Quantum Information and AdS/CFT

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Outline

1. Introduction

2. Symmetry-resolved Entanglement in AdS3/CFT2

   S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
   K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

   Suting Zhao, Christian Northe, Konstantin Weisenberger, RM, 2202.11111

3. Large N Limit of Quantum Complexity as Two-dimensional hydrodynamics
   P. Basteiro, J. Erdmenger, P. Fries, F. Goth, I. Matthaiakakis, RM, 2109.01152

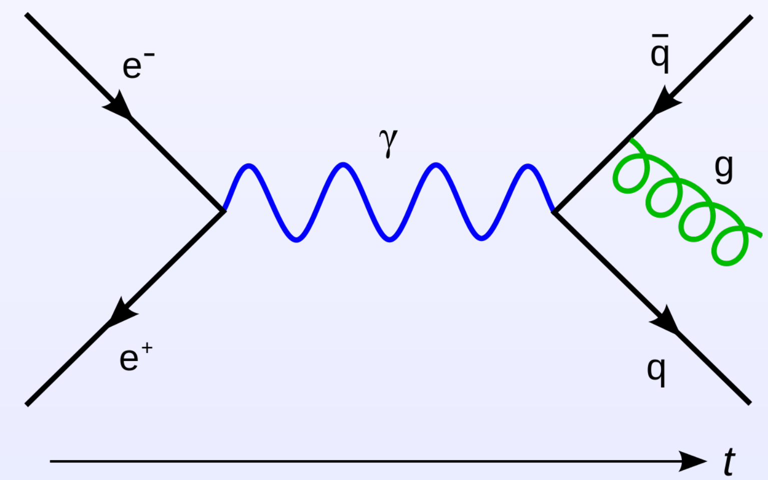
Quantum Information, QFT and Gravity

Gravity

AdS/CFT

Maldacena 1997

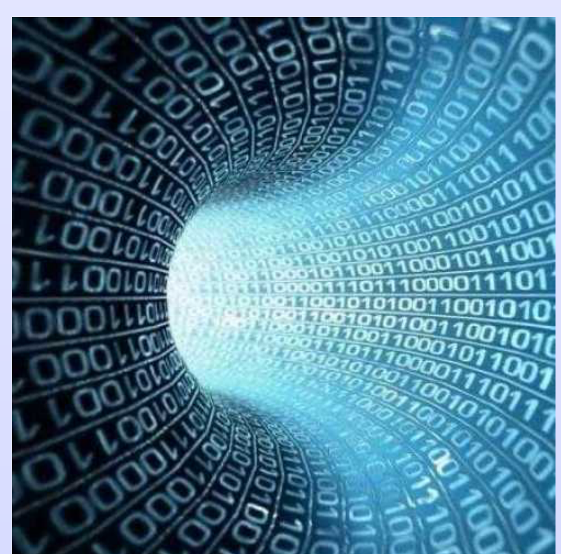
QFT



AdS/CFT

(this talk)

Quantum Information



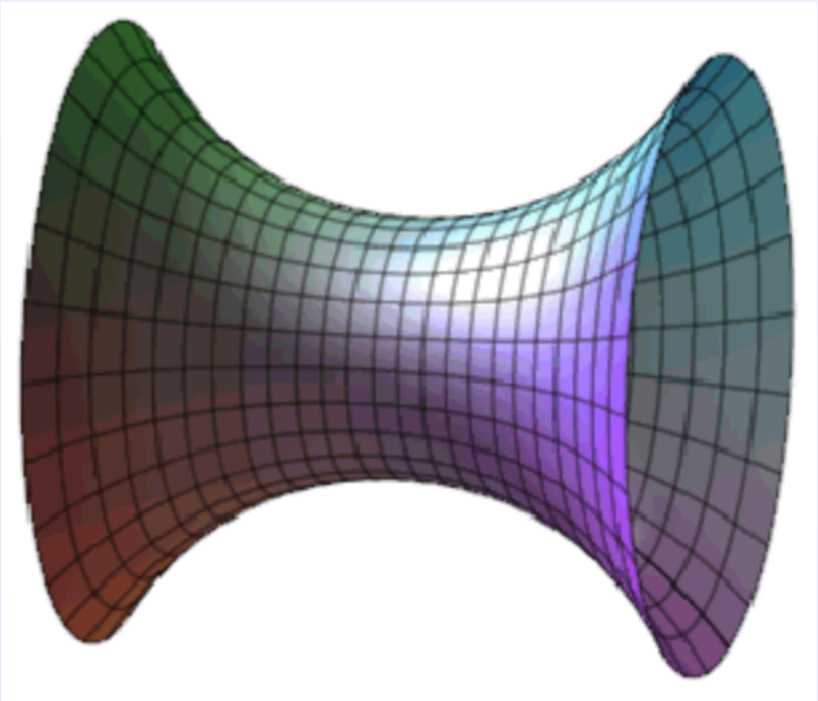
Ryu, Takayanagi 2006

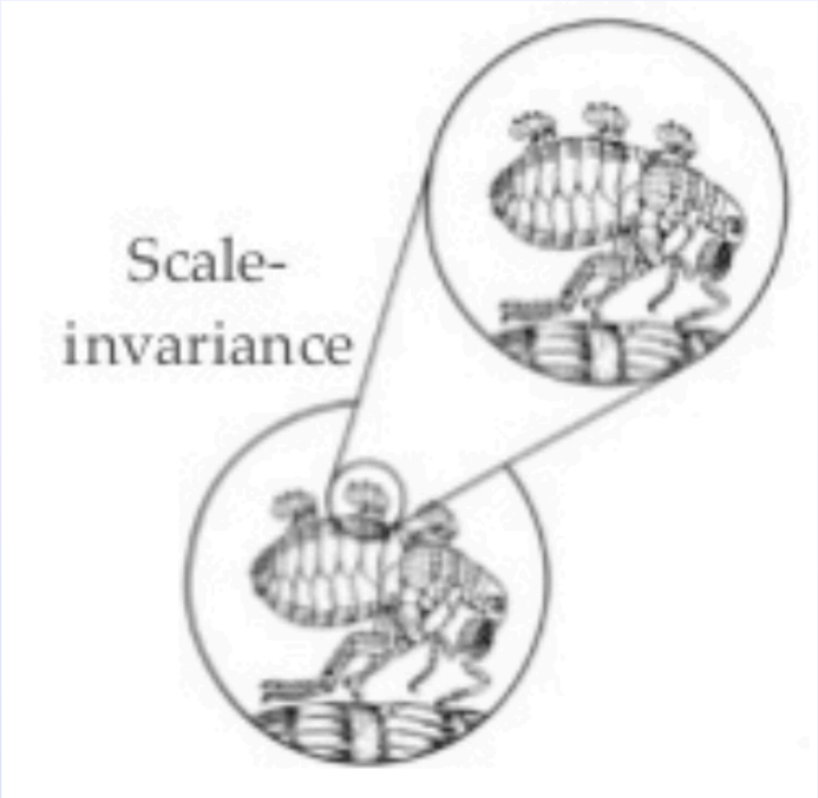
The AdS/CFT Correspondence

Maldacena 1997

Quantum Gravity in
in d+1 dimensional
Anti de Sitter space

Conformal Field Theory
in d dimensions





Type IIB String Theory
on AdS5 x S5

N=4 SYM Theory

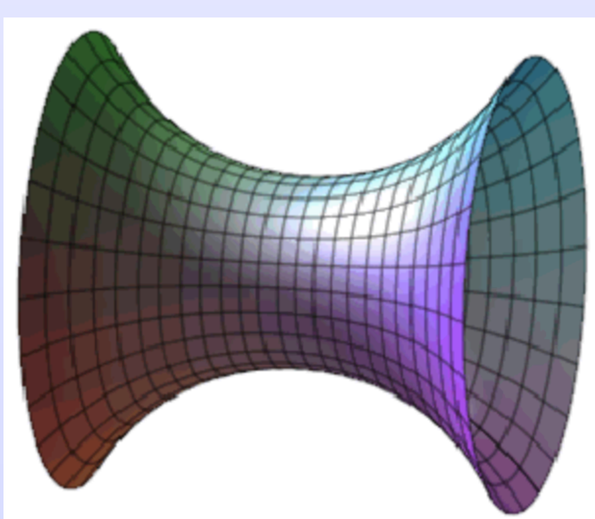
AdS/CFT in different limits

Anti de Sitter
Quantum Gravity

Semiclassical gravity
$G_N \ll 1$

Small curvatures

$L \sim \lambda \gg 1$



“Large N”

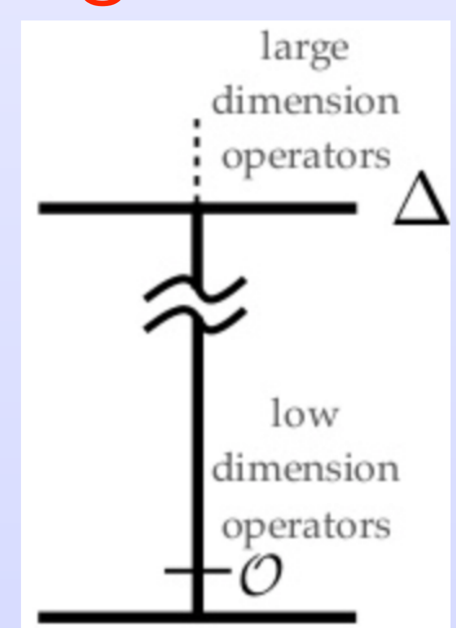
“Large λ”

Strong-weak
coupling duality

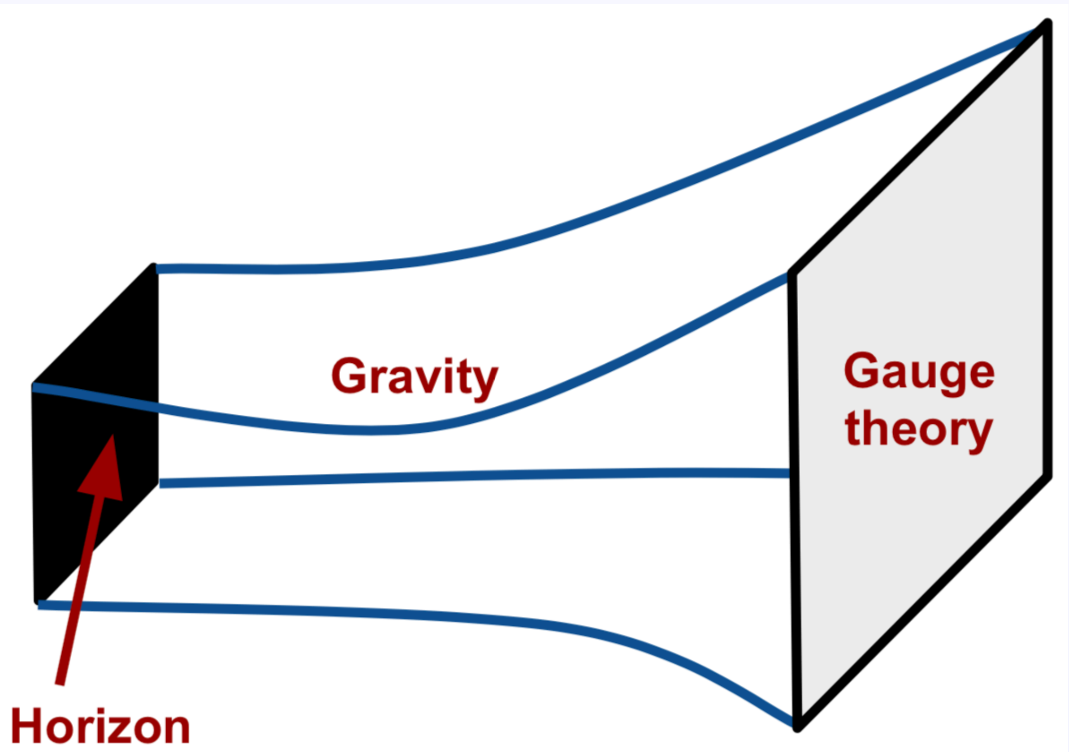
Conformal Field Theory

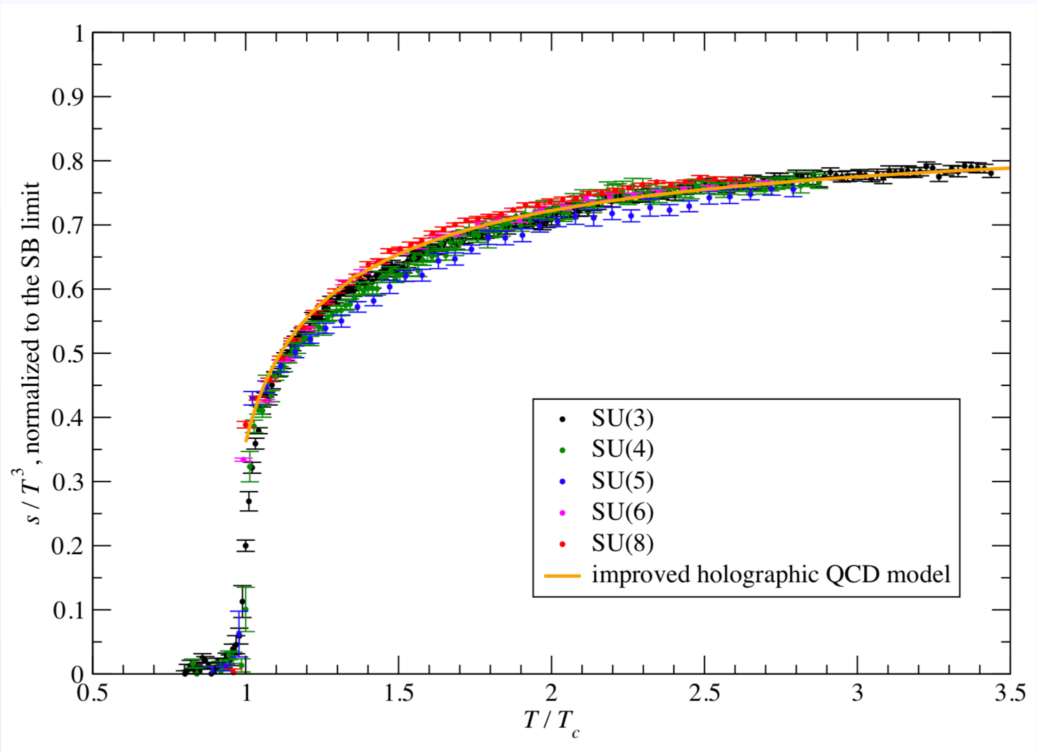
Many degrees of
freedom
(large central charge)

