Mixed QCD-electroweak corrections to Drell-Yan production

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Delto, Jaquier, Melnikov, RR [hep-ph/1909.08428]

Buccioni, Caola, Delto, Jaquier, Melnikov, RR, [hep-ph/2005.10221]

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR [hep-ph/2009.10386, hep-ph/2103.02671]

Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile [hep-ph/2203.11237]

University of Genova

12 May 2022

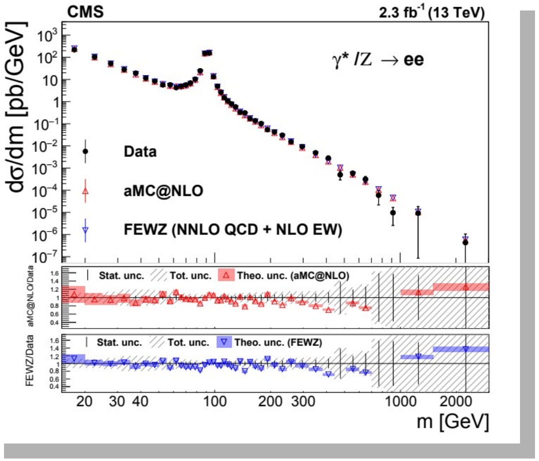
Outline

- Introduction and Motivation
- Mixed QCD-EW corrections to onshell vector boson production
- Impact of mixed QCD-EW corrections on the W-mass measurement
- Mixed QCD-EW corrections to dilepton production
- Conclusions

Introduction and Motivation

Drell-Yan production at the LHC

- Drell-Yan production $pp \to V^* \to \ell\bar{\ell}$ is one of the keystone processes at the LHC:
  - Calibrating detectors;
  - Determining pdfs;
  - Measuring fundamental parameters of SM, e.g. m_W;
  - Searching for BSM physics at high energies;
  - ….
- Lots of events with clear experimental signatures – extremely well-controlled experimentally.



Precision Predictions for Drell-Yan Production

Fixed-order predictions: expand partonic cross section in strong and electroweak couplings:

$$\hat{\sigma}_{ij} = \hat{\sigma}_{ij}^{(0,0)} + \alpha_s \hat{\sigma}_{ij}^{(1,0)} + \alpha_s^2 \hat{\sigma}_{ij}^{(2,0)} + \alpha_s^3 \hat{\sigma}_{ij}^{(3,0)} + \ldots + \alpha\, \hat{\sigma}_{ij}^{(0,1)} + \alpha_s \alpha\, \hat{\sigma}_{ij}^{(1,1)} + \ldots$$

LO | NLO QCD | NNLO QCD | N3LO QCD | NLO EW | mixed QCD-EW

Recent advances:

- N3LO QCD corrections

  [Dulat, Duhr, Mistlberger (‘20); Duhr, Mistlberger (‘21); Chen, Gehrmann, Glover, Huss, Yang, Zhu (‘21); Camarda, Cieri, Ferrara (‘22); Chen *et al.* (‘22)]

- Mixed QCD-EW corrections

  [Bonciani, Buccioni, Mondini, Vicini (‘17); De Florian, Der, Fabre (‘18); Delto, Jaquier, Melnikov, R.R. (‘19); Bonciani, Buccioni, Rana, Triscari, Vicini (‘19); Buccioni *et al.* (‘20); Cieri, De Florian, Der, Mazzitelli (‘20); Bonciani, Buccioni, Rana, Vicini (‘20); Behring *et al.* (‘20); Buonocore, Grazzini, Kallweit, Savoini, Tramontano (‘21); Bonciani *et al.* (‘21); Armadillo *et al.* (‘22)]

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- Couplings $\alpha_s \sim 0.1$; $\alpha \sim 0.01$
- Mixed QCD-EW corrections $\sim \mathcal{O}(\alpha_s \alpha) \sim 0.1\%$
- Experimental precision usually percent level.
- *Why do we need this level of precision in theoretical predictions?*

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- *Why do we need this level of precision in theoretical predictions?*

1. Target precision is sometimes below percent level: m_W measurement. ← Onshell vector boson production + decay
2. QCD-EW corrections enhanced at high energies. ← Dilepton production

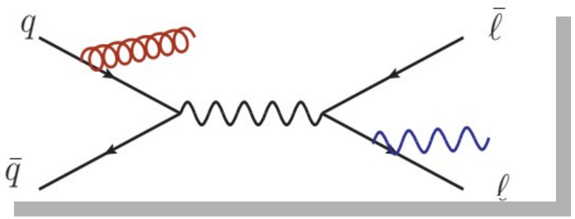
Mixed QCD-EW corrections to onshell vector boson production

QCD-EW corrections to onshell vector boson production

- Consider onshell vector boson production and decay $pp \to V \to \ell\bar{\ell}$
- In resonance region, QCD-EW corrections to production and decay processes can be treated separately.

[Dittmaier, Huss, Schwinn ('14, '15)]

QCD (production) x EW (decay)



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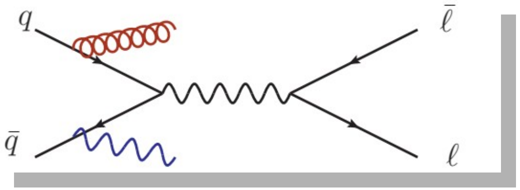

Non-factorizable resonant contributions (not shown) suppressed by Γ_V / m_V

[Dittmaier, Huss, Schwinn ('14, '15)]

QCD-EW corrections to onshell vector boson production

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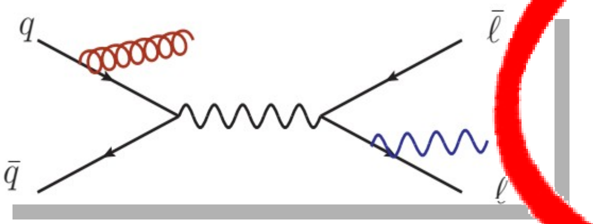
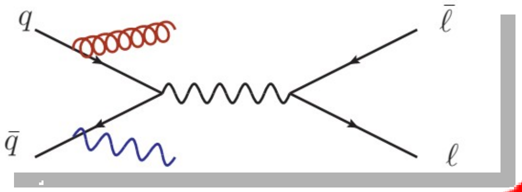


QCD x EW (production)



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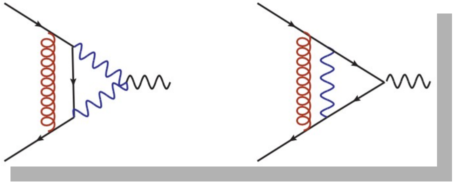
QCD-EW corrections to onshell vector boson production

- Two major challenges in computing higher order corrections:
  1. Loop amplitudes
  2. Handling infrared singularities

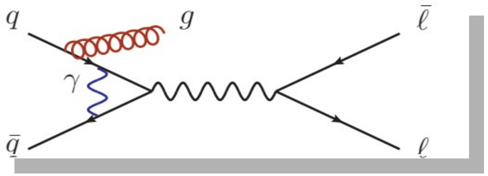
- Loop amplitudes:
  - Two-loop form factors



    - Known for Z production. [Kotikov, Kühn, Veretin ('08)]
    - Computed for first time for W production.

    [Behring *et al.* ('20)]

  - One-loop real-virtual amplitudes



    - Standard one-loop programs, e.g. `OpenLoops`

    [Cascioli, Maierhöfer, Pozzorini ('12); Buccioni, Pozzorini, Zoller ('18); Buccioni *et al.* ('19)]

QCD-EW corrections to onshell vector boson production

- Two major challenges in computing higher order corrections:
  1. Loop amplitudes
  2. Handling infrared singularities

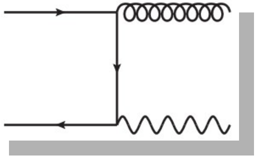
- Infrared singularities with different origins appear simultaneously:
  - *Virtual* photons;
  - *Virtual* partons;
  - *Unresolved real* photons;
  - *Unresolved real* partons.

