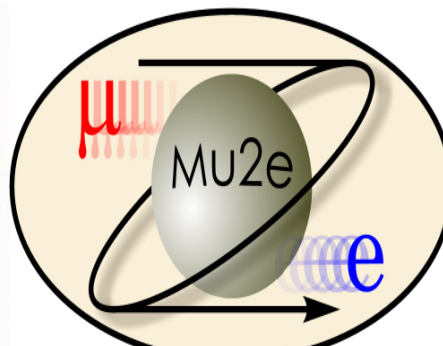


# Mu2e : Developing the Track Selection Algorithm

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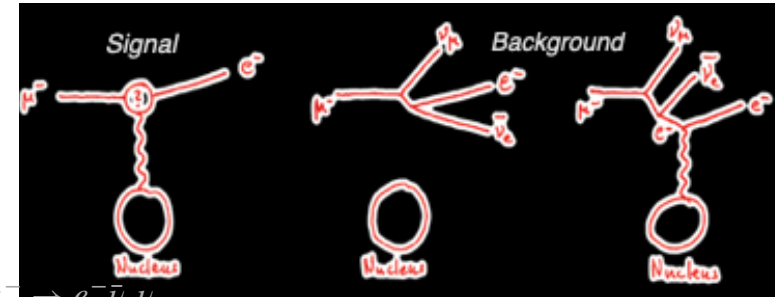


# Overview

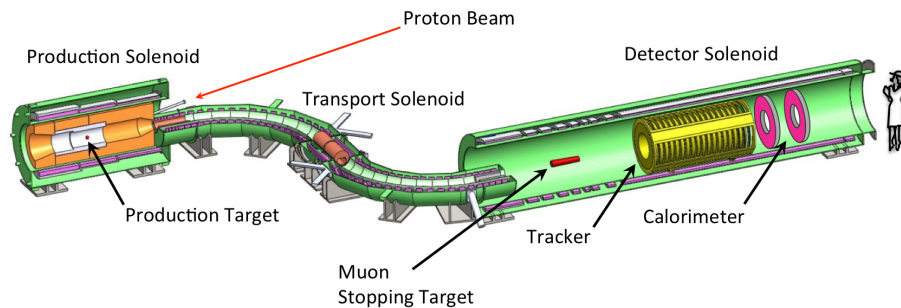
- Mu2e will search for the neutrino-less, coherent conversion  $\mu \rightarrow e$  in the field of an Al nucleus, an example of the Charged Lepton Flavour Violation (CLFV) processes, by measuring the ratio :

$$R_{\mu e} = \frac{\mu^- + N(Z, A) \rightarrow e^- + N(Z, A)}{\mu^- + N(Z, A) \rightarrow \nu_\mu + N(Z - 1, A)}$$

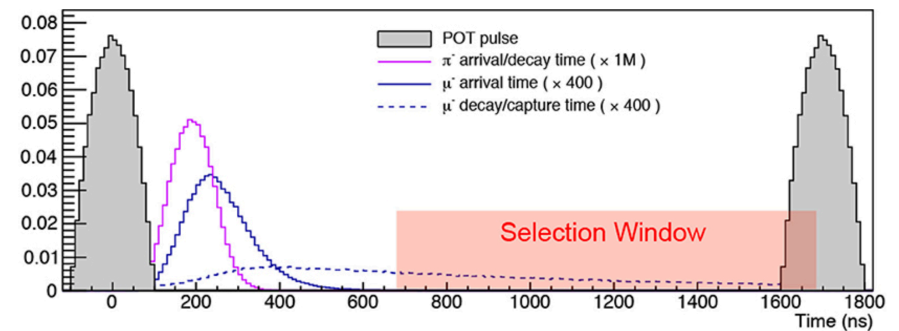
- Signal : Monochromatic Electron of energy  $E_e = m_\mu - E_b - E_{recoil} \approx 104.97 \text{ MeV}$
- Goal : Single-event sensitivity of  $3 * 10^{-17}$
- Possible Background(s) :
  - Intrinsic background : Decay of muons in the atomic orbit in the stopping target (DIO)  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$
  - Beam related : Radiative Pion Capture  $\pi^- N \rightarrow \gamma N'$  or Radiative Muon Capture  $\mu^- N \rightarrow \gamma N' \nu_\mu$
  - Cosmic Rays



- The primary, pulsed 8 GeV proton beam with a pulse width of 250 ns FWHM with 1695 ns spacing will collide with the Tungsten Production target in the **Production Solenoid (PS)** and produce pions.
- The backward-going pions decay into muons which spiral through the S-shaped **Transport Solenoid (TS)**. Positive charges are filtered out by a collimator in the TS.
- The  $\mu^-$  beam will collide with the Al stopping target in the **Detector Solenoid (DS)**, where the conversion process may occur. The negative  $\vec{B}$  gradient guides the particles from the upstream (4.6T) to the downstream (1T) end of the last solenoid.



Mu2e Solenoid System

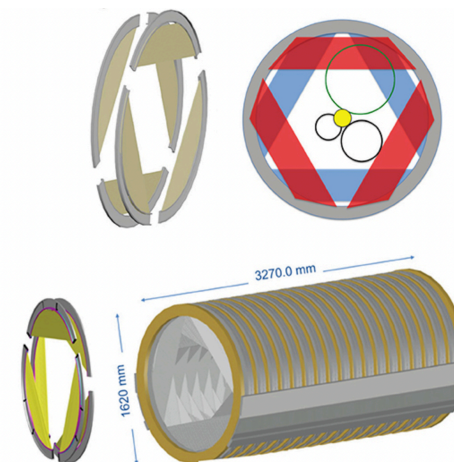


Mu2e beam timing and Event window

# Mu2e Detectors

1. Straw-tube Tracker : High precision measurements of electrons with energy  $> 53$  MeV with a momentum resolution  $< 200$  KeV at 105 MeV are made using a low-mass, straw drift tube, annular tracker.

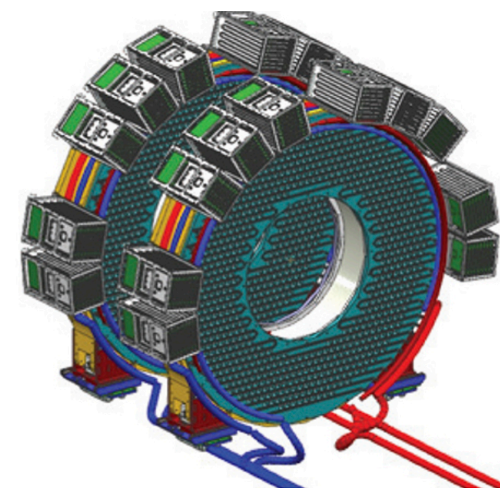
It consists of 18 stations, evenly spaced along its whole length of 3m. Each station is made of two planes (36 planes total) and each plane consists of 6 panels (216 panels total) rotated by  $30^\circ$ , on two faces of a support ring; there are three panels per face. Groups of 96 straws are assembled into panels.



Mu2e tracker : The upper left picture shows panels assembled into a plane and a station. The assembled tracker is shown in the bottom figure.

2. Calorimeter : Two disks made of CsI crystals form the calorimeter that measures the energy of the electron leaving the stopping target. In addition to providing precise timing ( $\sigma_t \approx 100$ ps) and particle identification, the locations of showers in the calorimeter seed the tracker.

