari, Rocco, 2019; Oleari, Rocco 2020.

- Mass effects

  Liu, Neubert 2019; Liu, Mecaj, Neubert, Wang, Fleming, 2020;
  Liu, Mecaj, Neubert, Wang, 2020;
  Anastasiou, Penin, 2020.

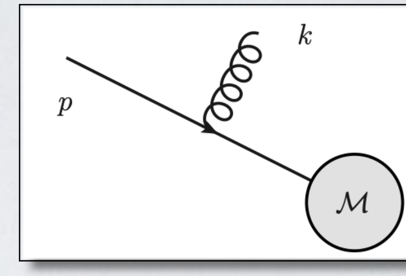
- K+G and RGE equations

  Ajjath, Mukherjee, Ravindran, Sankar, Tiwari, 2020, 2021.

And many more!

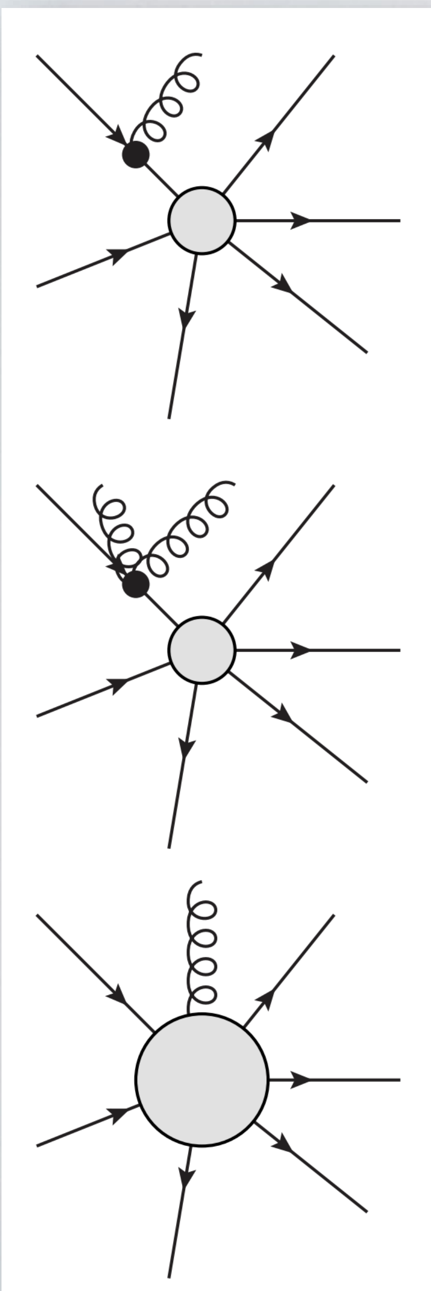
FACTORIZATION OF SOFT GLUONS BEYOND LP

- Soft gluon emission at LP: eikonal emission.



$$\sim \mathcal{M}\,\frac{p^\mu}{p\cdot k}\,T^A\,u(p)$$

- Beyond LP one needs to consider several effects:

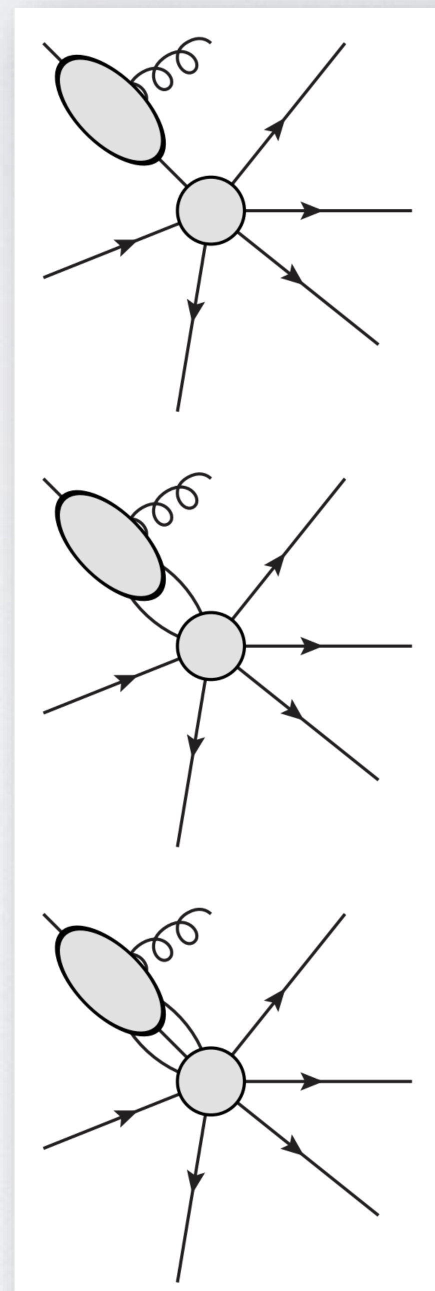


- Emission of soft gluons beyond the eikonal approximation, for instance sensitive to the spin of the emitting particle

Laenen, Magnea, Stavenga, White, 2009, 2010; Bonocore, Laenen, Magnea, LV, White, 2016.

- The soft emission resolve the hard interaction (LBK theorem)

Low 1958, Burnett,Kroll 1968



- Emission of soft gluons from a cluster of collinear particles: one finds several types of "radiative jets".

Del Duca 1990;

Bonocore, Laenen, Magnea, Melville, LV, White, 2015,2016;

Gervais 2017;

Laenen, Sinninghe-Damsté, LV, Waalewijn, Zoppi, 2020

Obtained factorization theorems incorporating these structures within a diagrammatic and an effective field theory approach.

FACTORIZATION AND RESUMMATION AT NLP

- Effective field theories (Soft-collinear effective field theory, SCET) provide a systematic tool for describing the factorization of soft and collinear radiation.
- The hard scattering kernel is described in terms of effective operators; Momentum modes in the theory are integrated out, giving rise to short-distance coefficients.
- I have derived factorization theorems for DY, DIS and Higgs production at NLP.

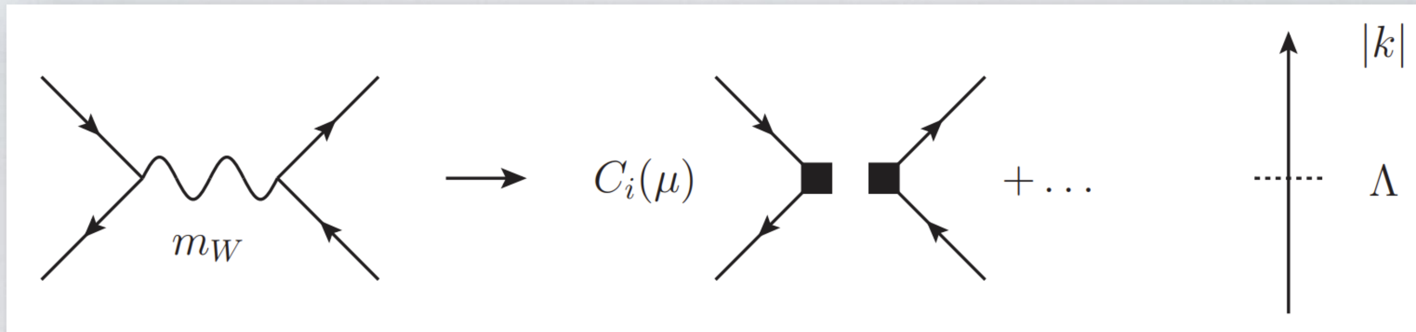
$$\hat{\sigma}_{q\bar{q}}^{\mathrm{NLP}} = \sum_{\mathrm{terms}} \left[C \otimes J \otimes \bar{J}\right] \otimes S.$$