- Insight from NNLO QCD: treatment of IR singularities with non-trivial structure.

Infrared singularities in QCD

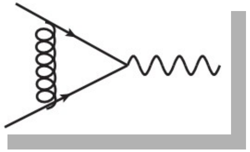
Higher order corrections contain IR singularities from soft and/or collinear radiation.

- Real corrections
  - Integrate over phase space of radiated parton:



$$\longrightarrow \int |\mathcal{M}|^2 F_J d\phi_g \text{ diverges}$$

- Virtual corrections
  - Explicit IR singularities from loop integration



$$\longrightarrow \mathcal{M}_{1-\text{loop}} = \frac{c_{-2}}{\epsilon^2} + \frac{c_{-1}}{\epsilon} + c_0$$

- Singularities unphysical, guaranteed to cancel in sum (KLN theorem).
- Cancellation only manifest after integrating over full phase space of emitted parton:

  → lose kinematic information.

Subtracting IR singularities in QCD

Subtraction scheme: extract singularities without integrating over full phase space of radiated parton:

- Singularities manifest as poles in $1/\epsilon$ cancel against poles in virtual correction → finite fully differential result.

$$\int |\mathcal{M}|^2 F_J d\phi_d = \int \left(|\mathcal{M}_J|^2 F_J - S\right) d\phi_4 + \int S d\phi_d$$

Finite;

integrate in 4-dim.

Counterterm;

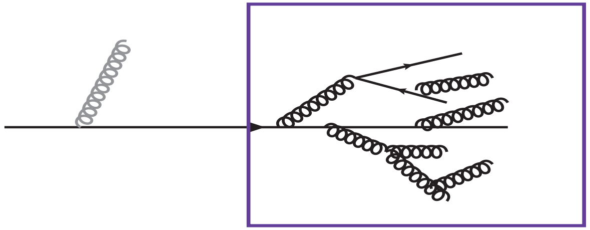
Explicit singularities

- Subtractions at NLO fully solved. [Catani, Seymour ('96); Frixione, Kunszt, Signer ('96, '97)]
- More complicated singularity structure at NNLO – constructing NNLO subtraction schemes is an active area of research.

Nested soft-collinear subtractions

[Caola, Melnikov, R.R. (2017)]

- Extension of FKS subtraction to NNLO.
- Exploits color coherence of onshell, gauge-invariant amplitudes
  - ➢ Used in resummation & parton showers; not manifest in subtractions.



- Soft gluon cannot resolve details of collinear splittings; only sensitive to total color charge.

No overlap between soft and collinear limits – treated independently:

- Energies and angles decouple.
- Regularize soft singularities first, then collinear singularities – iterative subtraction of divergences.

Nested soft-collinear subtractions

- Overlapping soft singularities separated by energy ordering.
- Overlapping collinear singularities separated using partitioning and sectoring of phase space. [Czakon ('10, '11)]
  - Natural splitting by rapidity.

- Fully local and fully analytic.

  [Caola, Melnikov, R.R. ('19); Asteriadis, Caola, Melnikov, R.R. ('19)]

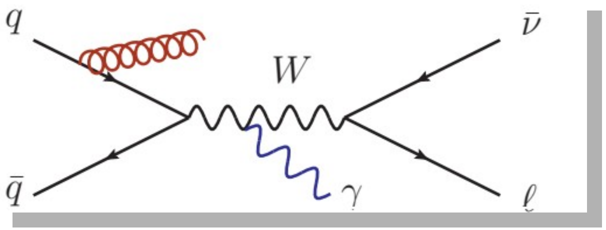
  [Delto, Frellesvig, Caola, Melnikov ('18); Delto, Melnikov ('19)]

- Clear physical origin of singularities (soft & collinear).
- Used for several phenomenological studies (VH, VBF, $gg \to H/A$, ...)
- Flexible → straightforward adaptation for mixed QCD-EW singularities.

QCD-EW corrections to onshell vector boson production

Consider onshell vector boson production $pp \to V \to \ell\bar{\ell}$

- Qualitatively new feature: photon radiated off W.
- Collinear limits regulated by W-mass, but soft limit is singular.
- Changes form of eikonal function in soft limit:



Soft gluon → $\mathrm{Eik}_g(p_1, p_2; p_g) = \dfrac{2C_F(p_1 \cdot p_2)}{(p_1 \cdot p_g)(p_2 \cdot p_g)}$

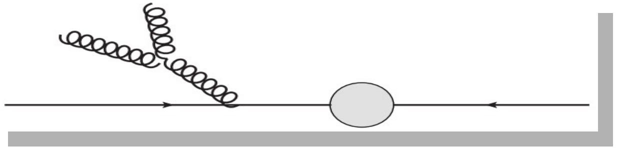
Soft photon → $$\mathrm{Eik}_\gamma(p_1, p_2, p_W; p_\gamma) = Q_u Q_d \frac{2(p_1 \cdot p_2)}{(p_1 \cdot p_\gamma)(p_2 \cdot p_\gamma)} - Q_W^2 \frac{p_W^2}{(p_W \cdot p_\gamma)^2} + Q_W \left(Q_u \frac{2(p_W \cdot p_1)}{(p_W \cdot p_\gamma)(p_1 \cdot p_\gamma)} - Q_d \frac{2(p_W \cdot p_2)}{(p_W \cdot p_\gamma)(p_2 \cdot p_\gamma)} \right)$$

➡ More complicated function, but method is conceptually unchanged!

QCD-EW corrections to onshell vector boson production

Can make subtraction scheme simpler:

- NNLO QCD: soft limits of gluons overlap → introduced energy ordering.
- Mixed QCD-EW: soft limits of gluons and photons are independent → no energy ordering needed.
- Soft subtraction: iterated NLO-like soft limits.

- Genuine NNLO-like singularities in collinear limits → require phase-space partitioning and sectoring.
- Fewer collinear limits, e.g.  disappears.

→ Fewer sectors required.

QCD-EW corrections to onshell vector boson production

Can make subtraction scheme simpler:

- NNLO QCD: s
- Mixed QCD ... g needed.
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→ Fewer se

Implemented this subtraction method to compute mixed QCD-EW corrections to production of onshell Z and W bosons.

[Delto, Jaquier, Melnikov, RR ('19);

Buccioni, Caola, Delto, Jaquier, Melnikov, RR, ('20)

Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, RR ('20)]

- How could these corrections impact the measurement of the W-mass?

Impact of mixed QCD-EW corrections on the W-mass measurement

W-mass measurements

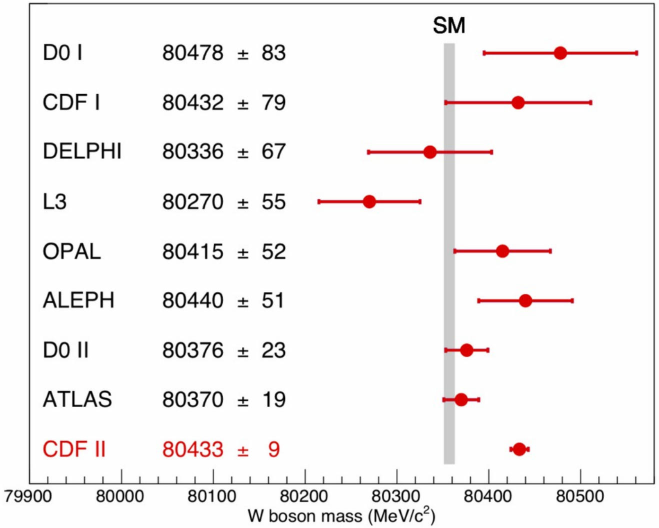
- SM is overconstrained – predict parameters of SM Lagrangian (e.g. m_W) using global electroweak fits.

$$\Rightarrow \quad m_W = 80.354 \pm 0.007 \text{ GeV}$$ [Gfitter Group: Haller *et al.* (‘18)]

- Comparison with direct measurements tests self-consistency of SM and probes BSM effects.

