Neutrino Mass Models:

how could they be constrained by results from neutrino oscillations?

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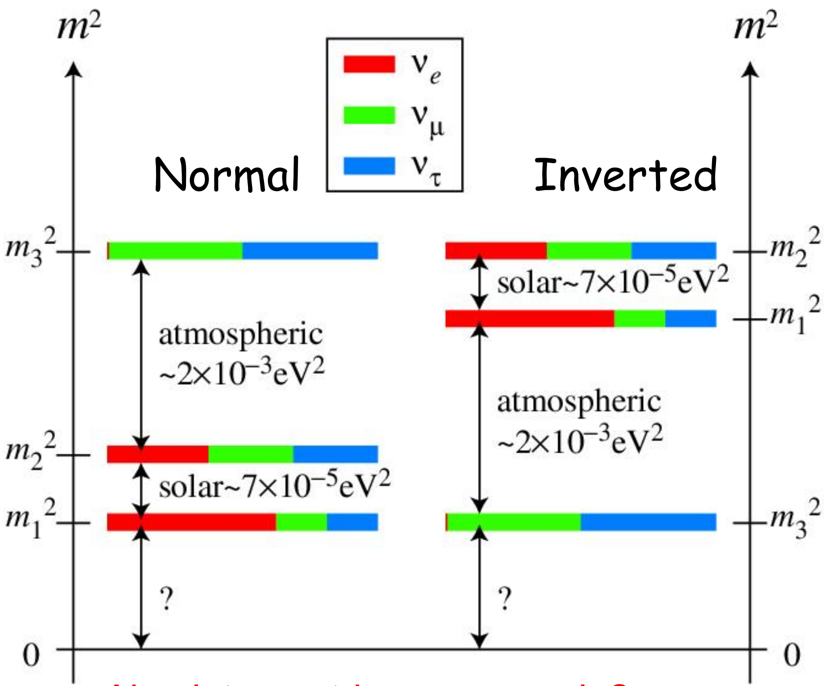
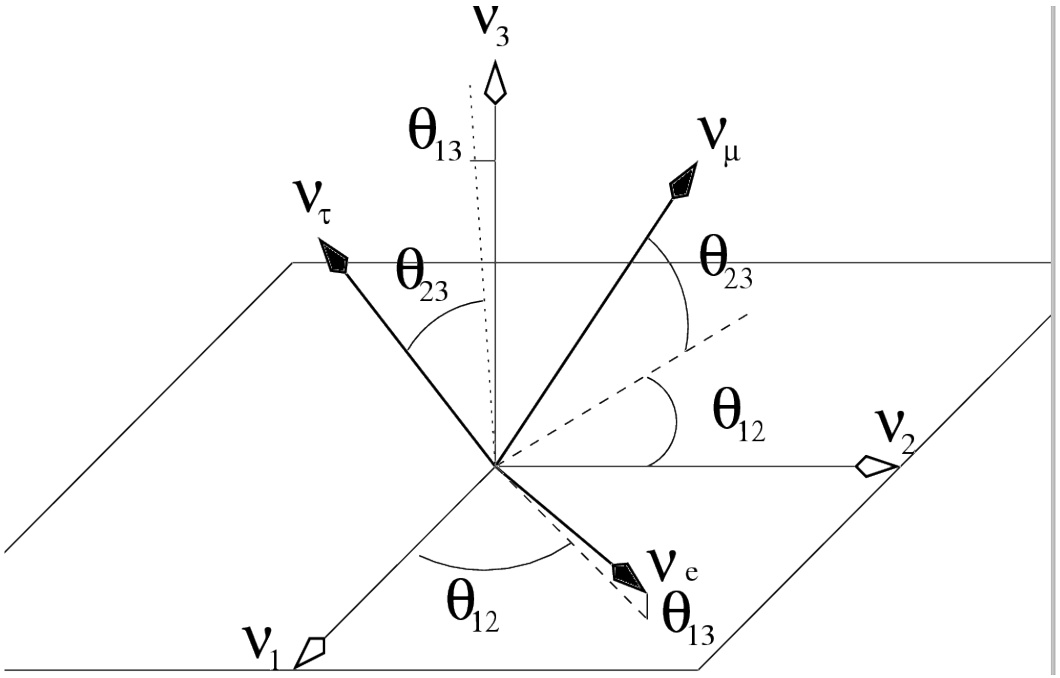
Three Neutrino Mass and Mixing





Absolute neutrino mass scale?

$$\Delta m^2_{21} = 7.59 \pm 0.20 \ \binom{+0.61}{-0.69} \times 10^{-5} \ \text{eV}^2$$

$$\Delta m^2_{31} = \begin{cases} -2.36 \pm 0.11 \ (\pm 0.37) \times 10^{-3} \ \text{eV}^2 \\ +2.46 \pm 0.12 \ (\pm 0.37) \times 10^{-3} \ \text{eV}^2 \end{cases}$$

Gonzalez-Garcia, Maltoni, Salvado arXiv:1001.4524

$$\theta_{12} = 34.4 \pm 1.0 \ \binom{+3.2}{-2.9}^\circ$$

$$\theta_{23} = 42.8 \, ^{+4.7}_{-2.9} \ \binom{+10.7}{-7.3}^\circ$$

$$\theta_{13} = 5.6 \, ^{+3.0}_{-2.7} \ (\leq 12.5)^\circ$$

Large solar and atmospheric angles have been **measured**

Small reactor angle is only **inferred**

some leading order approximations of interest

Feruglio

BIMAXIMAL [BM]

Altarelli, Feruglio, Merlo

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \quad \sin^2\vartheta_{13}^0 = 0 \quad \sin^2\vartheta_{12}^0 = \frac{1}{2}$$

Common to all these approaches

$$\vartheta_{12}^0 = 45^0$$

requires a correction of $O(\vartheta_C)$

$\vartheta_{13} \approx O(\vartheta_C)$ expected

[related to Quark-Lepton complementarity ? [Smirnov; Raidal; Minakata and Smirnov 2004]

$\vartheta_{12}+\vartheta_C \approx \pi/4$]

TRIBIMAXIMAL [TB]

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \quad \sin^2\vartheta_{13}^0 = 0 \quad \sin^2\vartheta_{12}^0 = \frac{1}{3}$$

$$\vartheta_{12}^0 = 35.26^0$$

"GOLDEN RATIO" [GR]

