PHYSICS OF 0νββ DECAY

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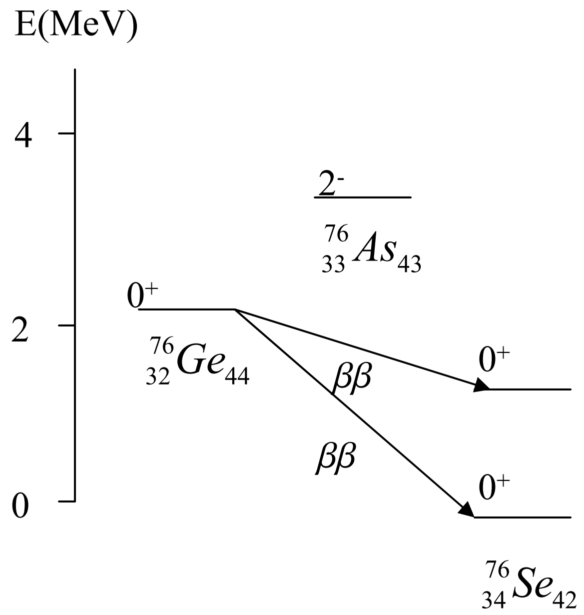
INTRODUCTION

Fundamental Process $0\nu\beta\beta$:

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} $$



Half-life for the process:

$$ \left[T_{1/2}^{0\nu\beta\beta}(0^{+} \rightarrow 0^{+}) \right]^{-1} = G_{0\nu} \left| M^{(0\nu)} \right|^{2} \left| f_{b}(m_{i}, U_{ei}) \right|^{2} $$

Beyond the standard model
(Particle physics)

Matrix elements
(Nuclear physics)

Phase-space factor
(Atomic physics)

Difficult calculation: Three different scales

1. Particle physics
   Transition operator inducing the decay

$$T(p) = H(p) f_b(m_i, U_{ei})$$

neutrino mixing matrix

neutrino masses

2. Nuclear physics
   Matrix elements

$$M^{(0\nu)} = \langle f | H(p) | i \rangle$$

3. Atomic physics
   Phase space factor

$$G_{0\nu} = G_{0\nu}(Q_{\beta\beta}, Z)$$

Charge of daughter nucleus

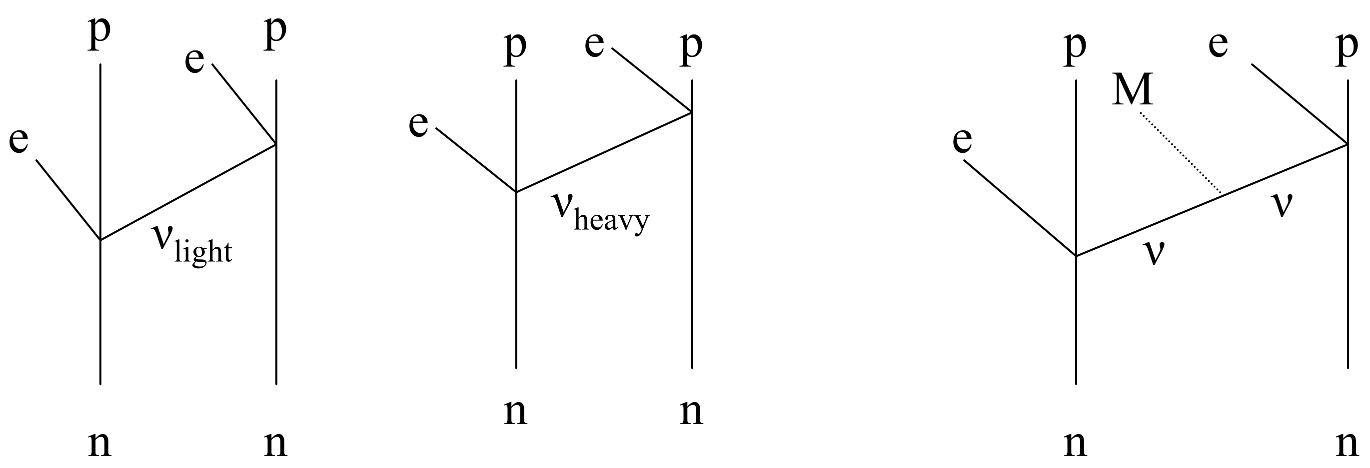
Q-value

$$Q_{\beta\beta} = E_i - E_f - 2m_e c^2$$

1. PARTICLE PHYSICS

The transition operator *T(p)* depends on the model of 0νββ decay.
Three scenarios have been considered [¶,§].

1 2 3



$m_{\nu light} \ll 1 MeV$ $m_{\nu heavy} \gg 1 GeV$

¶ T.Tomoda, Rep. Prog. Phys. 54, 53 (1991).

§ Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

After the discovery of neutrino oscillations, attention has been focused on the first scenario.

Brief review of theory of T(p). Scenario 1: Light neutrinos

Weak interaction Hamiltonian

$$H^{\beta} = \frac{G_F}{\sqrt{2}} \left[\overline{e} \gamma_{\mu} \left(1 - \gamma_5 \right) \nu_{eL} \right] J_L^{\mu\dagger} + h.c.$$

Nucleon current [§]

$$J_L^{\mu\dagger} = \overline{\Psi} \tau^{+} \left[g_V(q^2)\gamma^{\mu} - i g_M(q^2) \frac{\sigma^{\mu\nu}}{2m_p} q_{\nu} - g_A(q^2)\gamma^{\mu}\gamma_5 + g_P(q^2) q^{\mu}\gamma_5 \right] \Psi$$

vector — weak-magnetism — axial vector — induced pseudo-scalar

HOC (weak-magnetism, induced pseudo-scalar)

q^{μ} = momentum transferred from hadrons to leptons

[§] F. Šimkovic *et al.*, loc.cit.