The separation between the two disks is specifically chosen to be “half a wavelength” for the 105 MeV/c conversion electron in the 1 T field: if a conversion electron passes through the hole at the centre of the first disk, it will hit the second.

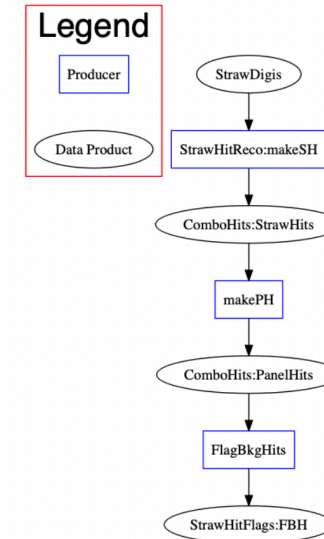


A schematic of the Mu2e calorimeter showing the two disks, location of readout modules, and part of the calibration system

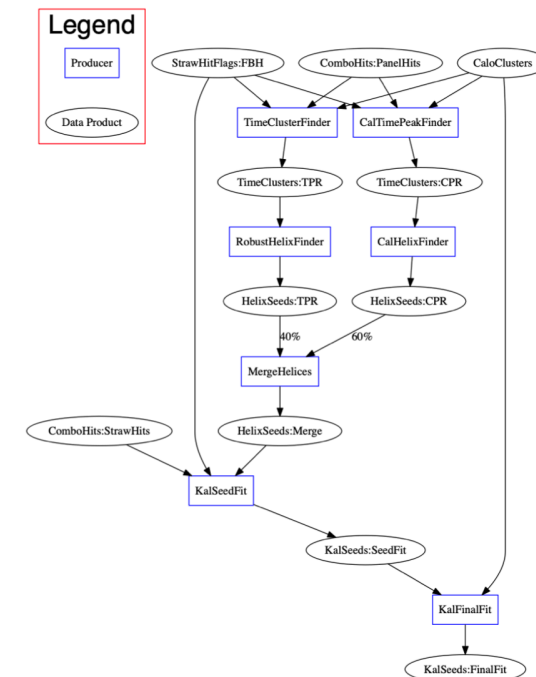
# Track Reconstruction Pattern Algorithms

- Mu2e follows a targeted reconstruction sequence where the particles are distinguished by charge (which affects the Helicity), mass and direction (up and downstream).
- MakeStrawHits : Applies calibration to the raw digis to convert them to physical times and energy deposition in standard units.
- The collection of **ComboHits** in the tracker and the possible simultaneous presence of **Clusters** in the calorimeter are the starting ingredients necessary to reconstruct the helices.
- Separate instances of the Downstream reconstruction are run for positive and negative charge and for the electron and muon masses.
- Each downstream sequence instance runs two pattern recognition algorithms, one based primarily on StrawHits (**TrkPatRec**), the other based on a combination of StrawHits and CaloClusters (**CalPatRec**).
- TrkPatRec can include CaloClusters, while CalPatRec requires them.
- Through both the algorithms we are trying to determine the helix parameters  $\vec{n} \equiv (d_0, \phi_0, \omega, z_0, \tan(\lambda))$  which define the trajectory.

## MDC2018 Straw Hit Reconstruction



## MDC2018 Downstream Track Reconstruction



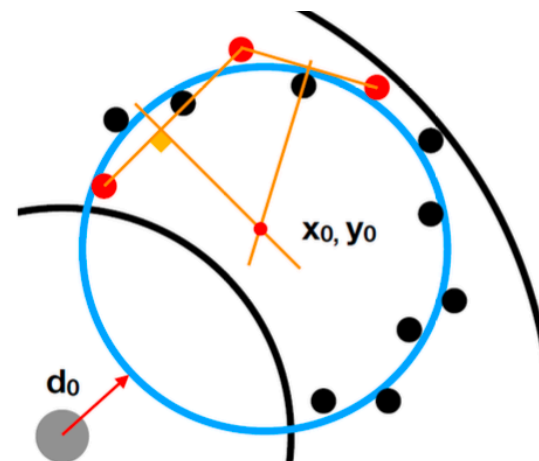
# TrkPatRec : Tracker “only” Pattern Recognition

- XY plane : To determine the optimal circle compatible with the hit distribution, a loop on all possible triplets of ComboHits belonging to the same TimeCluster is performed. The centre and radius of all possible circle combinations is determined and the median value is taken as the centre and radius of the helix.
- $\phi Z$  plane : Combinations of hits belonging to different panels are taken and  $1/\lambda = d\phi/dz$  is estimated as :

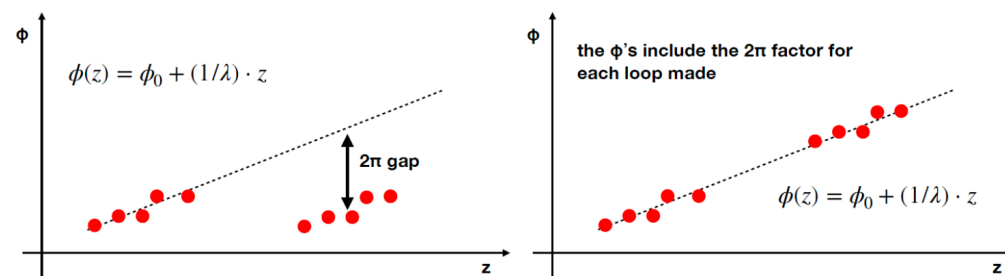
$$\frac{1}{\lambda_{i,j,k}} = \frac{\phi_j + 2\pi k - \phi_i}{z_j - z_i}$$

where i,j indicate the two different hits and k = number of full rotations.

- The peaks in the resulting distribution are used to assign hits to the corresponding k-th loop to resolve the  $2\pi$  ambiguity and obtain the helix  $d\phi/dz$  and  $\phi_0$  values.



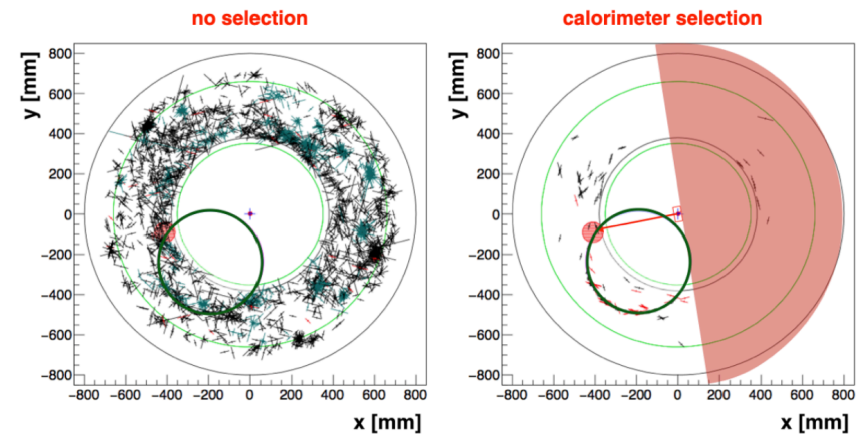
Circle fitting



2π ambiguity resolution

# CalPatRec : Calorimeter seeded Pattern Recognition

- The calorimeter clusters are used as seeds for the helix reconstruction.
- The cluster's time and position are used to filter the collection of ComboHits: the hits are required to be in a  $\pm 40$  ns window from the calorimeter cluster and in the same semi-plane.
- The algorithm takes the calorimeter cluster, one of the ComboHits and the solenoid centre as the starting points.
- The XY and  $\phi Z$  plane reconstruction is performed similar to the TrkPatRec but with increased sophistication.