Few low-lying operators
Strong Interactions



AdS Black Hole Entropy





Black Hole Entropy = Thermal Entropy of QFT

$$S_{BH} = \frac{A}{4G_N}$$

$$S_{th} = -\frac{\partial F_{th}}{\partial T}$$

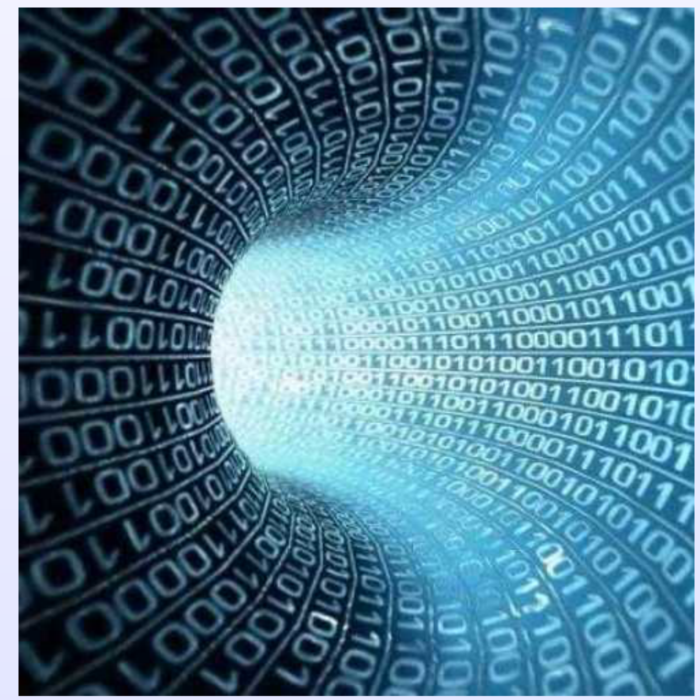
$$s_{\mathcal{N}=4} = \frac{\pi^2}{2} N^2 T^3 = \frac{3}{4} s_{\text{free}}$$

Bekenstein-Hawking,
Witten hep-th/9803131

Burgess, Constable, Myers 1999
Data from Panero 0907.3719

Quantum Information and Black Holes

Black Holes geometrize quantum information



$$S_{BH} = \frac{A}{4G_N}$$

$$S = -k_B \mathrm{Tr} \rho \log \rho$$

Bekenstein & Hawking 1970s

Boltzmann & Gibbs 1870s
Shannon 1948, Jaynes 1957

Quantum Information and AdS/CFT

Emergence of holographic dimension from quantum information?

Building up spacetime with quantum entanglement

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Abstract

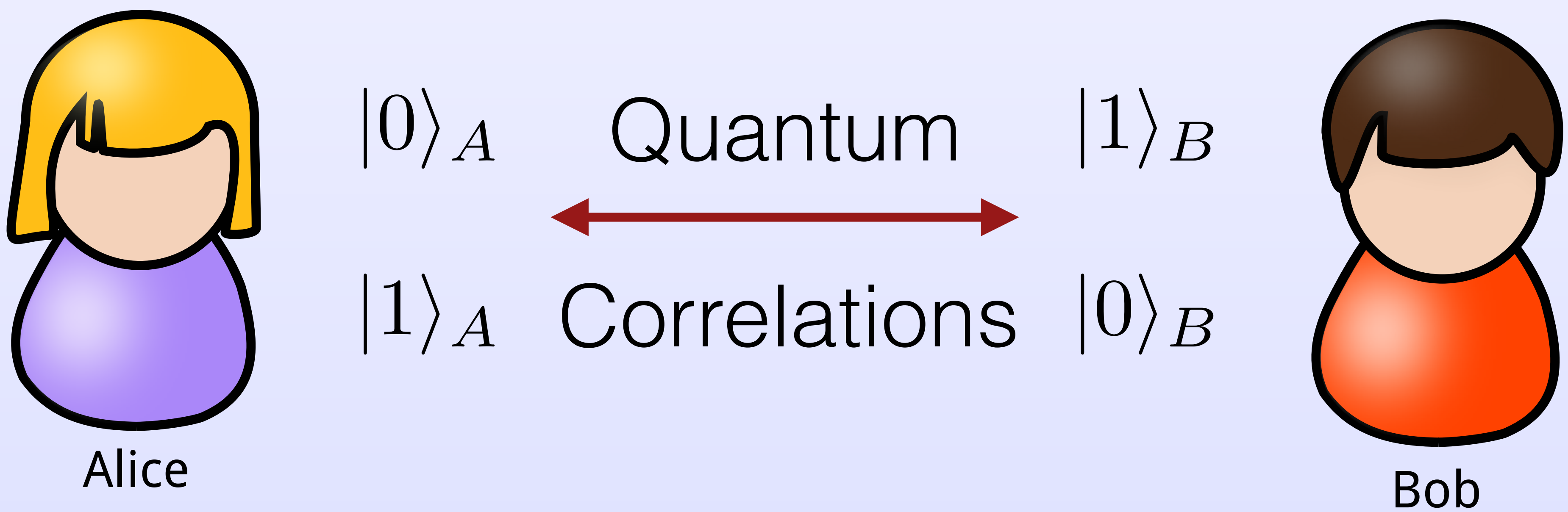
In this essay, we argue that the emergence of classically connected spacetimes is intimately related to the quantum entanglement of degrees of freedom in a non-perturbative description of quantum gravity. Disentangling the degrees of freedom associated with two regions of spacetime results in these regions pulling apart and pinching off from each other in a way that can be quantified by standard measures of entanglement.

arXiv:1005.3035 [hep-th]

Quantum Information

Stored in quantum entanglement

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \times |1\rangle_B + |1\rangle_A \times |0\rangle_B\right)$$

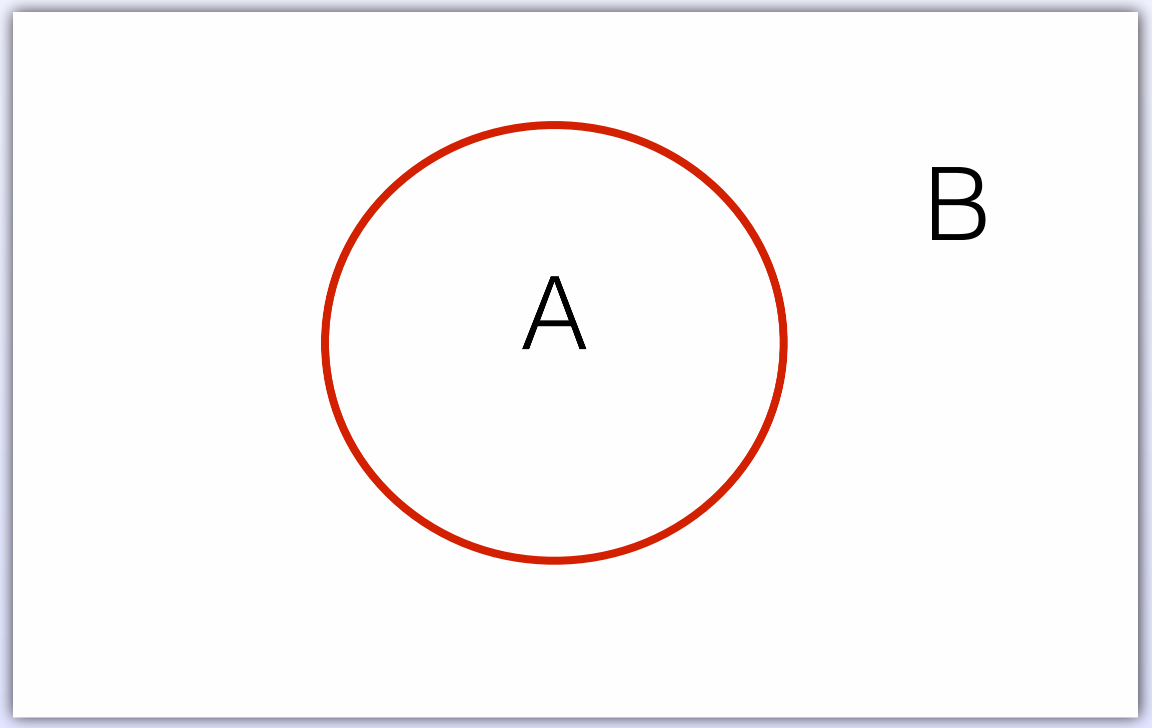


Measures of quantum entanglement?

Entanglement Entropy

Measure of bipartite quantum entanglement

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



$$\rho_A = \mathrm{Tr}_B\,(\rho)$$

$$S(A) = -\mathrm{Tr}\,(\rho_A \log \rho_A)$$

$$(k_B = 1)$$

e.g. EPR State:

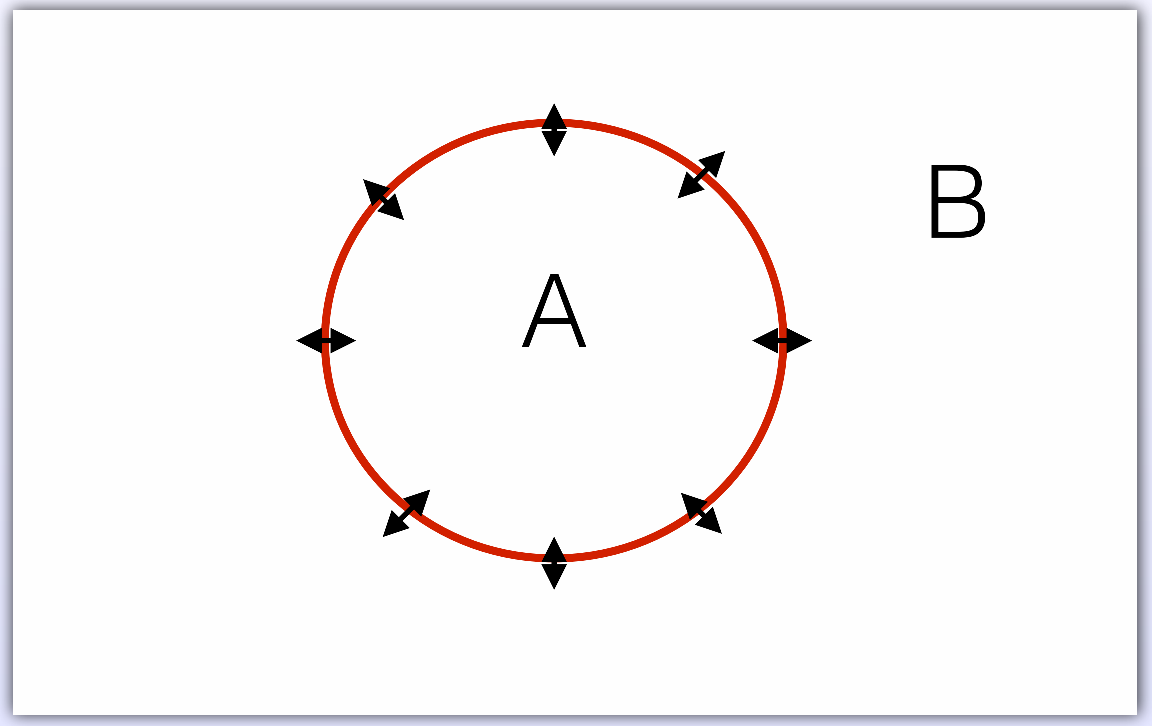
$$\rho_A = \tfrac{1}{2}\,(|0\rangle_A\langle 0|_A) + |1\rangle_A\langle 1|_A)$$