From the weak interaction Hamiltonian, H^{β}, and the weak nucleon current, J^{μ}, one finds the transition operator, $T(p)$, which can be written as

$$T(p) = H(p)\frac{\langle m_\nu \rangle}{m_e} \qquad f_b = \frac{\langle m_\nu \rangle}{m_e}$$

with $p = |\vec{q}|$ and

$$\langle m_\nu \rangle = \sum_{k=ligth} (U_{ek})^2 m_k$$

The average neutrino mass is constrained by atmospheric, solar, reactor and accelerator neutrino oscillation experiments §

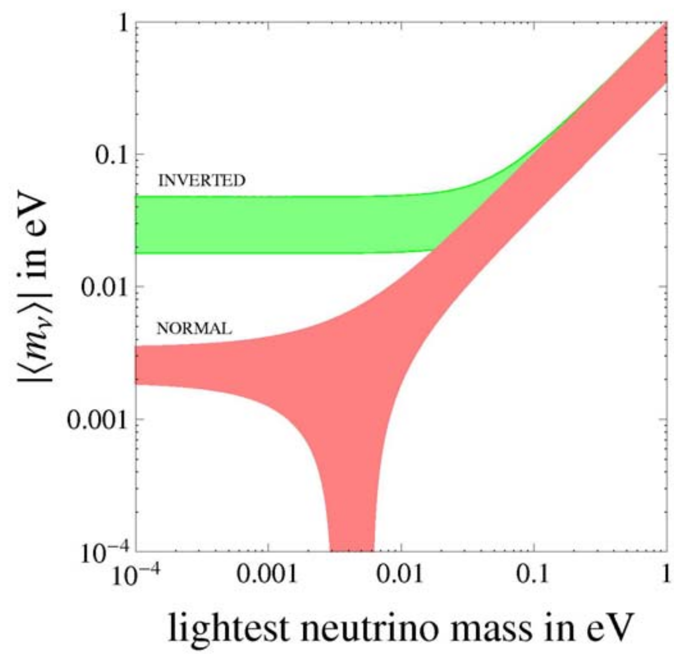
$$\langle m_\nu \rangle = \left| c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3} \right|$$

$$c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}, \varphi_{2,3} = [0, 2\pi]$$

$$\left(m_1^2, m_2^2, m_3^2\right) = \frac{m_1^2 + m_2^2}{2} + \left(-\frac{\delta m^2}{2}, +\frac{\delta m^2}{2}, \pm\Delta m^2\right)$$

$$\sin^2\theta_{12} = 0.312, \sin^2\theta_{13} = 0.016, \sin^2\theta_{23} = 0.466$$

$$\delta m^2 = 7.67\times 10^{-5} eV^2, \Delta m^2 = 2.39\times 10^{-3} eV^2$$



§ G.L. Fogli *et al.*, Phys. Rev. D75, 053001(2007); D78, 033010 (2008).

Brief theory of H(p). Scenario 1

To lowest order and in momentum space, *H(p)*, can be written as

$$H(p) = \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p) \vec{\sigma}_n \bullet \vec{\sigma}_{n'}]$$

Higher order corrections (HOC) induce a tensor term, and modify the Fermi and Gamow-Teller terms, producing an operator [§]

$$H(p) = \tau_n^+ \tau_{n'}^+ [-h^F(p) + h^{GT}(p) \vec{\sigma}_n \bullet \vec{\sigma}_{n'} + h^T(p) S^p_{nn'}]$$

[The general formulation of Tomoda [¶] includes more terms, nine in all, 3GT, 3F, 1T, one pseudoscalar (P) and one recoil (R).]

[§] F. Šimkovic *et al.*, Phys. Rev. C60, 055502 (1999).

[¶] T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

The form factors $h^{F,GT,T}(p)$ are given by:

$$h^{F,GT,T}(p) = v(p)\tilde{h}^{F,GT,T}(p)$$

with

$$v(p) = \frac{2}{\pi}\frac{1}{p\left(p+\tilde{A}\right)}$$

$\tilde{A}$=closure energy=1.12$A^{1/2}$(MeV)

called neutrino "potential", and $\tilde{h}(p)$ listed by Šimkovic *et al.* §

The finite nucleon size (FNS) is taken into account by taking the coupling constants, g_V and g_A, momentum dependent

$$g_V(p^2) = g_V \frac{1}{\left(1+\frac{p^2}{M_V^2}\right)^2}$$

$$g_A(p^2) = g_A \frac{1}{\left(1+\frac{p^2}{M_A^2}\right)^2}$$

$$g_V = 1; M_V^2 = 0.71\left(GeV/c^2\right)^2$$

$$g_A = 1.25; M_A^2 = 1.09\left(GeV/c^2\right)^2$$

Short range correlations (SRC) are taken into account by convoluting the "potential" *v(p)* with the Jastrow function *j(p)*

$$u(p) = \int v(p-p')j(p')dp'$$

§ F. Šimkovic, *loc.cit.*

[Note: Tomoda's form factors are slightly different from Šimkovic. His formulation is in coordinate space, i.e. the form factors are the Fourier transform of those given above.]

Brief theory of $T_h(p)$. Scenario 2: Heavy neutrinos [§]

$$T_h(p) = H_h(p) m_e \left\langle m_\nu^{-1} \right\rangle \qquad f_b = m_e \left\langle \frac{1}{m_\nu} \right\rangle$$

$$\left\langle m_\nu^{-1} \right\rangle = \sum_{k=heavy} \left(U_{ek} \right)^2 \frac{1}{m_k}$$

Brief theory of $H_h(p)$

Same as for scenario 1 but with neutrino "potential"

$$v(p) = \frac{2}{\pi} \frac{1}{m_e^2}$$

[§ Constraints on the average inverse heavy neutrino mass are model dependent. For a recent constraint, see V. Tello *et al*, arXiv:1011.3522v1 [hep-ph] 15 Nov 2010.]

2. NUCLEAR PHYSICS

Calculation of the "nuclear matrix elements" $M^{(0\nu)}$

$$\tilde{M}^{(0\nu)} = g_A^2 M^{(0\nu)}$$

$$M^{(0\nu)} \equiv M_{GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_F^{(0\nu)} + M_T^{(0\nu)}$$

Calculations up to 2008:

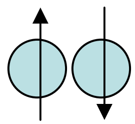
1. Quasi-particle random phase approximation (QRPA).
   Limitations: Results depend on fine-tuning of the interaction, especially near the spherical-deformed transition, for example ^{150}Nd.
2. Shell model (SM).
   Limitations: Cannot address nuclei with many particles in the valence shells, for example ^{150}Nd, due to the exploding size of the Hamiltonian matrices ($>10^9$).

Recent advances >2009:

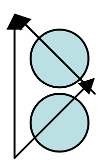
3. Development of a program to compute $0\nu\beta\beta$ (and $2\nu\beta\beta$) nuclear matrix elements in the closure approximation within the framework of the microscopic Interacting Boson Model (IBM-2). This approach can be used for any nucleus.

THE INTERACTING BOSON MODEL/ THE INTERACTING BOSON FERMION MODEL

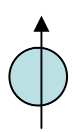
A model of even-even nuclei in terms of correlated pairs of protons and neutrons with angular momentum J=0,2 treated as bosons (s_π,d_π and s_ν,d_ν), called IBM-2 [¶].
A model of odd-even or odd-odd nuclei in terms of correlated pairs (bosons) and unpaired particles, $a_{j\pi}$ and $a_{j\nu}$, (fermions), called IBFM-2 [§].