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \quad \sin^2\vartheta_{13}^0 = 0 \quad \tan\vartheta_{12}^0 = \frac{1}{\phi}$$

$$\phi \equiv \frac{1+\sqrt{5}}{2} \qquad \vartheta_{12}^0 = 31.72^0$$

[also $\cos\vartheta_{12} = \varphi/2$ $\vartheta_{12} = 36^0$ Rodejohann 08105239]

agreement of ϑ_{12} suggests that only tiny corrections $[O(\vartheta_C^2)]$ are tolerated

$\vartheta_{13} \approx O(\vartheta_C^2)$ expected

Tri-bimaximal

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott

TB angles $\theta_{12} = 35^\circ, \quad \theta_{23} = 45^\circ, \quad \theta_{13} = 0^\circ.$

c.f. data $\theta_{12} = 34.4 \pm 1.0 \quad \theta_{23} = 42.8^{+4.7}_{-2.9} \quad \theta_{13} = 5.6^{+3.0}_{-2.7}$

Current data is consistent with TB mixing but the (one sigma) error bars are large and there is a hint for θ_{13}

Discrete Family Symmetry

Consider the TB Neutrino Mass Matrix

$$M_{TB}^{\nu} = U_{TB}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{TB}^{T}$$

$$M^E = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} = T M^E T^\dagger$$

$$M_{TB}^{\nu} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

$$\Phi_1 = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

TB Neutrino Mass Matrix is invariant under a discrete $Z_2^S \times Z_2^U$ group generated by S,U

$$M_{TB}^{\nu} = S M_{TB}^{\nu} S^T$$

$$M_{TB}^{\nu} = U M_{TB}^{\nu} U^T$$

Family Symmetry G_{Fam}

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$S, T, U \rightarrow S_4$$

$$S, T \rightarrow A_4$$

Direct Models

G_{Fam}

S, U broken but T preserved by flavon VEV ϕ_T

T broken but S,U preserved by flavons ϕ_S, ϕ_U

Charged Lepton Sector

U_{TB}

Neutrino Sector

$$\mathcal{L}^{Yuk} \sim \psi(\phi_T + \phi_I)\psi^c H \ ,$$

$$\mathcal{L}^{Maj} \sim \psi(\phi_S + \phi_U + \phi_I)\psi H H$$

Indirect Models

G_{Fam}

G_{Fam} broken by flavon VEVs

$$\langle\phi_1\rangle = v_1\Phi_1, \quad \langle\phi_2\rangle = v_2\Phi_2, \quad \langle\phi_3\rangle = v_3\Phi_3$$

$$\Phi_1 = \frac{1}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

Accidental $Z_2^S \times Z_2^U$ symmetry emerges

$$\mathcal{L}^{Maj} \sim \psi(\phi_1\phi_1^T + \phi_2\phi_2^T + \phi_3\phi_3^T)\psi H H$$

$$M^{\nu}_{TB} = m_1\Phi_1\Phi_1^T + m_2\Phi_2\Phi_2^T + m_3\Phi_3\Phi_3^T$$

N.B. Indirect models have flavon VEVs aligned along the columns of U

Vacuum alignment

With Driving fields (SUSY F-terms)

Altarelli, Feruglio

$$M(\varphi_T \varphi_0^T) + g(\varphi_0^T \varphi_T \varphi_T) + g_1(\varphi_0^S \varphi_S \varphi_S)$$

$$\phi_S \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \phi_T \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Without Driving fields (SUSY D-terms)

SFK, Malinsky

$$\Delta V = \kappa \sum \phi^{i\dagger} \phi^i \phi^{i\dagger} \phi^i + \lambda |\phi_2^\dagger . \phi_3|^2$$

$$\phi_2 \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \phi_3 \sim \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

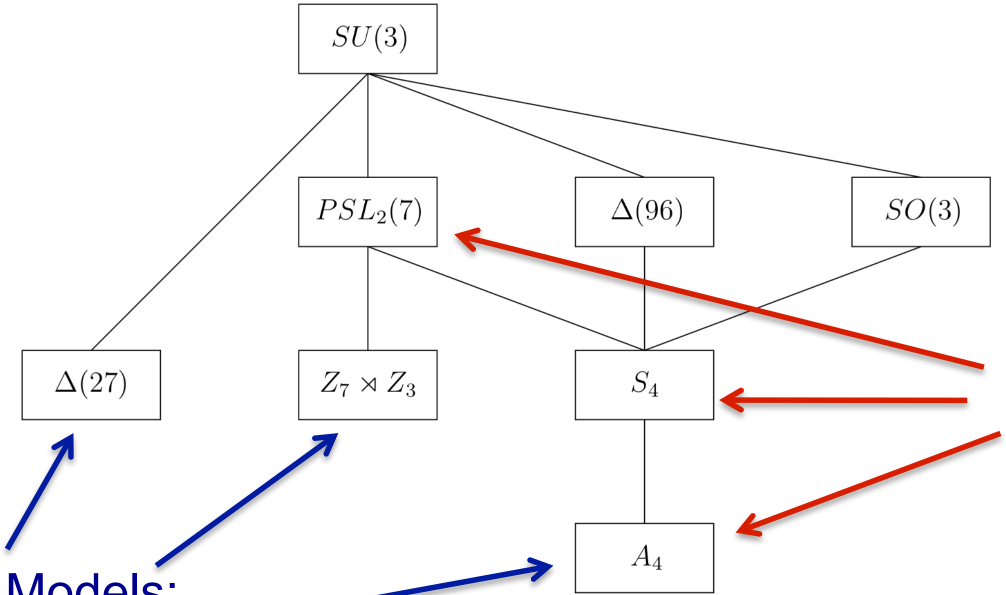
Orbifold boundary conditions

Kobayashi, Omura, Yoshioka; Burrows, SFK

$$\varphi_S(-z_1) = P_2 \varphi_S(z_1)$$
$$\varphi_T(\omega z_2) = P_3 \varphi_T(z_2)$$

$$\phi_S \sim \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \phi_T \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Discrete Groups



Direct Models:
G_{Fam} must contain A_4 as a subgroup

Indirect Models:
G_{Fam} may or may not contain A_4 as a subgroup

↓

In general G_{Fam} may be $\Delta(3n^2)$ or $\Delta(6n^2)$

Luhn, SFK

Altarelli, Feruglio 1002.0211
(and references therein)

~~TBM~~ (rows D_3 to A_5); TBM (rows A_4 to T_7)