[e.g. Bjørn, Trott (‘16)]

[A. Kotwal (‘22)]



- Tension between new CDF measurement and other measurements.
- Higher precision desirable.
- Uncertainty dominated by physics modeling.

Experimental measurements

- W mass directly measured in $pp \to W \to \ell\nu$
- Template fit: simulate data for different values of W-mass and fit to data.
- Three most relevant observables: $p_{T,\ell},\ p_{T,\mathrm{miss}},\ m_{T,W}$
- Strongest pull from $p_{T,\ell}$, also most sensitive to higher order corrections.

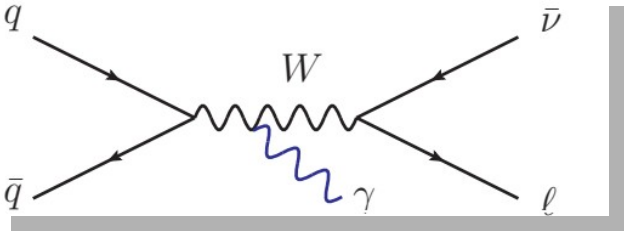
[Carloni Calame *et al.* (‘16)].

- Uncontrolled non-perturbative effects enter at the level of $\Lambda_{\mathrm{QCD}}/Q \sim 0.01$

  → theoretical predictions not reliable at the desired precision of 0.1 per mille.
- Use excellent control of process $pp \to Z \to \ell\bar{\ell}$ to calibrate detector response, tune generators, and verify results.

W-mass measurements

- Implicit assumption: higher-order corrections to W and Z production strongly correlated.
- Reasonable for QCD corrections:
  - Minor differences: pdfs, masses, helicity structures, …
- EW corrections: qualitatively different – *W* charged, can radiate:



- Mixed QCD-EW corrections potentially decorrelated.
- Possible impact on W-mass measurements at desired precision.

Impact on W mass determination

- ***Estimate*** effect of QCD-EW corrections on W mass measurement, due to decorrelations between *Z* and *W* production.
- Correlation between average transverse momentum of leptons and mass of boson:

$$\frac{m_W}{m_Z} = \frac{\langle p_{T,l}^W \rangle}{\langle p_{T,l}^Z \rangle} \Rightarrow m_W^{\text{meas.}} = m_Z \frac{\langle p_{T,l}^{W,\text{meas.}} \rangle}{\langle p_{T,l}^{Z,\text{meas.}} \rangle} C_{\text{th.}}$$

- Theoretical correction: assume input masses, compute W-mass, and compare with input W-mass.

$$\Rightarrow C_{\text{th.}} = \frac{m_W^{\text{in}}}{m_Z^{\text{in}}} \frac{\langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

→ estimate impact of decorrelations in *W* and *Z* spectra from higher order corrections:

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

[Behring *et al.* ('21)].

Impact on W mass determination

Shifts in W-mass, inclusive:

- NLO EW: $\Delta m_W = 1 \text{ MeV}$
- QCD-EW: $\Delta m_W = -7 \text{ MeV}$

$\sqrt{s} = 13 \text{ TeV}$

G_μ scheme

$m_Z = 91.1876 \text{ GeV}$

$m_W = 80.398 \text{ GeV}$

$m_t = 173.2 \text{ GeV}$

$m_H = 125 \text{ GeV}$

$G_F = 1.16339 \cdot 10^{-5} \text{ GeV}^{-2}$

NNPDF31_luxQED

$\mu_R = \mu_F = m_V/2$

➔ Impact of QCD-EW corrections larger than NLO EW:

- NLO EW corrections suppressed in G_μ scheme.
- NLO EW corrections more correlated between *W* and *Z* production.

$$\frac{\delta m_W^{\text{meas.}}}{m_W^{\text{meas.}}} = \frac{\delta C_{\text{th.}}}{C_{\text{th.}}} = \frac{\delta \langle p_{T,l}^{Z,\text{th.}} \rangle}{\langle p_{T,l}^{Z,\text{th.}} \rangle} - \frac{\delta \langle p_{T,l}^{W,\text{th.}} \rangle}{\langle p_{T,l}^{W,\text{th.}} \rangle}$$

NLO EW: $\Delta m_W = -31 \text{ MeV} + 32 \text{ MeV}$

QCD-EW: $\Delta m_W = +54 \text{ MeV} - 61 \text{ MeV}$

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Impact on W mass determination

Shifts in W-mass: fiducial setup

- Inclusive setup: $\Delta m_W = -7\ \mathrm{MeV}$
- "ATLAS" cuts: $\Delta m_W = -17\ \mathrm{MeV}$
- "Tuned" cuts: $\Delta m_W = -1\ \mathrm{MeV}$

→ Cuts can have dramatic impact: shifts vary by factor of ~20.

  - "ATLAS" cuts have stronger cuts on leptons from (lighter) *W* than from *Z* → decorrelation.

→ QCD-EW shifts potentially relevant for target precision of 8 MeV.

$p_{T,\ell}^{Z} > 25\ \mathrm{GeV}; |\eta_\ell^{Z}| < 2.4$

"ATLAS" cuts: $\mathrm{p}_{T,\ell}^{W} > 30\ \mathrm{GeV}; \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 30\ \mathrm{GeV}; |\eta_\ell^{\mathrm{W}}| < 2.4.$

"Tuned" cuts: $\mathrm{p}_{T,\ell}^{W} > 25.44\ \mathrm{GeV}; \mathrm{p}_{\mathrm{T,miss}}^{\mathrm{W}} > 25.44\ \mathrm{GeV}; |\eta_\ell^{\mathrm{W}}| < 2.4.$

Interpretation

- These results are **estimates** of impact of QCD-EW corrections on W-mass measurements at the LHC.
- Indicate that QCD-EW corrections could be relevant for 0.1 permille precision on W-mass measurements.
- Further investigations are essential:
  - What is the impact when using the full transverse momentum spectrum?
  - What is the impact on other observables?
  - How well are these captured with standard experimental simulation tools?
  - How reliable are these results – do we need to include parton showers to handle multiple photon emissions?
  - …

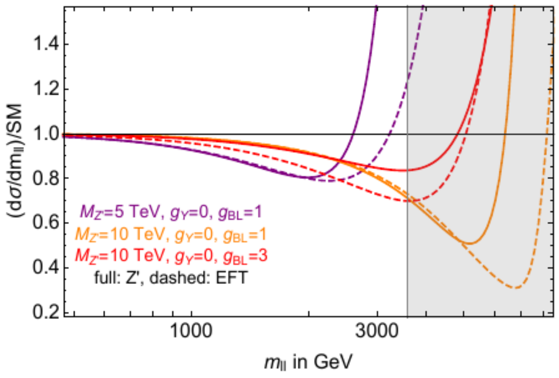
Mixed QCD-EW corrections to dilepton production

QCD-EW corrections to dilepton production

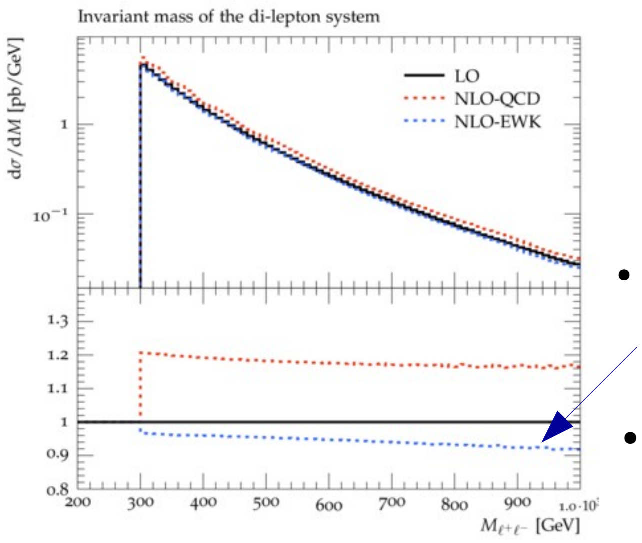
Consider $pp \to V^* \to \ell\bar{\ell}$ and focus on high-mass tail.