Semi-Plane selection based on the CaloCluster position

# MergeHelices : Algorithm to select between the CalPatRec and TrkPatRec helices

- If helices from either of the input sources share 10 or more hits the one with the most hits (or best fit chi-squared) is kept in the MergeHelices output and moves ahead in the reconstruction.
- The selection criteria at present is :
  1. If one of the helix candidates has a CaloCluster associated with it and the other does not, the helix with the CaloCluster is selected.
  2. If both the candidates have CaloCluster then the the helix with the greater number of hits is selected.
  3. If the number of hits is equal then the chi-square of the helices are compared and the helix with the lower chi-square value is selected.

# Issues with the present Track Selection algorithm

- The algorithm assumes that the helix with the greater number of hits is the better one which may not always be true like due to the presence of accidental hits : background hits mistaken for conversion electron hits leading to wrong track identification and reconstruction.
- The chi-square parameter which is used to select the better helix are computed through different algorithms in TrkPatRec and CalPatRec. This introduces a bias in this selection parameter.
- Duplication of tracks in some events : a single particle is found as two tracks. At present, there is no system to veto the fake track.
- Aim :
  1. Remove the bias due to the different chi-squared computation methods in TrkPatRec and CalPatRec.
  2. The comparison assumes that “more the hits better the helix”. We have altered this to “if the difference of hits between the helices  $< n$ ”, select the helix with the lower chi-squared value. The cases with  $n$  varying from 1 to 5 are studied.



# Uniform $\chi^2$ calculation

- The  $\chi^2$  calculation method followed in the CalPatRec algorithm was adopted for the TrkPatRec helix as well. It is a least square fit based  $\chi^2$  minimisation approach [ [http://www.hep.ph.ic.ac.uk/~hallg/UA9/Karimaki\\_1991.pdf](http://www.hep.ph.ic.ac.uk/~hallg/UA9/Karimaki_1991.pdf)].

1. Circle fit :

$$\chi_{XY}^2 = \sum_i (r_i^2 - r^2)^2 / \sigma_{r_i}^4$$

Assuming  $\Delta r/r \ll 1$ ,

$$\chi_{XY}^2 = (2r)^2 \sum_i (r_i - r)^2 / \sigma_{r_i}^4$$

where  $r_i = \sqrt{(x_o - x_i)^2 + (y_o - y_i)^2}$ ,  $(x_o, y_o)$  is the helix centre,  $(x_i, y_i)$  is the straw hit position in the transverse plane and  $\sigma_{r_i}$  is the error on  $r_i$ .

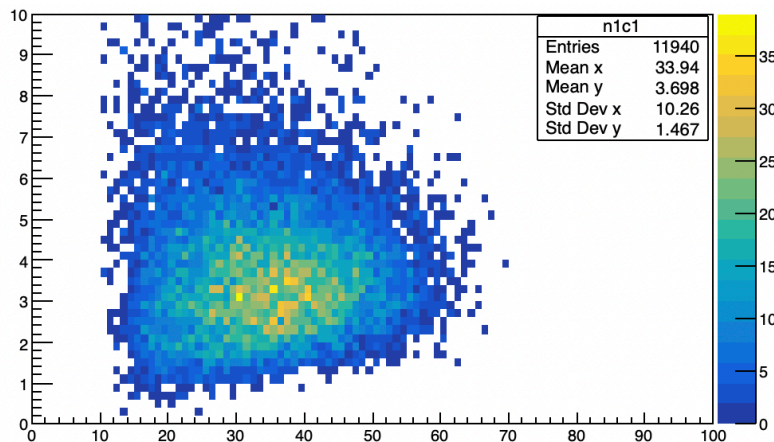
2.  $\phi$ -z fit : The relation between  $\phi$  and z is linear, so the  $\chi^2$  minimisation is :

$$\chi_{\phi z}^2 = \sum_i (\phi_i - \phi)^2 / \sigma_{\phi_i}^2$$

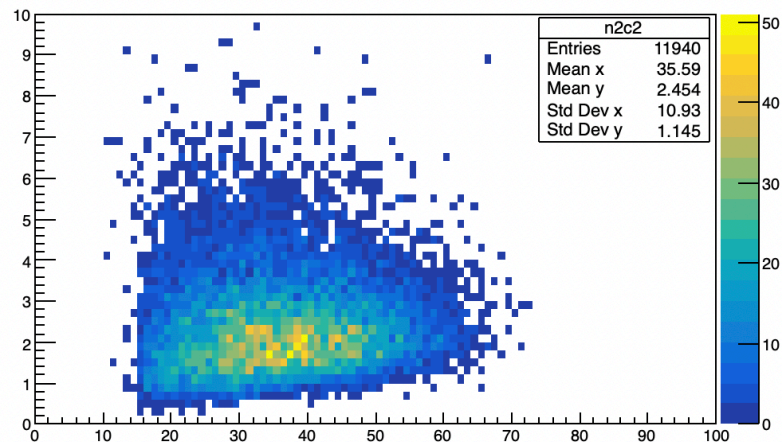
$$\chi_{\phi z}^2 = \sum_i (\phi_i - (\phi_0 + (z_i - z_0)d\phi/dz))^2 / \sigma_{\phi_i}^2$$

# Original algorithm

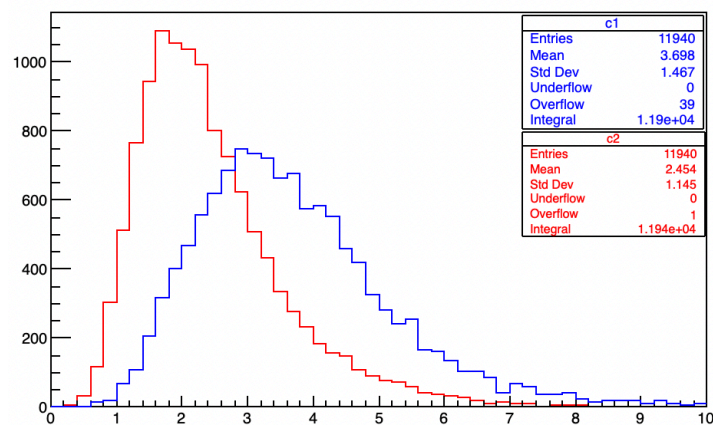
- Data = Pure conversion electrons
- Number of events = 14538
- Histograms are filled for events where there is a candidate present from TrkPatRec and CalPatRec each



