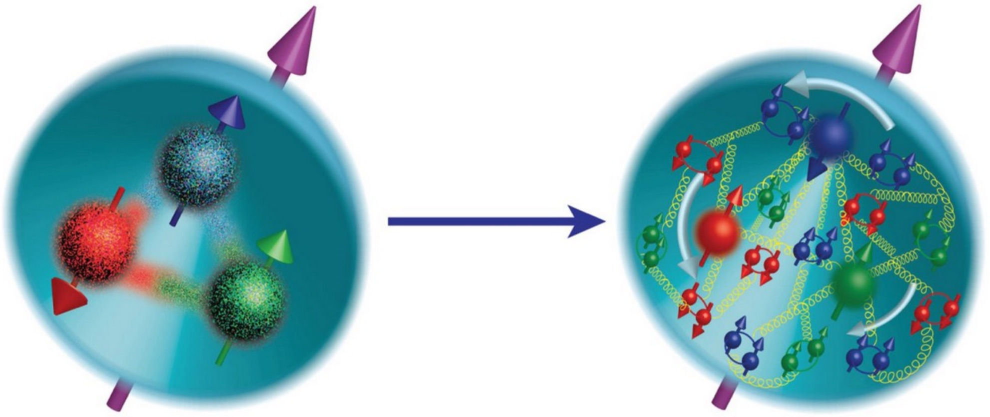

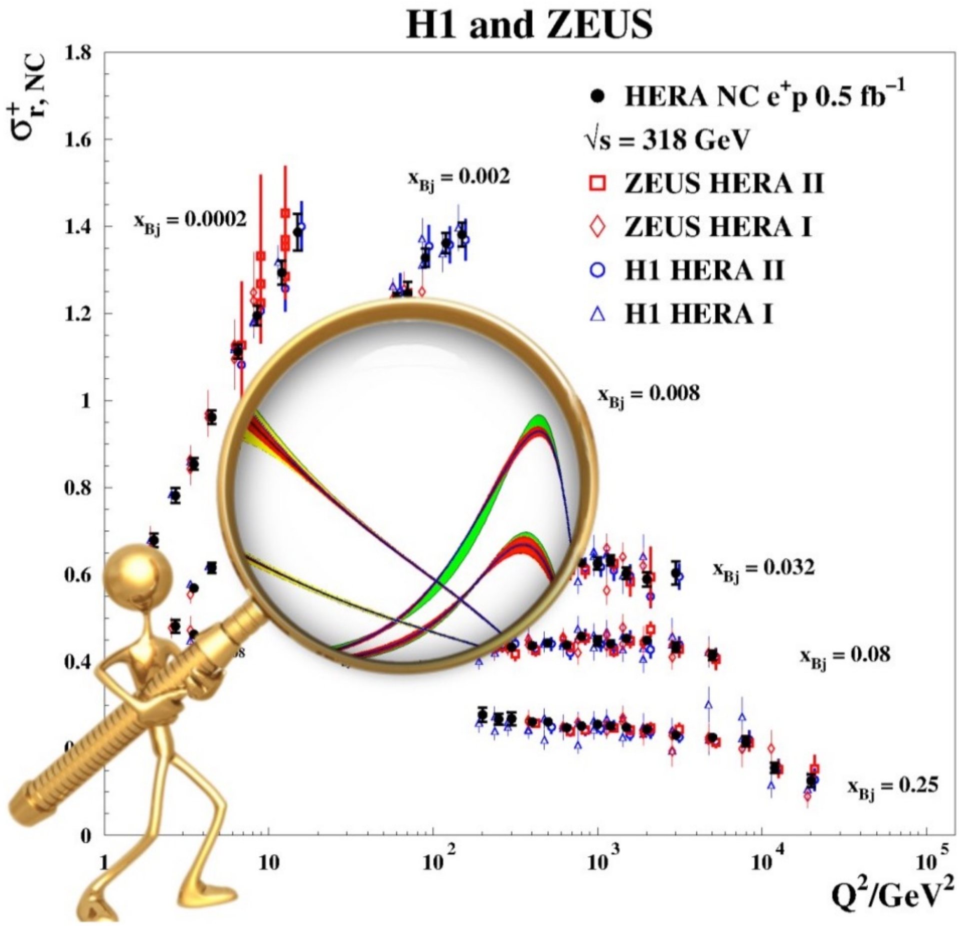

Il Deep Inelastic Scattering: da SLAC-MIT all'Electron Ion Collider

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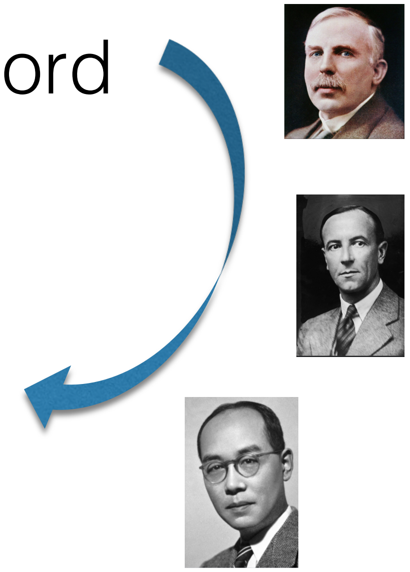
Contenuto

- I. Esperimenti a targhetta fissa
- II. HERA
- III. Electron Ion Collider

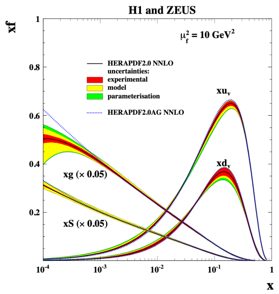
Nelle slide di back-up:
Preistoria (1909-1960)

- Diffusione Rutherford
- Nucleo atomico
- Protoni e neutroni
- Forza forte

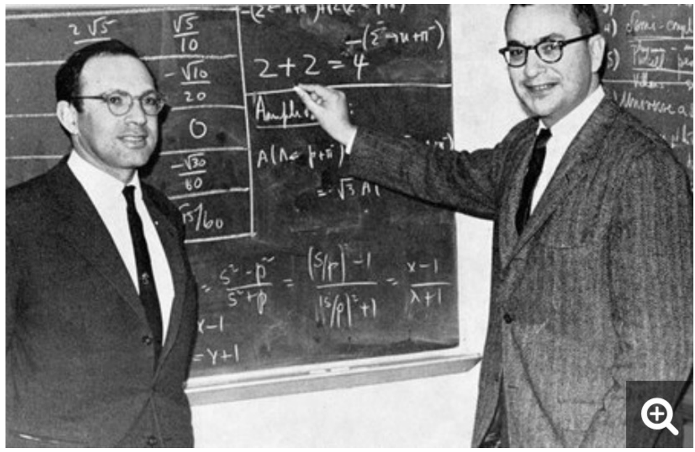


I. Esperimenti a targhetta fissa

- Quark model
- SLAC-MIT
- Gargamelle
- pQCD



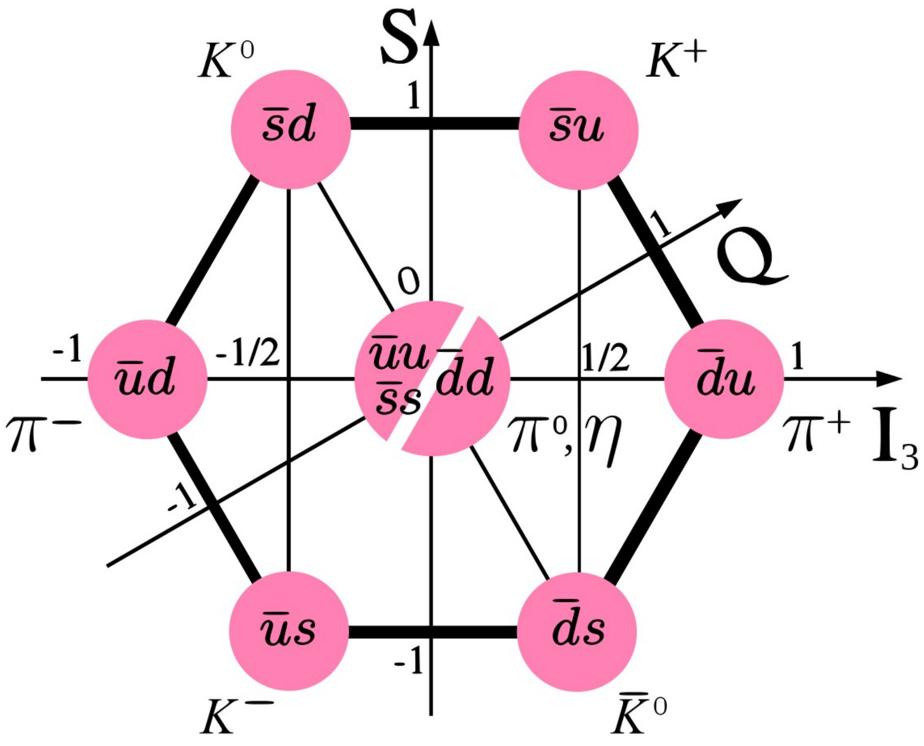
La via dell'ottetto

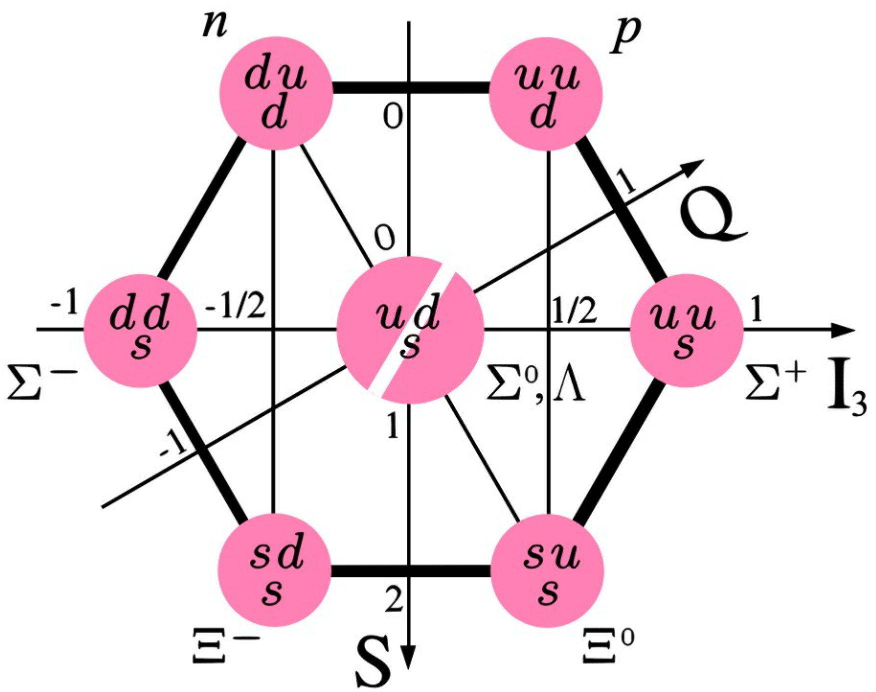


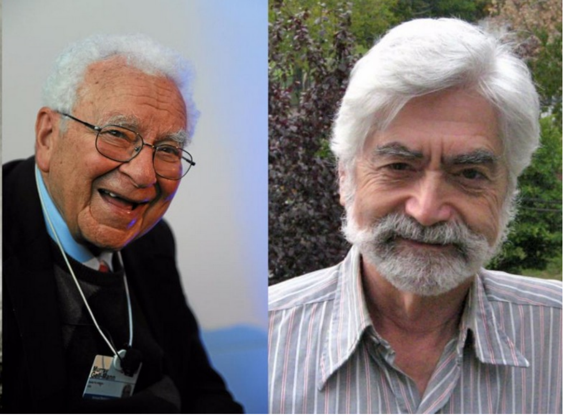
All'inizio degli anni 1960 molte particelle (mesoni e barioni) erano stato scoperte grazie all'utilizzo delle macchine acceleratrici e nuovi rivelatori.

Nel 1961 M. Gell-mann e Y. Ne'eman introducono una schema per la classificazione dei Mesoni e Barioni in famiglie/multipletti.

Framework Matematico: Rappresentazioni irr. del gruppo SU(3)

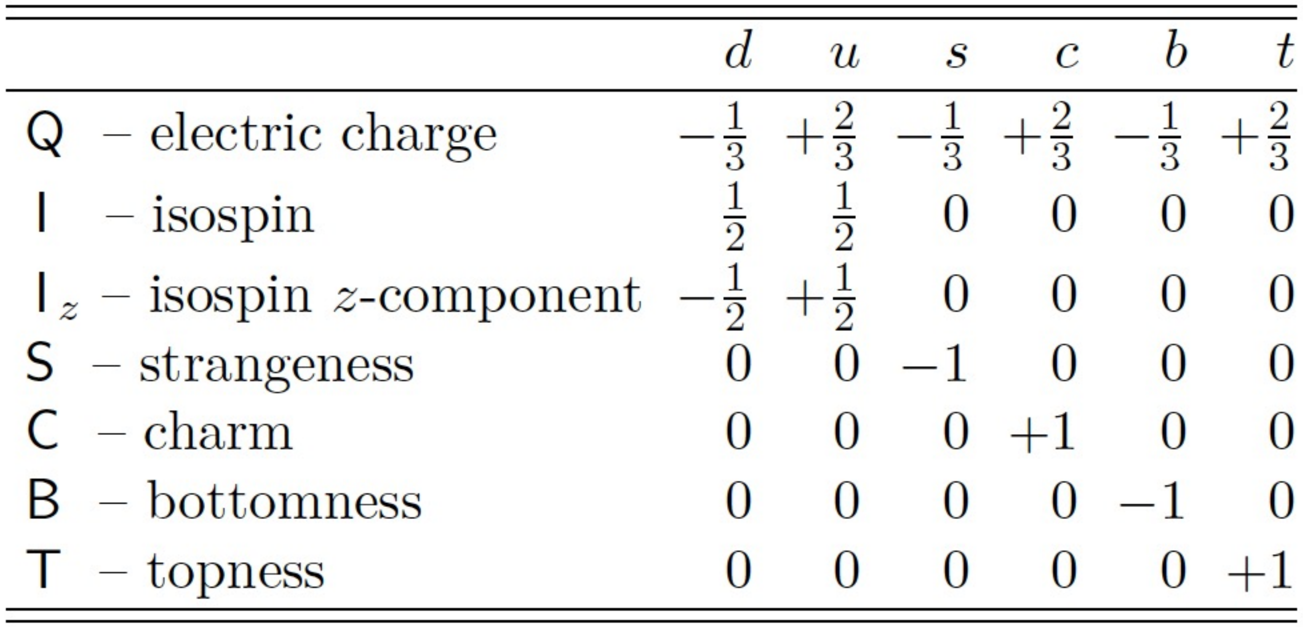




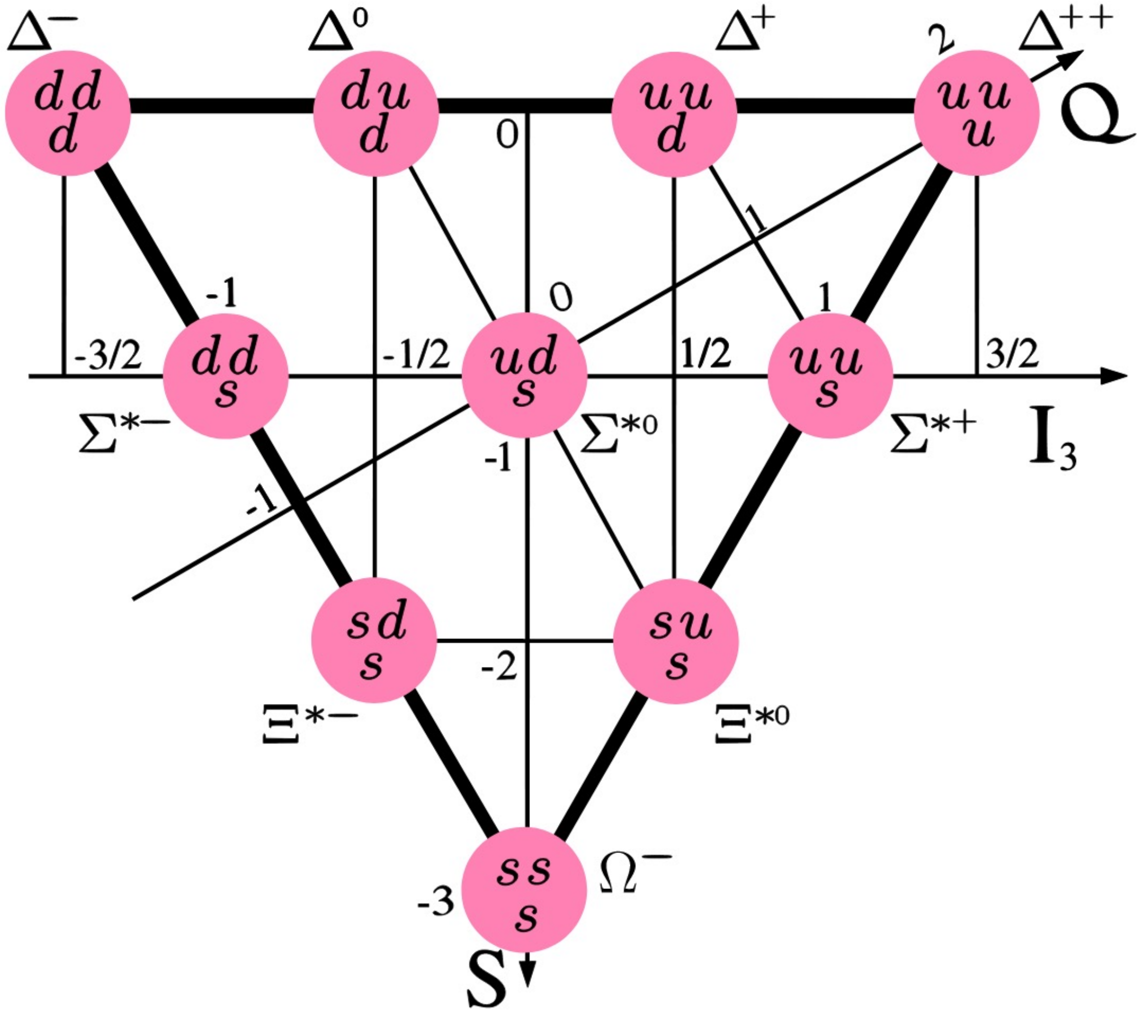


Il modello a quark

1964: Gell-Mann and Zweig propongono l'esistenza di particella elementari di spin ½ e carica frazionaria (i quarks). Tre tipi di quarks (up, down e strange) ed i corrispondenti anti-quark spiegano le regolarita' osservate negli adroni.



	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	−1	0	0	0
C – charm	0	0	0	+1	0	0
B – bottomness	0	0	0	0	−1	0
T – topness	0	0	0	0	0	+1



Modello descrittivo e parzialmente predittivo (es. masse, momenti magnetici)

I quark sono reali?

Interpretazione prevalente (~1965s) :

- Il modello a quark e' solo un **utile schema organizzativo per la spettroscopia adronica**;
- Le particelle hanno una "sottostruttura diffusa" ma non "costituenti elementari"

(nuclear democracy – Bootstrap model)

"…the idea that mesons and baryons are made primarily of quarks and gluons is hard to believe…"

M. Gell-mann 1966

"Additional data are necessary and very welcome to destroy the picture of elementary constituents."

J. Bjorken 1967

"I think Professor Bjorken and I constructed the sum rules in the hope of destroying the quark model."

K. Gottfried 1967

"Of course the whole quark idea is ill founded."

J.J. Kokkedee 1967

SLAC ed il “Progetto-M”

Il 10 Aprile 1956, lo staff di Stanford staff si riunisce in casa di W. Panofsky per discutere la proposta di R. Hofstadter di costruire un acceleratore lineare 10 volte piu’ potente di Mark III. Al progetto viene assegnato il nome provvisorio “M(onster) Project” perche’ si stimava che l’acceleratore dovesse essere lungo 2 miglia e raggiungere una energía di 20 GeV!!

- 1957 viene presentato un progetto dettagliato
- 1959 Eisenhower dice si
- 1961 Il congreso approva il progetto ($114 Milioni di dollari)

Nel 1962 inizia la costruzione

Durante i lavori di scavo viene ritrovato lo scheletro di un mammifero vissuto nel Miocene, (*Paleoparadoxia*), che popolava quella regione14 milioni di anni fa...

CIT-SLAC-MIT

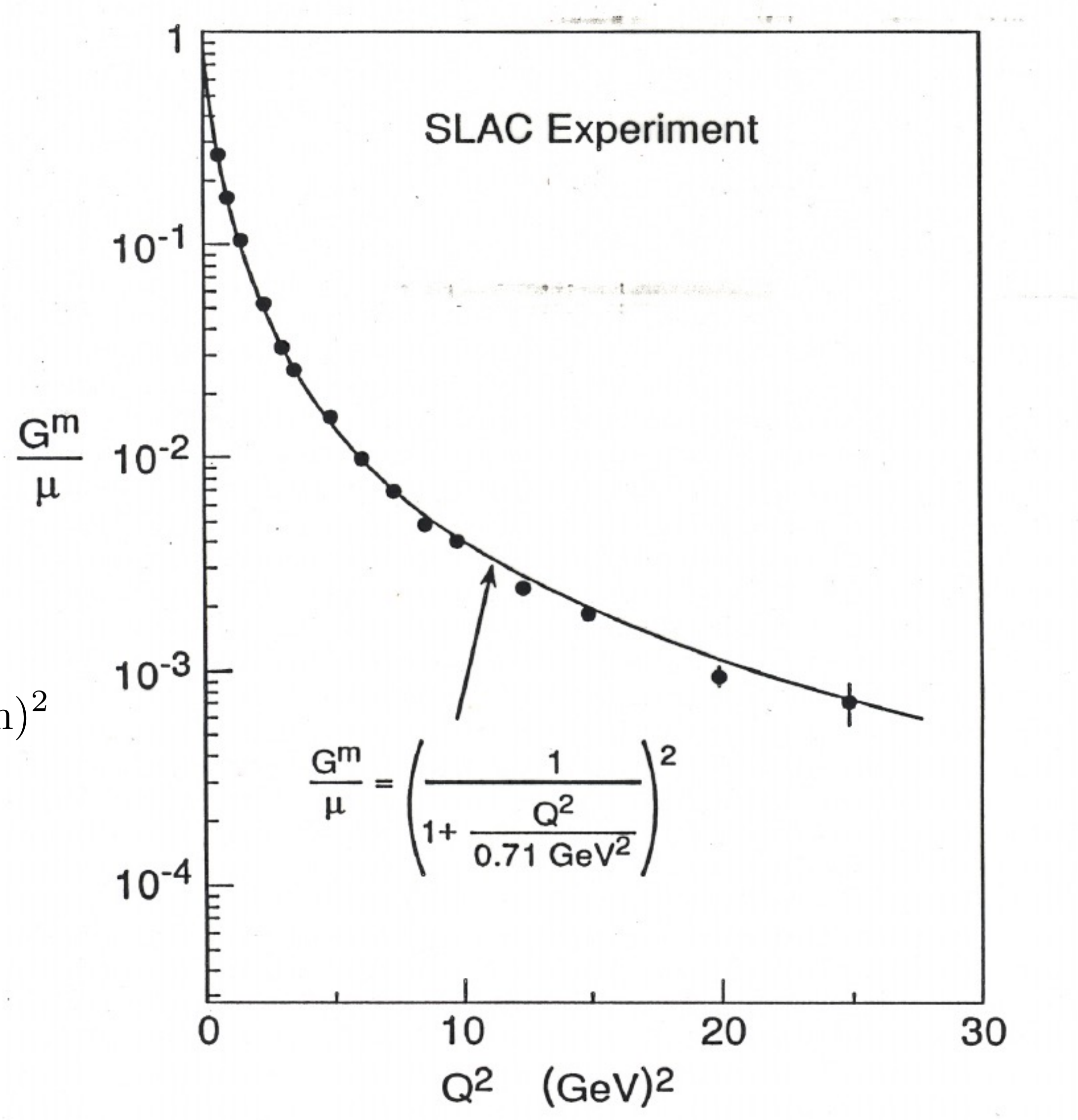
SLAC inizia la sua fase operativa nel 1966

La collaborazione CIT-SLAC-MIT realizza lo spettrometro per elettroni per studiare la struttura del protone attraverso il processo di diffusione elastica e-p

L'elettrone e' una "sonda" ideale:
- particella "elementare"
- interagisce elettromagneticamente (QED)

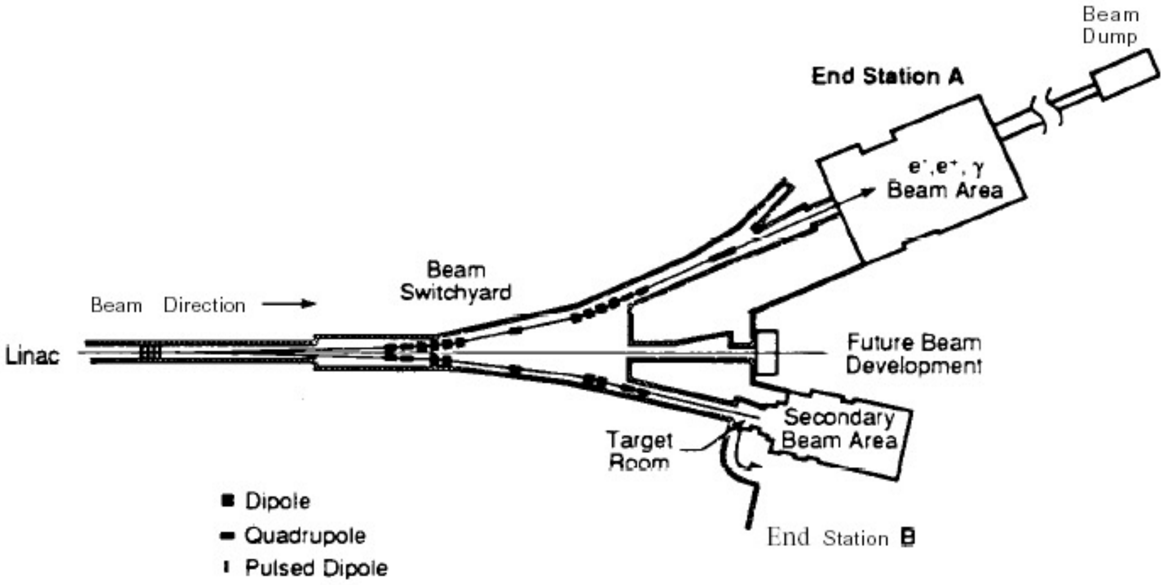
Fattore di forma magnetico ⟹

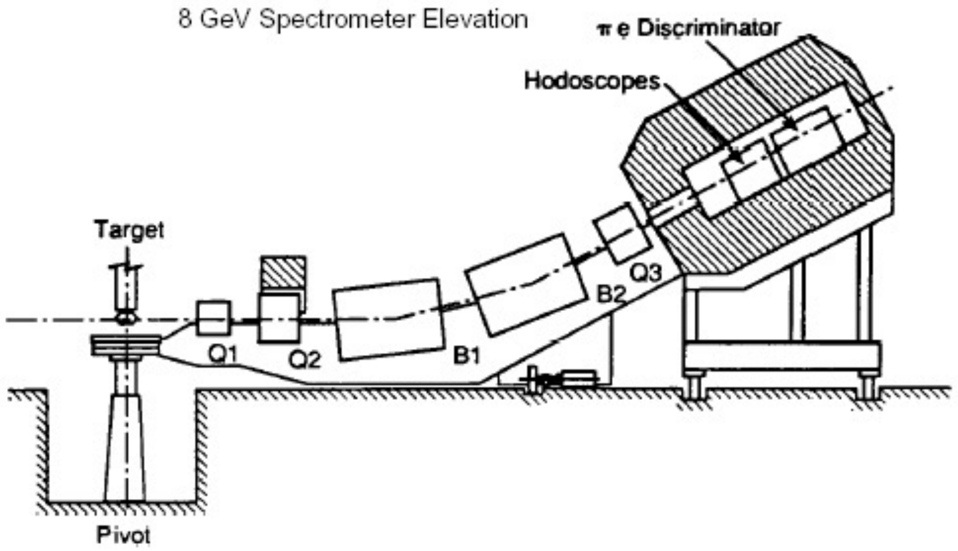
$$< r^2 >= \int d^3 r r^2 \rho(\mathbf{r}) = -6 \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0} = ((0.81 \pm 0.04) \cdot 10^{-13}\ \mathrm{cm})^2$$

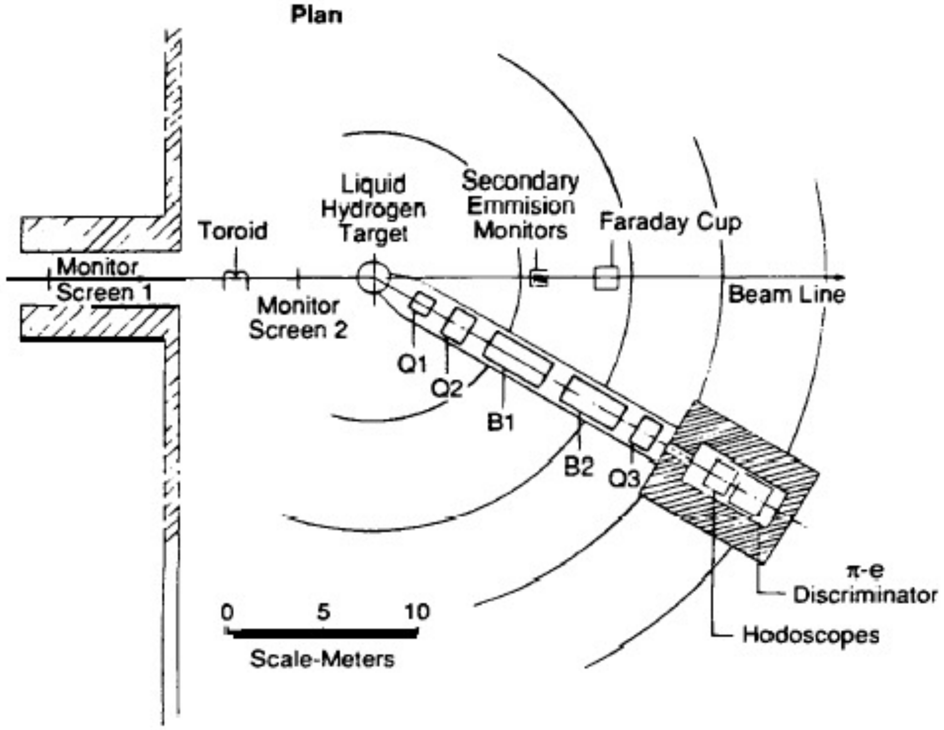


SLAC-MIT

….sotto la direzione di Taylor, Friedman e Kendall

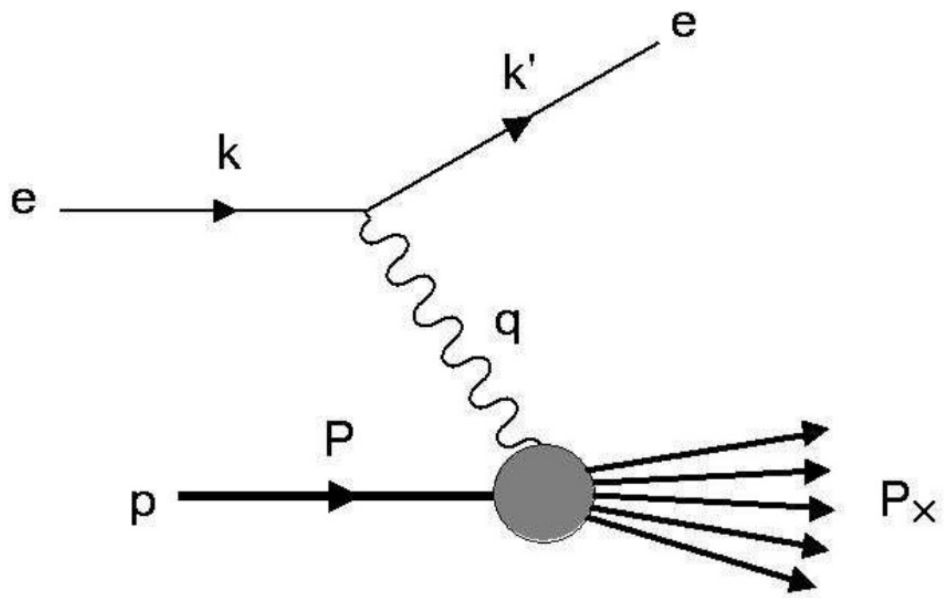






SLAC-MIT

Nel 1967 la collaborazione SLAC-MIT inizia lo studio sistematico della diffusione anelastica:



$$Q^2 = -(k-k')^2 = 4EE'\sin^2(\theta/2)$$

$$\omega = \frac{1}{x} = \frac{2m_p\nu}{Q^2}$$

$$\nu = \frac{P\cdot q}{m_p} = E - E'$$

$$W^2 = P_X^2 = 2m_p\nu + m_p^2 - Q^2$$

diffusione elastica vs anelastica:

La diffusione elastica fornisce informazioni sulle distribuzioni di carica elettrica e momento magnetico “mediate sul tempo”

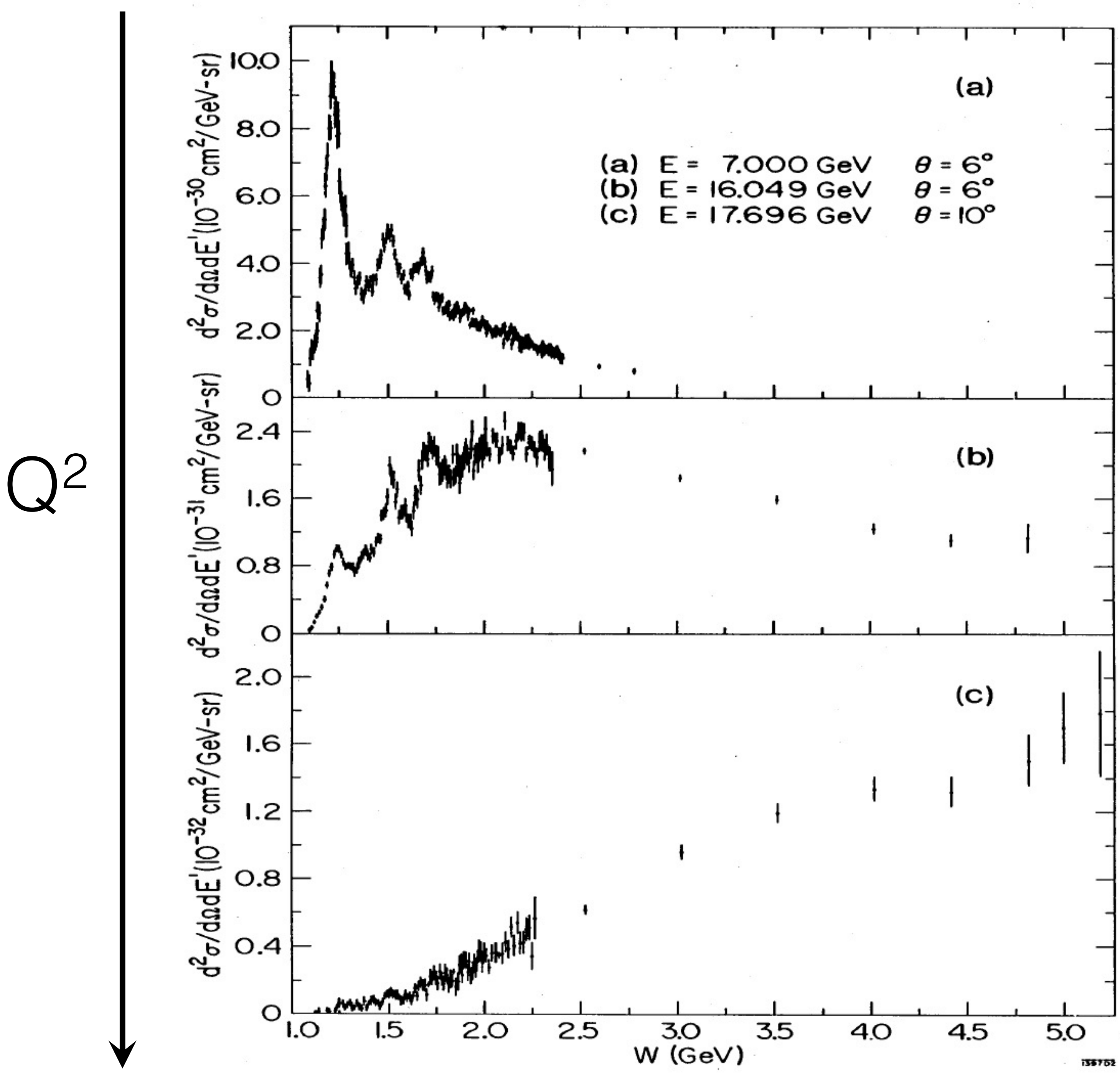
La diffuzione anelastica ci fornisce un’ “istantanea” della struttura del protone

$$\Delta t = \frac{h}{\Delta E}$$

$$\Delta E = 2\ \mathrm{GeV} \to \Delta t = 3\cdot 10^{-25}\ \mathrm{s}$$

$$\to \Delta x \simeq 10^{-16}\ \mathrm{m} \quad (v=c)$$

SLAC MIT

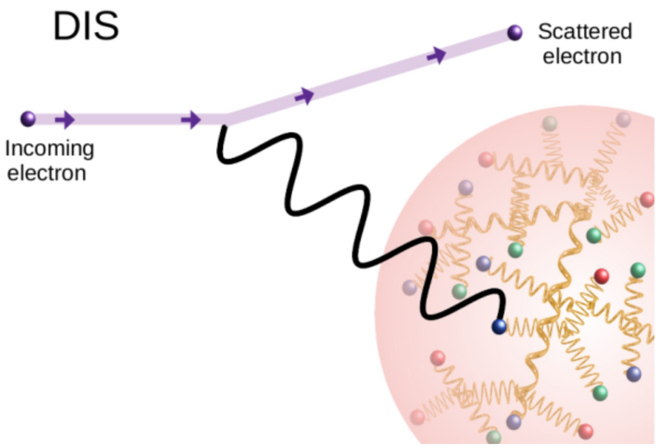


Si decide di studiare la regione del continuo.