Group	d	Irr. Repr.'s	Presentation	Ref.'s
$D_3 \sim S_3$	6	$1,\ 1',\ 2$	$A^3 = B^2 = (AB)^2 = 1$	[i]
D_4	8	$1_1, ...1_4, 2$	$A^4 = B^2 = (AB)^2 = 1$	[ii]
D_7	14	$1,\ 1',\ 2,\ 2',\ 2''$	$A^7 = B^2 = (AB)^2 = 1$	[iii]
$A_5 \sim PSL_2(5)$	60	$1,\ 3,\ 3',\ 4,\ 5$	$A^3 = B^2 = (BA)^5 = 1$	[v]
A_4	12	$1,\ 1',\ 1'',\ 3$	$A^3 = B^2 = (AB)^3 = 1$	[iv]
T'	24	$1,\ 1',\ 1'',\ 2,\ 2',\ 2'',\ 3$	$A^3 = (AB)^3 = R^2 = 1,\ B^2 = R$	[vi]
S_4	24	$1,\ 1',\ 2,\ 3,\ 3'$	$BM: A^4 = B^2 = (AB)^3 = 1$ $TB: A^3 = B^4 = (BA^2)^2 = 1$	[vii]
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, ...1_9, 3, \bar{3}$		[viii]
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$	$A^3 = B^2 = (BA)^7 = (B^{-1}A^{-1}BA)^4 = 1$	[ix]
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$	$A^7 = B^3 = 1,\ AB = BA^4$	[x]

See-saw mechanism

Many direct and indirect models use the type I see-saw mechanism

$$M^{\nu} = M_D \left(\frac{1}{M_R} \right) M_D^T$$

Large M_R implies small M^{ν}

In the diagonal M_R basis it is a remarkable fact that in all family symmetry models (e.g. A_4) the columns of the Dirac mass matrix M_D are proportional to the columns of the PMNS matrix U

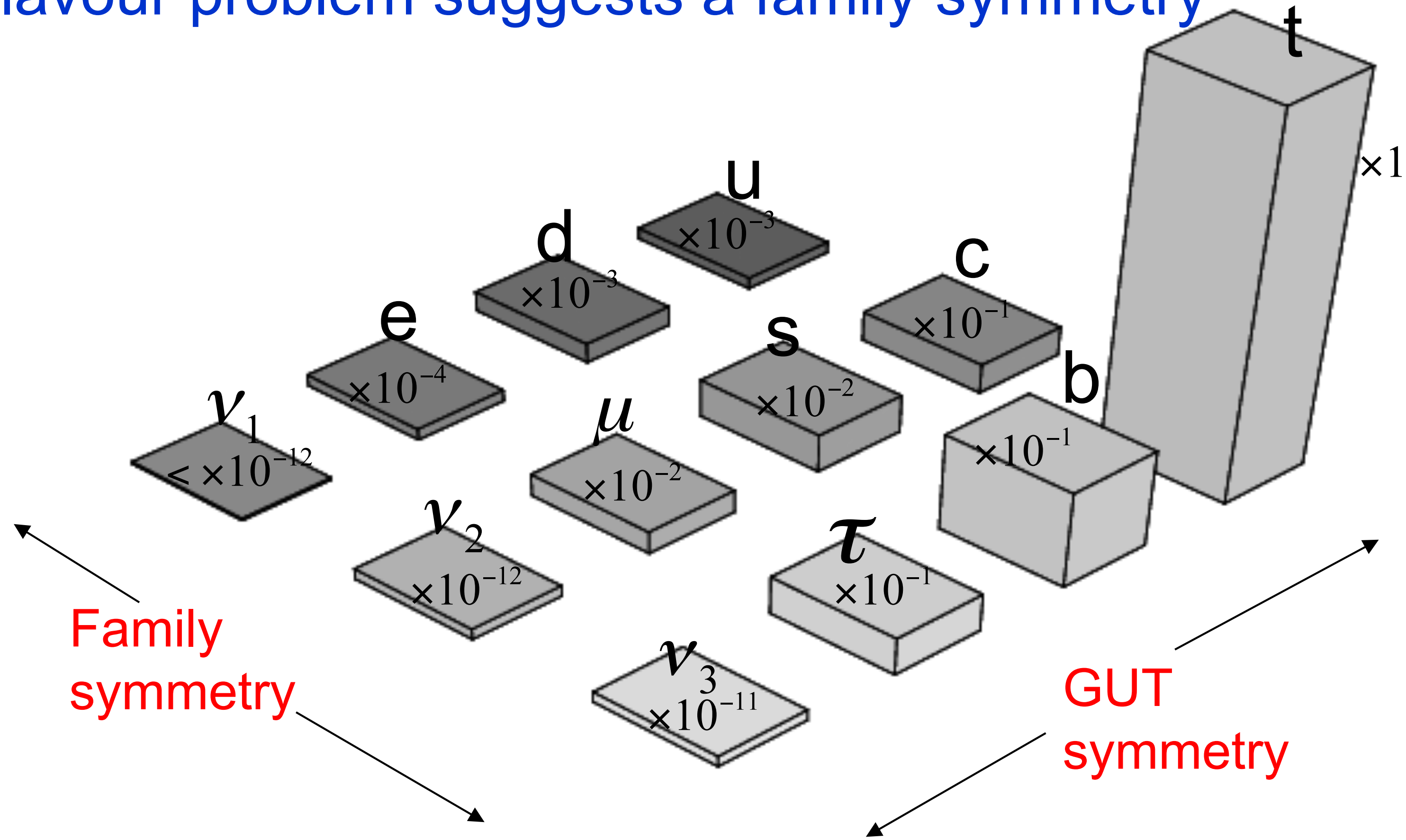
$$m_{D1}^{TB} = \frac{a_1}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad m_{D2}^{TB} = \frac{a_2}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad m_{D3}^{TB} = \frac{a_3}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

This property is called Form Dominance since it implies that M^{ν} is form diagonalisable, i.e. the mixing angles are determined independently of the neutrino masses