Chi2 v/s Nhits TrkPatRec



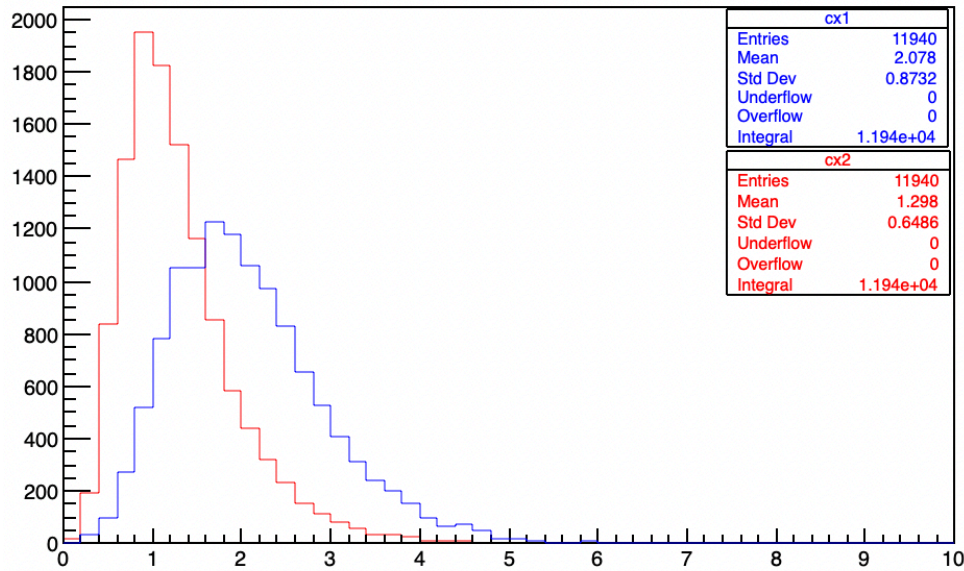
Chi2 v/s Nhits CalPatRec



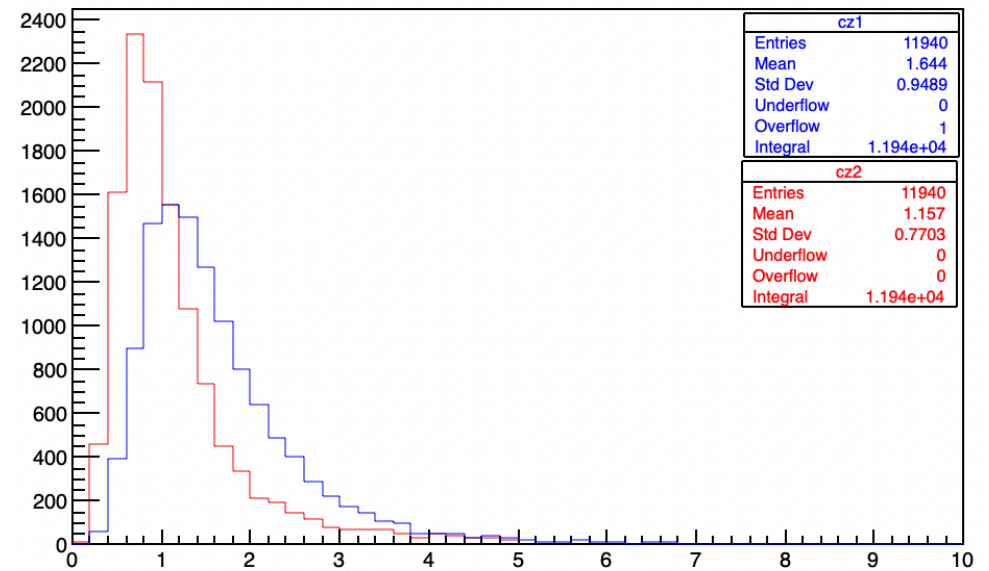
Chi2 (Chi2XY + Chi2ZPhi) Blue = TrkPatrec Red = CalPatRec

# Original algorithm

- Number of events = 14538
- Histograms are filled for events where there is a candidate present from TrkPatRec and CalPatRec each

