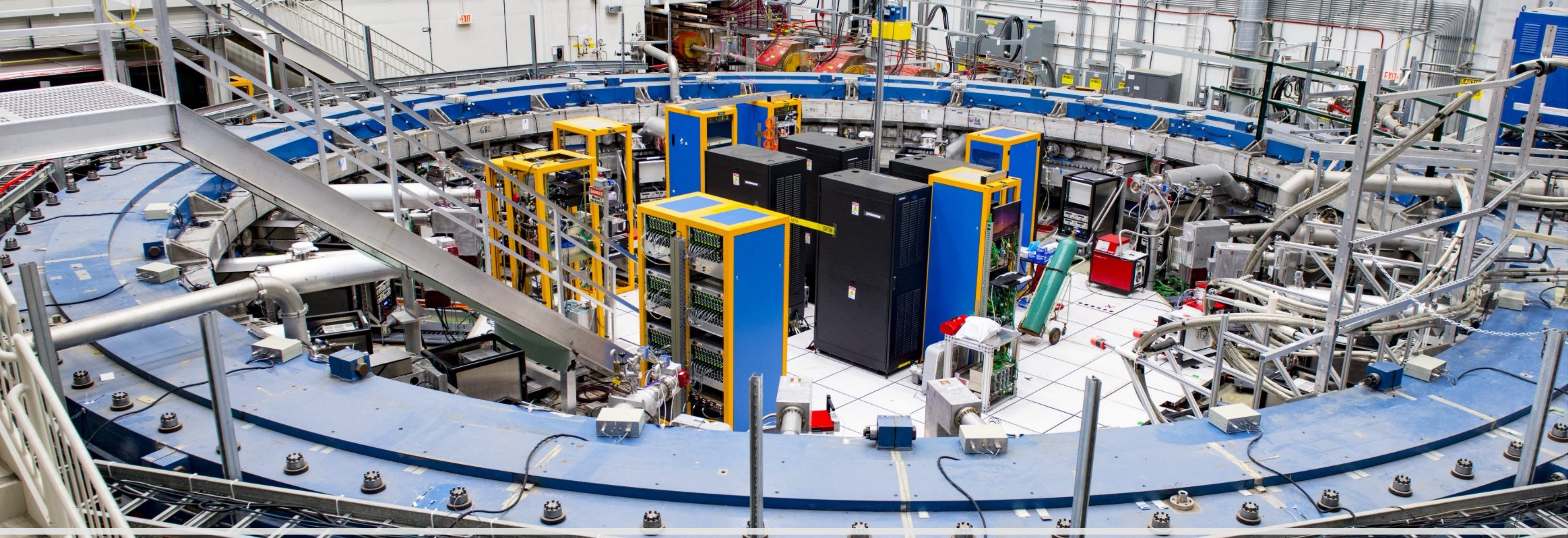


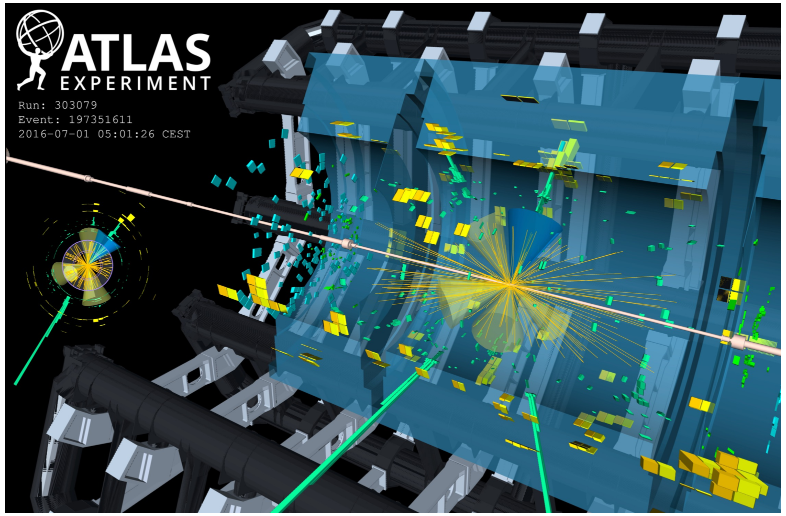
Beam dynamics at Muon g-2: a different approach to HEP experiment

E. Bottalico

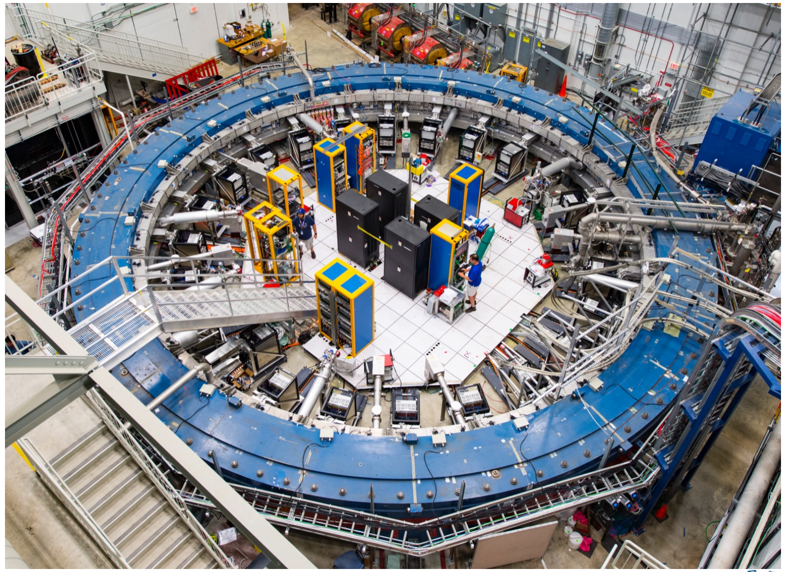
Fermilab 2022 Summer Students School – 19 July 2022

Muon g-2: an atypical HEP experiment

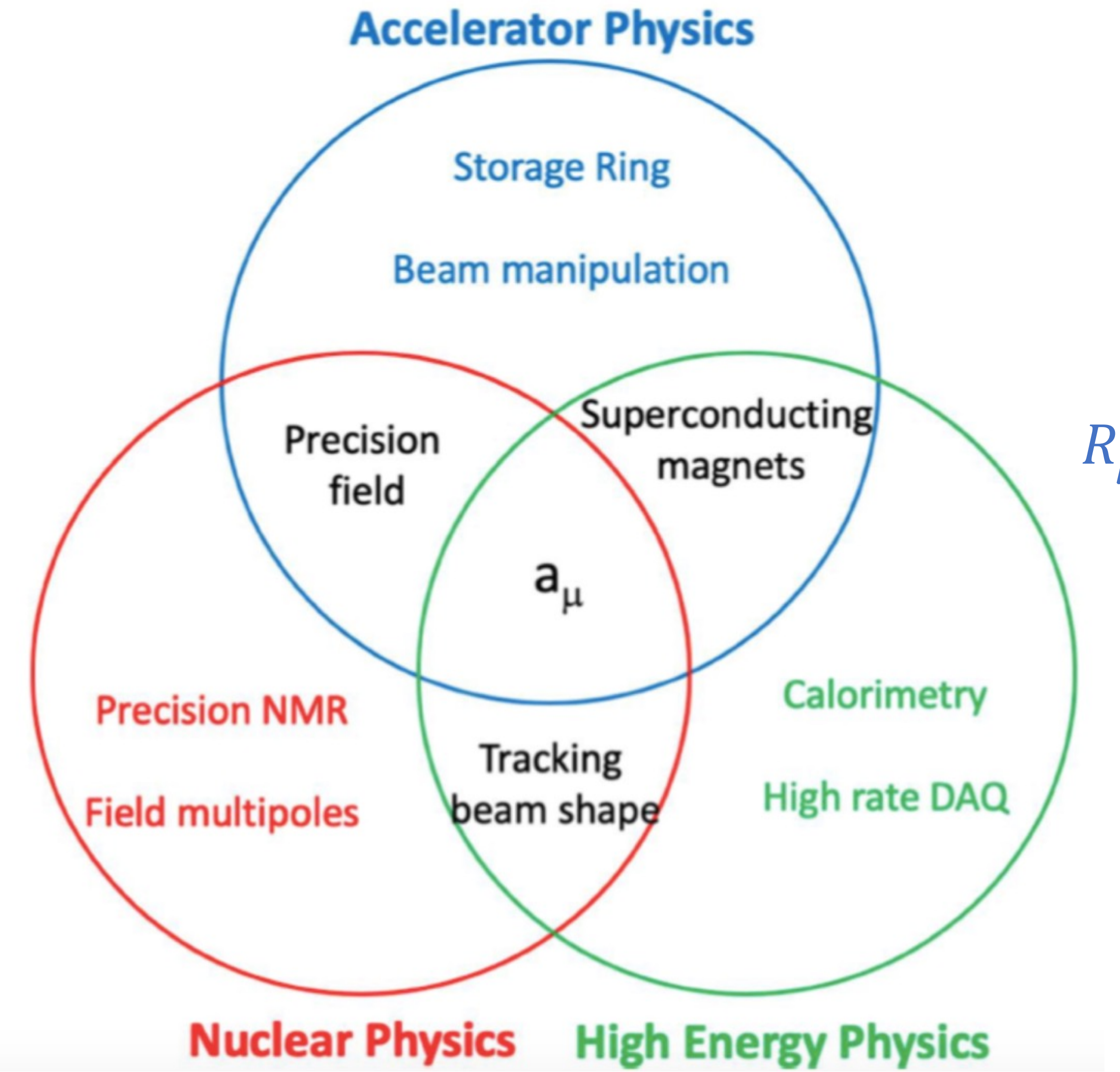
- At collider experiment in the analysis what matter is the rate of interaction. The beam motion is important to get an high rate, but doesn't have a part in the analysis.



