OBSERVING THE RINGDOWN:
ON THE DETECTABILITY OF HIGHER MODES

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IN SUPREMÆ DIGNITATIS • 1343 •

OCTOBER 5, 2022

THIRD GRAVI-GAMMA WORKSHOP

1. RINGDOWN
2. HIGHER MODES DETECTABILITY
3. TEOBPM ANALYSIS ON GWTC-3

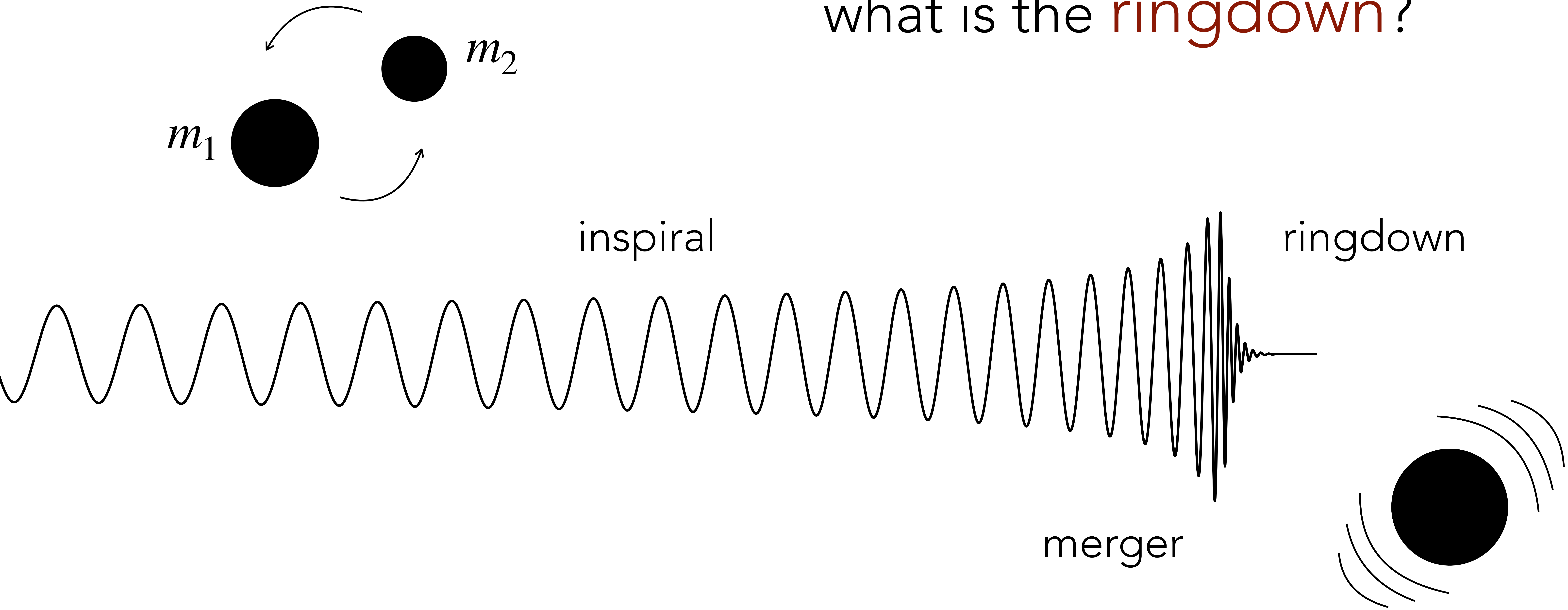
1. RINGDOWN

RINGDOWN BASICS

what is the ringdown?

RINGDOWN BASICS

what is the ringdown?



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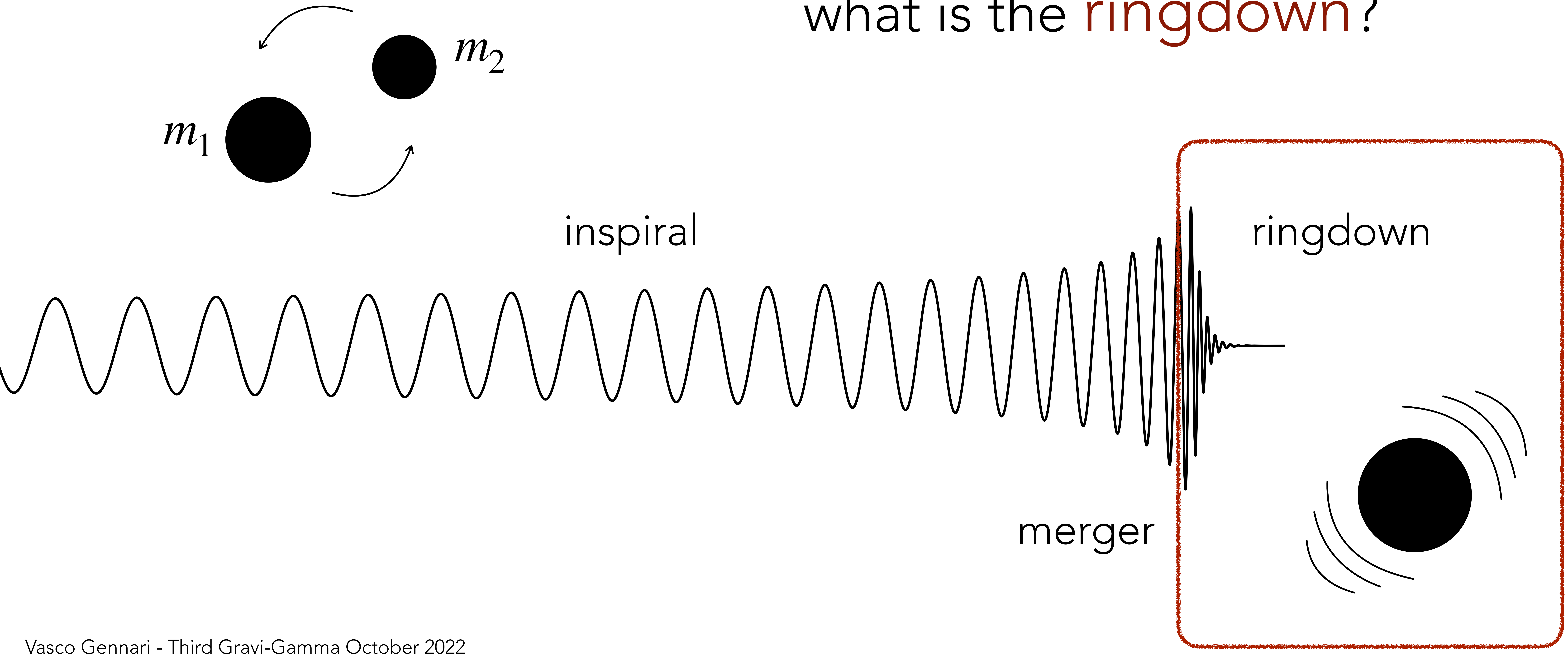
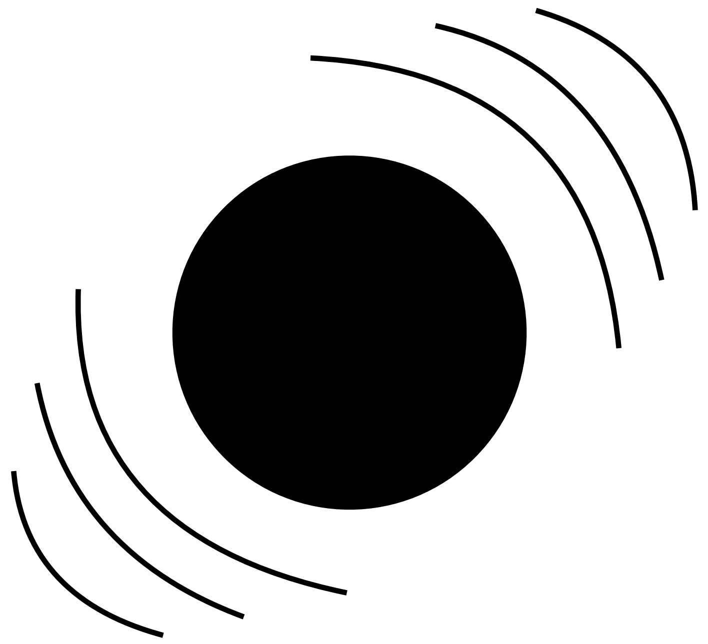


RINGDOWN WAVEFORM

BH linear perturbation theory predicts:



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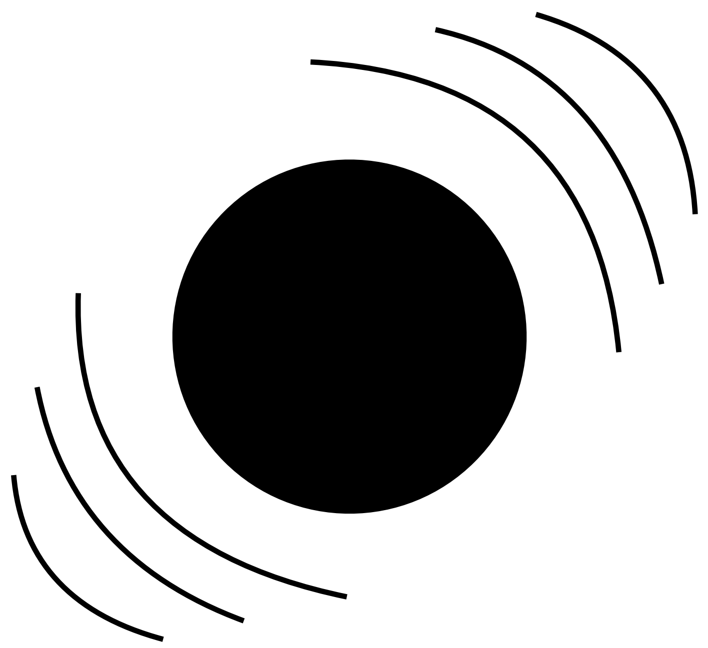