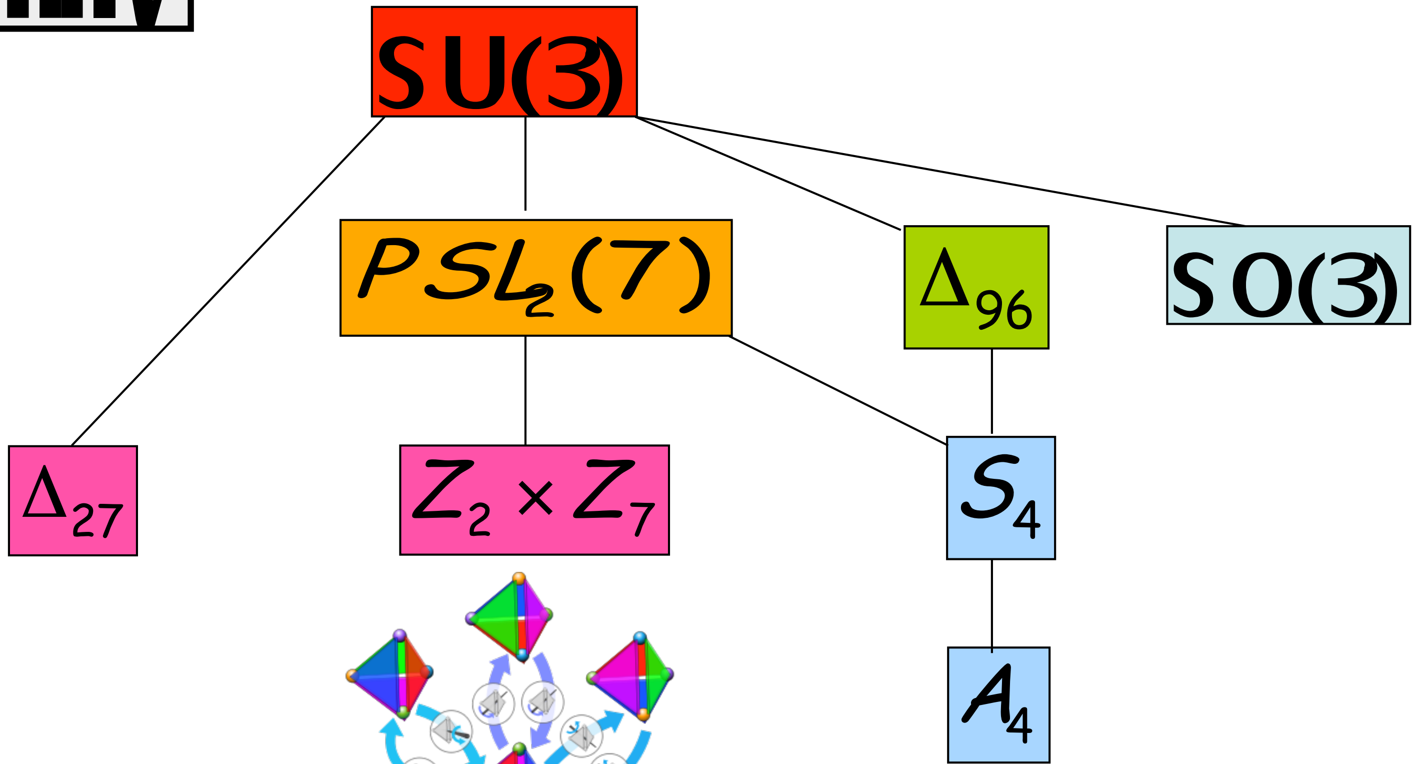
Chen, SFK

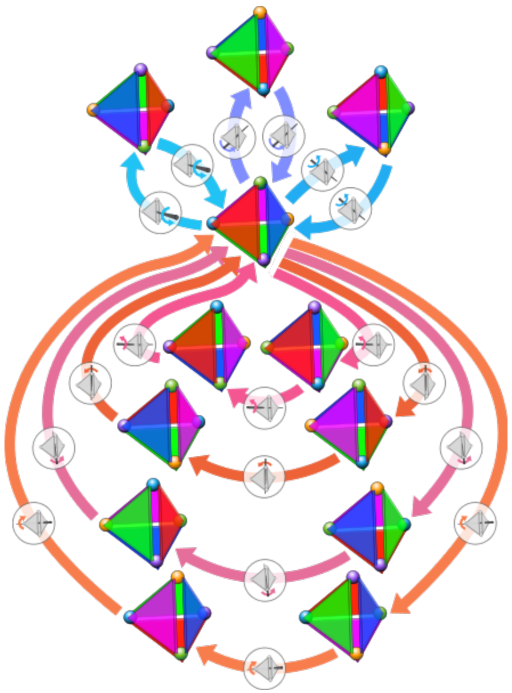
GUTs and Family Symmetry

- ❖See-saw naturally suggests a high scale → GUT
- ❖Flavour problem suggests a family symmetry