$$S(A) = \log 2 = S(B)$$

Entanglement Entropy in QFT

Area law in local relativistic QFTs

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$



$$\rho_A = \mathrm{Tr}_B\,(\rho)$$

$$S(A) = \gamma\,\frac{\mathrm{Area}(\partial A)}{\epsilon^{d-2}} + \ldots$$

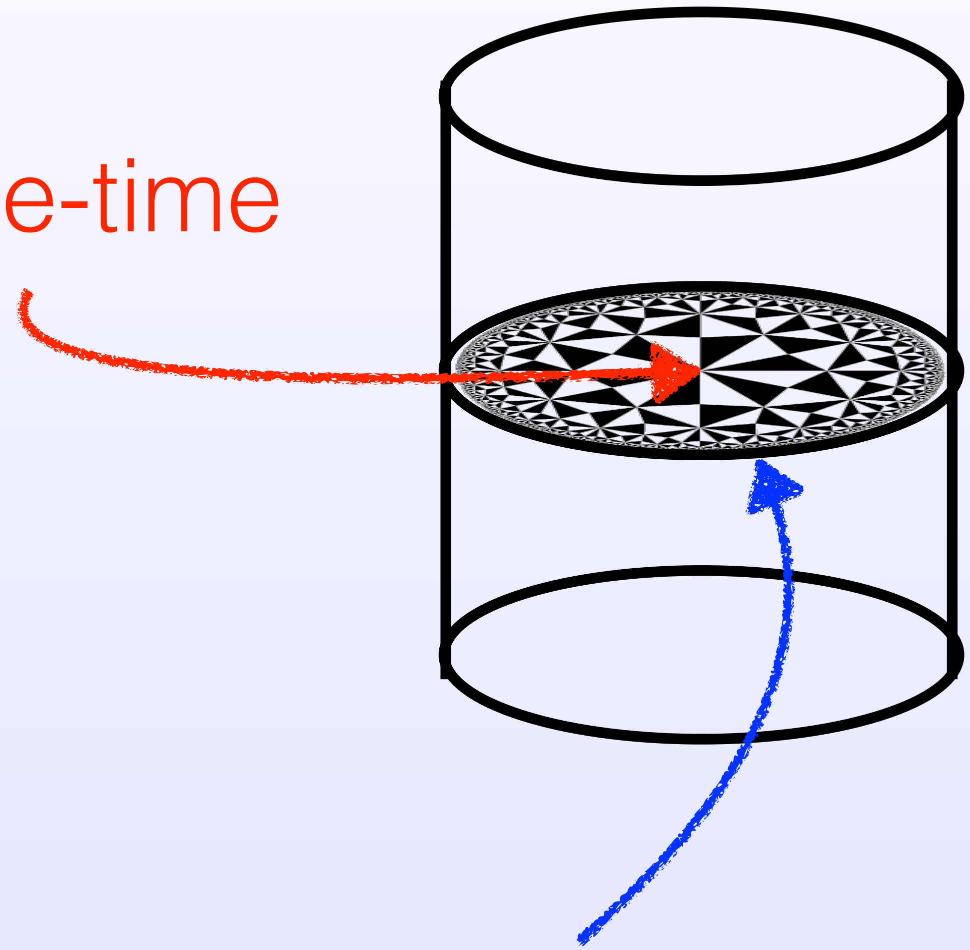
Universal Terms

$$S_{\mathcal{A}} = \begin{cases} a_{d-2}\left(\frac{L}{\epsilon}\right)^{d-2} + a_{d-4}\left(\frac{L}{\epsilon}\right)^{d-4} + \cdots + a_1\,\frac{L}{\epsilon} + (-1)^{\frac{d-1}{2}}\,\mathsf{S}_{\mathcal{A}} + \mathcal{O}(\epsilon)\,, & d \text{ odd} \\ a_{d-2}\left(\frac{L}{\epsilon}\right)^{d-2} + a_{d-4}\left(\frac{L}{\epsilon}\right)^{d-4} + \cdots + (-1)^{\frac{d-2}{2}}\,\mathsf{S}_{\mathcal{A}}\,\log\left(\frac{L}{\epsilon}\right) + \mathcal{O}(\epsilon^0)\,, & d \text{ even} \end{cases}$$

Casini & Huerta 2004, 2006, Myers & Sinha 2010

AdS3/CFT2 Correspondence

AdS3 space-time



3D Einstein-Hilbert gravity

$$S_g = \frac{1}{16\pi G_3} \int d^3x \sqrt{g} \left(R + \frac{2}{L^2}\right)$$

L… Curvature Radius of AdS3 space-time

Two-dimensional relativistic conformal field theory:

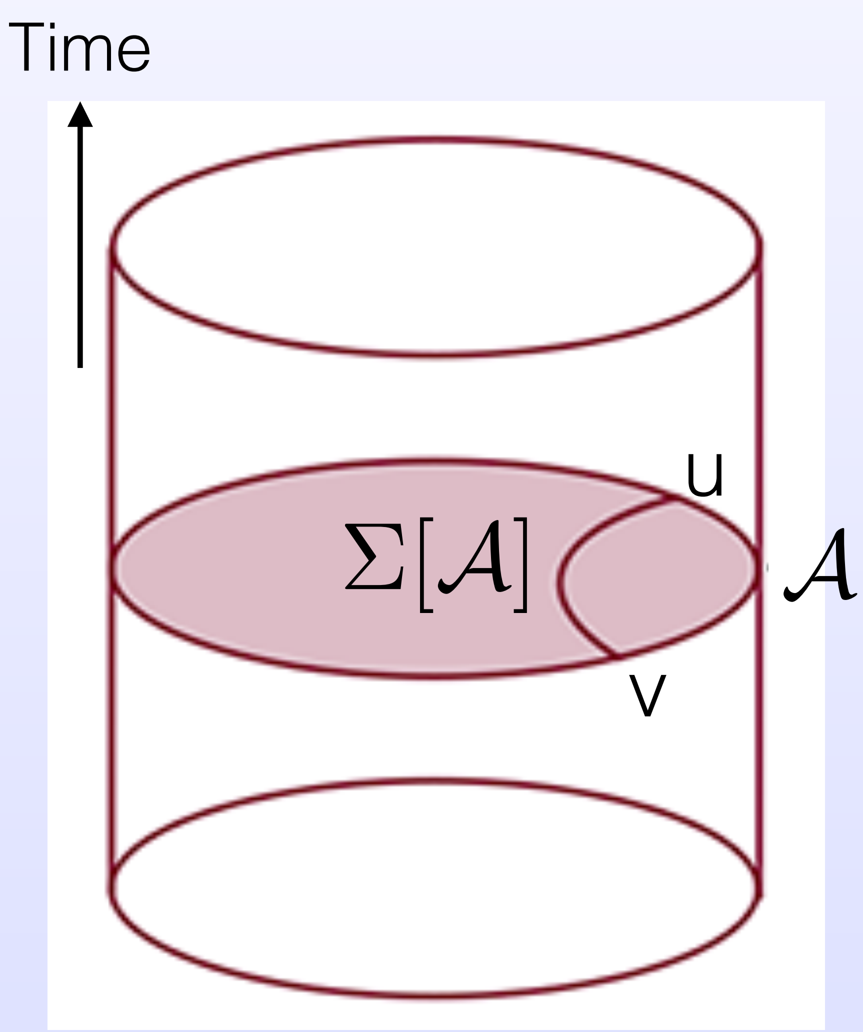
$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}\left(n^3 - n\right)\delta_{n+m,0}$$

To suppress quantum gravity effects: $c = \frac{3L}{2G_3} \gg 1$

Brown, Hennaux 1986

Entanglement Entropy in AdS3/CFT2

Minimal length curve (geodesic)
anchored at the ends of the entangling interval



$$S(\mathcal{A}) = \frac{\mathrm{Length}(\Sigma[\mathcal{A}])}{4G_3}$$

$$c = \frac{3L}{2G_3} \gg 1$$

L… Curvature Radius of AdS3 space-time

For ground state of 2D CFTs:

$$S(\mathcal{A}) = \frac{c}{3}\log\frac{|v-u|}{\epsilon}$$

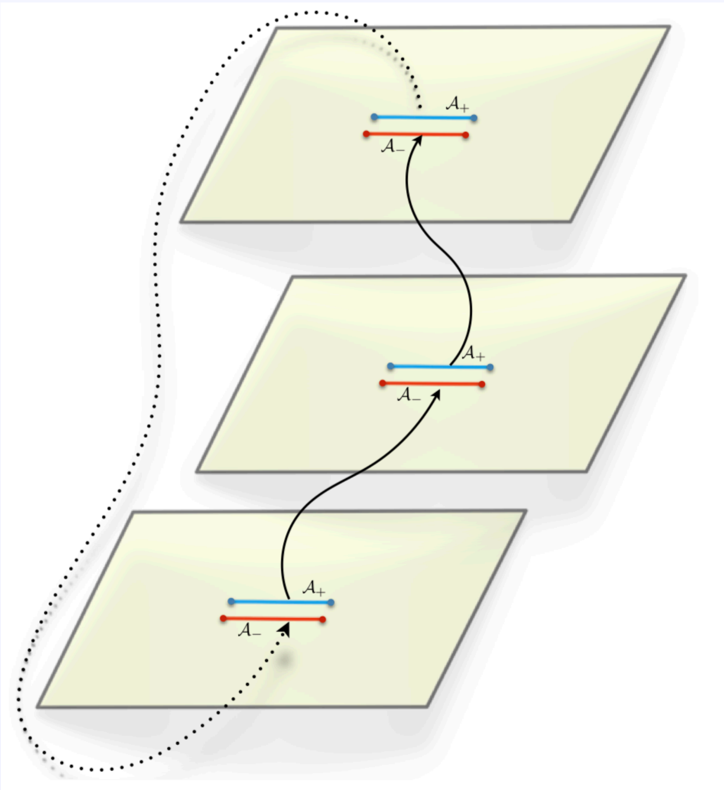
ϵ … Short Distance Cutoff

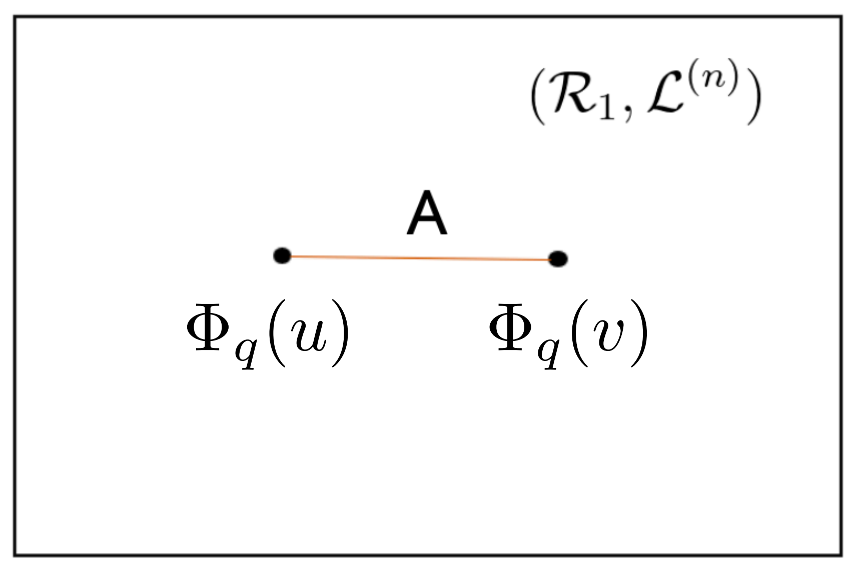
Ryu, Takayanagi 2006
Hubeny-Rangamani-Takayanagi 2007

Holzhey-Larsen-Wilczek 1994,
Cardy-Calabrese 2004

Entanglement Entropy in 2D CFT

Exact Result for a 2D relativistic CFT





$$S_q(A) = \frac{\log \mathrm{Tr}\left(\rho_A^q\right)}{1-q}$$

$$S_q(A) = \frac{1}{1-q} \log \left(\frac{Z[B_q])}{Z[B]^q} \right)$$

$$\frac{Z[B_q])}{Z[B]^q} = \left(\langle \Phi_q(v) \Phi_q(u) \rangle\right)^q$$

$$\Delta(q) = \frac{c}{12}\left(1 - \frac{1}{q^2}\right)$$

$$S(A) = \lim_{q \to 1} S_q(A)$$

$$S = \frac{c}{3} \log \left(\frac{|v-u|}{a} \right)$$

Holzhey-Larsen-Wilczek 1994 , Cardy-Calabrese 2004

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector, e.g. U(1)

charge operator Q $\quad Q = Q_{\mathcal{A}} \oplus Q_{\mathcal{B}}.$

eigenstate of Q, $\quad [\rho, Q] = 0. \quad \rightarrow \quad [\rho_{\mathcal{A}}, Q_{\mathcal{A}}] = 0.$

Block decomposition: $\quad \rho_{\mathcal{A}} = \oplus_q \rho_{\mathcal{A}}(q)$

Symmetry Resolved Renyi and Entanglement Entropy:

$$S_n(q) = \frac{1}{1-n} \log \mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)^n \qquad P_{\mathcal{A}}(q) = \frac{\mathrm{Tr}\rho_{\mathcal{A}}(q)}{\mathrm{Tr}\rho_{\mathcal{A}}} = \mathrm{Tr}\rho_{\mathcal{A}}(q)$$

$$S_1(q) = \lim_{n\to 1} S_n(q) = -\mathrm{Tr}\left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \log \frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)$$