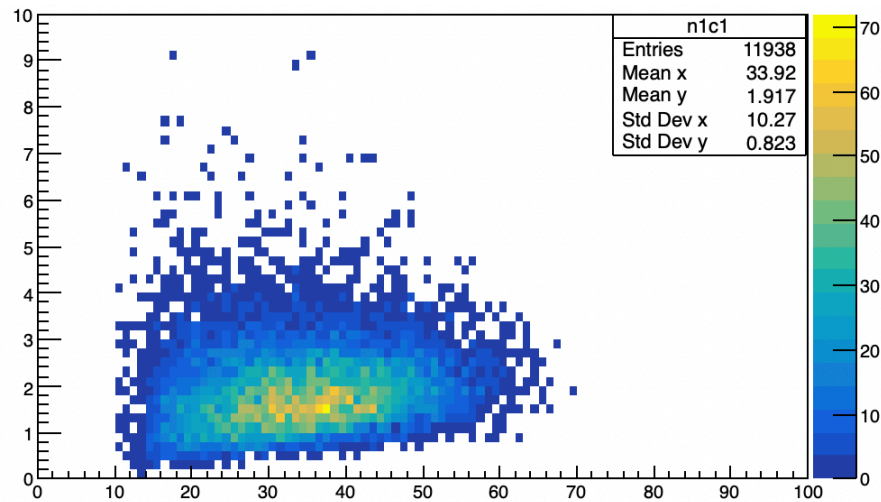


Chi2XY blue = TrkPatRec red = CalPatRec

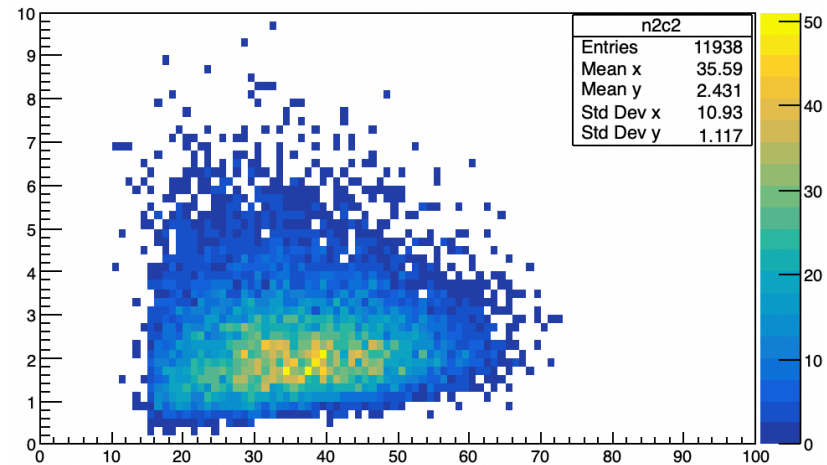


Chi2ZPhi blue = TrkPatRec red = CalPatRec

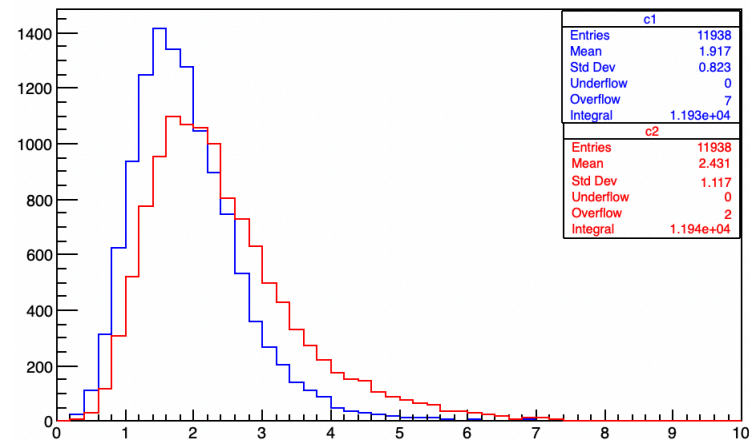
# New $\chi^2$ for TrkPatRec helices



Chi2 v/s Nhits TrkPatRec

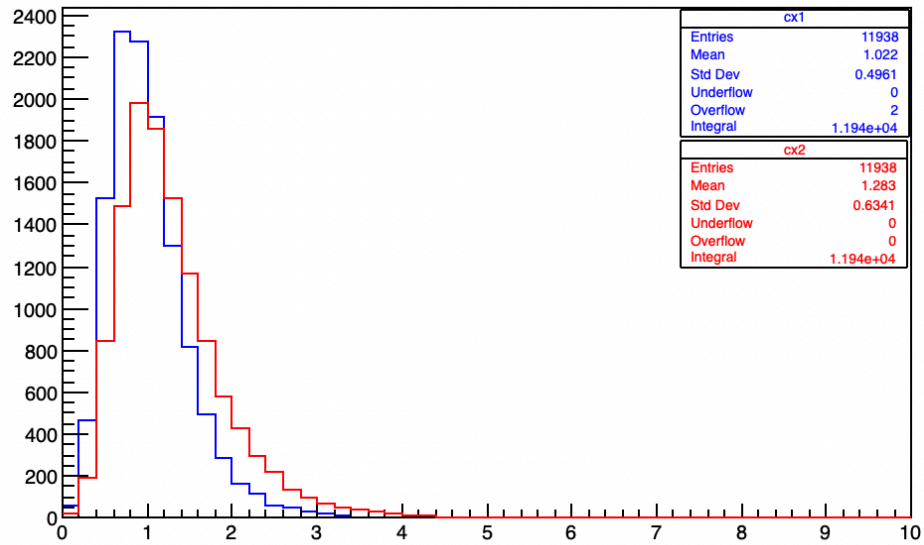


Chi2 v/s Nhits CalPatRec

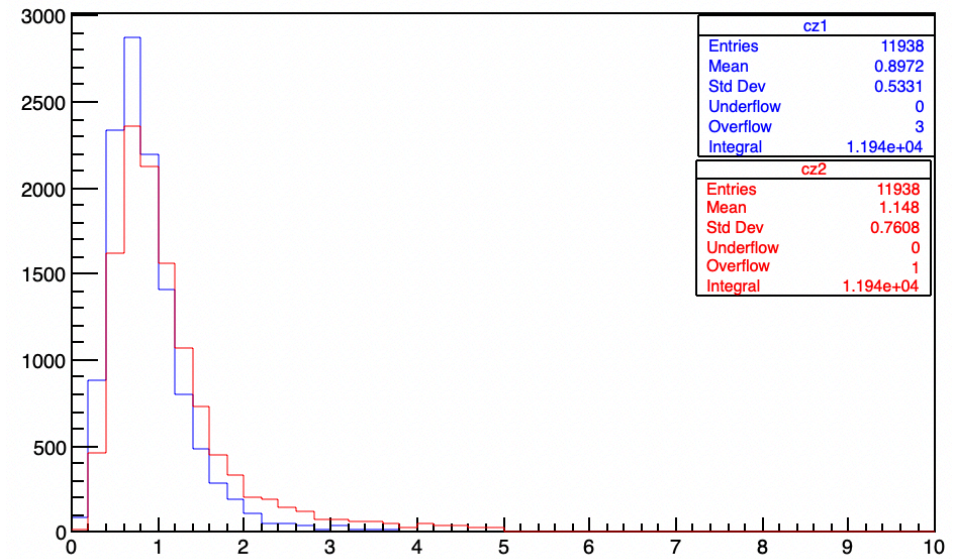


Chi2 (Chi2XY + Chi2ZPhi) Blue = TrkPatrec  
Red = CalPatRec

# New $\chi^2$ for TrkPatRec helices



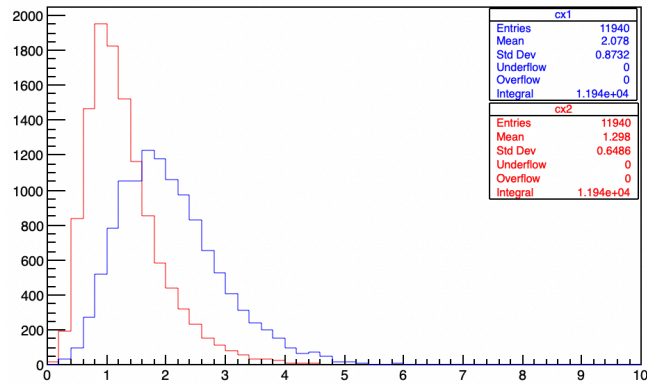
Chi2XY Blue = TrkPatrec Red = CalPatRec



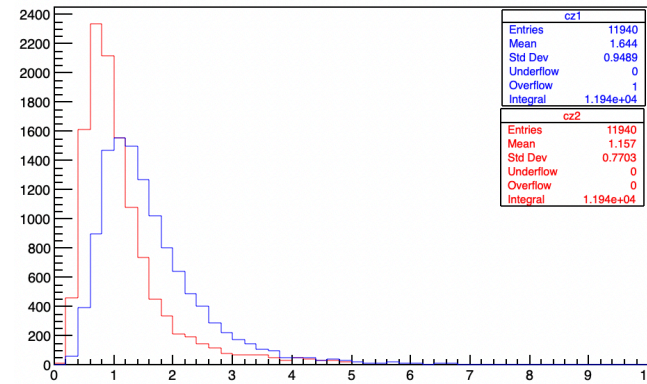
Chi2ZPhi Blue = TrkPatrec Red = CalPatRec