- Muon g-2 experiment is different, we accumulate muon and we observe their motion, that can change the way we measure their decays.



Muon g-2: an atypical HEP experiment



$$R'_\mu \approx \frac{f_{clock}\overbrace{\omega_a^m}^{\text{HEP}}(1+\overbrace{C_e+C_p+C_{ml}+C_{pa}}^{\text{Accelerator Physics}})}{f_{calib} < \underbrace{\omega'_p(x,y,\phi)}_{\text{Nuclear Physics}} \times M(x,y,\phi) > (1+B_k+B_q)}$$

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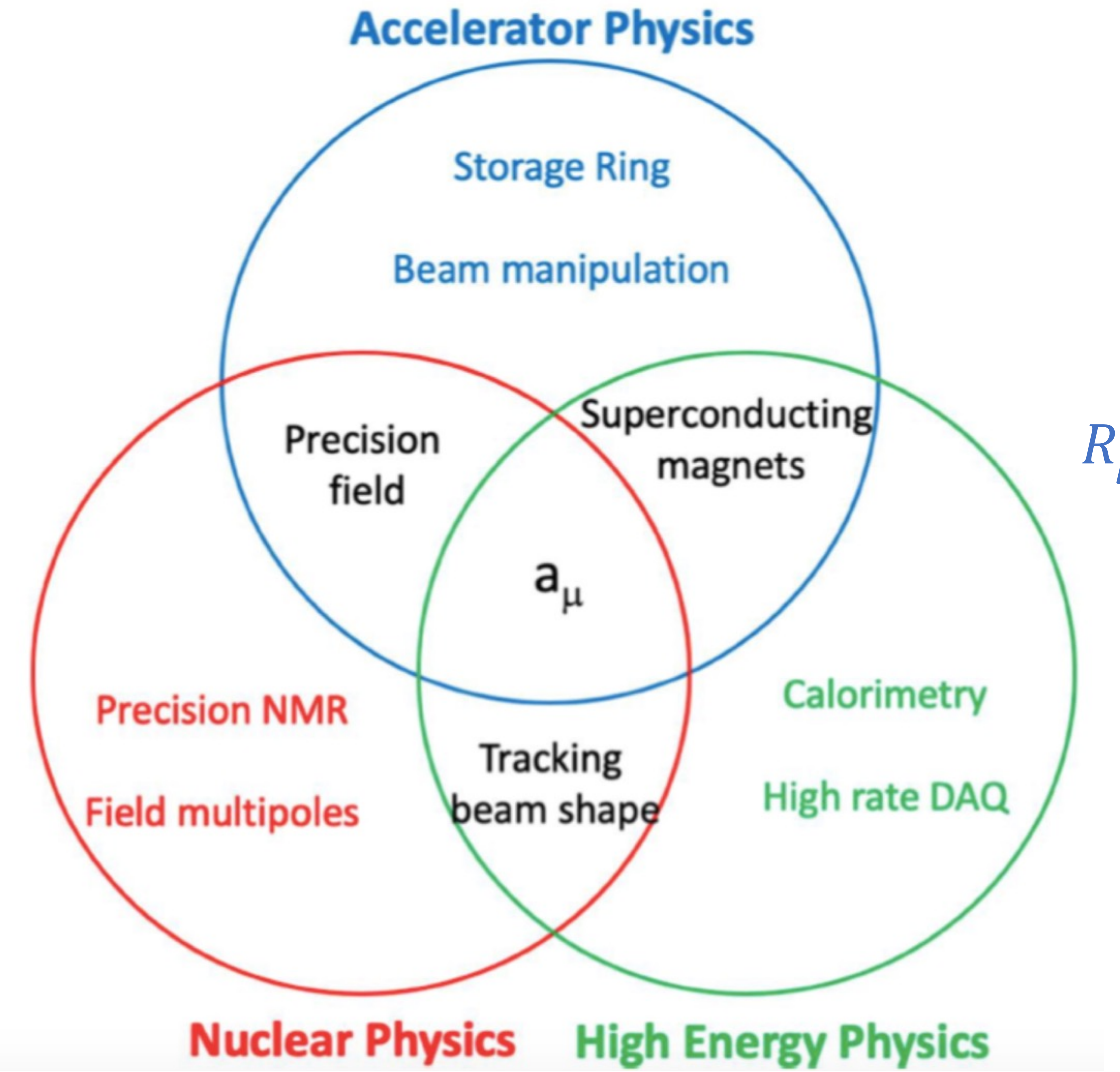

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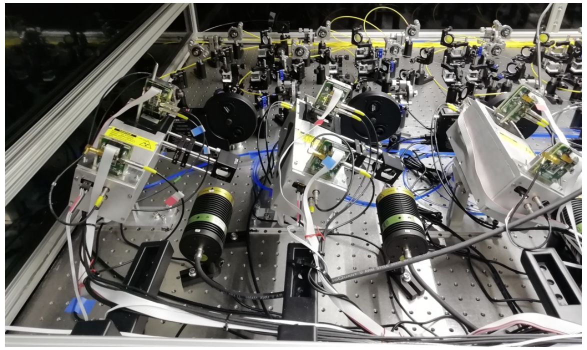
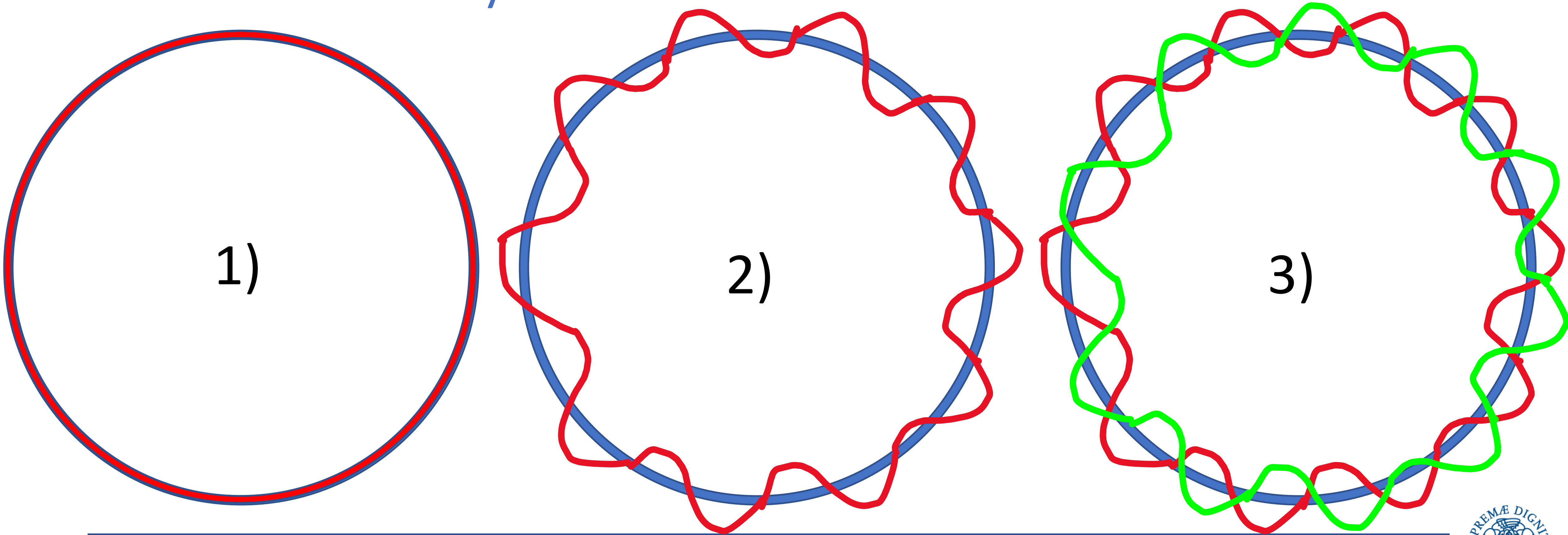