- Indirect probe of New Physics at very high scales.

[cf. Rizzo ('09); Greljo, Marzocca ('17); Alioli, Farina, Pappadopulo, Ruderman ('17)]



[Alioli, Farina, Pappadopulo, Ruderman ('17)]



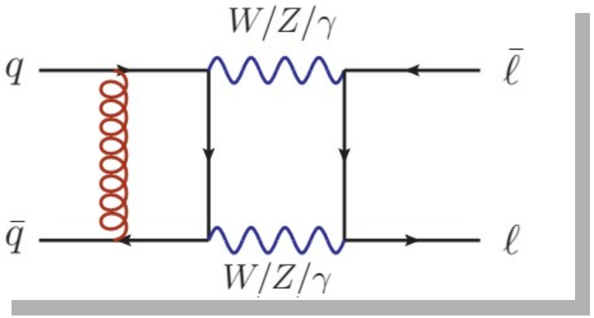
- EW corrections enhanced by Sudakov logarithms

$\log^2\left(s/m_W^2\right) \simeq 25$ for s ~ 1 TeV.

- Cannot separate into production and decay – consider corrections to complete dilepton production process!

QCD-EW corrections to dilepton production

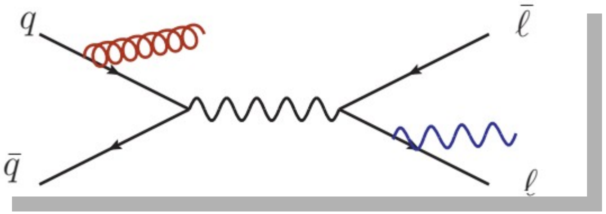
- Two-loop amplitudes are extremely challenging!

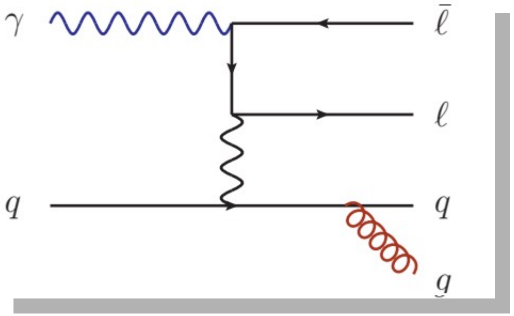


- Several energy scales.
- Recent computations:

  [**Heller, von Manteuffel, Schabinger, Spiesberger ('20);** Bonciani *et al.* ('21); Armadillo, Bonciani, Devoto, Rana, Vicini ('22)]

- IR singularities arising from emission of photons from leptons as well as quarks.



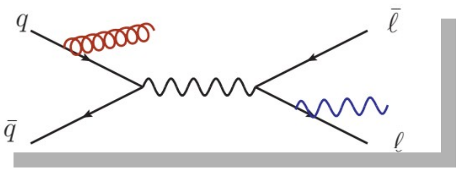


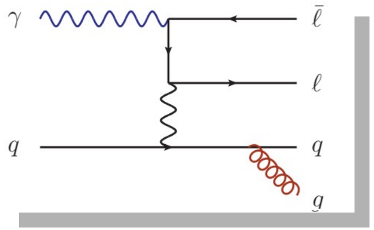
QCD-EW IR singularities in dilepton production

Three new complications arise in regulating IR singularities:

1. Photons radiated from leptons lead to singularities in soft and collinear limits.

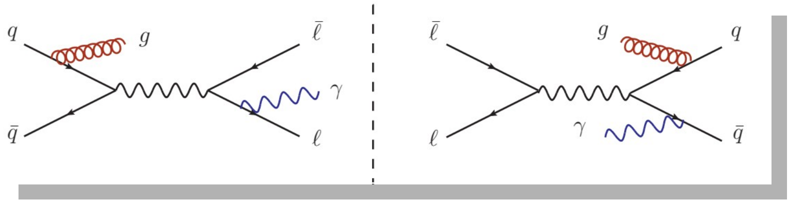
2. New diagrams from initial state photons.





→ extend partitions to isolate singularities from configurations with collinear photons and leptons

3. Interference between “initial-initial” and “initial-final” corrections



- Only singular for soft photons.
- Soft singularities can be treated as iterated NLO-like singularities.

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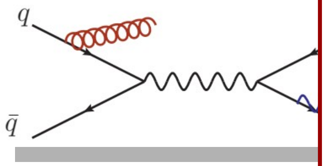

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ations with
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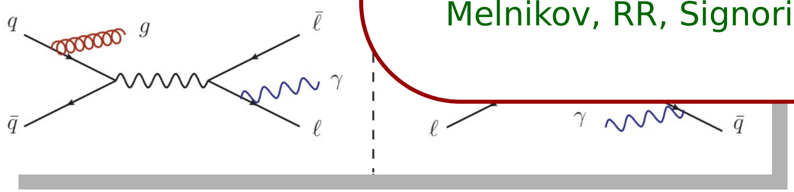
Extending subtraction method to dilepton production is conceptually straightforward (in practice quite intricate).

→ compact expressions for integrated subtraction counterterms.

→ results for $pp \to \ell^- \ell^+$

[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, RR, Signorile-Signorile ('22)]

3. Interference be



- Soft singularities can be treated as iterated NLO-like singularities.

Phenomenological results: Fiducial cross section

LO Corrections $\sim \mathcal{O}(\alpha_s^i \alpha^j)$

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
$q\bar{q}$	1561.42	340.31	−49.907	44.60	−16.80
$\gamma\gamma$	59.645		3.166		
qg		0.060		−32.66	1.03
$q\gamma$			−0.305		−0.207
$g\gamma$					0.2668
gg				1.934	
sum	1621.06	340.37	−47.046	13.88	−15.71

- LHC 13.6 TeV
- `NNPDF31_nnlo_as_0118_luxqed`
- G_μ input scheme for EW parameters.
- Massless leptons, clustered with photons if $\Delta R_{\ell\gamma} < 0.1$ ("lepton jets")
- $\mu_R = \mu_F = \mu = m_{\ell\ell}/2$
- $m_{\ell\ell} > 200$ GeV

  $p_{T,\ell^\pm} > 30$ GeV

  $|y_{\ell^\pm}| < 2.5$

$\sqrt{p_{T,\ell^+} p_{T,\ell^-}} > 35$ GeV

Phenomenological results: Fiducial cross section

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sum	1621.06	340.37	−47.046	13.88	−15.71

Impact of corrections:

- NLO QCD: +20%
- NLO EW: -3%
- NNLO QCD: +0.9%

Compatible with sizes of couplings

$\alpha_s \sim 0.1; \qquad \alpha \sim 0.01$

Phenomenological results: Fiducial cross section

LO — Corrections $\sim \mathcal{O}(\alpha_s^i \alpha^j)$

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- Cancellation between partonic channels at NNLO QCD
  - ➔ NNLO QCD corrections slightly suppressed
- Fairly large photon-initiated contribution at LO.