J=0 s-boson



J=2 d-boson



Unpaired fermions

[¶] F. Iachello and A. Arima, *The Interacting Boson Model*, Cambridge University Press, 1987.

[§] F.Iachello and P. Van Isacker, *The Interacting Boson Fermion Model*, Cambridge University Press, 1991.

EVALUATION OF MATRIX ELEMENTS IN IBM-2 ¶

All matrix elements, F, GT and T, can be calculated at once using the compact expression:

$$V^{(\lambda)}_{s_1,s_2} = \frac{1}{2}\sum_{n,n'} \tau^+_n \tau^+_{n'} \left[\Sigma^{(s_1)}_n \times \Sigma^{(s_2)}_{n'}\right]^{(\lambda)} \cdot V(r_{nn'}) C^{(\lambda)}(\Omega_{nn'})$$

$$\lambda = 0, s_1 = s_2 = 0(F)$$
$$\lambda = 0, s_1 = s_2 = 1(GT)$$
$$\lambda = 2, s_1 = s_2 = 1(T)$$

In second quantized form:

$$V^{(\lambda)}_{s_1,s_2} = -\frac{1}{4}\sum_{j_1 j_2}\sum_{j'_1 j'_2}\sum_{J} (-1)^J \sqrt{1+(-1)^J \delta_{j_1 j_2}} \sqrt{1+(-1)^J \delta_{j'_1 j'_2}}$$
$$\times G^{(\lambda)}_{s_1 s_2}(j_1 j_2 j'_1 j'_2; J) \left[\left(\pi^\dagger_{j_1} \times \pi^\dagger_{j_2}\right)^{(J)} \cdot \left(\tilde{\nu}_{j'_1} \times \tilde{\nu}_{j'_2}\right)^{(J)}\right]$$

Creates a pair of protons with angular momentum J

Annihilates a pair of neutrons with angular momentum J

¶ J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).

The fermion operator V is then mapped onto the boson space by using:

$$\left(\pi_j^\dagger \times \pi_j^\dagger\right)^{(0)} \mapsto A_\pi(j) s_\pi^\dagger$$

$$\left(\pi_j^\dagger \times \pi_{j'}^\dagger\right)_M^{(2)} \mapsto B_\pi(j, j') d_{\pi,M}^\dagger$$

$$V_{s_1 s_2}^{(\lambda)} \mapsto -\frac{1}{2} \sum_{j_1} \sum_{j'_1} G_{s_1 s_2}^{(\lambda)}\left(j_1 j_1 j'_1\, j'_1; 0\right) A_\pi(j_1) A_\nu(j'_1) s_\pi^\dagger \cdot \tilde{s}_\nu$$

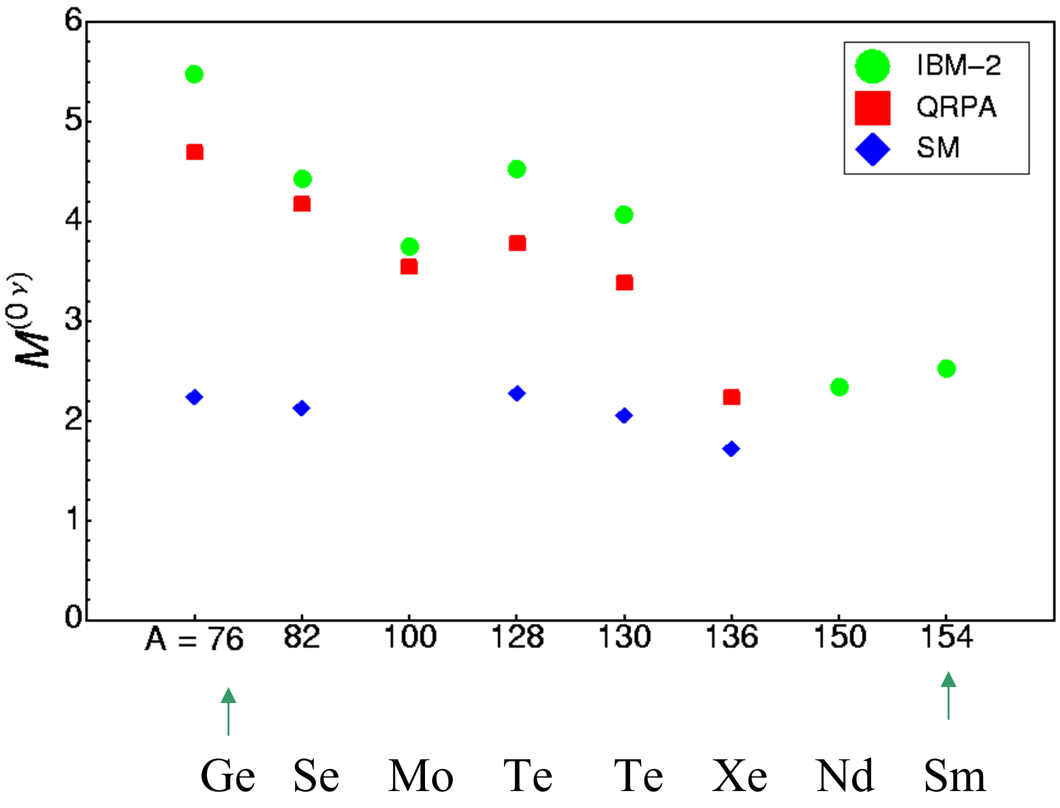
$$-\frac{1}{4} \sum_{j_1 j_2} \sum_{j'_1 j'_2} \sqrt{1+\delta_{j_1 j_2}} \sqrt{1+\delta_{j'_1 j'_2}}\, G_{s_1 s_2}^{(\lambda)}(j_1 j_2 j'_1\, j'_2; 2) B_\pi(j_1, j_2) B_\nu(j'_1, j'_2) d_\pi^\dagger \cdot \tilde{d}_\nu$$

The coefficients A, B are obtained by means of the so-called OAI mapping procedure §

Matrix elements of the mapped operators are then evaluated with realistic wave functions of the initial and final nuclei either taken from the literature, when available, or obtained from a fit to the observed energies and other properties.

§ T. Otsuka, A. Arima and F. Iachello, Nucl. Phys. A309, 1 (1978).

RESULTS FOR THE MATRIX ELEMENTS (2009)



IBM-2 from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009), g_A=1.25, Jastrow SRC.
QRPA from F. Šimkovic, A Faessler, V. Rodin, P. Vogel, and J. Engel, Phys. Rev. C77, 045503 (2008), with g_A=1.25, Jastrow SRC.
SM from E. Caurier, J. Menendez, F. Nowacki, and A. Poves, Phys. Rev. Lett. 100, 052503 (2008).