Optics with laser system

How does the beam move?

We know that charged particle stored in a magnetic field start moving on machine plane, oscillating with a frequency equal to cyclotron period, but is it just that?

Place your bet!!!



Betatron oscillation

Betatron oscillation is given by the magnetic field recall force.

$$F(r) = \frac{m\gamma v^2}{r} - qvB(r)$$

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r can be rewritten as: $\frac{1}{r} = \frac{1}{R+x} = \frac{1}{R}\frac{1}{1+\frac{x}{R}} \approx \frac{1}{R}\left(1 - \frac{x_0}{R}\right)$

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Given the x component of the force:

$$F_x(z) = qv_zB_y \longrightarrow B_y(x) \cong B_y(R) + \left.\frac{\partial B_y(r)}{\partial x}\right|_{r=R} \cdot x$$

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$$B_y(x) \cong B_y(R) + \frac{\partial B_y(r)}{\partial x}\Big|_{r=R} \cdot x = B_y(R)[1 - \underbrace{\frac{R}{B_y(R)}\frac{\partial B_y(R)}{\partial x}}_{\text{Field index } n} \cdot \frac{x}{R}]$$

Betatron oscillation

Lorentz force can be written as:

$$F_r(x) = \frac{m\gamma v^2}{R}\left(1 - \frac{x}{R}\right) - qvB_y(R)\left(1 - n\frac{x}{R}\right)$$

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The force can be rewritten:

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Writing $F_r(x) = \gamma m\frac{d^2x}{dt^2}$ and simplifying:

$$\frac{d^2x}{dt^2} = -\frac{v^2}{R^2}(1-n)x = -\omega_C^2(1-n)x \;\rightarrow \boxed{\boldsymbol{\omega_{BO} = \omega_C\sqrt{1-n}}}$$

How does the beam move?

This derivation gives us 3 important information:

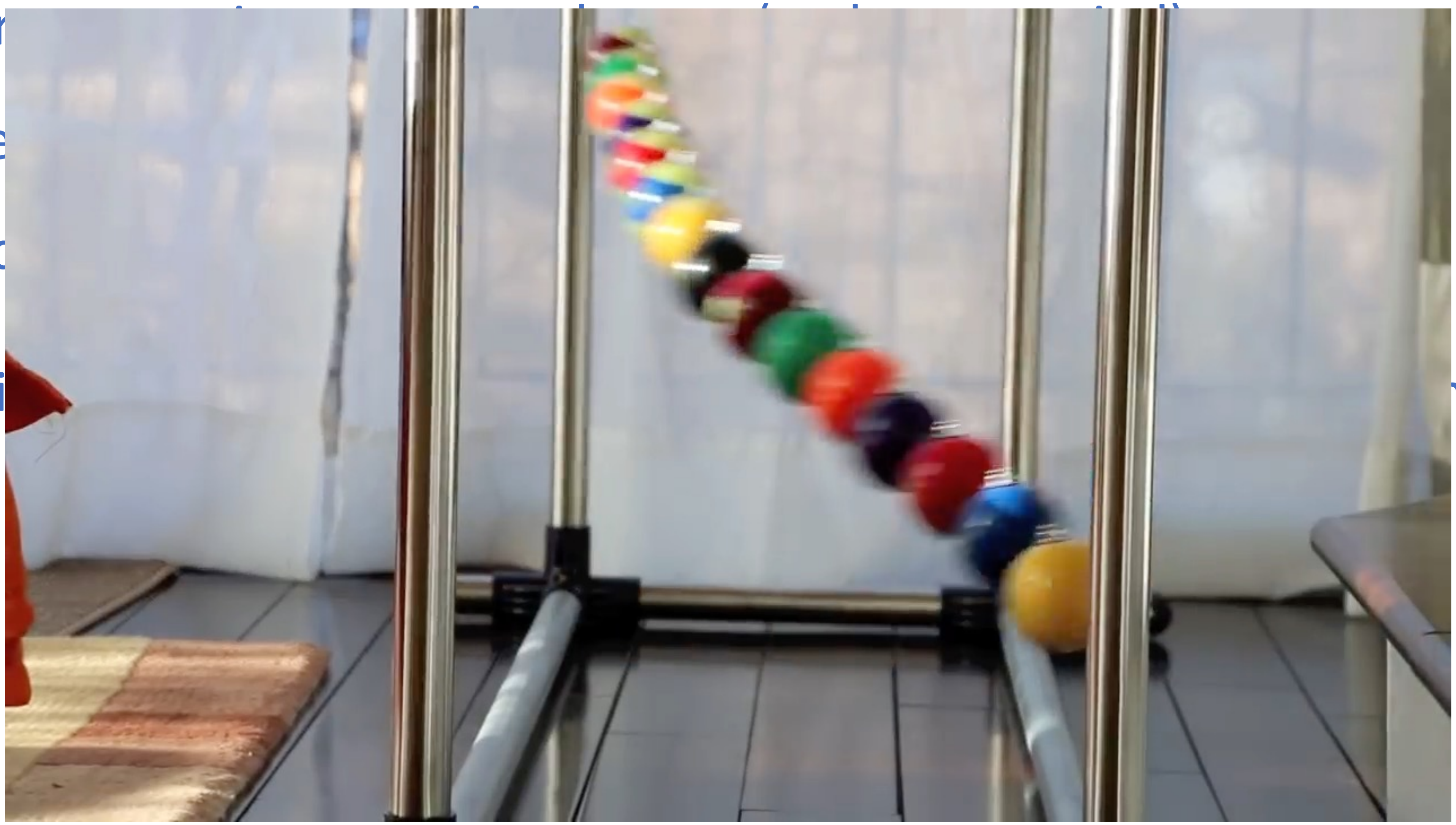
1. Betatron frequency is proportional to ω_C (cyclotron period)
2. The n-index should be within [0, 1] $\rightarrow \sqrt{1-n} \rightarrow n < 1$
3. Muon inside the ring oscillate along the radial (and vertical) position.

Since muon has different momentum $\omega_C = \frac{c\beta}{R+x}$ has also different ω_{BO}, their motion look like that:

How does the beam move?