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sum of different
modes of vibration

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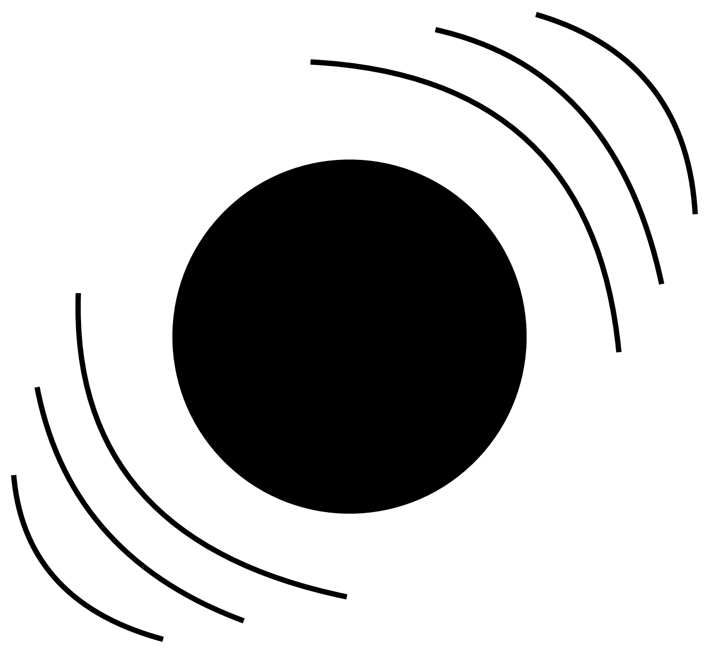

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sum of different modes of vibration

amplitude

RINGDOWN WAVEFORM

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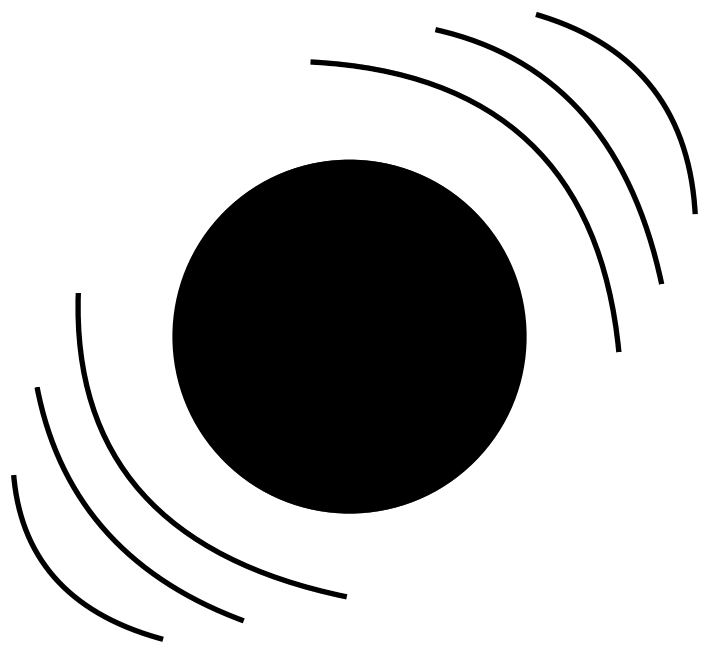

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exponentially damped harmonic oscillations

RINGDOWN WAVEFORM

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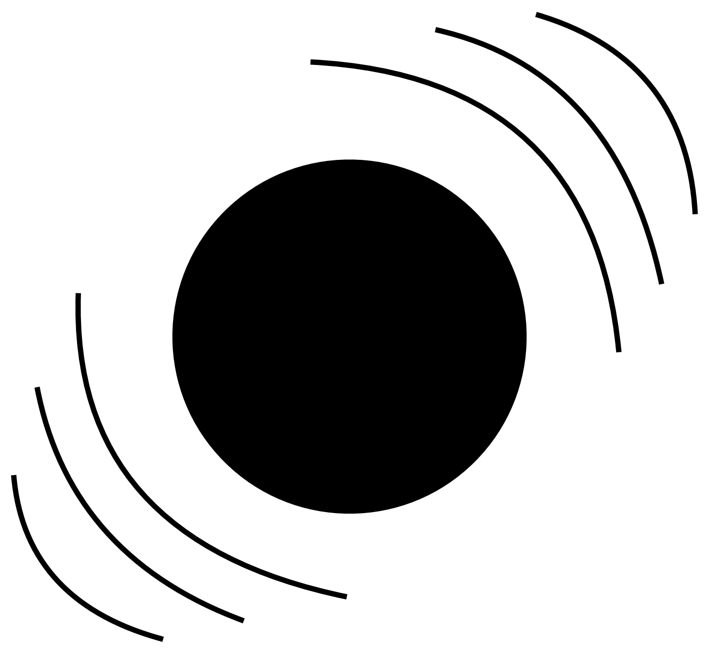


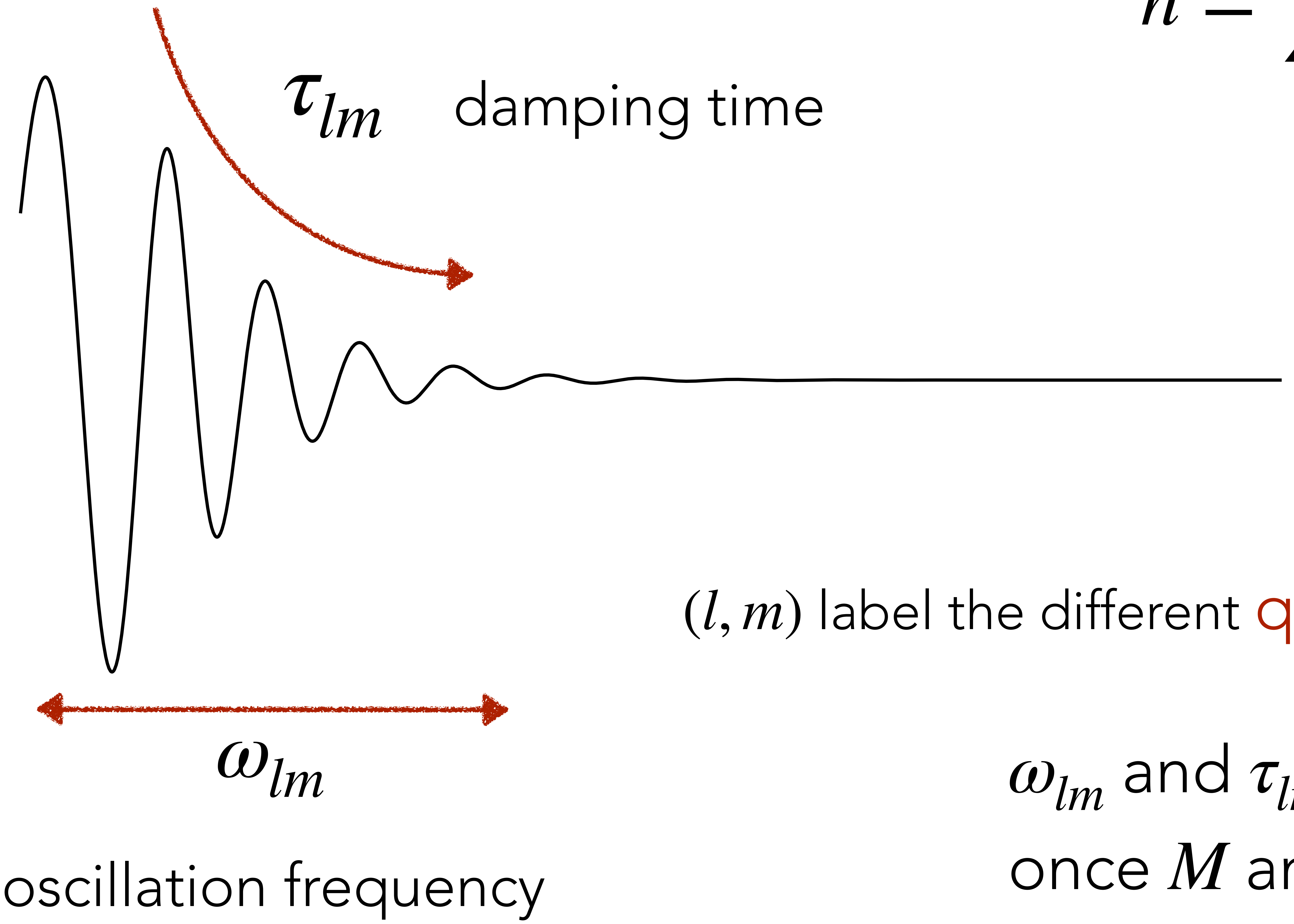
$$h = \sum_{lm} A_{lm} \, e^{-i\omega_{lm}t - t/\tau_{lm}} \, {}_2Y_{lm}$$