Phenomenological results: Fiducial cross section

LO — Corrections $\sim \mathcal{O}(\alpha_s^i \alpha^j)$

σ[fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$
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- Mixed QCD-EW corrections: **-1%**
  - Much larger than permille corrections expected at $\mathcal{O}(\alpha_s \alpha)$
  - Larger in magnitude than NNLO QCD corrections.
  - About 1/3 of NLO EW corrections.
- Earlier calculation of mixed QCD-EW corrections using massive leptons.

  [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini ('21)].

- Results show same qualitative features.

Phenomenological Results: Fiducial cross section

- Fiducial cross section to NNLO QCD + NLO EW:

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%}\ \text{fb}$$

- Theoretical uncertainty:
  - Vary scale μ by factor of 2 in either direction.
  - Change input scheme for EW parameters to $\alpha(m_Z)$-scheme.
  - Take envelope of these results.
- Mixed QCD-EW corrections (~ -1%) comparable to theoretical uncertainty.
- Including mixed QCD-EW corrections decreases theoretical uncertainty (mainly through decreasing dependence on EW input scheme)

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%}\ \text{fb}$$

(*) Uncertainties from pdfs not included

Phenomenological Results: Cross sections in mass windows

- At high energies, EW corrections dominated by universal Sudakov logarithms.
- Look at fiducial cross section in 4 mass windows:

$$\Phi^{(1)}: \ 200 \text{ GeV} < m_{\ell\ell} < 300 \text{ GeV},$$
$$\Phi^{(2)}: \ 300 \text{ GeV} < m_{\ell\ell} < 500 \text{ GeV},$$
$$\Phi^{(3)}: \ 500 \text{ GeV} < m_{\ell\ell} < 1.5 \text{ TeV},$$
$$\Phi^{(4)}: \ 1.5 \text{ TeV} < m_{\ell\ell} < \infty.$$

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$	$\sigma_{\text{QCD}\times\text{EW}}$
$\Phi^{(1)}$	1169.8	254.3	-30.98	10.18	-10.74	-6.734	$1392.6^{+0.75\%}_{-0\%}$
$\Phi^{(2)}$	368.29	71.91	-11.891	2.85	-4.05	-2.321	$427.1^{+0.41\%}_{-0.02\%}$
$\Phi^{(3)}$	82.08	14.31	-4.094	0.691	-1.01	-0.7137	$91.98^{+0.22\%}_{-0.14\%}$
$\Phi^{(4)} \times 10$	9.107	1.577	-1.124	0.146	-0.206	-0.1946	$9.500^{+0\%}_{-0.97\%}$

Phenomenological Results: Cross sections in mass windows

- NLO EW corrections: ~ 3% in low-mass window, 12% in high-mass window
- QCD-EW corrections: ~1% in low-mass window, 2% in high-mass window
- EW corrections enhanced at high invariant mass, as expected from Sudakov logs.
- Can QCD-EW corrections be described by factorized QCD and EW corrections?

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$	$\sigma_{\text{QCD}\times\text{EW}}$
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Phenomenological Results: Cross sections in mass windows

Factorized approximation:

$$\delta\sigma^{(1,1)}_{\text{fact.}} = \frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \times \frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \times \sigma^{(0,0)}$$

- Very good approximation in high mass bin $m_{\ell\ell} \geq 1.5$ TeV
- Underestimates mixed QCD-EW corrections for all lower bins.
- Factorized approx. provides Sudakov logarithms that are dominant in TeV range.

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$	$\sigma_{\text{QCD}\times\text{EW}}$
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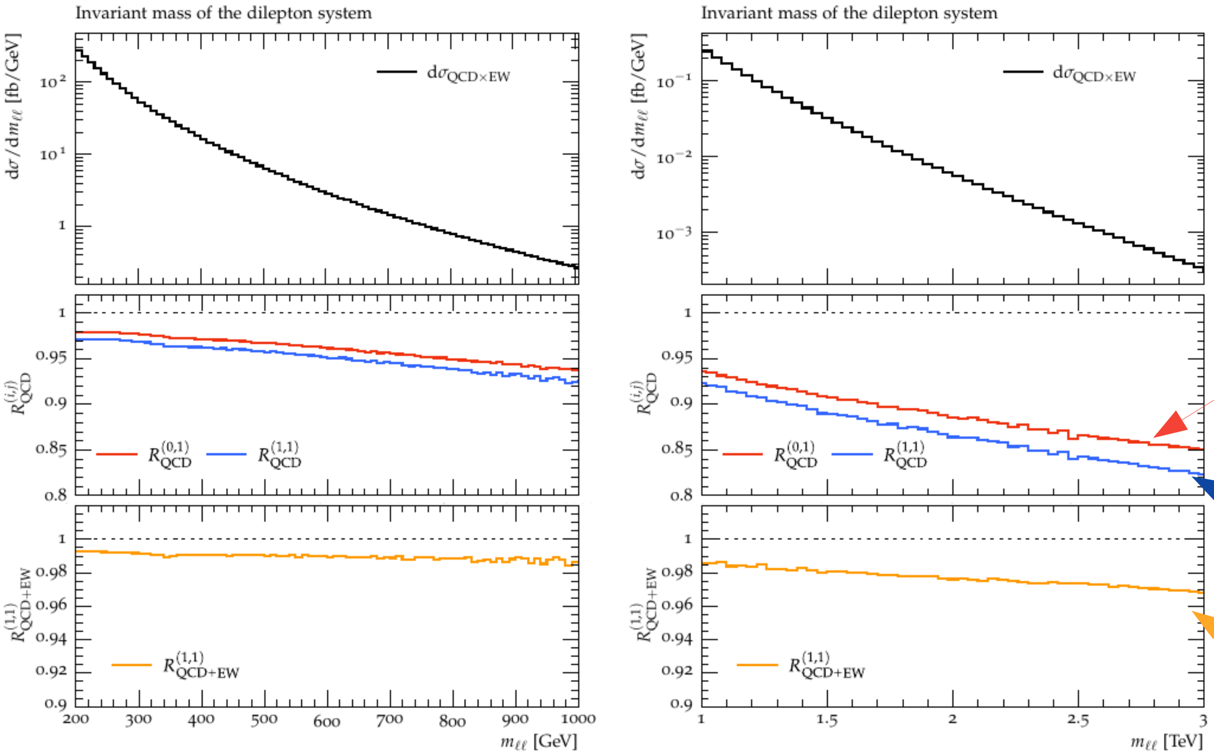
Phenomenological results: Distributions