Due risultati inattesi:

- Bjorken scaling
- Leggera dipendenza da Q^2

Bjorken scaling



Sezione d'urto:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_M \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2(\theta/2) \right]$$

Bjorken scaling:

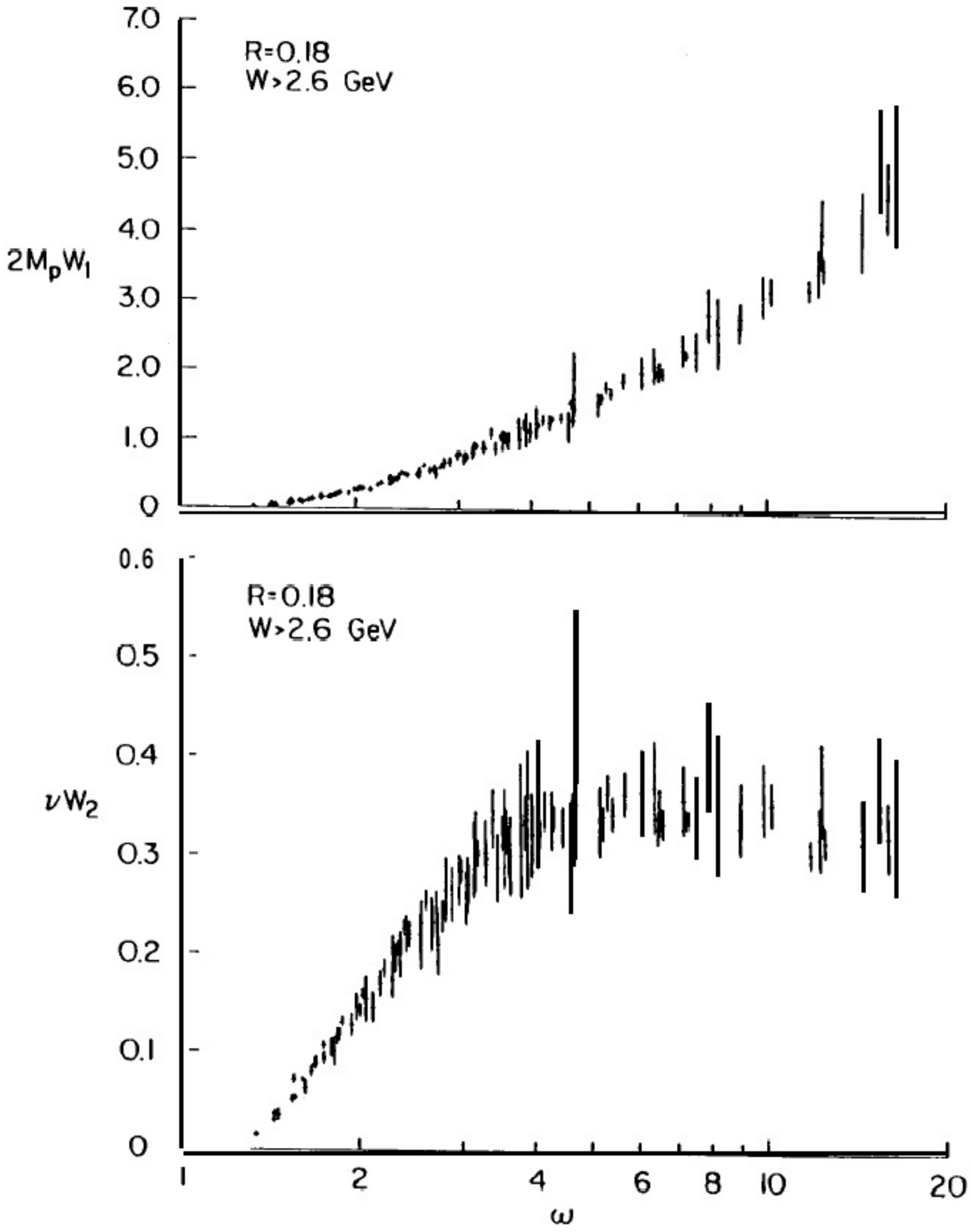
$$\text{per } \nu \to \infty \text{ , } Q^2 \to \infty \text{ (con } \omega = \frac{2m_p\nu}{Q^2} \text{ fissato)}$$

$$\nu W_2(\nu, Q^2) \to F_2(\omega)$$

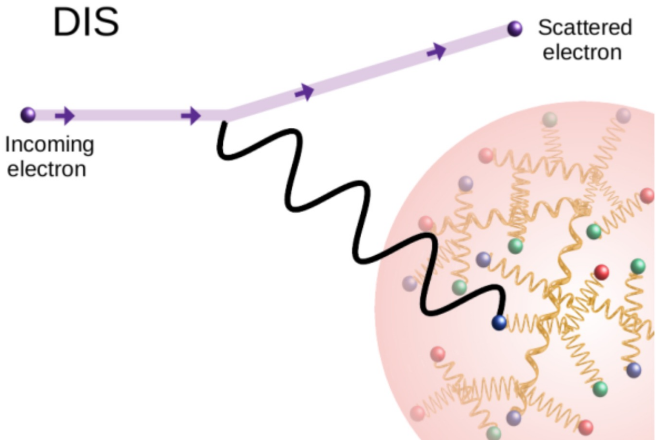
$$2m_p W_1(\nu, Q^2) \to F_1(\omega)$$

$$W > 2.6 \text{ GeV}$$

$$2 < Q^2 < 20 \text{ GeV}^2$$

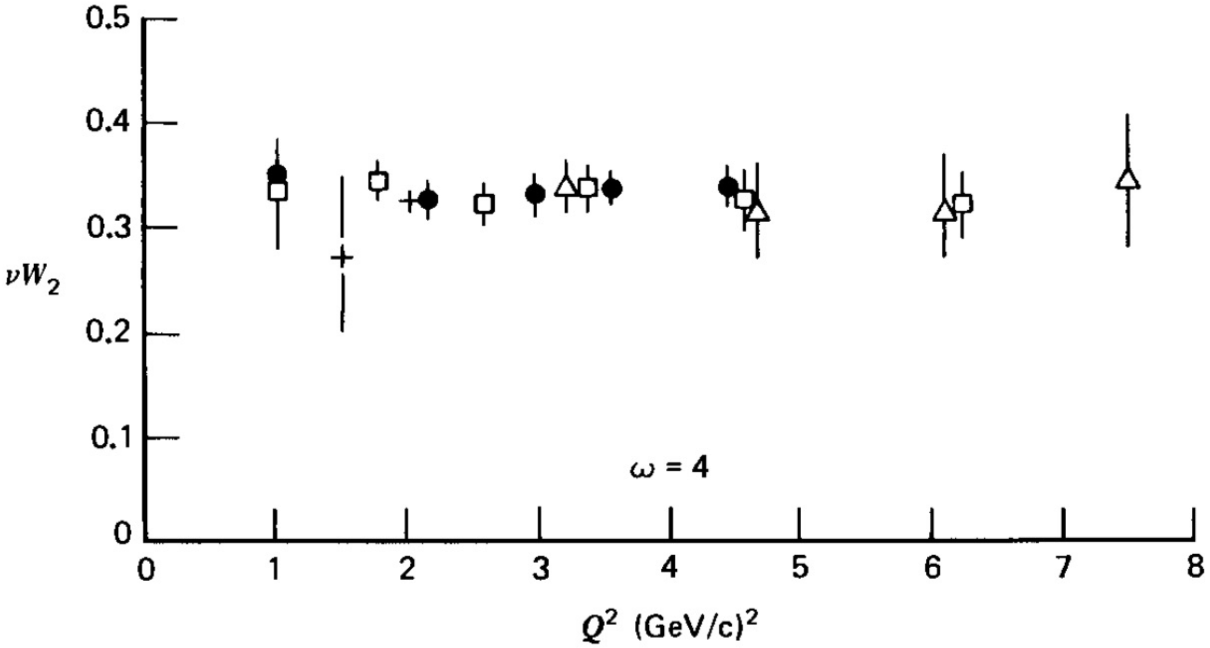


Bjorken scaling



Sezione d'urto:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_M \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2(\theta/2)\right]$$



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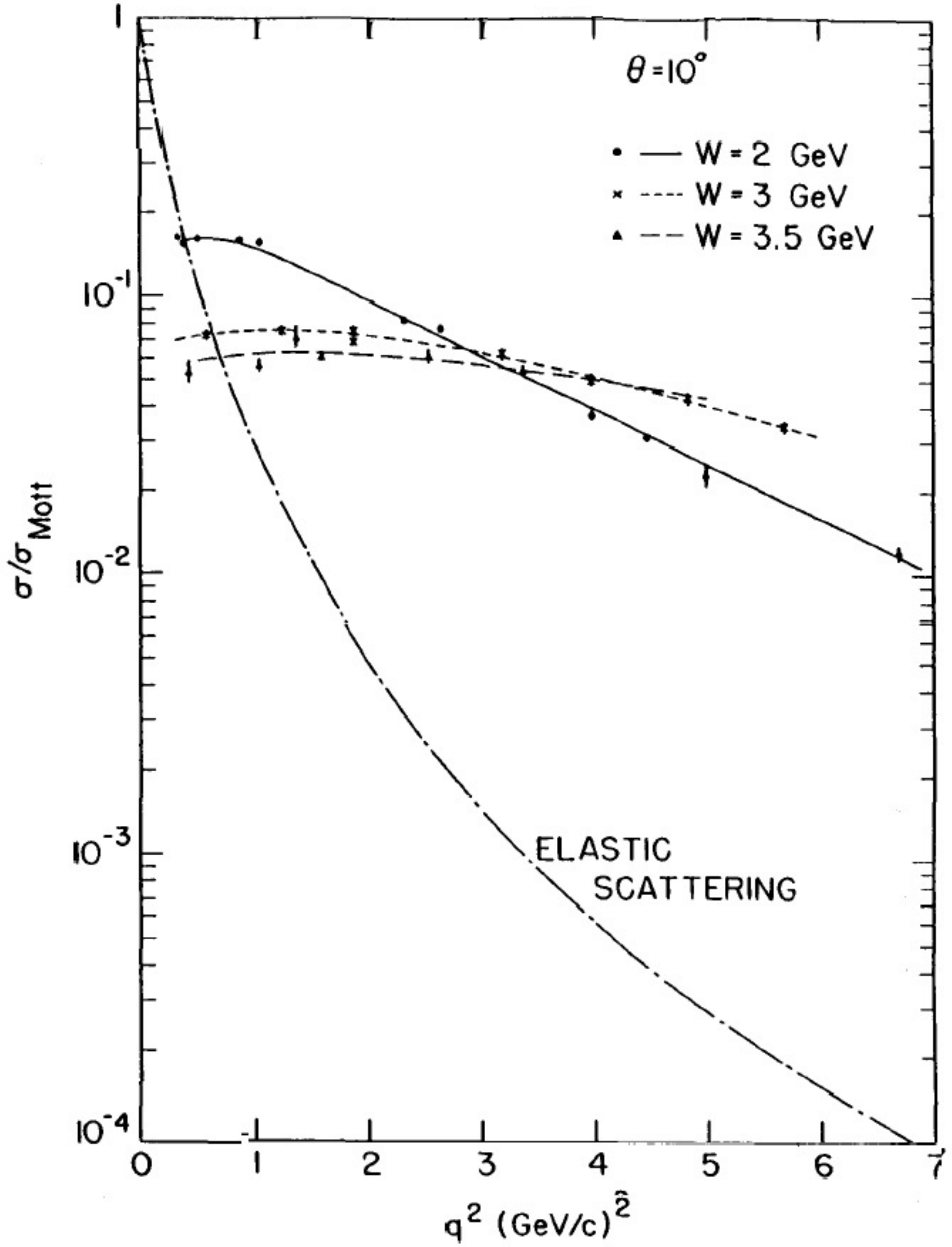
$$\nu W_2(\nu, Q^2) \to F_2(\omega)$$

$$2m_pW_1(\nu, Q^2) \to F_1(\omega)$$

Q^2 (in)dependence



Leggera dipendenza da Q^2
(rispetto all diffusione elastica)

La misura suggerisce la possibile
esistenza di costituenti "puntiformi" del protone

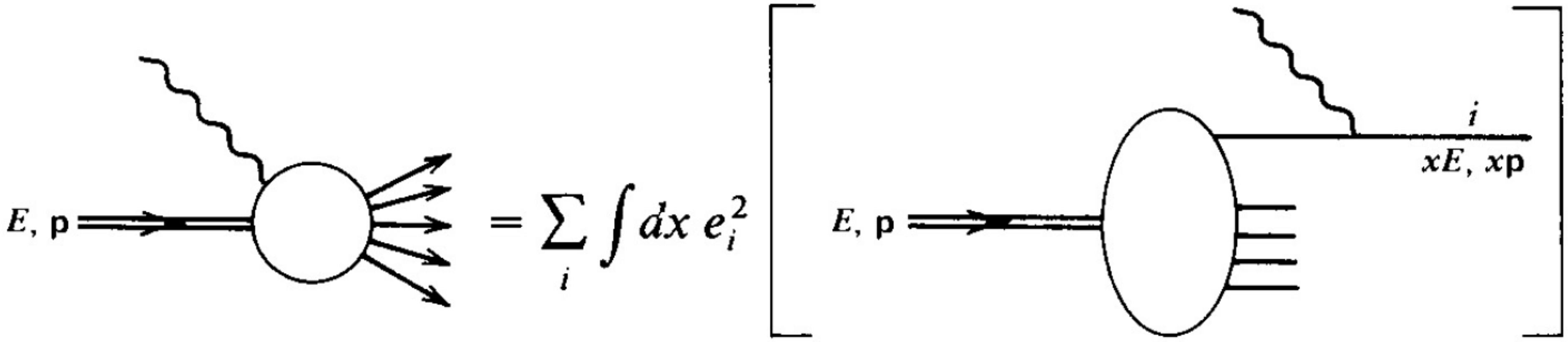
$$F(\mathbf{q}) = \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3 r$$

$$\text{se } \rho(\mathbf{r}) = \delta(\mathbf{r}) \rightarrow F(\mathbf{q}) = 1$$

"Pointlike" ⟹ weak Q^2 dependence

Il modello a partoni

Feynman (1969): il protone e' una collezione di particelle ed il fotone virtuale interagisce con il singolo partone. La sezione d'urto e' la somma incoerente delle sezioni d'urto fotone-partone



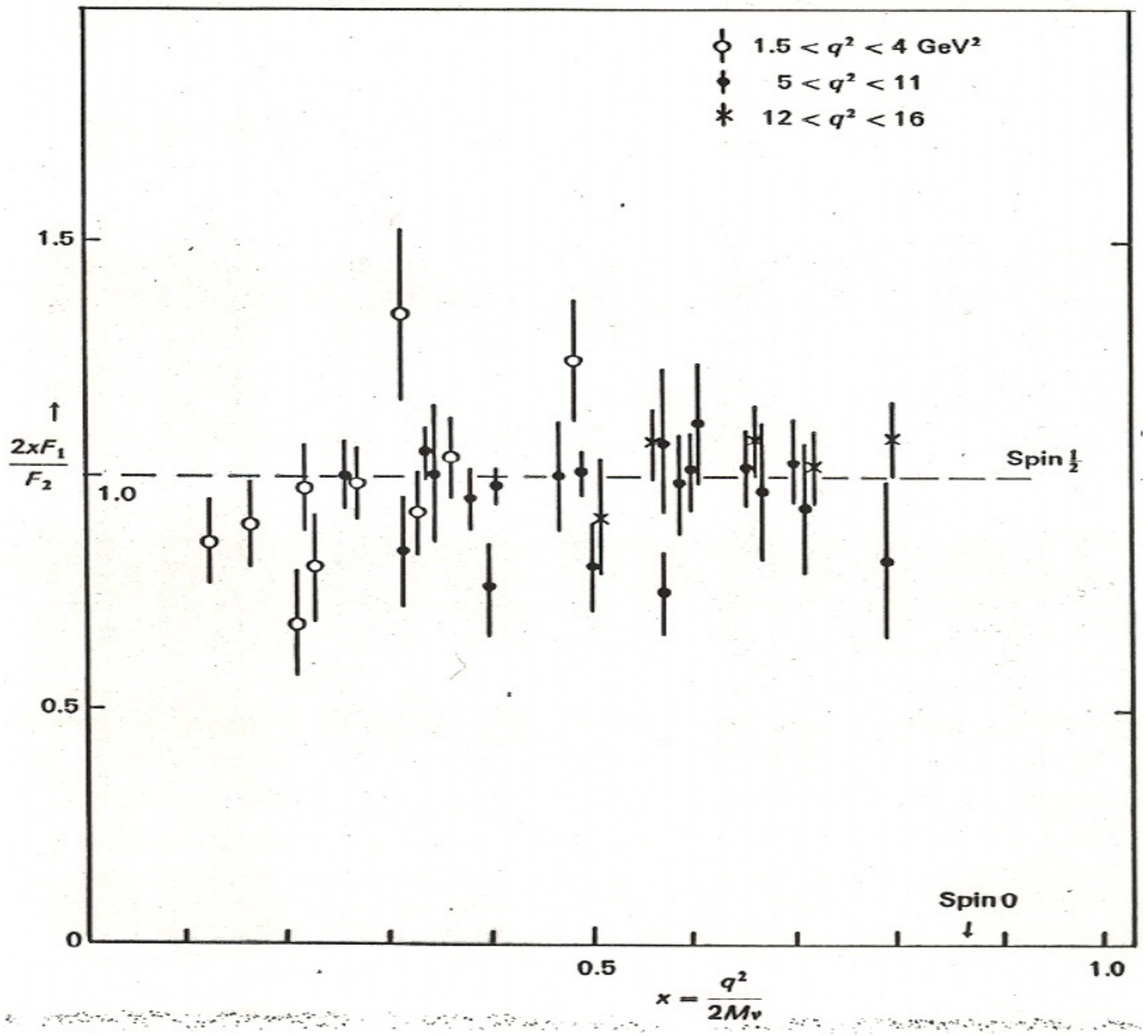
$$\nu W_2(\nu, Q^2) \to F_2(x) = \sum_i e_i^2 x f_i(x)$$

$$m_p W_1(\nu, Q^2) \to F_1(x) = \frac{1}{2x} F_2(x)$$

Partoni e quarks

Possiamo identificare i partoni con quark?
- lo spin deve essere pari a 1/2
- la carica elettrica deve essere frazionaria (+2/3 , -1/3)

Per lo spin una prima risposta venne dalla verifica sperimentale della relazione di Callan-Gross



$$R = \frac{\sigma_L}{\sigma_T} = \frac{W_2}{W_1}\left(1 + \frac{\nu^2}{q^2}\right) - 1$$

Per spin 1/2 e nella "DIS Region"

$$R \to 0 \quad ; \quad F_2(x) = 2xF_1(x)$$

Partoni e quarks

Per la carica elettrica frazionaria (+2/3 , -1/3) le prime indicazioni vennero dalla “F_2 sum rule”:

$$\frac{1}{2}\int \frac{[\nu W_2^p(\omega)+\nu W_2^n(\omega)]}{\omega^2}d\omega =$$

$$\frac{1}{2}\int [F_2^p(x)+F_2^n(x)]dx = \frac{Q_u^2+Q_d^2}{2}\int x[u_p(x)+\bar{u}_p(x)+d_p(x)+\bar{d}_p(x)]dx$$

$$\frac{1}{2}\int [F_2^p(x)+F_2^n(x)]dx = \left(\frac{Q_u^2+Q_d^2}{2}\right)(?) = \frac{5}{18}\cdot(?) \simeq 0.28\cdot(?)$$

Risultato sperimentale (SLAC-MIT):

$$\frac{1}{2}\int [F_2^p(x)+F_2^n(x)]dx = 0.14 \pm 0.005$$

Conclusione: consistente con il modello a quark se i quark/antiquarks trasportano il 50% della quantita’ di moto del protone ed i gluoni la restante parte

Gargamelle

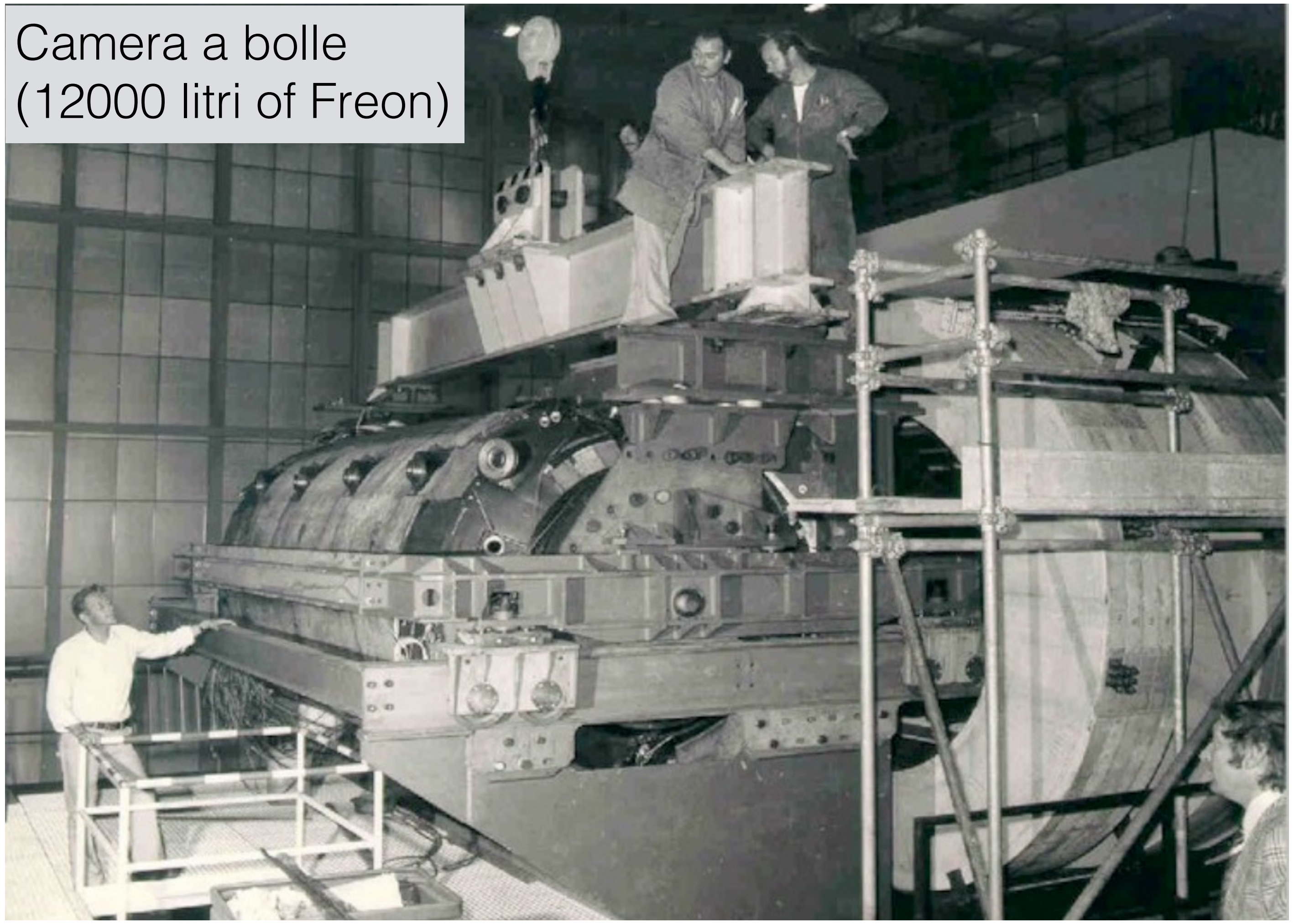
Possiamo identificare i partoni con quark?
- lo spin devre essere pari a 1/2
- la carica frazionaria deve essere frazionaria (+2/3 , -1/3) ?

Ulteriore conferme vennero dai primi **confronti tra i dati relativi alla diffusione di elettroni e neutrini**

Esperimento Gargamelle
(CERN 24 GeV PS Synchroton)

Processi:

$$\nu_\mu + N \to \mu^- + X$$
$$\bar{\nu}_\mu + N \to \mu^+ + X$$


Camera a bolle
(12000 litri of Freon)

Gargamelle

Possiamo identificare i partoni con quark?
- lo spin devre essere pari a 1/2
- la carica frazionaria deve essere frazionaria (+2/3 , -1/3) ?

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Esperimento Gargamelle
(CERN 24 GeV PS Synchroton)

Processi:

$$\nu_\mu + N \rightarrow \mu^- + X$$
$$\bar{\nu}_\mu + N \rightarrow \mu^+ + X$$

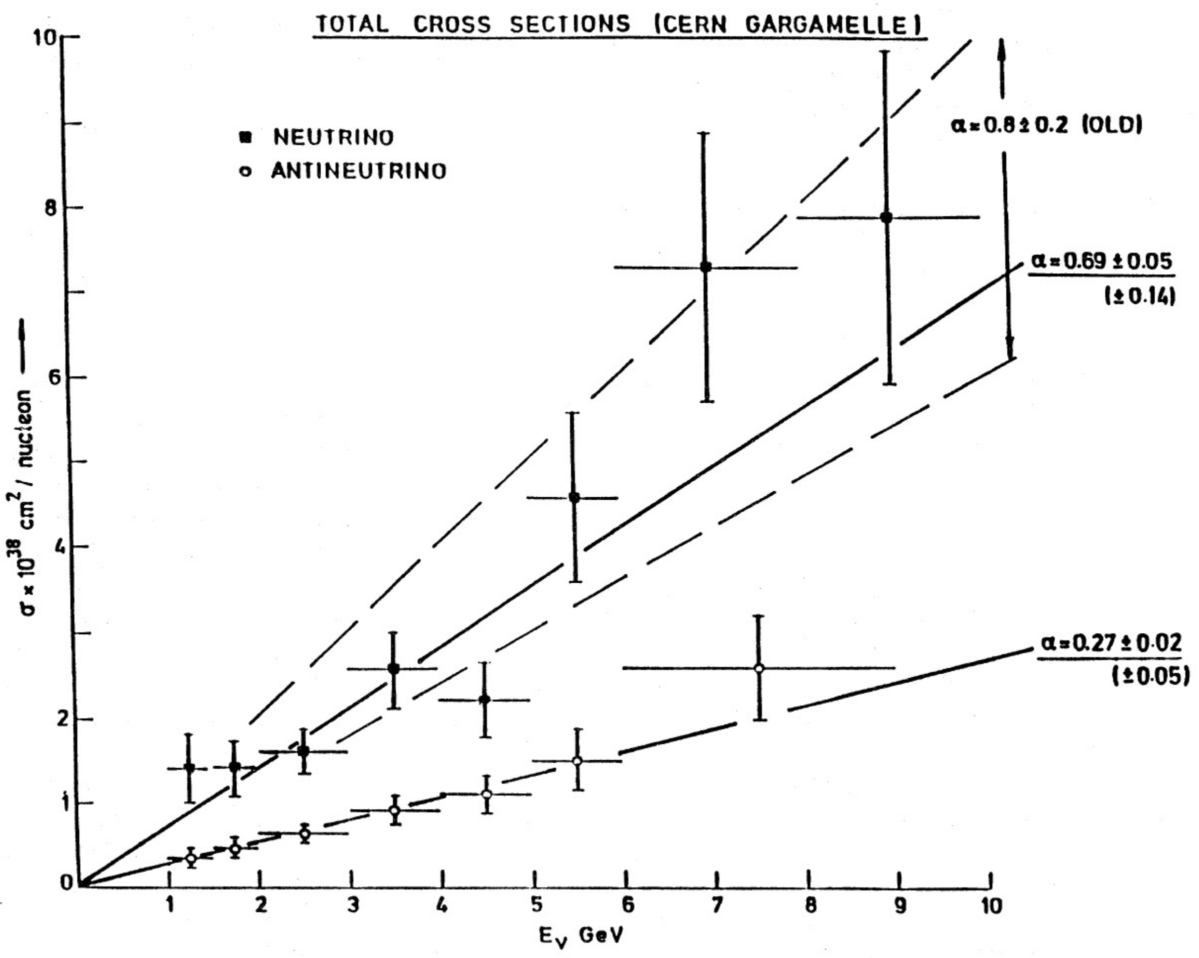
Neutrino

Andamento lineare di $\sigma_{tot}(\nu_\mu)$, $(\sigma_{tot}(\bar{\nu}_\mu))$ fornisce un ulteriore conferma all'esistenza di costituenti puntiformi nel protone ed neutrone

Sezioni d'urto ("scaling region"):

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dx\,dy} = \frac{G^2 ME}{\pi} \left[(1-y)F_2(x) + \frac{y^2}{2}\left[2xF_1(x)\right] \mp y\left(1-\frac{y}{2}\right)xF_3(x)\right],$$

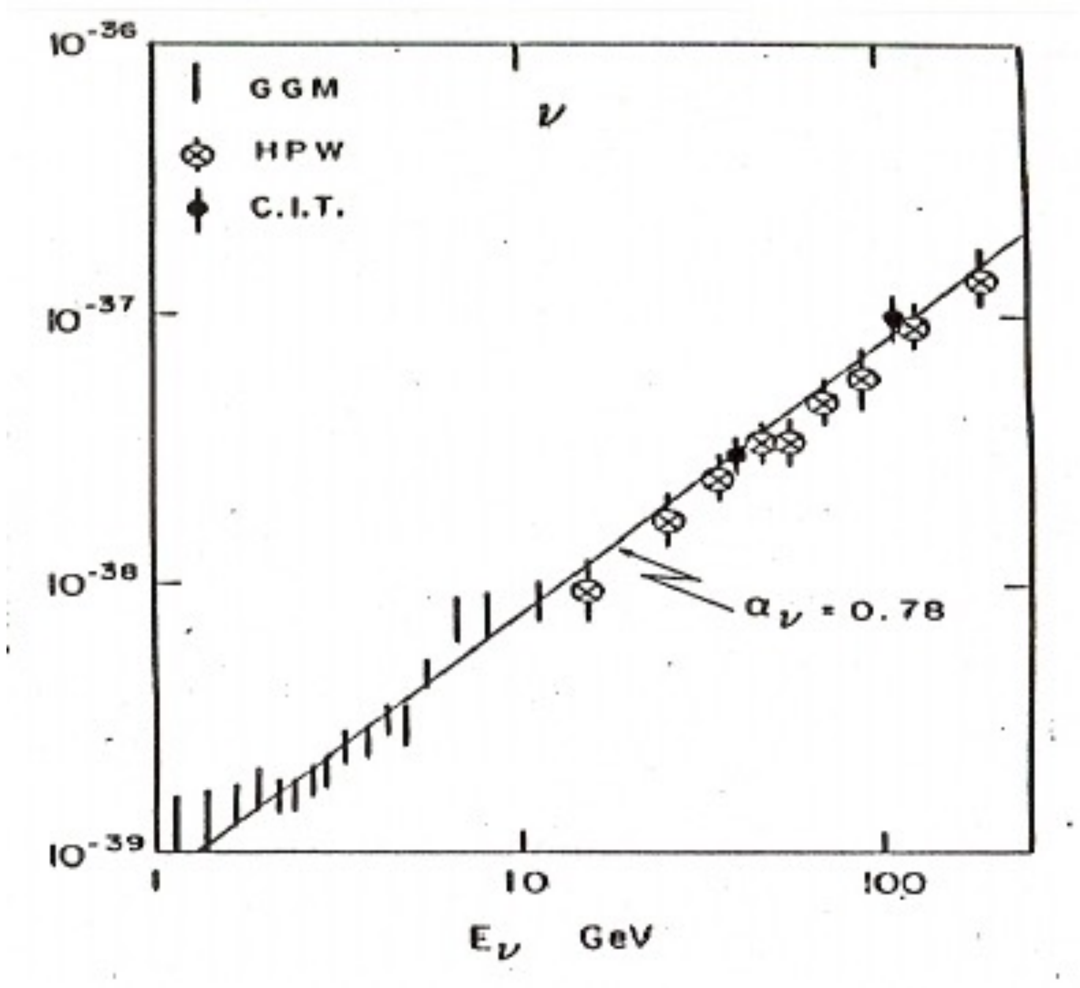
$$\sigma_{tot}(\nu_\mu) = \alpha_\nu E$$
$$\sigma_{tot}(\bar{\nu}_\mu) = \alpha_{\bar{\nu}} E$$

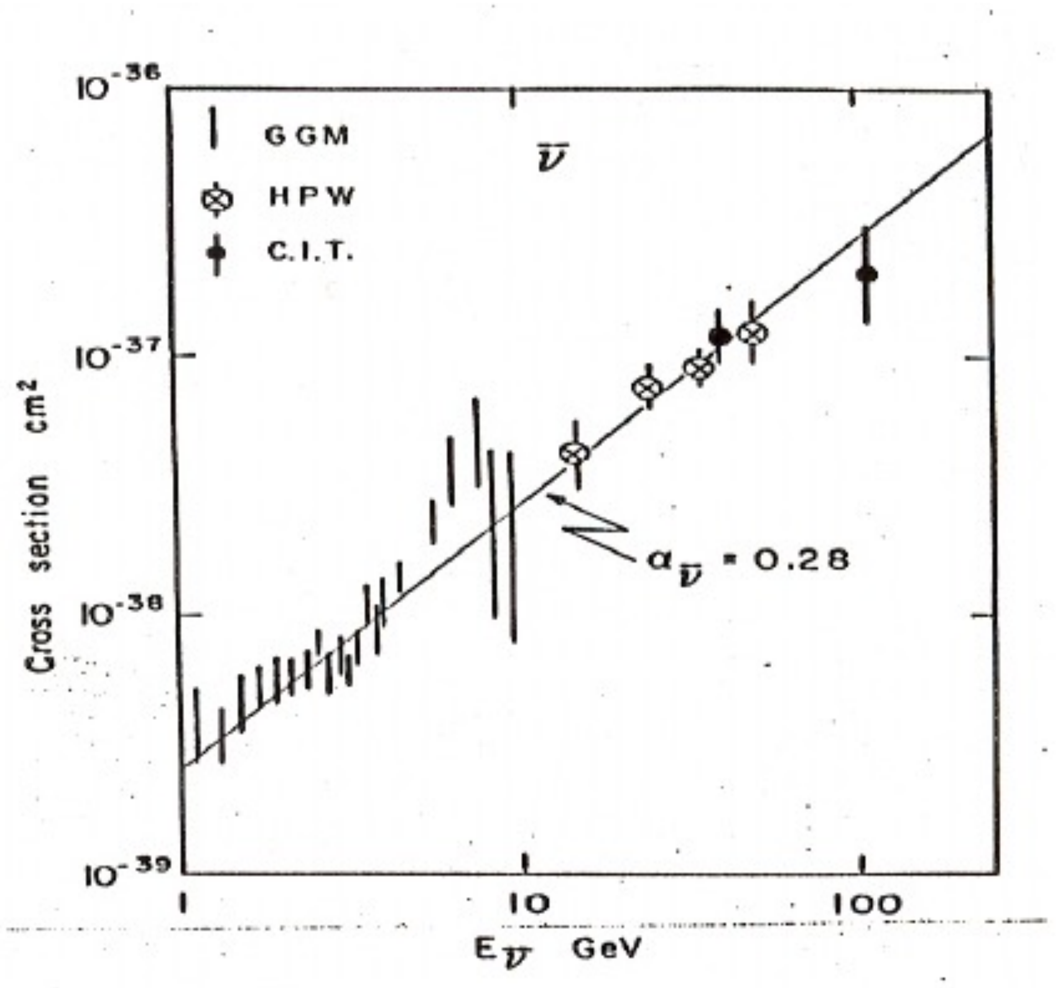


ICHEP 1972
Perkins "...the preliminary data on the cross sections provide an astonishing verification for the Gell-Mann/Zweig quark model of hadrons."

Neutrino

Andamento lineare di $\sigma_{tot}(\nu_\mu)$, $(\sigma_{tot}(\bar{\nu}_\mu))$ fornisce un ulteriore conferma all’esistenza di costituenti puntiformi nel protone ed neutrone





ICHEP 1972

Perkins "...the preliminary data on the cross sections provide an astonishing verification for the Gell-Mann/Zweig quark model of hadrons.”

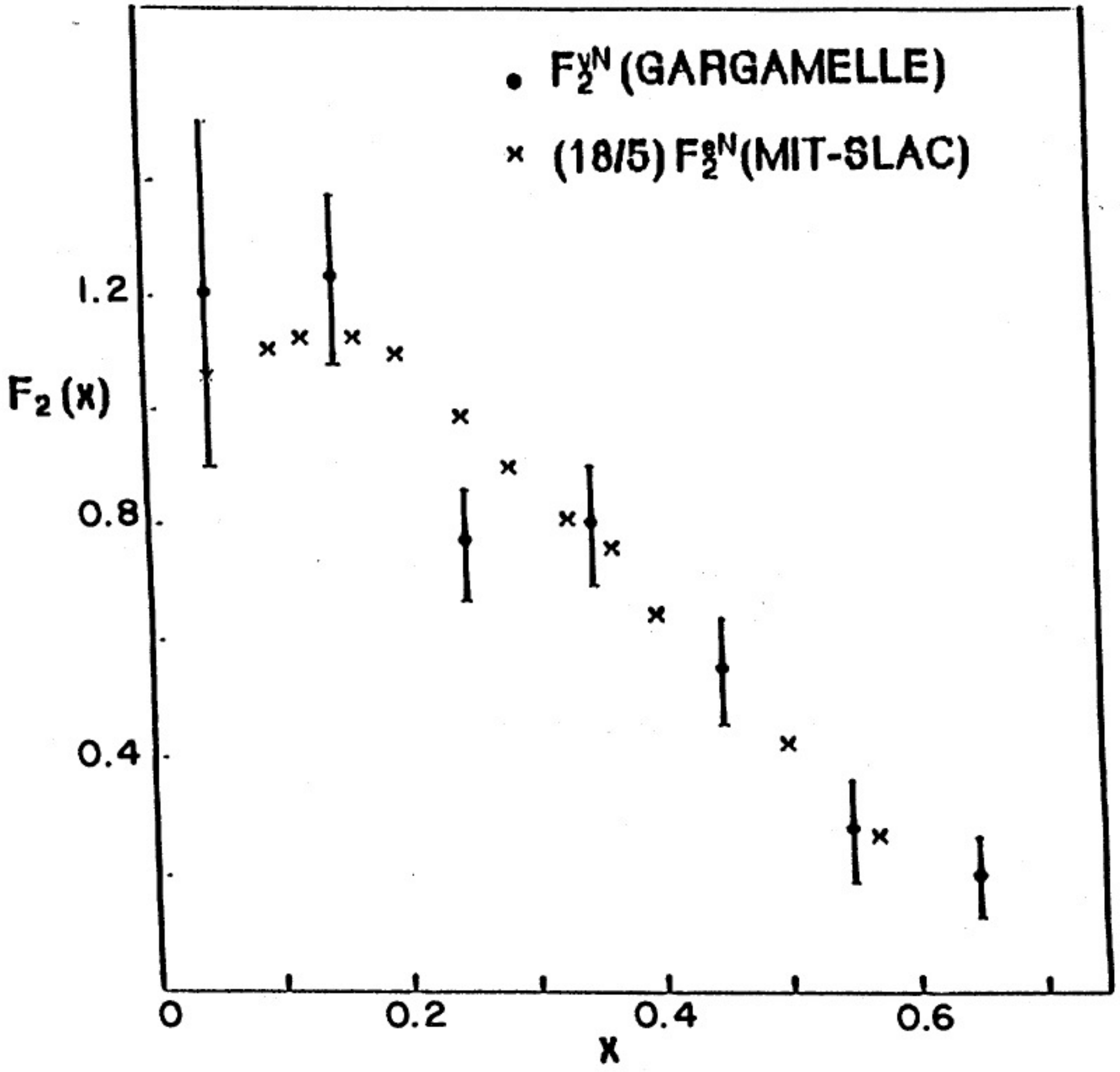
Carica Partoni

I partoni hanno carica frazionaria (+2/3 , -1/3) ?
La risposta venne dal primo confronto tra i dati relativi alla diffusione di elettroni e neutrini

$$\frac{\frac{1}{2}\int\left[F_2^{\nu n}(x)+F_2^{\nu p}(x)\right]dx}{\frac{1}{2}\int\left[F_2^{en}(x)+F_2^{ep}(x)\right]dx}=\frac{2}{(Q_u^2+Q_d^2)}=\frac{2}{(2/3)^2+(1/3)^2}=\frac{18}{5}=3.6$$

Risultato sperimentale (SLAC-MIT, Gargamelle): 3.4 ± 0.7

Altri risultati con i neutrini



Primo confronto $F_2^{\nu N}$ vs F_2^{eN}

$$F_2^{\nu N} \sim (\frac{18}{5}) F_2^{eN}$$

Altri risultati con i neutrini

$$\frac{1}{2}\int [F_2^{\nu p}(x) + F_2^{\nu n}(x)]\,dx = \int x[u_p(x) + \bar{u}_p(x) + d_p(x) + \bar{d}_p(x)]dx$$

Risultato sperimentale (Gargamelle): 0.49 ± 0.7

50% della quantita' di moto del protone e' trasportata dai quarks.
In accordo con i risultati SLAC-MIT

$$\frac{1}{2}\int [F_3^{\nu p}(x) + F_3^{\nu n}(x)]\,dx = \text{Nr. quark di valenza}$$

Risultato sperimentale (Gargamelle): 3.2 ± 0.6

Consistente con il Modello a quark.