- $\sum_{lm}$: sum of different modes of vibration
- A_{lm}: amplitude
- $e^{-i\omega_{lm}t - t/\tau_{lm}}$: exponentially damped harmonic oscillations
- ${}_2Y_{lm}$: (spin-weighted) spherical harmonics

1. QUASINORMAL MODES

$$h = \sum_{lm} A_{lm} \, e^{-i\omega_{lm}t - t/\tau_{lm}} \, {}_2Y_{lm}$$

exponentially damped harmonic oscillations



(l, m) label the different quasinormal modes (QNMs)

ω_{lm} and τ_{lm} are known once M and a are fixed

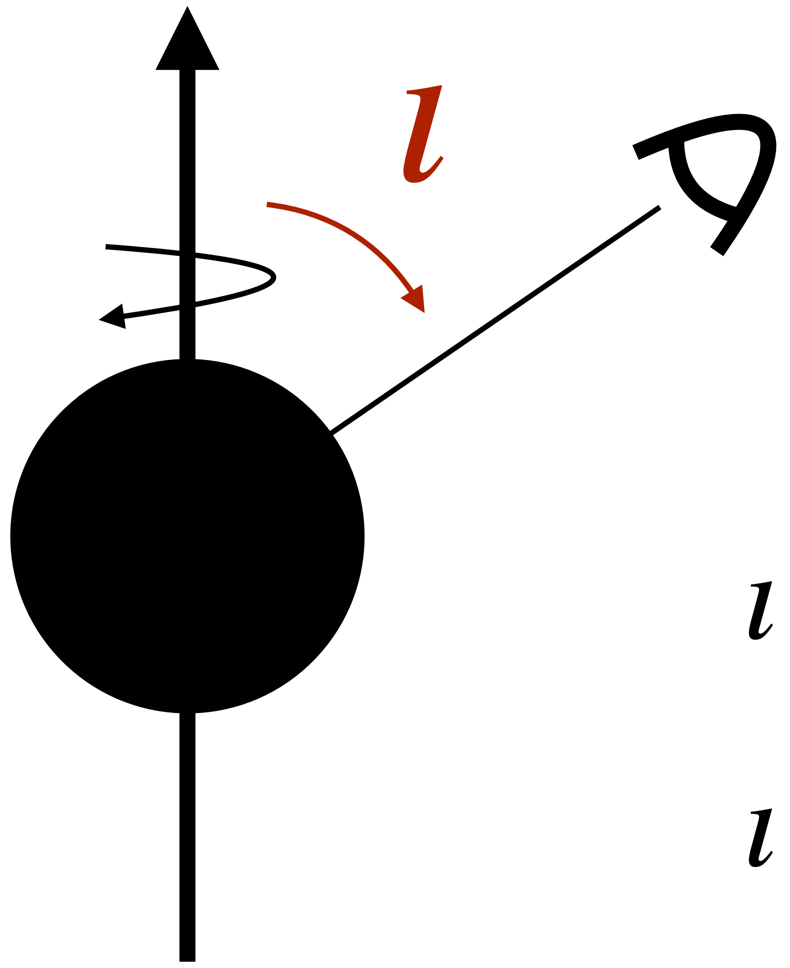
2. SW SPHERICAL HARM.