Matrix elements in dimensionless units.

ERROR ANALYSIS (IBM-2)

Estimated sensitivity to input parameter changes:

1.	Single-particle energies [¶,§]	10%
2.	Strength of surface delta interaction	5%
3.	Oscillator parameter	5%
4.	Closure energy	5%

Estimated sensitivity to model assumptions:

1.	Truncation to S, D space	1% (spherical nuclei)-10% (deformed nuclei)
2.	Isospin purity	1%(GT)-20%(F)-1%(T)

Estimated sensitivity to operator assumptions:

1.	Form of the operator	5%
2.	Finite nuclear size (FNS)	2%
3.	Short range correlations (SRC)	2%

[¶] This point has been emphasized by J. Suhonen and O. Civitarese, Phys. Lett. B668, 277 (2008).

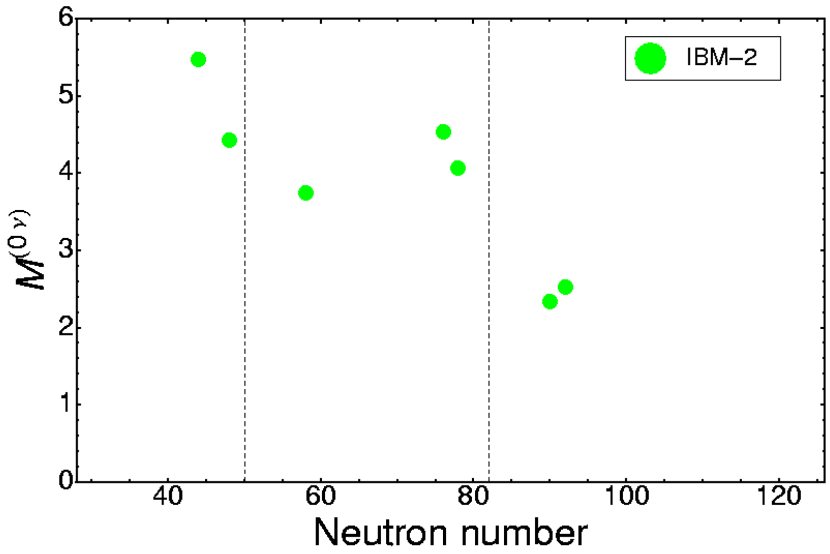
[§] New experiments are being done to check the single particle levels in Ge, Se and Te, J.P. Schiffer *et al.*, Phys. Rev. Lett. **100**, 112501 (2008).

MAIN FEATURES OF IBM-2 CALCULATIONS

1. Shell effects

Neutron number dependence

This is a major effect: The matrix elements are small at the closed shells



2. Deformation effects

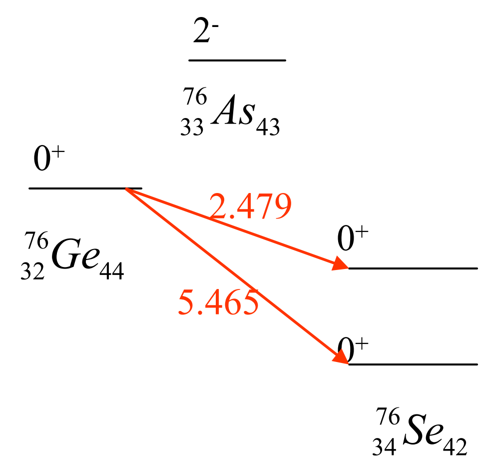
Estimated from comparison between a GS calculation (only S pairs) and a full IBM-2 calculation (S and D pairs).

Deformation effects always decrease the matrix elements:

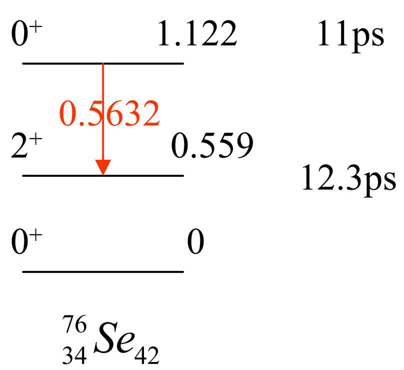
^{76}Ge	-19%
^{128}Te	-26%
^{154}Sm	-32%

MATRIX ELEMENTS TO FIRST EXCITED 0^+ STATE

^{76}Ge	2.479
^{82}Se	1.247
^{100}Mo	0.419
^{128}Te	3.243
^{130}Te	3.090
^{150}Nd	0.395
^{154}Sm	0.021