This derivation gives us 3 important information:

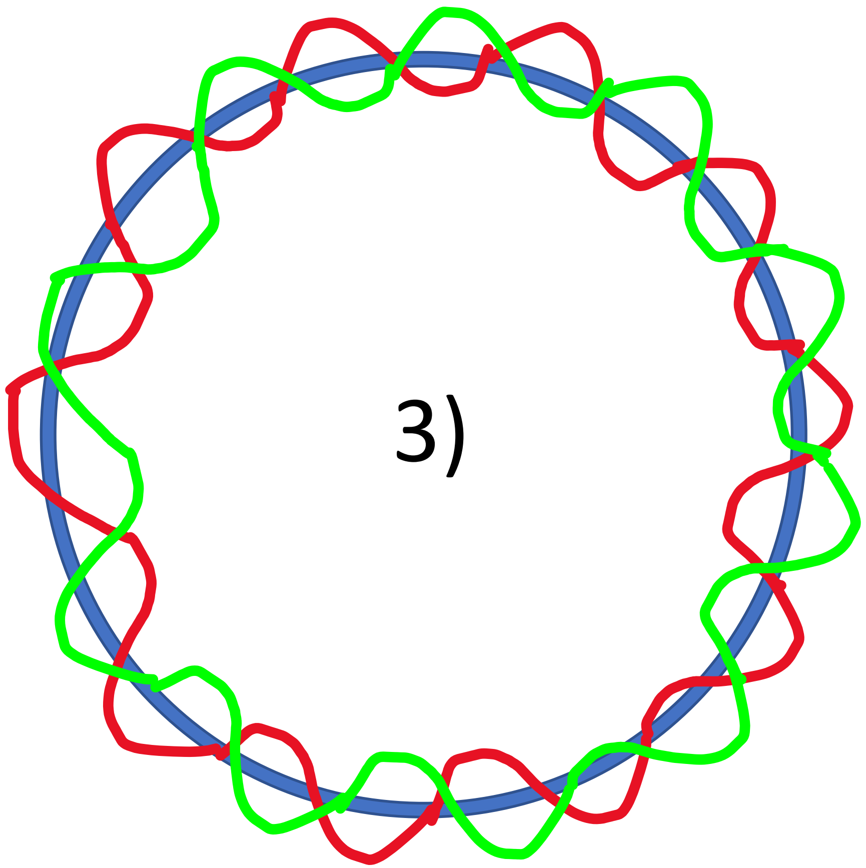
1. Betatron fr
2. The n-inde
3. Muon insic



Since muon has di motion look like that:

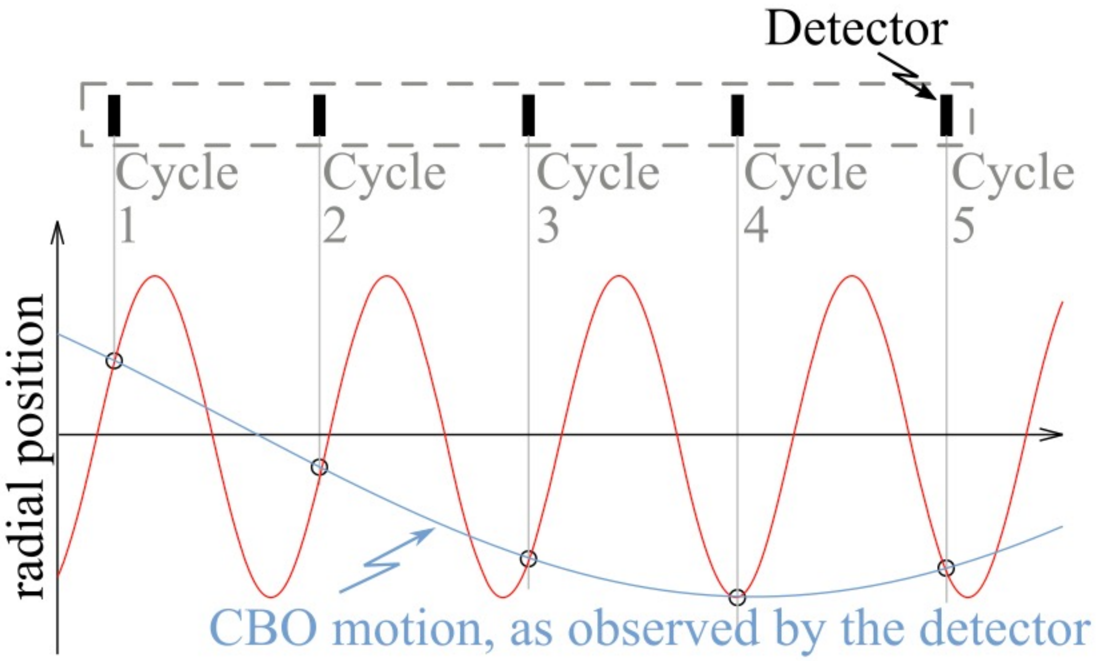
How does the beam move?

So the winner is 3) the motion is even more complex!!!



Detector effect – CBO oscillation

- The beam is measured by detectors, calorimeters and trackers.
- The $\omega_{BO} \neq \omega_C$, so calorimeters see a different phase at each turn, measuring an oscillation called Coherent Betatron Oscillation (CBO), given by $\omega_{CBO} = \omega_C - \omega_{BO}$