$$R_{\rm QCD}^{(0,1)} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)}}$$

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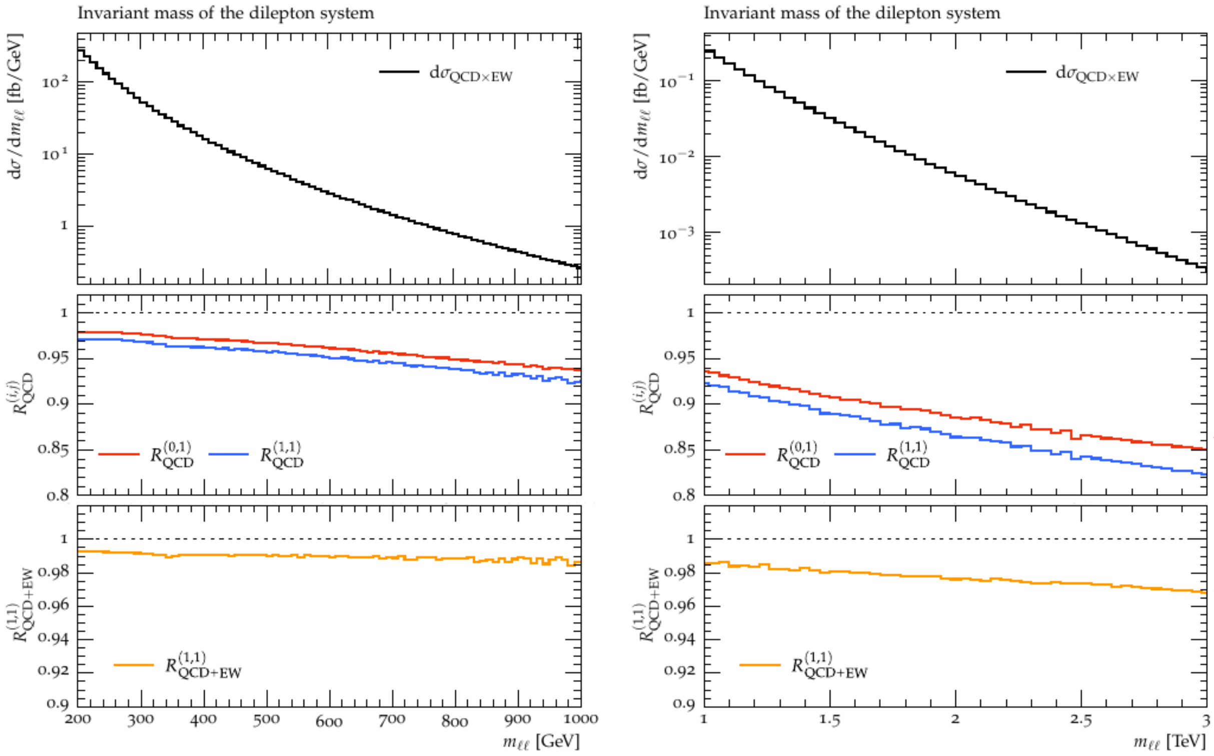
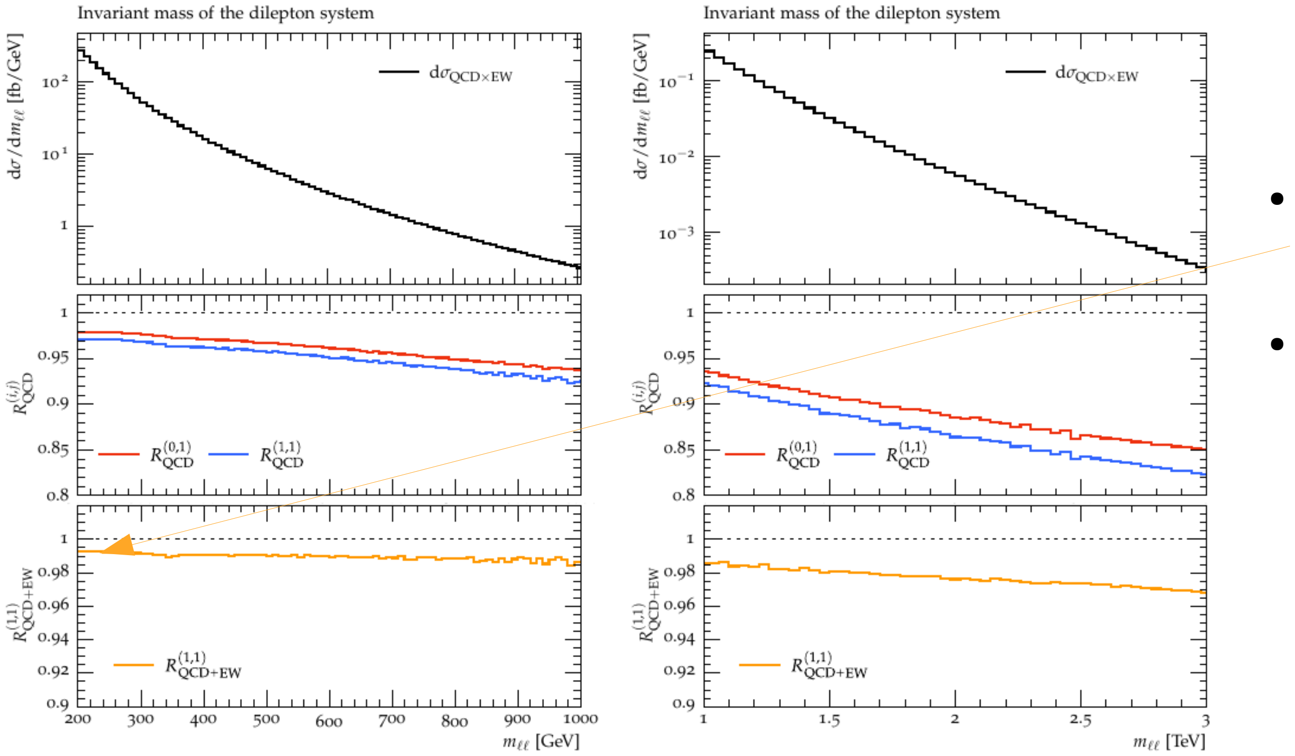
$$R_{\rm QCD+EW}^{(1,1)} = R_{\rm QCD}^{(1,1)}/R_{\rm QCD}^{(0,1)} = \frac{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(1,1)}}{\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)}}$$

Phenomenological results: Distributions



- NLO EW corrections grow substantially with invariant mass
  - -15% at 3 TeV
- Mixed QCD-EW corrections largely follow shape of NLO EW corrections.
- But do have some additional dependence on invariant mass:
  - -3% at 3 TeV
- Consistent with Sudakov logarithms

Phenomenological results: Distributions

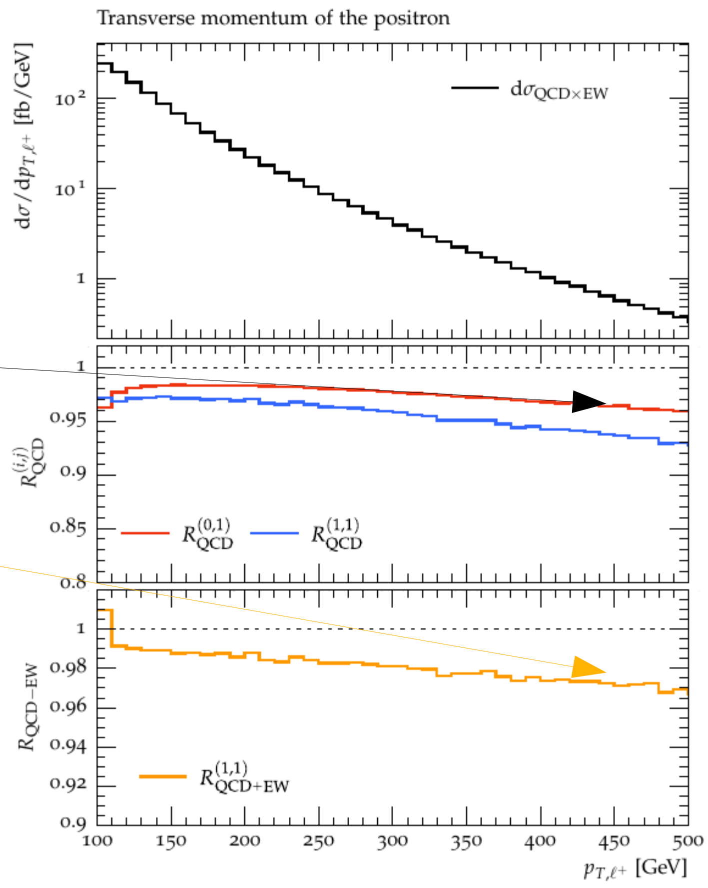


- At $m_{\ell\ell} \simeq 200$ GeV, QCD-EW corrections are ~ -1%
- Relatively large size of mixed QCD-EW corrections to fiducial cross sections not due to Sudakov logarithms!