# Comparison of Original and New $\chi^2$ distributions

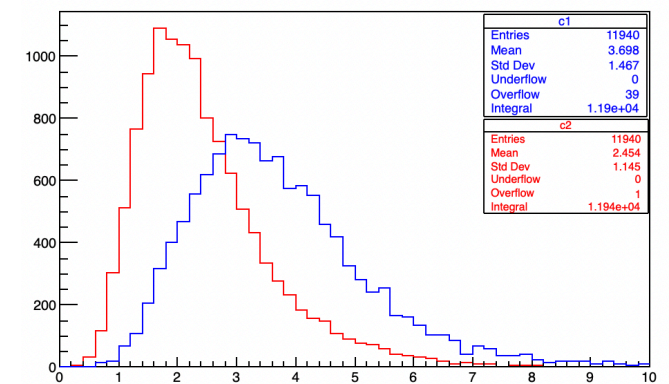
Blue = TrkPatRec Red = CalPatRec



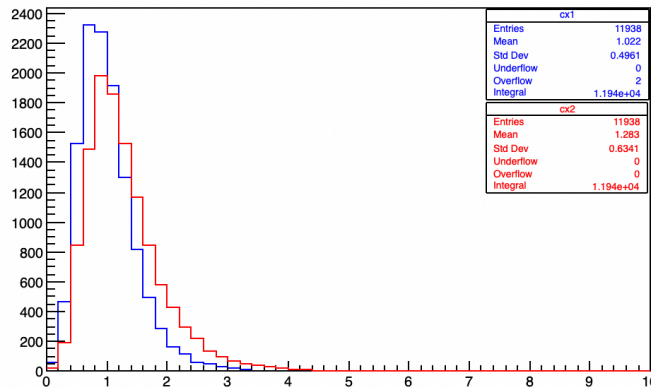
Original Chi2XY



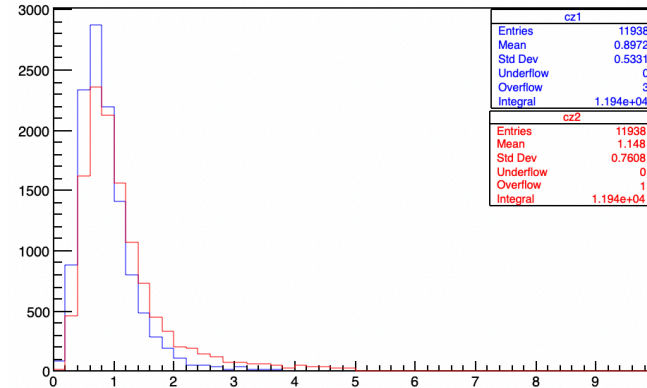
Original Chi2ZPhi



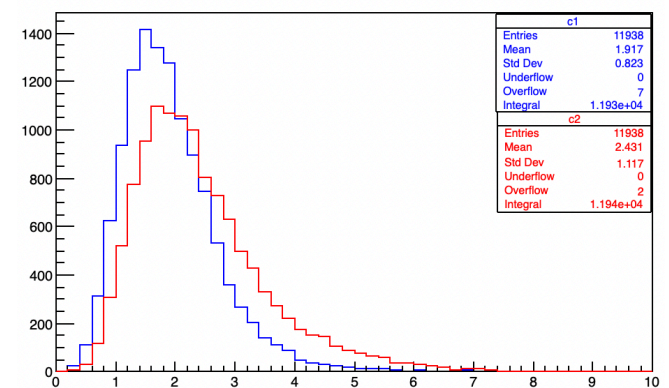
Original Chi2



New Chi2XY



New Chi2ZPhi



New Chi2

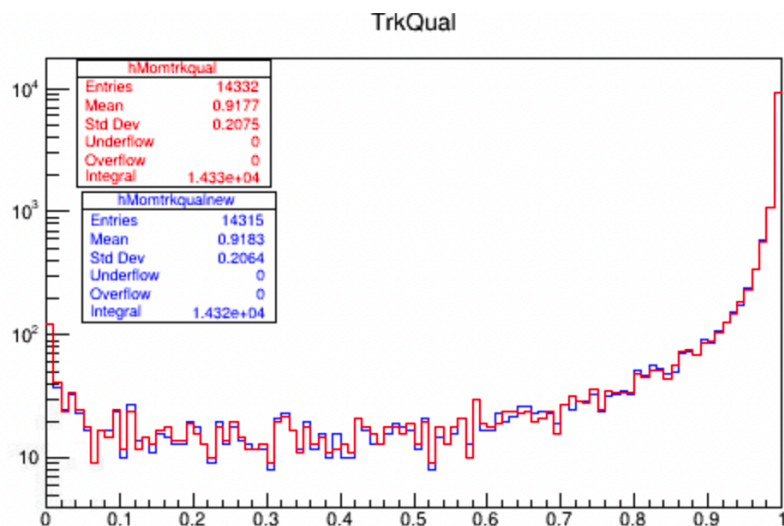
# Statistics

- Originally, about 13% of the helices selected are from the TPR algorithm.
- With the introduction of the uniform  $\chi^2$  calculation, about 28 % of the selected helices are from TPR.
- With the **New5** algorithm, about 66 % of the selected helices are from TPR.

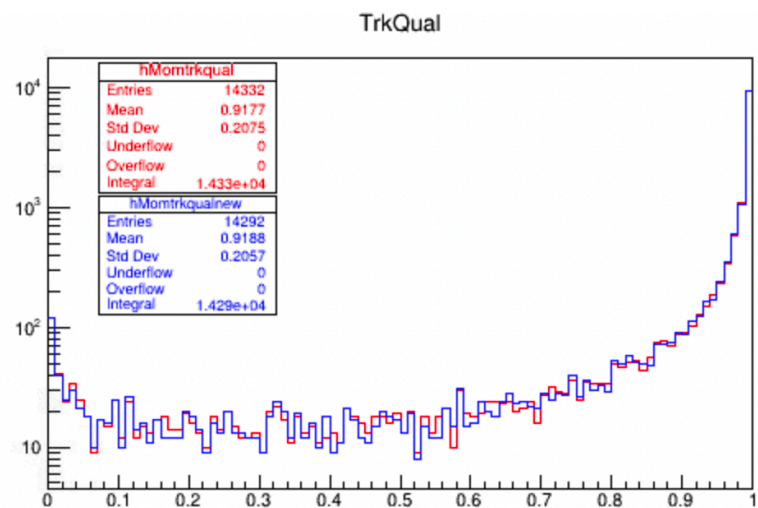
Algorithm	TrkPatRec Helices selected	CalPatRec helices selected	Total number of helices selected at compareHelices( )
Original	1539	10380	11919
New 0	3402	8517	11919
New 1	4799	7120	11919
New 2	6044	5875	11919
New 3	6943	4976	11919
New 4	7560	4359	11919
New 5	7952	3967	11919