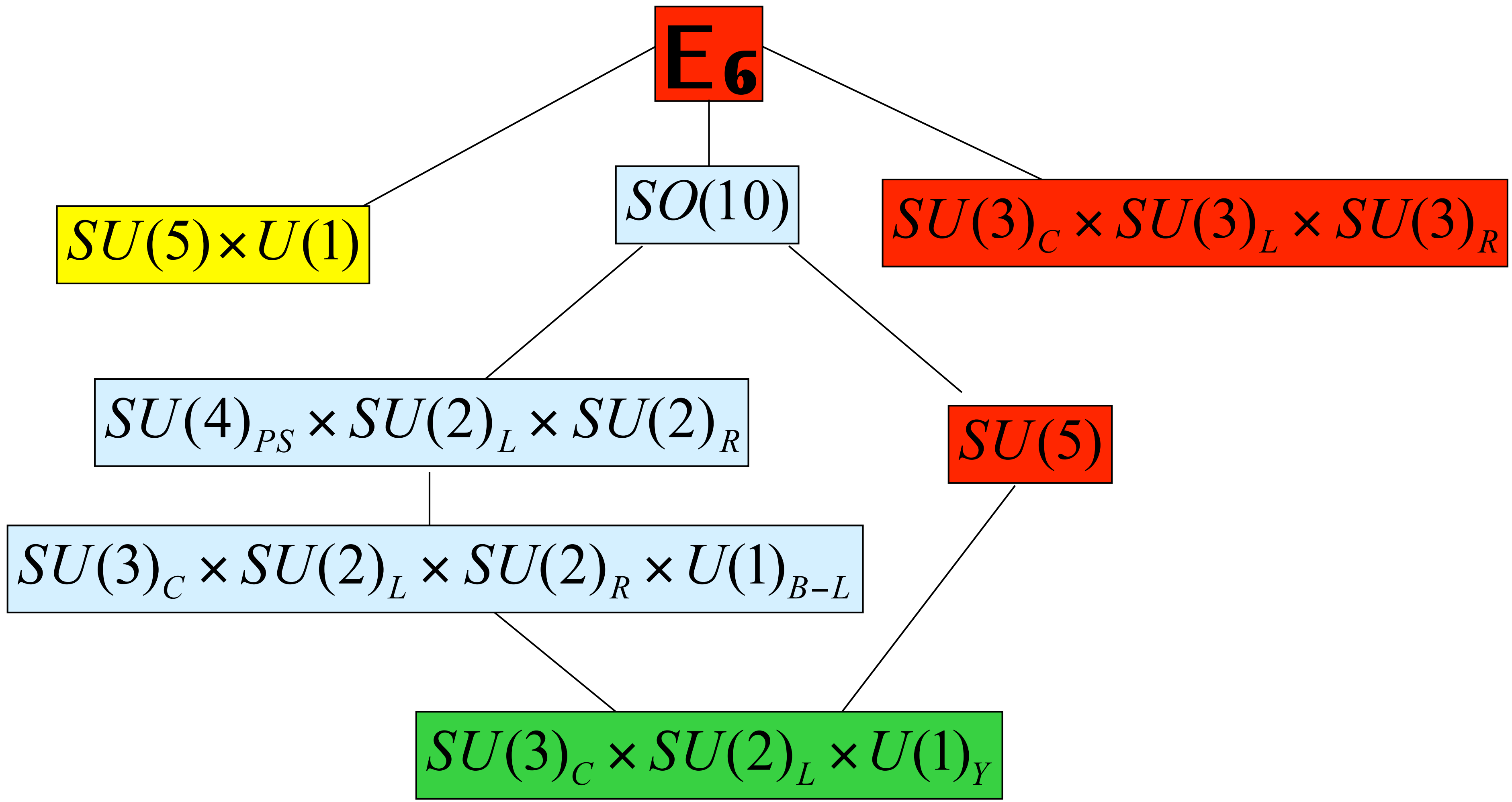
G_F Family



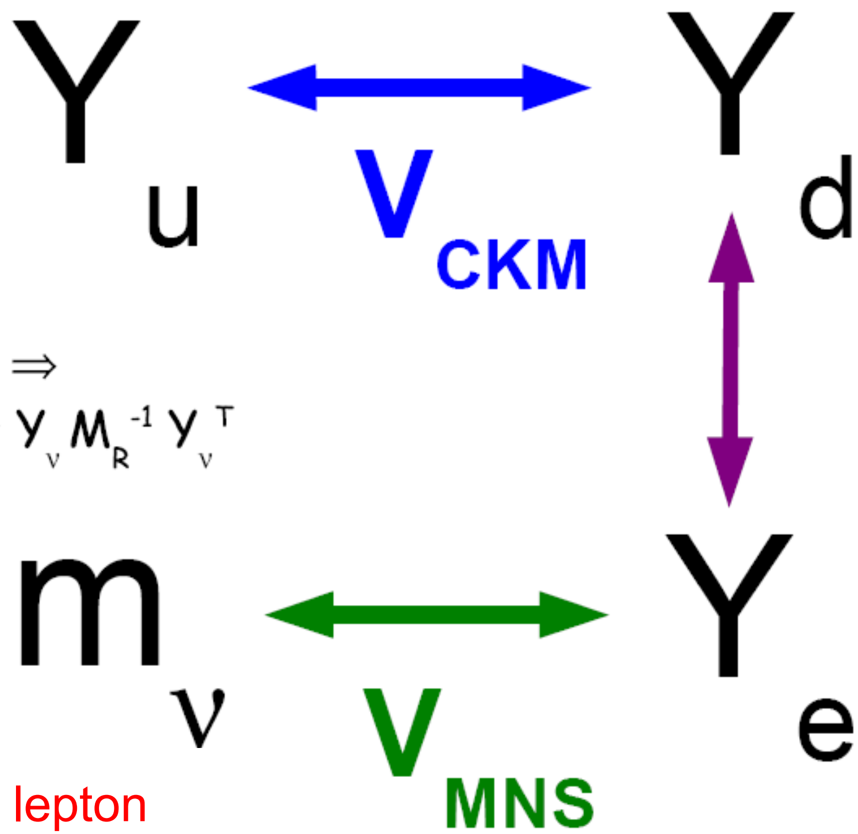


e.g. A_4 is the symmetry of the tetrahedron

G_{GUT}


E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C\times SU(3)_L\times SU(3)_R$
$SU(4)_{PS}\times SU(2)_L\times SU(2)_R$
$SU(5)$
$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$
$SU(3)_C\times SU(2)_L\times U(1)_Y$

GUT relations and sum rule



Class of models: $\theta_{12}^{d} \gg \theta_{12}^{u}$

$\theta_{12}^{d} \approx \theta_C$

See-saw $\Rightarrow$

$m_\nu = v_{EW}^2 \, Y_\nu M_R^{-1} Y_\nu^T$

GUT relation

$$\theta_{12}^{e} \approx \frac{\theta_{12}^{d}}{3} \approx \frac{\theta_C}{3}$$

TB + charged lepton corrections + GUTs leads to predictions: $\theta_{13} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^\circ, \quad \theta_{12} \approx 35.3^o + \theta_{13}\cos\delta$

Bjorken, SFK, Pakvasa…

Sum Rule

SFK, Antusch, Masina

Lesson: TB mixing can never be exact in GUT models

Tri-bimaximal deviations

SFK, Parke, Pakvasa et al…

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

$$0.07 < r < 0.21, \quad -0.05 < s < 0.003, \quad -0.09 < a < 0.04$$

r = reactor s = solar a = atmospheric

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

GUT sum rule predictions recast as:

$$s \approx r \cos \delta \qquad r \approx \lambda / 3$$

Solar Reactor CP phase Reactor Wolfenstein

Quark CP violation very well known

Wolfenstein

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$

$\lambda \approx 0.226 \quad A \approx 0.81 \quad \rho \approx 0.13 \quad \eta \approx 0.35$