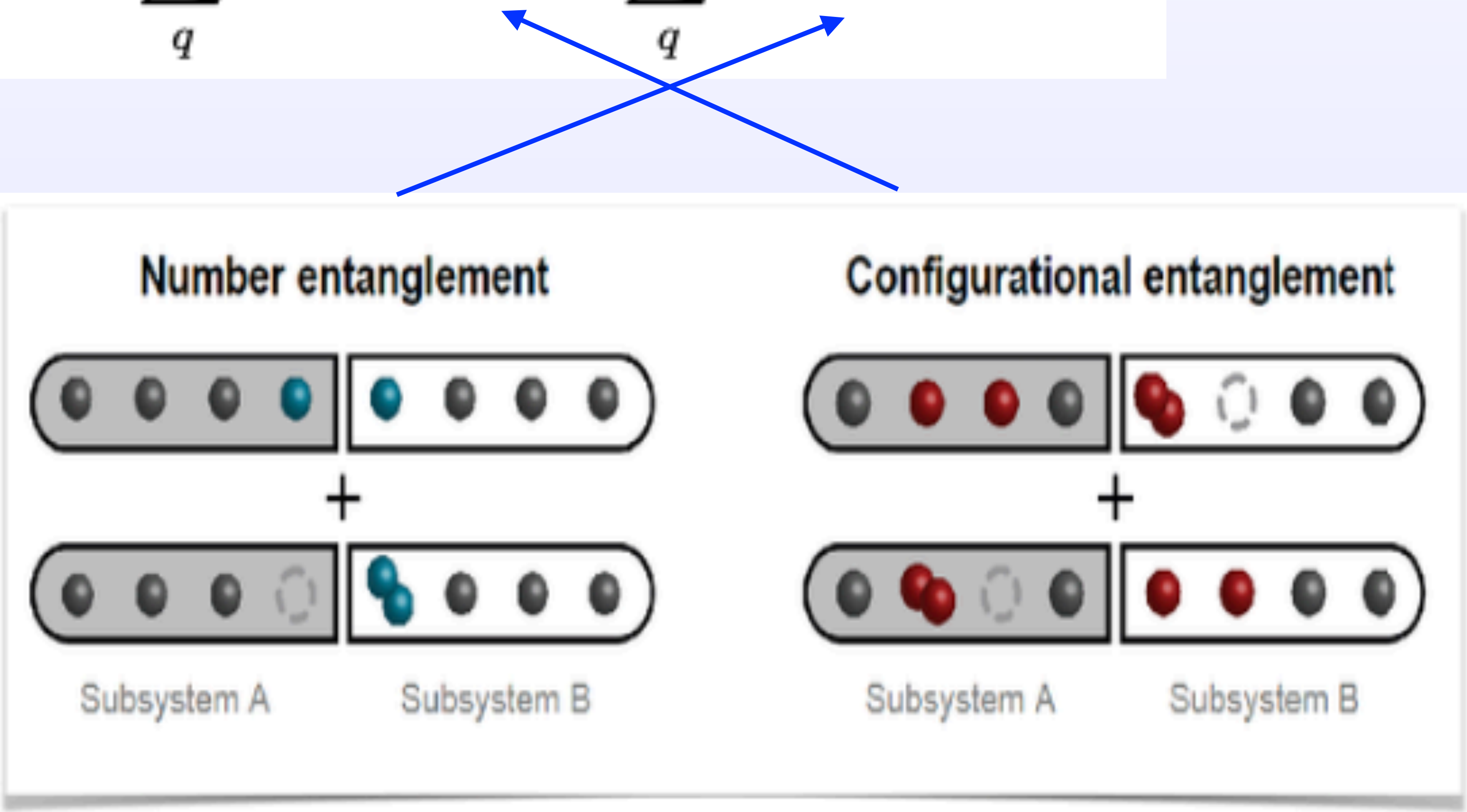
Sela, Goldstein PRL 2018

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector

$$S_1 = \sum_q P_A(q) S_1(q) - \sum_q P_A(q) \log P_A(q)$$

$$P_A(q) = \frac{\mathrm{Tr}\rho_A(q)}{\mathrm{Tr}\rho_A} = \mathrm{Tr}\rho_A(q)$$



A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman,S. Choi, V. Khemani, J. Leonard, and M. Greirer, Probing entanglement in a many-body localized system, Science 364, 6437 (2019).

Example: 2-Qubit system

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \times |1\rangle_B + |1\rangle_A \times |0\rangle_B\right)$$

$$\rho_A = \frac{1}{2}\left(|0\rangle_A\langle 0|_A) + |1\rangle_A\langle 1|_A\right)$$

Conserved Charge: Occupation number (0 or 1)

$$\rho_A(0) = |0\rangle_A\langle 0|_A \qquad \rho_A(1) = |1\rangle_A\langle 1|_A$$

$$P_A(0) = P_A(1) = \frac{1}{2}$$
$$S_1(0) = S_1(1) = 0$$

Only number entanglement (classical charge correlations)

No configurational entanglement (quantum correlations)

U(1) Kac-Moody CFTs

Holographic CFT with U(1) conserved current: $c = \frac{3L}{2G_3} \gg 1$

$$J(z) = \sum_{n=-\infty}^{\infty} \frac{J_n}{z^{n+1}}$$

Conserved current

$$T(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

Energy-Momentum-Tensor

U(1)k Kac-Moody algebra at level k

$$[J_n, J_m] = \frac{1}{2} n k \delta_{m+n}$$
$$[L_n, J_m] = -m J_{n+m}$$
$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12}\left(n^3 - n\right) \delta_{n+m,0}$$

S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

AdS$_3$ dual to U(1)$_k$ Kac-Moody CFT

3D Einstein-Hilbert gravity

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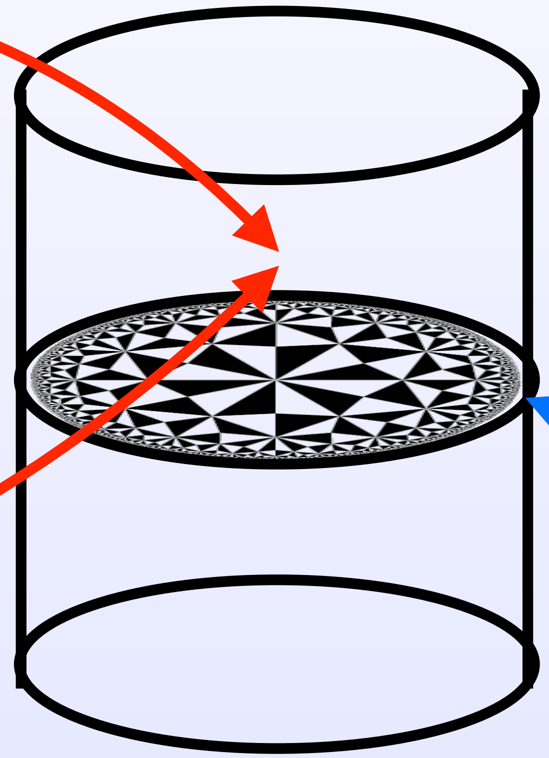
L… Curvature Radius of AdS3 space-time

To suppress quantum gravity effects: $c = \frac{3L}{2G_3} \gg 1$



U(1)$_k$ Chern-Simons theory

$$S_{CS} = \frac{ik}{8\pi}\int A\wedge dA$$

k… Chern-Simons level

Asymptotic symmetry analysis:

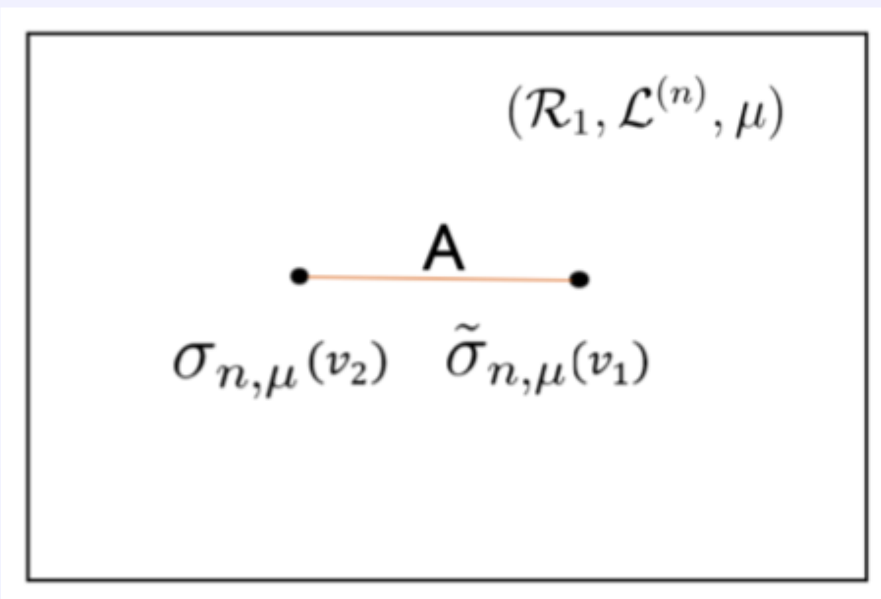
Boundary theory has U(1)$_k$ Kac-Moody symmetry

P. Kraus arXiv:hep-th/0609074

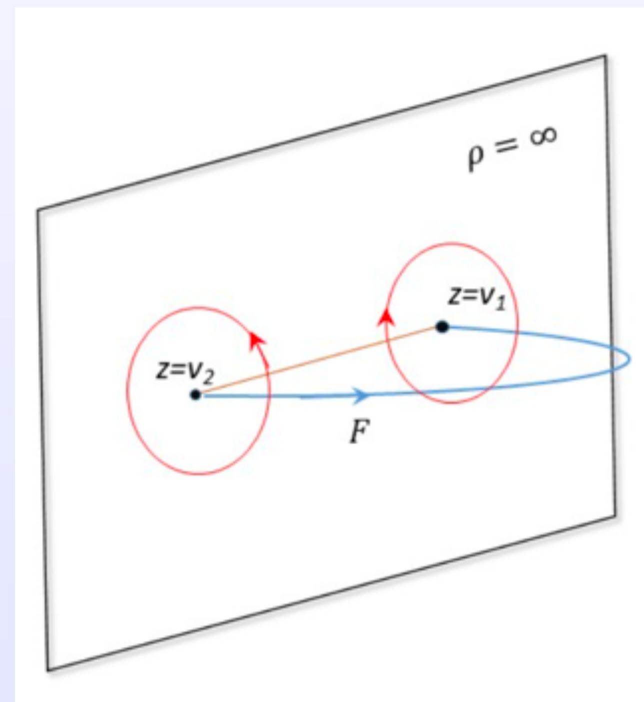
Single Interval SREE in AdS_3/CFT_2

Charged twist operator induces flux:

Sela, Goldstein PRL 2018



Wilson line following the Ryu-Takayanagi geodesic



Suting Zhao

CFT_2 result matches AdS_3 result for c>>1

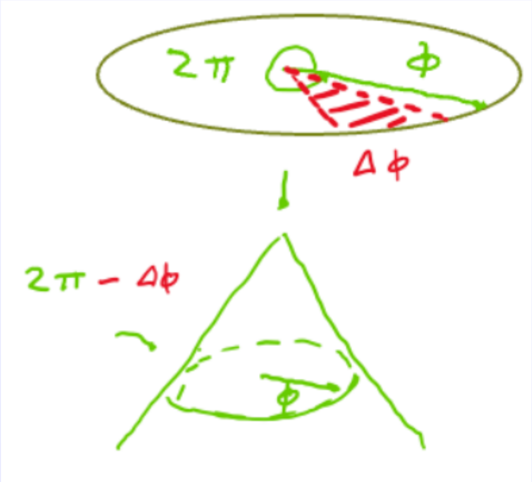
$$S_1(q) = \frac{c}{6}\ell - \frac{1}{2}\log\left(\frac{k\ell}{2\pi}\right) \quad \text{with} \quad \ell = 2\log\frac{|v_1 - v_2|}{\epsilon}$$

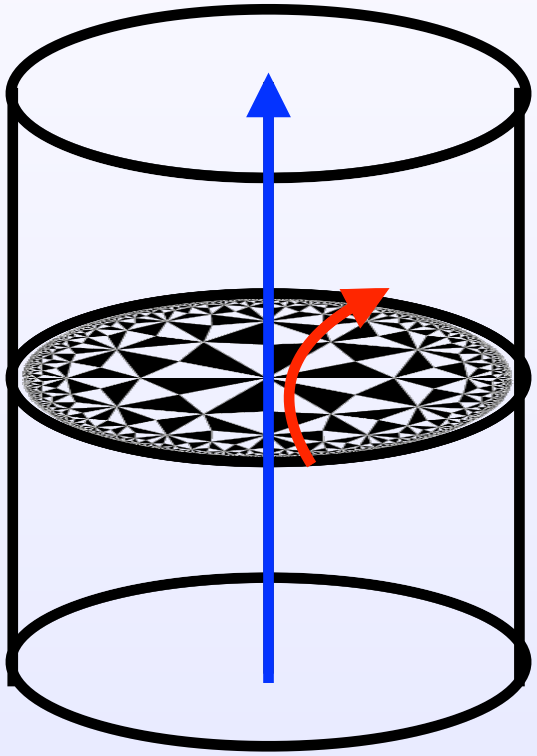
Equipartiton of entanglement!