$$h = \sum_{lm} A_{lm} \; e^{-i\omega_{lm}t - t/\tau_{lm}} \; {}_2Y_{lm}$$

(spin-weighted) spherical harmonics

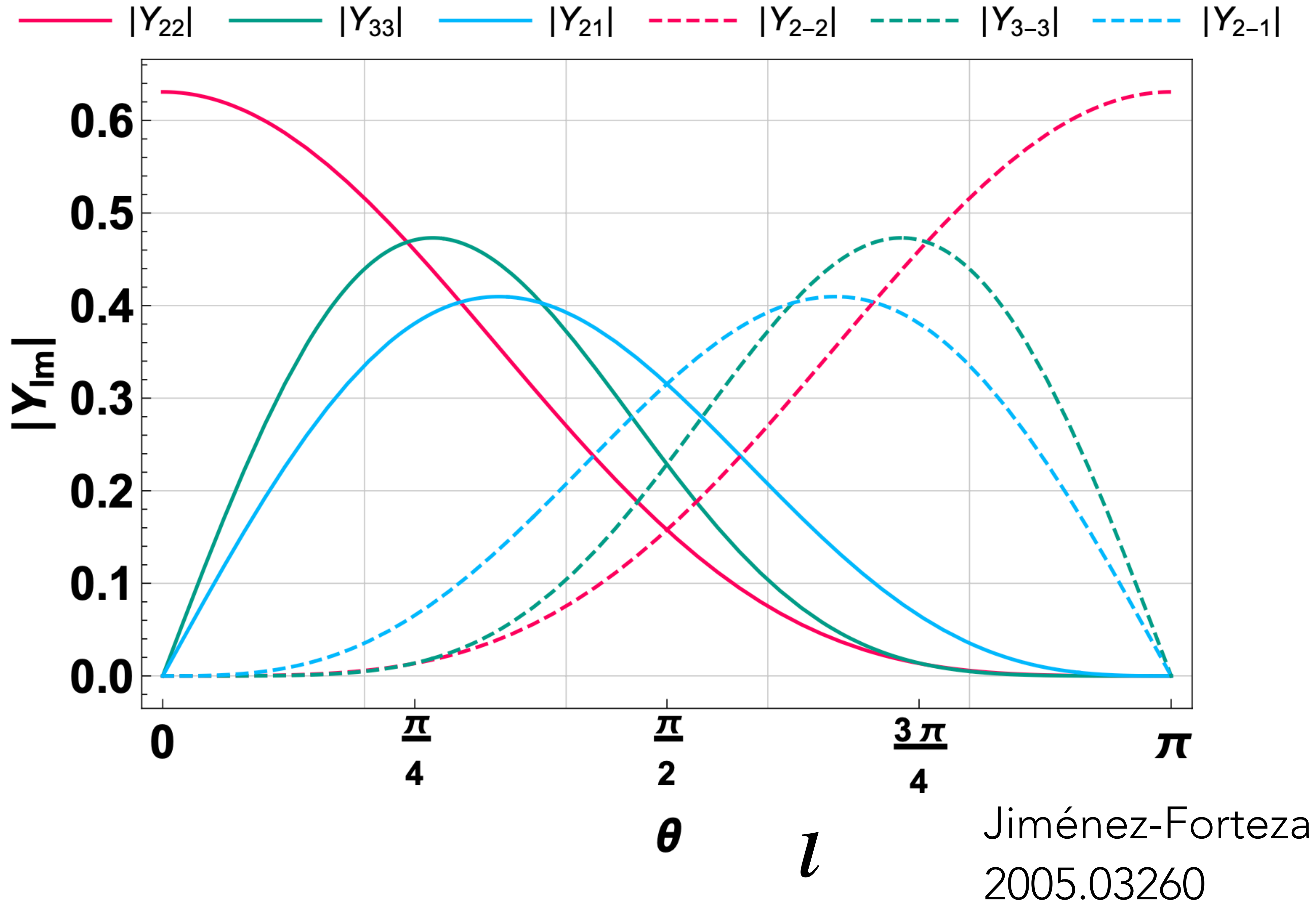
$${}_2Y_{lm}(\iota, \varphi)$$

ι is the inclination



$\iota = 0$ face-off

$\iota = \pi/2$ edge-on



Jiménez-Forteza et al. (2020) 2005.03260

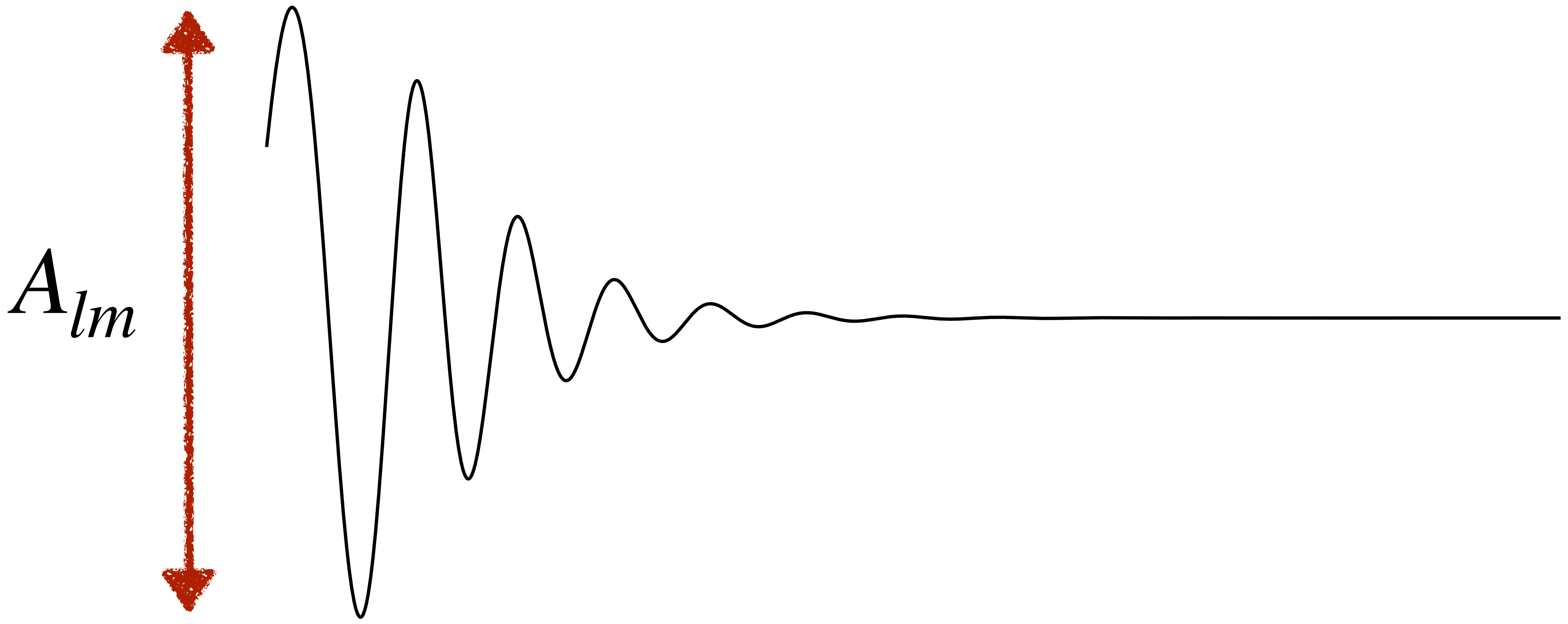
3. QNMS AMPLITUDE

$$h = \sum_{lm} A_{lm} \, e^{-i\omega_{lm}t - t/\tau_{lm}} \; {}_2Y_{lm}$$

amplitude

A_{lm} depend on the specific process that perturbs the BH

A_{lm} are not known analytically



TEOBPM includes A_{lm} in the model

(informed on NR simulations)

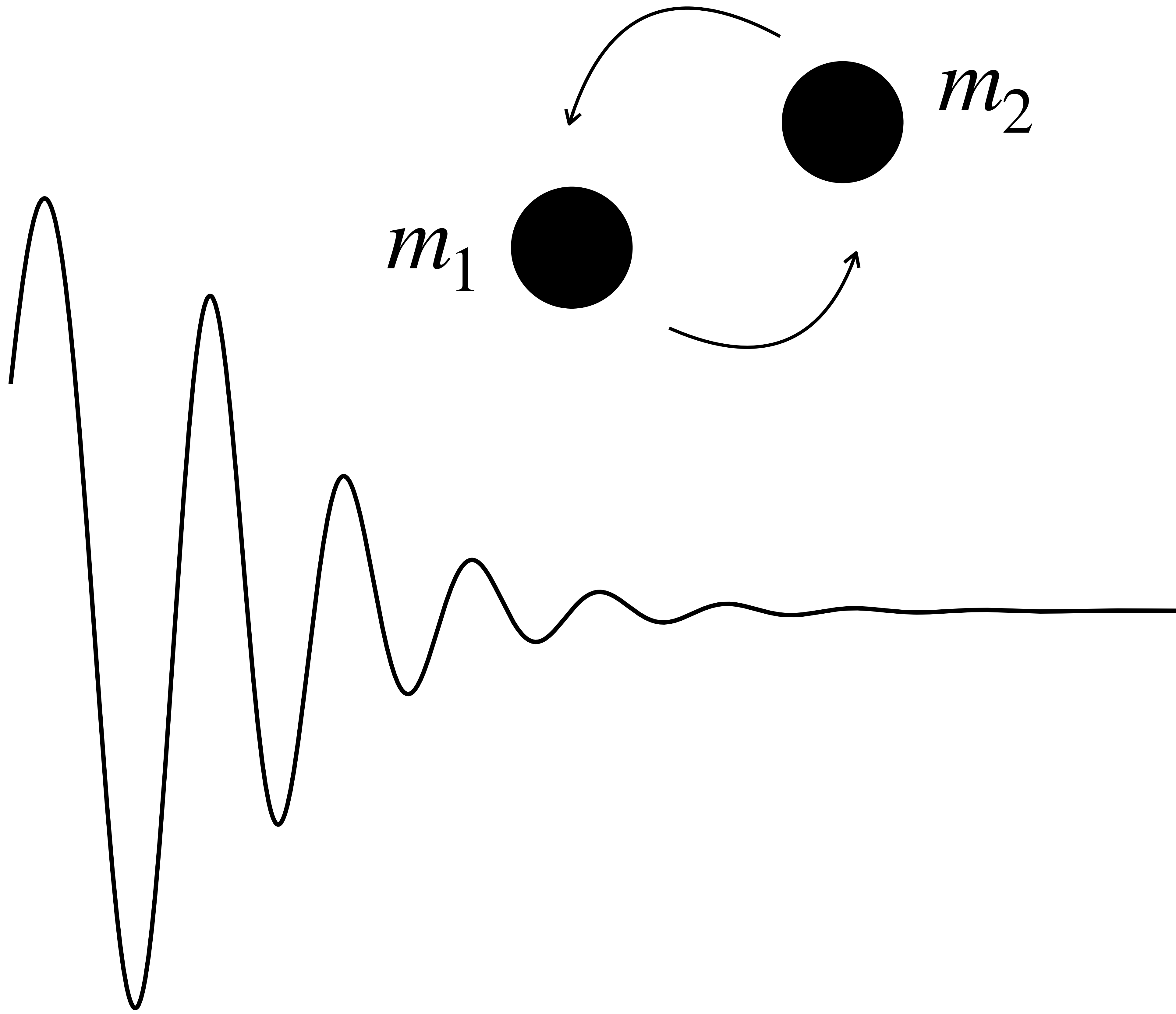
TEOBPM: Damour, Nagar (2014)
1406.0401

2. HIGHER MODES DETECTABILITY

HIGHER MODES I

which modes are observable in the ringdown?

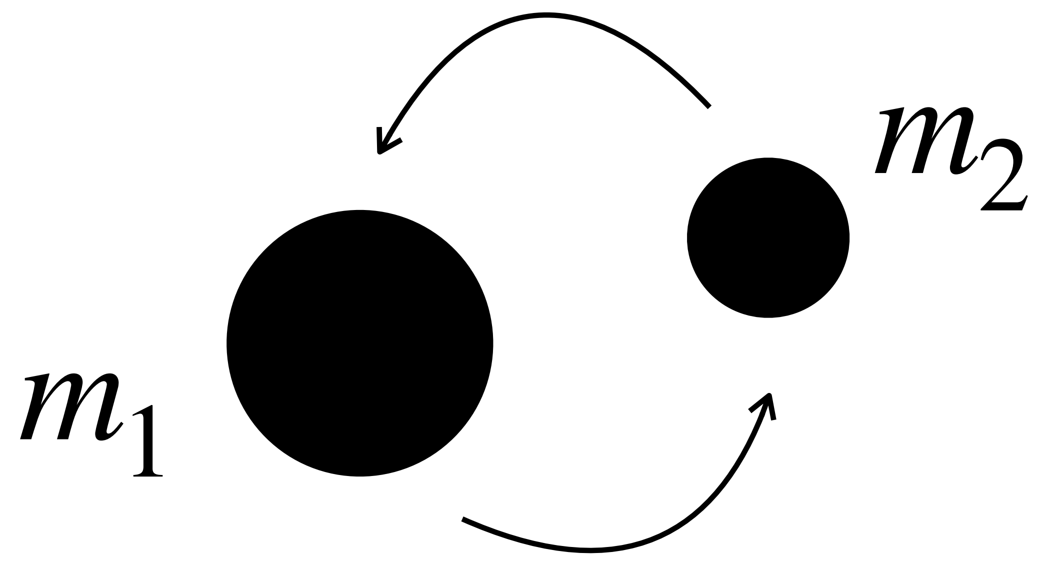
- $(2,2)$ is the fundamental mode
- $(l, m) \neq (2,2)$ are higher modes (HMs)



for quasi-circular BHs with equal masses $m_1 \simeq m_2$:

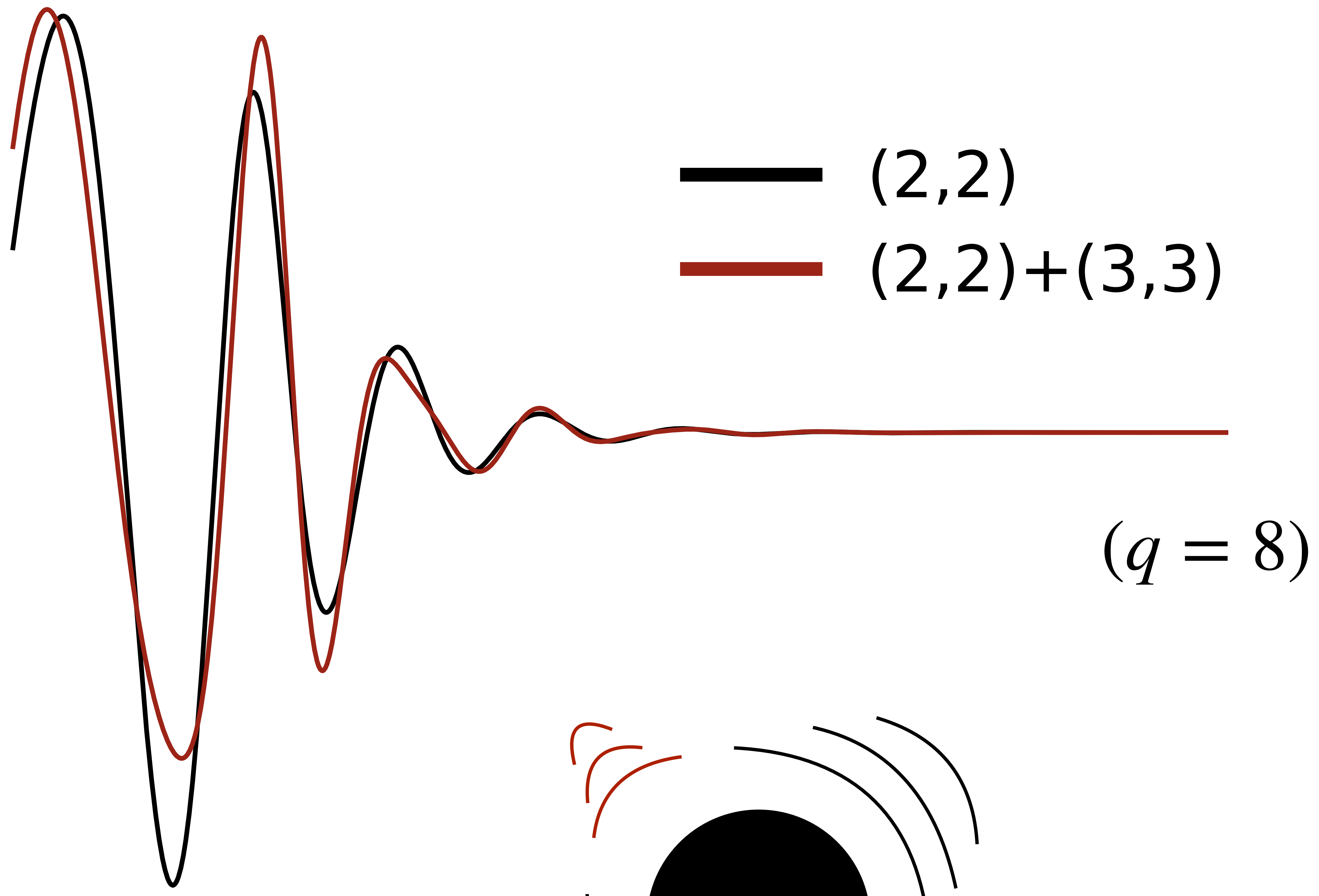
- dominant contribution $(2,2)$
- subdominant contribution $(3,3), (2,1), (4,4)$

HIGHER MODES II



HMs can be excited by :

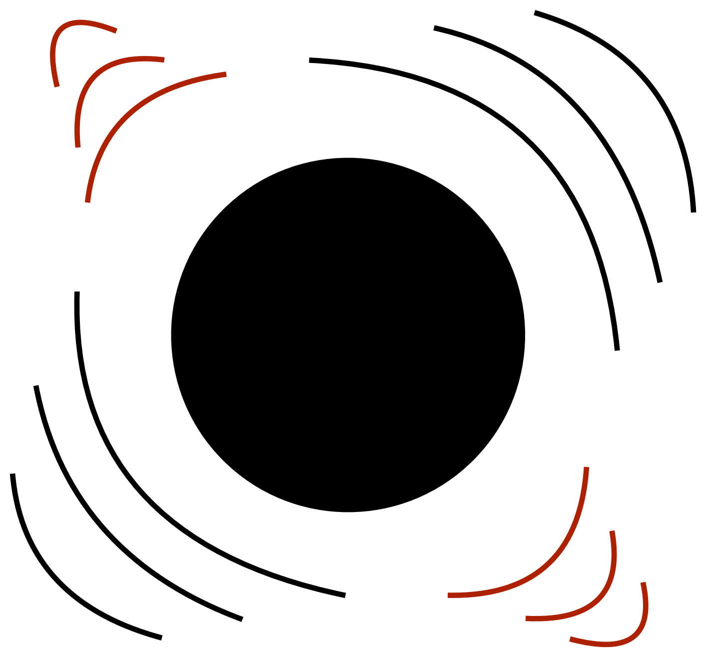
- increasing the mass ratio $q \equiv m_1/m_2$
- increasing the spins χ_1 and χ_2



are detectable?

first, how we can detect them

→ Bayes factor B

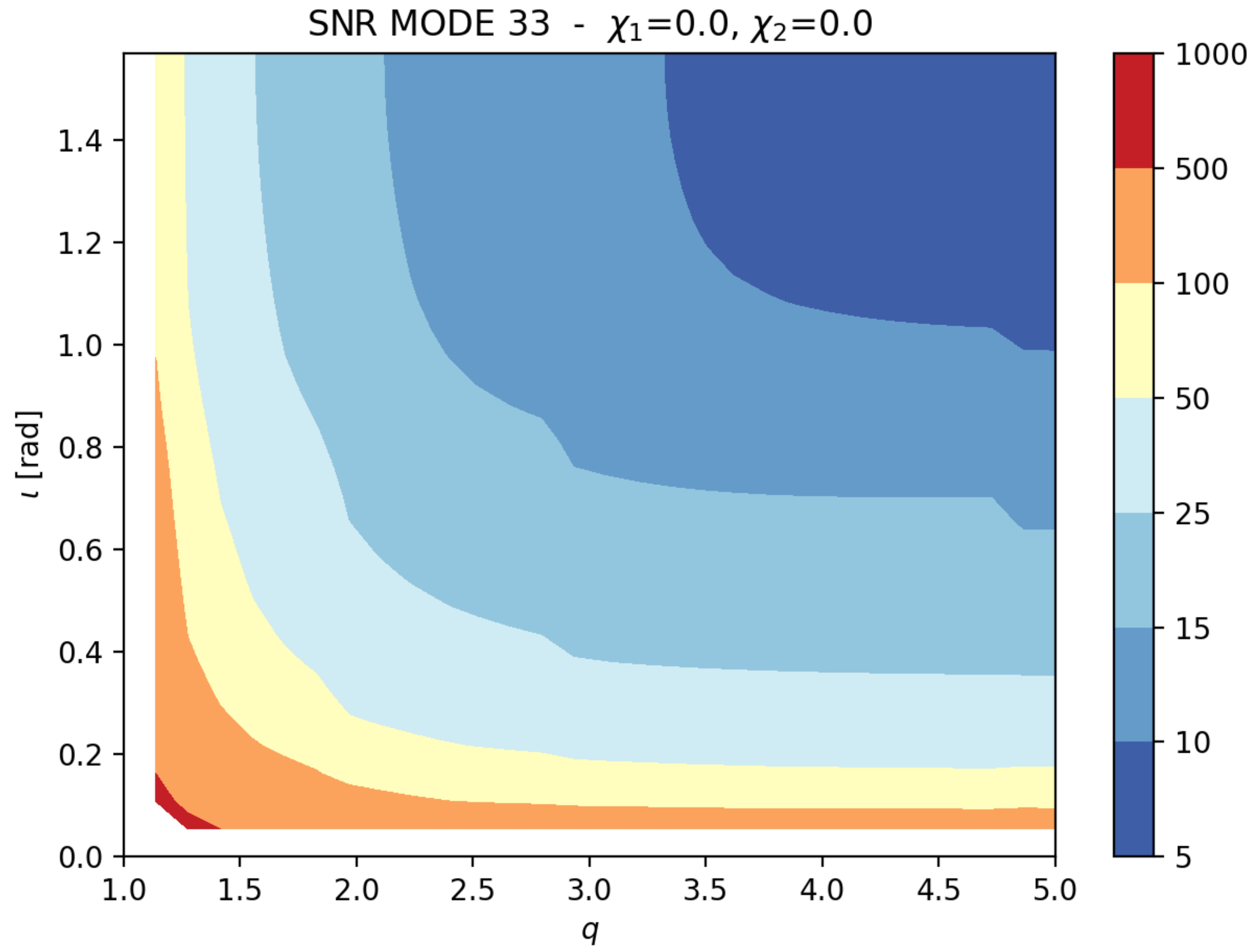


HMS DETECTABILITY II

$$\ln B \simeq \frac{1}{2}(1 - FF^2)\, SNR^2$$

SNR needed to detect the $(3,3)$

(with $\ln B = 5$)