# Track quality assessment

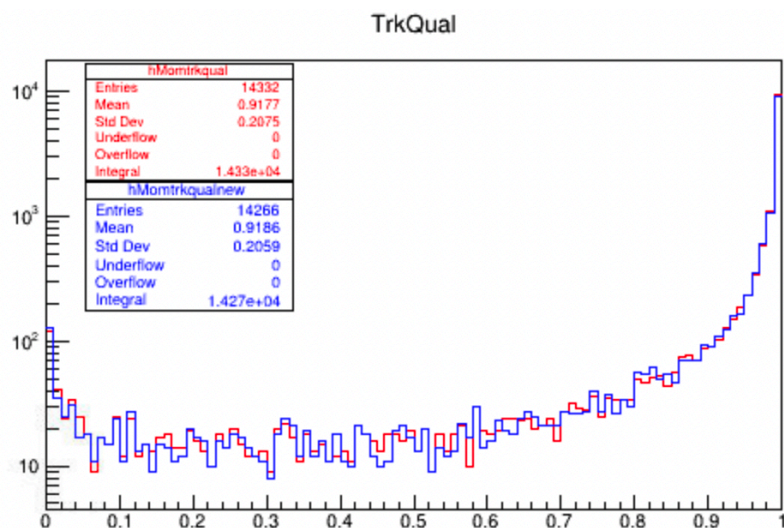
Red = Original Blue = New



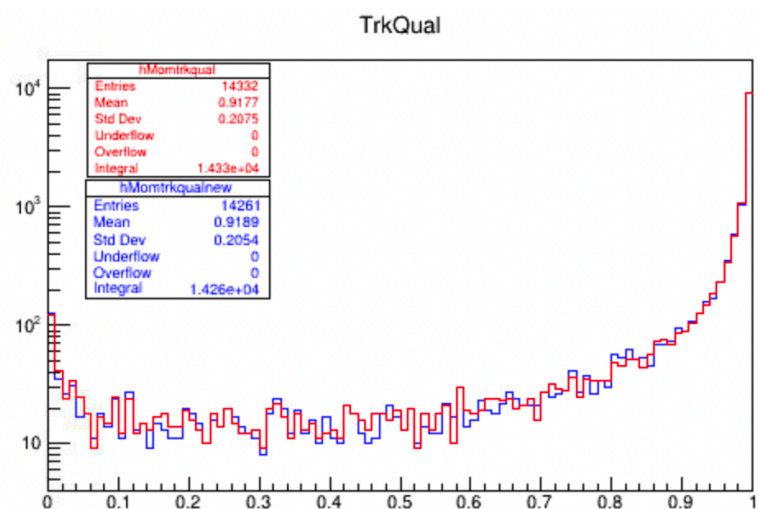
New 0



New 2



New 4



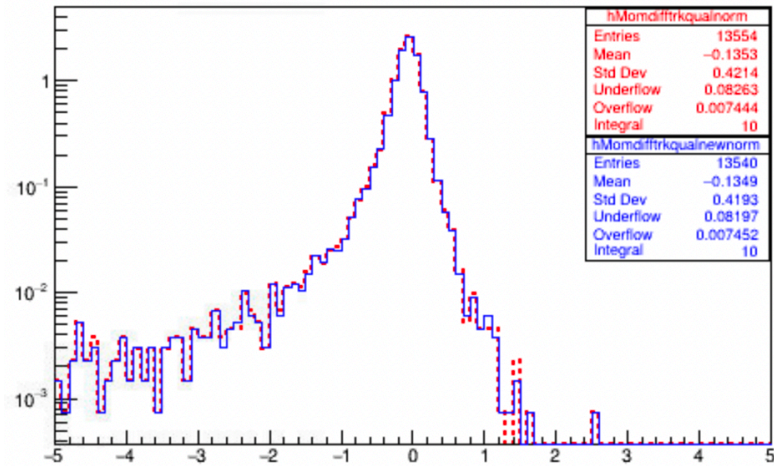
New 5



# Track quality assessment

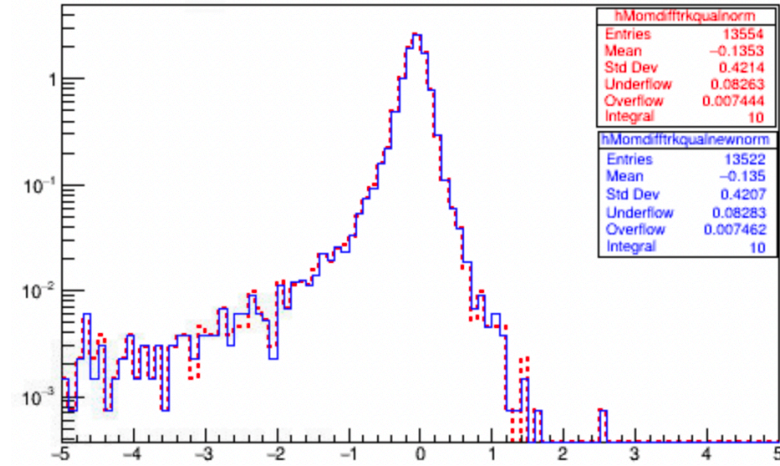
Red = Original Blue = New

Reconstructed - True Momentum, TrkQual > 0.4



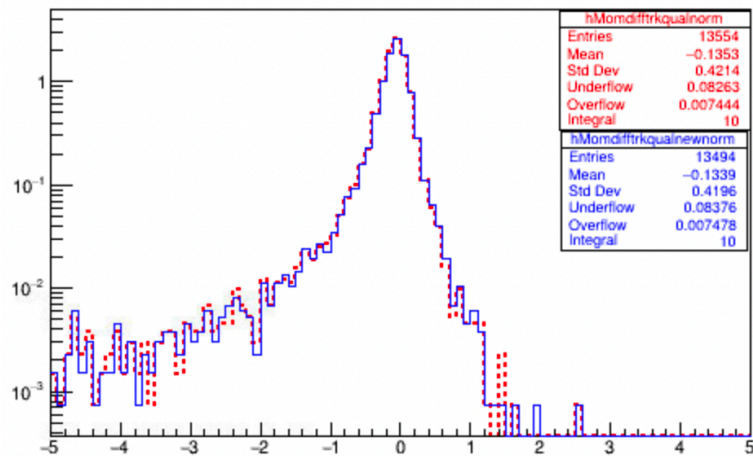
New 0

Reconstructed - True Momentum, TrkQual > 0.4



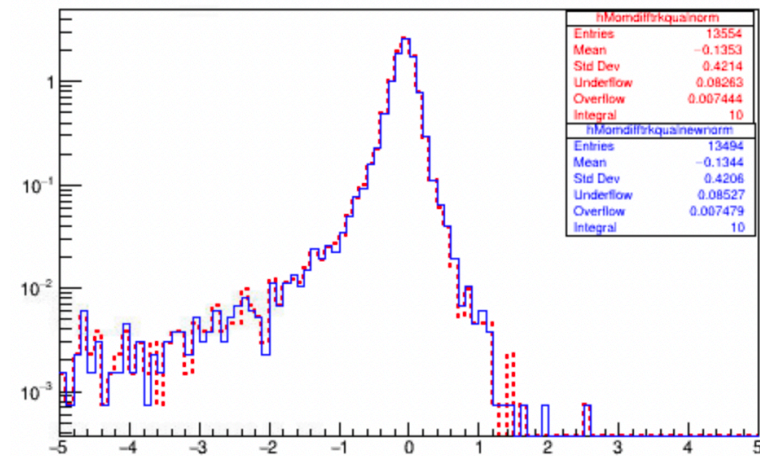
New 2

Reconstructed - True Momentum, TrkQual > 0.4



New 4

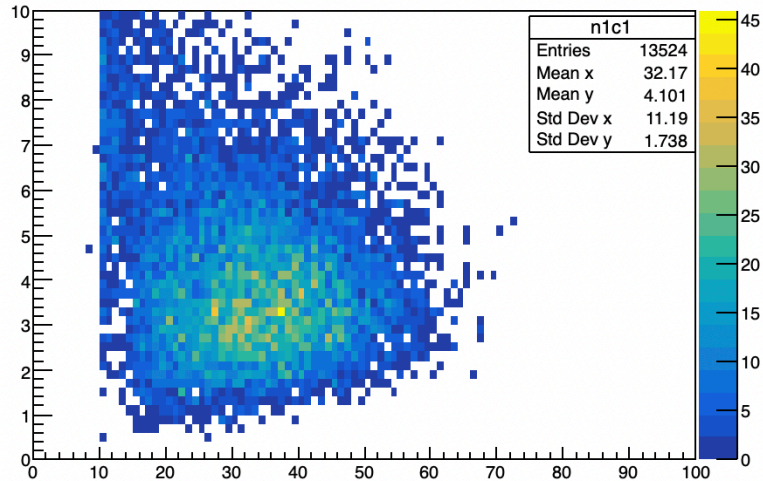
Reconstructed - True Momentum, TrkQual > 0.4