CromoDinamica Quantistica (QCD)

$$\mathcal{L} = -\frac{1}{4}F^A_{\alpha\beta}F^{\alpha\beta}_A + \sum_{\text{flavours}} \bar{q}_a(i\not{D} - m)_{ab}q_b + \mathcal{L}_{\text{gauge-fixing}}$$

$$F^A_{\alpha\beta} = \partial_\alpha \mathcal{A}^A_\beta - \partial_\beta \mathcal{A}^A_\alpha - g f^{ABC}\mathcal{A}^B_\alpha \mathcal{A}^C_\beta$$

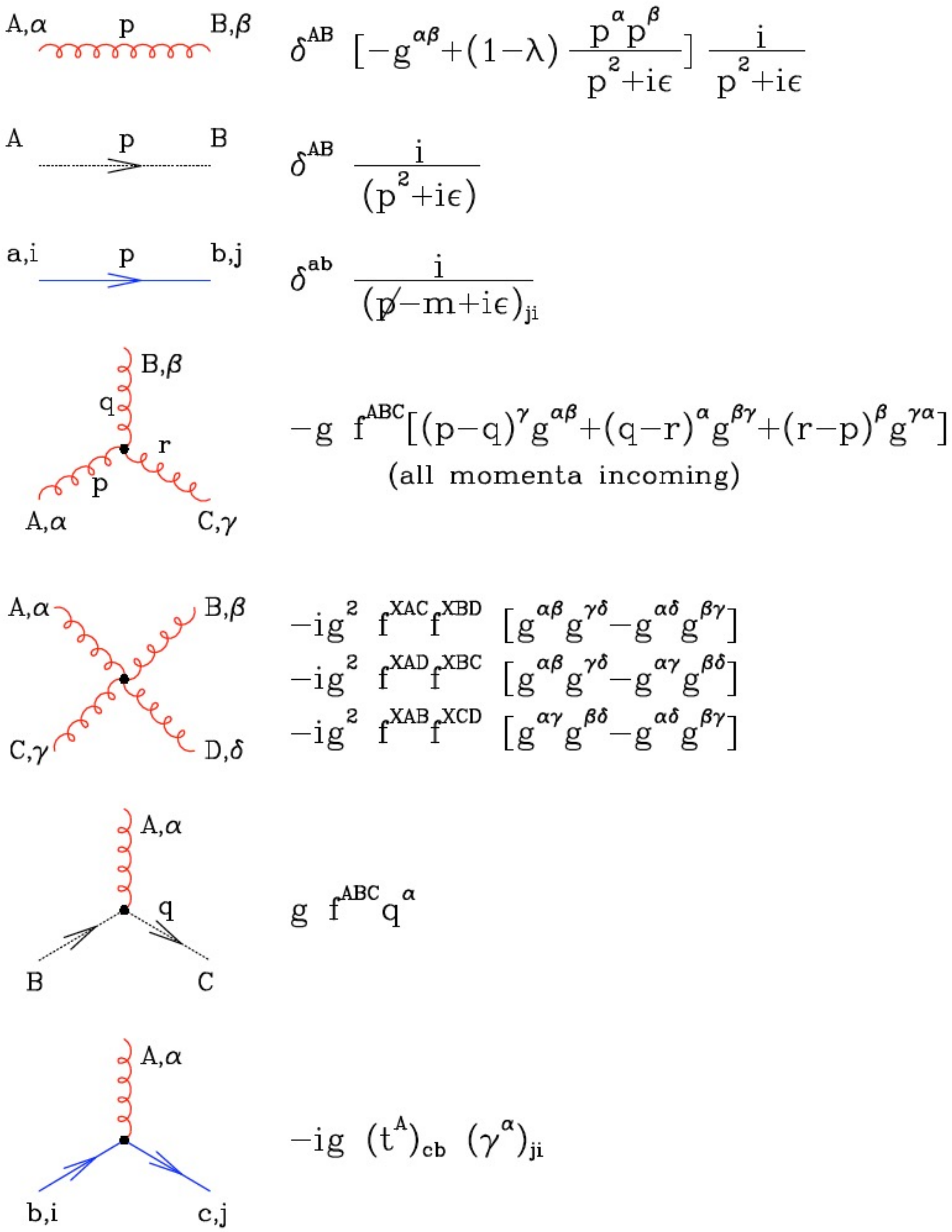
$$(D_\alpha)_{ab} \quad = \quad \partial_\alpha \delta_{ab} + ig\left(t^C \mathcal{A}^C_\alpha\right)_{ab}$$

$$\frac{\partial \alpha_S(Q)}{\partial \tau} = \beta(\alpha_S(Q)) \qquad \tau = \ln\left(\frac{Q^2}{\mu^2}\right)$$

$$\beta(\alpha_S) = -b\alpha_S^2(1 + b'\alpha_S) + \mathcal{O}(\alpha_S^4)$$

$$b = \frac{(11C_A - 2N_f)}{12\pi}, \quad b' = \frac{(17C_A^2 - 5C_A N_f - 3C_F N_f)}{2\pi(11C_A - 2N_f)}$$

$$\alpha_S(Q) = \frac{1}{b\ln(Q^2/\Lambda^2)}\left[1 - \frac{b'}{b}\frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right] \qquad \text{(NLO)}$$



$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^\alpha p^\beta}{p^2+i\epsilon}\right]\frac{i}{p^2+i\epsilon}$

$\delta^{AB}\frac{i}{(p^2+i\epsilon)}$

$\delta^{ab}\frac{i}{(\not{p}-m+i\epsilon)_{ji}}$

$-g\ f^{ABC}[(p-q)^\gamma g^{\alpha\beta}+(q-r)^\alpha g^{\beta\gamma}+(r-p)^\beta g^{\gamma\alpha}]$

(all momenta incoming)

$-ig^2\ f^{XAC}f^{XBD}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\delta}g^{\beta\gamma}]$

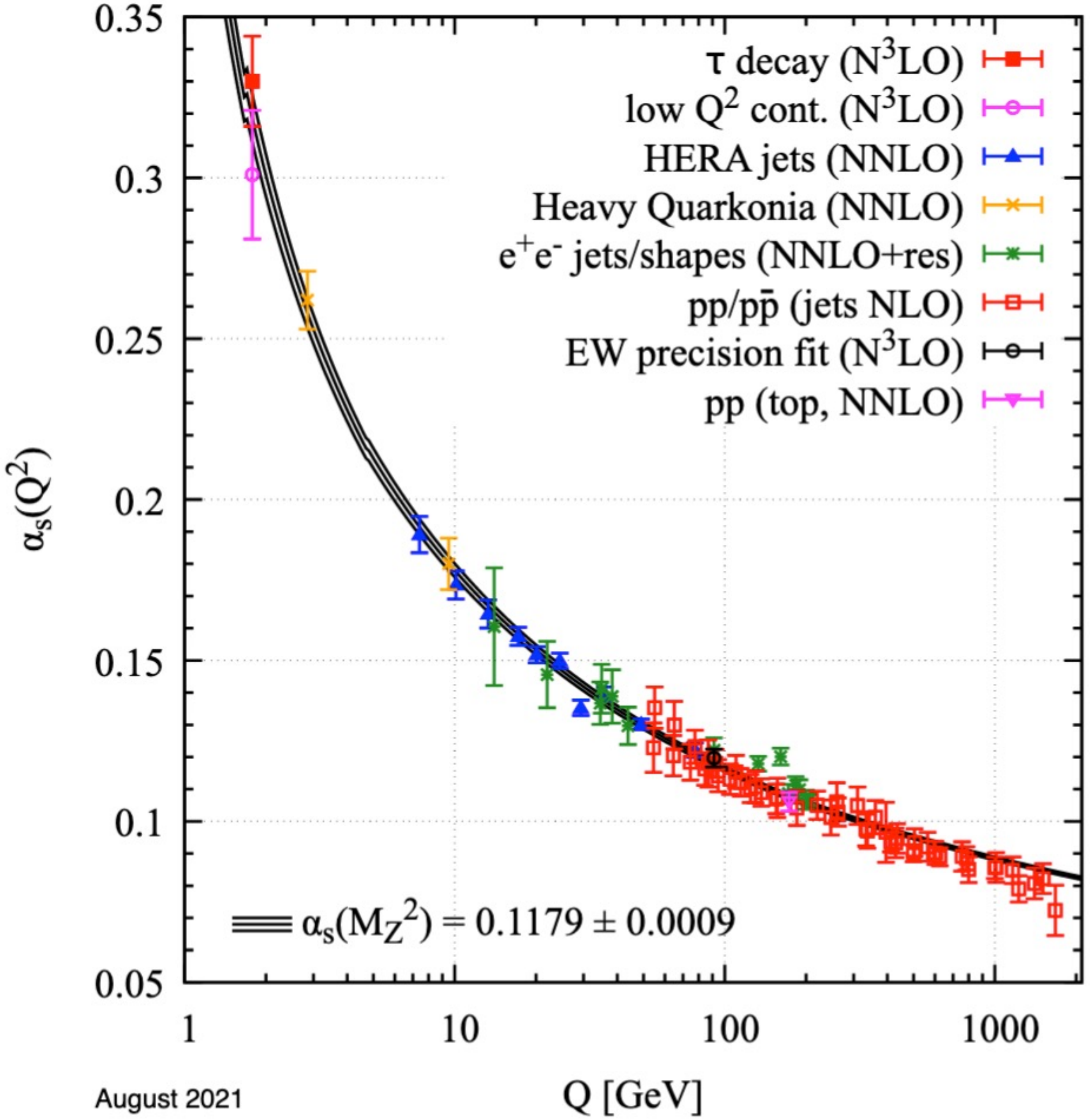
$-ig^2\ f^{XAD}f^{XBC}\ [g^{\alpha\beta}g^{\gamma\delta}-g^{\alpha\gamma}g^{\beta\delta}]$

$-ig^2\ f^{XAB}f^{XCD}\ [g^{\alpha\gamma}g^{\beta\delta}-g^{\alpha\delta}g^{\beta\gamma}]$

$g\ f^{ABC}q^\alpha$

$-ig\ (t^A)_{cb}\ (\gamma^\alpha)_{ji}$

“Running coupling constant”



DGLAP Equations

Equazioni di evoluzione di Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP)

$$\frac{dq_i(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[q_i(y,Q^2)P_{qq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{qg}\left(\frac{x}{y}\right)\right]$$

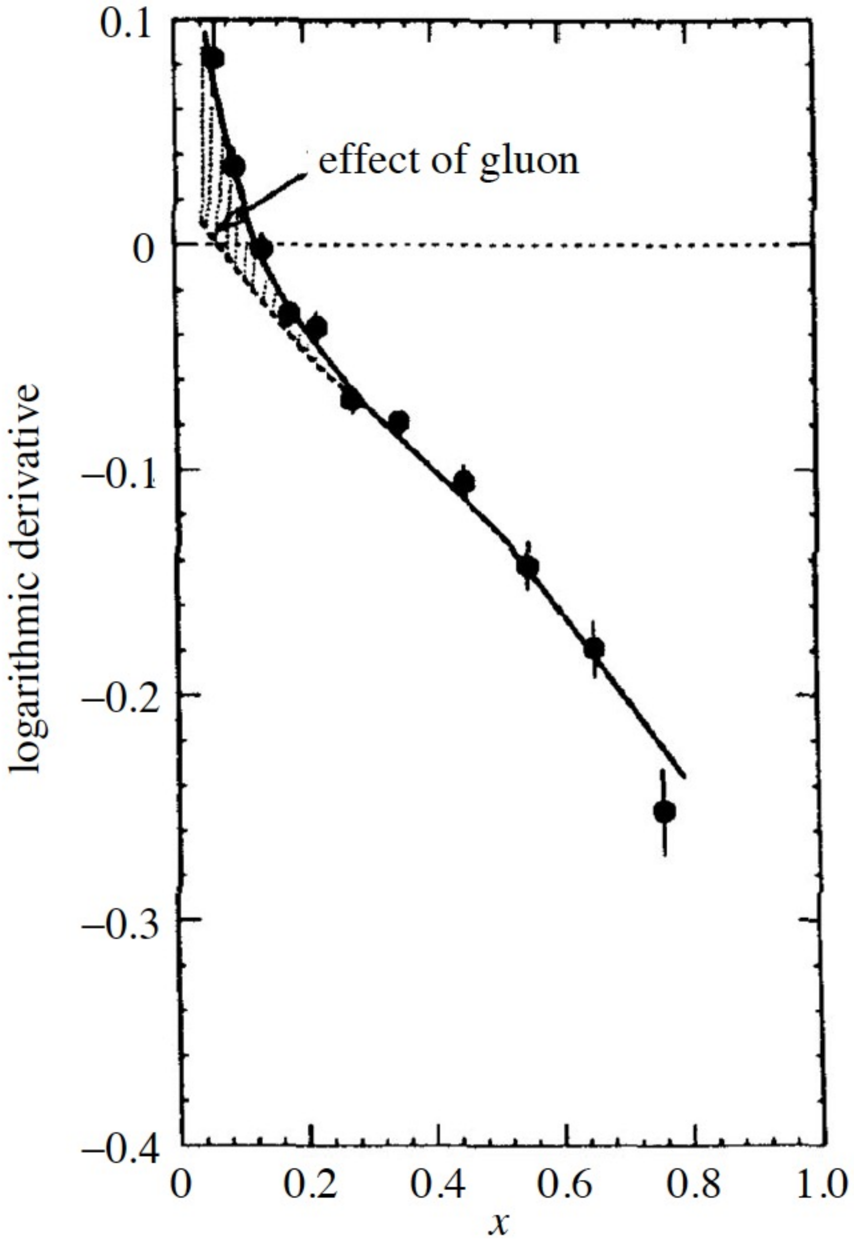
$$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[\sum_i q_i(y,Q^2)P_{gq}\left(\frac{x}{y}\right) + g(y,Q^2)P_{gg}\left(\frac{x}{y}\right)\right]$$

Splitting functions:

$$\begin{aligned}
P^{(0)}_{\mathrm{qq}}(z) &= \frac{4}{3}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\,\delta(1-z)\right] \\
P^{(0)}_{\mathrm{qg}}(z) &= \frac{1}{2}\left[z^2+(1-z)^2\right] \\
P^{(0)}_{\mathrm{gq}}(z) &= \frac{4}{3}\left[\frac{1+(1-z)^2}{z}\right] \\
P^{(0)}_{\mathrm{gg}}(z) &= 6\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \left(\frac{11}{12}-\frac{n_f}{18}\right)\delta(1-z)\right]
\end{aligned}$$

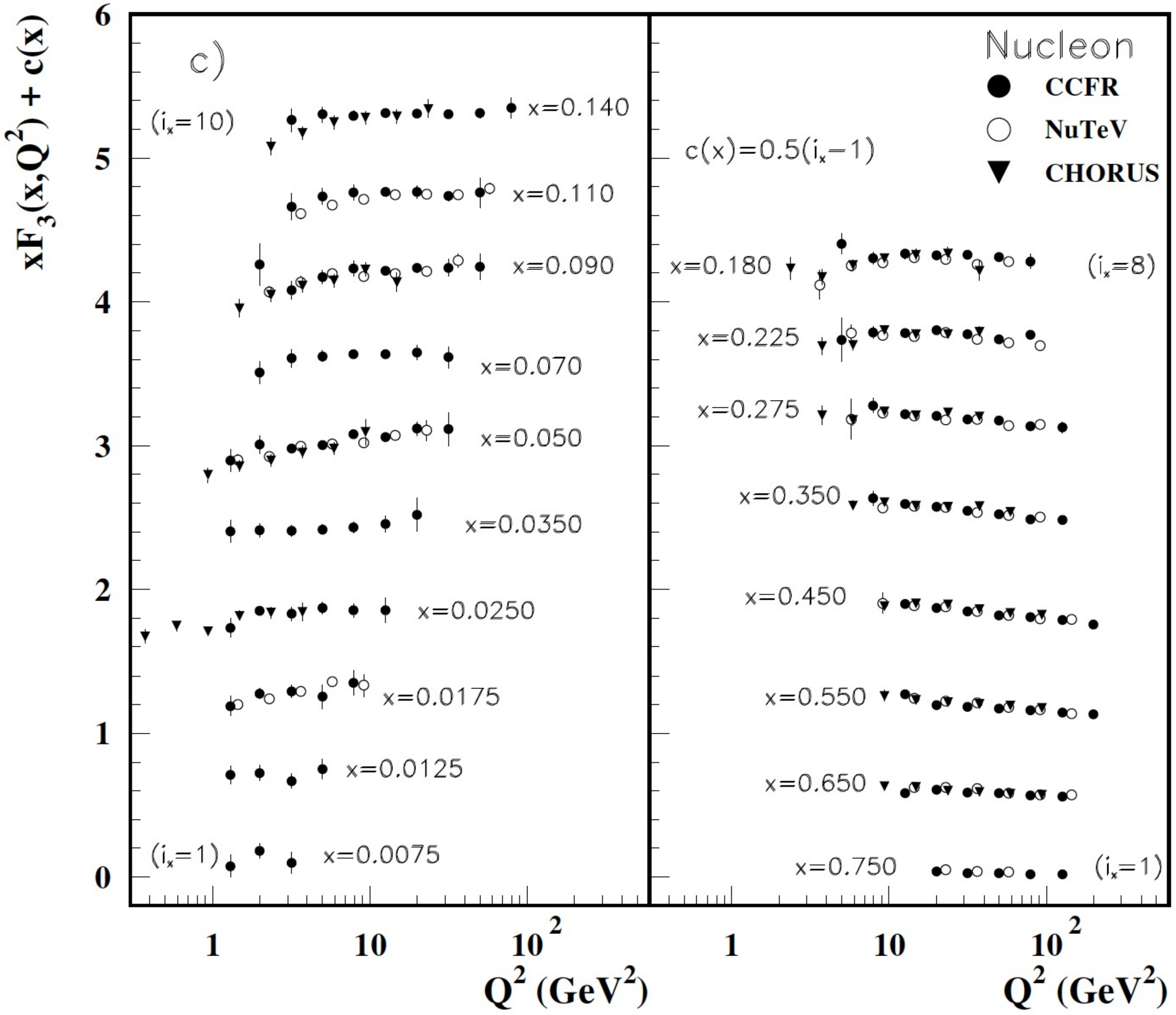
Scaling violation

$$\frac{\partial F_2}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{F_2(\xi,Q^2)P_{qq}\left(\frac{x}{\xi},\alpha_s\right) + 2fG(\xi,Q^2)P_{qg}\left(\frac{x}{\xi},\alpha_s\right)\right\}$$



Scaling violation

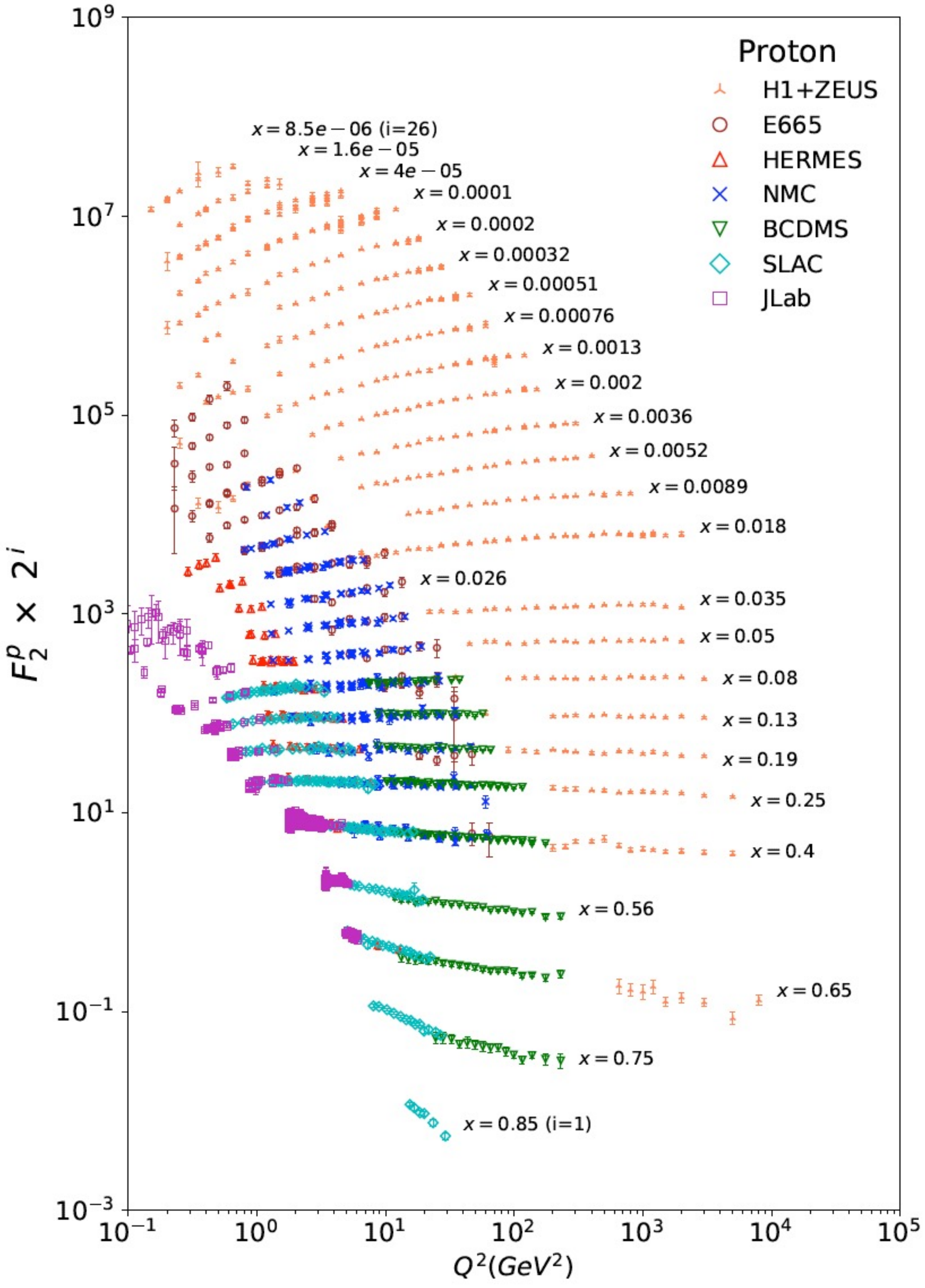
$$\frac{\partial xF_3}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{\xi F_3(\xi,Q^2)P_{qq}\left(\frac{x}{\xi},\alpha_s\right)\right\}$$

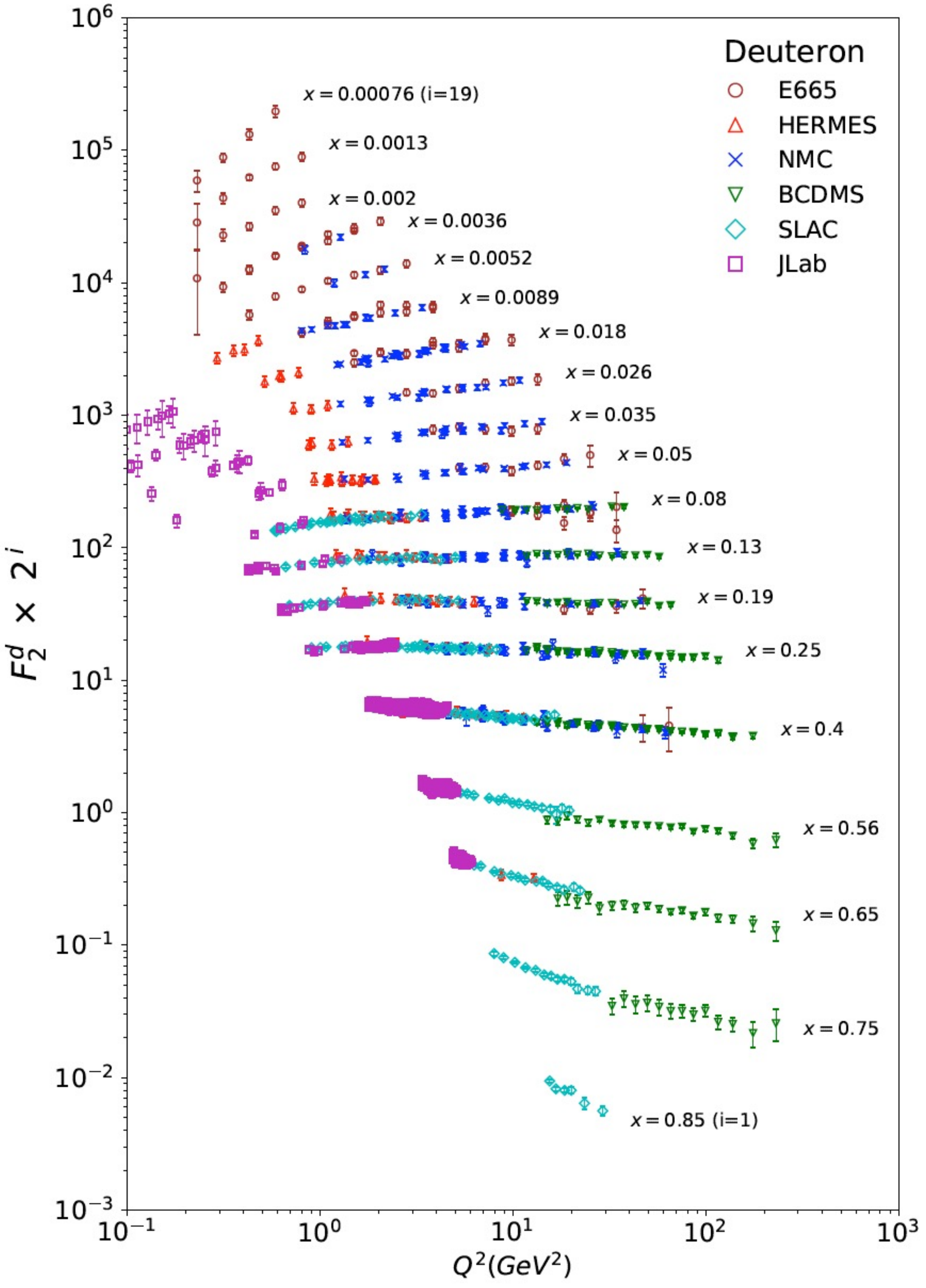


Altri esperimenti a targhetta fissa

Esperimenti con fasci di muoni: EMC, BFP, NMC, BCDMS, etc.

Esperimenti con fasci di neutrini: CCFR, HPWF, CDHSW, CHARM,WA24,WA21, etc.



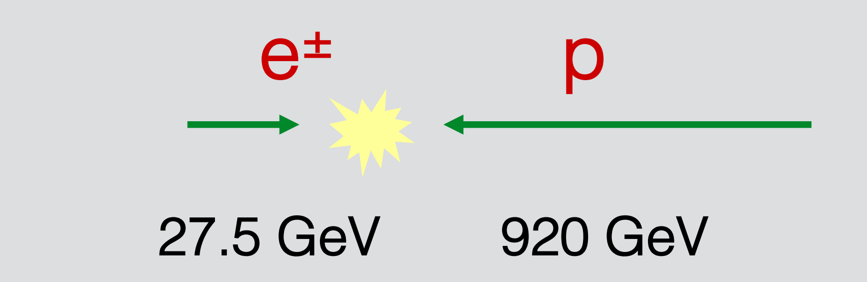


II. HERA

- H1-ZEUS
- High-lights
- LHC & PDFs

HERA

Il primo ed unico collisionatore e-p al mondo



$\sqrt{s}$ = 318 GeV

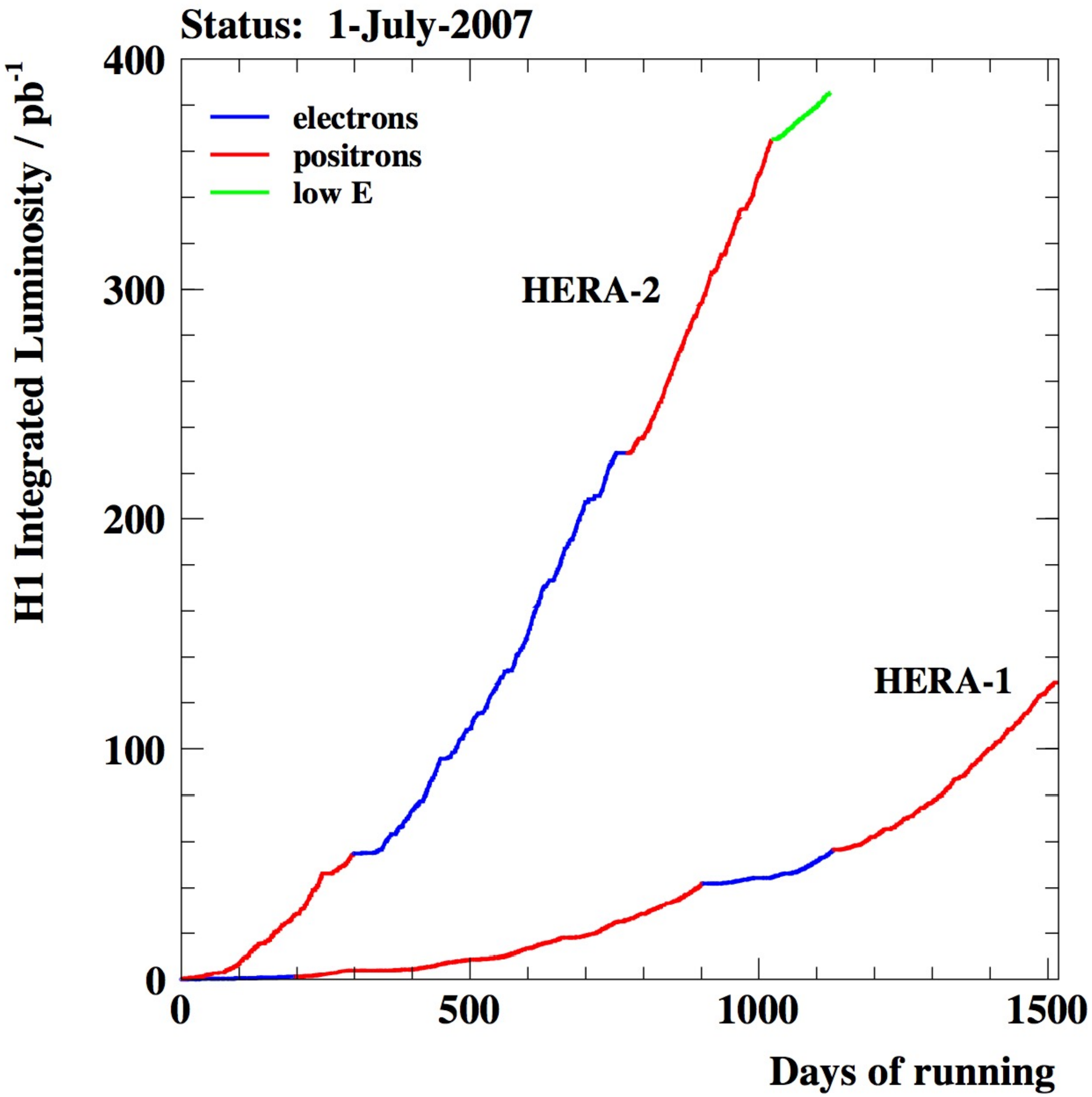
“Equivalente” ad un esperimento a targhetta fissa con $e^{\pm}$ di 50 TeV

HERA Operation

HERA-I (1992-2000)
Ee=27.6 GeV
Ep=820 & 920 GeV
L_{int} ~ 130 pb^{-1} per experiment
Mostly $e^{+}p$

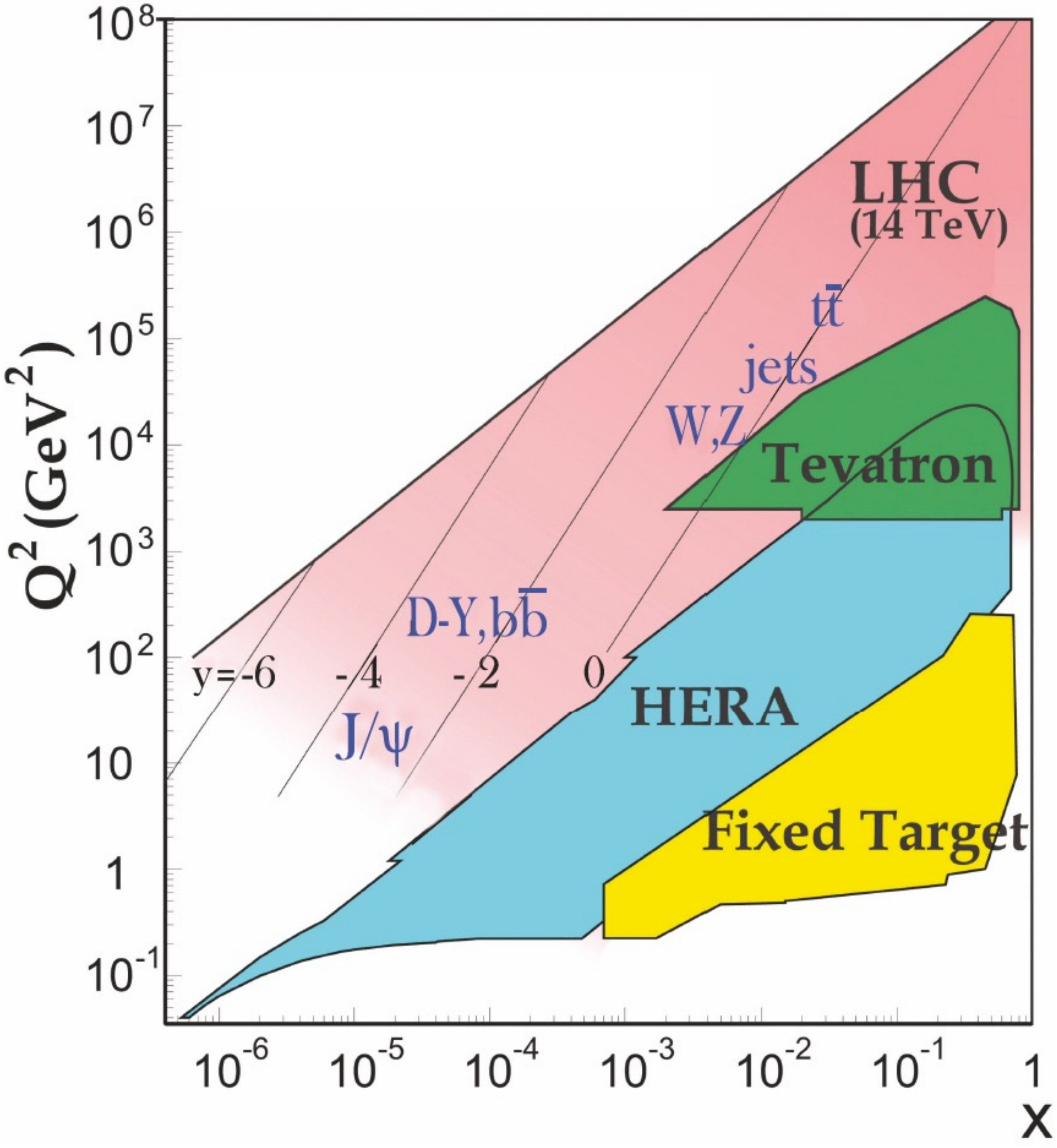
HERA-II (2003-2007)
Ee=27.6 GeV
Ep=920 GeV
L_{int} ~ 360 pb^{-1} per experiment
Longitudinally polarized lepton beams
Similar amounts of $e^{+}p$ and $e^{-}p$

Low Energy Run 2007
Ee=27.6 GeV
Ep=460 & 575 GeV
Runs at reduced $\sqrt{s}$:
225 GeV (LER), 252 (MER) GeV
Dedicated F_L measurements



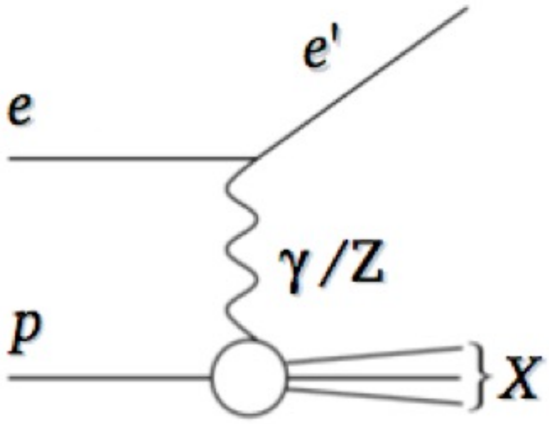
1 fb^{-1} Integrated lumi, H1+ZEUS

HERA Kinematic domain

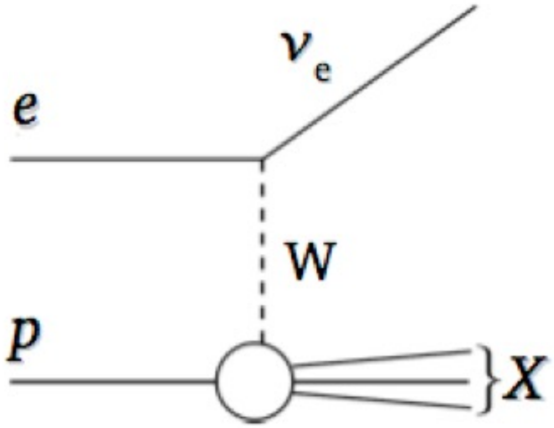


DIS processes and cross sections

$\mathrm{NC}: ep \rightarrow e'X$



$\mathrm{CC}: ep \rightarrow \nu_e X$



Kinematic variables:

- Virtuality exchanged boson

$$Q^2 = -q^2 = -(k-k')^2$$

- Bjorken scaling variable

$$x = \frac{Q^2}{2p \cdot q}$$

“Reduced” Cross sections

NC:

$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{d^2\sigma^{e^\pm p}_{\mathrm{NC}}}{dx dQ^2} \cdot \frac{Q^4 x}{2\pi\alpha^2 Y_+} = F_2 \mp \frac{Y_-}{Y_+} x F_3 - \frac{y^2}{Y_+} F_L$$

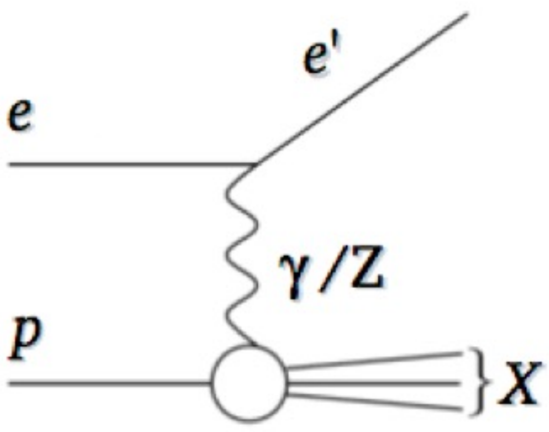
CC:

$$\sigma^{\pm}_{r,\mathrm{CC}} = \frac{d^2\sigma^{e^\pm p}_{\mathrm{CC}}}{dx dQ^2} \cdot \frac{2\pi x}{G_F^2} \left[\frac{M_W^2 + Q^2}{M_W^2}\right]^2 = \frac{1}{2}\left(Y_+ W_2^{\pm} \mp Y_- x W_3^{\pm} - y^2 W_L^{\pm}\right)$$

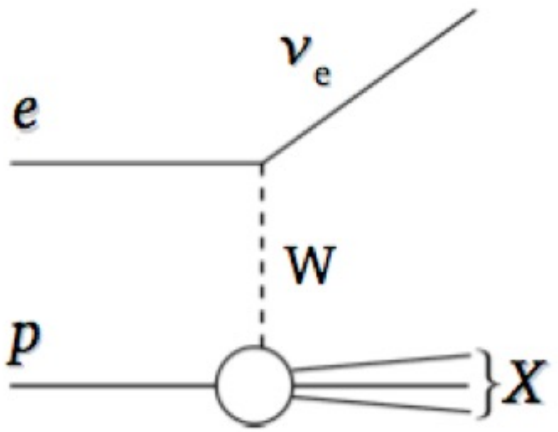
$$\text{with} \quad Y_{\pm} = 1 \pm (1-y)^2$$

DIS processes and cross sections

NC : $e\,p \to e'\,X$



CC : $e\,p \to \nu_e\,X$



Kinematic variables:

- Virtuality exchanged boson

$$Q^2 = -q^2 = -(k-k')^2$$

- Bjorken scaling variable

$$x = \frac{Q^2}{2p\cdot q}$$

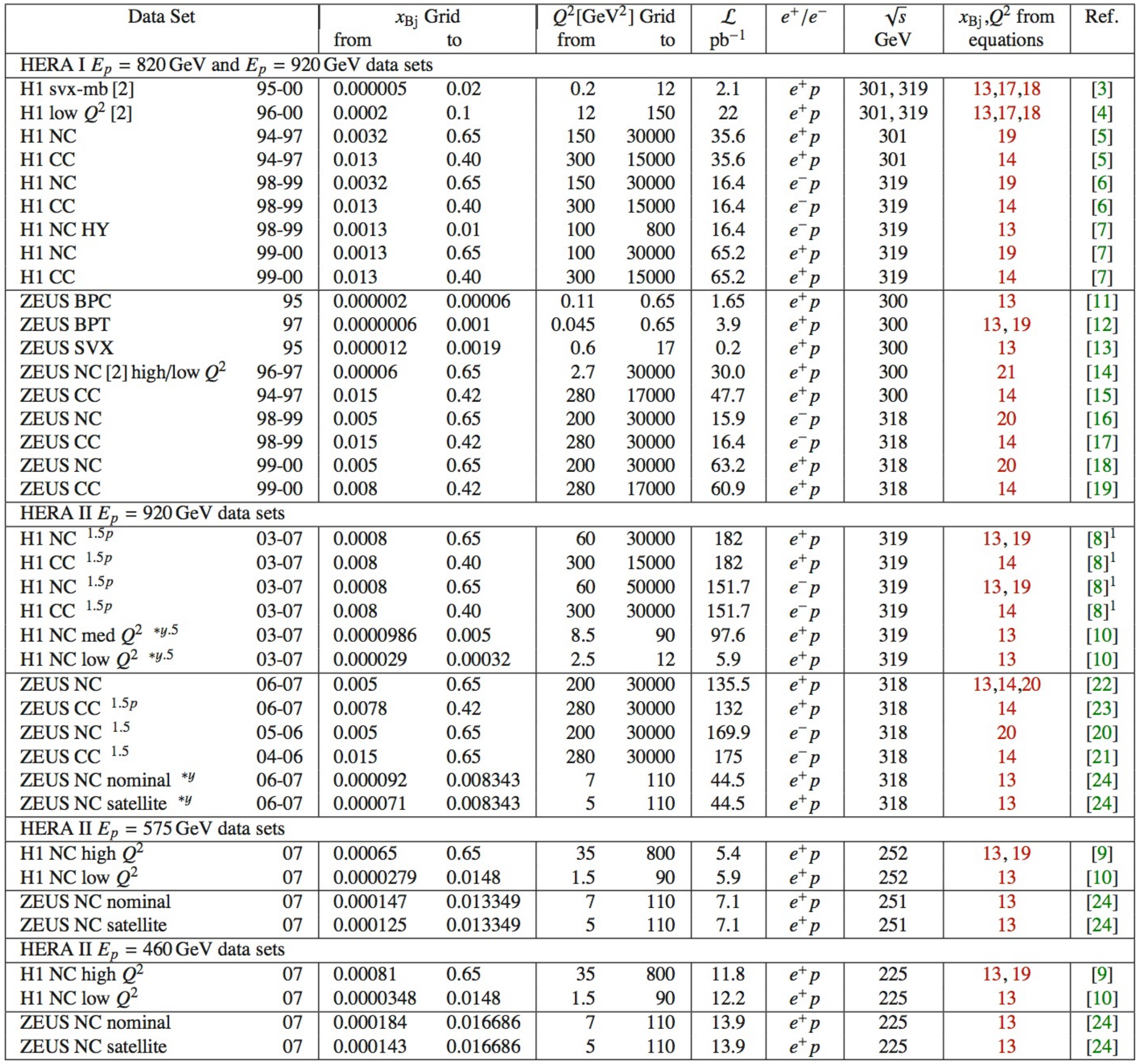
Structure Functions, PDFs and DGLAP evolution equations (LO, NLO and NNLO):

$$x^{-1}F_2(x,Q^2) = \sum_{i=q,g}\int_x^1 \frac{d\xi}{\xi} C_{2,i}\left(\frac{x}{\xi}, \alpha_s(\mu^2), \frac{\mu^2}{Q^2}\right) f_i(\xi,\mu^2)$$

$$\frac{d}{d\ln\mu^2} f_i(\xi,\mu^2) = \sum_k \left[P_{ik}(\alpha_s(\mu^2) \otimes f_k(\mu^2))\right](\xi)$$

Combination: Data sets

The HERA Legacy



Data Set		x_{Bj} Grid		Q^2[GeV^2] Grid		$\mathcal{L}$	e^+/e^-	$\sqrt{s}$	x_{Bj},Q^2 from	Ref.
		from	to	from	to	pb^{-1}		GeV	equations	
HERA I E_p = 820 GeV and E_p = 920 GeV data sets										
H1 svx-mb [2]	95-00	0.000005	0.02	0.2	12	2.1	e^+p	301, 319	13,17,18	[3]
H1 low Q^2 [2]	96-00	0.0002	0.1	12	150	22	e^+p	301, 319	13,17,18	[4]
H1 NC	94-97	0.0032	0.65	150	30000	35.6	e^+p	301	19	[5]
H1 CC	94-97	0.013	0.40	300	15000	35.6	e^+p	301	14	[5]
H1 NC	98-99	0.0032	0.65	150	30000	16.4	e^-p	319	19	[6]
H1 CC	98-99	0.013	0.40	300	15000	16.4	e^-p	319	14	[6]
H1 NC HY	98-99	0.0013	0.01	100	800	16.4	e^-p	319	13	[7]
H1 NC	99-00	0.0013	0.65	100	30000	65.2	e^+p	319	19	[7]
H1 CC	99-00	0.013	0.40	300	15000	65.2	e^+p	319	14	[7]
ZEUS BPC	95	0.000002	0.00006	0.11	0.65	1.65	e^+p	300	13	[11]
ZEUS BPT	97	0.0000006	0.001	0.045	0.65	3.9	e^+p	300	13, 19	[12]
ZEUS SVX	95	0.000012	0.0019	0.6	17	0.2	e^+p	300	13	[13]
ZEUS NC [2] high/low Q^2	96-97	0.00006	0.65	2.7	30000	30.0	e^+p	300	21	[14]
ZEUS CC	94-97	0.015	0.42	280	17000	47.7	e^+p	300	14	[15]
ZEUS NC	98-99	0.005	0.65	200	30000	15.9	e^-p	318	20	[16]
ZEUS CC	98-99	0.015	0.42	280	30000	16.4	e^-p	318	14	[17]
ZEUS NC	99-00	0.005	0.65	200	30000	63.2	e^+p	318	20	[18]
ZEUS CC	99-00	0.008	0.42	280	17000	60.9	e^+p	318	14	[19]
HERA II E_p = 920 GeV data sets										
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	30000	182	e^+p	319	13, 19	[8]^1
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	15000	182	e^+p	319	14	[8]^1
H1 NC $^{1.5p}$	03-07	0.0008	0.65	60	50000	151.7	e^-p	319	13, 19	[8]^1
H1 CC $^{1.5p}$	03-07	0.008	0.40	300	30000	151.7	e^-p	319	14	[8]^1
H1 NC med Q^2 $^{*y.5}$	03-07	0.0000986	0.005	8.5	90	97.6	e^+p	319	13	[10]
H1 NC low Q^2 $^{*y.5}$	03-07	0.000029	0.00032	2.5	12	5.9	e^+p	319	13	[10]
ZEUS NC	06-07	0.005	0.65	200	30000	135.5	e^+p	318	13,14,20	[22]
ZEUS CC $^{1.5p}$	06-07	0.0078	0.42	280	30000	132	e^+p	318	14	[23]
ZEUS NC $^{1.5}$	05-06	0.005	0.65	200	30000	169.9	e^-p	318	20	[20]
ZEUS CC $^{1.5}$	04-06	0.015	0.65	280	30000	175	e^-p	318	14	[21]
ZEUS NC nominal ^{*y}	06-07	0.000092	0.008343	7	110	44.5	e^+p	318	13	[24]
ZEUS NC satellite ^{*y}	06-07	0.000071	0.008343	5	110	44.5	e^+p	318	13	[24]
HERA II E_p = 575 GeV data sets										
H1 NC high Q^2	07	0.00065	0.65	35	800	5.4	e^+p	252	13, 19	[9]
H1 NC low Q^2	07	0.0000279	0.0148	1.5	90	5.9	e^+p	252	13	[10]
ZEUS NC nominal	07	0.000147	0.013349	7	110	7.1	e^+p	251	13	[24]
ZEUS NC satellite	07	0.000125	0.013349	5	110	7.1	e^+p	251	13	[24]
HERA II E_p = 460 GeV data sets										
H1 NC high Q^2	07	0.00081	0.65	35	800	11.8	e^+p	225	13, 19	[9]
H1 NC low Q^2	07	0.0000348	0.0148	1.5	90	12.2	e^+p	225	13	[10]
ZEUS NC nominal	07	0.000184	0.016686	7	110	13.9	e^+p	225	13	[24]
ZEUS NC satellite	07	0.000143	0.016686	5	110	13.9	e^+p	225	13	[24]