Beneke, Broggio, Garny, Jaskiewicz, Szafron, LV, Wang, 2018;
Beneke, Broggio, Jaskiewicz, LV, 2019;
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2020

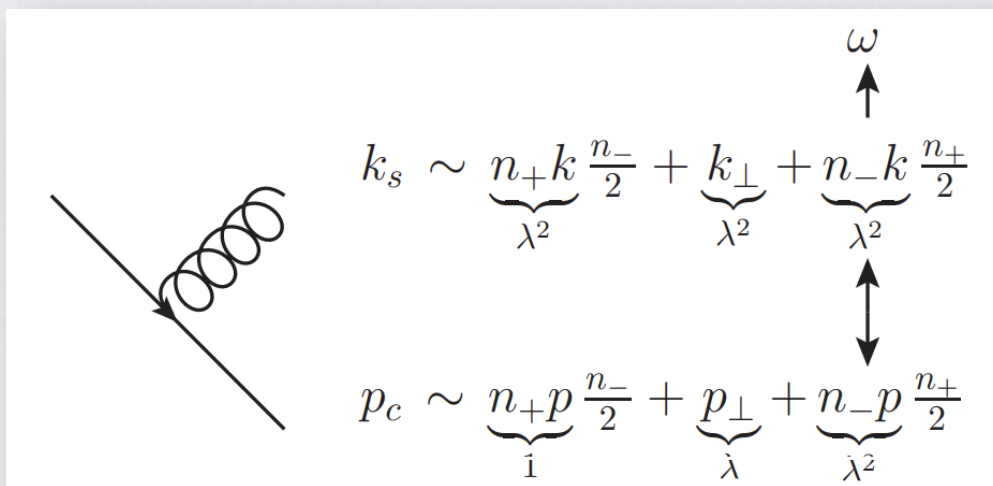
- I have obtained the first systematic resummation of large leading logarithms at NLP.
- Resummation of large logarithms beyond NLP LL is still an open problem, due to the appearance of endpoint divergences. Solving this problem has far more reaching consequences!

FACTORIZATION AND RESUMMATION AT NLP

- "Standard" EFTs:



- Non-local EFTs:

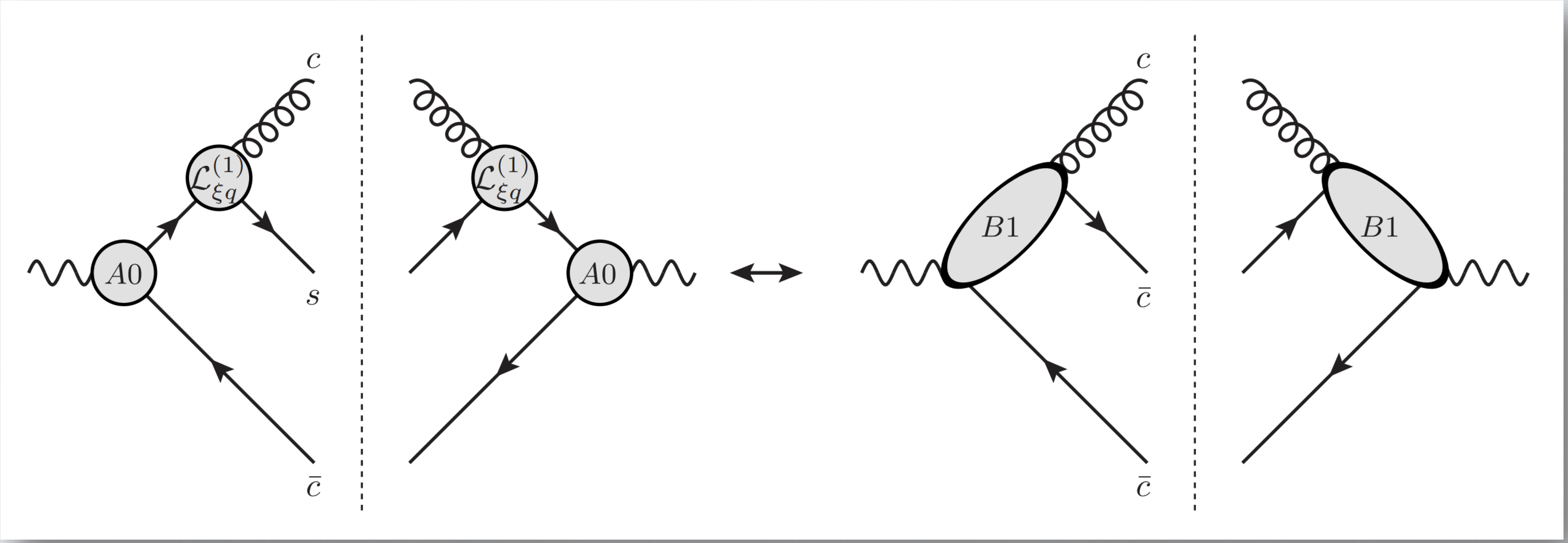


$$\int d\omega \; \underbrace{(n_+p\,\omega)^{-\epsilon}}_{\text{collinear piece}} \; \underbrace{\frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^{\epsilon}}}_{\text{soft piece}} \, .$$

- Non-local EFTs involve convolutions along the small momentum component; Beyond LP these convolution are in general divergent in d = 4, potentially spoil factorization!

FACTORIZATION AND RESUMMATION AT NLP

- Derived the first systematic treatment of endpoint divergences in SCET I by means of a re-factorization approach.