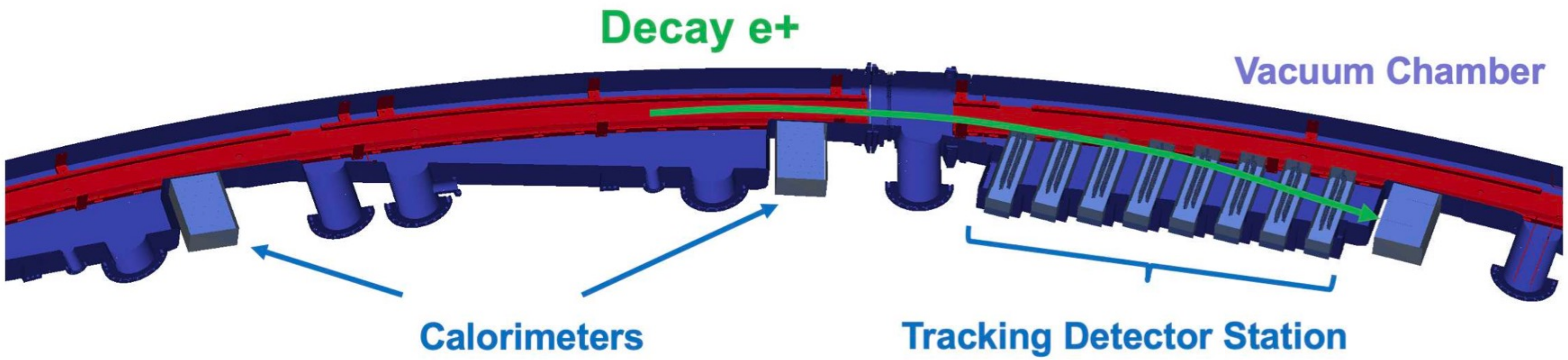


$$2\pi f_{CBO} = \omega_C - \omega_{BO} = \omega_C(1 - \sqrt{1-n})$$

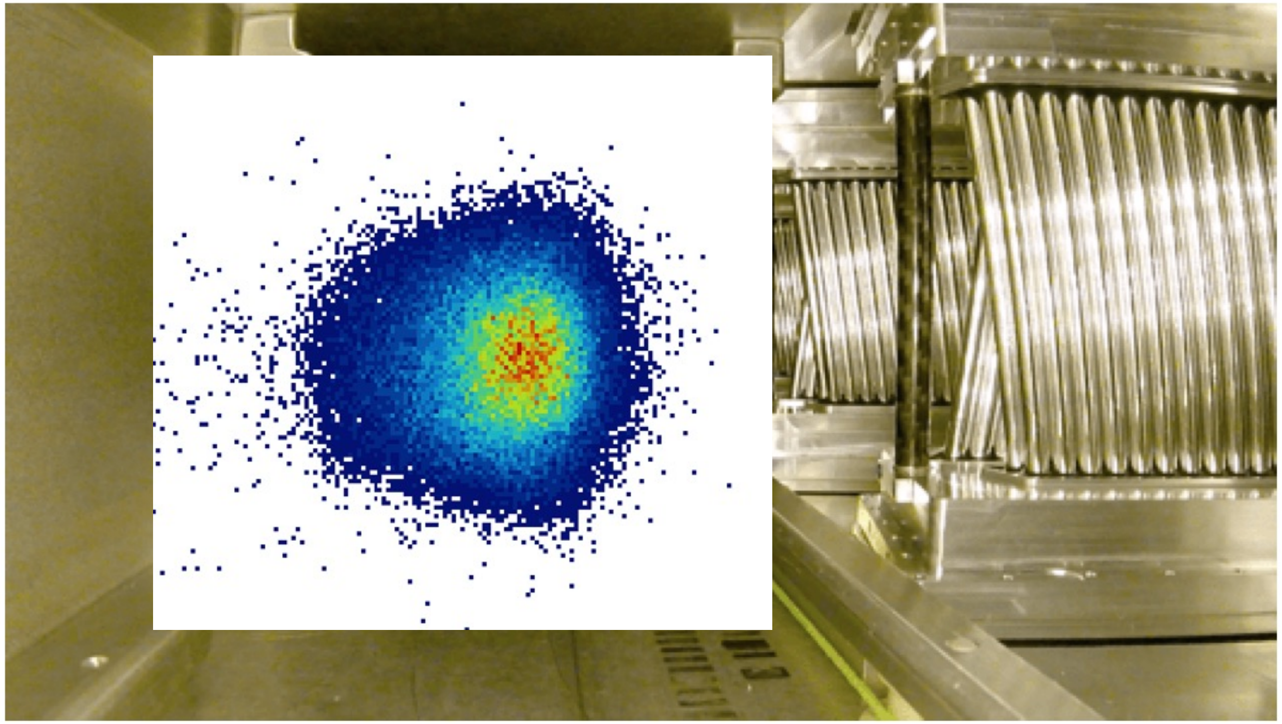
$$\omega_{CBO} = 2.34\, rad/\mu s$$

Where $\omega_C \sim 0.149\, ns$ and $n \sim 0.108$

Detector effect – Tracker detector

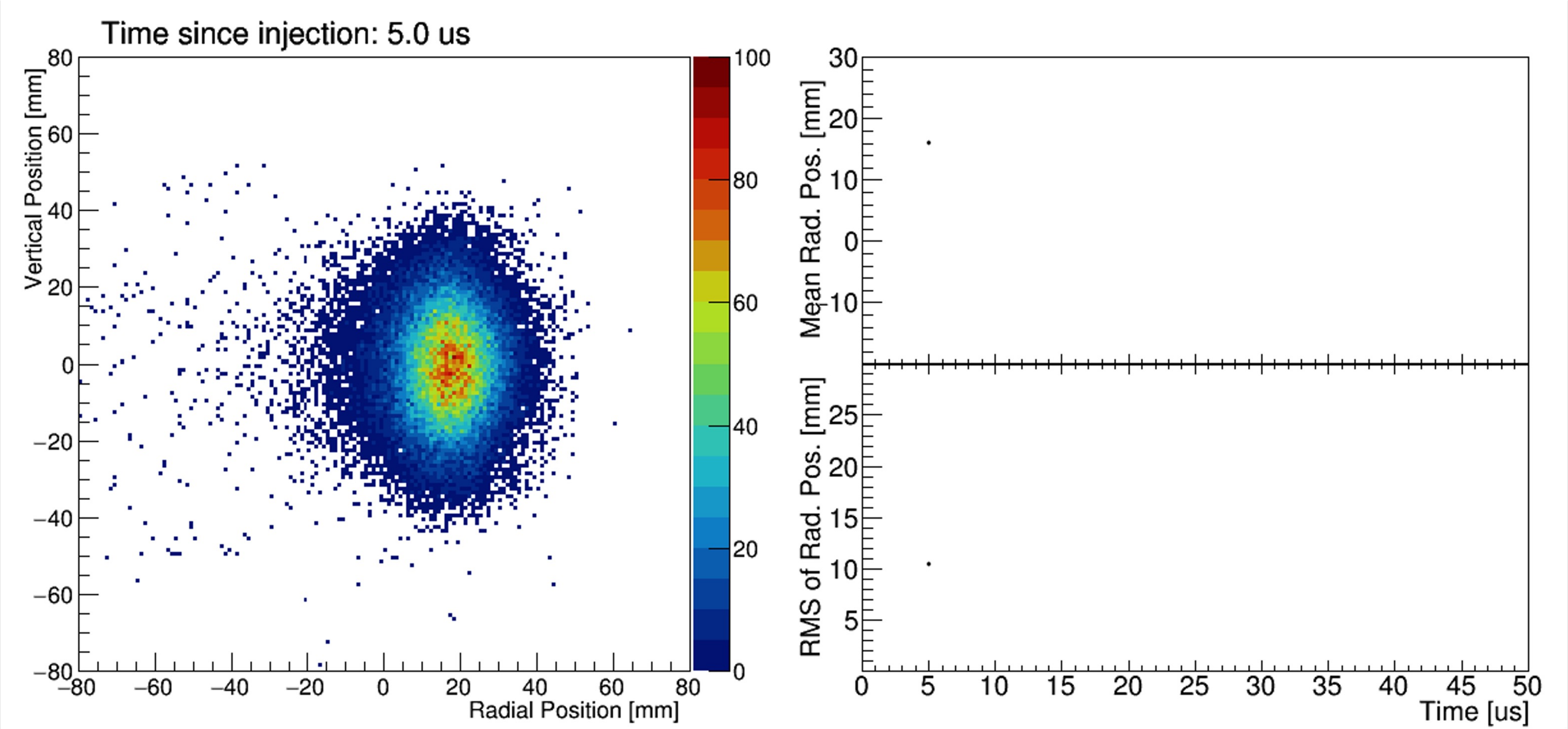


- 2 tracker station 90° apart
- 8 Identical tracker modules
- 32 straws each module grouped into 2 pairs of UV layers
- Straws argon ethane filled



Detector effect – CBO oscillation

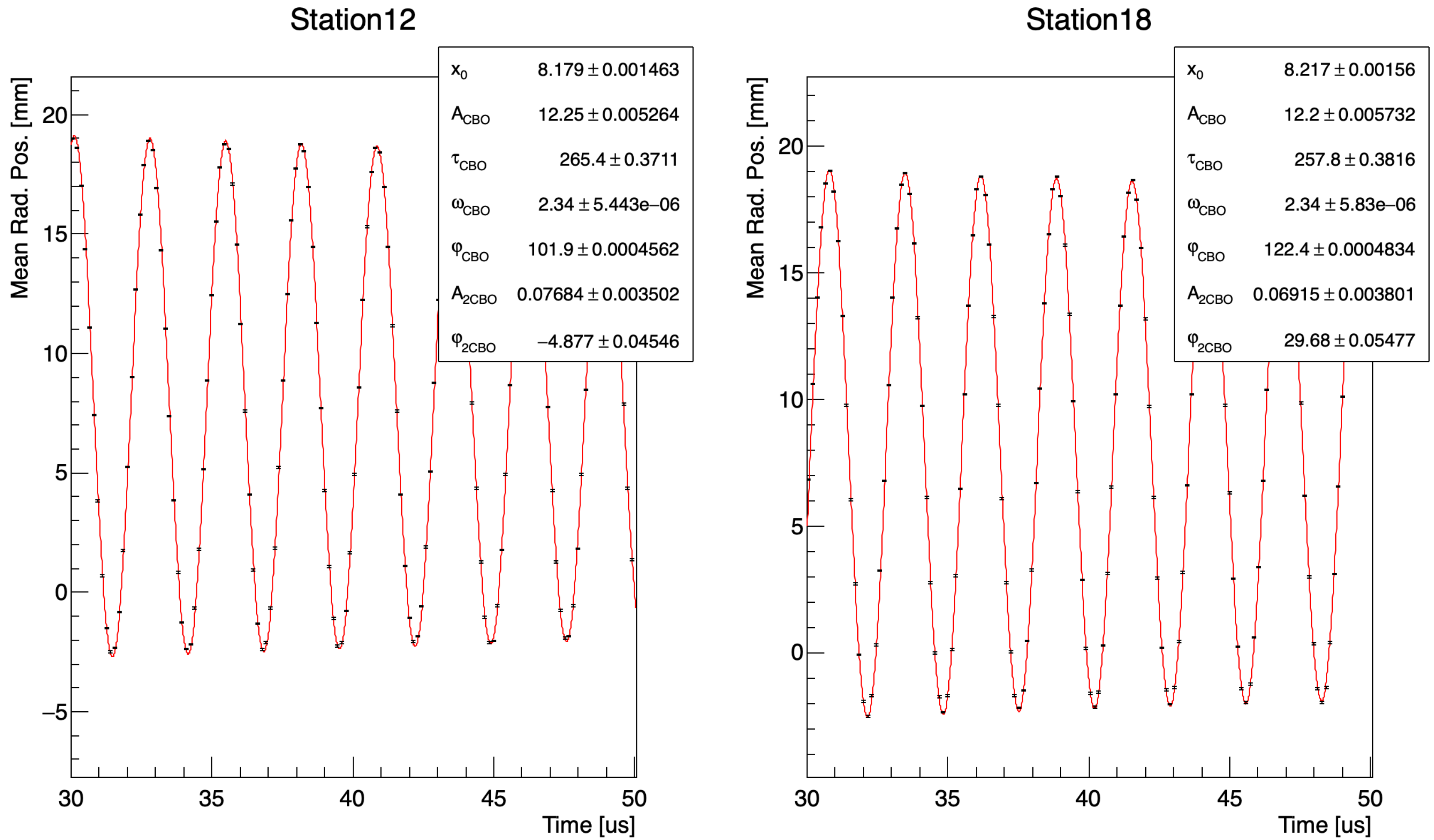
- This is the beam motion observed by the tracker.