Suting Zhao, Christian Northe, RM, JHEP 2021 (arXiv: 2012.11274)

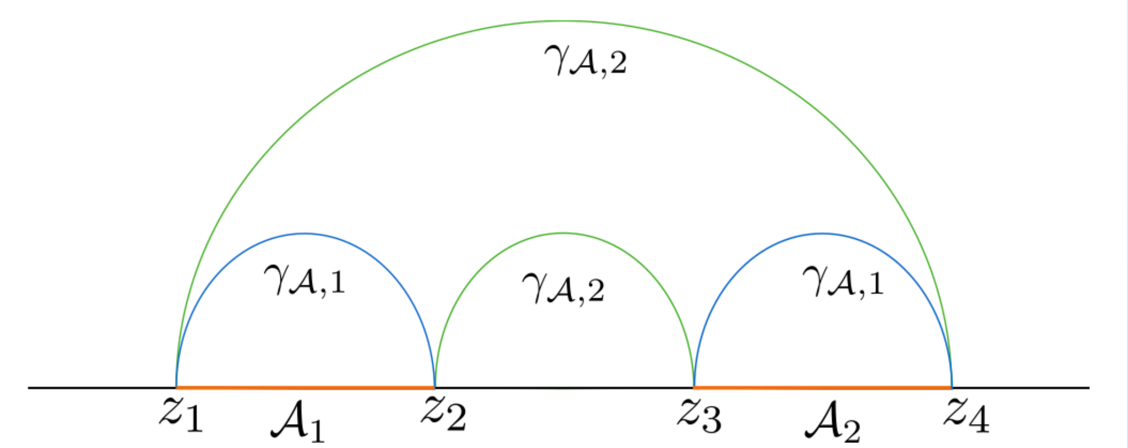
Further Checks

- Single interval with uncharged/charged heavy primary insertions





- Two intervals in the ground state



S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

Breakdown of Equipartition

SL(3,R) Higher Spin Gravity in 3D: W_3 symmetric CFT
Energy-momentum tensor plus Spin 3 current

$$T(z)W(w) = \frac{3W(w)}{(z-w)^2} + \frac{\partial W(w)}{z-w} + \cdots$$

Charged moments for a single interval:
Topological black hole grand canonical partition function
Perturbative result to quartic order in μ

$$\log \mathrm{Tr}\left(e^{-2\pi n\mathcal{H}+2\pi i\mu Q_{\mathcal{A}}}\right) = \frac{c\ell}{6n}\left(-\frac{1}{3}\frac{\mu^2}{n^4} + \frac{10}{27}\frac{\mu^4}{n^8} + \cdots\right)$$

Fourier transformation and taking the replica limit
yields breakdown of equipartition in SREE at large c.

Suting Zhao, Christian Northe, Konstantin Weisenberger, RM, 2202.11111

Thermal Entropy as Entanglement Entropy

Thermo field double state: Purification of the thermal state of a system with spectrum $\{E_n\}$

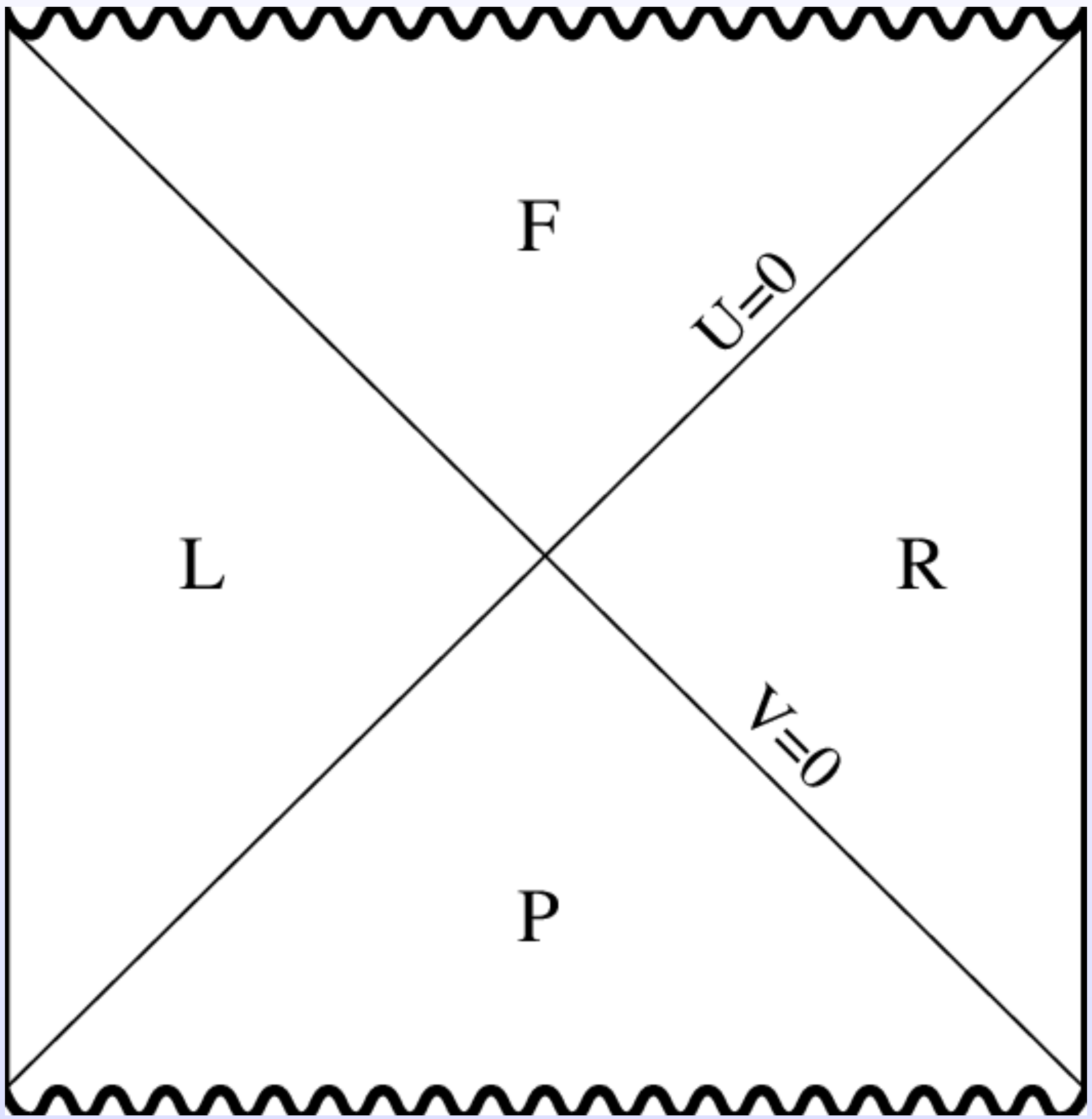
Take two copies of the system's Hilbert space:

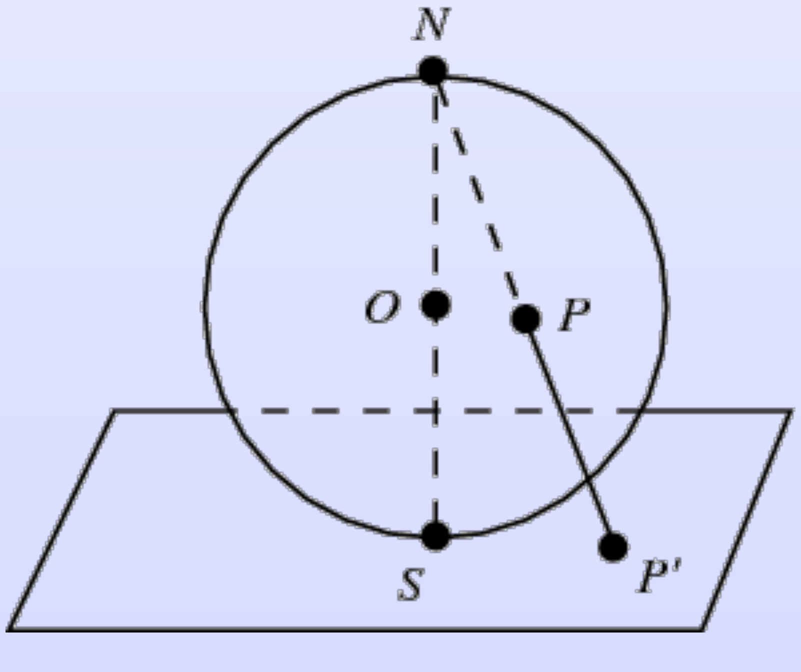
$$\mathcal{H} = \mathcal{H}_L \times \mathcal{H}_R$$

$$|\mathrm{TFD}\rangle \equiv \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

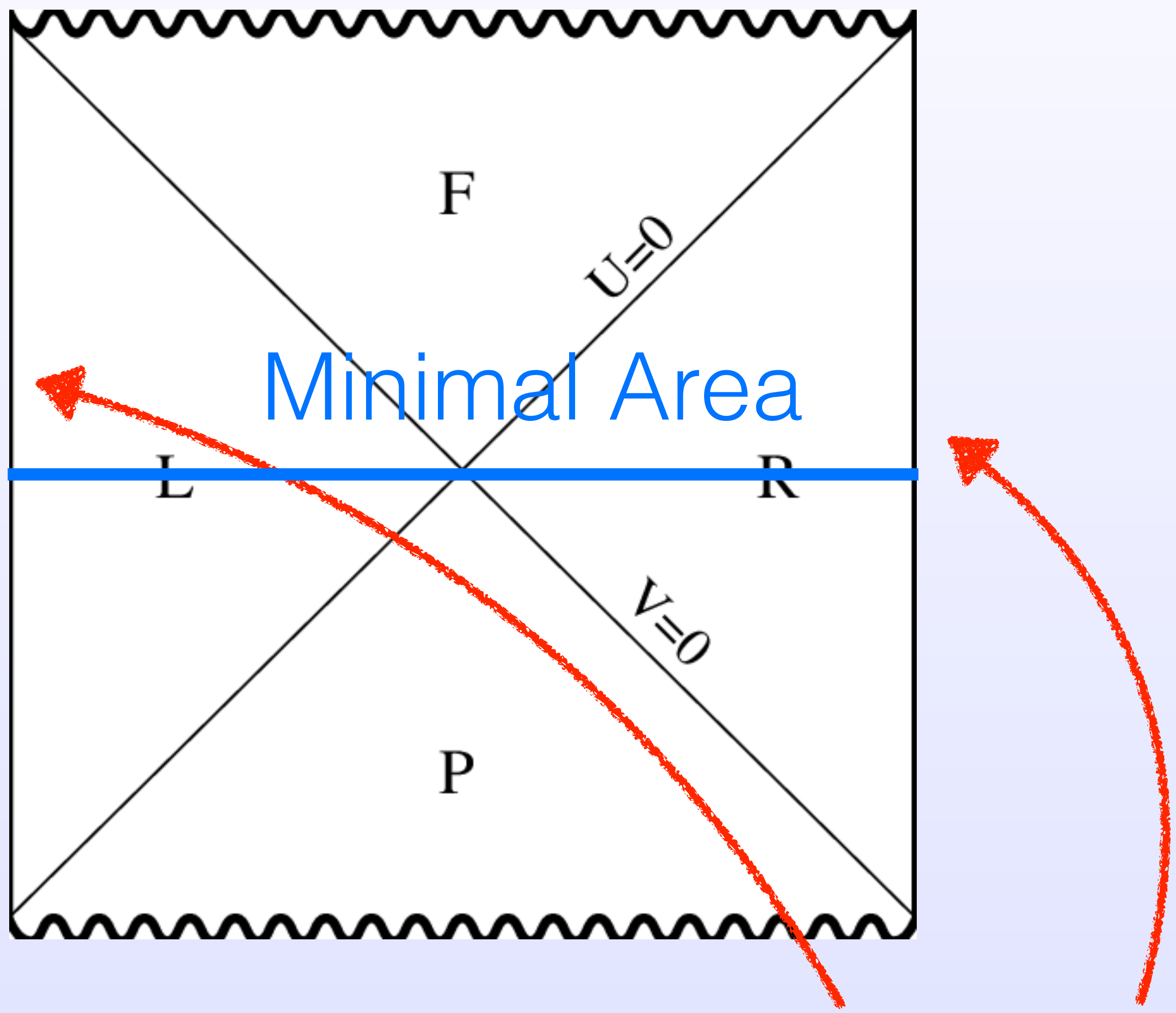
$$\rho_R = Tr_L |TFD\rangle\langle TFD| = \sum_n e^{-\beta E_n} |n\rangle_R \langle n| = \rho_{thermal}$$