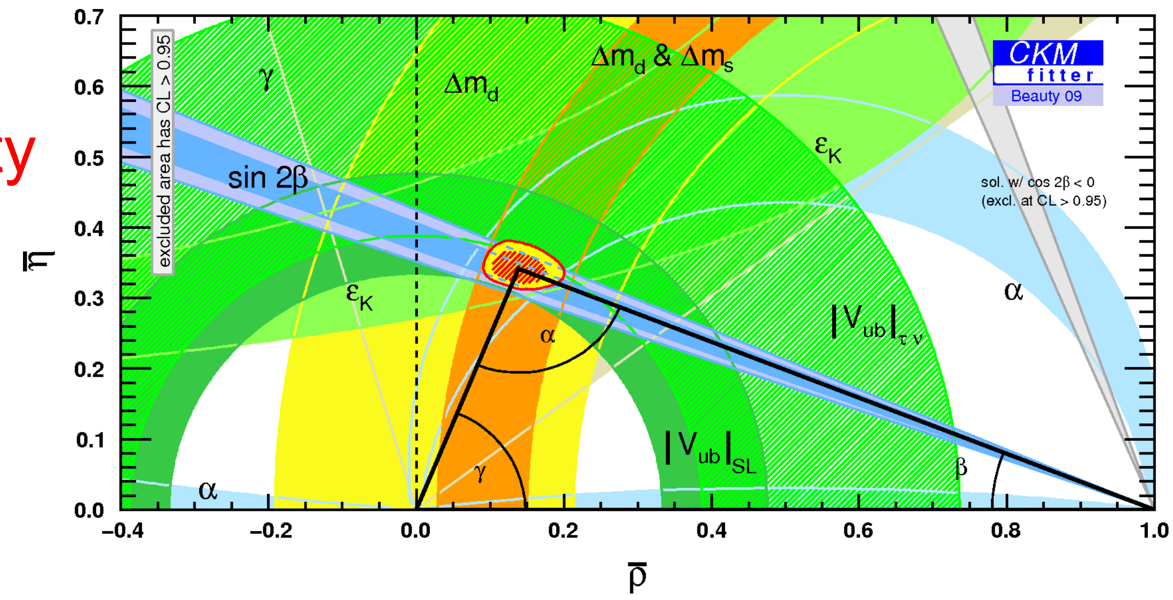
$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Right-angled Unitarity Triangle

– accident or hint?

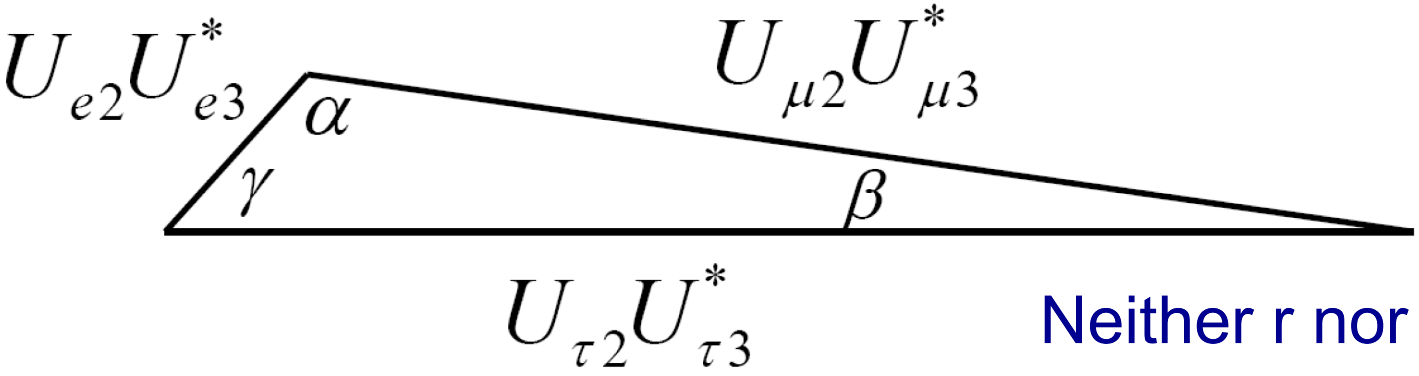
$\alpha \approx 90^o \pm 4^o$

$\delta_{CP} \approx \gamma \approx 70^o \pm 5^o$



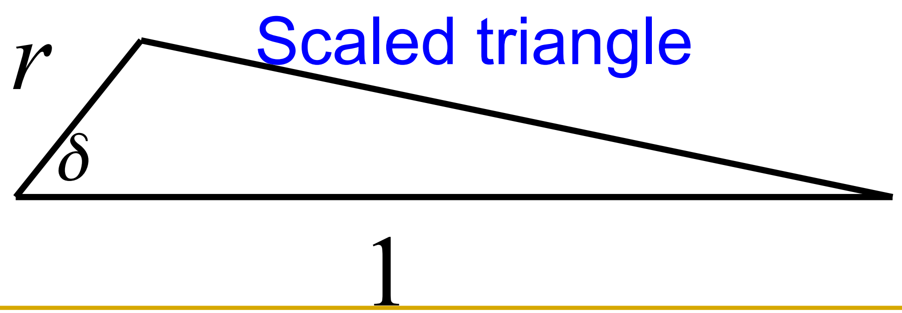
Leptonic CP violation is unknown

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$



$\nu_2 . \nu_3$

Neither r nor δ is measured



Two extreme possibilities:
a) UT = straight line
b) UT = right-angled triangle

SUSY GUT of Flavour S_4 x SU(5)

Hagedorn, SFK, Luhn

Field	T_3	T	F	N	H_5	$H_{\overline{5}}$	$H_{\overline{45}}$	Φ^u_2	$\widetilde{\Phi}^u_2$	Φ^d_3	$\widetilde{\Phi}^d_3$	Φ^d_2	$\Phi^\nu_{3'}$	Φ^ν_2	Φ^ν_1
$SU(5)$	$\mathbf{10}$	$\mathbf{10}$	$\overline{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\overline{\mathbf{45}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
S_4	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{3}'$	$\mathbf{2}$	$\mathbf{1}$
$U(1)$	0	x	y	$-y$	0	0	z	$-2x$	0	$-y$	$-x-y-2z$	z	$2y$	$2y$	$2y$

- F-term vacuum alignment studied to NLO
- Separate flavons in up, down and neutrino sectors distinguished by U(1)
- GST and GJ relations
- S_4 enforces precise TB neutrino mixing accurate to 0.1 per cent
- Accurate lepton mixing sum rules $s \approx r\cos\delta \quad r \approx \lambda/3 \quad a \approx -r^2/4$

Tri-bimaximal mixing & right unitarity triangles

Antusch, SFK, Luhn, Spinrath
(to appear)

S$_4$xSU(5) $\quad F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \sim \mathbf{3}\,, \quad T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \sim \mathbf{2}\,, \quad T_3 \sim \mathbf{1}\,.$

$$\begin{aligned} W_u &= T_3T_3H_5 + \frac{1}{M}TT\phi_2^u H_5 + \frac{1}{M^2}TT(\phi_{1'}^u)^2 H_5 + \frac{1}{M^3}TT(\phi_3^d)^2\phi_1^\nu H_5\,, \\ W_d &= \frac{1}{M}FT_3\phi_3^d H_5 + \frac{1}{M^2}(F\widetilde{\phi}_3^d)_1(T\phi_2^d)_1 H_{\overline{45}} + \frac{1}{M^2}(F\phi_3^d)_2(T\widetilde{\phi}_2^d)_2 H_5\,, \\ W_\nu &= FNH_5 + N(\phi_{3'}^\nu + \phi_2^\nu + \phi_1^\nu)N\,, \end{aligned}$$