H1 & ZEUS have now published all their inclusive measurements (1992-2007)
- HERA-I
- HERA-II measurements at high-Q^2
- HERA-II measurements at reduced $\sqrt{s}$

$0.6 \times 10^{-6} < x_{Bj} < 0.65$, $0.045 < Q^2 < 50000$

41 data sets are combined:
- NC & CC cross sections
- e^+p and e^-p scattering
- 4 different $\sqrt{s}$ (318, 301, 252 and 225 GeV)

2927 data points

1307 combined points

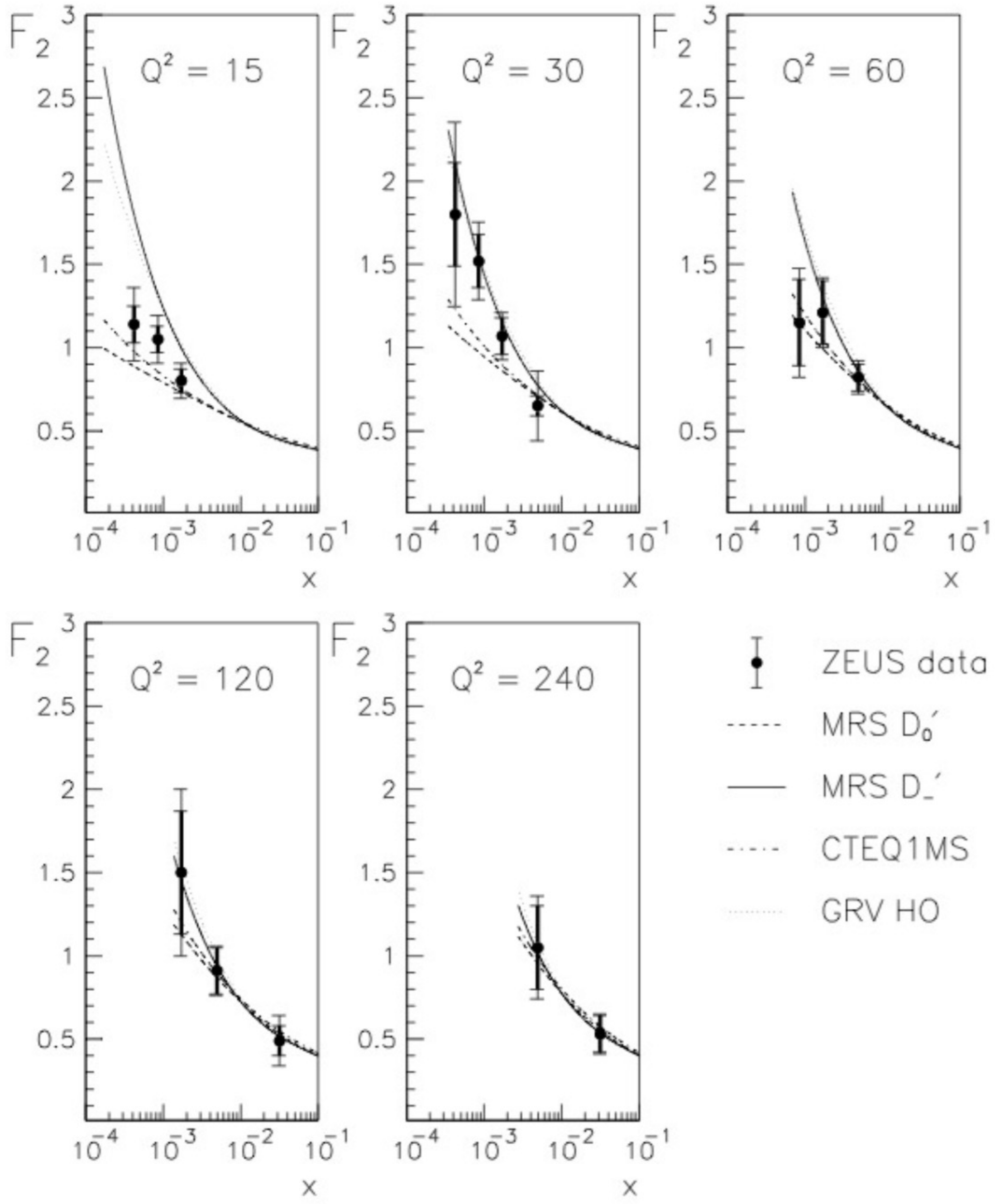
In typical cases 3 to 6 measurements contribute to a combined result

NC e^+p accuracy reaches ~1%

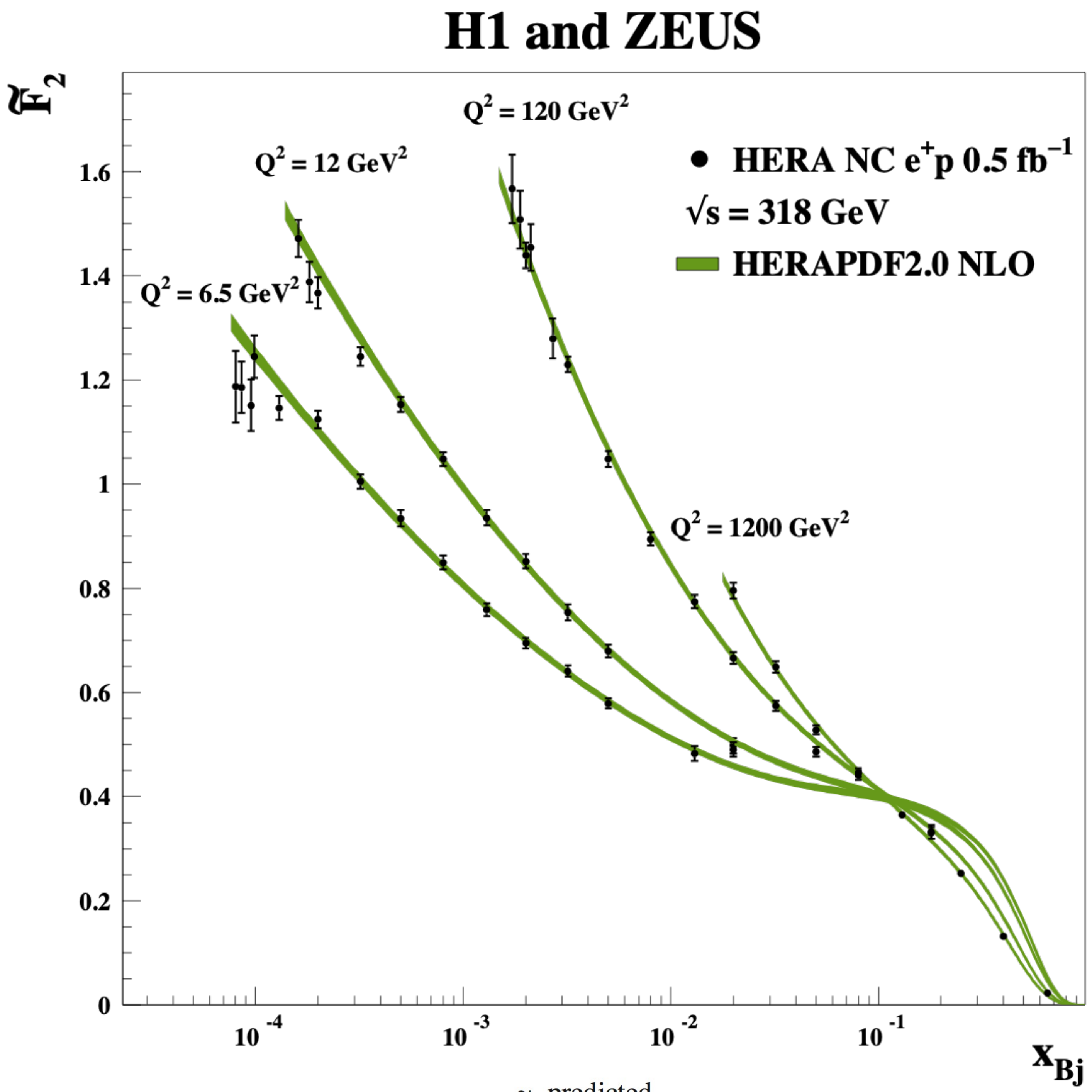
The usage of different reconstruction techniques and the differences in the strengths of the detector components of the two experiments lead to a substantial reduction of the systematic uncertainties of the combined cross sections.

Rise of F_2

HERA F_2 (1993)



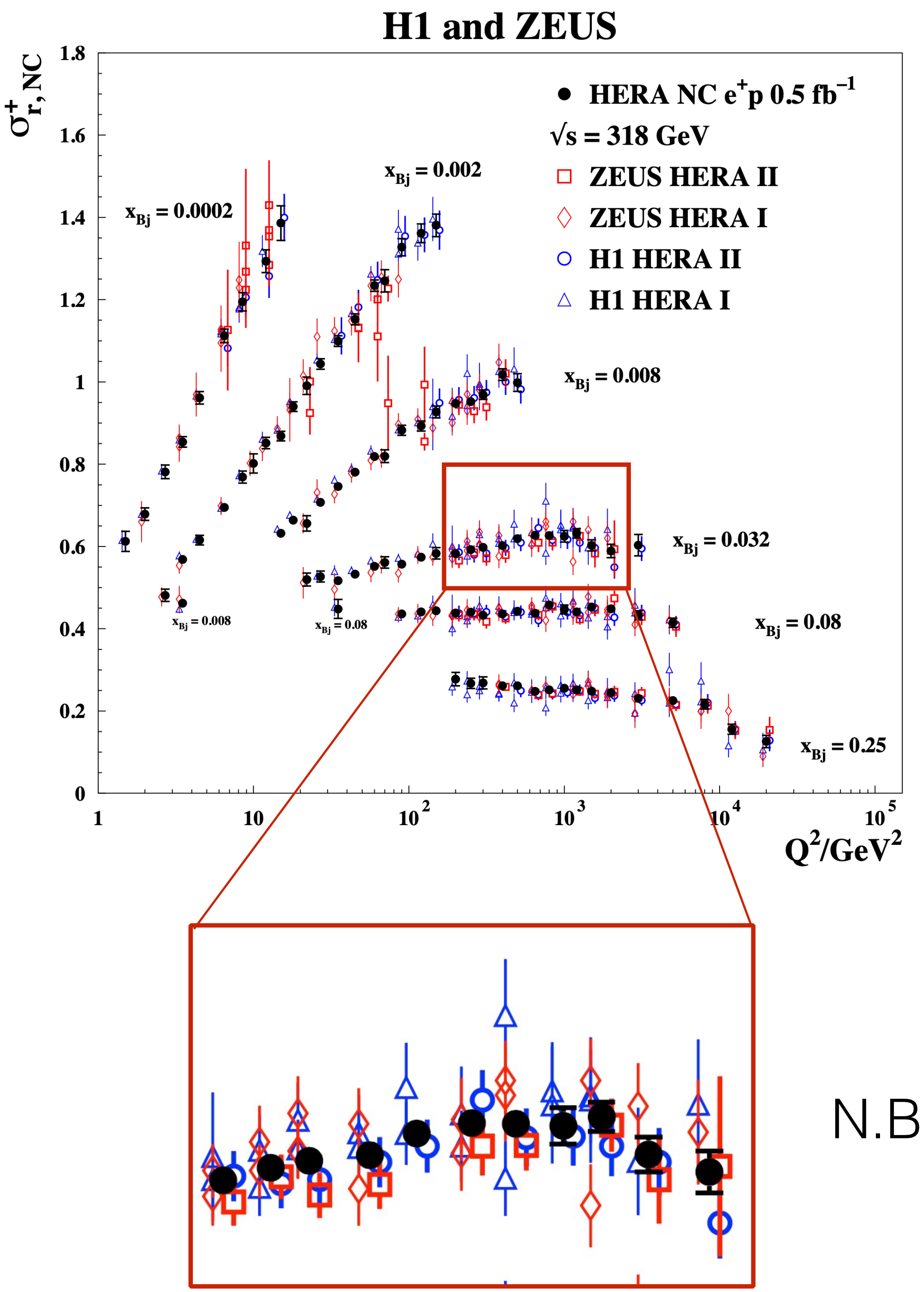
HERA F_2 (2015)

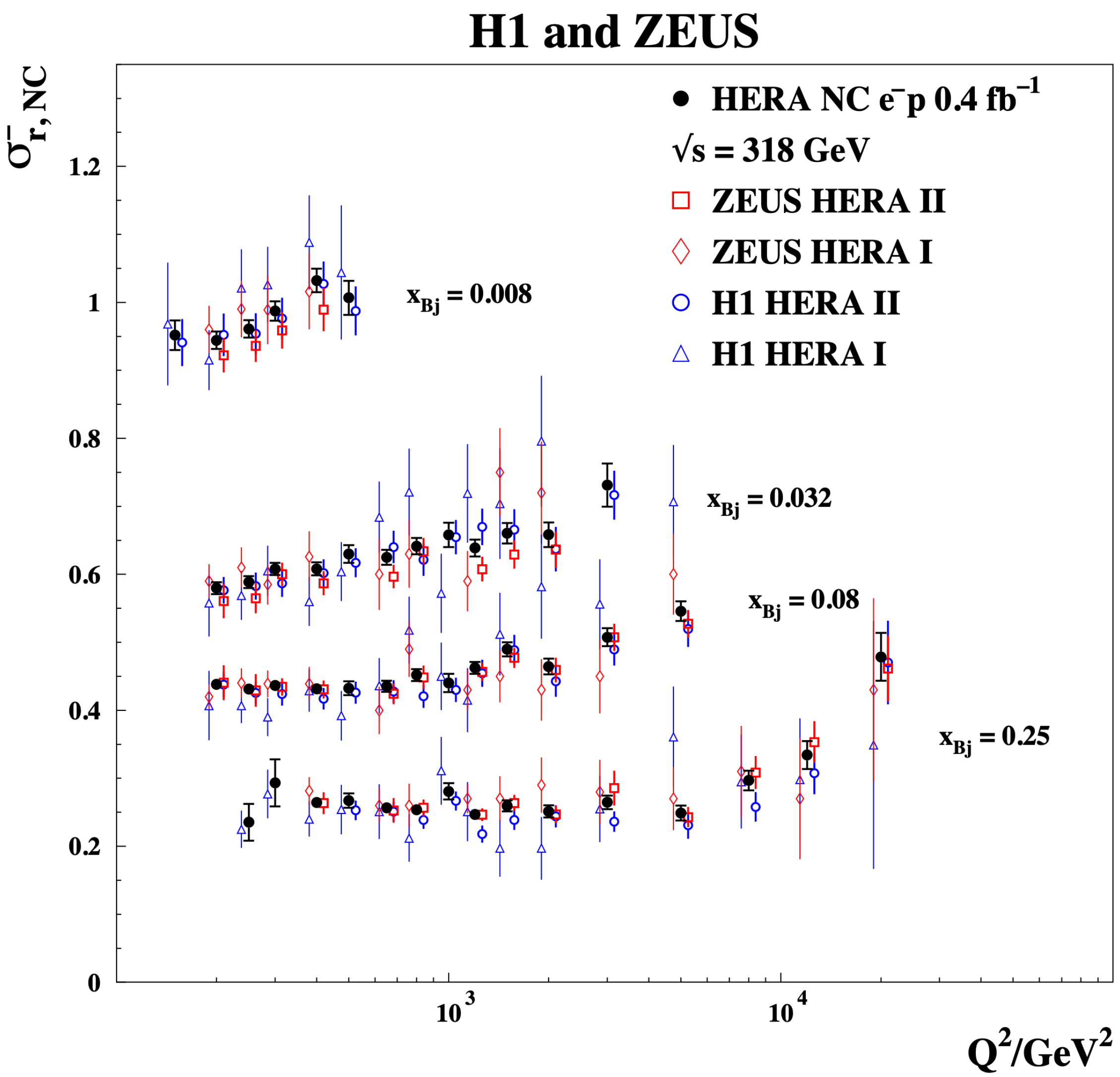


$$\tilde{F}_2 = \sigma^{\pm}_{r,\mathrm{NC}} \cdot \frac{\tilde{F}_2^{\,\text{predicted}}}{\sigma^{\pm\,\text{predicted}}_{r,\mathrm{NC}}} = \sigma^{\pm}_{r,\mathrm{NC}} \cdot (1 + C_F)$$

Combination: Results

NC e±p , √s =318 GeV

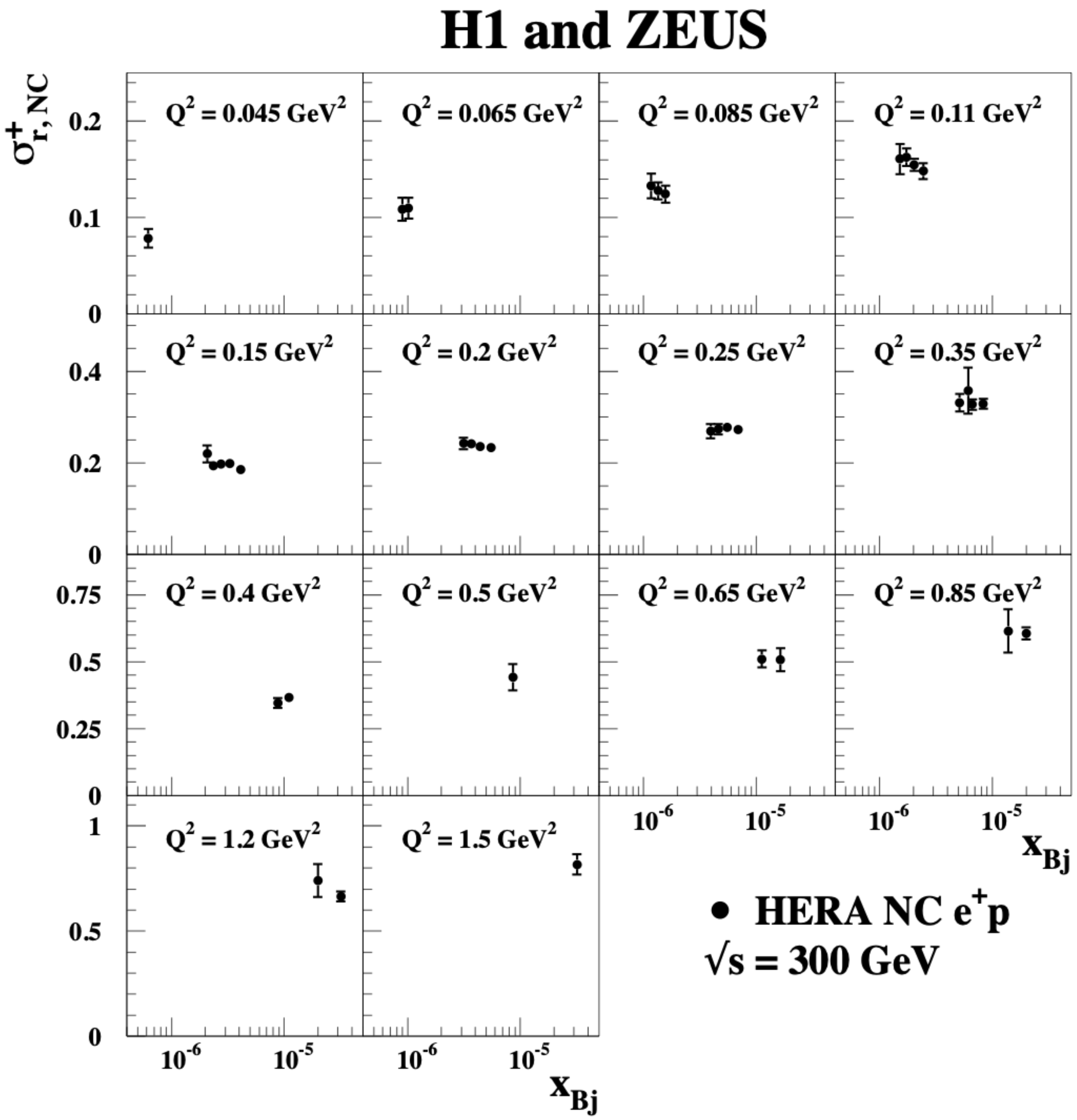


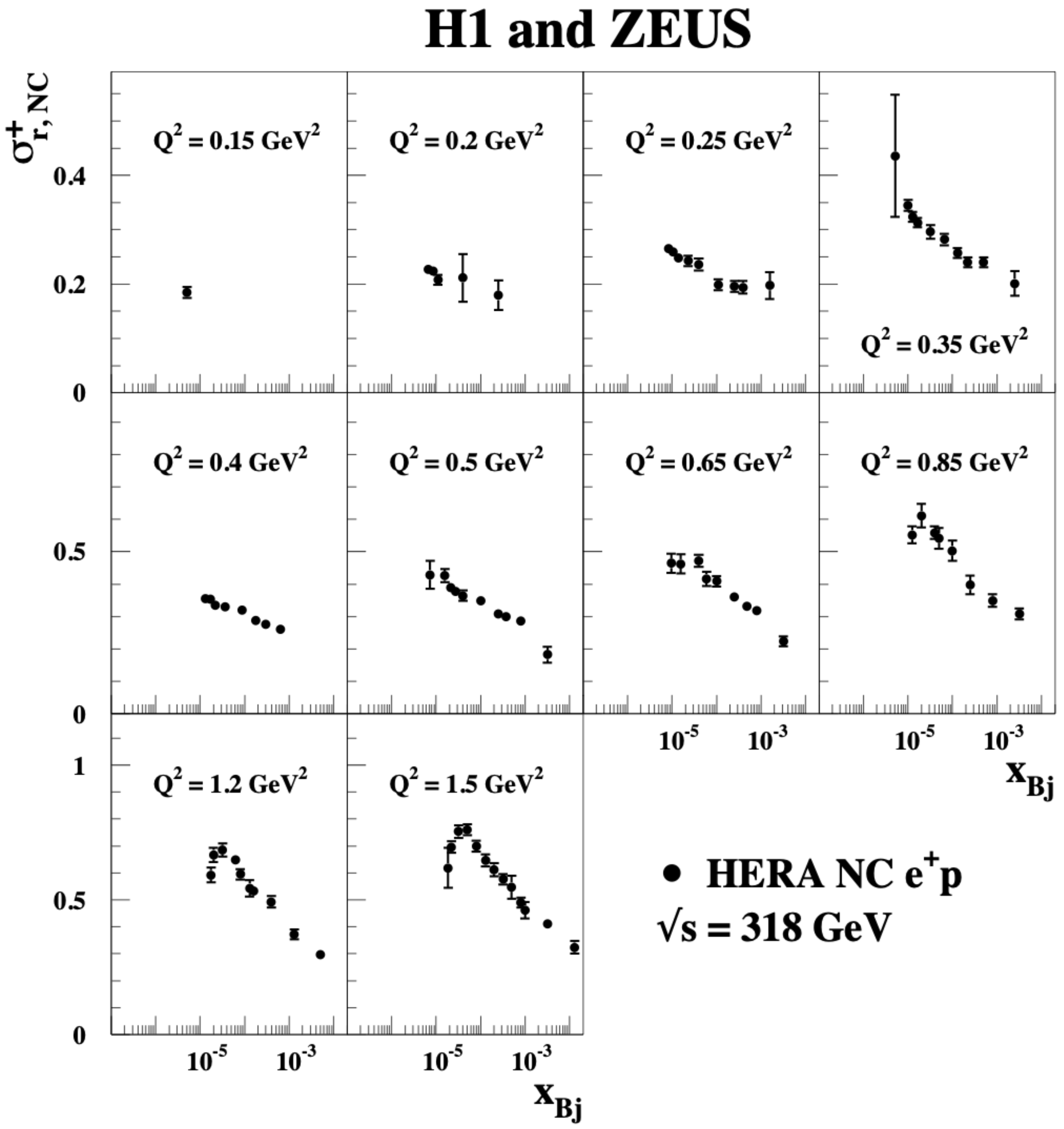


N.B. only a few representative x_{Bj} bins are shown

Combination: Results

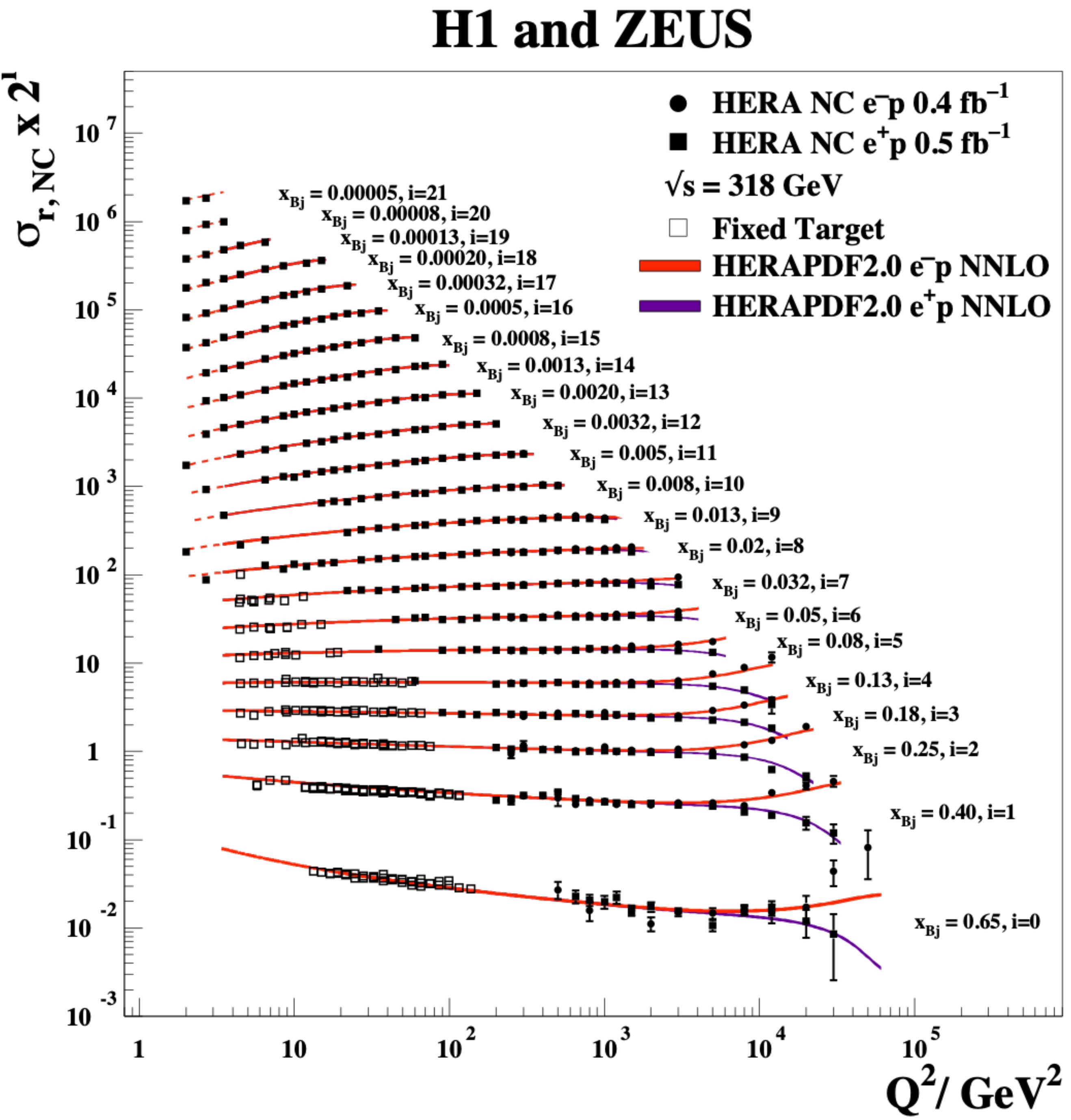
Very low Q^2 and low x_{Bj} data $\sqrt{s}$ =300, 318 GeV





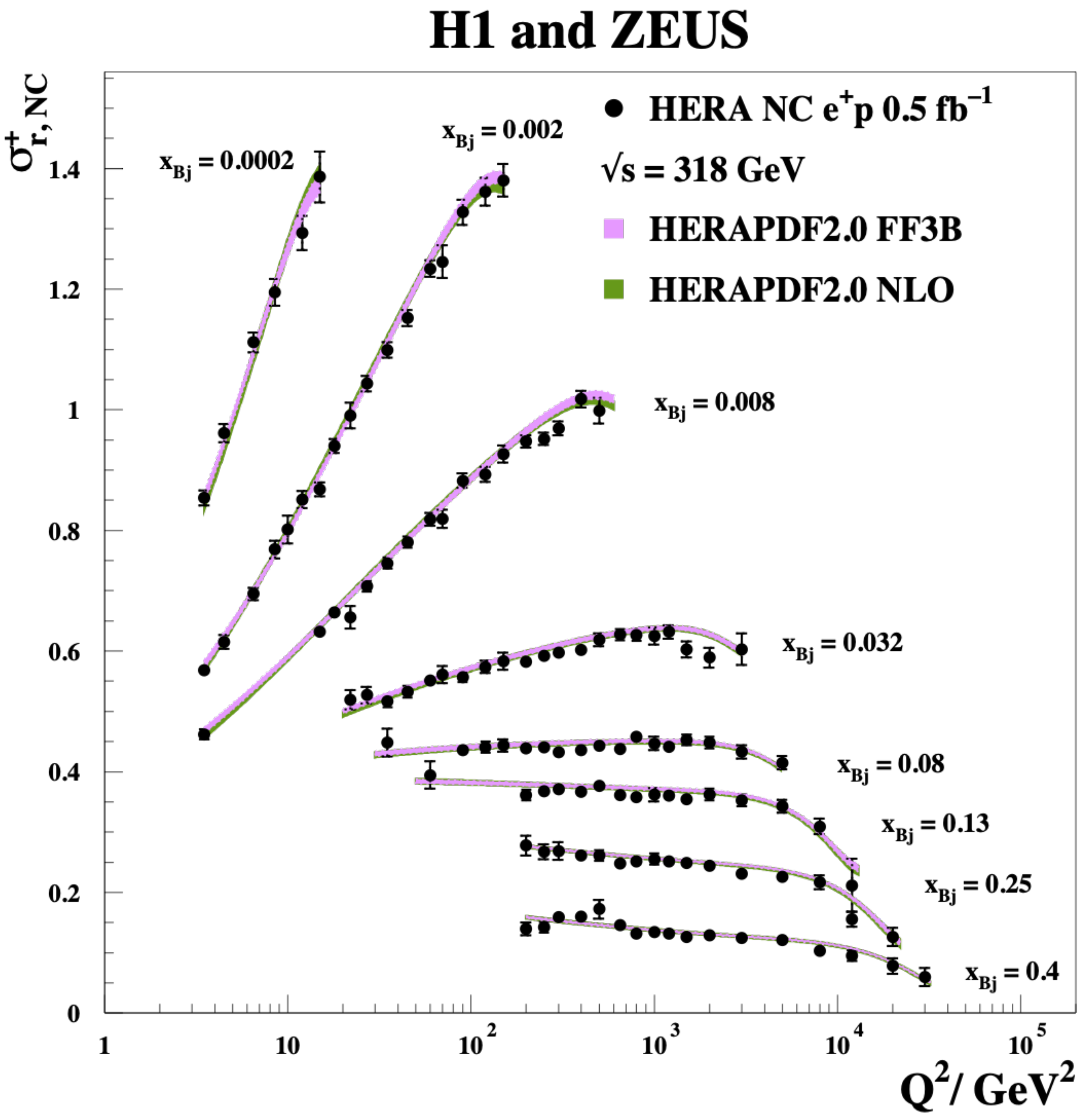
- A very important data sample for QCD studies at low-x_{Bj}
- Interesting also for dipole/saturation models and higher-twist studies

Scaling violations



Textbook plots showing with great precision scaling violations patterns (and EW effects at high-Q^2 and high-x)

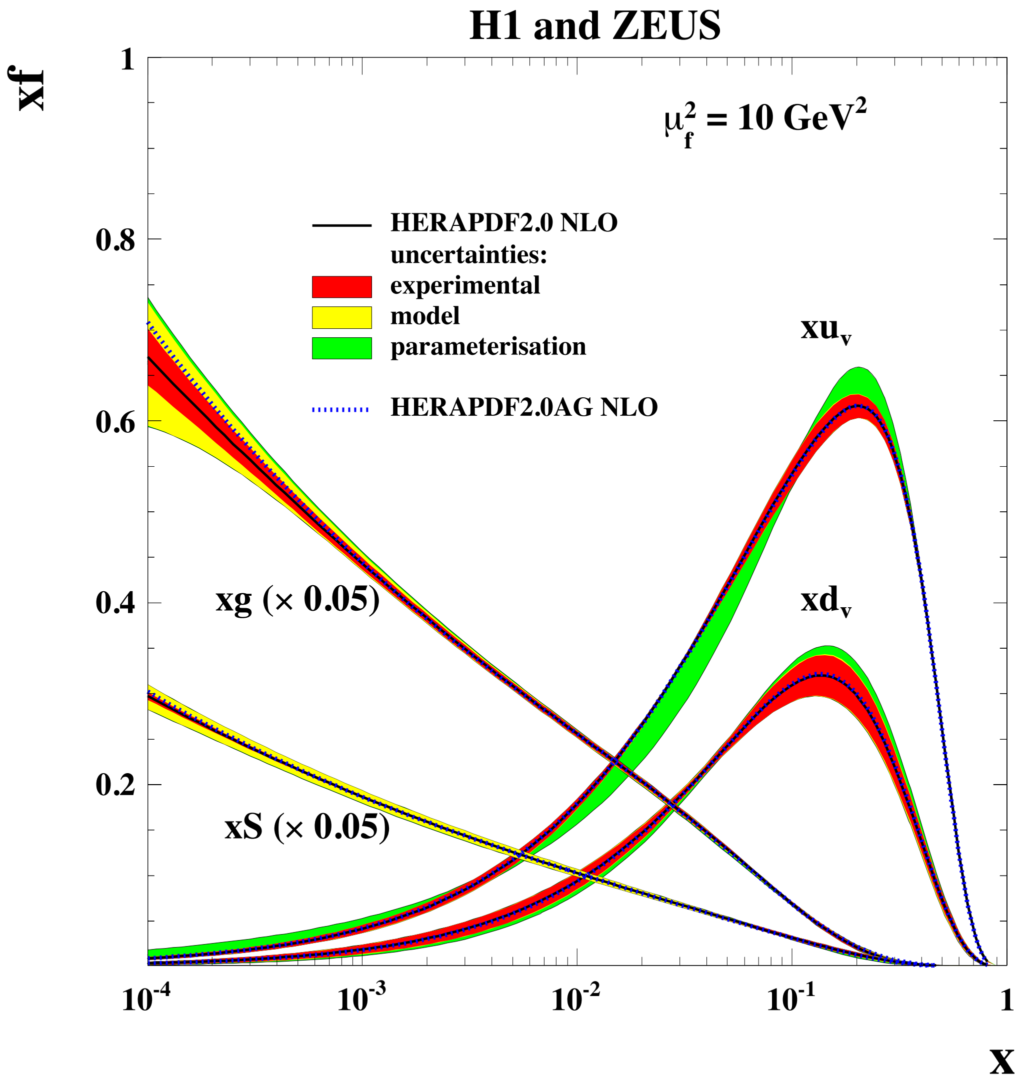
Scaling violations



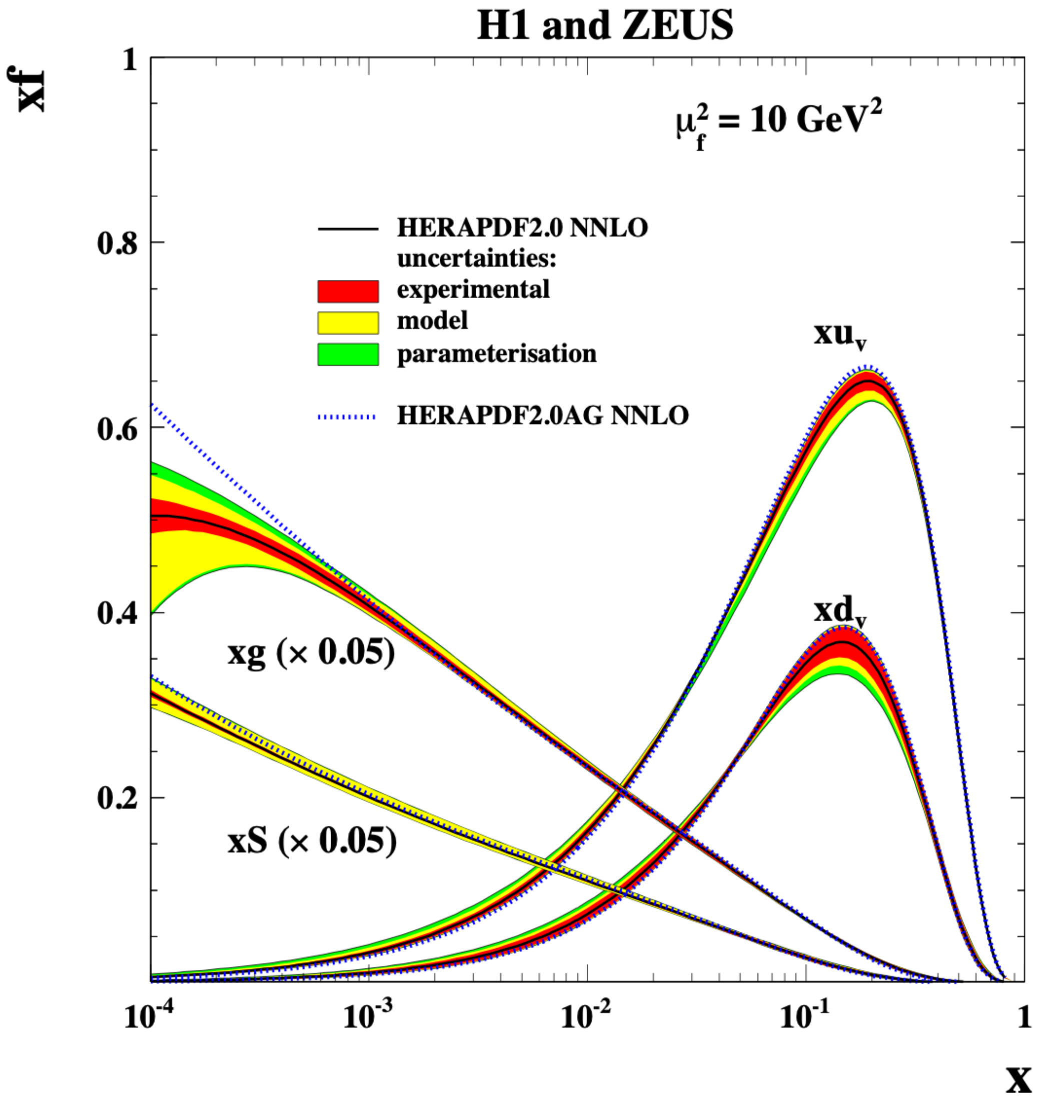
HERAPDF2.0 NLO (VFNS) vs HERAPDF2.0FF3B (FFNS)