Phenomenological results: Distributions

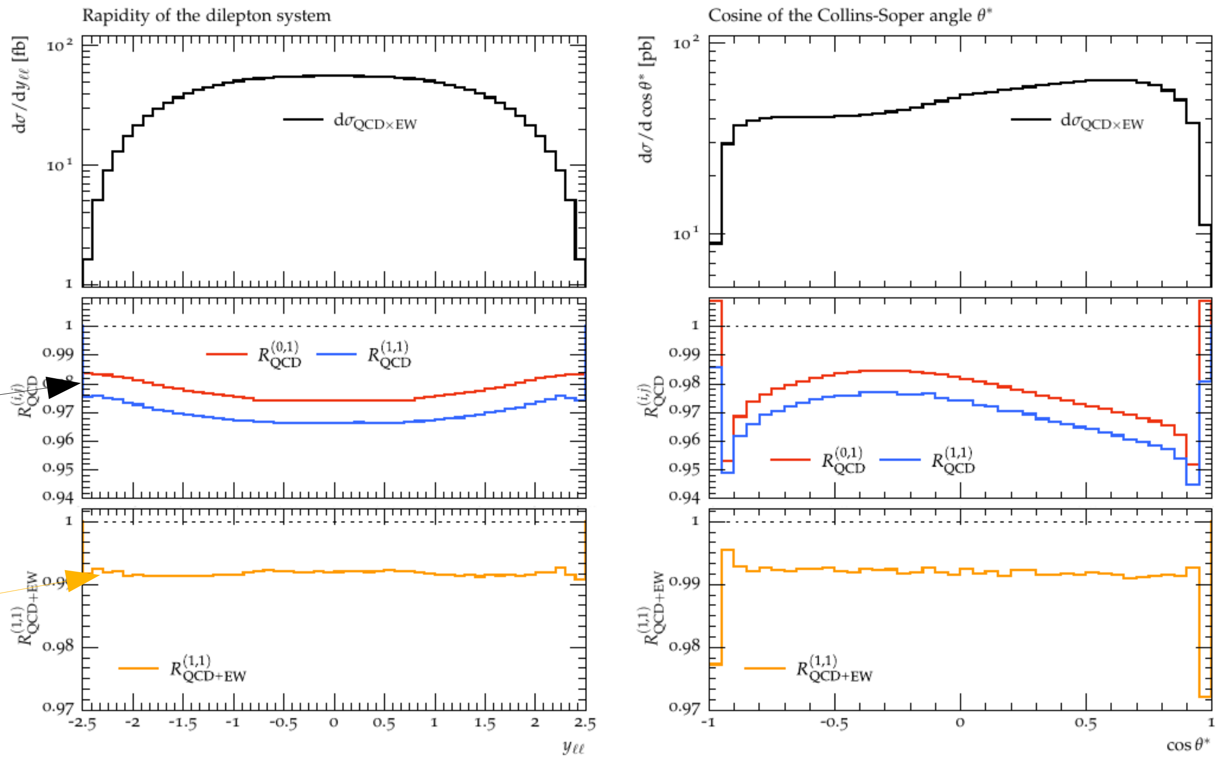
Similar pattern for p_{T,ℓ^+} :

- NLO EW and QCD-EW corrections become more important at high transverse mass.
- QCD-EW corrections have slightly stronger dependence on transverse momentum compared to NLO EW corrections.
- Reach ~ -3% at $p_{T,\ell^+} \simeq 500\ \mathrm{GeV}$



Phenomenological results: Distributions

- NLO EW corrections to angular and rapidity distributions show minor shape changes.
- Mixed QCD-EW corrections very flat.



Conclusions (I)

- Mixed QCD-EW corrections in Drell-Yan production are important for:
  - Precision determination of the W-mass;
  - Searches for NP in the high energy regime.
- Nested soft-collinear subtraction scheme can be extended in a straightforward manner to treat IR singularities appearing in mixed QCD-EW corrections to DY.
- Allowed calculation of mixed QCD-EW corrections to onshell Z and W production.
- Estimated impact on measurement of W-mass at LHC ~ 10 MeV.
  - Strongly cut-dependent.
  - Potentially relevant for target uncertainty of 0.1 per mille.
  - Further investigations needed.

Conclusions (II)

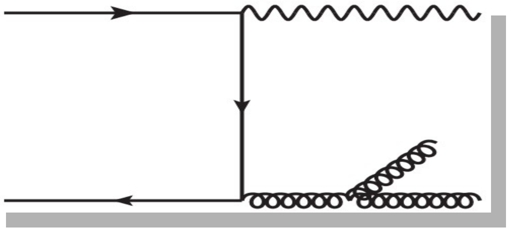
- Calculation of mixed QCD-EW corrections to massless dilepton production:
  - Mixed QCD-EW corrections ~ -1% – larger than naive power counting at $\mathcal{O}(\alpha\alpha_s)$
  - Increase with energy to ~ -3% at $m_{\ell\ell} \sim 3\ \mathrm{TeV}$
  - Well-approximated by factorized QCD and EW corrections in TeV regime.
  - Relatively flat corrections to angular and rapidity distributions.

THANK YOU FOR YOUR ATTENTION

BACKUP SLIDES

IR singularities at NNLO in QCD

Consider double-real emissions in vector boson production: $q\bar{q} \to V + g + g$



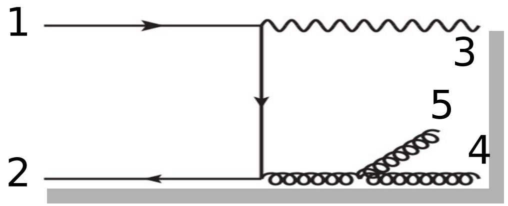
Singularities arise when:

- *Either* gluon or *both* gluons→ soft.
- *Either* gluon or *both* gluons→ collinear to either initial state quark.
- Gluons → collinear to each other.

- Any combination of above – overlapping singularities.
  - Can approach each limit in different ways.
- Need to separate the singularities.
- Multiple approaches: qT, N-jettiness, antennas, STRIPPER, CoLoRFulNNLO, Projection-to-Born, **nested soft-collinear subtraction**, geometric subtraction, local analytic subtraction, unsubraction, Loop-Tree Duality, …

Nested soft-collinear subtractions

- Consider partonic process $q(p_1)\bar{q}(p_2) \to V(p_3)g(p_4)g(p_5)$



- Define

$$F_{LM}(1,2,4,5) = \mathrm{dLips}_V \, |\mathcal{M}(1,2,4,5,V)|^2 \, \mathcal{F}_{\mathrm{kin}}(1,2,4,5,V)$$

$$\Rightarrow \quad 2s \cdot \mathrm{d}\sigma^{\mathrm{RR}} = 1/2! \int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5) \qquad [\mathrm{d}g_i] = \frac{\mathrm{d}^{d-1}p_i}{(2\pi)^d 2E_i}\theta(E_{\mathrm{max}} - E_i)$$

- Overlapping double-soft and single-soft singularities:

$$E_4, E_5 \to 0; \qquad E_4 \to 0; \qquad E_5 \to 0.$$

- Order energies: $E_4 > E_5$ → soft singularities: either double soft or g_5 soft.