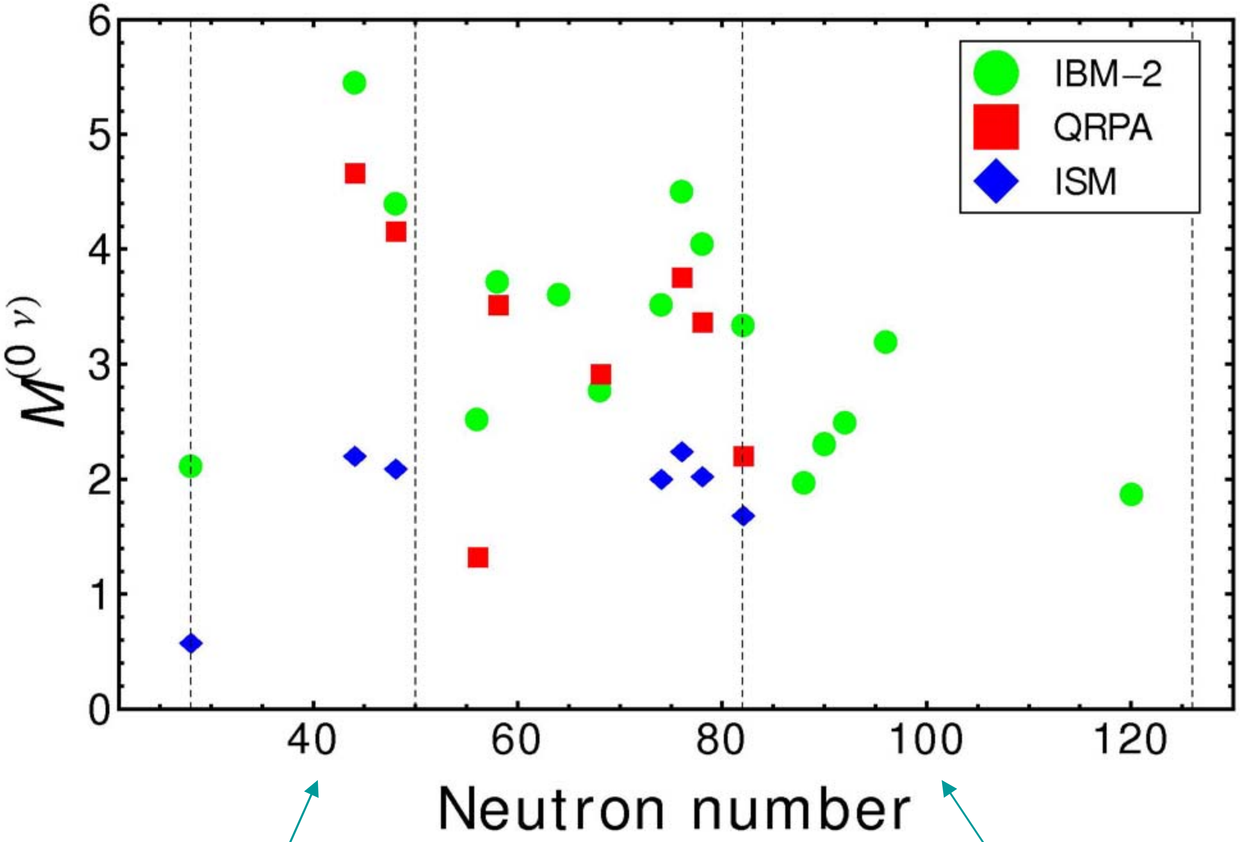


[In some cases, the matrix elements are large. Although the kinematical factor hinders the decay to the excited state, large matrix elements offer the possibility of a direct detection, by looking at the γ-ray de-exciting the 0^+ level.]



RECENT IBM-2 RESULTS (FEB 2011)

(2011) ^{48}Ca ^{96}Zr ^{110}Pd ^{116}Cd ^{124}Sn ^{136}Xe ^{148}Nd ^{160}Gd ^{198}Pt



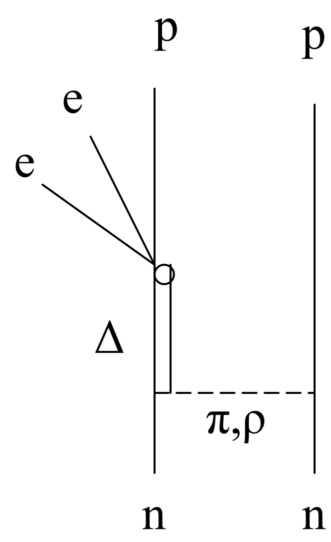
(2009) ^{76}Ge ^{82}Se ^{100}Mo ^{128}Te ^{130}Te ^{150}Nd ^{154}Sm

THE NEXT IMPORTANT PROBLEM: RENORMALIZATION OF G_A

After agreeing on the nuclear matrix elements, one should consider the next important problem, i.e.,
Renormalization of the axial vector coupling constant g_A in nuclei.

A well know problem for single β decay where $g_{A, eff} \sim 0.7 g_A$

A crucial problem for extraction of the neutrino mass.
g_A appears to the fourth power in the half-life!



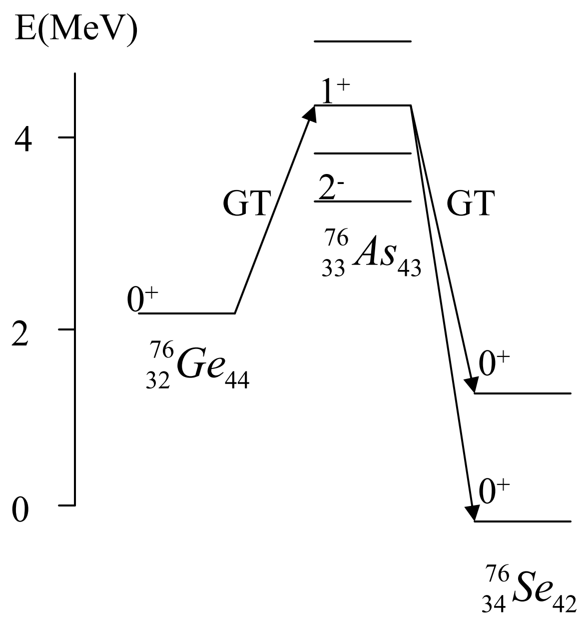
Origin of the renormalization:

1. Limited model space
2. Missing hadronic degrees of freedom, Δ,…

This is a difficult problem to solve: for case 1 we are limited by the size of the matrices (>10^9); for case 2 we are limited by a detailed knowledge of the decay process.
It can only be solved indirectly (by studying 2νββ).

Fundamental Process $2\nu\beta\beta$:

$${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+2}Y_{N-2} + 2e^{-} + 2\overline{\nu}$$



We have done a calculation of 2νββ in the closure approximation and find a renormalization of $g_{A,eff} \sim 0.7 g_A$.
However, the closure approximation may not be good for 2νββ (only a selected number of states contributes to the decay). The average neutrino momentum is of the order of 10MeV. We have therefore started a full scale calculation.
Also, the renormalization effects could be different in 0νββ than in 2νββ.

[The calculation is similar to that of 0νββ except that the neutrino "potential" is different.]

$$v^{(2\nu)}(p) = \frac{\delta(p)}{p^2}$$

3. ATOMIC PHYSICS

For an extraction of the neutrino mass and for estimates of the half-life we also need the phase-space factor $G_{0\nu}$. A general relativistic formulation was given by Tomoda ¶ and results for selected cases tabulated. Also, a calculation of phase-space factors is reported in the book of Boehm and Vogel § where a complete tabulation is given. The two calculations differ by a factor of 4, but this is due to a different normalization of the NME. Once this factor is included, the two calculations agree.

$$G_{0\nu} = \frac{F_{11}^{(0)}}{4R^2}$$

$$R = 1.2\,A^{1/3}\,fm$$

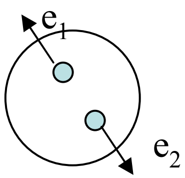
$$F_{11}^{(0)} = F_{11}^{(0)}\left(Q_{\beta\beta}, Z_d\right)$$

[Also some calculations use 1.1 instead of 1.2.]

¶ T. Tomoda, *loc.cit.*

§ F. Bohm and P. Vogel, *Physics of massive neutrinos*, Cambridge University Press, 1987.