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Bayes factor

fitting factor
(mismatch)

signal-to-noise ratio

SNR in RD for loud events is $\sim 10 - 15$

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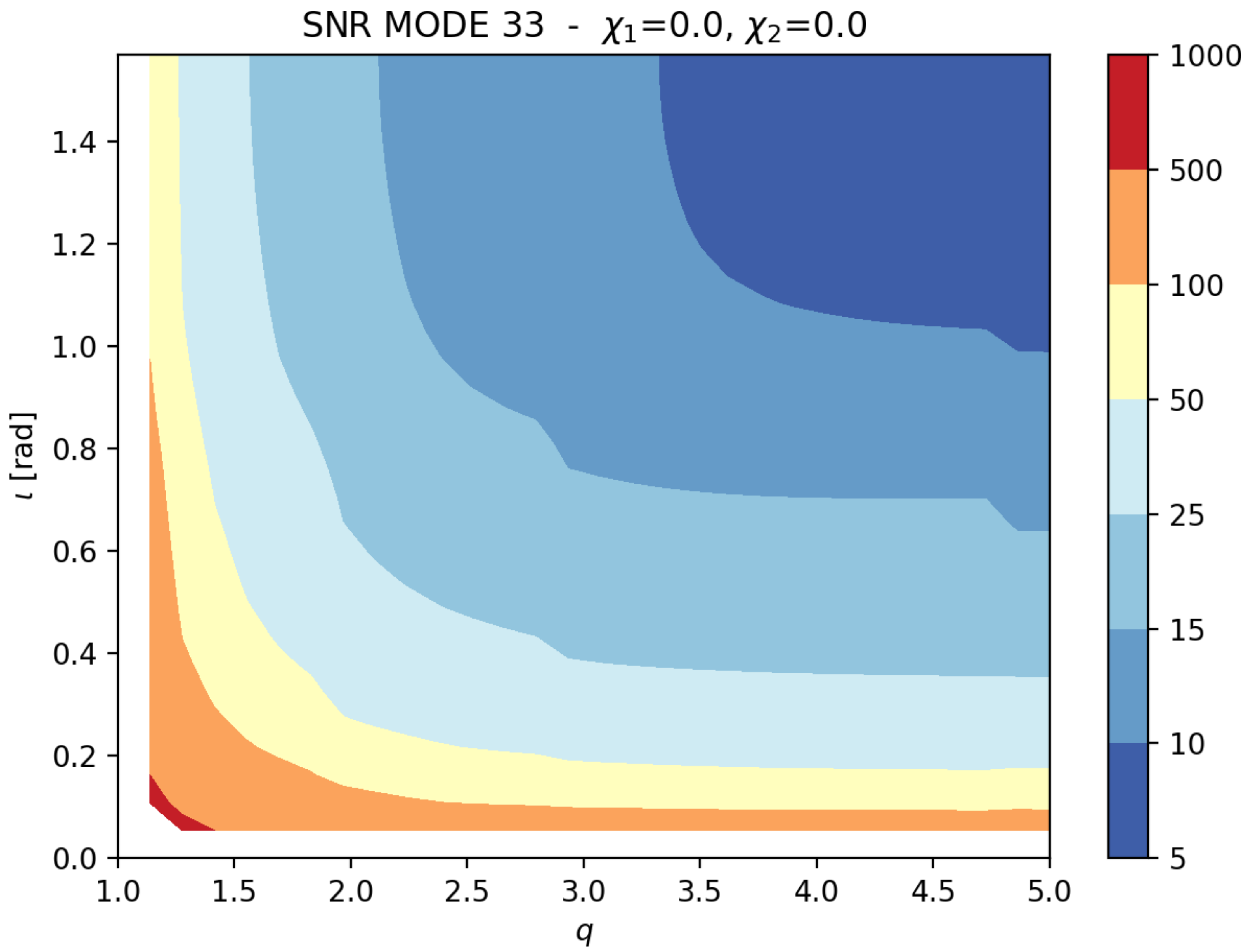

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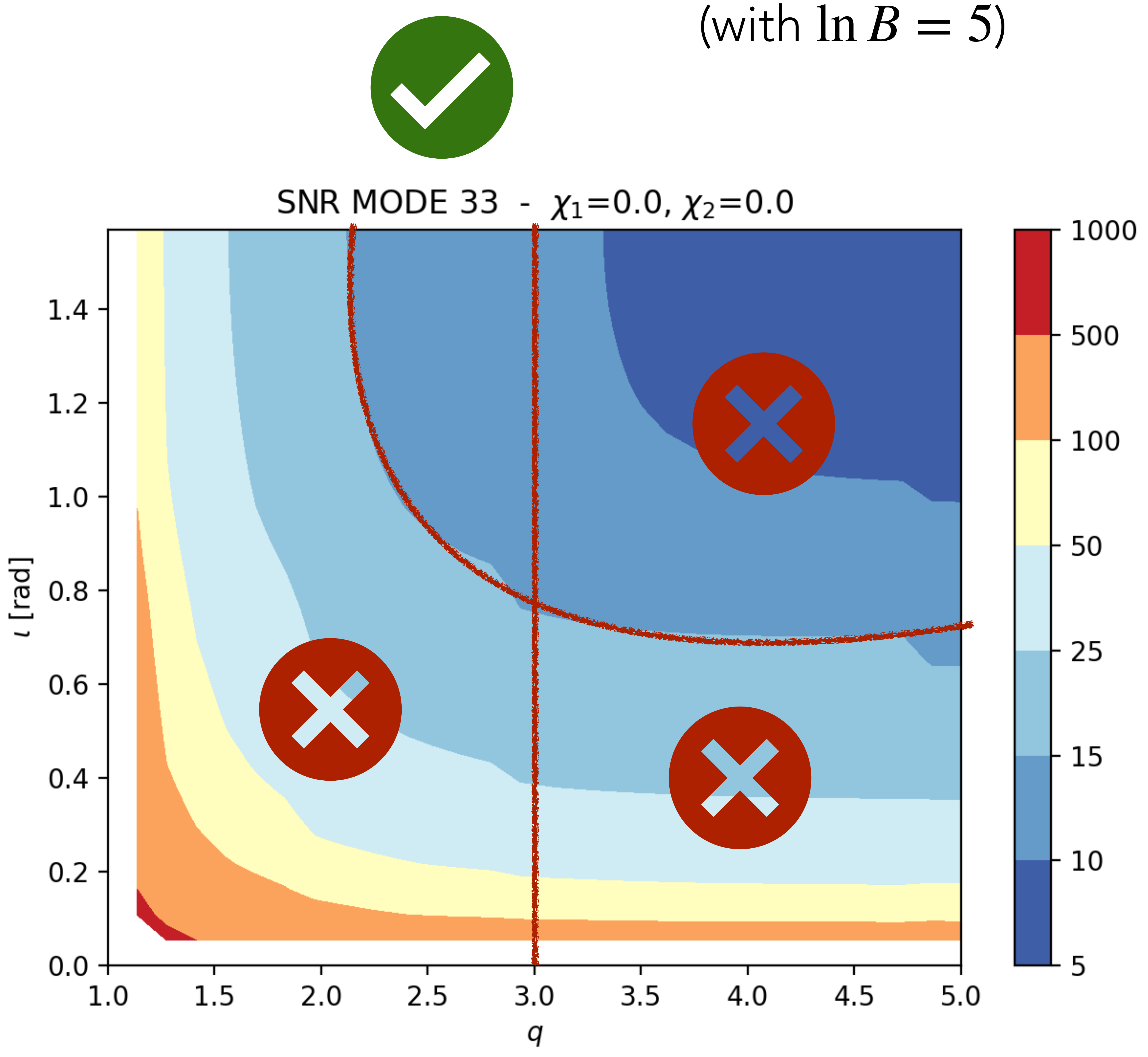

3. TEOBPM ANALYSIS ON GWTC-3

OBSERVATION OF HMS WITH TEOBPM

results are in general consistent with LVK

… except for one event GW190521B

GW150914

	m_1 $[\mathrm{M}_\odot]$	m_2 $[\mathrm{M}_\odot]$	d_L [Mpc]	$\mathrm{SNR}^{\mathrm{net}}_{\mathrm{opt}}$
LVK	$35.6^{+4.7}_{-3.1}$	$30.6^{+3.0}_{-4.4}$	440^{+150}_{-170}	$26.0^{+0.1}_{-0.2}$
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but RD is weakly measured