Nested soft-collinear subtractions

$$\Rightarrow \quad 2s \cdot \mathrm{d}\sigma^{\mathrm{RR}} = 1/2! \int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5)$$

$$= \int [\mathrm{d}g_4][\mathrm{d}g_5] \theta(E_4 - E_5) F_{LM}(1,2,4,5) \equiv \langle F_{LM}(1,2,4,5) \rangle$$

- Regulate the soft singularities:

$$\langle F_{LM}(1,2,4,5) \rangle = \langle \mathbb{S} F_{LM}(1,2,4,5) \rangle + \langle (I - \mathbb{S}) F_{LM}(1,2,4,5) \rangle$$

$$= \langle \mathbb{S} F_{LM}(1,2,4,5) \rangle + \langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \rangle$$ Double- and single-soft counterterms

$$+ \langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \rangle .$$

Soft-subtracted term – still has (overlapping) collinear singularities

$\mathbb{S}$: Extracts double-soft limit

S_5 : Extracts $E_5 \to 0$ limit

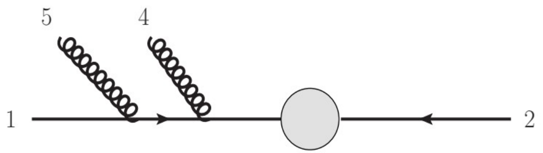
Phase-space partitioning

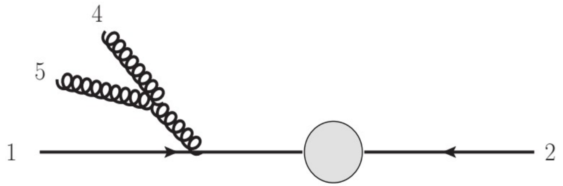
Separate overlapping collinear limits in two stages:

1. Introduce phase-space partitions $1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$

$$C_{42} w^{14,15} = C_{52} w^{14,15} = 0 \;\Rightarrow\; w^{14,15} \text{ contains } C_{41},\; C_{51},\; C_{45}$$

Triple collinear partition

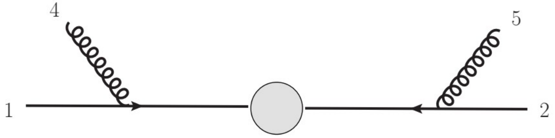




and

$$C_{42} w^{14,25} = C_{51} w^{14,25} = C_{45} w^{14,25} = 0 \;\Rightarrow\; w^{14,25} \text{ contains } C_{41},\; C_{52}$$

Double collinear partition



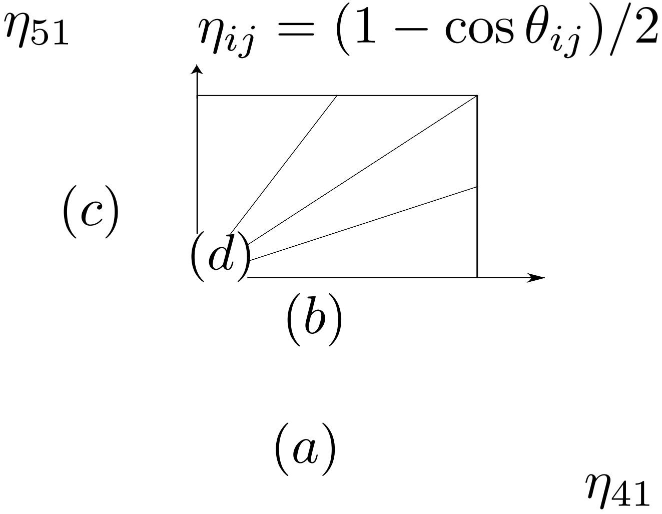
Sector Decomposition

2. Sector decomposition to remove remaining overlapping singularities in triple collinear partitions.

- Define angular ordering to separate singularities.

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$

$$\eta_{ij} = (1 - \cos\theta_{ij})/2$$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Achieved using angular phase-space parametrization [Czakon ('10, '11)].

Removing collinear singularities

Separates collinear limits – subtract iteratively from soft-regulated term

$$\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \rangle =$$

$$\langle F_{LM}^{s_r c_s}(1,2,4,5) \rangle + \langle F_{LM}^{s_r c_t}(1,2,4,5) \rangle + \langle F_{LM}^{s_r c_r}(1,2,4,5) \rangle$$

(Soft-regulated) single and triple collinear counterterms.

Fully subtracted term – finite

Integrate four singular counterterms

$$\langle \mathbb{S} F_{LM}(1,2,4,5) \rangle \ \langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \rangle \ \langle F_{LM}^{s_r c_s}(1,2,4,5) \rangle \ \langle F_{LM}^{s_r c_t}(1,2,4,5) \rangle$$

over unresolved phase space :

- cancel IR poles against loop amplitudes;
- Finite remainder: subtraction counterterm.

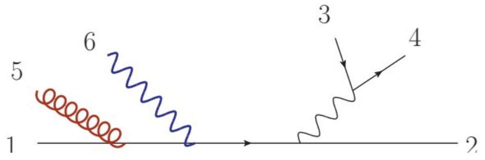
Phase space partitioning for dilepton production

$$q(p_1)\bar{q}(p_2) \to e^-(p_3)e^+(p_4)g(p_5)\gamma(p_6)$$

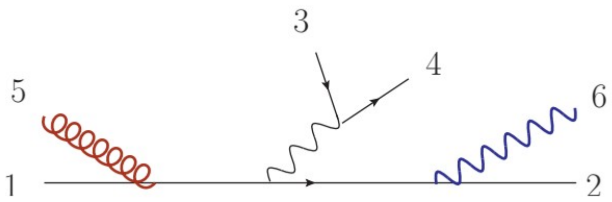
- Partitioning:

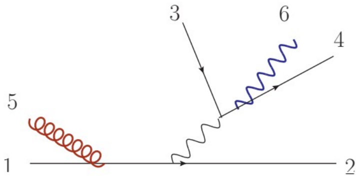
$$1 = w^{15,16} + w^{25,26} + w^{15,26} + w^{16,25} + w^{15,36} + w^{15,46} + w^{25,36} + w^{15,46}$$

- Triple collinear sectors



- Double collinear sectors

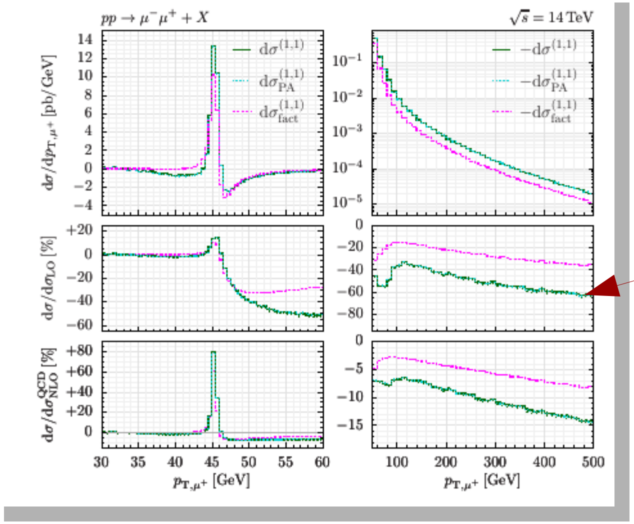




- Additional partitions have only double collinear limits
  - ➢ ~ NLO x NLO – simple!

QCD-EW corrections to dilepton production

- First mixed QCD-EW corrections to dilepton production presented by [Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini ('21)].
- Two-loop amplitudes evaluated with help of semi-analytic method. [cf. Armadillo *et al.* ('22)]
- IR singularities regulated by qT subtractions as implemented in MATRIX.
  [Grazzini, Kallweit, Wiesemann ('17)]



- Results for massive leptons (collinear singularities regulated by mass):
  - Fiducial cross section increased by 0.5% relative to LO.
  - Larger impact at high-pT: -60% correction
  - High invariant mass: correction ~ -1.5%.
  - Factorized approximation works well at Jacobian peak, fails at higher pT.