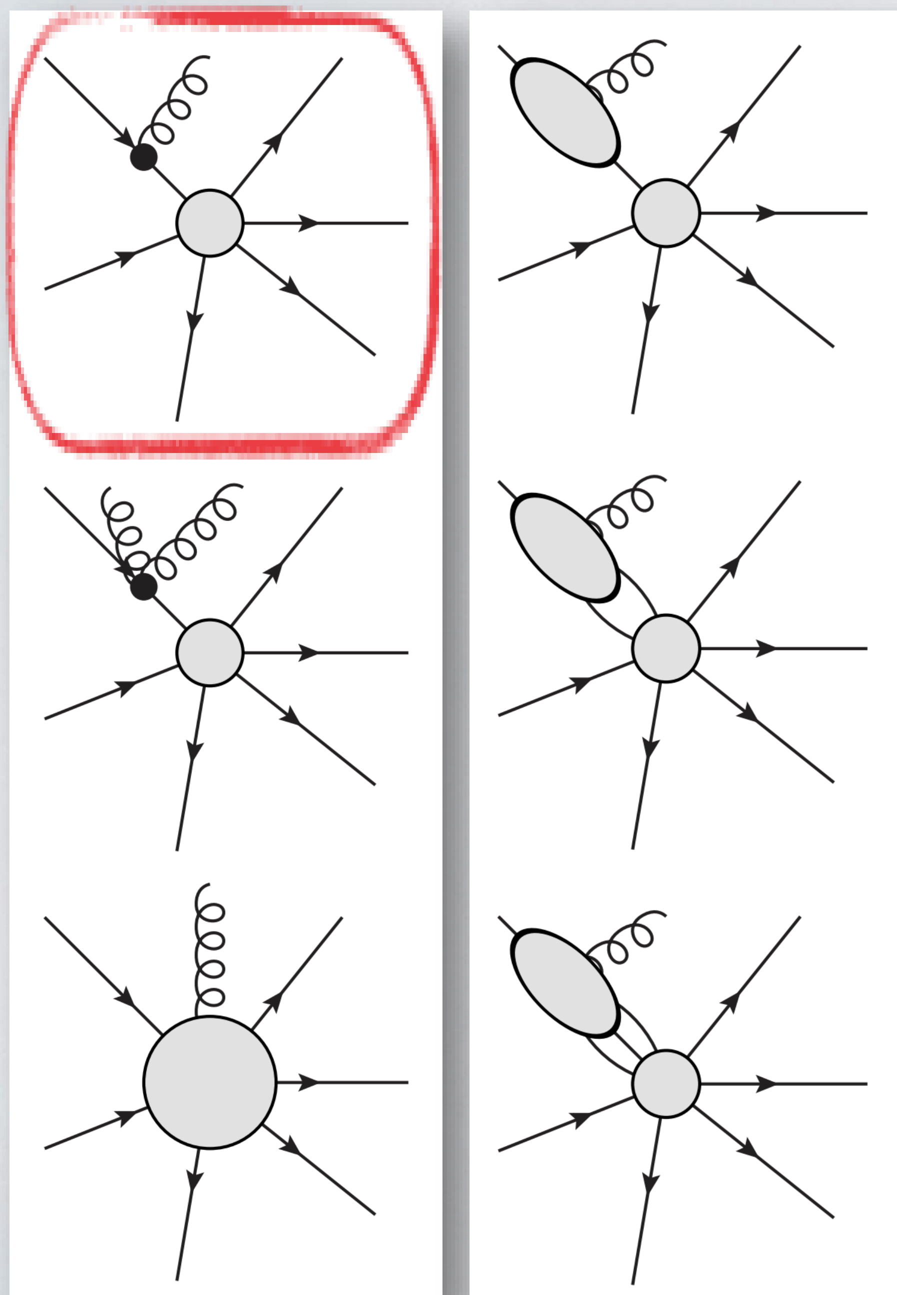


Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang, 2022
(see also Liu, Mecaj, Neubert, Wang, 2020, 2021)

- It opens up the way to a consistent treatment of endpoint divergences in effective field theories.

RESUMMATION OF LEADING LOGS AT NLP

- Restricting to leading logarithmic accuracy at NLP, one has to consider less terms. The factorization theorem simplifies and resummation becomes easier:



Leading production channels:

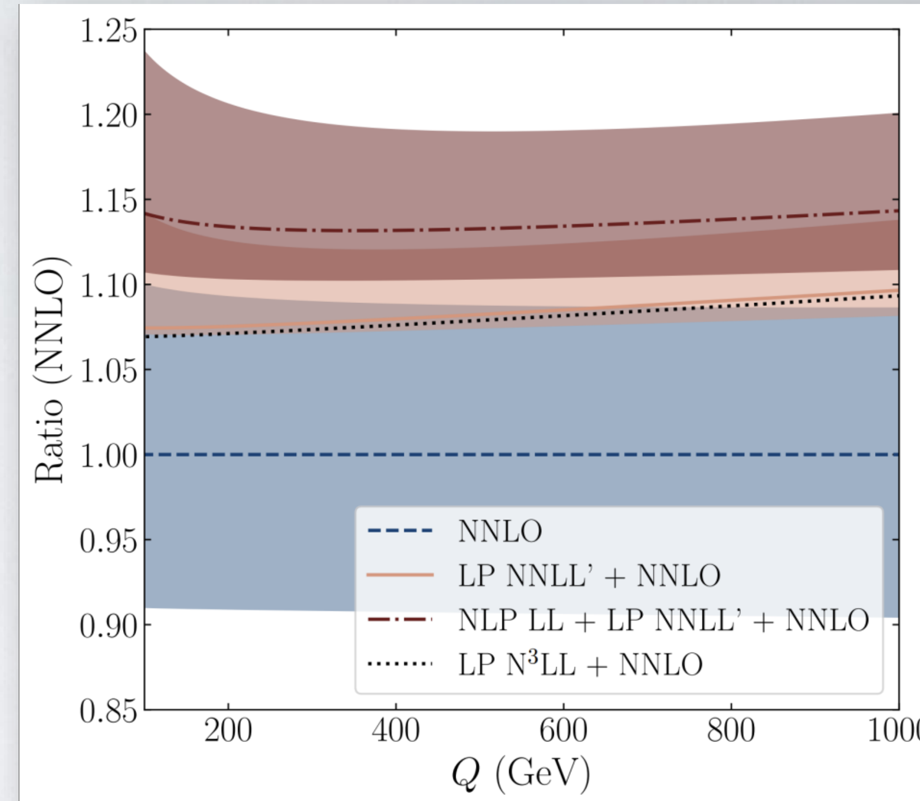
Beneke, Garny, Jaskiewicz, Szafron, LV, Wang, 2019;