HERAPDF2.0: NLO and NNLO PDFs

NLO

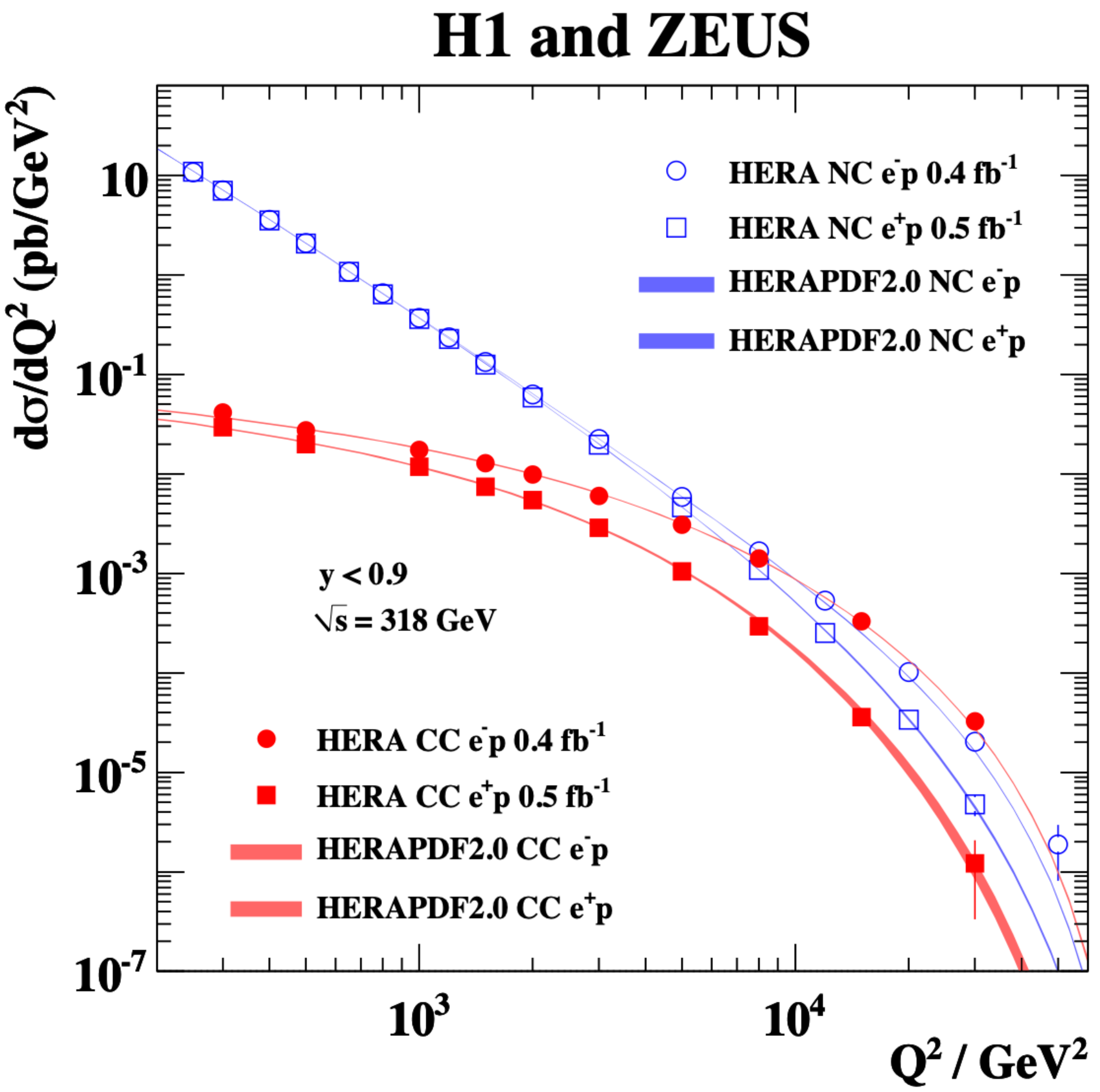


NNLO



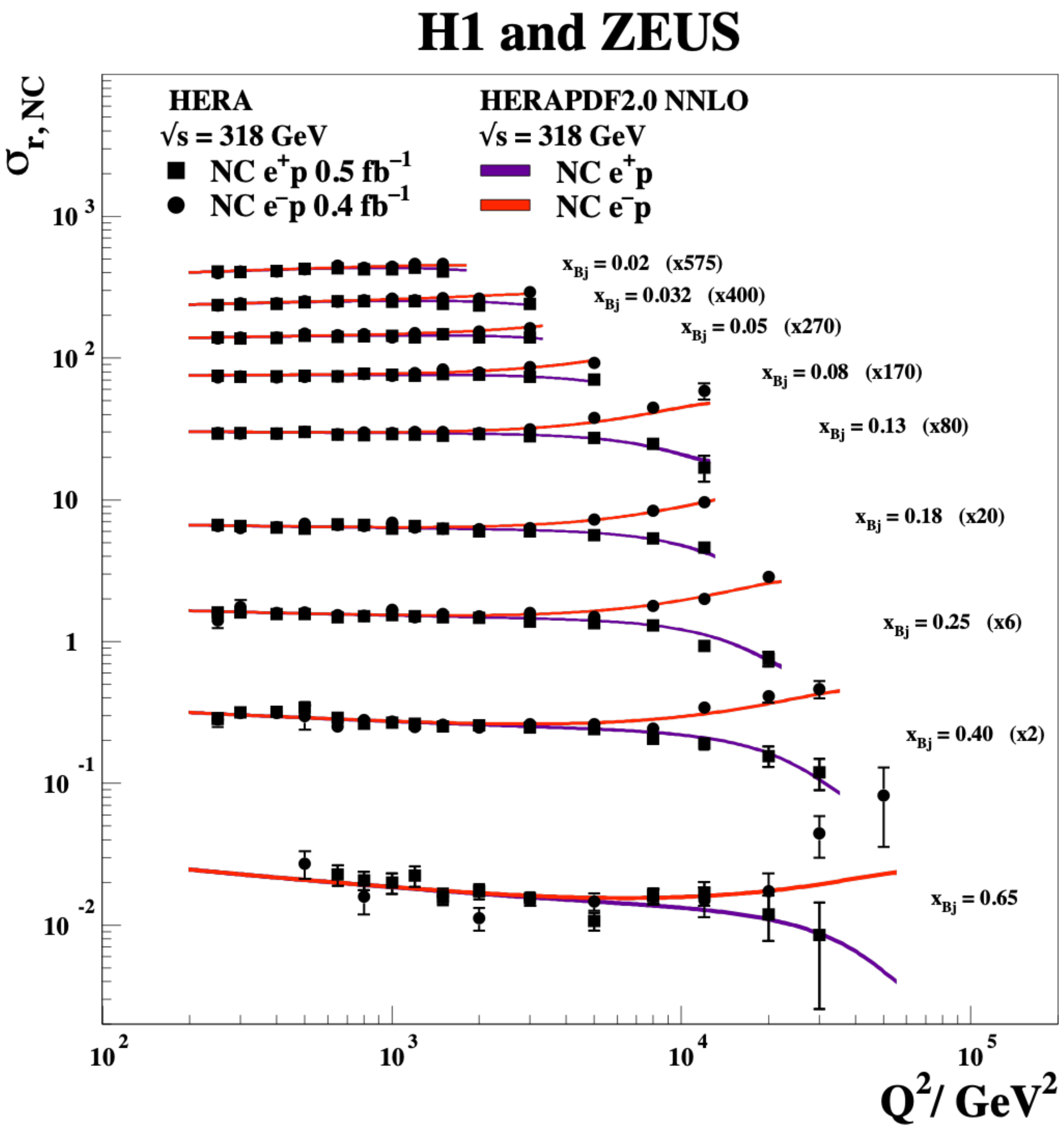
NNLO vs NLO:
- gluon ceases to rise at low-x
- sea at low-x somewhat steeper w.r.t. NLO

Electroweak Unification

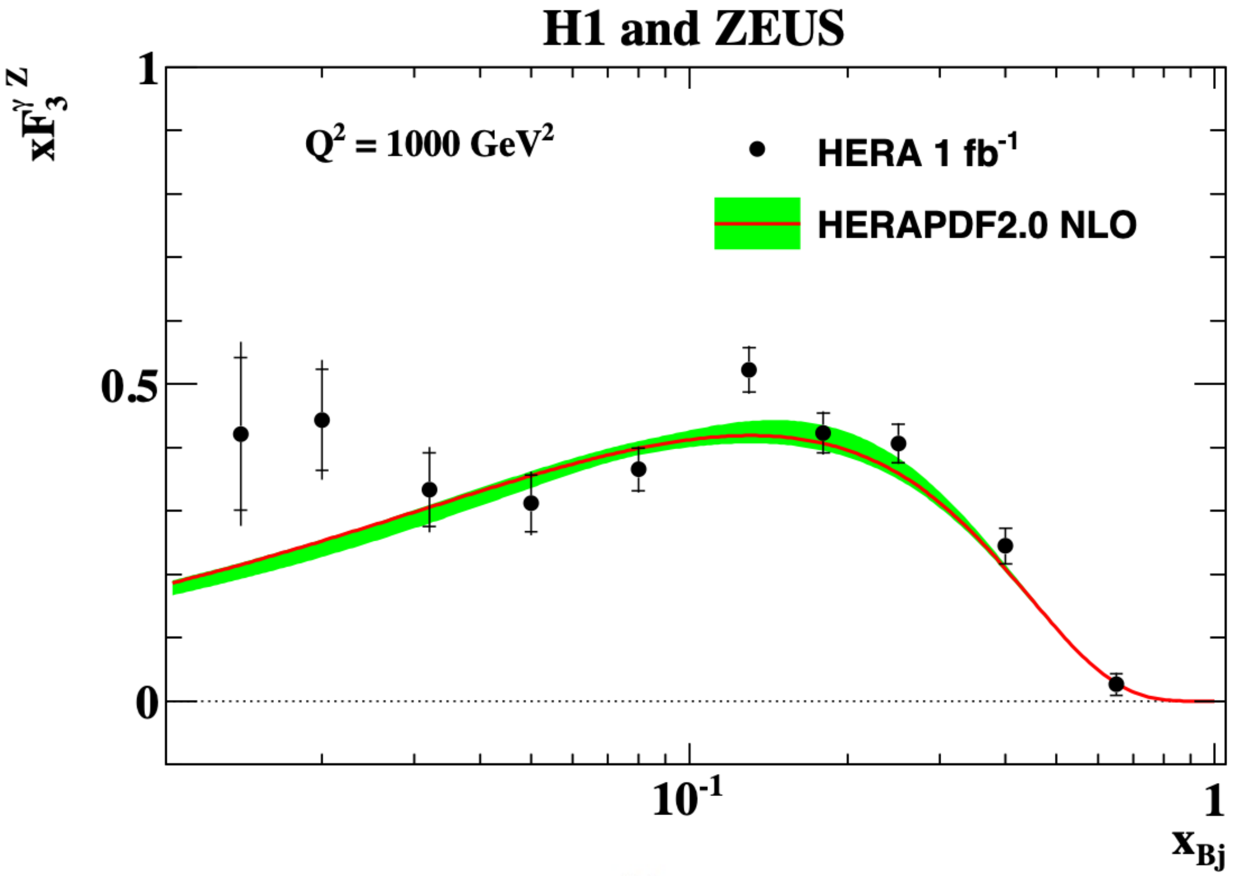


A beautiful textbook plot that shows the unification of the NC and CC interactions at the EW scale

Electroweak effects and $xF_3^{\gamma Z}$



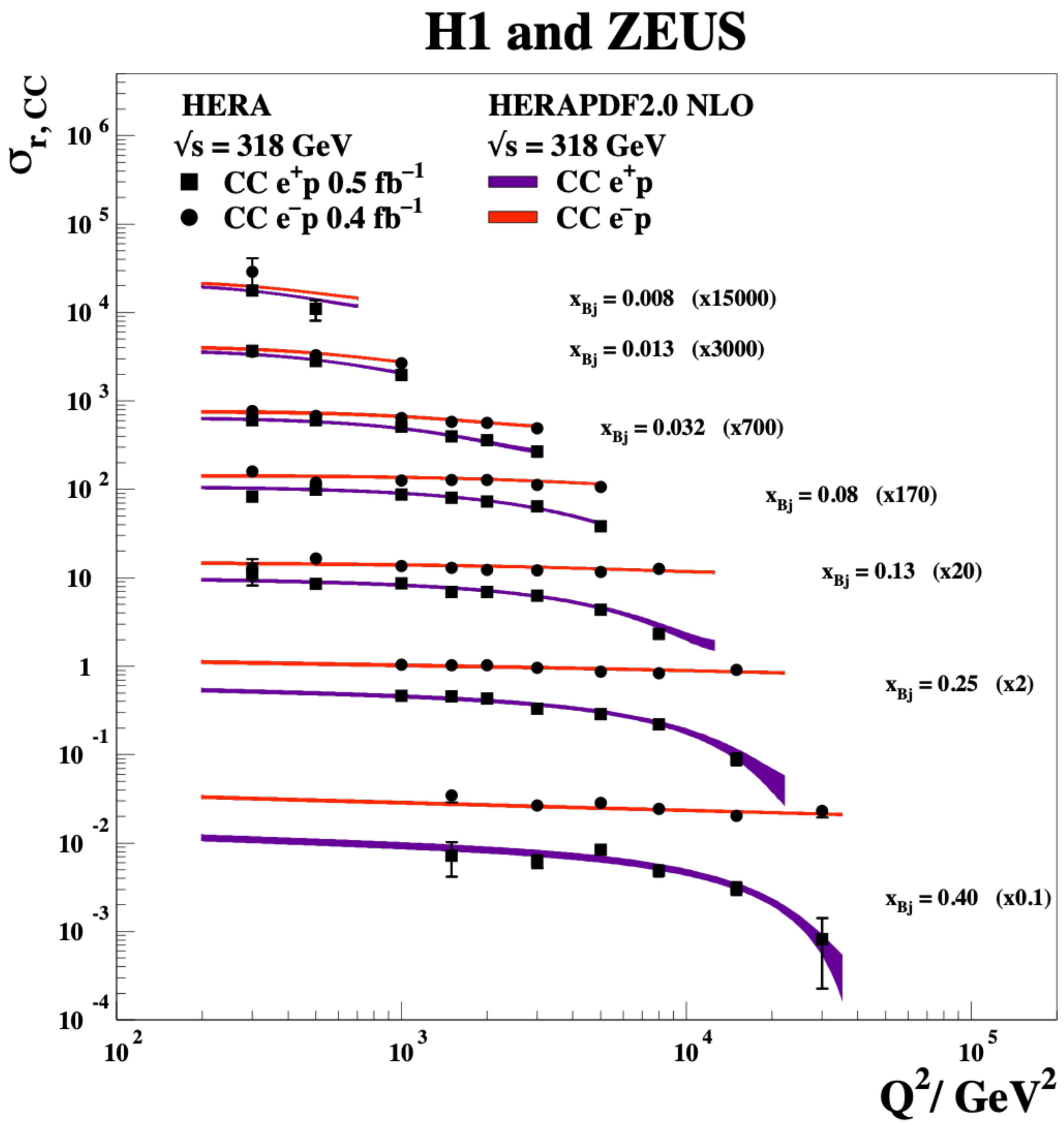
$$x\tilde{F}_3 = \frac{Y_+}{2Y_-}(\sigma^-_{r,\mathrm{NC}} - \sigma^+_{r,\mathrm{NC}})$$



$$xF_3^{\gamma Z} \approx \frac{x}{3}(2u_v + d_v)$$

- Best determination of $xF_3^{\gamma Z}$
- Parity violation demonstrated down to scale of 10^{-18} m

Helicity effects in CC interactions



Reminder:

$$\sigma^{+}_{r,\mathrm{CC}} \approx (x\bar{U} + (1-y)^2 xD)$$

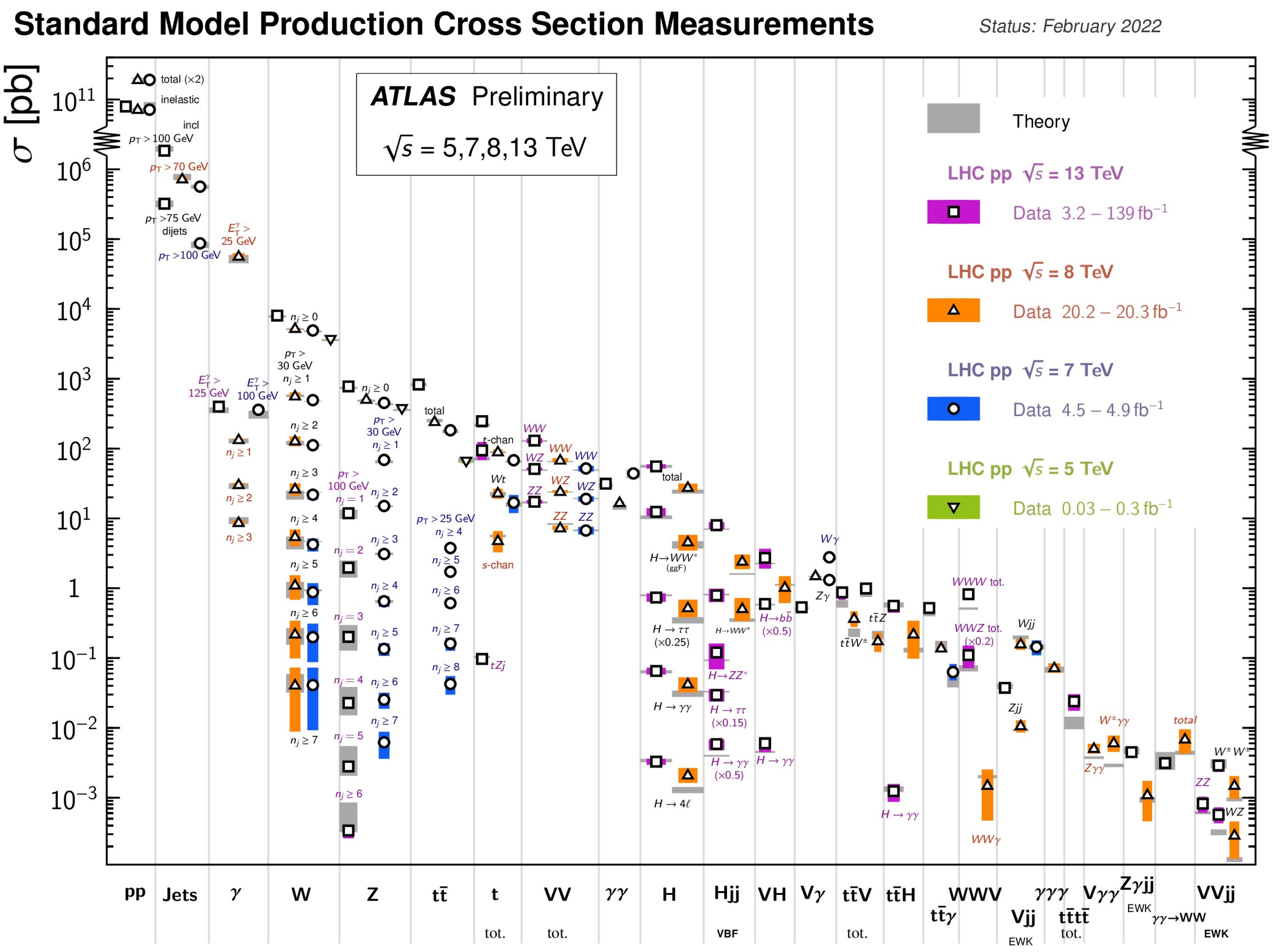
$$\sigma^{-}_{r,\mathrm{CC}} \approx (xU + (1-y)^2 x\bar{D})$$

The helicity factor $(1-y)^2$ affects differently the $e^{\pm}p$ CC cross sections:

- The $e^{+}p$ cross section is suppressed at high-y (high-Q^2)
- The $e^{-}p$ cross section is almost unaffected

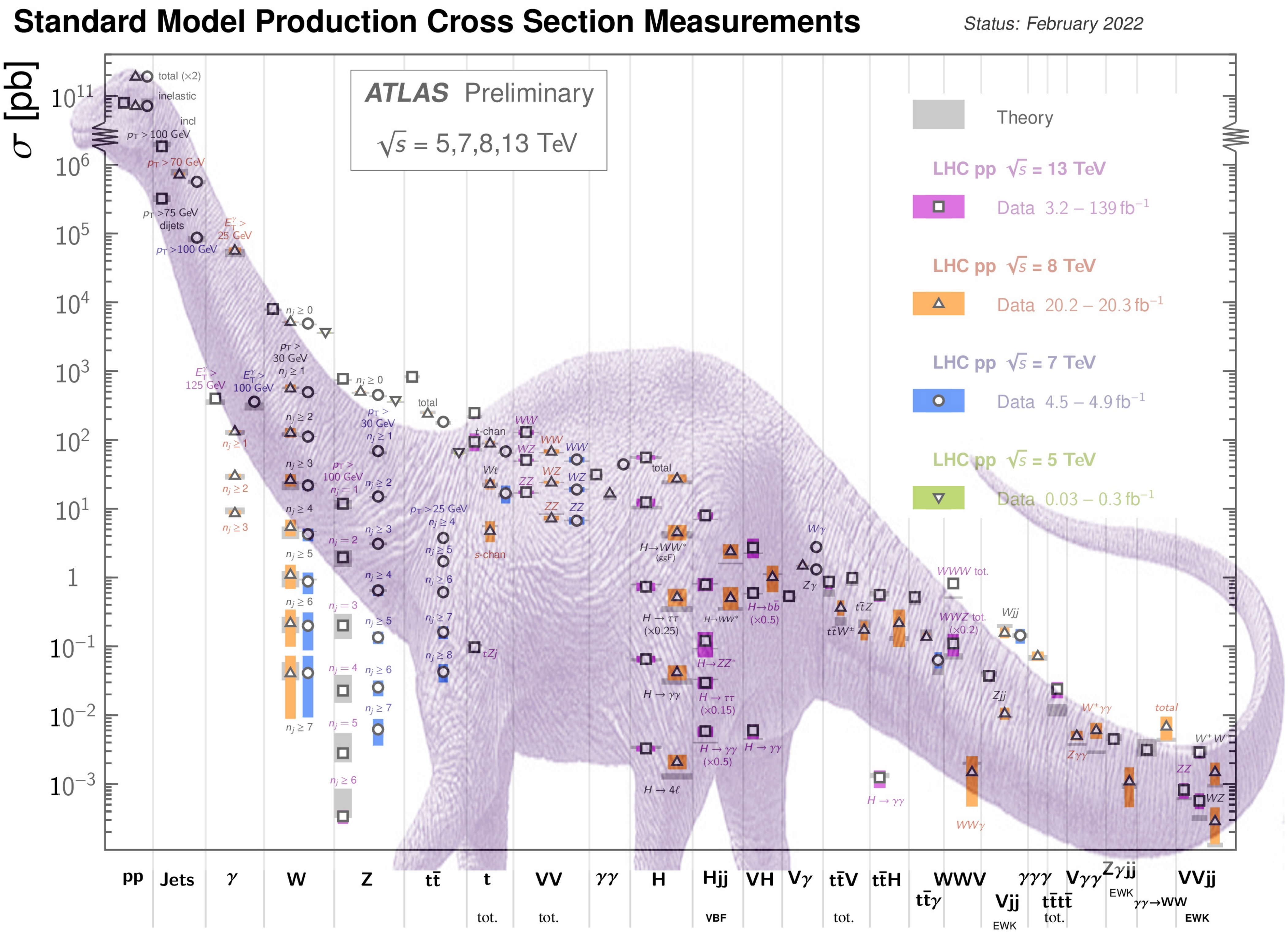
The precision of the CC cross sections at high-Q^2 allow the study of these helicity effects.

Dati LHC e PDFs


Standard Model Production Cross Section Measurements

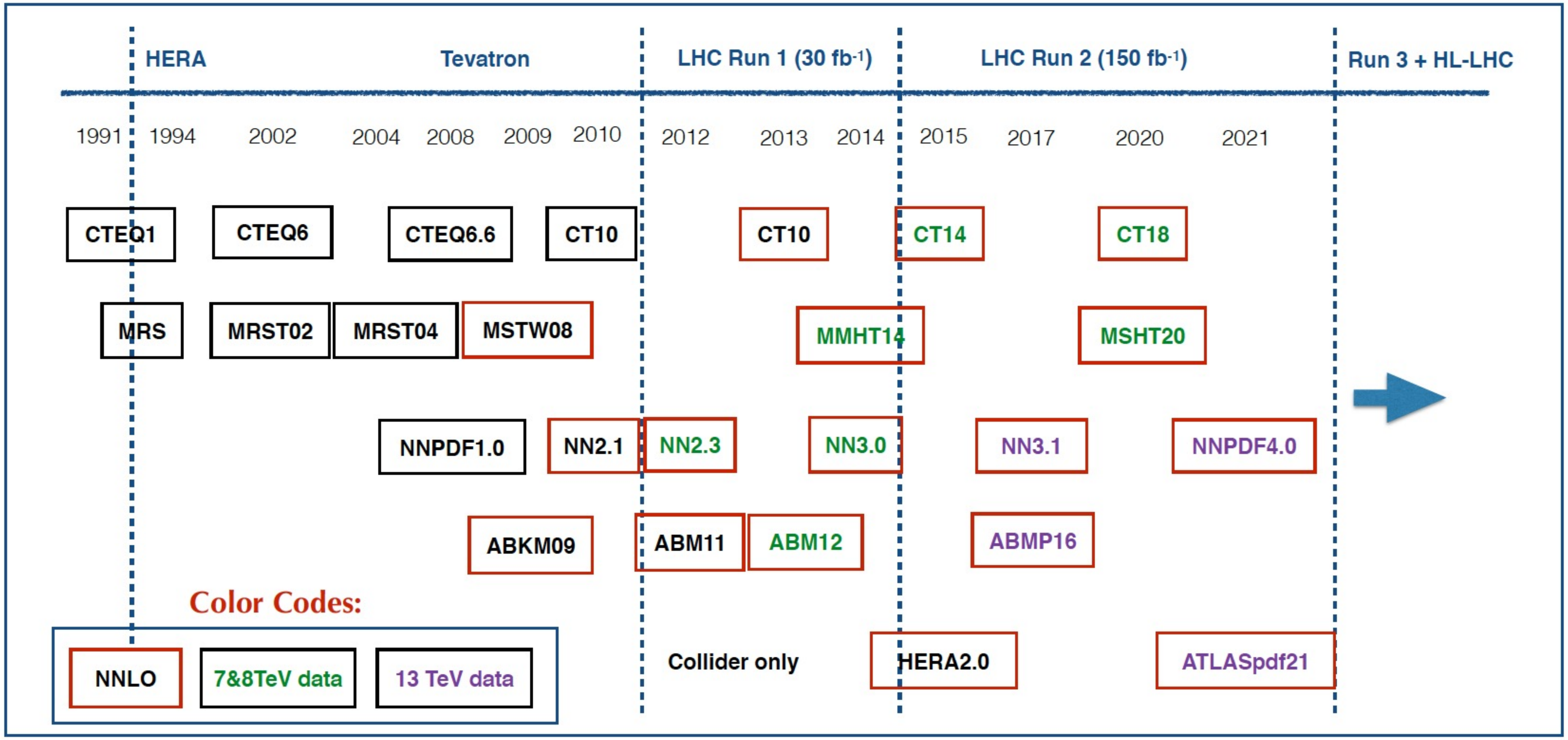
Status: February 2022

Dati LHC e PDFs



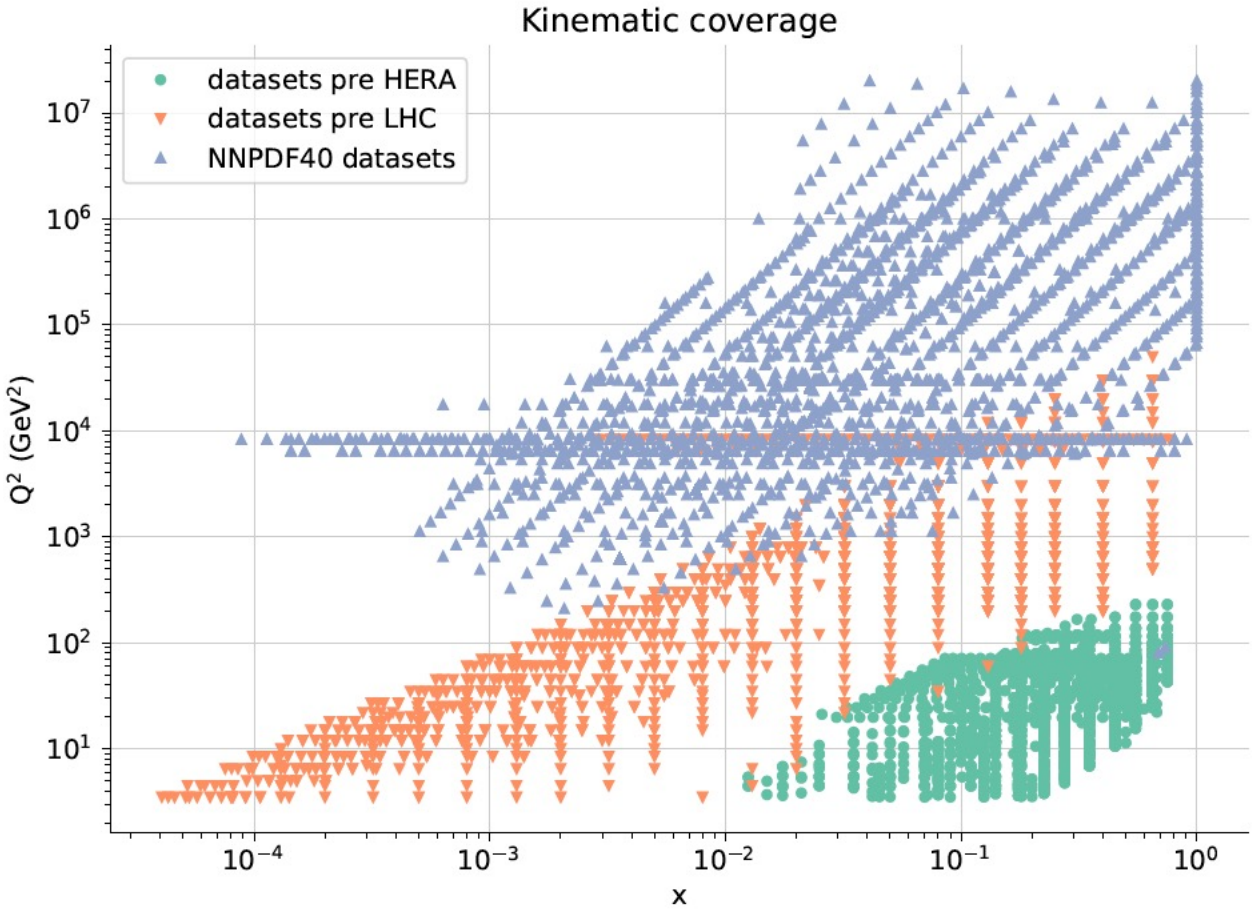
Dati LHC e PDF

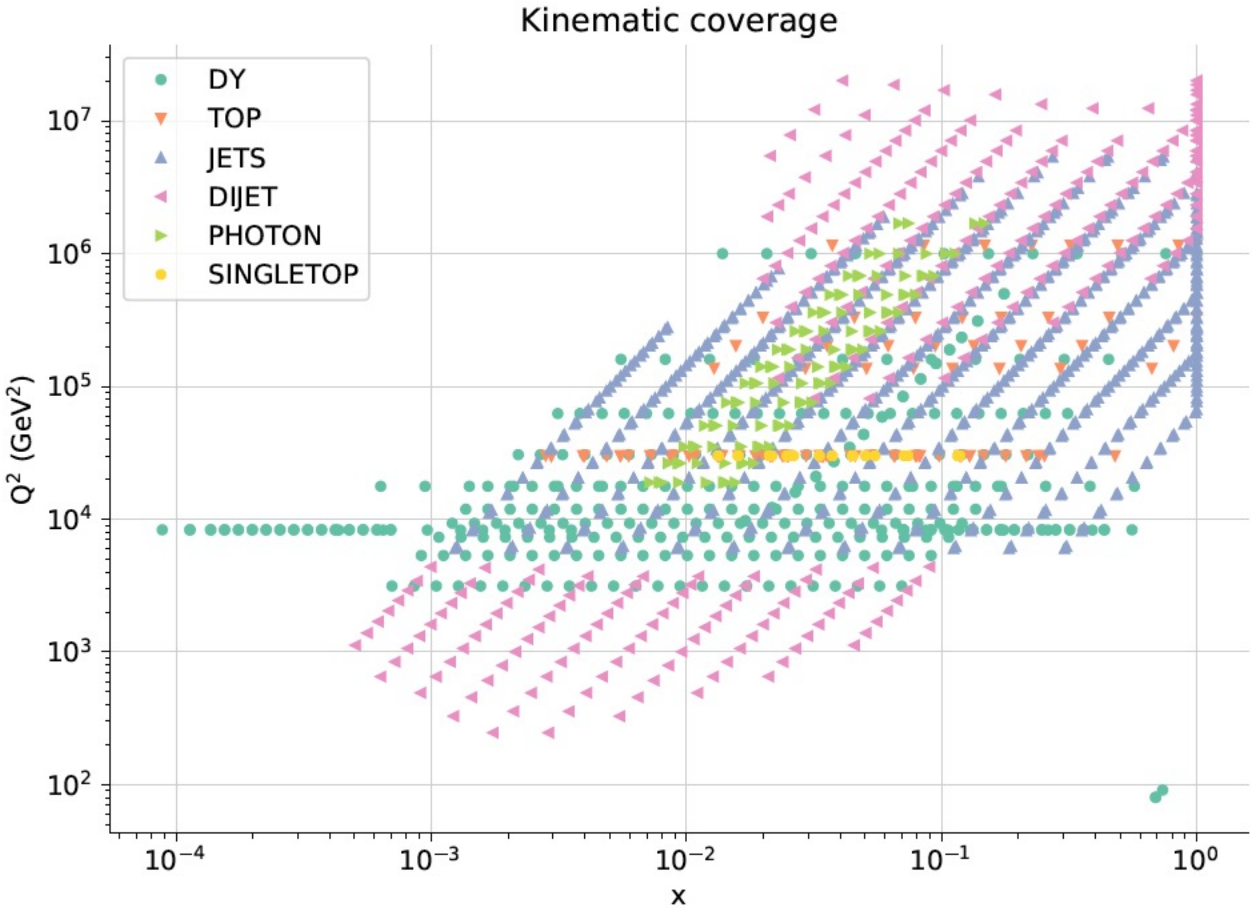
Gruppi attivi nelle analisi DGLAP:



Dati LHC e PDF: NNPDF4.0

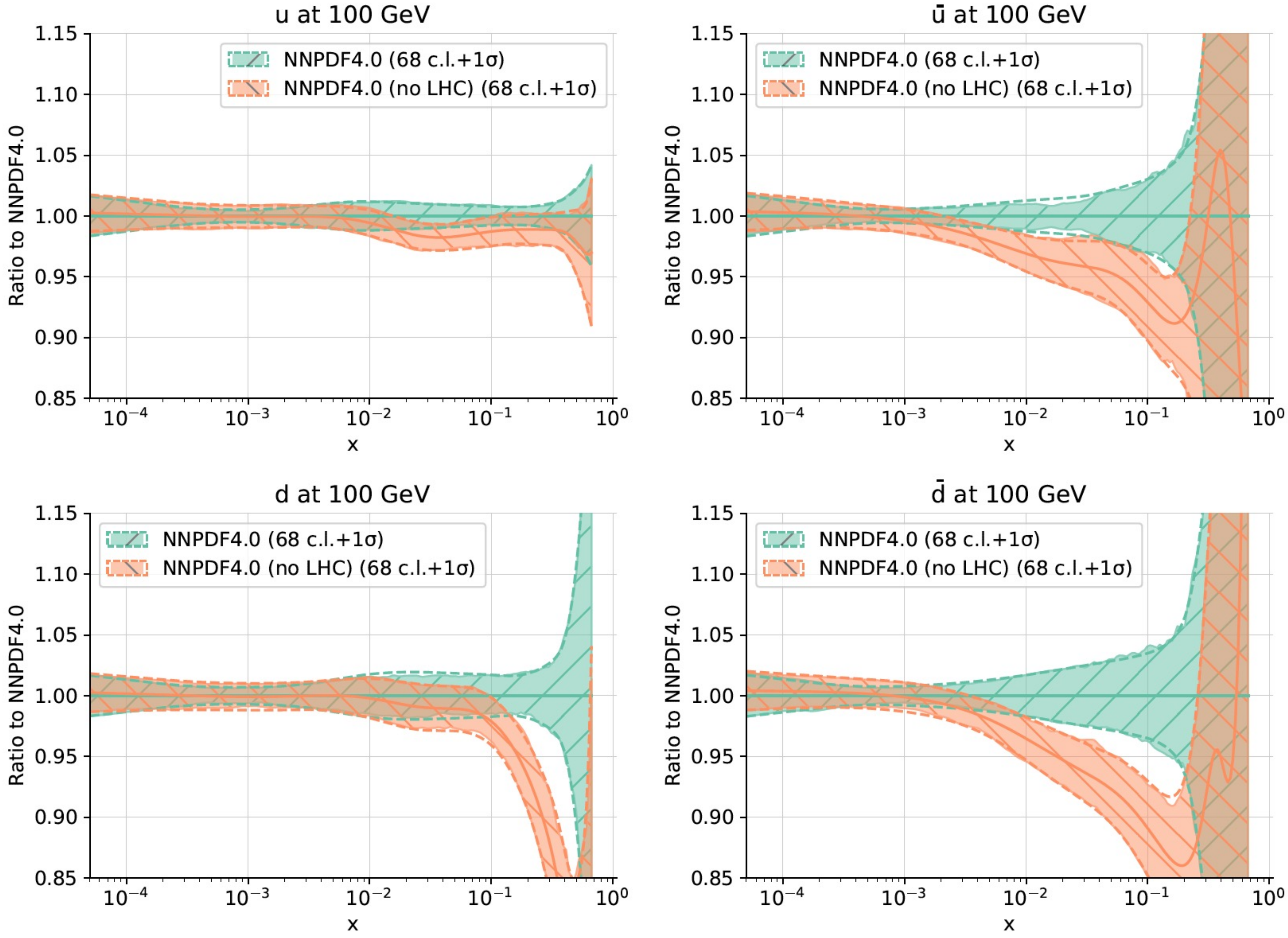
Regione cinematica ed osservabili:





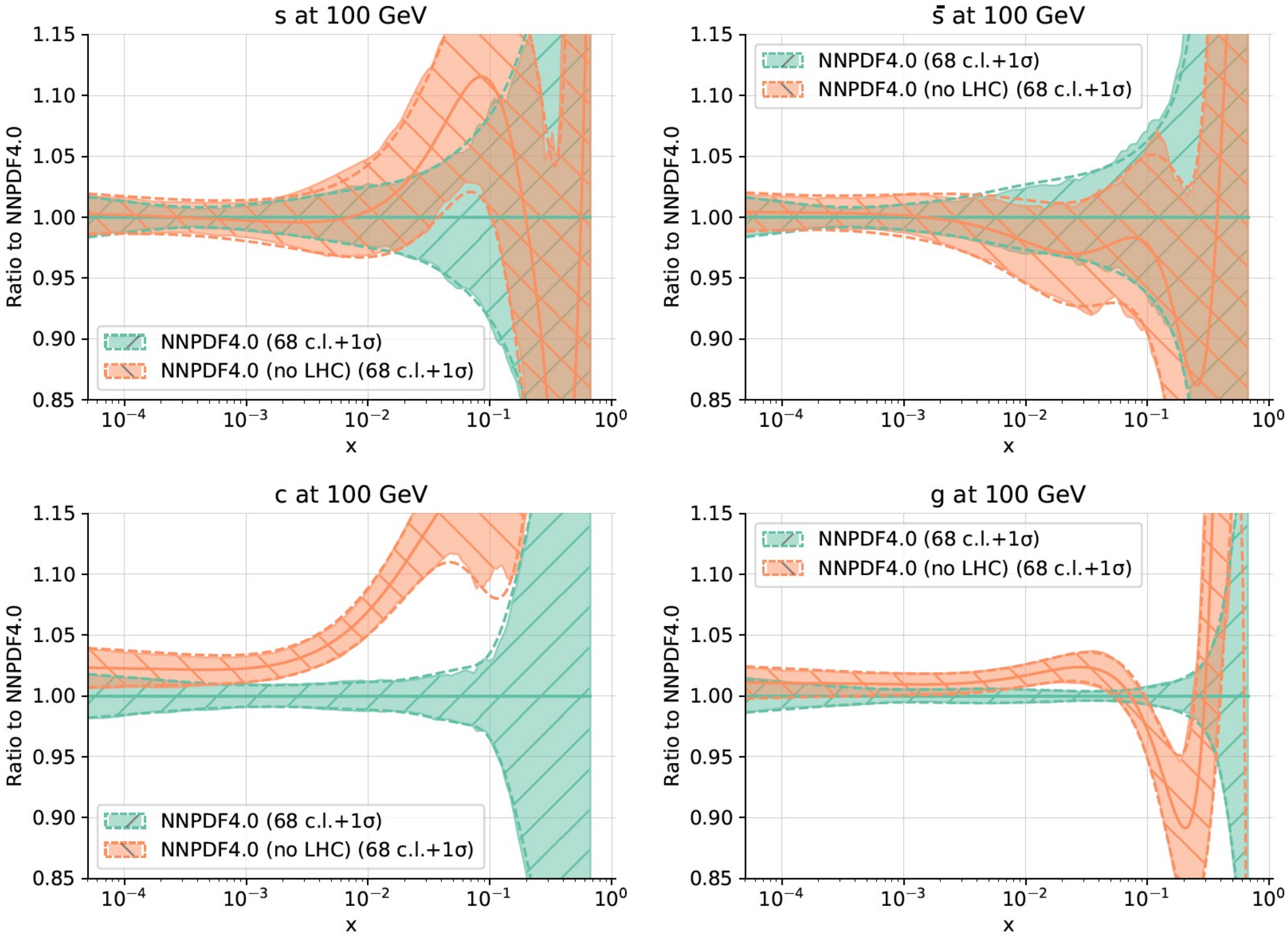
Dati LHC e PDF: NNPDF4.0

Impatto dati LHC:



Dati LHC e PDFs: NNPDF4.0

Impatto dati LHC:



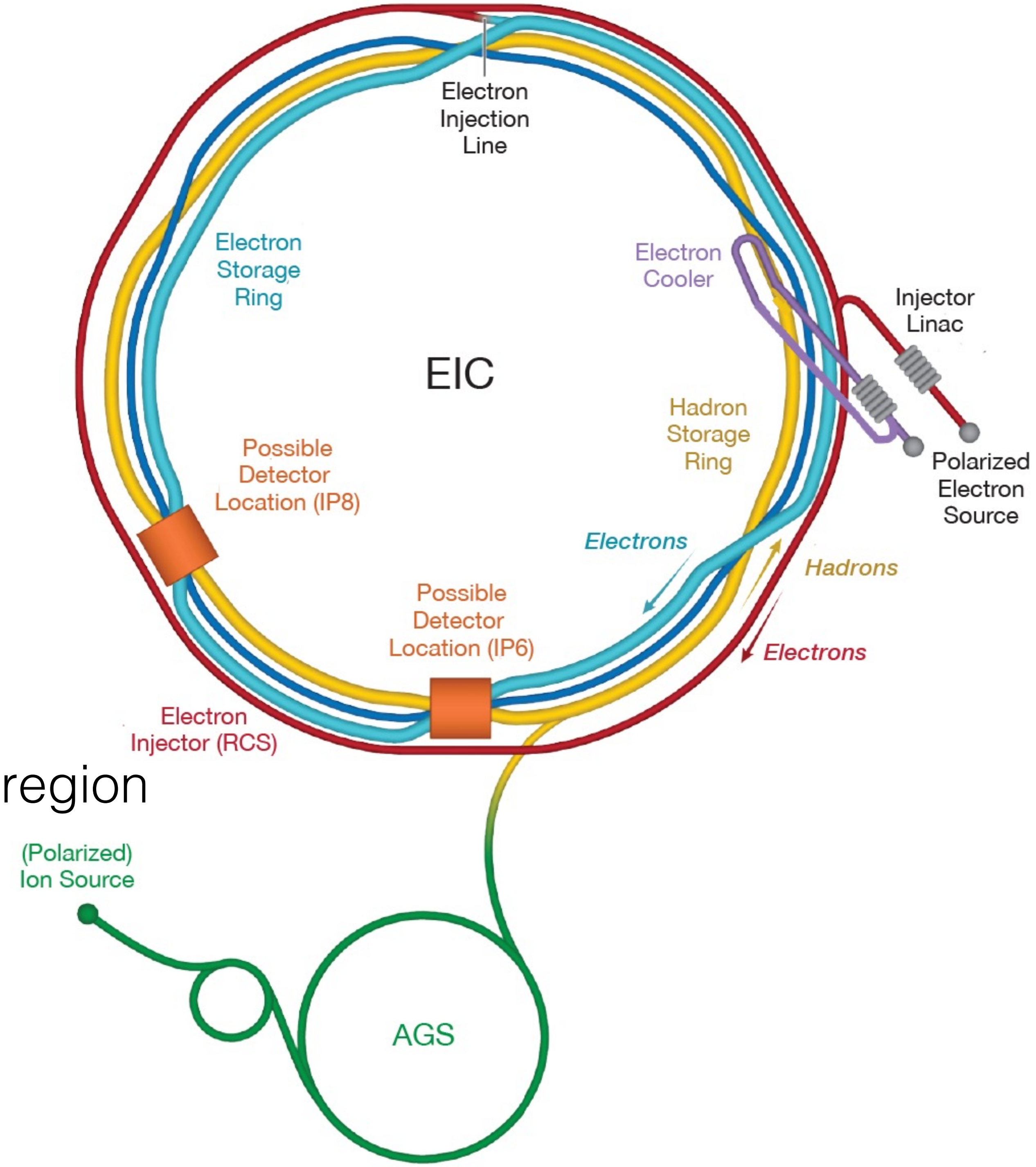
III. EIC

- Acceleratore
- Obiettivi scientifici
- Esempi

EIC

The main design requirements of the EIC:

- Highly polarized electron (70%) and proton (70%) beams
- Ion beams from deuterons to heavy nuclei
- Variable e+p center-of-mass energies from 20-100 GeV, upgradable to 140 GeV
- High collision electron-nucleon luminosity 10^{33}-10^{34} cm^{-2} s^{-1}
- Possibility to have more than one interaction region



Obiettivi scientifici

Dare risposte alle “key scientific questions” dello Yellow report :

- In che modo le proprietà nucleoniche come massa e spin emergono dai partoni e dalle loro interazioni?