RD not informative

no HMs in GW190521A

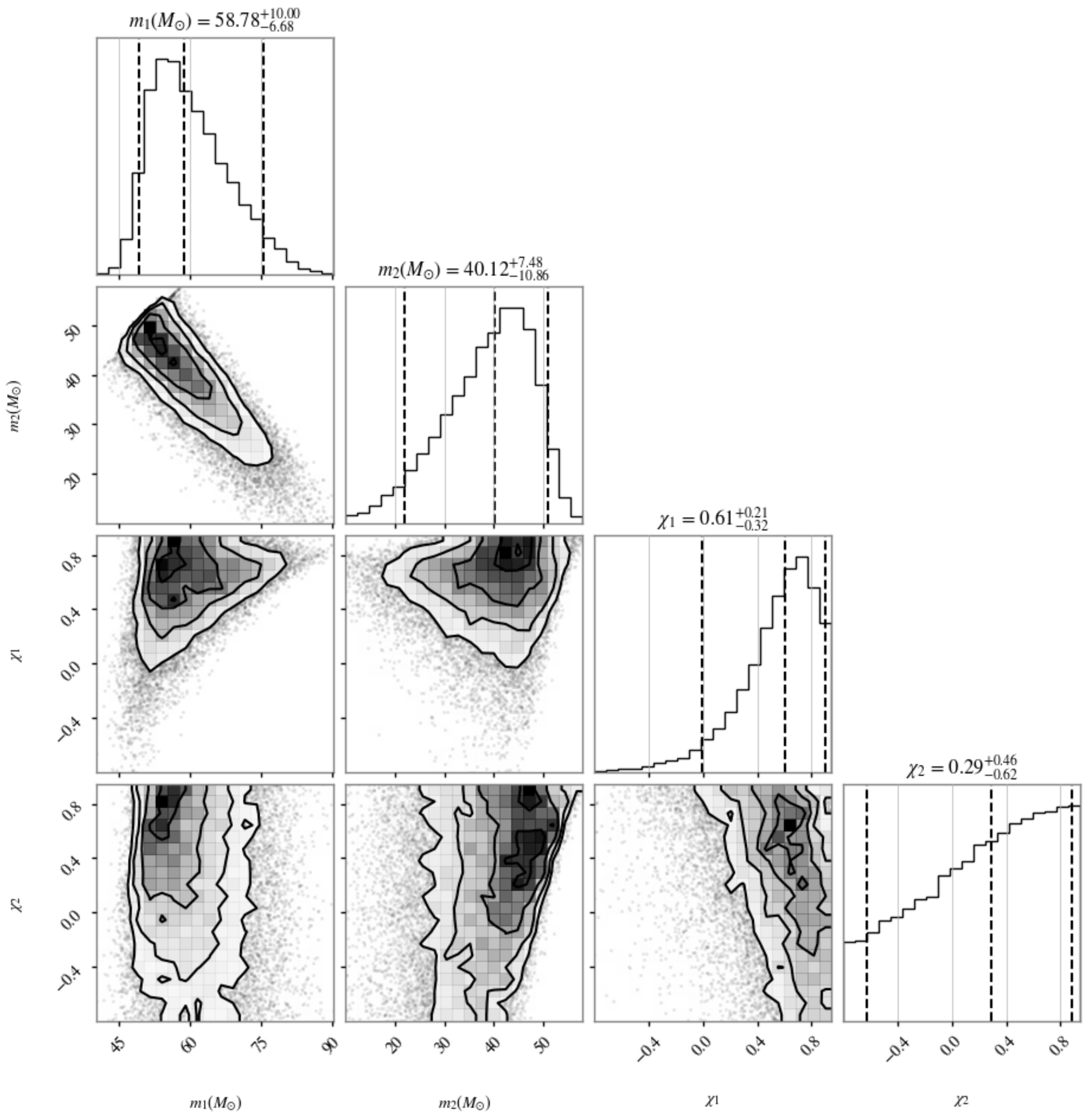
$\ln B_{33,22} = 0.13$

HMs in GW190521B

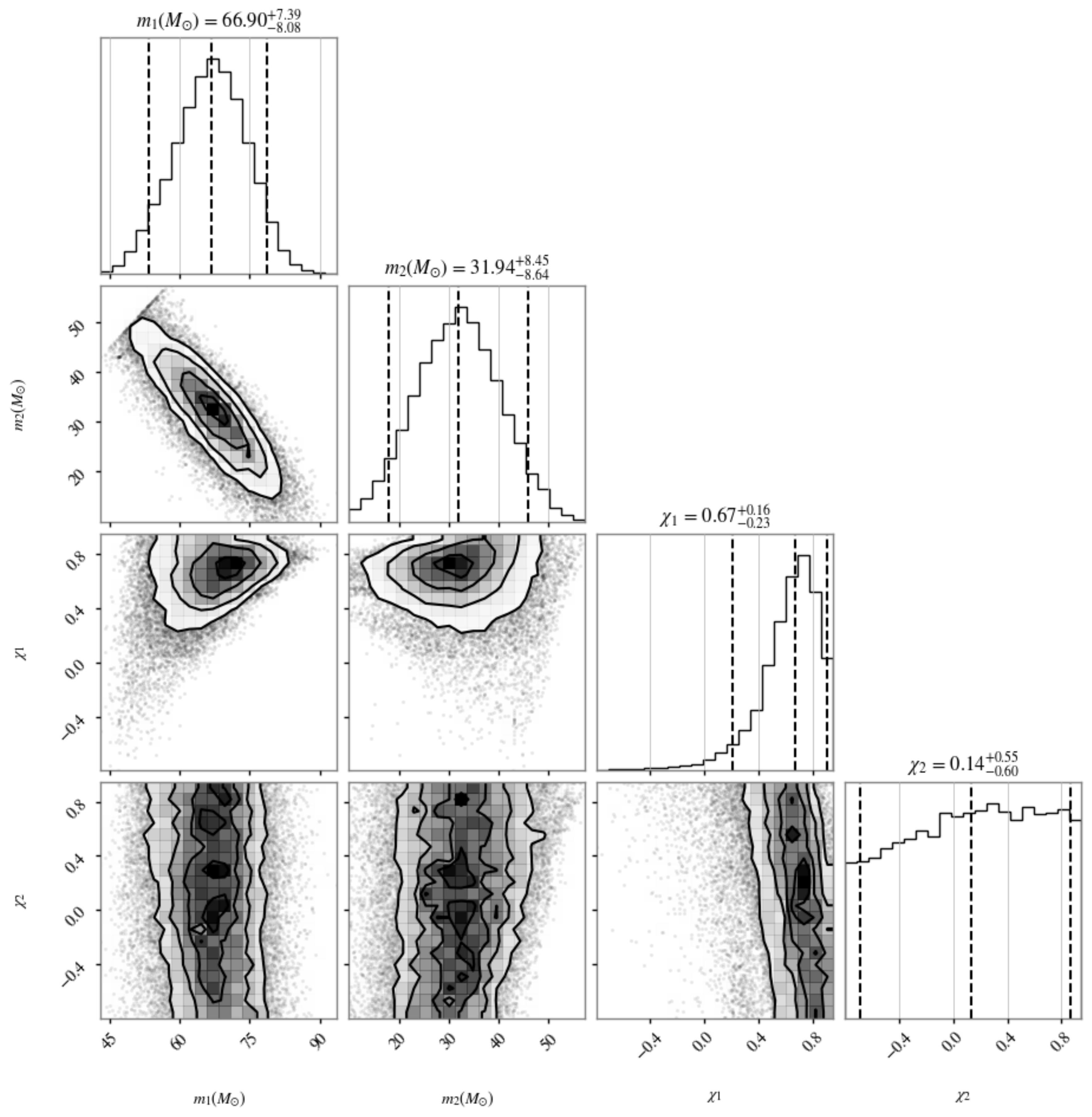
$\ln B_{33,22} = 2.03$

GW190521B

(2,2)



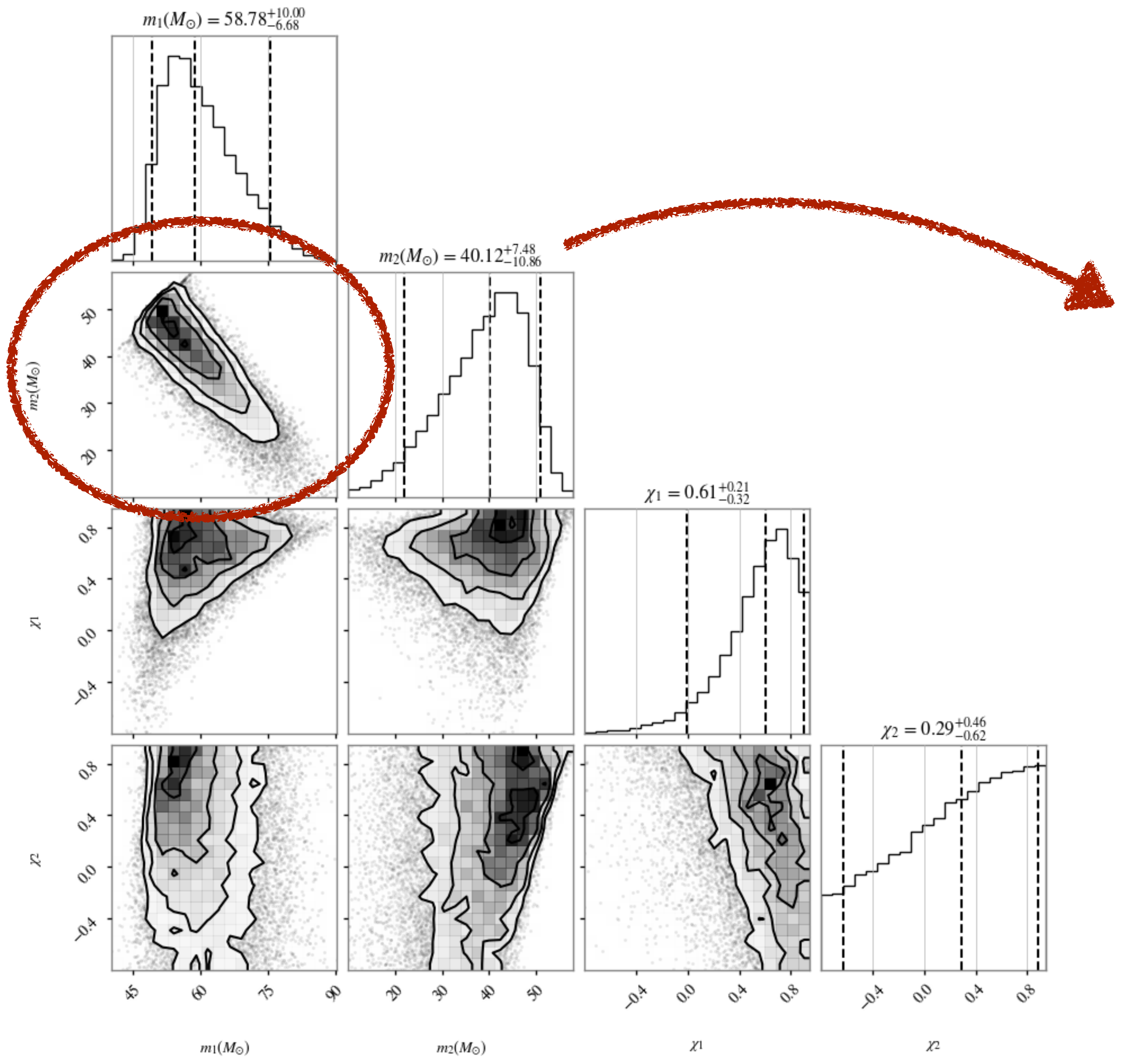
(2,2) + (3,3)