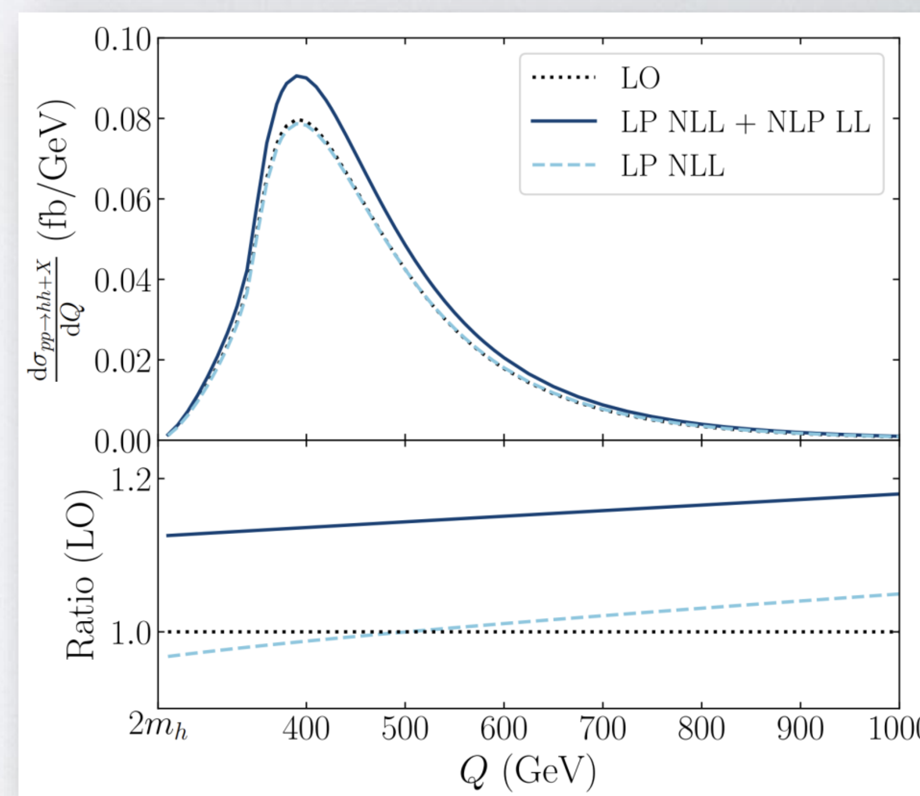
Van Beekveld, Laenen, Sinninghe-Damsté, LV, 2021;

Quark-gluon production channel:

Beneke, Garny, Jaskiewicz, Strohm, Szafron, LV, Wang, 2020;

Van Beekveld, LV, White 2021.





OUTLOOK

- I work on the development of analytic tools for precision in particle physics.
- I focus on
  - method of expansion by momentum regions;
  - diagrammatic and effective field theory methods for resummation of large logarithms at next-to-leading power;
  - analytic structure of scattering amplitudes:
    - calculation of scattering amplitudes to high-loop order in the high-energy limit;
    - determination of the structure of infrared divergences.
- In all these topics I developed new approaches that allows us to significantly extend our knowledge in the field.
- Most of the tools I am working on are general and have applications not only in collider physics. A few examples:
  - Precision in flavour physics;
  - Scattering amplitudes in gravity and the double copy.