- Come sono distribuiti i partoni all'interno del nucleone sia in quantità di moto che in posizione spaziale?

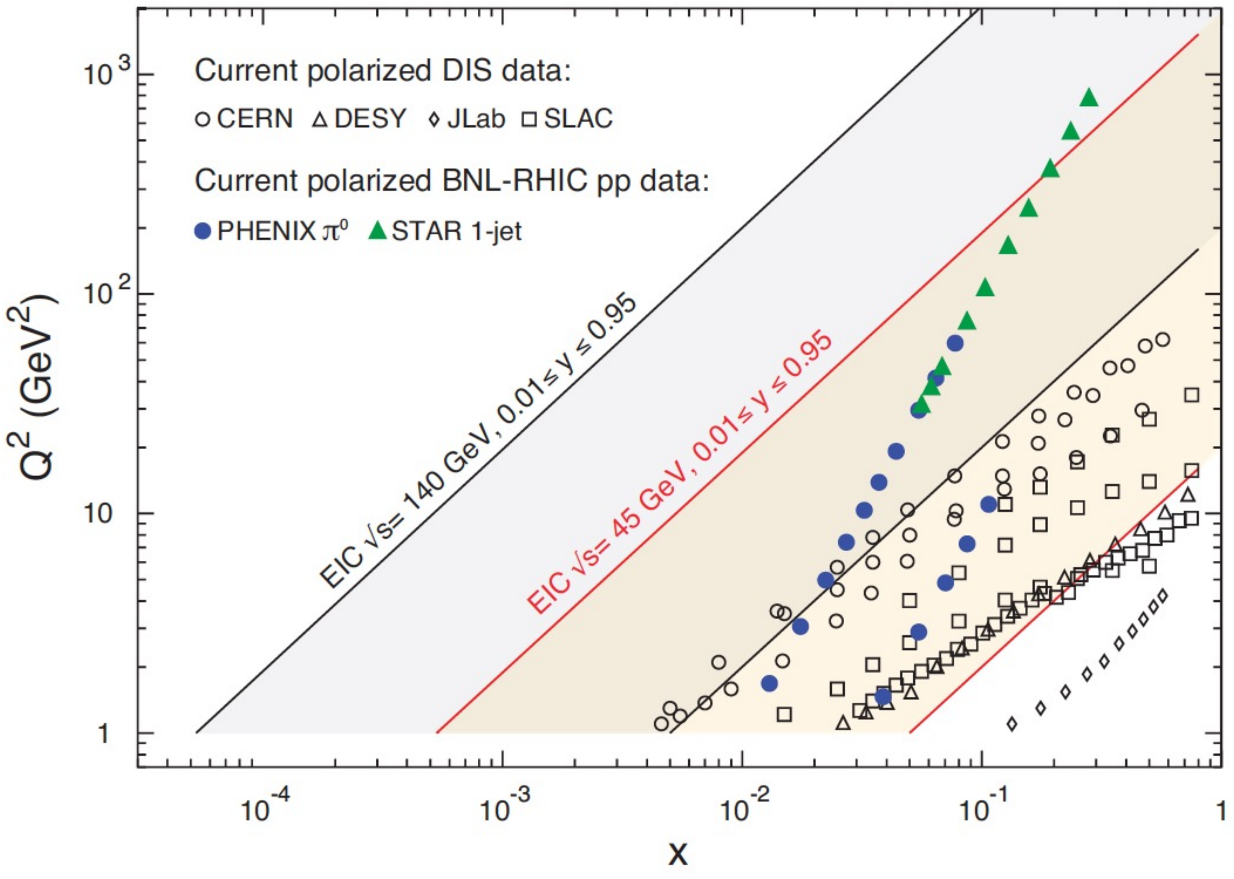
- In che modo i quark, i gluoni e i jet interagiscono con il mezzo nucleare?
  In che modo gli stati adronici confinati emergono da questi quark e gluoni?
  In che modo le interazioni quark-gluone creano il legame nucleare?

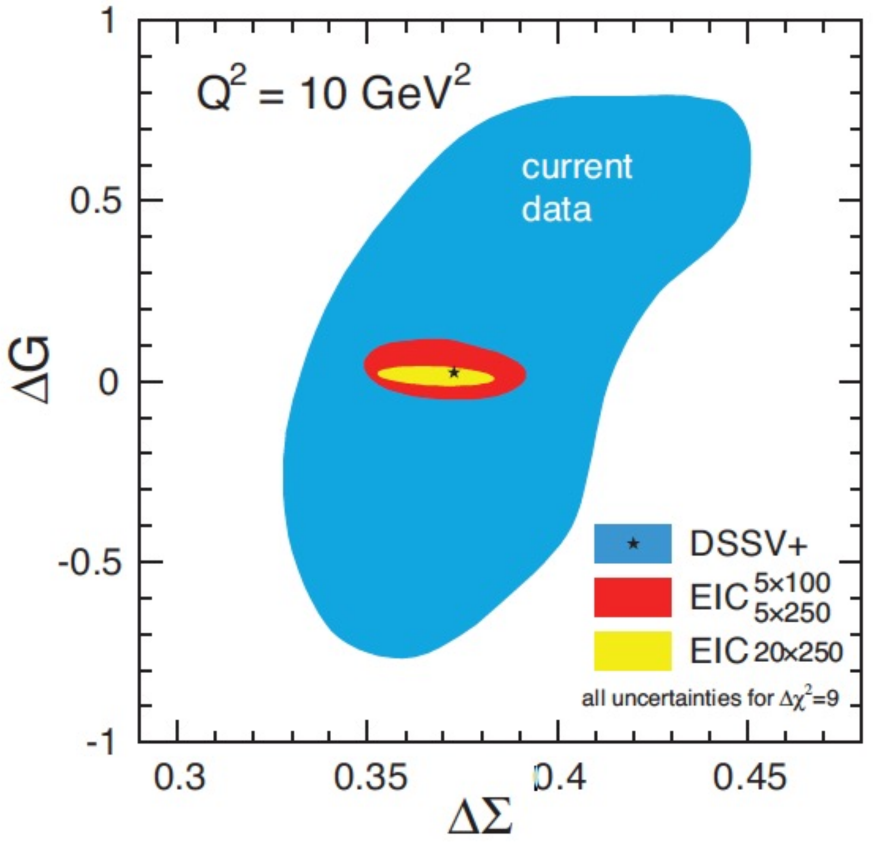
- In che modo un ambiente nucleare denso influisce sulla dinamica dei quark e gluoni, le loro correlazioni e le loro interazioni? Cosa succede alla densita’ gluonica nei nei nuclei?

Lo spin del protone

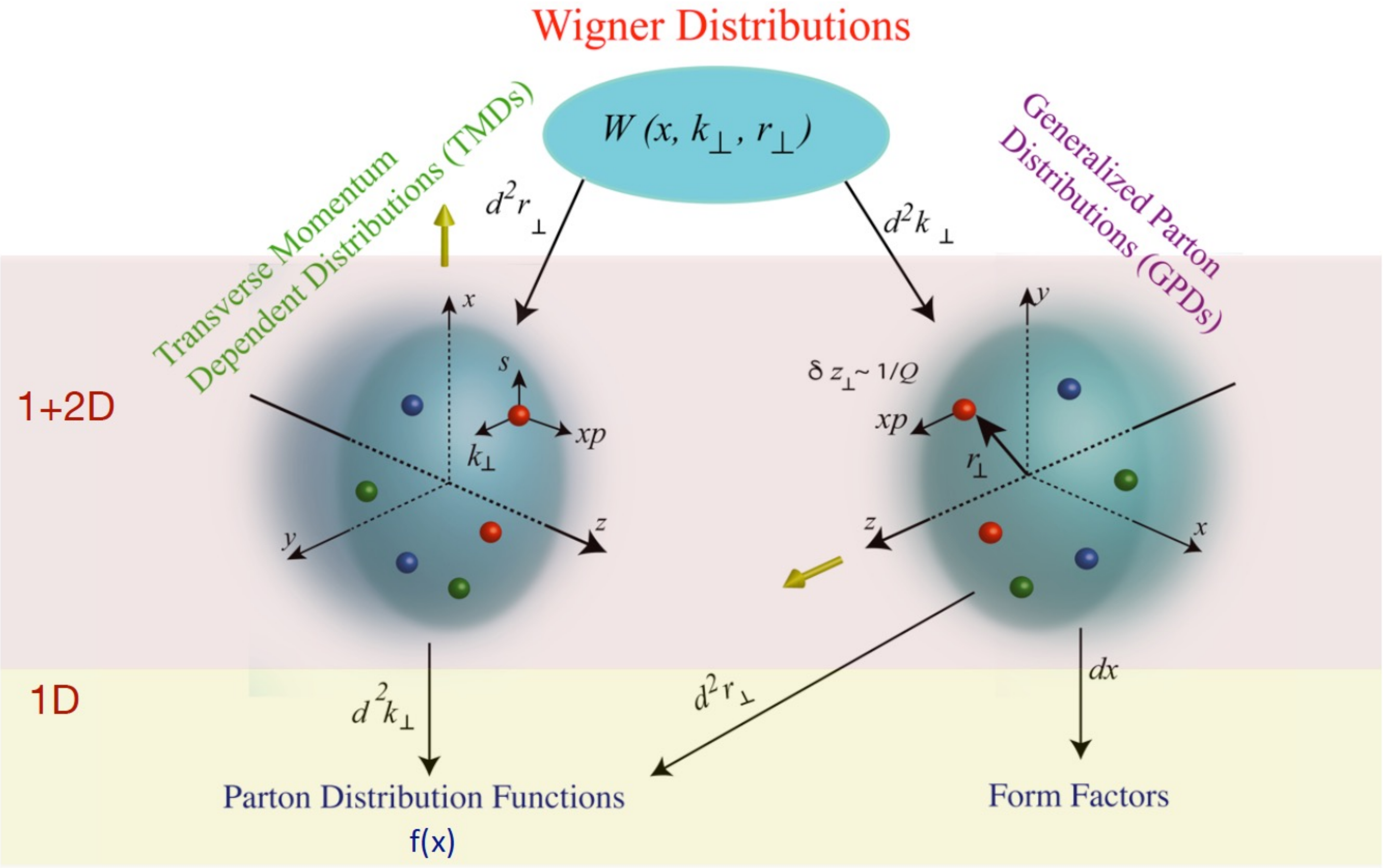
Qual’e’ il contributo dei quark e gluoni allo spin del protone?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma(\mu) + \Delta G(\mu) + L_{Q+G}(\mu)$$

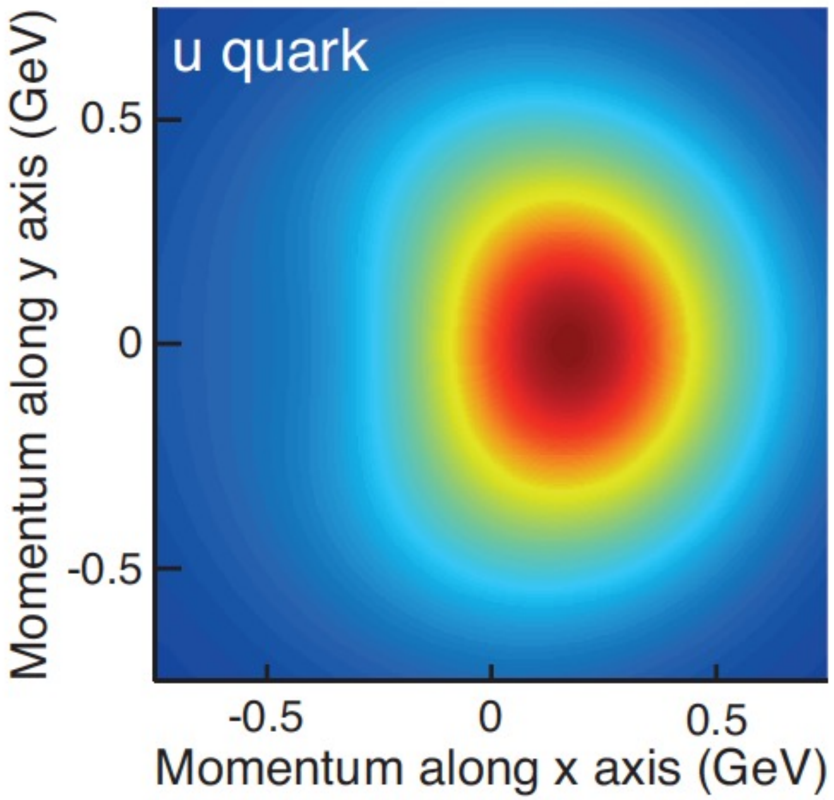


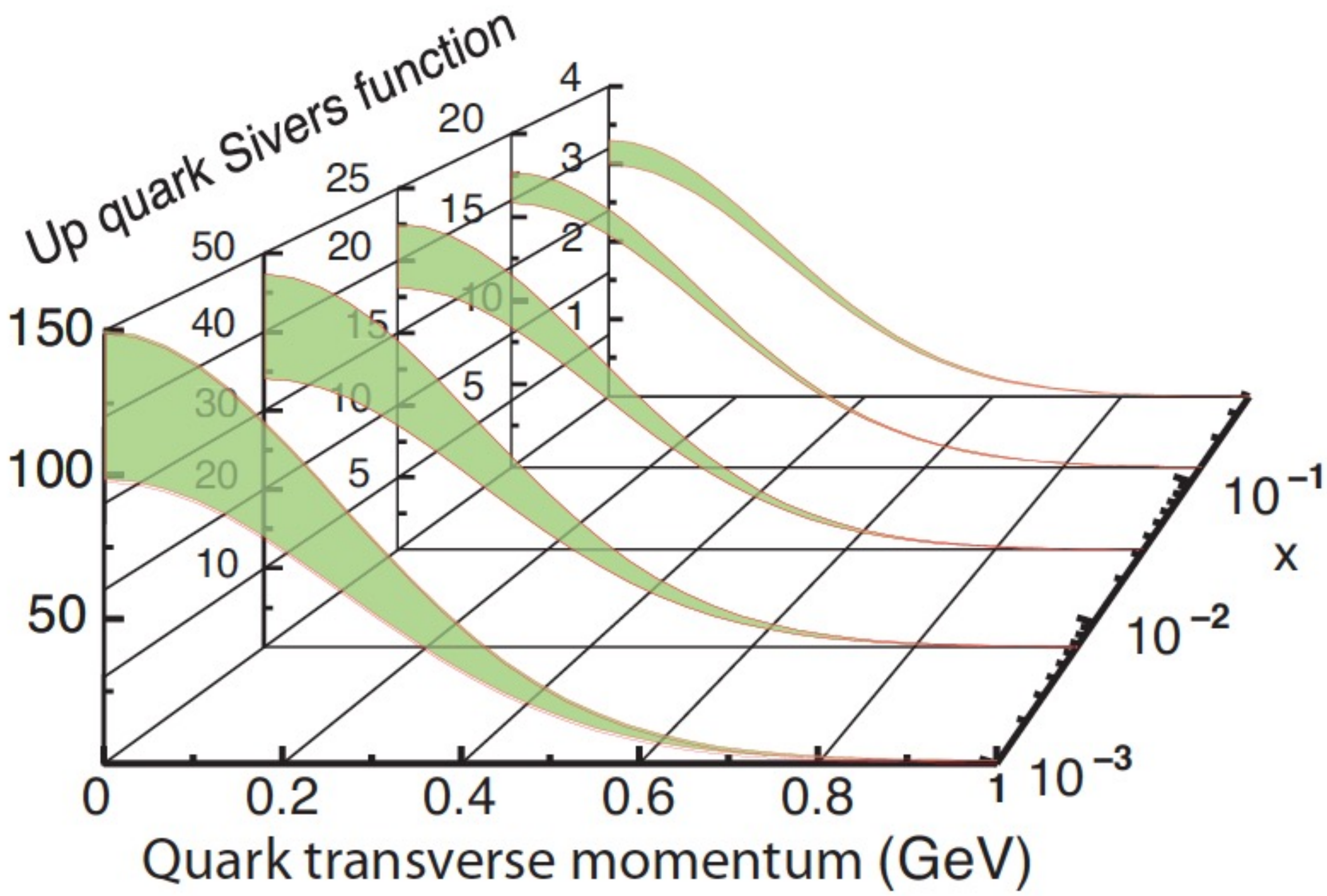


3D Nucleon Imaging: Wigner functions, TMDs & GPDs

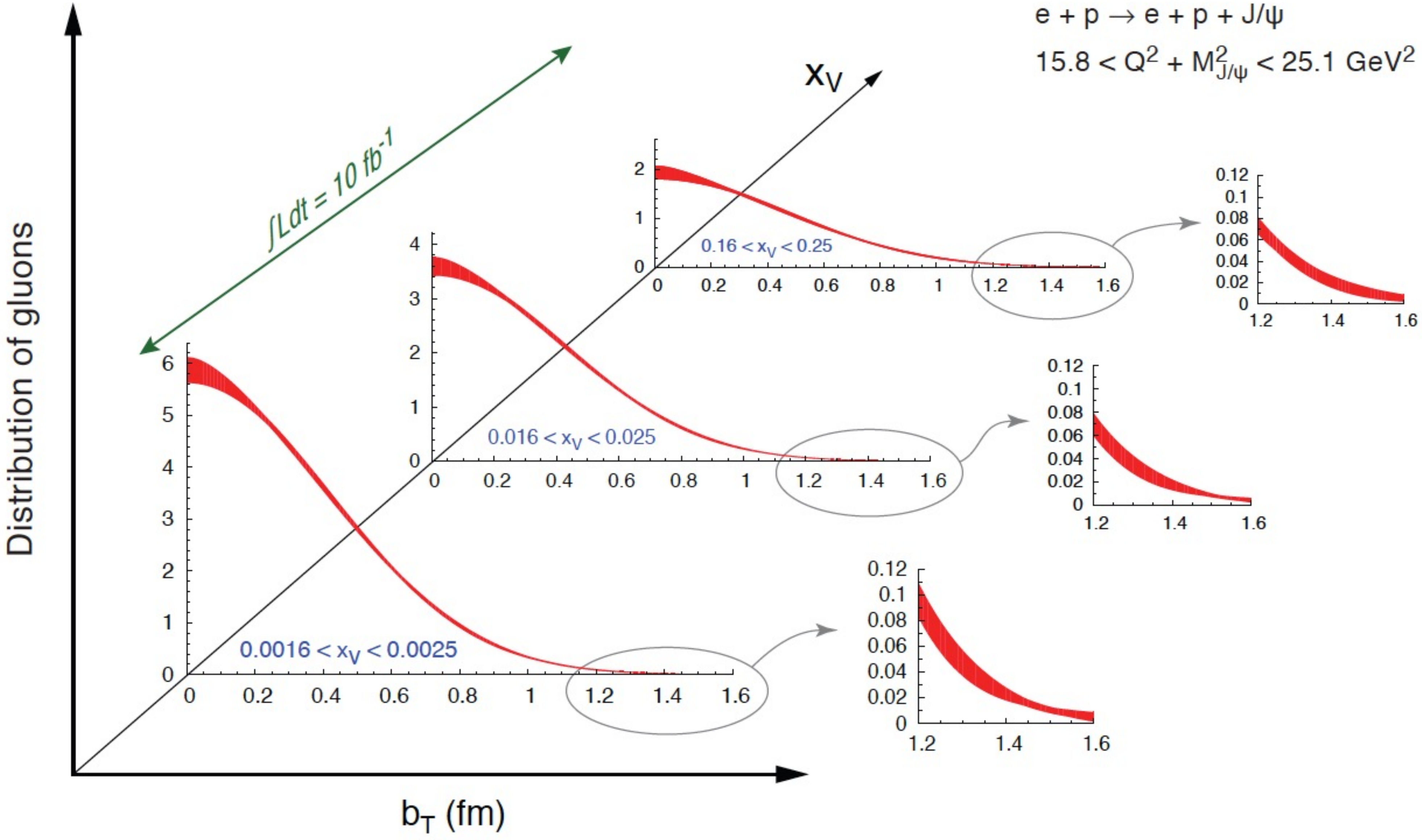


TMDs


u quark
Momentum along y axis (GeV)
0.5
0
-0.5
-0.5
0
0.5
Momentum along x axis (GeV)


Up quark Sivers function
150
100
50
50
40
30
20
10
25
20
15
10
5
20
15
10
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
Quark transverse momentum (GeV)
10^{-1}
x
10^{-2}
10^{-3}

GPDs

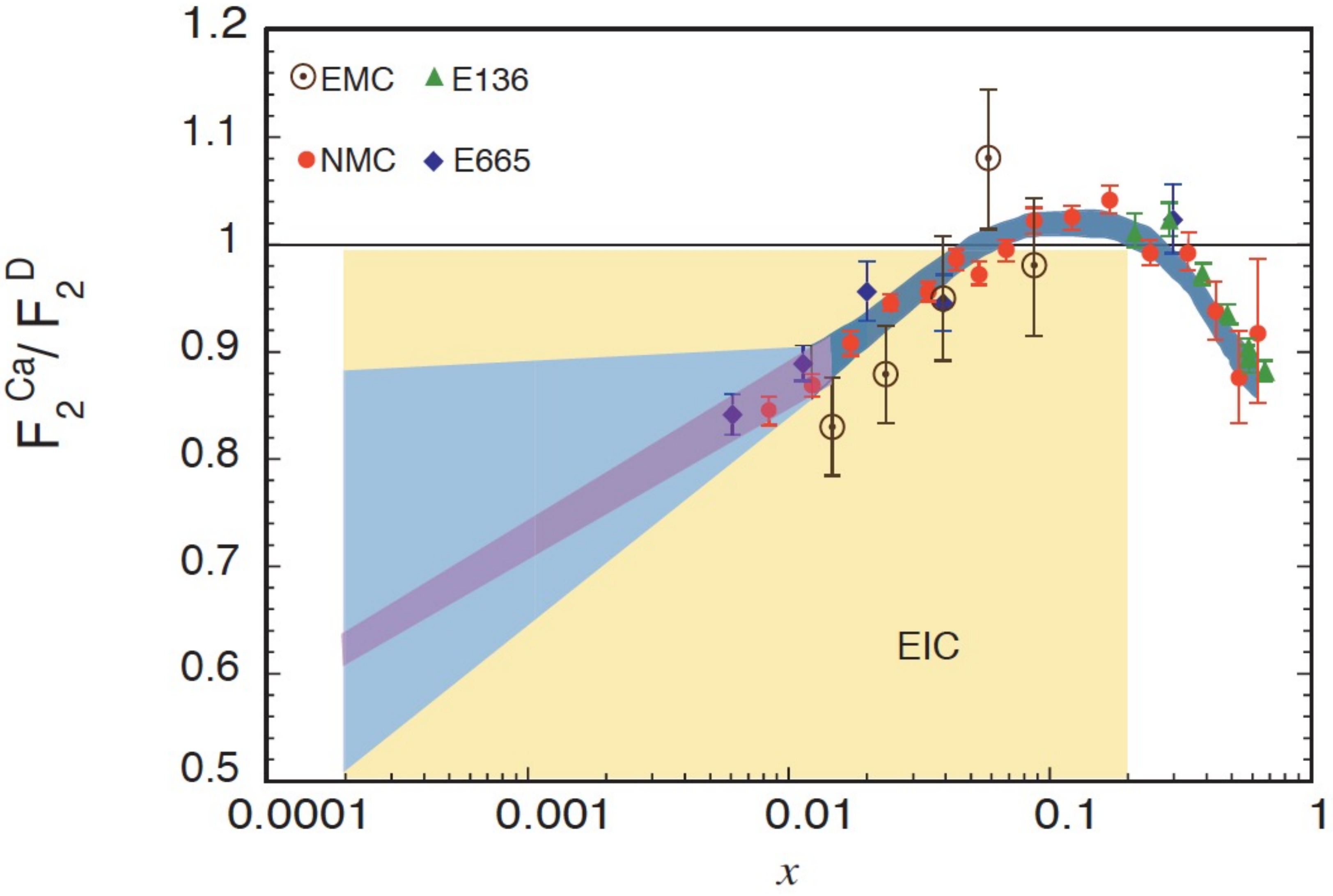


e + p → e + p + J/ψ
15.8 < Q² + M²_J/ψ < 25.1 GeV²
∫Ldt = 10 fb⁻¹
Distribution of gluons
b_T (fm)
x_V
0.0016 < x_V < 0.0025
0.016 < x_V < 0.025
0.16 < x_V < 0.25
0
1
2
3
4
5
6
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.02
0.04
0.06
0.08
0.1
0.12

Nuclear PDFs

I quark e i gluoni in un nucleo sono confinati all'interno dei singoli nucleoni o l'ambiente nucleare influisce in modo significativo sulla loro distribuzione?

Il rapporto misurato delle funzioni della struttura nucleare e del nucleone mostra una soppressione a basso x.



Conclusioni

I processi di Deep Inelastic Scattering (DIS) hanno giocato e svolgono ancora un ruolo molto importante per la nostra comprensione della QCD e della struttura del nucleone.

I passati esperimenti di DIS hanno dimostrato come ogniqualvolta una significativa innovazione e' stata introdotta (energia, natura delle particelle impegate, polarizzazione, ecc) questa si e' tradotta in un significativo progresso delle nostre conoscenze.

L' Electron Ion Collider rappresenta nel medio termine una straordinaria opportunita' per comprendere alcuni dei fondamentali problemi aperti della QCD.

Sara', come al solito, soprattutto l'entusiamo dei giovani a guidare questa nuova avventura scientifica. Siete interessati a partecipare?

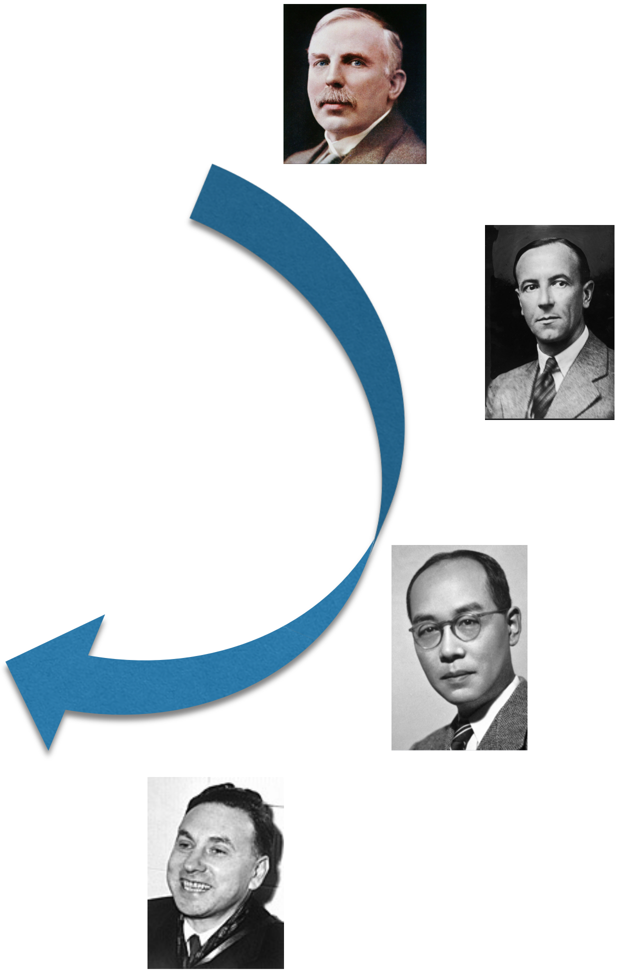
Conclusioni

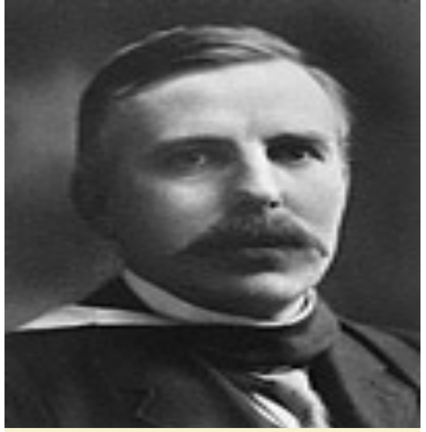
WE WANT YOU!

Back-up

Preistoria (1909-1960)

- Diffusione Rutherford
- Nucleo atomico
- Protoni e neutroni
- Forza forte
- Fattori di Forma



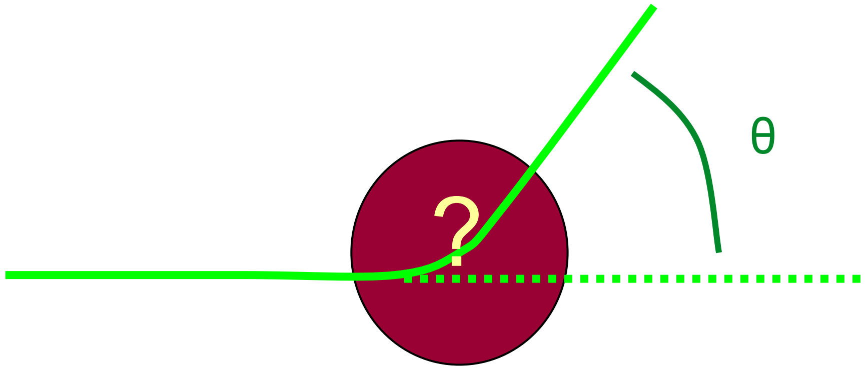


Nobel prize 1908

Diffusione Rutherford

"The most famous fixed-target experiment of all time:"

Rutherford ci ha insegnato la lezione più importante:
l'utilizzo del **processo di diffusione** per indagare la struttura della materia



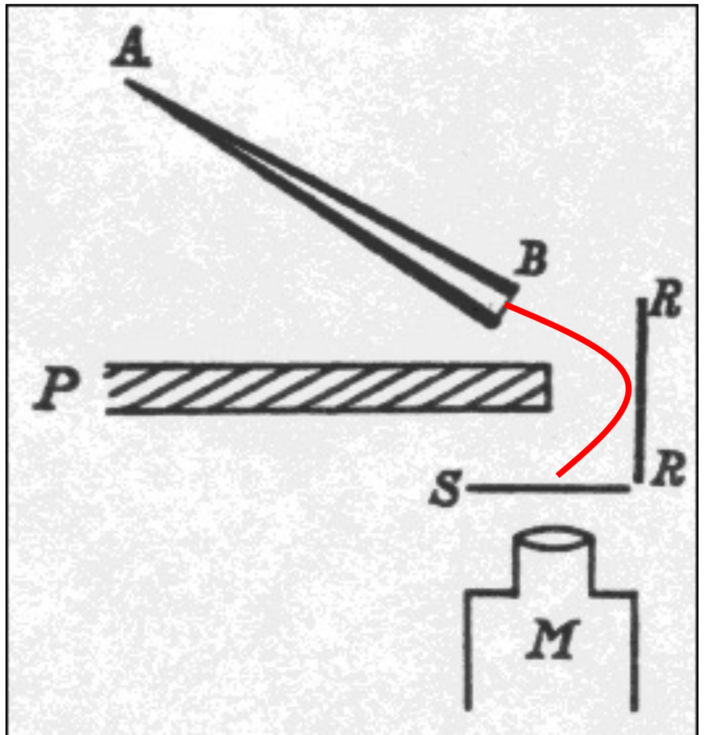


Fig. 1.

H. Geiger e E. Marsden osservano la diffusione ad alto angolo ($\theta > 90^0$) di particelle alpha deflesse da una sottile Lamina d'oro

Proc.Roy. Soc. A **82**, 495,1909

Rutherford interpreta i risultati come la dovuti alla diffusione delle particella alpha con una carica centrale massiva

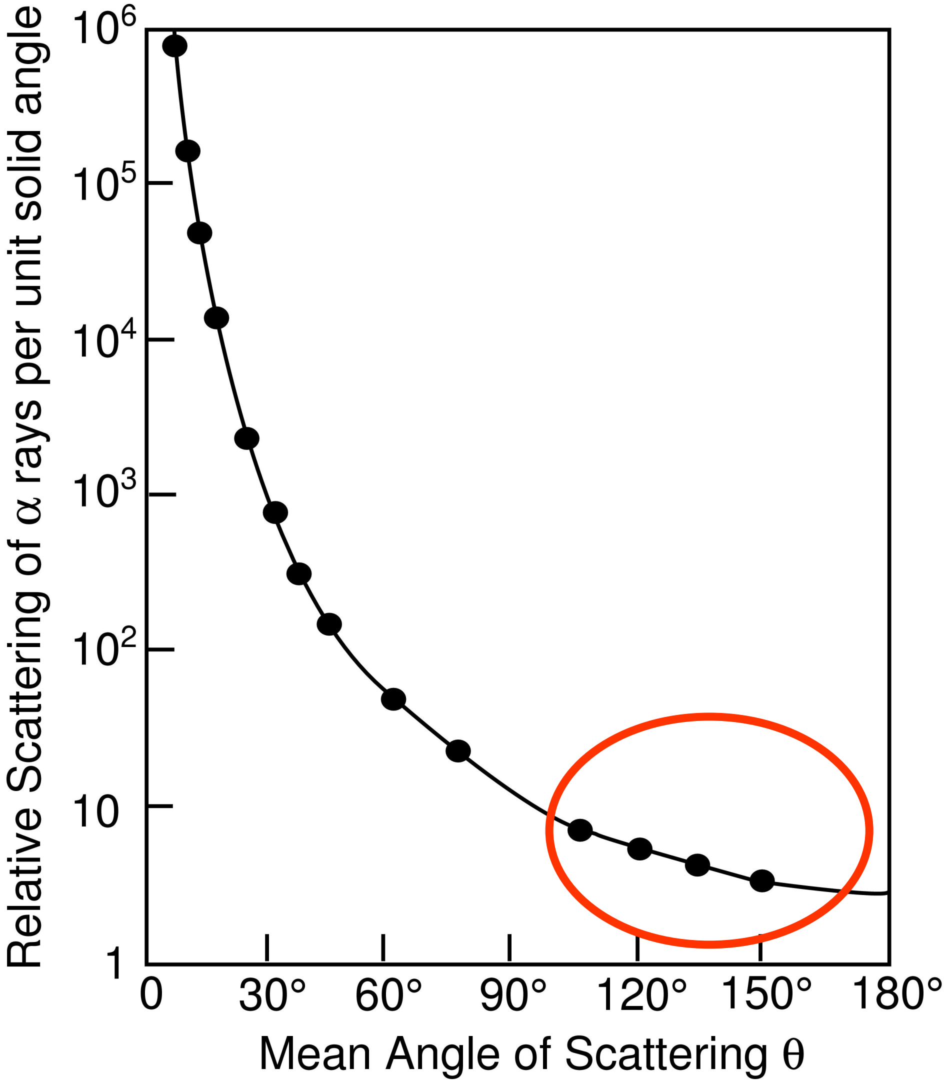
$$\sigma(\theta) = \frac{z^2 Z^2 e^4}{16 E^2} \frac{1}{\sin^4 \frac{1}{2}\theta}$$

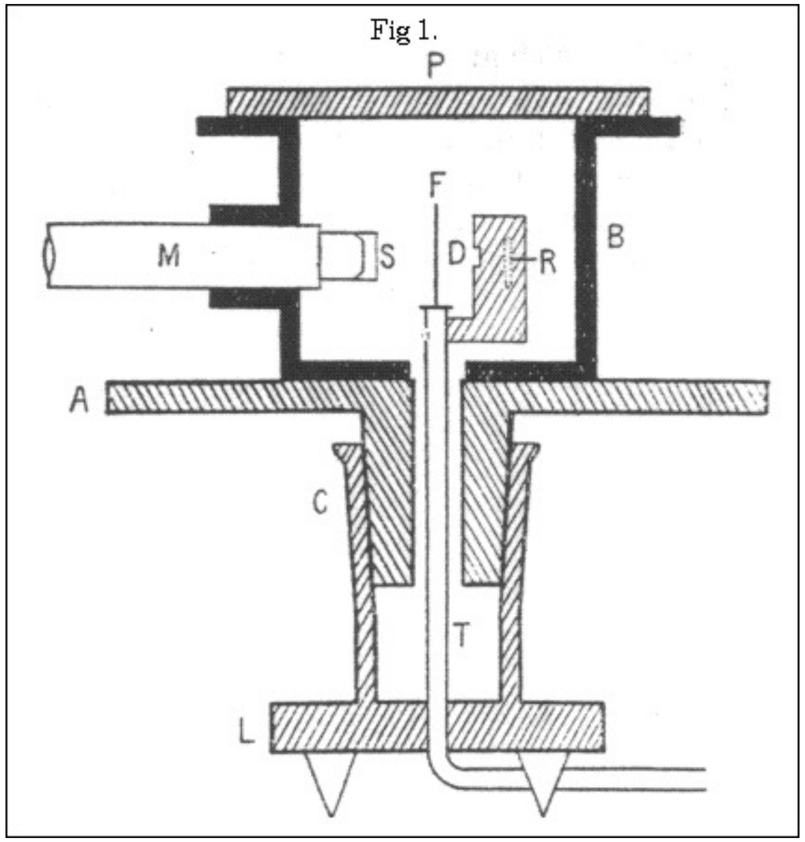
Phil. Mag. **21**,669,1911

Diffusione Rutherford

Nel 1913 Geiger e Marsden verificano con successo (per nuclei ad alto Z) le previsioni della teoria di Rutherford.