Purely real or imaginary vacuum alignments

$$\langle \phi_2^u \rangle \sim \lambda^4 M \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle \phi_{1'}^u \rangle \sim \lambda^3 M\,,$$

$$\langle \phi_3^d \rangle \sim \lambda^2 M \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \langle \widetilde{\phi}_3^d \rangle \sim \lambda^3 M \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \quad \langle \phi_2^d \rangle \sim \lambda M \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \langle \widetilde{\phi}_2^d \rangle \sim \lambda^3 M \begin{pmatrix} \mathrm{i} \\ \mathrm{i} \end{pmatrix},$$

$$\langle \phi_1^\nu \rangle \sim \lambda^4 M\,, \quad \langle \phi_2^\nu \rangle \sim \lambda^4 M \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \langle \phi_{3'}^\nu \rangle \sim \lambda^4 M \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},$$

Fritzsch type quark mass matrices

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u\,, \qquad M_d \sim \begin{pmatrix} 0 & \mathrm{i}\lambda^5 & 0 \\ \mathrm{i}\lambda^5 & \lambda^4 & 2\lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d\,,$$

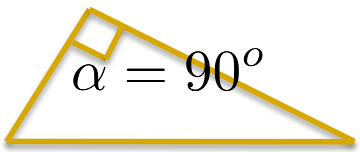
Phase sum rule $\quad \alpha \approx \delta_{12}^d - \delta_{12}^u$

Neutrino mass matrices

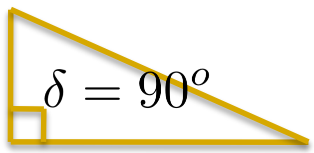
$$M_R = \begin{pmatrix} \alpha\varphi_1^\nu + 2\gamma\varphi_{3'}^\nu & \beta\varphi_2^\nu - \gamma\varphi_{3'}^\nu & \beta\varphi_2^\nu - \gamma\varphi_{3'}^\nu \\ \beta\varphi_2^\nu - \gamma\varphi_{3'}^\nu & \beta\varphi_2^\nu + 2\gamma\varphi_{3'}^\nu & \alpha\varphi_1^\nu - \gamma\varphi_{3'}^\nu \\ \beta\varphi_2^\nu - \gamma\varphi_{3'}^\nu & \alpha\varphi_1^\nu - \gamma\varphi_{3'}^\nu & \beta\varphi_2^\nu + 2\gamma\varphi_{3'}^\nu \end{pmatrix}$$

$$M_D = y_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u$$

Quark unitarity triangle



Lepton unitarity triangle



$$r \approx \frac{\lambda}{3} \qquad s \approx a \approx 0$$

How could neutrino mass models be constrained by results from neutrino oscillations?

Since the SUSY GUTs of Flavour predict approximate TB mixing with $\theta_{13} \approx 3^o$ they would be excluded if reactor angle is large

Spectrum of alternatives →

Anarchy
No symmetry, all angles are large

Allows r,s,a to be large with r typically too large

Semi-Anarchy
U(1) family symmetry to explain small reactor and quark angles

Allows r,s,a to all be large with r acceptable

Indirect models
Non-Abelian Family Symmetry is broken but accidental neutrino symmetry arises

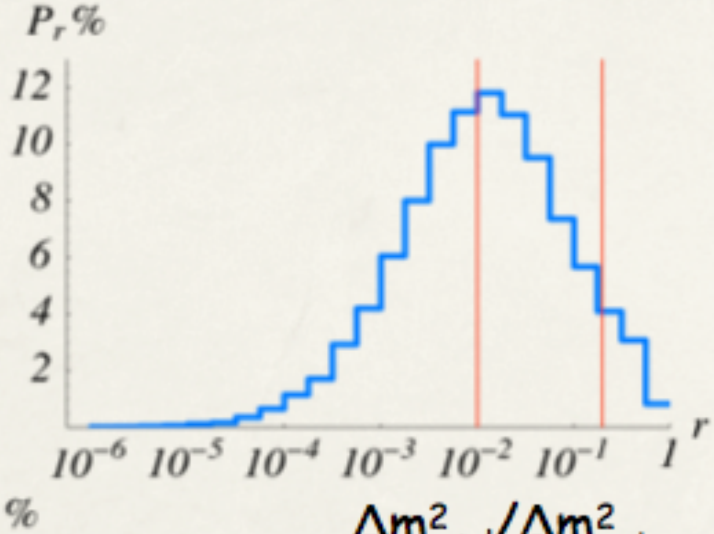
Vacuum misalignment allows one or more of r,s,a to be large

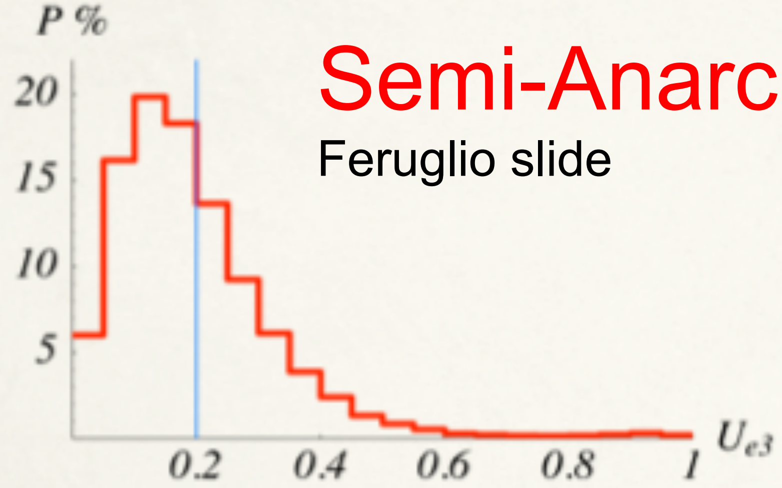
Direct models
Non-Abelian Family Symmetry is partly unbroken in the neutrino sector