Brief review of theory of $F_{11}^{(0)}$



$$F_{11}^{(0)} \propto \left|\psi_{e_1}(0)\psi_{e_2}(0)\right|^2$$

Scattering electron wave functions at the nucleus

Non-relativistic: $\left|\psi(0)\right|^2 = \dfrac{2\pi y}{1-e^{-2\pi y}}$ $\quad y = \dfrac{(Z\alpha)}{(v/c)}$

Relativistic: $\left|\psi(0)\right|^2$ diverges

Regularization: uniform charge distribution with $R = 1.2A^{1/3}(fm)$

Dependence on Z: $\approx (Z\alpha)^{\beta}, \beta \geq 3$

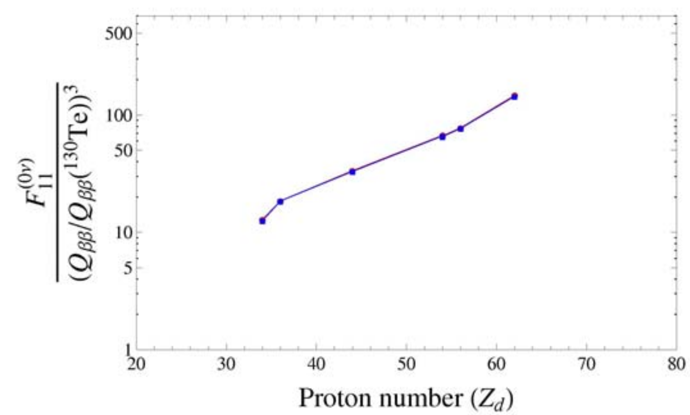
Tomoda ¶ solved the Dirac equation numerically for a uniform distribution

Simple parametrization of Tomoda's results

$$F_{11}^{(0)} = C\left(\frac{Q_{\beta\beta}}{Q_{\beta\beta(^{130}Te)}}\right)^3 \left(\frac{Z}{54}\right)^{\beta}$$

$$C = 66(10^{-13}y^{-1}fm^2), \beta = 3-4$$

[For ^{150}Nd decay, the wave function is already highly relativistic, 62/137~0.45]



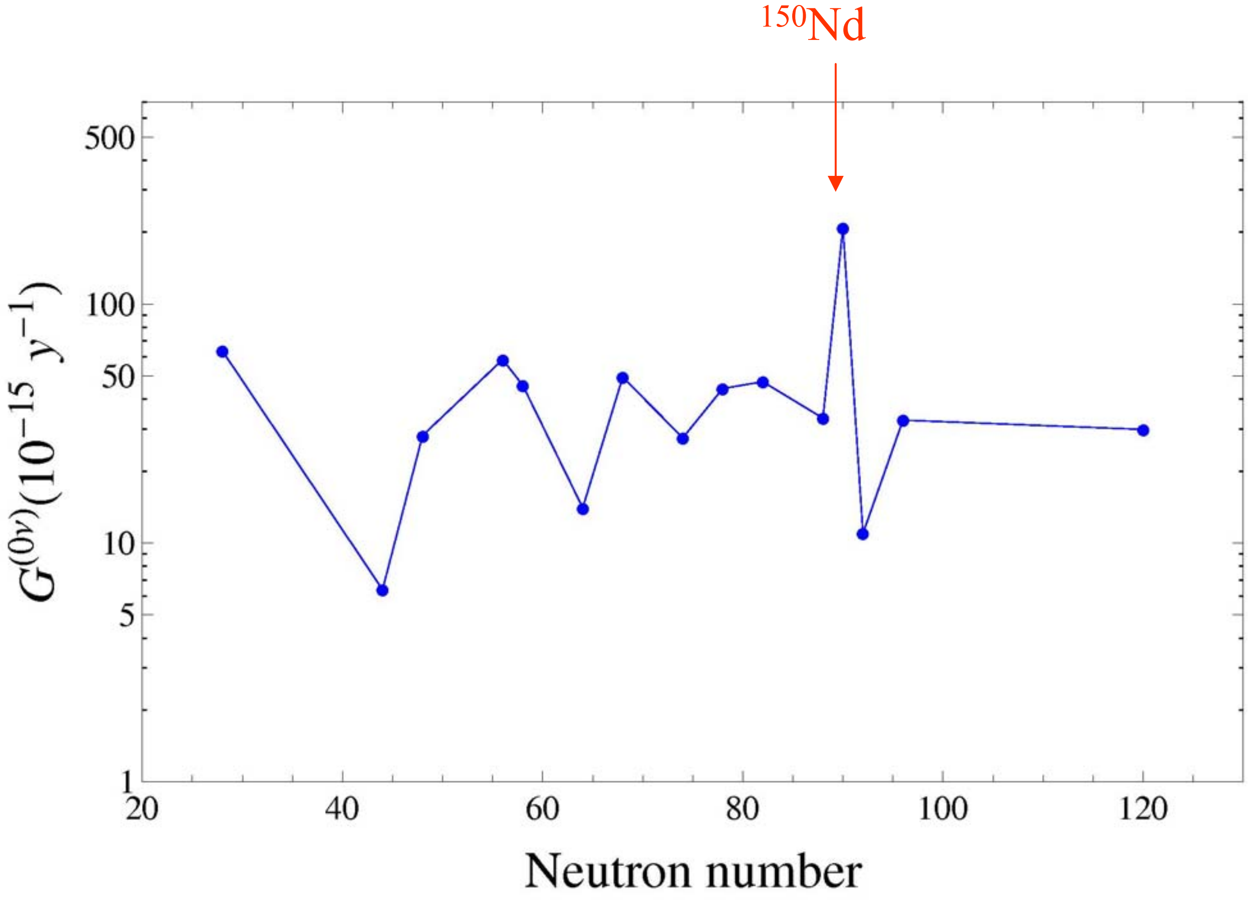
[Because of the complex nature of this calculation and of the resulting strong dependence on Z we are doing a new and independent calculation of $F_{11}^{(0)}$, including finite size and electron screening. Expected to be ready by June 2011.]

¶ T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).