Detector effect – CBO oscillation

- This is the beam motion observed by the tracker.

$$x(t) = x_0 + A_{CBO} \cdot e^{-\frac{t}{\tau_{CBO}}} \cdot \cos(\omega_{CBO} t + \varphi_{CBO}) + A_{2CBO} \cdot e^{-\frac{2t}{\tau_{CBO}}} \cdot \cos(2\omega_{CBO} t + \varphi_{2CBO})$$

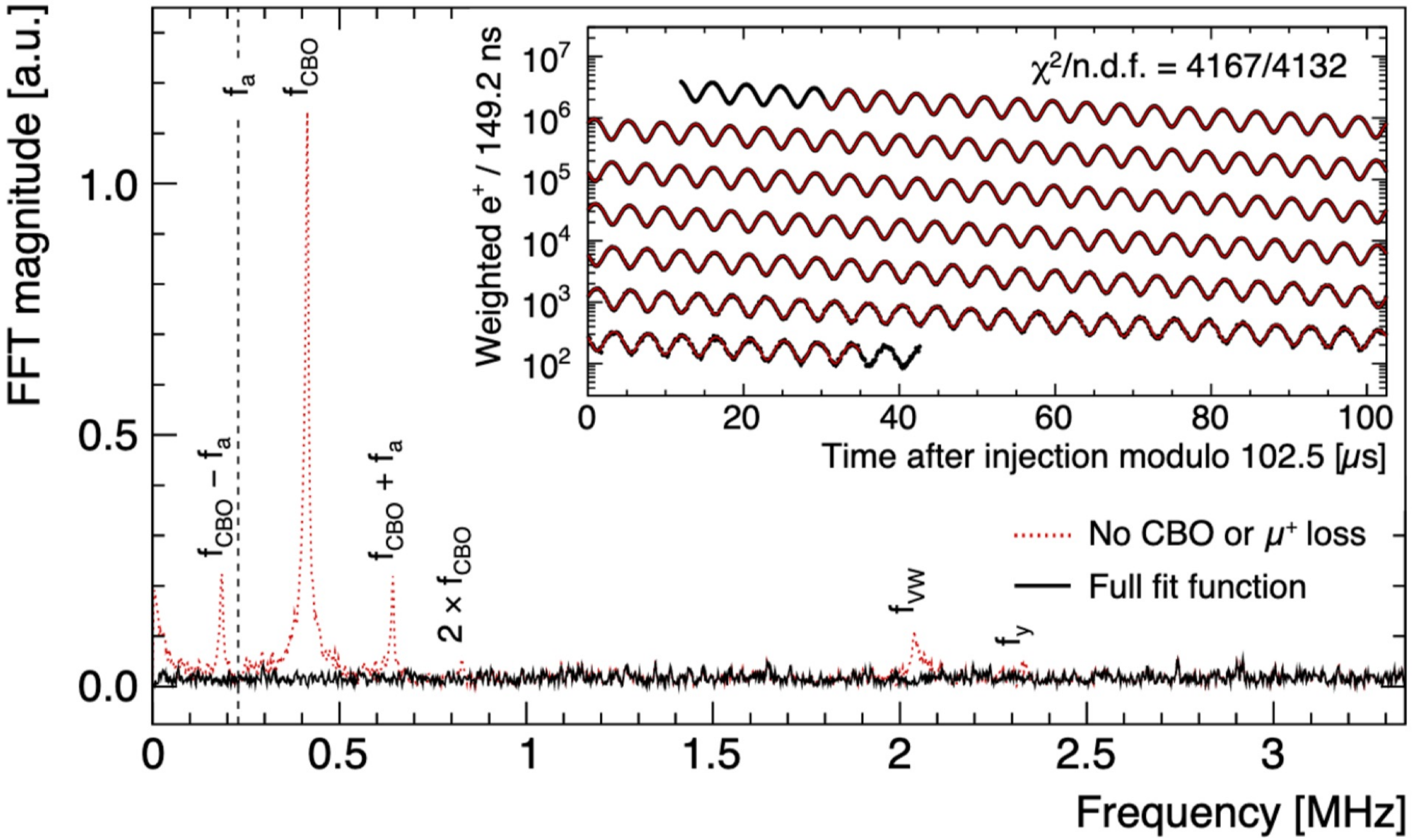


VADEMECUM

- Remember in g-2:
  - Everything which changes during the *fill* (700μs) ☹️
  - Everything which changes within hours/days/months 😕
  - Everything which never change 😃

Beam dynamics correction to $\boldsymbol{\omega_a}$

- Even though beam motion is taken into account from the fit function containing more then 20 parameters.
- The beam motion introduces some early-to-late effect which biases ω_a.



Beam dynamics correction to $\boldsymbol{\omega_a}$

- To take into account this variation, we correct the measured ω_a value from the fit using the following formula.

$$R'_\mu \approx \frac{f_{clock}\,\omega_a^m\left(1 + \boldsymbol{C_e} + \boldsymbol{C_p} + \boldsymbol{C_{ml}} + \boldsymbol{C_{pa}}\right)}{f_{calib} < \omega'_p(x,y,\phi)\times M(x,y,\phi) > \left(1 + B_k + B_q\right)}$$

Beam dynamics correction to $\boldsymbol{\omega_a}$: C_e

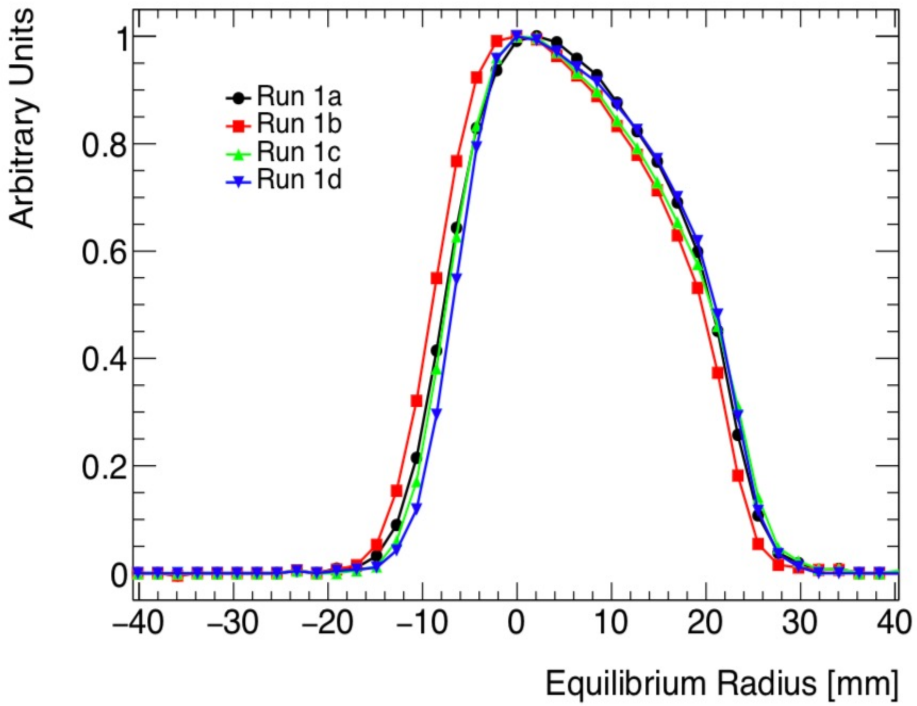
Considering the extended expression of the spin precession frequency in a magnetic field:

$$\overrightarrow{\omega_a} = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta}\times\vec{E}) - a_\mu \left(\frac{\gamma}{\gamma + 1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