Phil. Mag. **25**, 604,1913

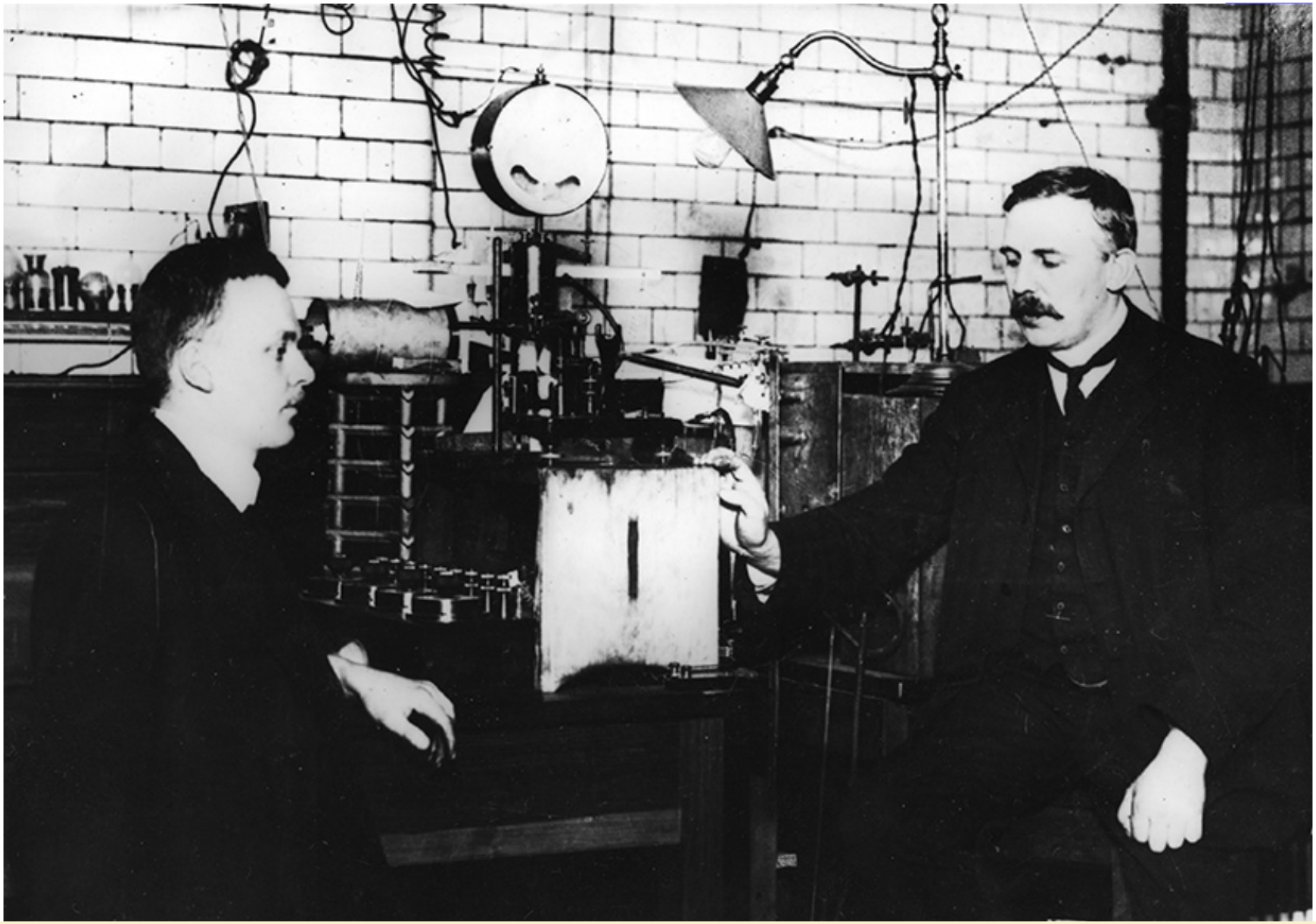




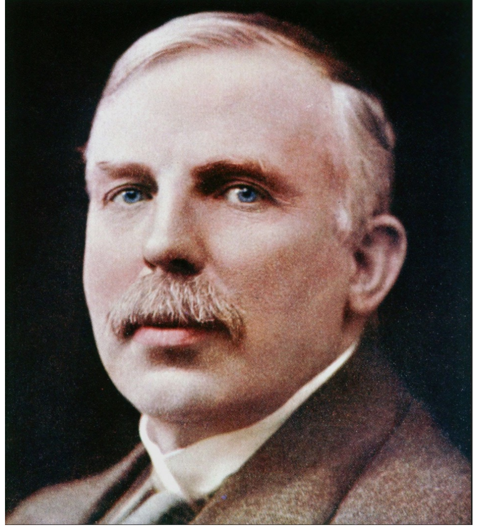
Scoperta del nucleo atomico

N.Bohr Old Quantum theory

Diffusione Rutherford



"It was quite the most incredible event that has ever happened to me in my life.
It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper
and it came back and hit you."



Scoperta del protone

Nel 1917 Rutherford intraprende una serie di esperimenti sulla collisione di particelle alpha con atomi leggeri.

1919 – Scoperta del protone (“H-atom”)

Prima rezione nucleare

$$\alpha + {}^{14}C \rightarrow {}^{17}O + p$$

Phil. Mag. **37**, 581,1919

Ne 1921 Chadwick e Bieler ottengono le prime evidenze di deviazioni dalla legge di diffusione di Rutherford

$$\alpha + {}^{1}H \rightarrow {}^{1}H + \alpha$$

Phil. Mag. **42**, 923,1921

> “The present experiments do not seem to throw any light on the nature of the law of variation of the forces at the seat of an electric charge, but merely show that the forces are of very great intensity…It is our task to find some field of force which will reproduce these effects.”
>
> **Chadwick, Bieler** Phil. Mag. **42**, 923,1921

Nobel prize
1935

Scoperta del neutrone

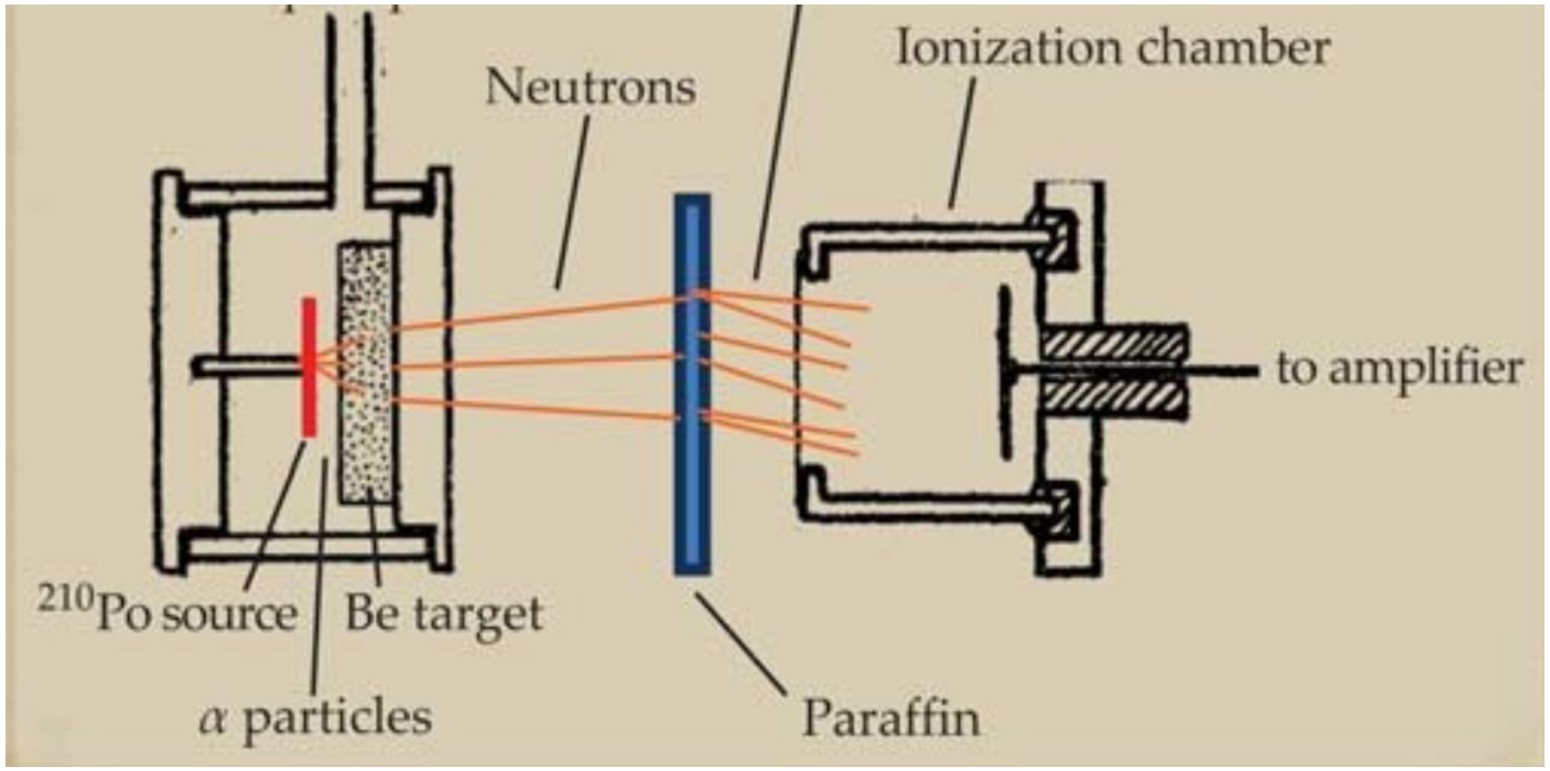
> "I think we shall have to make a real search for the neutron."
> Chadwick letter to Rutherford, 1924

Nel 1932, stimolato da precedenti esperimenti di Bothe-Becker ed in particolare di Joliot-Curie, J. Chadwick scopre il neutrone

$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

Proc. Roy. Soc. A **136**, 692,1932

Nature **129**, 312,1932



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Nature **129**, 312,1932

Scoperta del decadimento $\boldsymbol{\beta}^{+}$

Nobel prize 1935

$$\alpha + {}^{27}\mathrm{Al} \rightarrow {}^{30}\mathrm{P} + n$$

$$\searrow$$

$${}^{30}\mathrm{Si} + e^{+} + (\nu_e)$$

$$\alpha + {}^{10}\mathrm{Bi} \rightarrow {}^{13}\mathrm{N} + n$$

$$\searrow$$

$${}^{13}\mathrm{C} + e^{+} + (\nu_e)$$

Ernest Rutherford

"In our laboratory today we live in an atmosphere deemed with the flying fragments of exploding atoms and on this occasion I wish to say a few words on the methods and ideas employed to break up atoms and realize…the old dream of alchemists of transmutation of one element into another..."

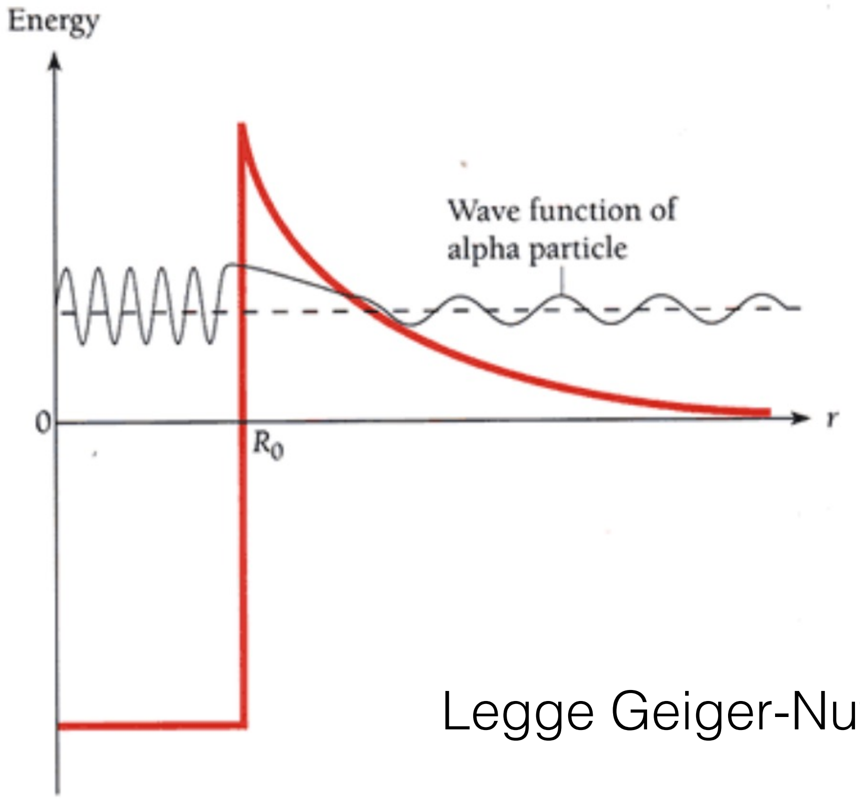
You tube Link. : https://www.youtube.com/watch?v=zBHD8ksx_Sg

Meccanica Quantistica e QED

La meccanica quantistica si sviluppa rapidamente nel periodo 1924-1927

In ambito nucleare:
- 1928: teoria del decadimento alpha
  (G. Gamow, R.W. Gurney &E.U. Condon)

Relativita' speciale:
- 1920-30 Legame energia di legame - stabilita'
  (B formula - Aston – Einstein $E_0=mc^2$)



Legge Geiger-Nuttal


Nobel prize 1933

1927-1930 A. P. Dirac pone le basi della meccanica quantistica relativistica e dell'elettrodinamica quantistica (QED)

Primi passi della teoria quantistica di campo

= > Spin e momento di dipolo magnetico dell'elettrone:

$$\vec{\mu} = -\left(\frac{g\mu_B}{\hbar}\right)\vec{S} \quad \text{con} \quad \mu_B = \frac{e\hbar}{2m_e}$$

Magnetone di Bohr

Nobel prize 1943

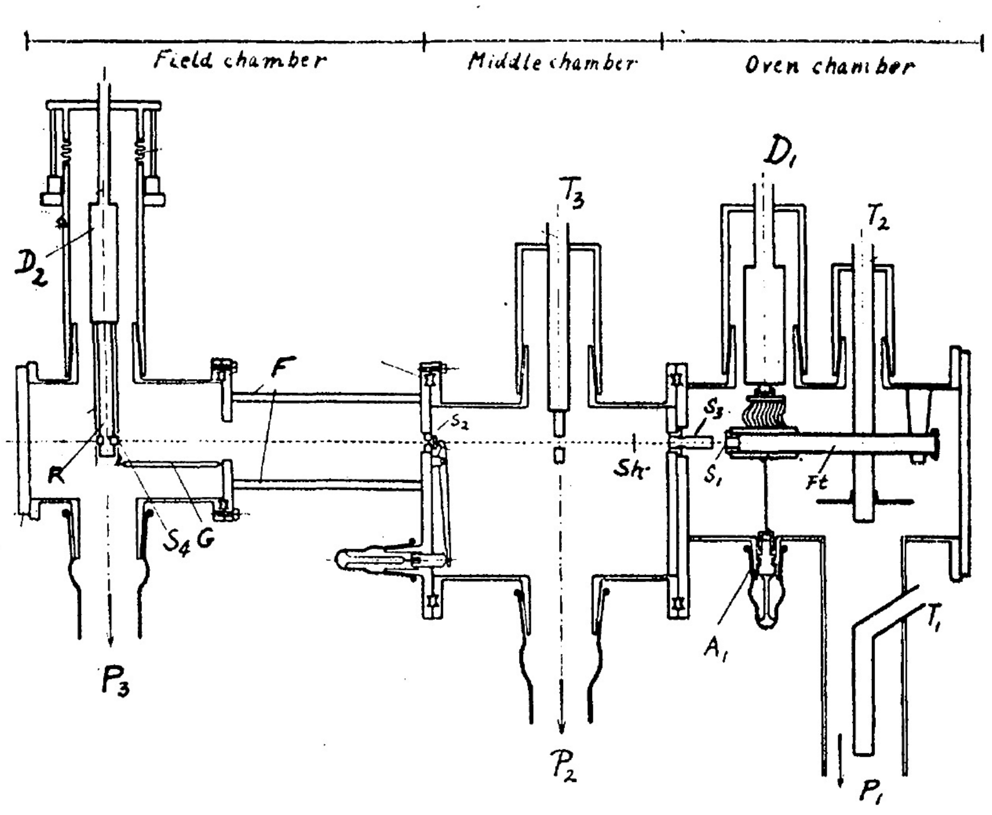
Momento magnetico del protone

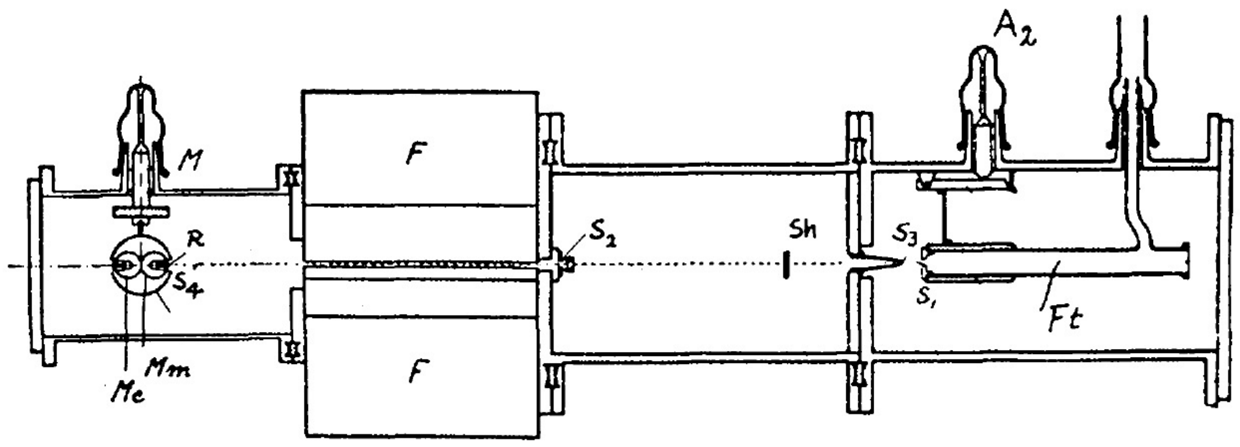
Nel 1933 O. Stern, I. Esterman e R. Frish presentano la prima determinazione del momento magnetico del protone utilizzando la deflessione in campo B (non omogeneo) di un fascio molecolare (H_2).

$$\mu_p = 2.5\ \mu_N\ (\pm 10\%) \quad \text{con} \quad \mu_N = \frac{e\hbar}{2m_p}$$

Z. Phy. **85**, 4,1933

Z. Phy. **85**, 17,1933





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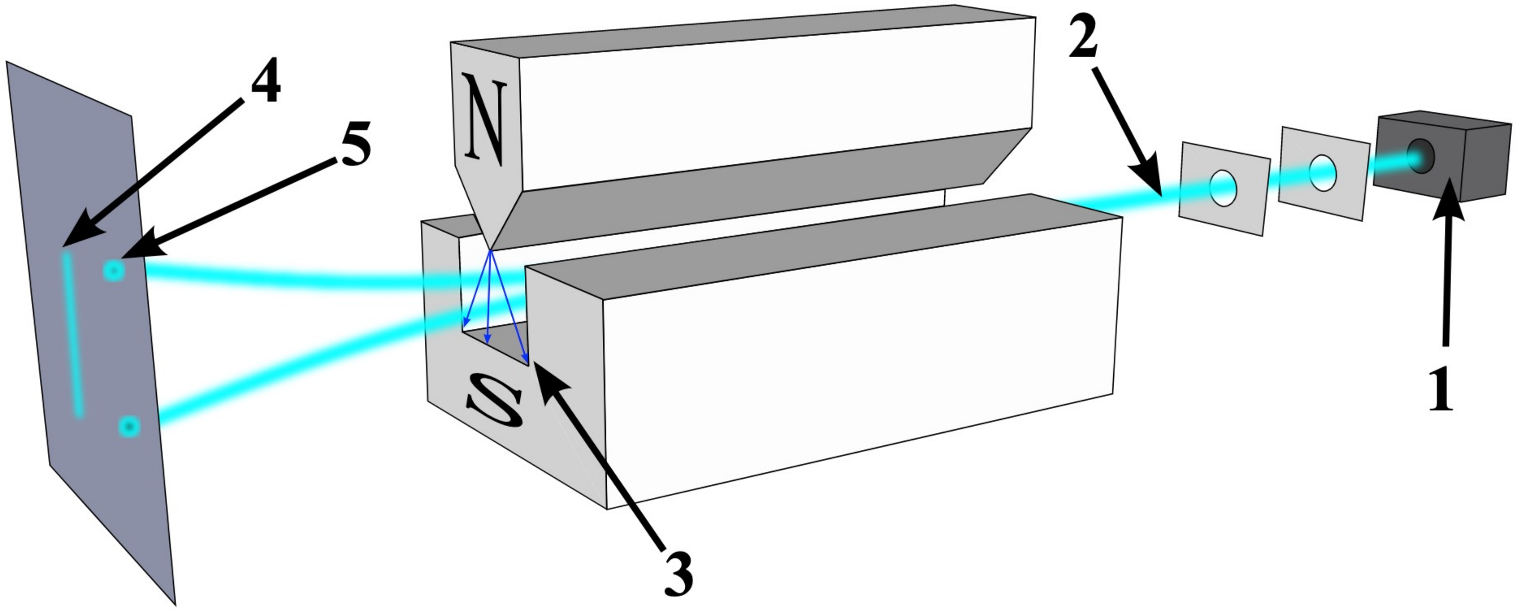
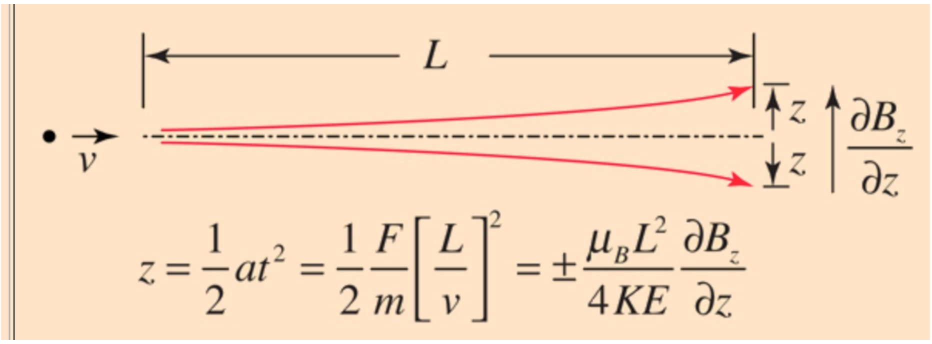


$$\vec{F}(\mathbf{r}) = \mu_k \vec{\nabla} B_k(\mathbf{r})$$



$$z = \frac{1}{2}at^2 = \frac{1}{2}\frac{F}{m}\left[\frac{L}{v}\right]^2 = \pm\frac{\mu_B L^2}{4KE}\frac{\partial B_z}{\partial z}$$

Nobel prize 1944

Momento magnetico del protone e del neutrone

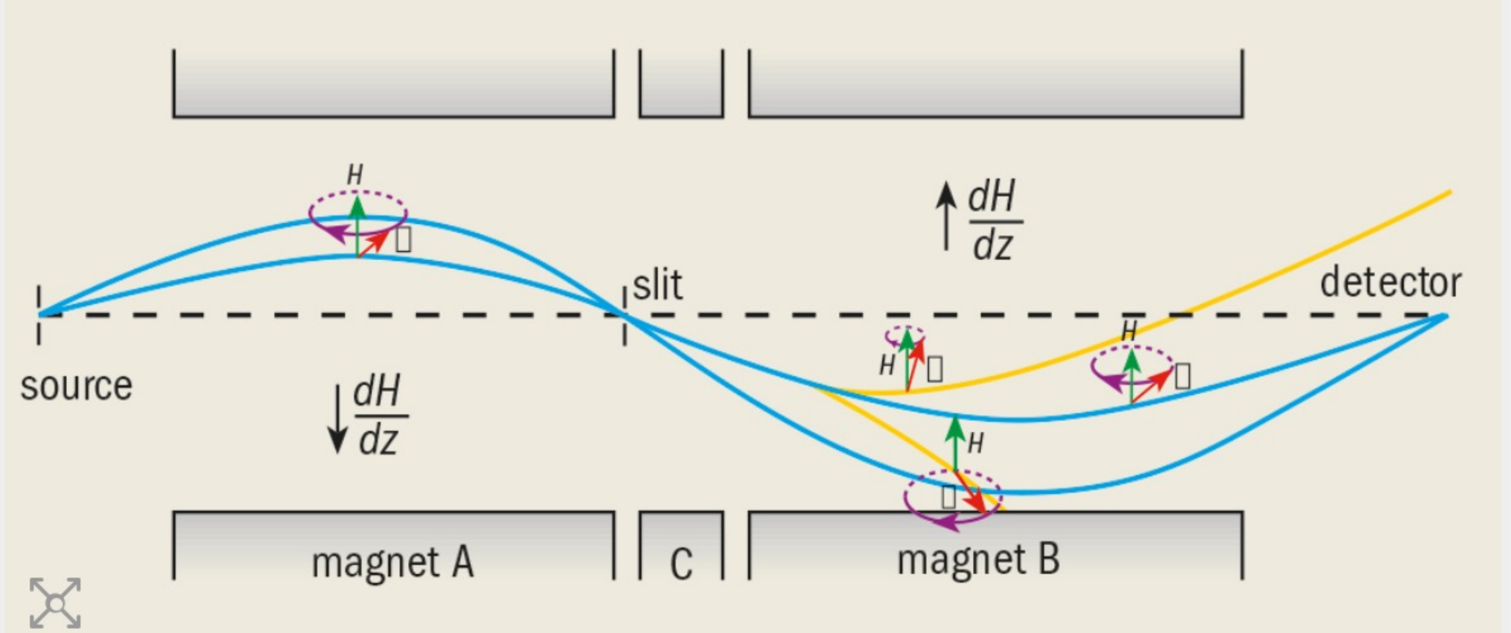
Nel 1939 I.I. Rabi e coll. migliorano (con una nuova tecnica) la misura del momento magnetico del protone e del deutone.

$$\mu_p = 2.785\mu_N \pm 0.02$$

$$\mu_D = 0.855\mu_N \pm 0.006 \; (\rightarrow \mu_n = -1.93\mu_N)$$

Phys. Rev. **56**, 728,1939

Il protone ed il neutrone non sono particelle elementari!



Nobel prize
1933

La Forza Forte e la Fisica Nucleare

Z. Phys. **82** (1933) 137

1932-33: Heisenberg pubblica tre articoli che marcano l'inizio di uno studio teorico sistematico dell'interazione forte
"*Über den Bau der Atomkern*" -> "forza nucleare di scambio" ("Platzwechsel")

1933-36 Importanti contributi di Maiorana, Breit, Wigner, Cassen & Condon

Z. Phys. **82** (1933) 137

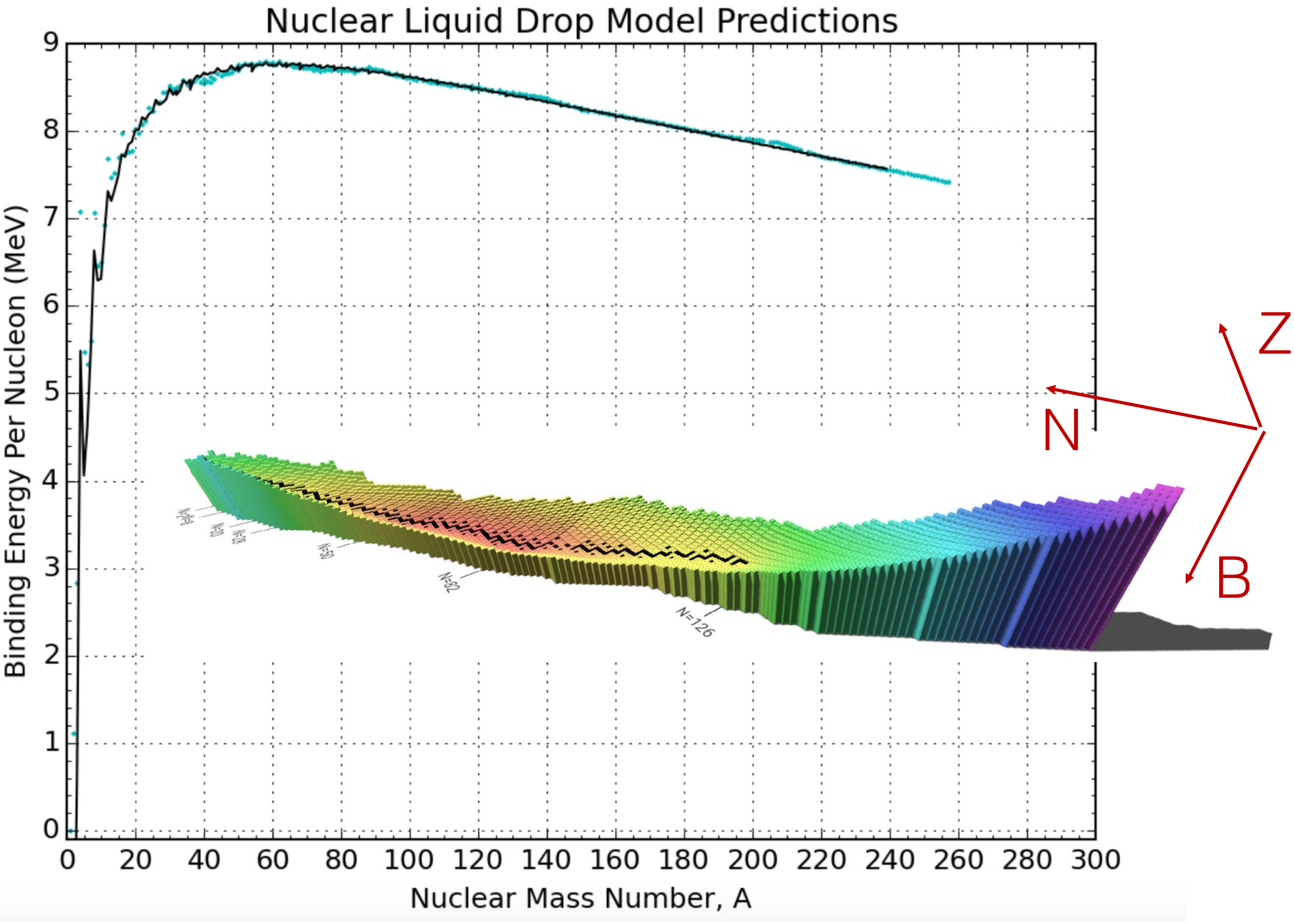
- Forza a corto raggio (< 10^{-15} m)
- Saturazione (B ~ A)
- Indipendenza di carica (p-p, n-n e p-n)
- Isospin

Formula di Weizsäcker

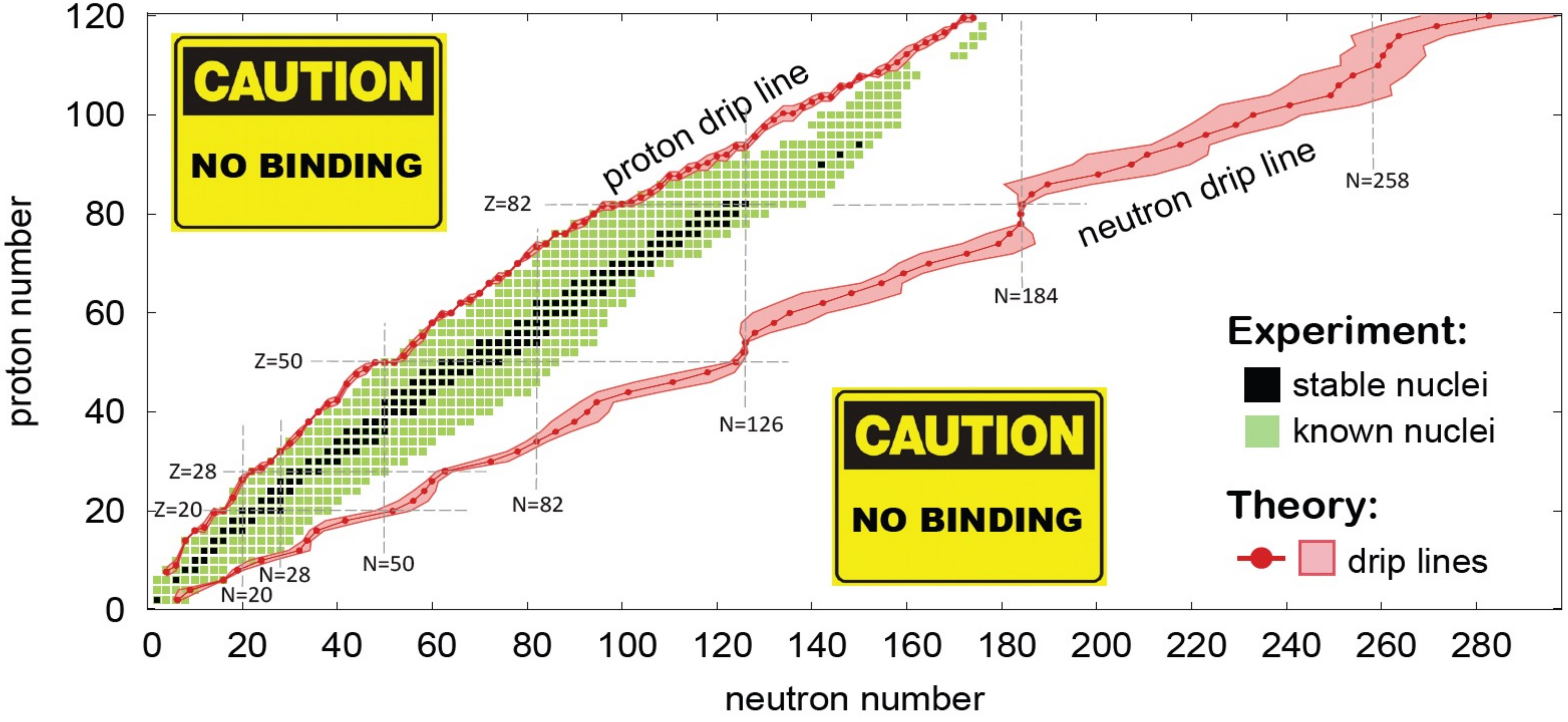
$$E = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A,Z)$$

Raggio nucleare $R = r_0 A^{1/3}$

La Forza Forte e la Fisica Nucleare



Nuclear Landscape

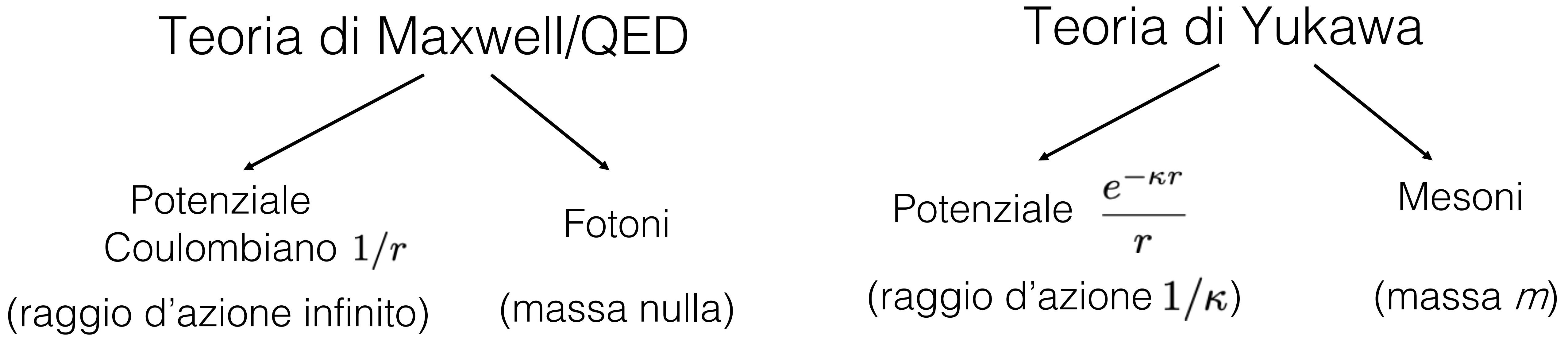


Nobel prize
1949

Yukawa

Nel 1935 Yukawa, per descrivere le interazioni forti, introduce una analogia tra la forza elettromagnetica e la forza forte.

Proc. **17, 48** (1935)



Teoria di Maxwell/QED

Potenziale Coulombiano $1/r$

(raggio d'azione infinito)

Fotoni

(massa nulla)

Teoria di Yukawa

Potenziale $\frac{e^{-\kappa r}}{r}$

(raggio d'azione $1/\kappa$)

Mesoni

(massa *m*)

Stima della massa del mesone: $m \sim 200\ m_e$

Il "mesone di Yukawa" venne inizialmente identificato con il muone ("mesone μ") e successivamente, grazie anche al fondamentale lavoro di Conversi-Pancini-Piccioni , con i pioni $\pi^{\pm}, \pi^0$

Nobel prize
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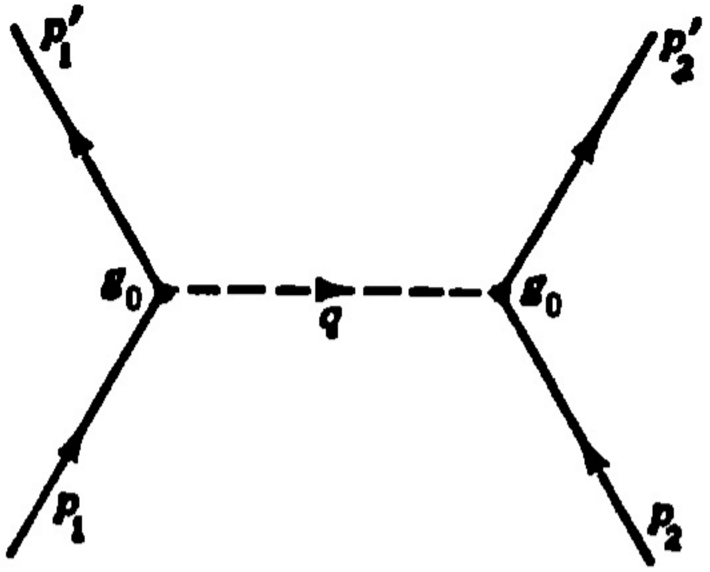


$$\mathcal{M} \sim \frac{g_0^2}{q^2 - \mu^2 + i\epsilon} \xrightarrow{\text{NR}} \frac{g_0^2}{|\mathbf{q}|^2 + \mu^2}$$

$$\frac{g_0^2}{|\mathbf{q}|^2 + \mu^2} \overset{\text{FT}}{\longleftrightarrow} V(r) \sim \; g_0^2 \frac{e^{-\mu r}}{r}$$

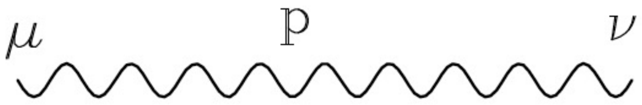
Nobel prize 1965

Elettrodinamica Quantistica

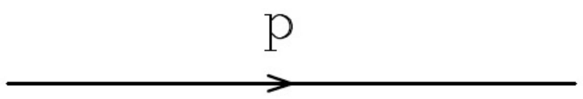
Nel periodo 1947-1949 viene conpletato il programma di rinormalizzazione dell'Elettrodinamica Quantistica (Tomonaga, Schwinger, Feynman).

Feynmann introduce un nuovo linguaggio

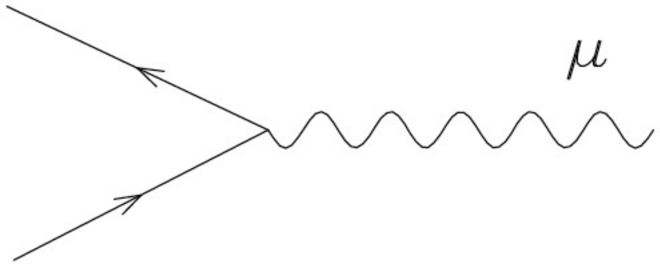
$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi - e\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



$$= \quad -i\frac{g_{\mu\nu}}{p^2 + i\epsilon} \text{ (Feynman gauge)}$$



$$= \quad \frac{i}{\not{p} - m + i\epsilon} = i\frac{\not{p} + m}{p^2 - m^2 + i\epsilon}$$



$$= \quad -ie\gamma_\mu Q$$

Spinori e 4-vettori polarizzazione:

$u, \bar{u}, v, \bar{v}$ e $\epsilon^\mu, {\epsilon^\mu}^*$

Nobel prize 1965

Elettrodinamica Quantistica

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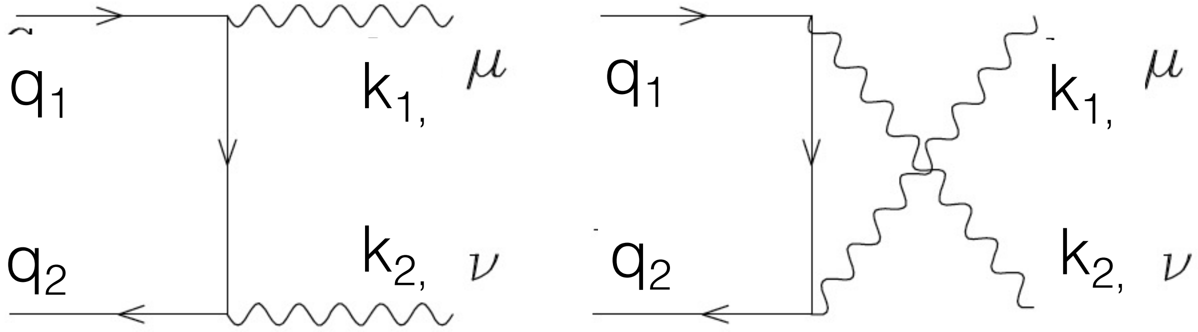
Example: $e^+e^- \rightarrow \gamma\gamma$



$$\mathcal{M} = \bar{v}(q_2, s_2)(-ie\gamma_\mu)\frac{i}{(\not{q}_1 - \not{k}_1 - m + i\epsilon)}(-ie\gamma_\nu)u(q_1, s_1)$$

$$\epsilon^{\nu *}(k_1, \lambda_1)\epsilon^{\mu *}(k_2, \lambda_2) +$$

$$+\bar{v}(q_2, s_2)(-ie\gamma_\nu)\frac{i}{(\not{q}_1 - \not{k}_2 - m + i\epsilon)}(-ie\gamma_\mu)u(q_1, s_1)$$

$$\epsilon^{\nu *}(k_1, \lambda_1)\epsilon^{\mu *}(k_2, \lambda_2)$$

$$\overline{\sum}|\mathcal{M}|^2 = 2e^4\left[\frac{(q_1k_2)}{(q_1k_1)} + \frac{(q_1k_1)}{(q_1k_2)} + 2m^2\left(\frac{1}{(q_1k_1)} + \frac{1}{(q_1k_2)}\right) + \right.$$

$$\left. -m^4\left(\frac{1}{(q_1k_1)} + \frac{1}{(q_1k_2)}\right)^2\right]$$

Perturbative meson field theory?