PHASE SPACE FACTORS

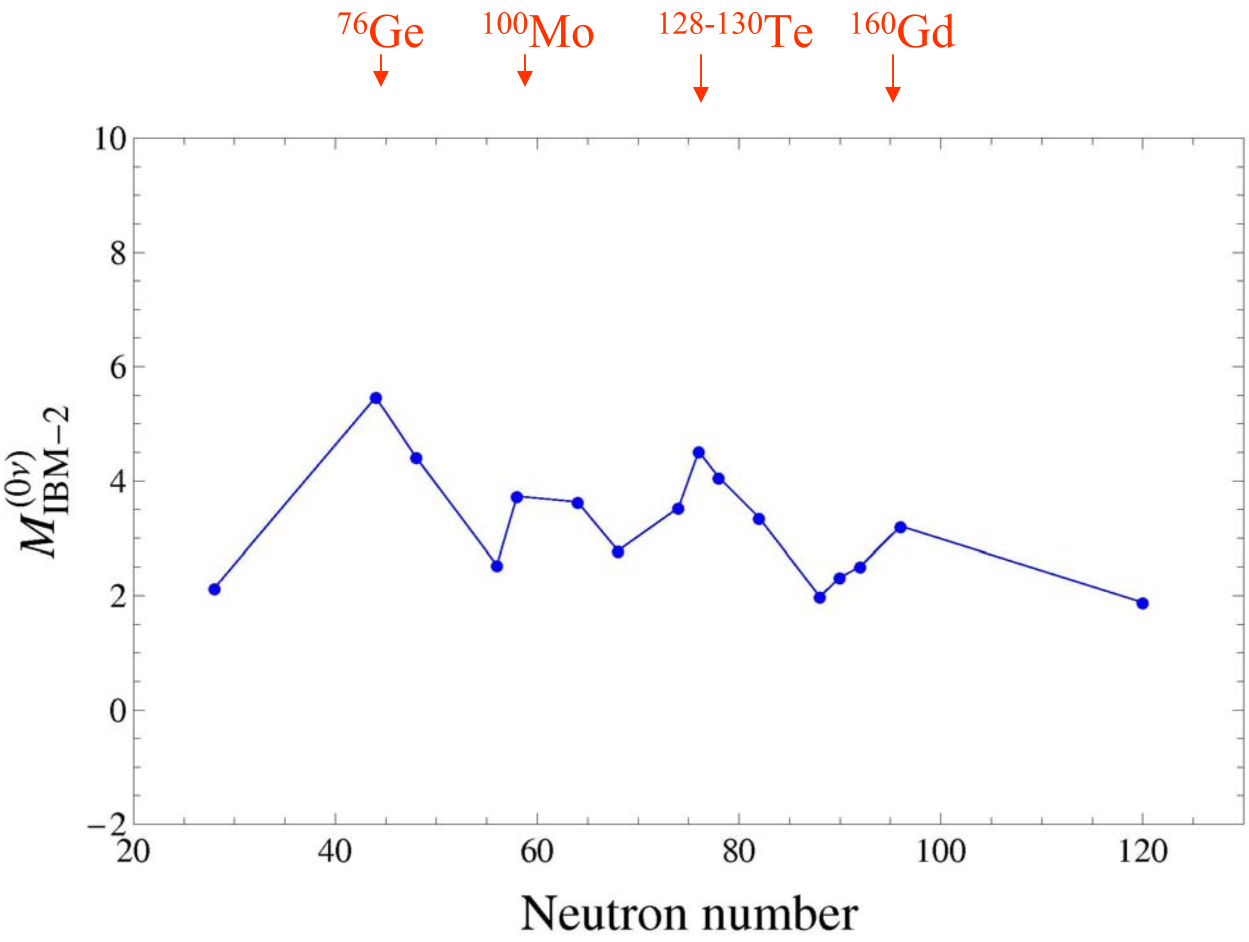
$$G_{0\nu} = \frac{F_{11}^{(0)}}{4R^2}$$

$$R = 1.2A^{1/3}\,fm$$



From F. Bohm and P. Vogel, *loc. cit.*

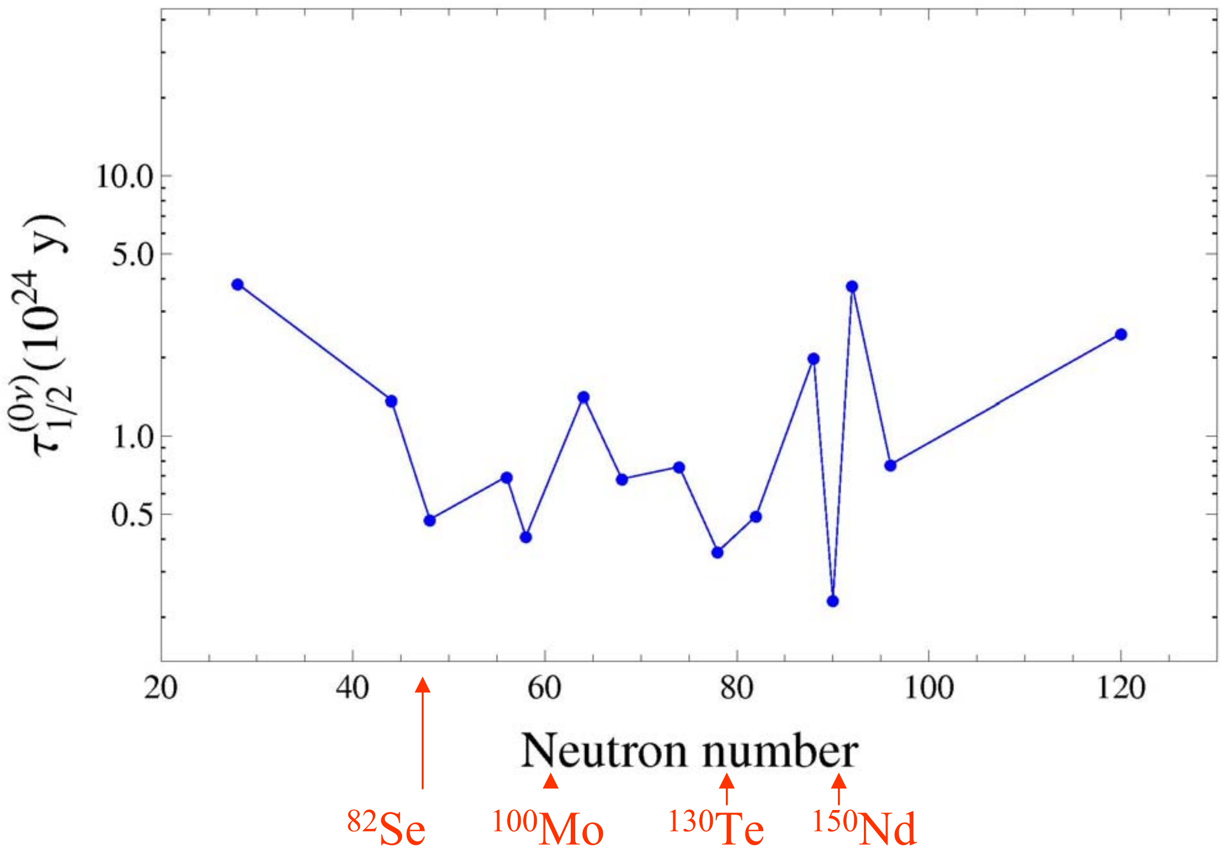
NUCLEAR MATRIX ELEMENTS (IBM-2)



From J. Barea and F. Iachello, *loc. cit.* and to be published.

[Estimated error of the IBM-2 calculation 25%, except for ^{48}Ca where the estimated error is 50%. QRPA calculation is within error of IBM-2. SM is a factor of two smaller, but its dependence on N is exactly the same as in IBM-2.]