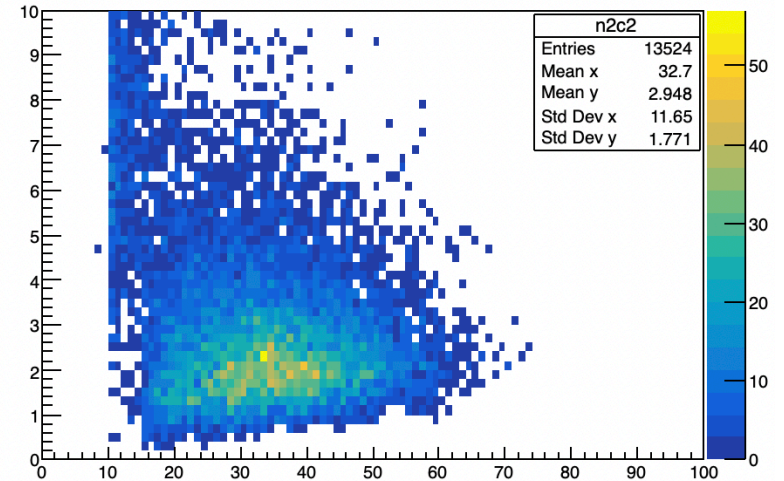
New 5

# Original algorithm

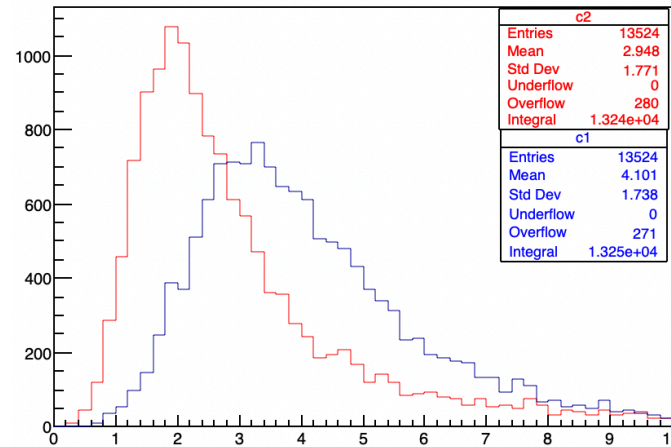
- Mixed Data
- Number of events = 12939
- Histograms are filled for events where there is a candidate present from TrkPatRec and CalPatRec each



Chi2 v/s Nhits TrkPatRec



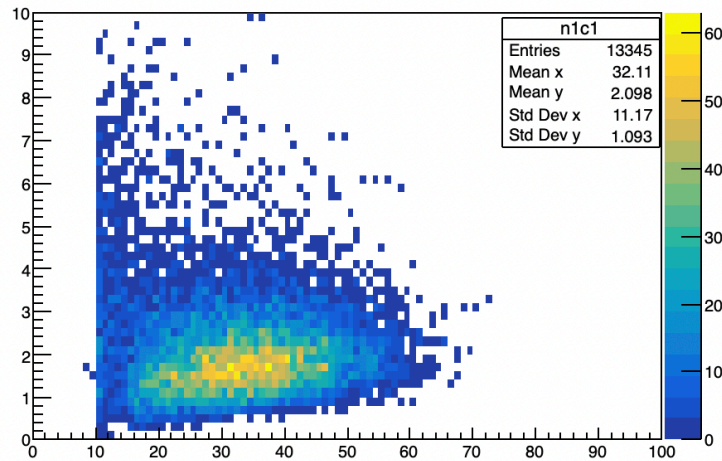
Chi2 v/s Nhits CalPatRec



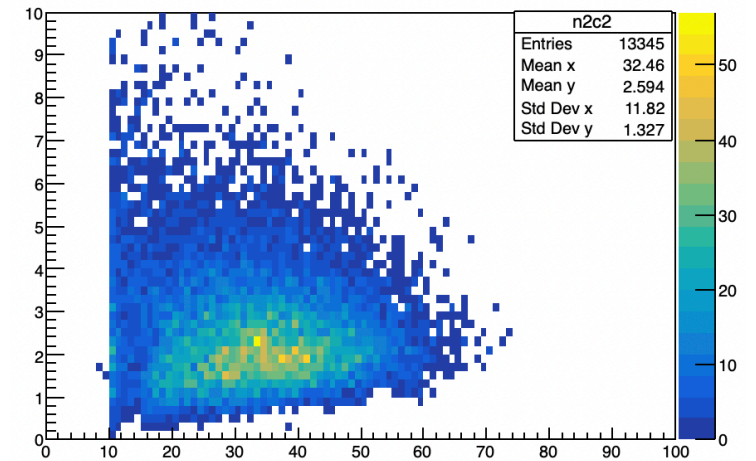
Chi2 (Chi2XY + Chi2ZPhi) Blue =  
TrkPatrec Red = CalPatRec

# New algorithm

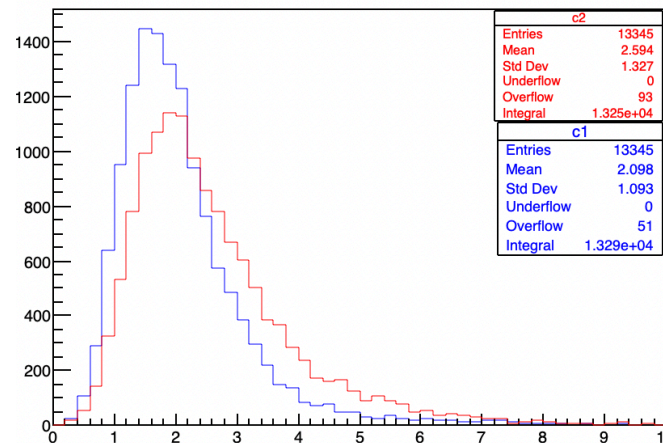
- Mixed Data
- Number of events = 12939
- Histograms are filled for events where there is a candidate present from TrkPatRec and CalPatRec each



Chi2 v/s Nhits TrkPatRec



Chi2 v/s Nhits TrkPatRec



Chi2 (Chi2XY + Chi2ZPhi) Blue =  
TrkPatrec Red = CalPatRec

# Future tasks

- Identify the cause for the reduction in the number of tracks as we move from New0 to New5 algorithm. All the helices are present at the MergeHelices stage of reconstruction.
- Complete the same tests for the Mixed data (Conversion electron + background tracks)
- In the study so far of the Mixed data, we have noticed that in about 10% of the events, multiple helix candidates ( sometimes up to 5) of signal track are saved for further reconstruction.
- Develop a method to identify events with fake tracks and remove them
- Improve and optimise the helix selection further to achieve maximum reconstruction efficiency of conversion electron.
- Fix the uniform  $\chi^2$  calculation. For instance, we are still deciding if the CaloCluster should be included as a hit during the  $\chi^2$  calculation.

# $\chi^2$ computation differences

- **TrkPatRec** :

1. Circle fit :  $\chi_{XY}^2$ :

$$rres = \sqrt{werr^2 \cdot rwdot^2 + terr^2 \cdot (1 - rwdot^2)}$$

$$\chi_{XY}^2 = |dr/rres| * rpullScaleF$$

where werr and terr are position resolutions of the hit along and transverse to the wire respectively,

dr = difference between helix radius and circle centre to hit distance, rpullScaleF = 1.414

2.  $\chi_{\phi Z}^2$

$$\chi_{\phi Z}^2 = dwire^2/wres^2 + dtrans^2/wtres^2$$

where

$$dwire = |dh \cdot Dot(wdir)|, dtrans = |dh \cdot Dot(wtdir)|$$

dwire = projection of the hit along the wire direction, dtrans = transverse projection, wres and wtres are the total resolutions computed along and transverse to the wire respectively,

- **CalPatRec** : Least square fit based  $\chi^2$  minimisation approach [ [http://www.hep.ph.ic.ac.uk/~hallg/UA9/Karimaki\\_1991.pdf](http://www.hep.ph.ic.ac.uk/~hallg/UA9/Karimaki_1991.pdf)]. The code used is <https://github.com/Mu2e/Offline/blob/main/Mu2eUtilities/src/LsqSums4.cc>

$$\chi^2 = \sum_{i=1}^N \left( \frac{y_i - f(x_i)}{\sigma_i} \right)^2$$

1. Circle fit :

$$\chi_{XY}^2 = \sum_i (r_i^2 - r^2)^2 / \sigma_{r_i}^4$$

Assuming  $\Delta r/r \ll 1$ ,

$$\chi_{XY}^2 = (2r)^2 \sum_i (r_i - r)^2 / \sigma_{r_i}^4$$