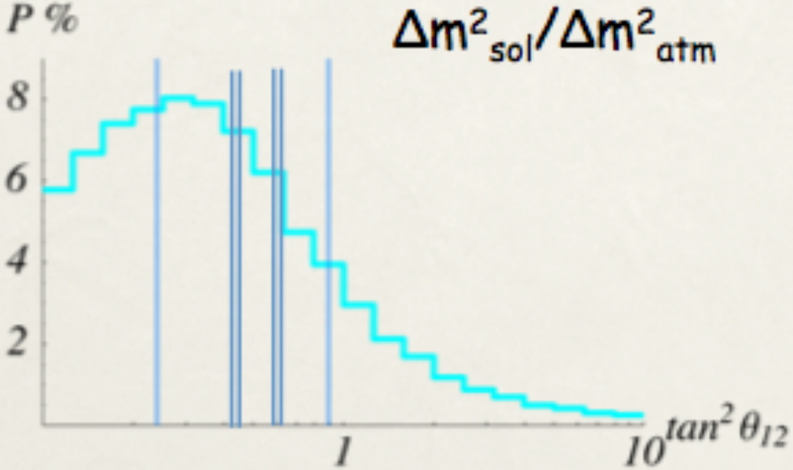
BM allows r and s large ~ λ (typically s too large) with small a

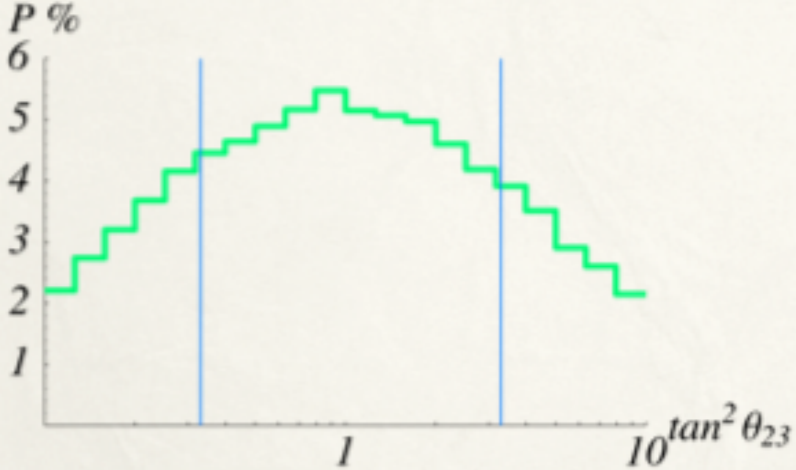
Semi-Anarchy

Feruglio slide









[Altarelli,F,Masina 0210342]

Ass

$U(1)_{FN}$

FN(L) = (2,0,0)
FN(N^c) = (1,-1,0)

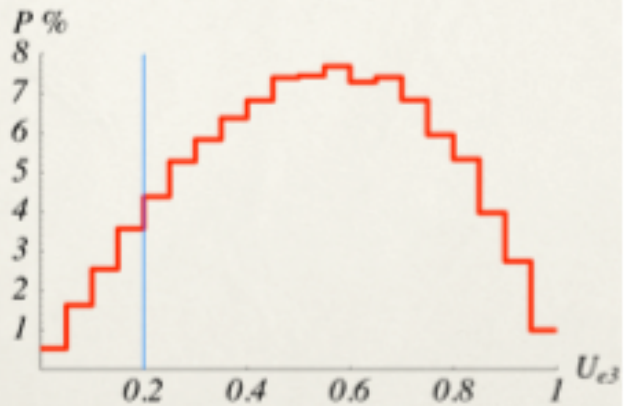
See also Hirsch, SFK 0102103

λ=0.35

Hall, Murayama, Weiner

cfr. Anarchy

FN(L) = (0,0,0)
FN(N^c) = (0,0,0)



Indirect Models with vacuum misalignment

$$\mathcal{L}^{Maj} \sim L\left(\frac{\phi_1\phi_1^T}{M_1}+\frac{\phi_2\phi_2^T}{M_2}+\frac{\phi_3\phi_3^T}{M_3}\right)LHH$$

SFK 1011.6167

For a normal hierarchy m_1 ➔ 0 the first term decouples and we expand

$$\langle\phi_2\rangle \propto \frac{a_2}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix}+\frac{\alpha_{21}}{\sqrt{6}}\begin{pmatrix}2\\-1\\1\end{pmatrix}+\frac{\alpha_{22}}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix}+\frac{\alpha_{23}}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$\langle\phi_3\rangle \propto \frac{a_3}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}+\frac{\alpha_{31}}{\sqrt{6}}\begin{pmatrix}2\\-1\\1\end{pmatrix}+\frac{\alpha_{32}}{\sqrt{3}}\begin{pmatrix}1\\1\\-1\end{pmatrix}+\frac{\alpha_{33}}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix},$$

Constrained Sequential Dominance (TB mixing)

Vacuum misalignment corrections α_{ij} small

Simple vacuum misalignments lead to interesting predictions

$$\langle\phi_3^{a=0}\rangle \propto \frac{a_2+\alpha_{33}}{\sqrt{2}}\begin{pmatrix}re^{-i\delta}\\1\\1\end{pmatrix}$$

Predicts s=a=0 with r≠0

$$\langle\phi_3^{r=0}\rangle \propto \frac{a_3+\alpha_{33}}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}+\sqrt{\frac{3}{2}}\alpha_{32}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

Predicts s=r=0 with a≠0

IMPORTANT TO MEASURE r,s,a TB DEVIATIONS

$$\langle\phi_3^{trimax}\rangle \propto \frac{a_3}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix}+\sqrt{\frac{2}{3}}\alpha_{31}\begin{pmatrix}1\\-1\\0\end{pmatrix}$$

Predicts tri-maximal mixing ($1/\sqrt{3}$ in 2nd column of PMNS)

Conclusion

- The lepton mixing angles suggest a simple tri-bimaximal mixing pattern

- This in turn suggests an underlying discrete family symmetry which is spontaneously broken by flavons with particular vacuum alignments

- The see-saw mechanism then has a simple property called Form Dominance (Dirac columns proportional to PMNS columns in diagonal RHN basis)

- See-saw mechanism also suggests a high scale in nature as in GUTs: SU(5) since SO(10) seems difficult (see Altarelli,Blankenburg; SFK, Luhn)

- Family symmetry GUT models predict small deviations from TB mixing with a reactor angle $\theta_{13} \approx 3^o$ and a solar angle $\theta_{12} \approx 35.3^o + \theta_{13}\cos\delta$ with latest models predicting right-angled unitarity triangles e.g. $\delta = 90^o$

- If the reactor angle is measured to be larger then we must consider a spectrum of alternative models: anarchy, semi-anarchy, indirect models with vacuum misalignment, direct models with BM, or some new idea or ingredient…

Take home message to experimental colleagues:

Important to measure r, s, a and δ where

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

So far they could all be zero

$$0.07 < r < 0.21, \quad -0.05 < s < 0.003, \quad -0.09 < a < 0.04$$

The job is not done until all the deviations from Tri-bimaximal mixing angles are measured (not just measurement of reactor angle)