Global structure of AdS Schwarzschild BH





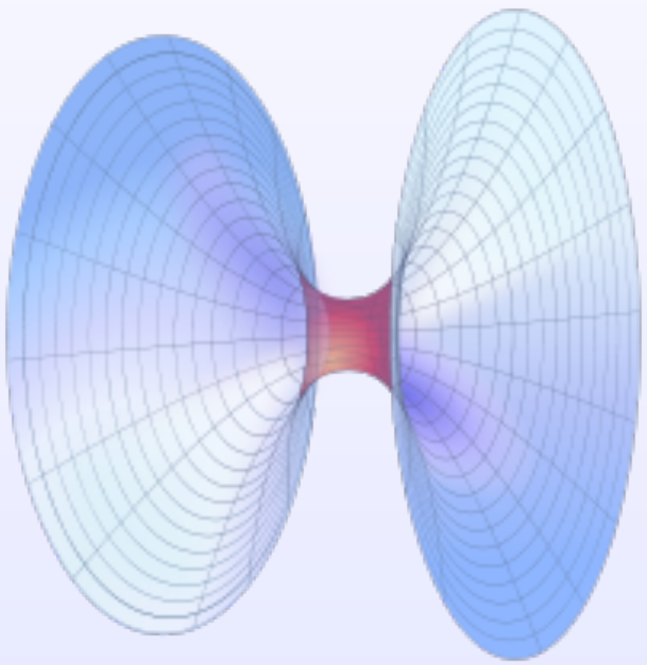
BH Entropy as Entanglement Entropy



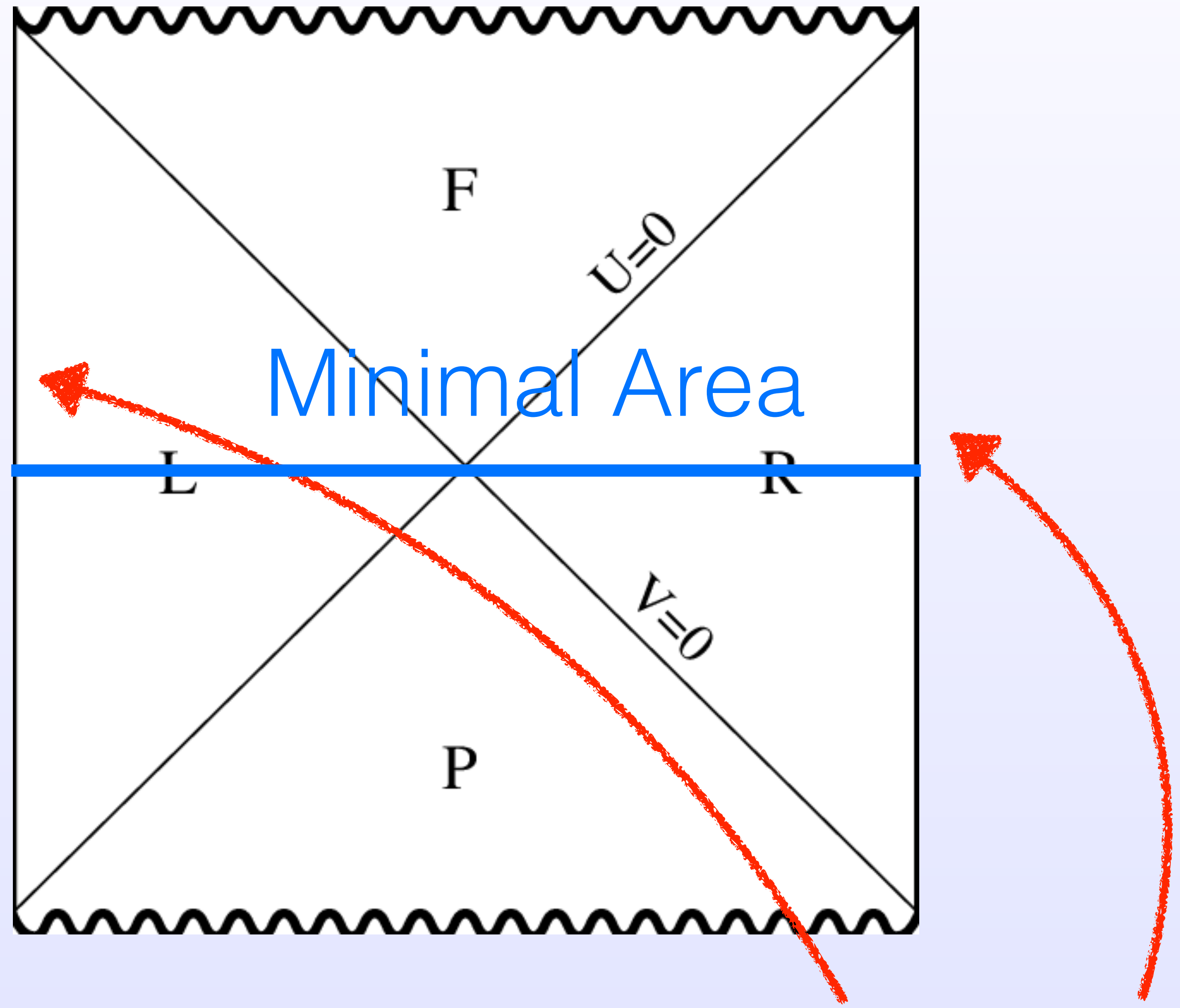
$$|\mathrm{TFD}\rangle \equiv \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

Maldacena 2001

EPR = ER



Wormhole
(Einstein-Rosen)



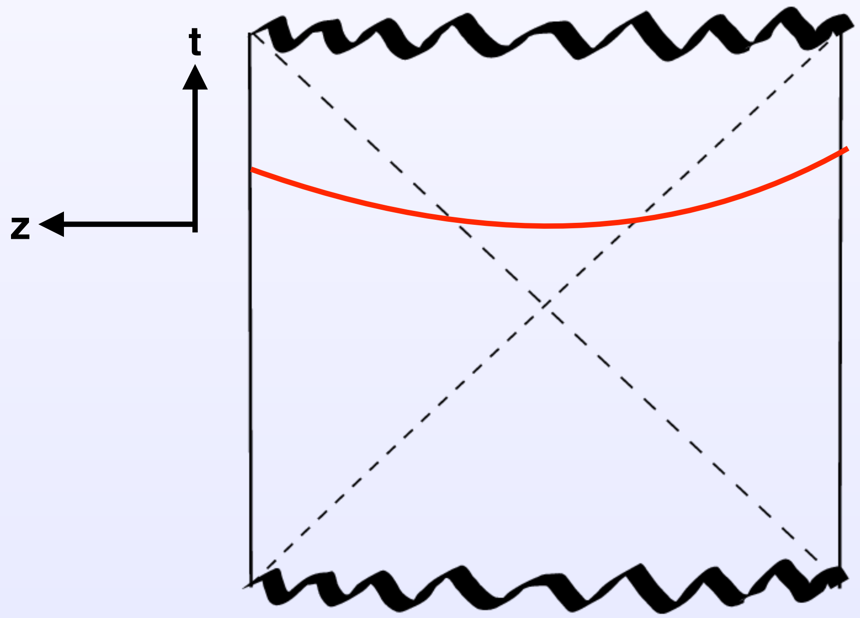
$$|\mathrm{TFD}\rangle \equiv \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |n\rangle_R$$

Quantum Entanglement: EPR correlations

Maldacena, Susskind 2013

Quantum Complexity in AdS/CFT

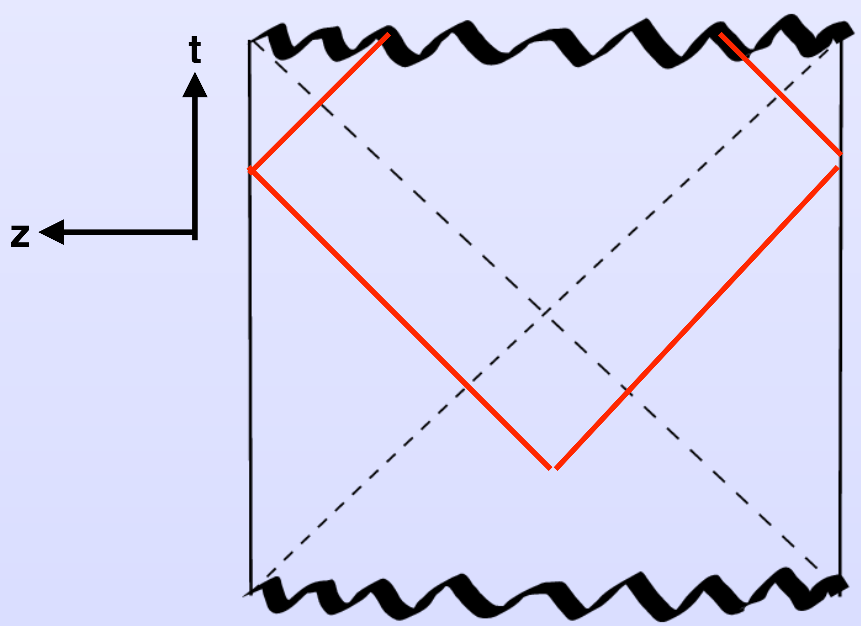
Complexity = Volume



$$\mathcal{C} = \frac{\mathrm{Vol}(\Sigma)}{L_{AdS} G_N}$$

Stanford & Susskind 2014

Complexity = Action



$$\mathcal{C} = S_{grav}\Big|_{\text{WdW Patch}}$$

Brown-Roberts-Susskind-Swingle-Zhao 2015

Quantum Complexity

How to build states given a set of unitary operators?

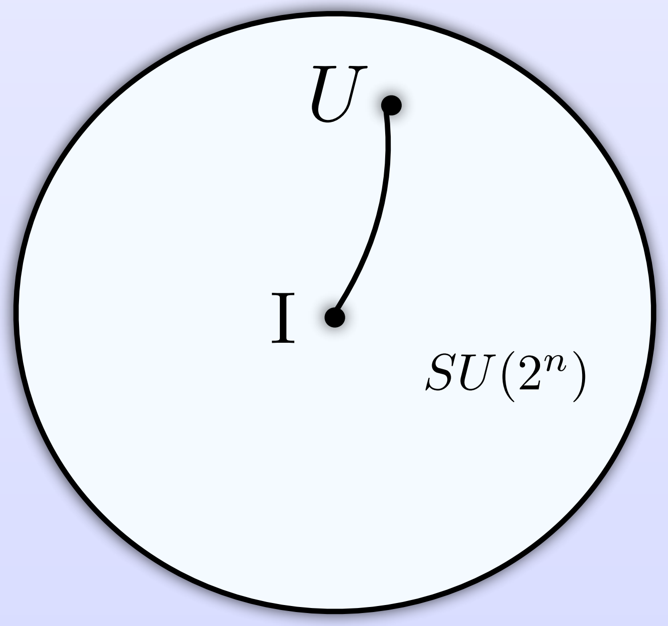
Hilbert Space with $\dim(\mathcal{H}) = 2^n$

Gates $\{U_i\}$

Complexity: Optimal Quantum Circuit $|\Psi\rangle = U|\Psi_0\rangle$

Minimal Number of Gates to build $U \in SU(2^n)$

Nielsen: Finsler Geometry & Hamiltonian Control Theory



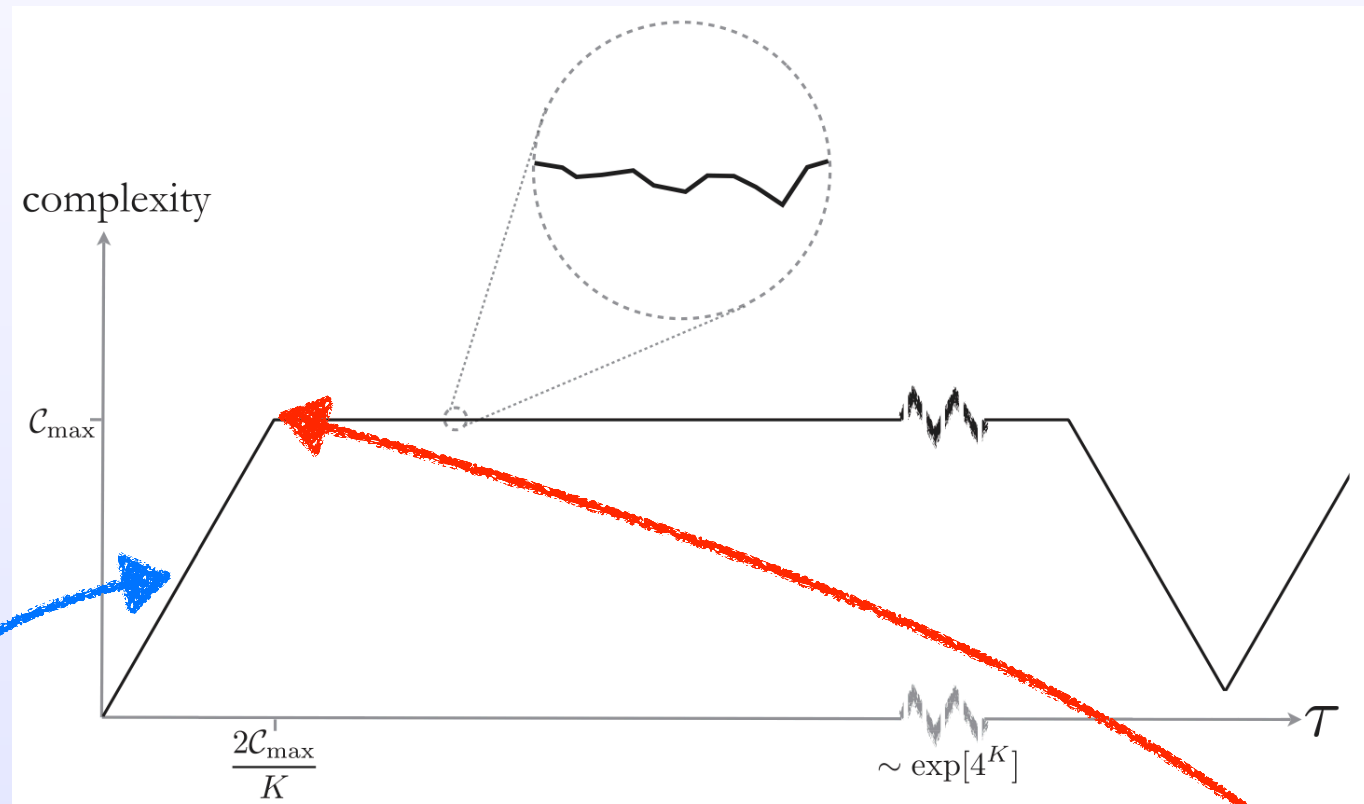
for $U(t) = e^{-iHt}$

$$\mathcal{C}(t) \sim nt$$

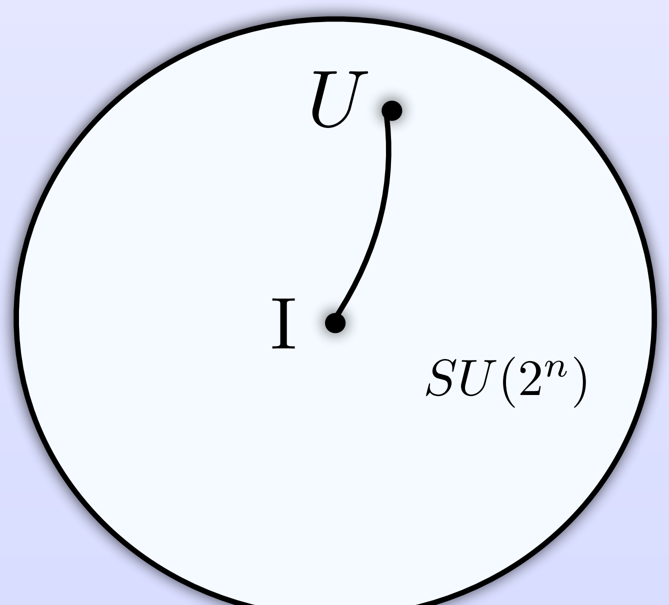
Nielsen (et.al.) 2005-2007

Time Evolution of Complexity

What are good holographic complexity measures?