$\boldsymbol{C_e}$: the Electric Field correction

$C_e = 2n(1-n)\beta^2 x_e^2/R_0^2$ is due to the equilibrium radii distribution .

$$\boxed{C_e \sim 490\ ppb}$$

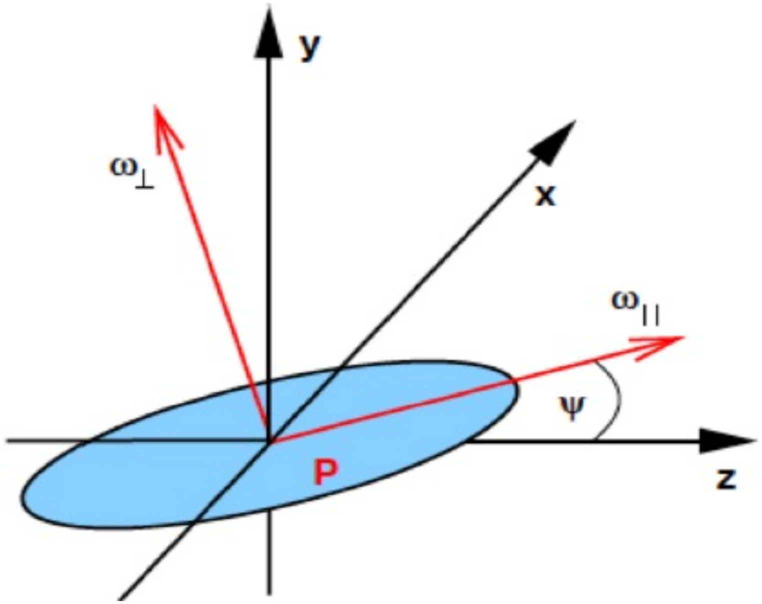


Beam dynamics correction to $\boldsymbol{\omega_a}$: C_p

Considering the extended expression of the spin precession frequency in a magnetic field:

$$\overrightarrow{\omega_a} = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)(\vec{\beta}\times\vec{E}) - a_\mu\left(\frac{\gamma}{\gamma + 1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right]$$

$$\boxed{C_p \sim 180\,ppb}$$



$\boldsymbol{C_p}$: the pitch correction

$C_p = n < A_y^2 > / 4R_0^2$

depends on vertical betatron oscillation (A_y).

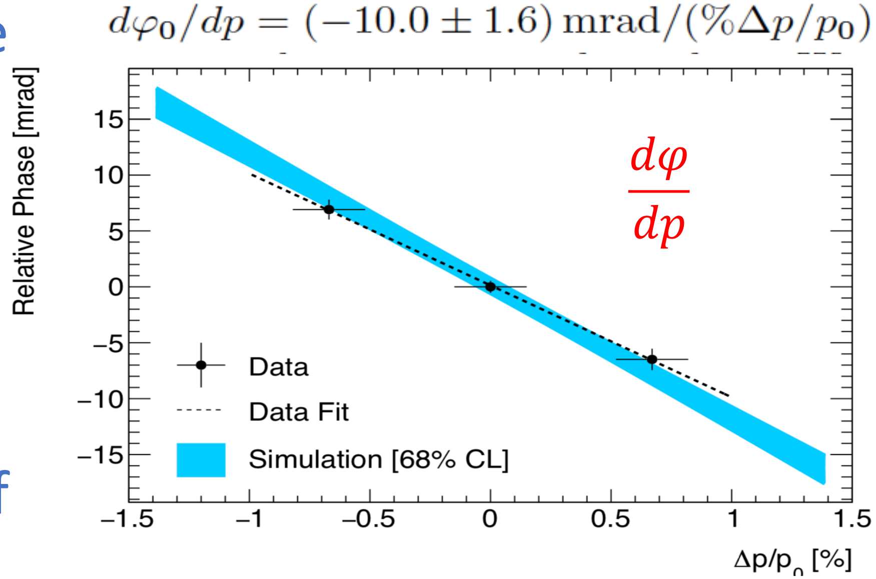
Beam dynamics correction to $\boldsymbol{\omega_a}$: C_{lm}

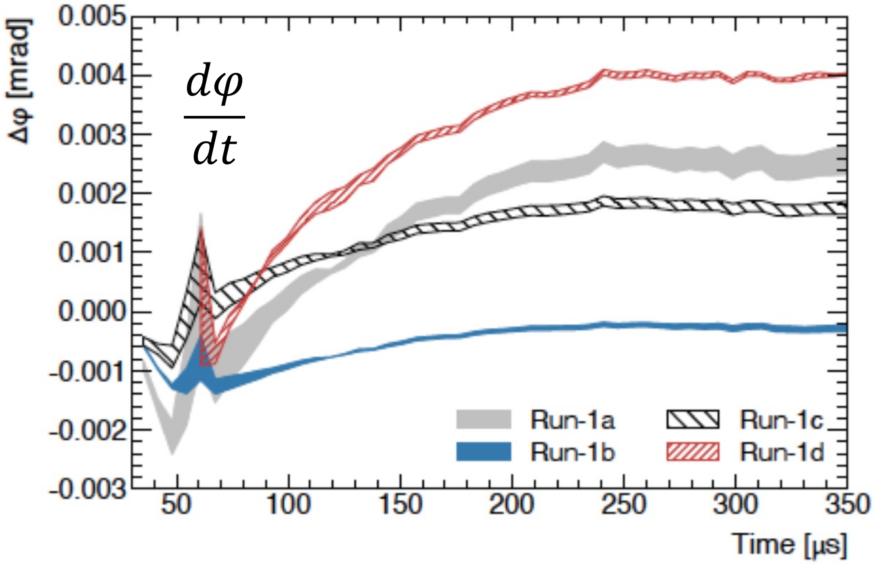
$\boldsymbol{C_{lm}}$: describes the motion introduced on ω_a phase due to the loss of muon during the *fill*. It's explained by:

1. Muons with different momentum has different phase;
2. The number of loss muon change as function of momentum.

$$\Delta\omega_a = \frac{d\varphi}{dt} = \overset{1)}{\frac{d\varphi}{dp}} \cdot \overset{2)}{\frac{dp}{dt}}$$

$$\boxed{C_{lm} < 20\ ppb}$$





Beam dynamics correction to ω_a: C_{pa}

C_{pa}: it is a Phase Acceptance effect. It is due to:

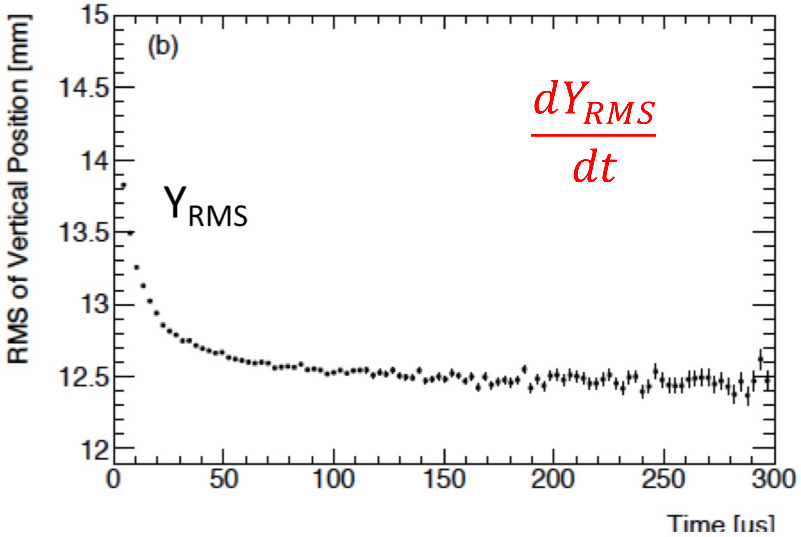
1. Beam variation during the fill *fill;*
2. Phase measured as function of the decay position.