Sulla base dei successi della QED vengono introdotte teorie di campo basate su particelle mediatrici scalari (e.g. i pioni)

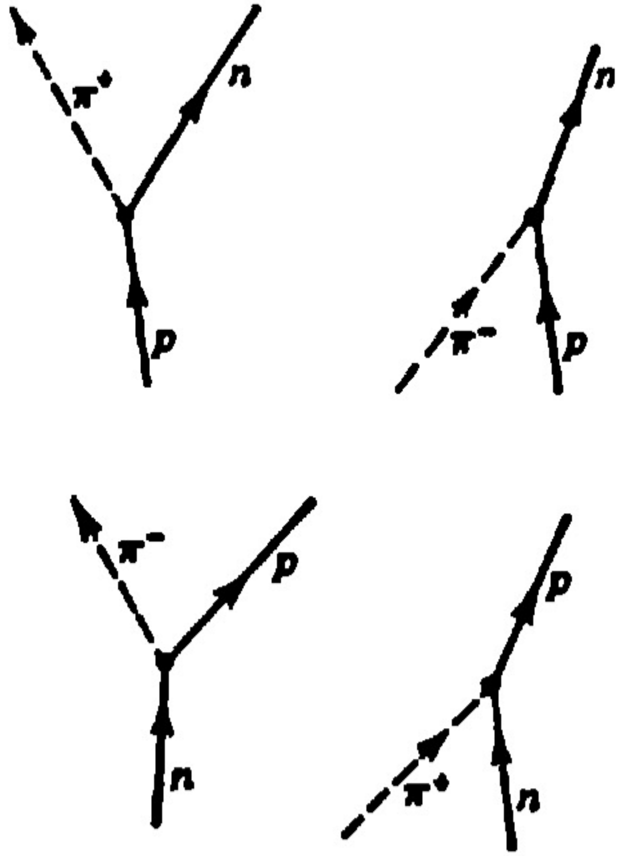
Pseudoscalar isotriplet with pseudoscalar coupling:

$$H_{int} = ig \int \left(\bar{\psi}(x)\vec{\tau}\gamma_5\psi(x) \cdot \vec{\pi}(x)\right) d\vec{x}$$

Creation and distruction operators of pions: $\pi^+ = \frac{\pi_1 + i\pi_2}{\sqrt{2}}, \quad \pi^- = \frac{\pi_1 - i\pi_2}{\sqrt{2}}, \quad \pi^0 = \pi_3$

Stime ottenute negli anni 1940: $\frac{g^2}{\hbar c} \sim 15 - 40$

Da confrontare con: $\alpha = \frac{e^2}{\hbar c} \simeq \frac{1}{137}$



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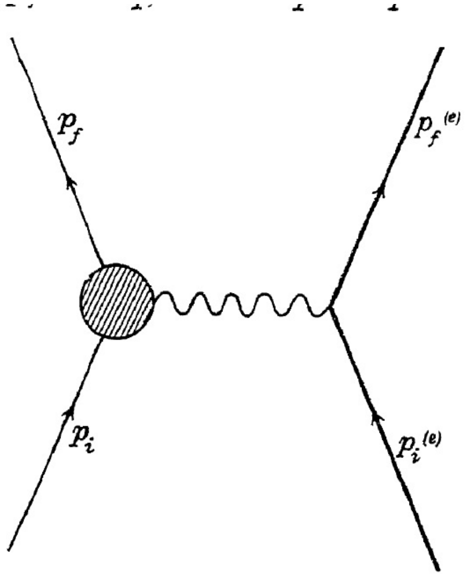
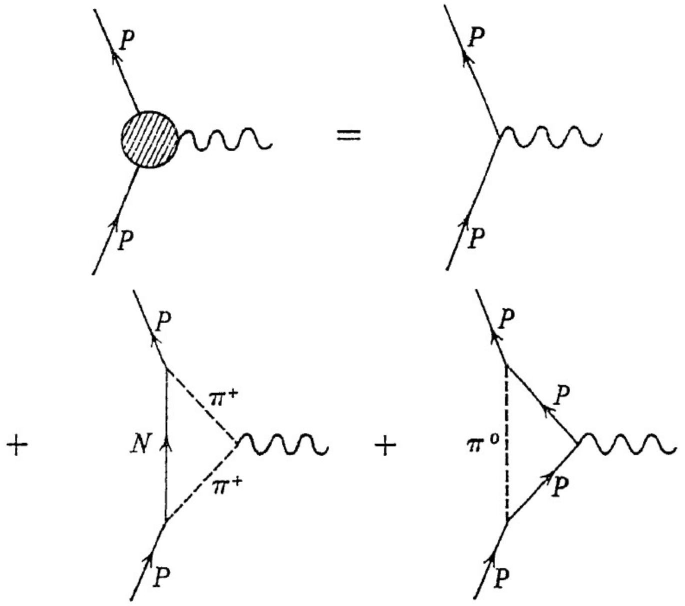




Proton magnetic moment: $\frac{g^2}{\hbar c} \simeq 52$

Neutron magnetic moment: $\frac{g^2}{\hbar c} \simeq 7$

Borowitz, Kohn – Phys.Rev. 76 818 (1949)

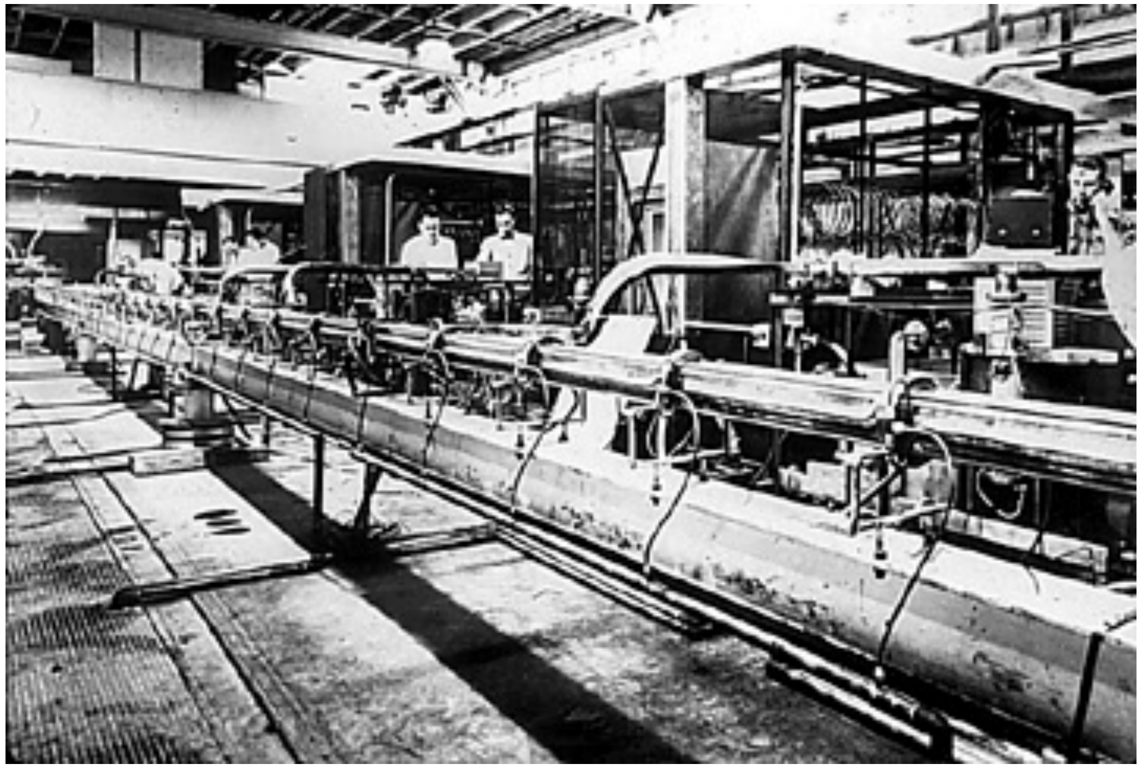
Linacs alla Stanford University

Nella meta' degli anni '30 i fratelli Varian ("research assistants" presso il dipartimento di micronde dell' Universita' di Stanford) sviluppano il "klystron" sfruttando una particulare cavita' elettromagnetica (Rhumbatron) ideata da W. Hansen.

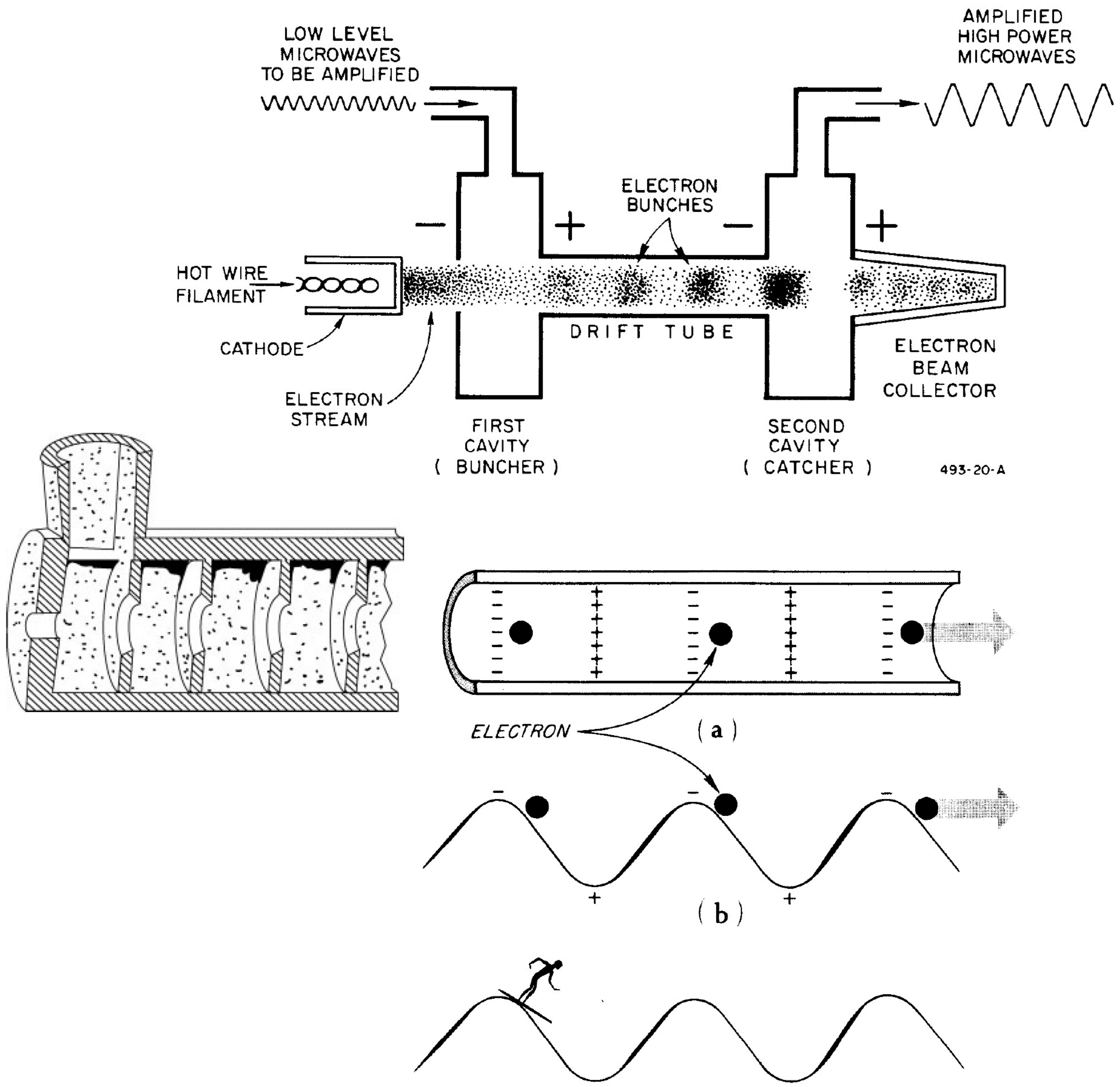
Questo dispositivo permettera' all' HEPL di Stanford di giocare un ruolo di assoluto primo piano nello sviluppo di acceleratori lineari…

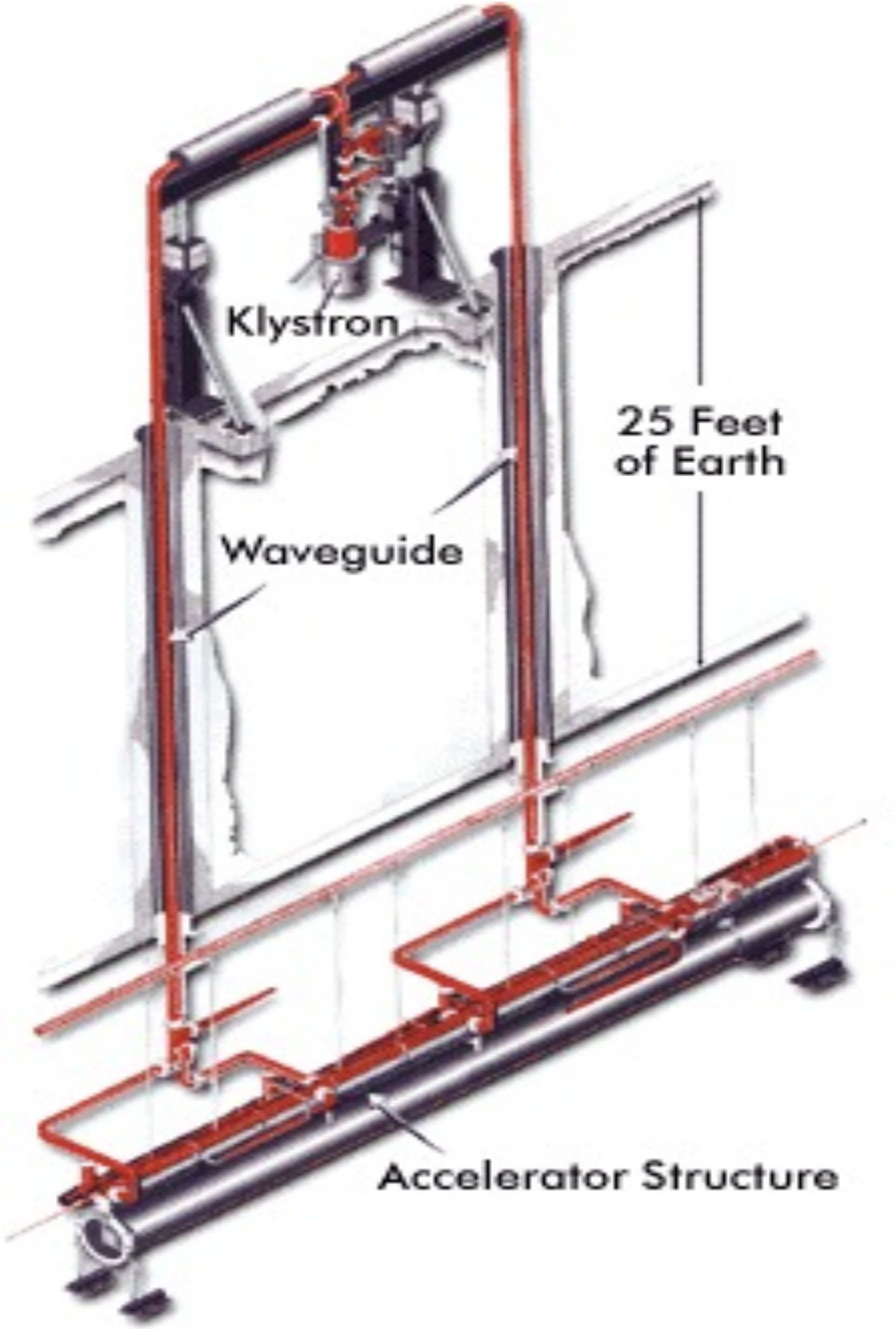
Sotto la direzione di E. Ginzton HEPL inizia la costruzione di una serie di linacs su "piccola scala" (MARK I,II,III…).

MARKIII sara' fondamentale per la realizzazione degli esperimenti di R. Hofstadter's sulla diffusione elástica e-N ed e-p.



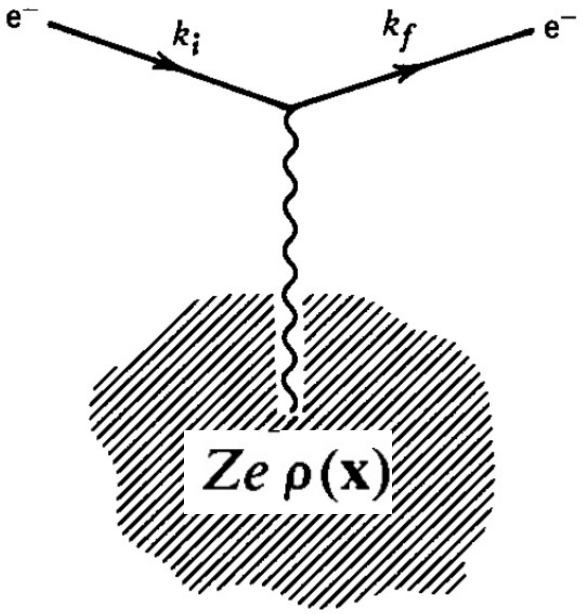
Klystron


LOW LEVEL MICROWAVES TO BE AMPLIFIED
AMPLIFIED HIGH POWER MICROWAVES
ELECTRON BUNCHES
HOT WIRE FILAMENT
CATHODE
ELECTRON STREAM
DRIFT TUBE
ELECTRON BEAM COLLECTOR
FIRST CAVITY (BUNCHER)
SECOND CAVITY (CATCHER)
493-20-A
ELECTRON
(a)
(b)


Klystron
Waveguide
25 Feet of Earth
Accelerator Structure

Fattore di forma nucleo

Stimulated by accelerators technology advances and fully muture QED various theoreticians (Rose (48), Elton(50)) started to calculate cross sections for elastic electron-Nucleus scattering



$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} |F(\mathbf{q})|^2$$

$$F(\mathbf{q}) = \int \rho(\mathbf{x}) e^{i\mathbf{q}\cdot\mathbf{x}}\, d^3x$$

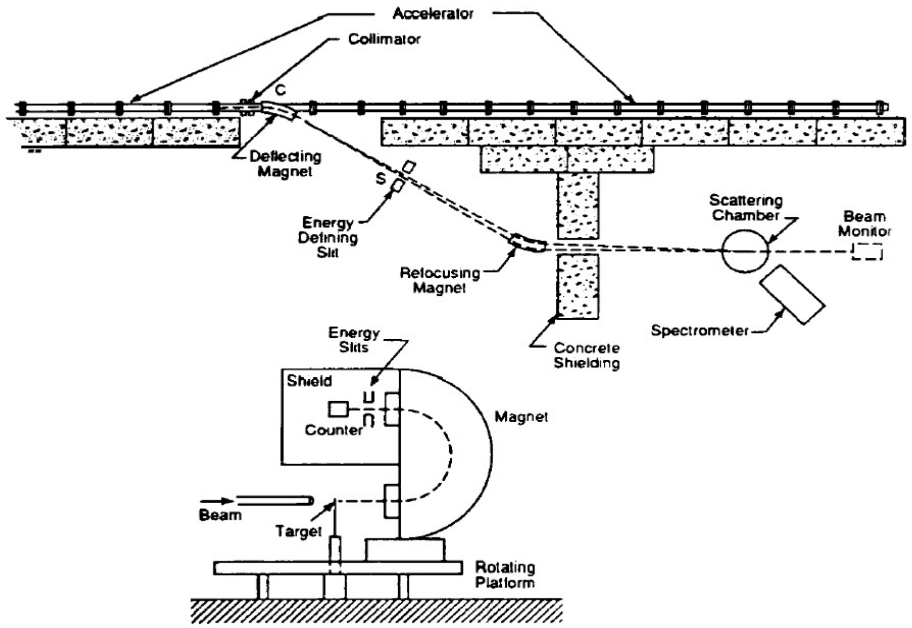
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{(Z\alpha)^2 E^2}{4k^4 \sin^4\frac{\theta}{2}}\left(1 - v^2 \sin^2\frac{\theta}{2}\right)$$

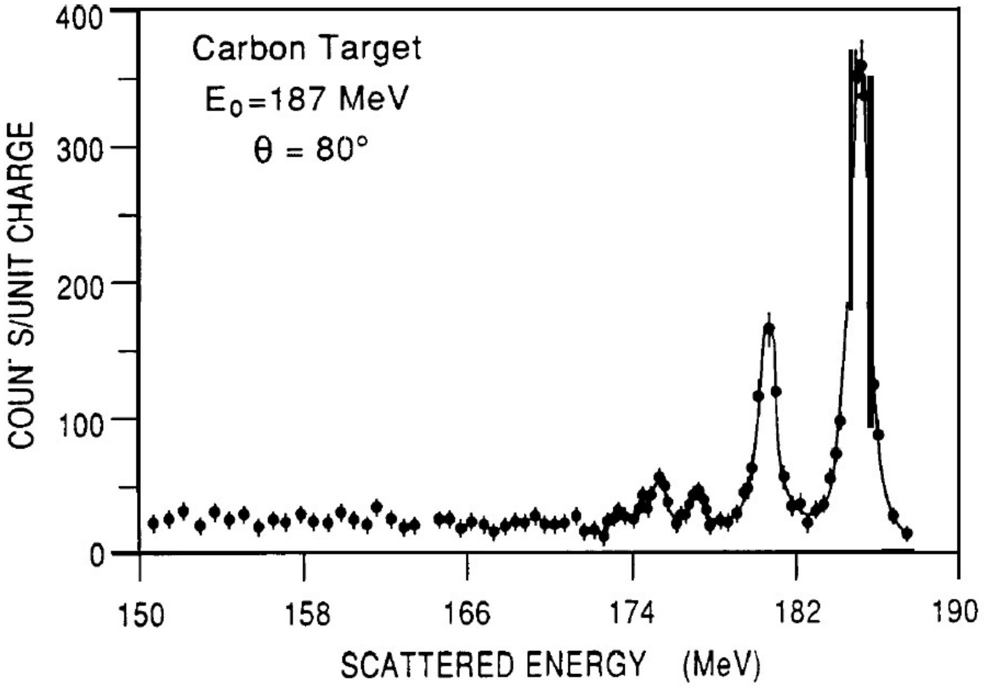
$$F(\mathbf{q}) = \int\left(1 + i\mathbf{q}\cdot\mathbf{x} - \frac{(\mathbf{q}\cdot\mathbf{x})^2}{2} + \cdots\right)\rho(\mathbf{x})\, d^3x$$
$$= 1 - \frac{1}{6}|\mathbf{q}|^2\langle r^2\rangle + \cdots,$$

$$r_{\text{m}}^2 = -6\,\frac{\mathrm{d}F(\mathbf{q})}{\mathrm{d}(|\mathbf{q}|^2)}\bigg|_{|\mathbf{q}|=0}.$$

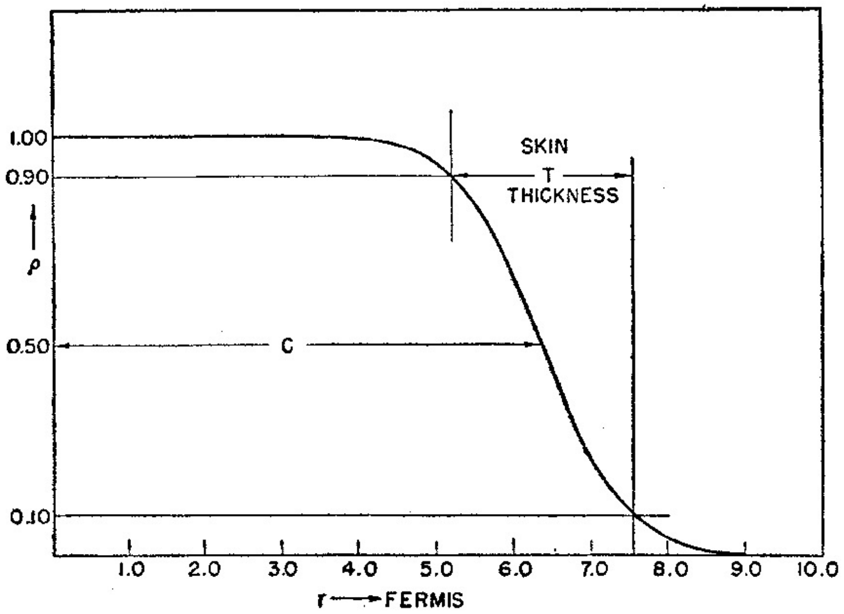
Nobel prize
1961

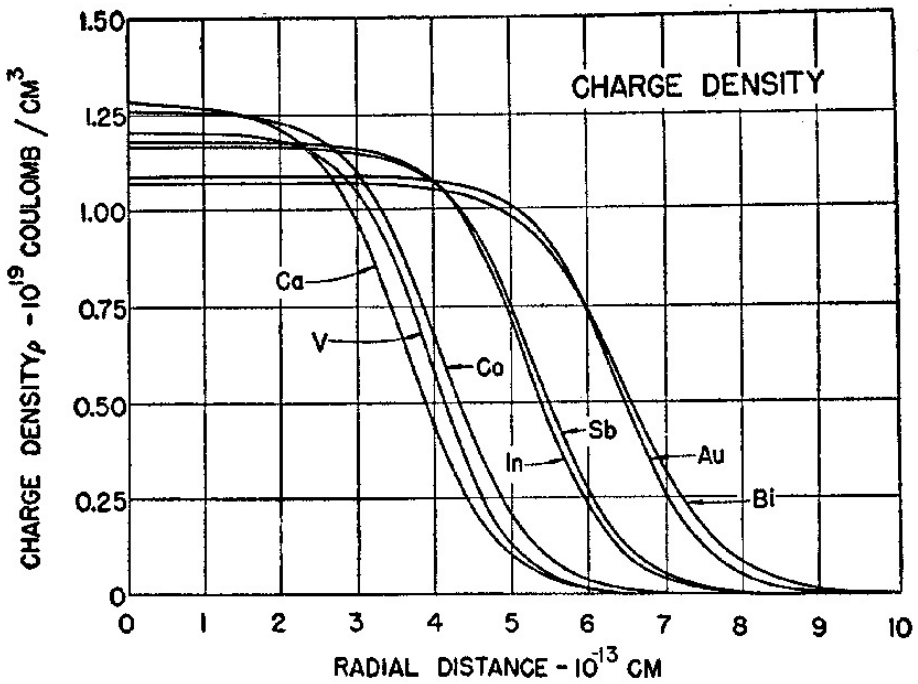
Fattore di forma nucleo





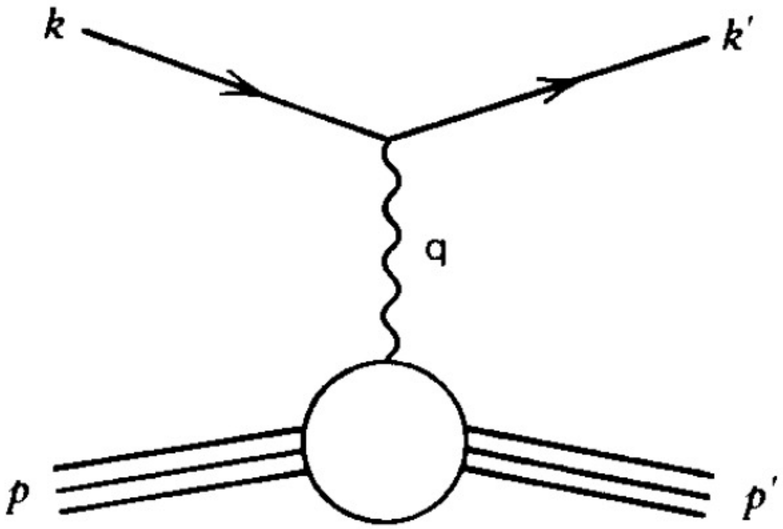
$$\rho(r)=\frac{\rho_1}{\exp[(r-c)/z_1]+1}.$$





Fattori di forma nucleoni

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{\cos^2 \theta/2}{1 + (2E/M) \sin^2 \theta/2}$$
$$\times \left\{ (F_1{}^p(t))^2 - \frac{t}{4M^2} \left(4M^2 (F_2{}^p(t))^2 + 2(F_1{}^p(t) + 2M\, F_2{}^p(t))^2 \tan^2 \frac{\theta}{2} \right) \right\}$$

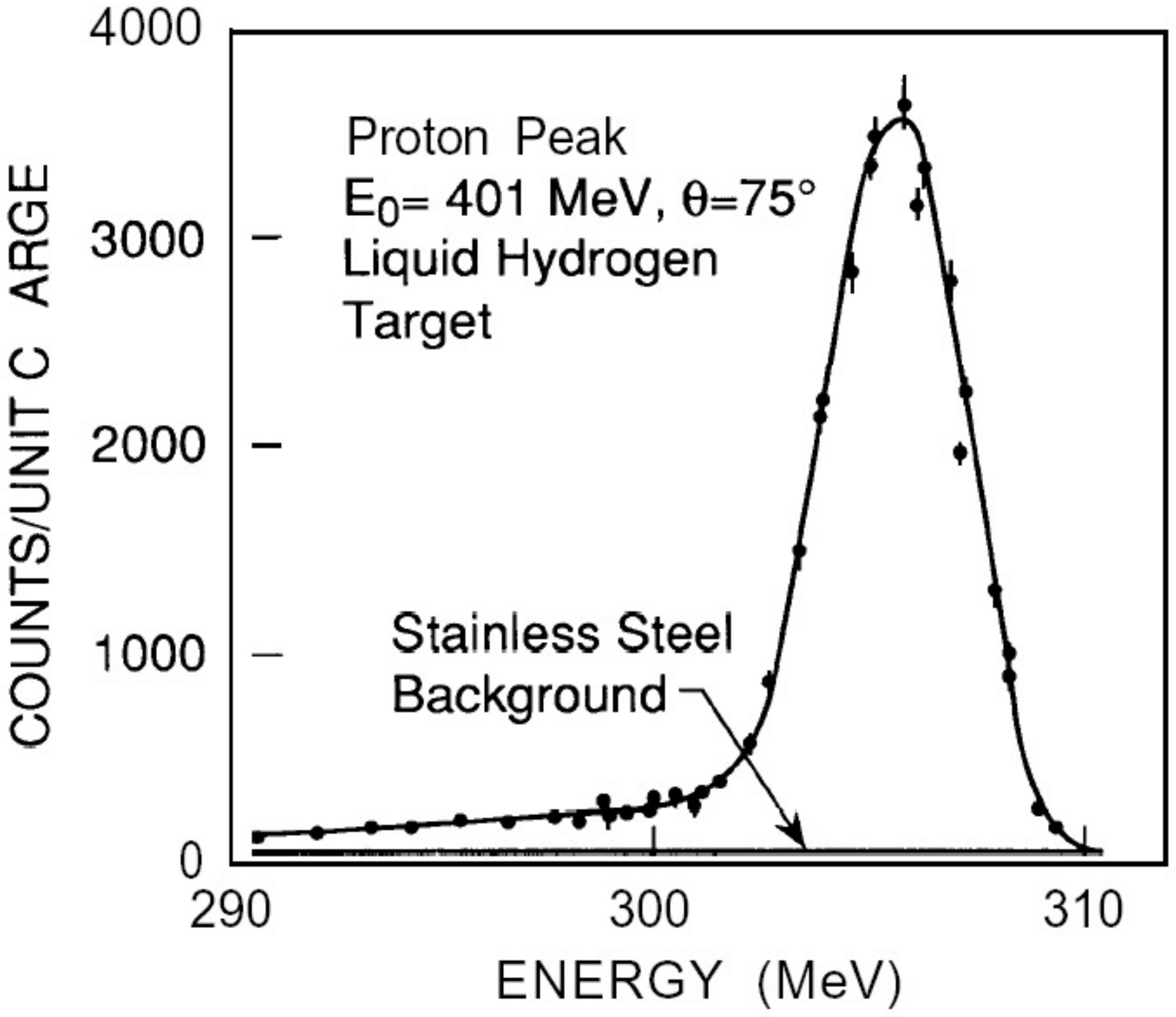


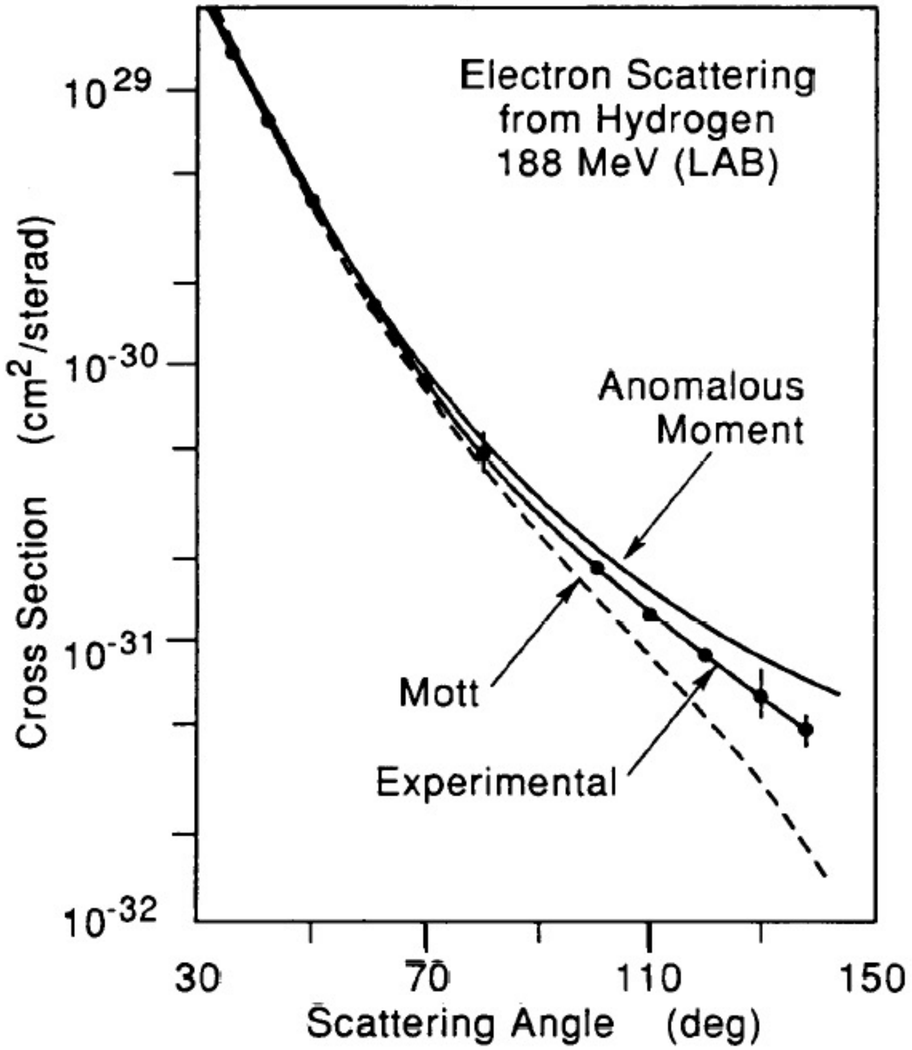
$$G_E{}^p(t) = F_1{}^p(t) + \frac{t}{2M} F_2{}^p(t)$$
$$G_M{}^p(t) = F_1{}^p(t) + 2M\, F_2{}^p(t)$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left\{ \frac{(G_E{}^p(t))^2 - \frac{t}{4M^2}(G_M{}^p(t))^2}{1 - t/4M^2} - \frac{t}{2M^2}(G_M{}^p(t))^2 \tan^2 \frac{\theta}{2} \right\}$$

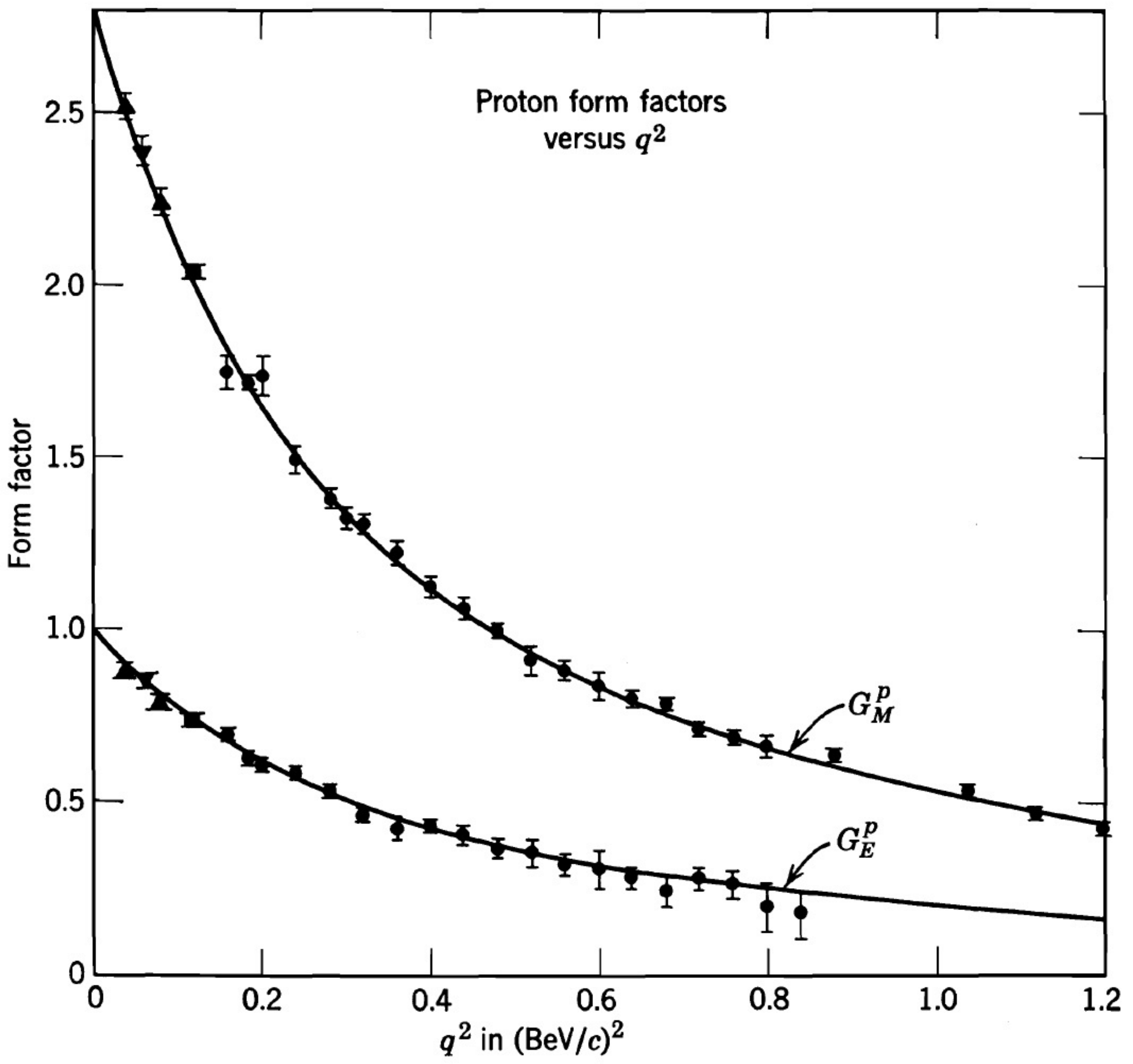
$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{\cos^2 \theta/2}{1 + (2E/M) \sin^2 \theta/2}$$

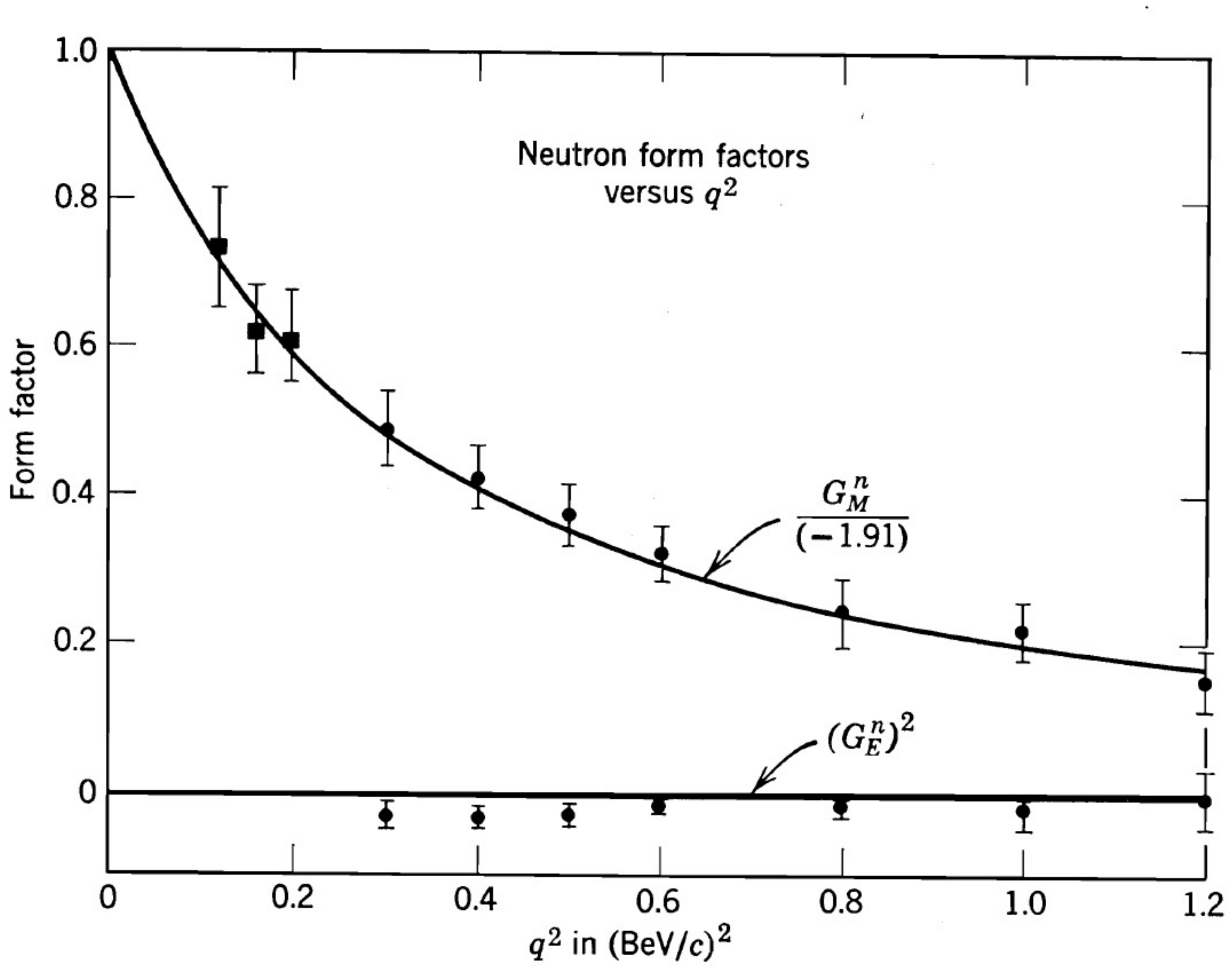
Fattori di forma nucleoni





Fattori di forma nucleoni





$$G_E{}^p(0) = 1$$
$$G_M{}^p(0) = 1 + \mu_p \cong 2.79$$

$$G_E{}^n(0) = 0$$
$$G_M{}^n(0) = \mu_n \cong -1.91$$

Nobel prize
1961

R. Hofstadter

"As we have seen, the proton and neutron, which were once thought to be elementary particles are now seen to be highly complex bodies. It is almost certain that physicists will subsequently investigate the constituent parts of the proton and neutron - the mesons of one sort or another. What will happen from that point on? One can only guess at future problems and future progress, but my personal convinction is that the search for ever-smaller and ever-more-fundamental particles will go on as Man retain the curiosity he has always demonstrated"

Nobel lecture 1961