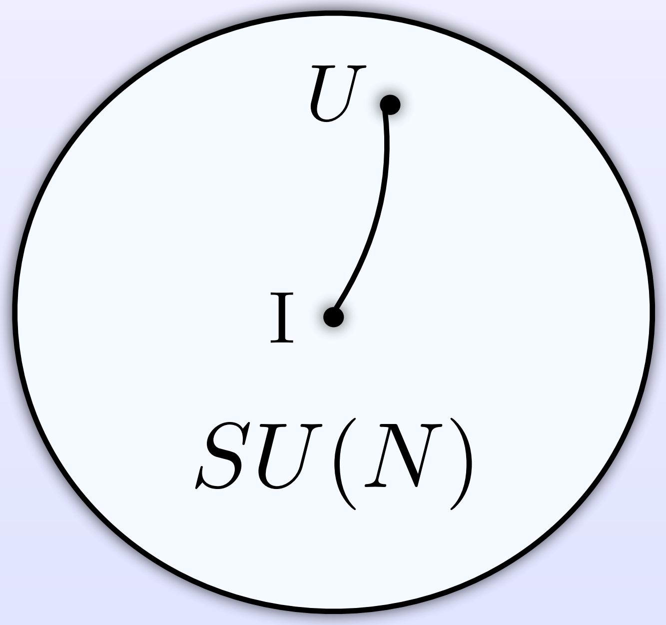
Ergodicity



Conjugate Points

A well-defined large N limit

We investigated curvatures of SU(N) in the large N limit with a novel choice of metric on SU(N)



P. Basteiro, J. Erdmenger, P. Fries, F. Goth, I. Matthaiakakis, RM, 2109.01152

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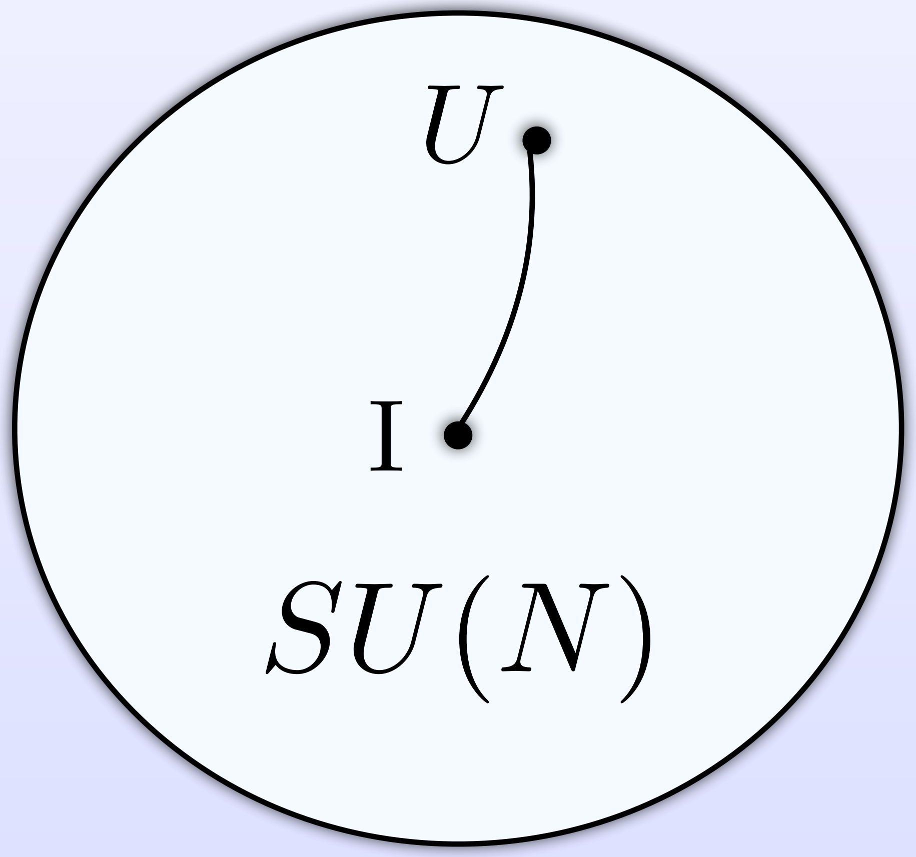

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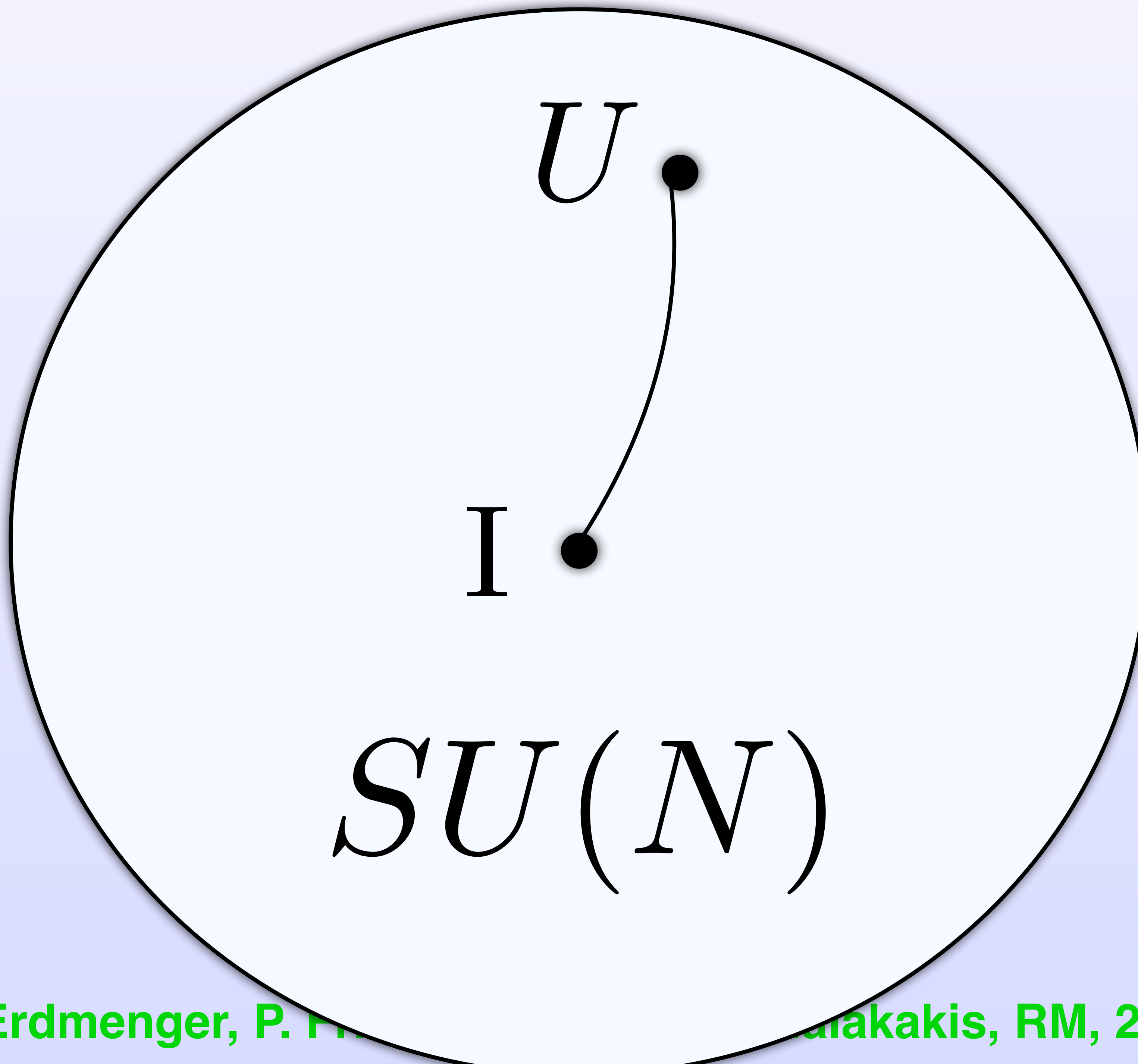
A well-defined large N limit

We investigated curvatures of SU(N) in the large N limit with a novel choice of metric on SU(N)



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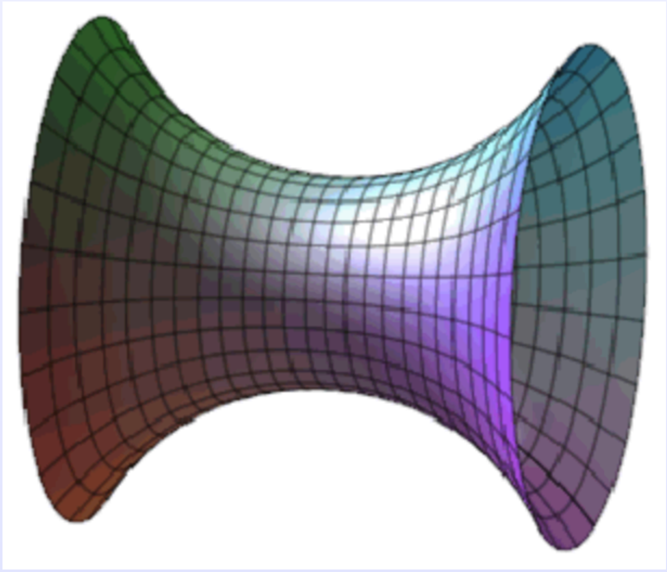
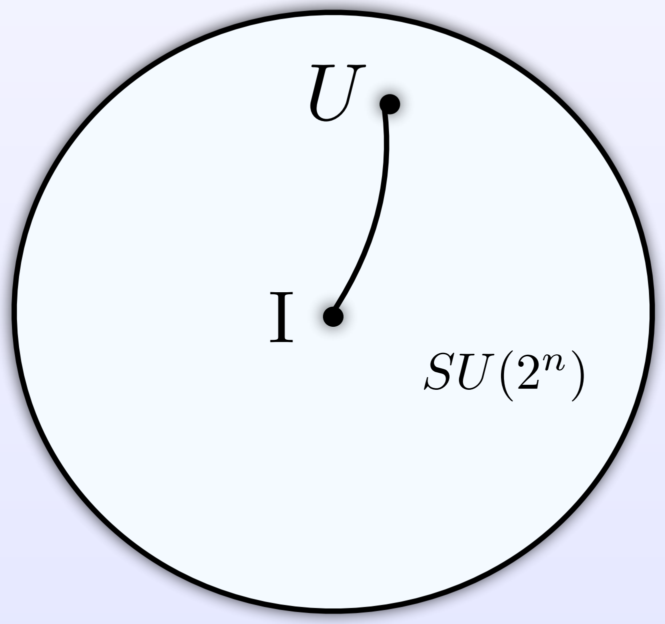
Why Curvatures?