FINAL RESULTS FOR HALF-LIFE



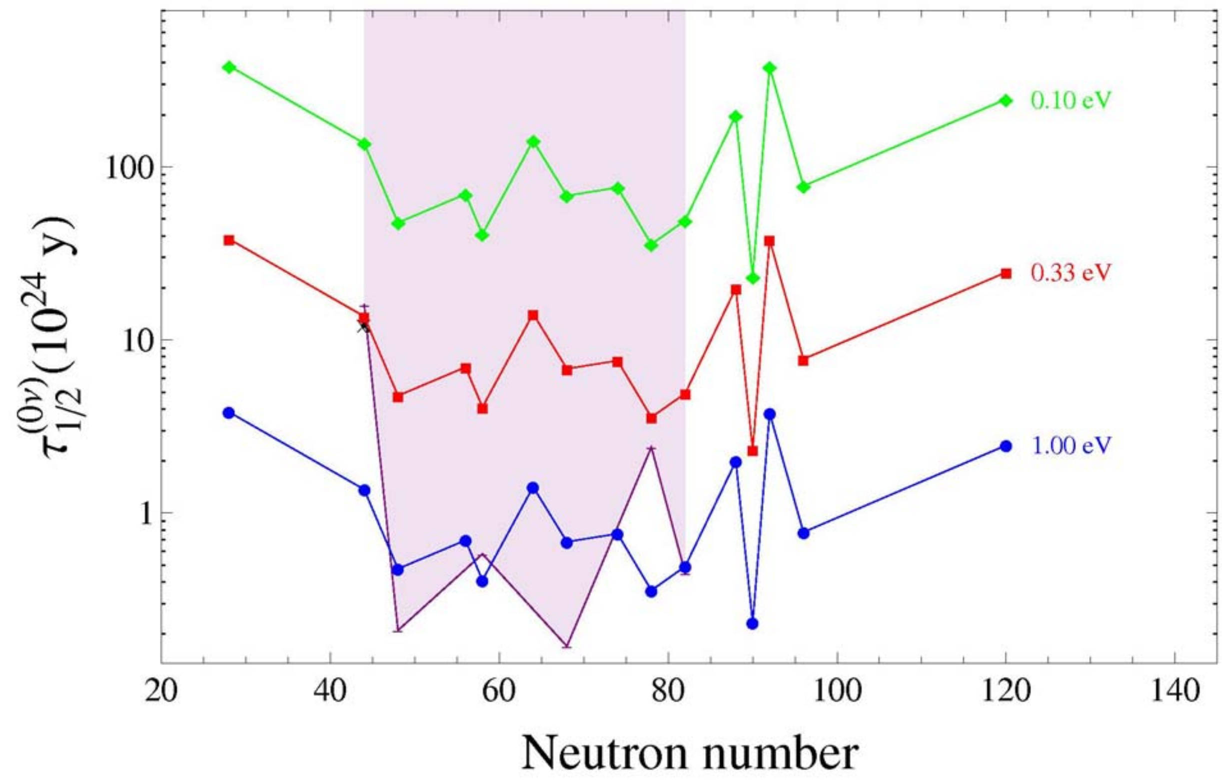
Neutrino mass
m_ν= 1 eV

g_A=1.25

Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009) and to be published.
Phase space factors from F. Boehm and P. Vogel, *loc. cit.*

LIMITS ON NEUTRINO MASS

$$\left[\tau_{1/2}^{(0\nu)}\right]^{-1} = \frac{F_{11}^{(0)}}{4R^2}\left|M^{(0\nu)}\right|^2\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2$$

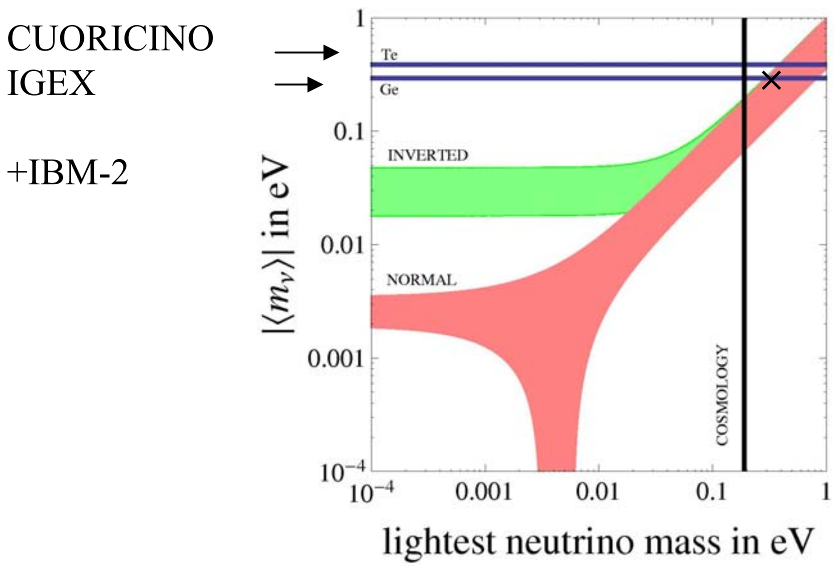


Neutrino mass

Theory: Nuclear matrix elements from J. Barea and F. Iachello, Phys. Rev. C79, 044301 (2009).
Phase space factors from T. Tomoda, Rep. Prog. Phys. 54, 53 (1991).
Experimental upper limits: from a compilation of A. Barabash, arXiv:hep-ex/0608054v1 23 Aug 2006.
Ge [IGEX], Se and Mo [NEMO-3], Cd [SOLOTVINO], Te [CUORICINO], Xe [DAMA].
× : from H.V. Klapdor-Kleingrothaus *et al.*, Phys. Lett. B586, 198 (2004).

CONCLUSIONS

- A new program (IBM-2) has been developed to calculate 0νββ-light,-heavy, 0νββM nuclear matrix elements in the closure approximation. The calculation for 0νββ-light in all nuclei of interest has been completed (February 2011). Matrix elements to the first excited 0^+ states have been calculated.
- A novel calculation of the phase space factors is in progress. This includes the angular correlation between the electrons and the single electron spectrum.
- A simultaneous calculation of 2νββ (not discussed here) has also been completed and will be published soon. This calculation is of importance also for 0νββ in order to estimate the background at the energy of the expected signal.
- The current limits on neutrino mass using IBM-2 matrix elements for ^{76}Ge and ^{130}Te and experimental results from IGEX and CUORICINO are given in the figure:



Cosmological constraint from G.L. Fogli, E. Lisi, *et al, loc.cit.*

$$\Sigma = m_1 + m_2 + m_3 < 0.19 eV$$

× H.V. Klapdor *et al, loc. cit.*