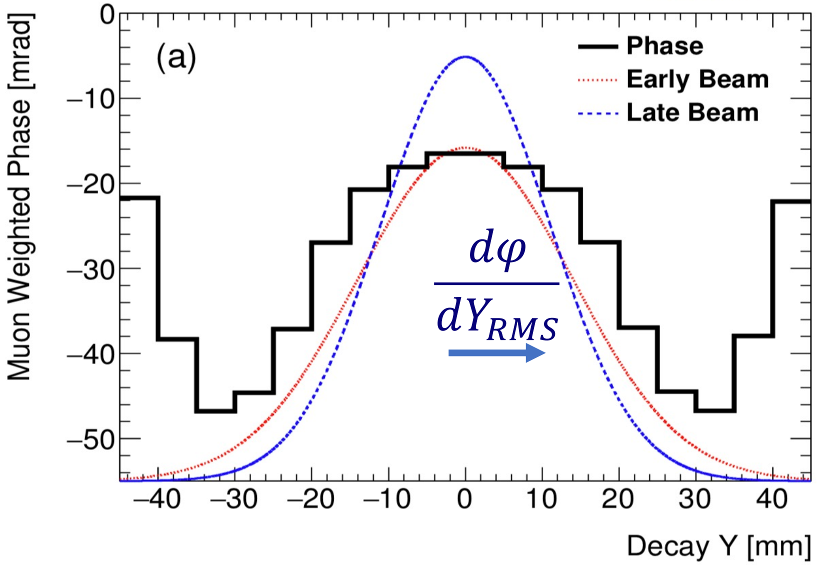
$$\Delta\omega_a = \frac{d\varphi}{dt} = \overset{1)}{\frac{dY_{RMS}}{dt}} \cdot \overset{2)}{\frac{d\varphi}{dY_{RMS}}}$$

The effect was large in Run1 due to *broken resistors*

$$\boxed{C_{pa} \sim 180\ ppb}$$

We expect a reduction in Run2/3 (~50ppb/ ~20ppb)

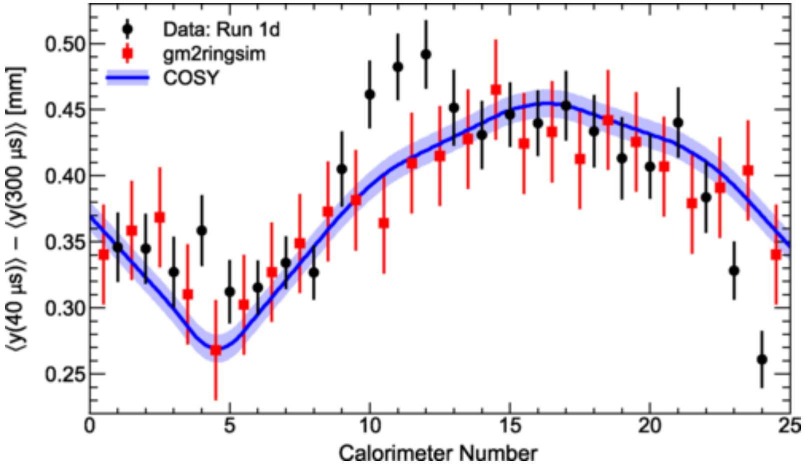




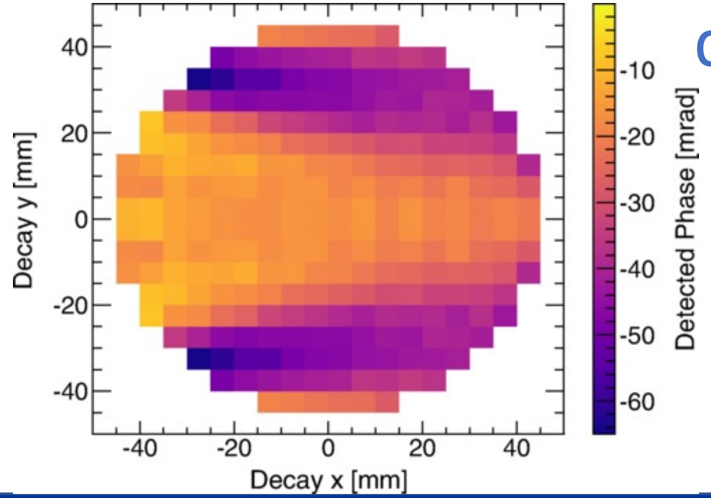
Simulation of the storage ring

- To compute with high precision the correction showed before, we use a Geant4 based simulation of the storage ring, from the injection up to detection.

Vertical distribution

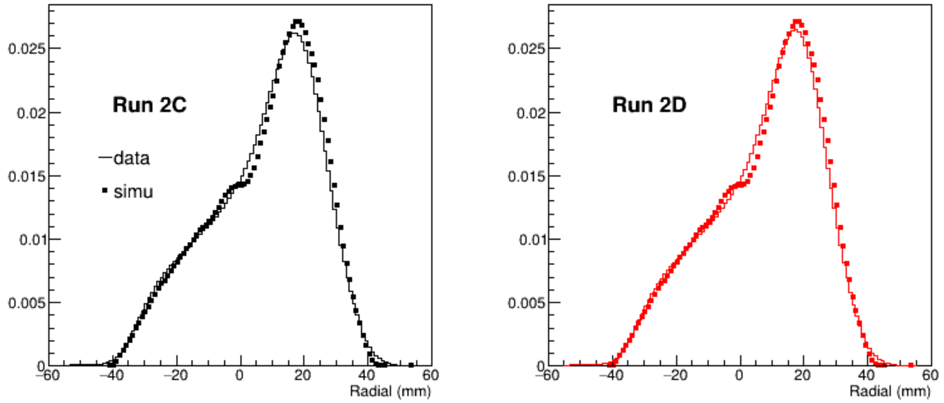


Muon Initial phase map

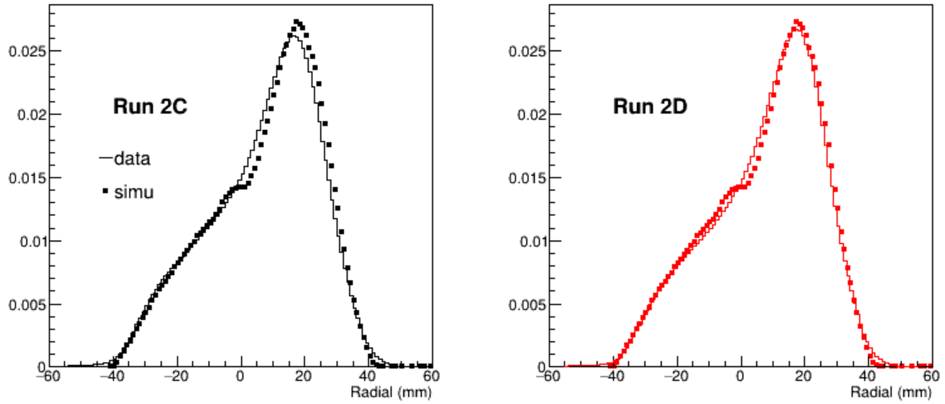


Radial distribution

STATION 12



STATION 18



Conclusion

- Beam motion inside the ring is complex and detectors measure it, but in turn, the detectors introduce effects given their acceptance.
- Beam dynamics affect not only the ω_a fit, but introduce biases that cannot be fitted from the wiggle plot.
- Thanks to both simulation and real data analysis, we are able to correct for all these effects.

“The closer you look the more there is to see”

F. Jegherlehner

Thank you!!!

- For any question or just to have a chat – elia.bottalico@phd.unipi.it