Both properties are related to the choice of cost function, i.e. the choice of metric on the group manifold

Ergodicity from Chaos:

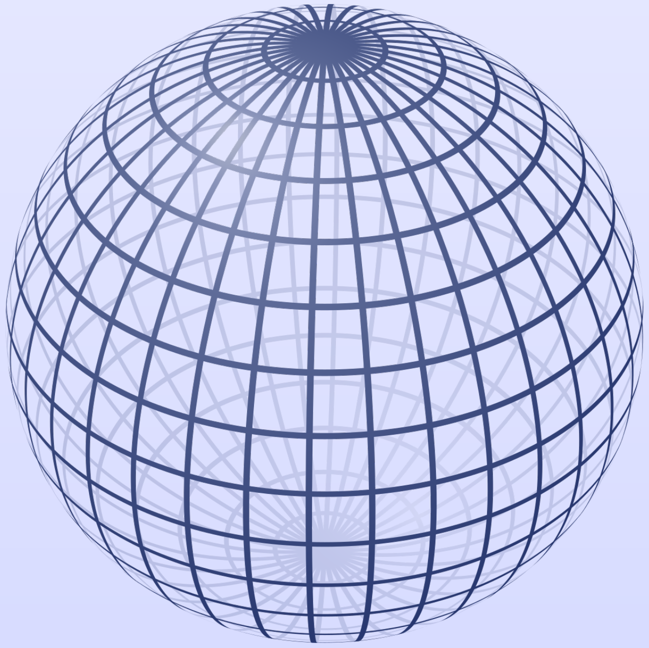
Needs negative sectional curvatures





Conjugate Points:

Existence needs some positive sectional curvatures



A well-defined large N limit

Our choice of metric admits a well-defined large N limit with finite sectional curvatures

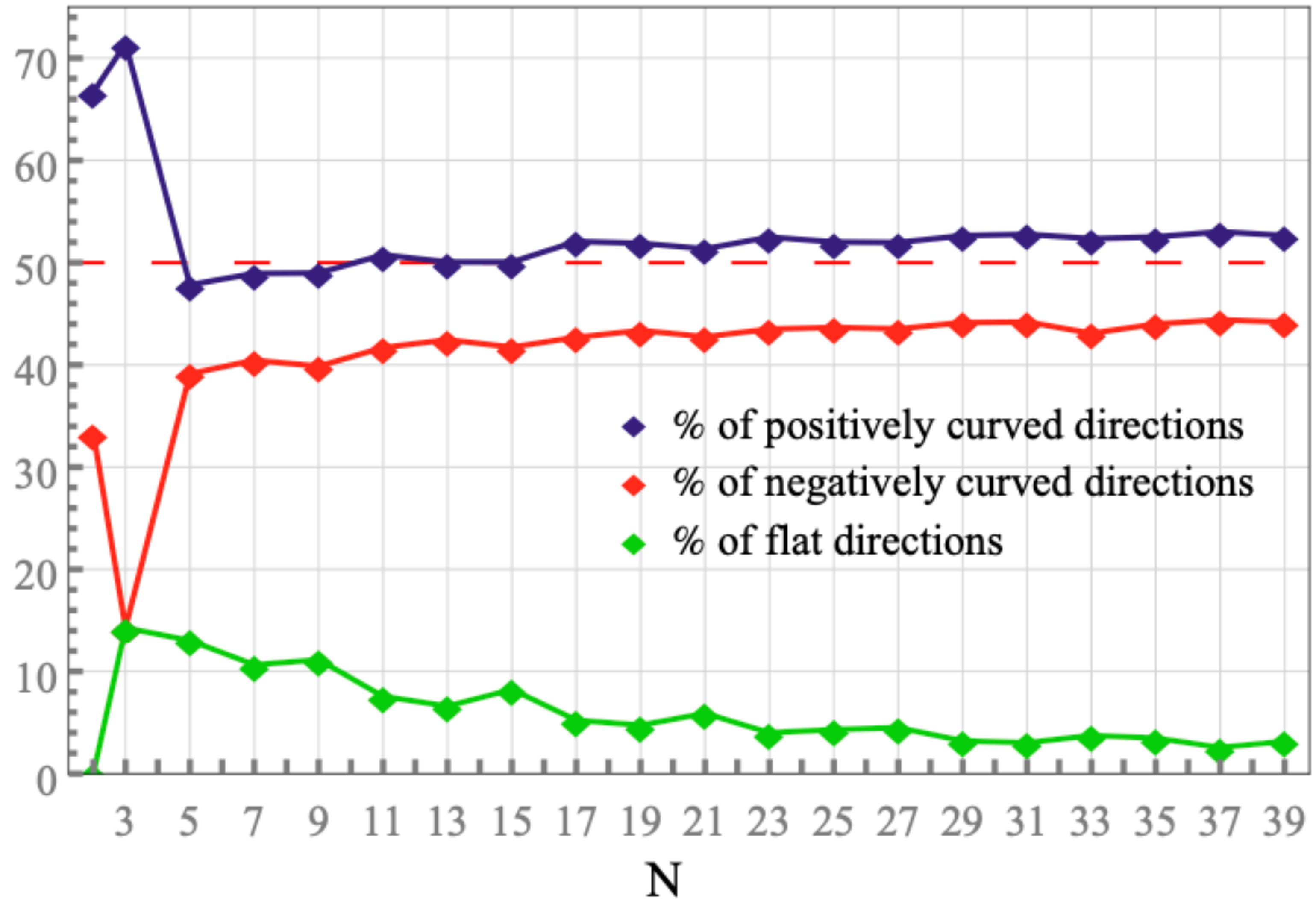


FIG. 3. Distribution of signs of sectional curvatures of $SU(N)$ in directions given by pairs of generators of the $\mathfrak{su}(N)$ algebra for $2 \leq N \leq 39$. The computations exhibited changes of less than 0.1% for large N, strongly suggesting that the percentages stabilize. Lines are a visual guide, not an interpolation. A slight majority of positively curved directions settles, but this is inconclusive with regards to the stability of geodesics on the manifold of unitaries.

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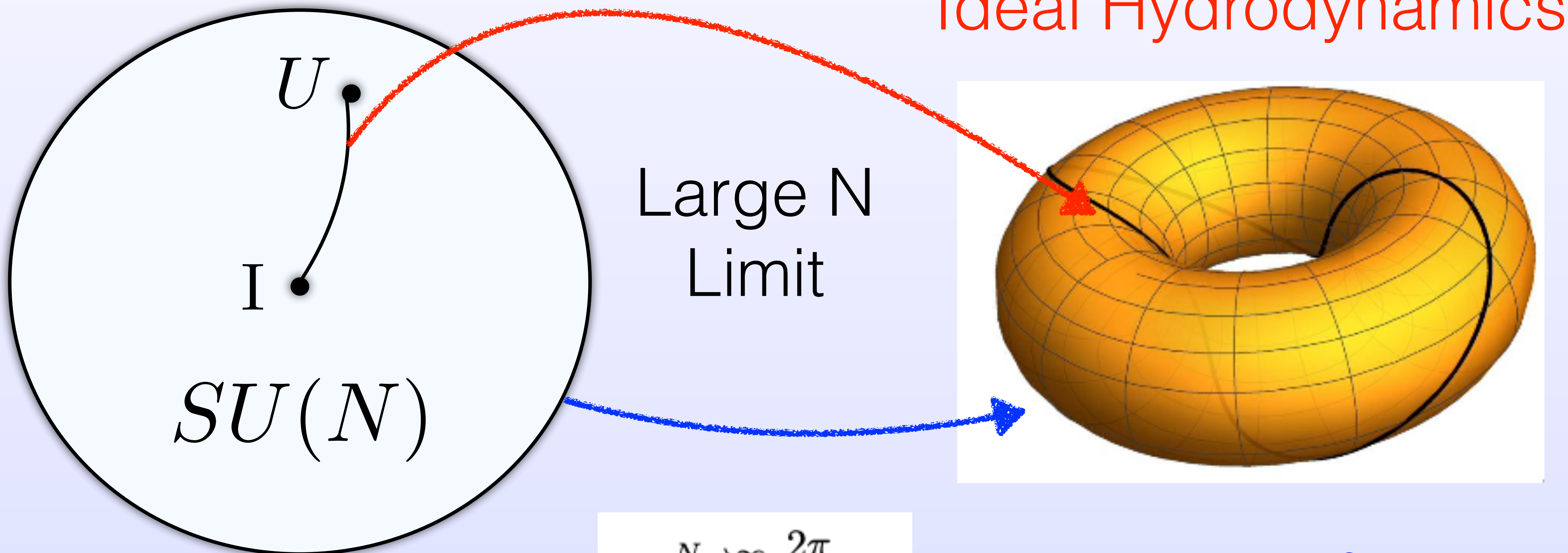
Complexity as Hydrodynamics

Our choice of metric admits a well-defined large N limit

Geodesic dynamics

Two-dimensional Ideal Hydrodynamics



$SU(N \to \infty)$

$$J_{\vec{m}} \overset{N\to\infty}{\longrightarrow} \frac{2\pi}{iN} X_{\vec{m}}$$

$\mathrm{sdiff}(T^2)$

$$[J_{\vec{m}}, J_{\vec{n}}] = -2i \sin\left(\frac{\pi}{N}(\vec{m}\times\vec{n})\right) J_{\vec{m}+\vec{n}}\,,$$

Sub-Riemannian Geometry

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Averaged Ricci Curvatures

Hydrodynamics: Negative Normalized Ricci Curvature

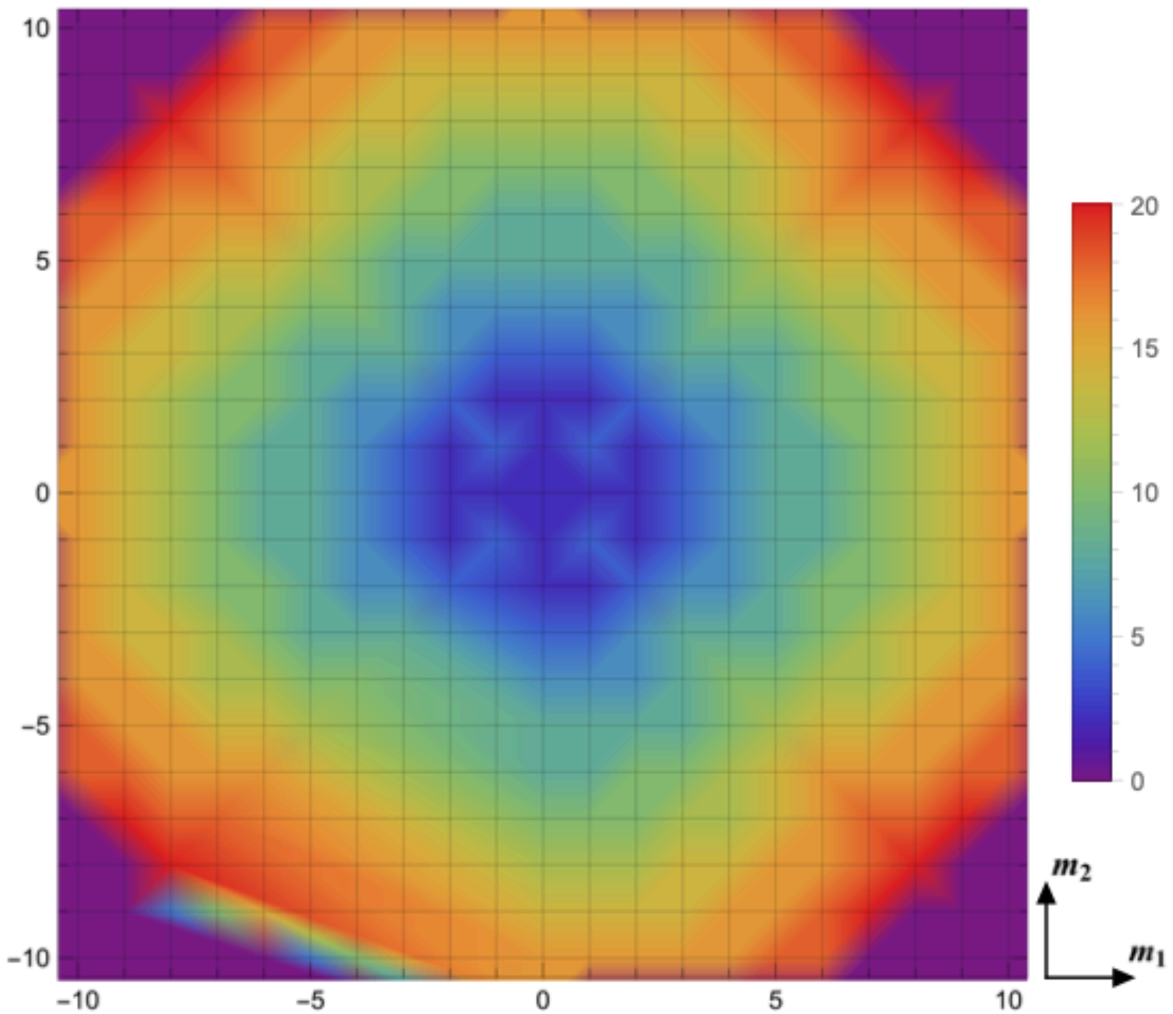

FIG. 1. Color density plot of the critical value N_c at which the normalized Ricci curvature of a given direction $\vec{m}$ in $\mathfrak{su}(N)$ becomes negative over the $\mathbb{Z}^2$ lattice, spanned by the vector components m_1, m_2. The interpolation between the integer points of the lattice is there to guide the eye. The color flare at the lower left corner is an artifact of this interpolation.

$$Ric(v) = \lim_{N\to\infty} \frac{1}{N^2-2} \sum_{\vec{m}} K(v, T_{\vec{m}})$$

$$Ric(v) \leq 0 \text{ for } \mathrm{SDiff}(\mathbb{T}^2)$$

Necessary for Ergodicity

A. M. Lukatskii, "On the curvature of the group of measure-preserving diffeomorphisms of an n-dimensional torus," *Russian Mathematical Surveys*, vol. 36, pp. 179–

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Conclusions & Outlook

- Gauge/gravity duality: New relations between gravity, black holes and quantum information

- Geometric concepts in AdS space: New insight into quantum information properties of strongly coupled holographic quantum systems and gravity
  E.g. Entanglement entropy, symmetry resolved entanglement, quantum complexity, wormholes

- New tests of "extended" AdS_3/CFT_2 correspondence
- First example of breakdown of equipartition at large c
- Large N qudit complexity as hydrodynamics

- Outlook:Improved understanding of both AdS Quantum Gravity and strongly interacting physics from quantum information