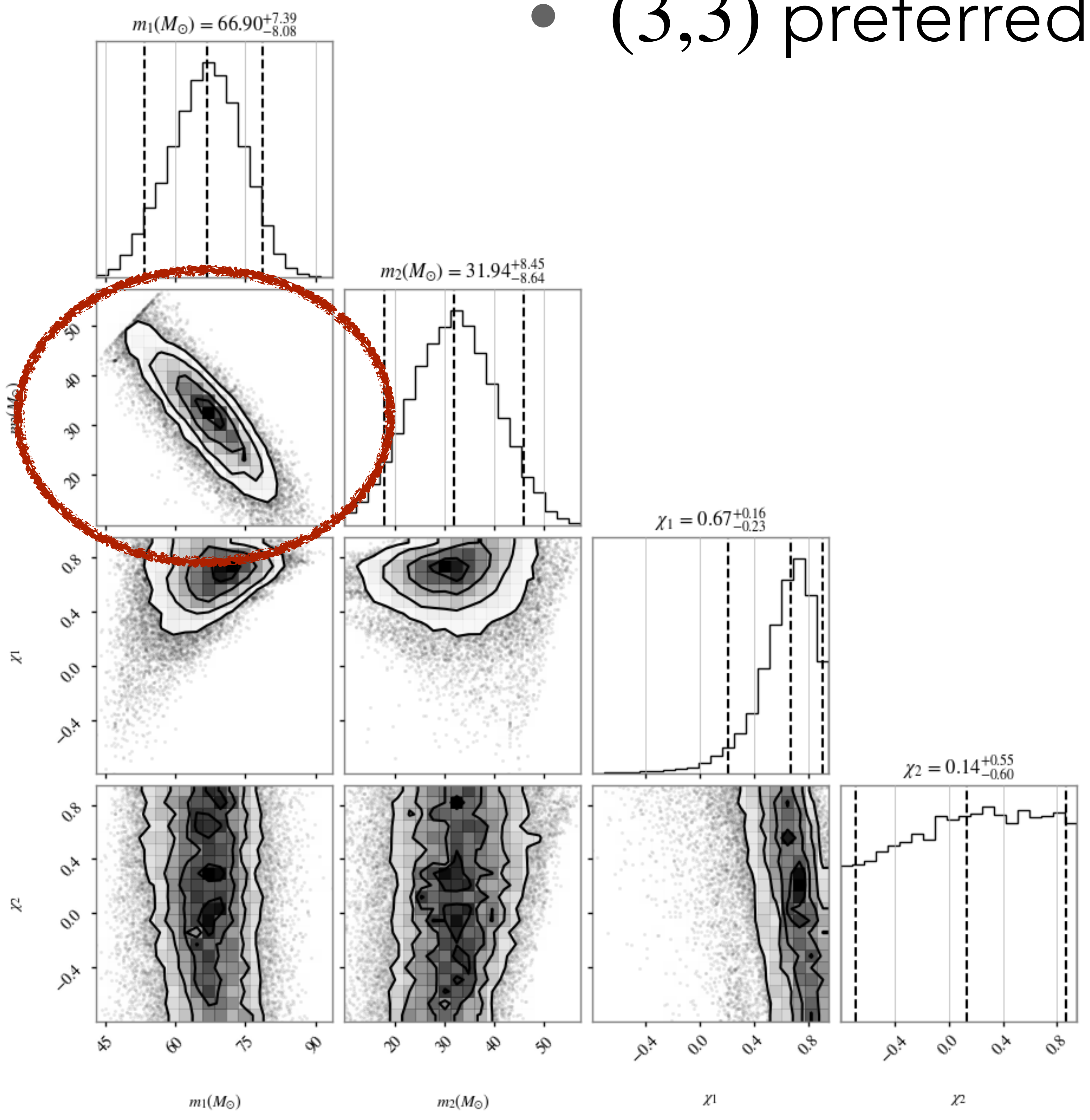
GW190521B

- change of posterior distributions
- (3,3) preferred with $B_{33,22} \simeq 8$

(2,2)



(2,2) + (3,3)

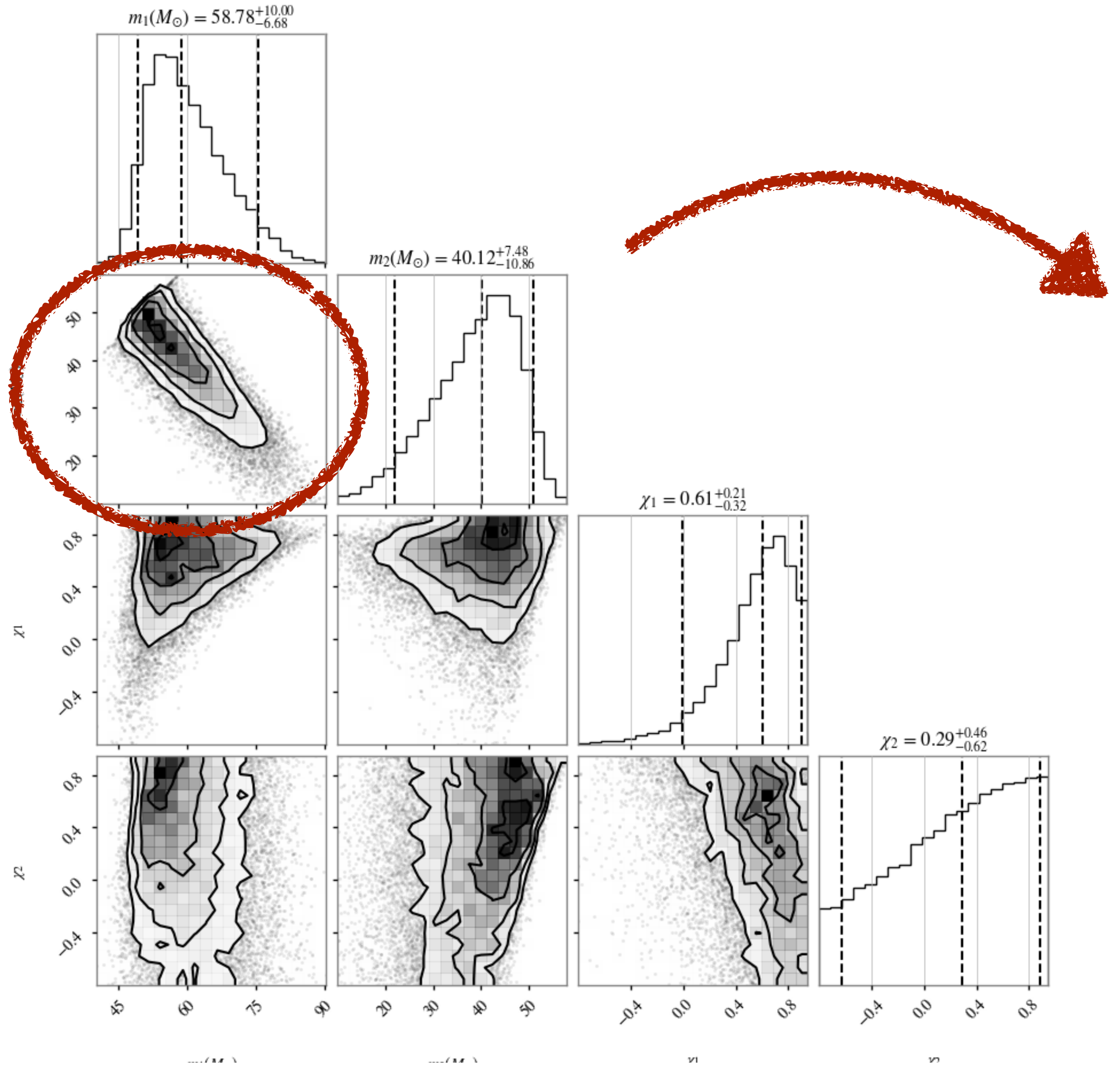


GW190521B

SNR needed to detect the (3,3) (with $\ln B = 2$)

- change of posterior distributions
- (3,3) preferred with $B_{33,22} \simeq 8$

(2,2)



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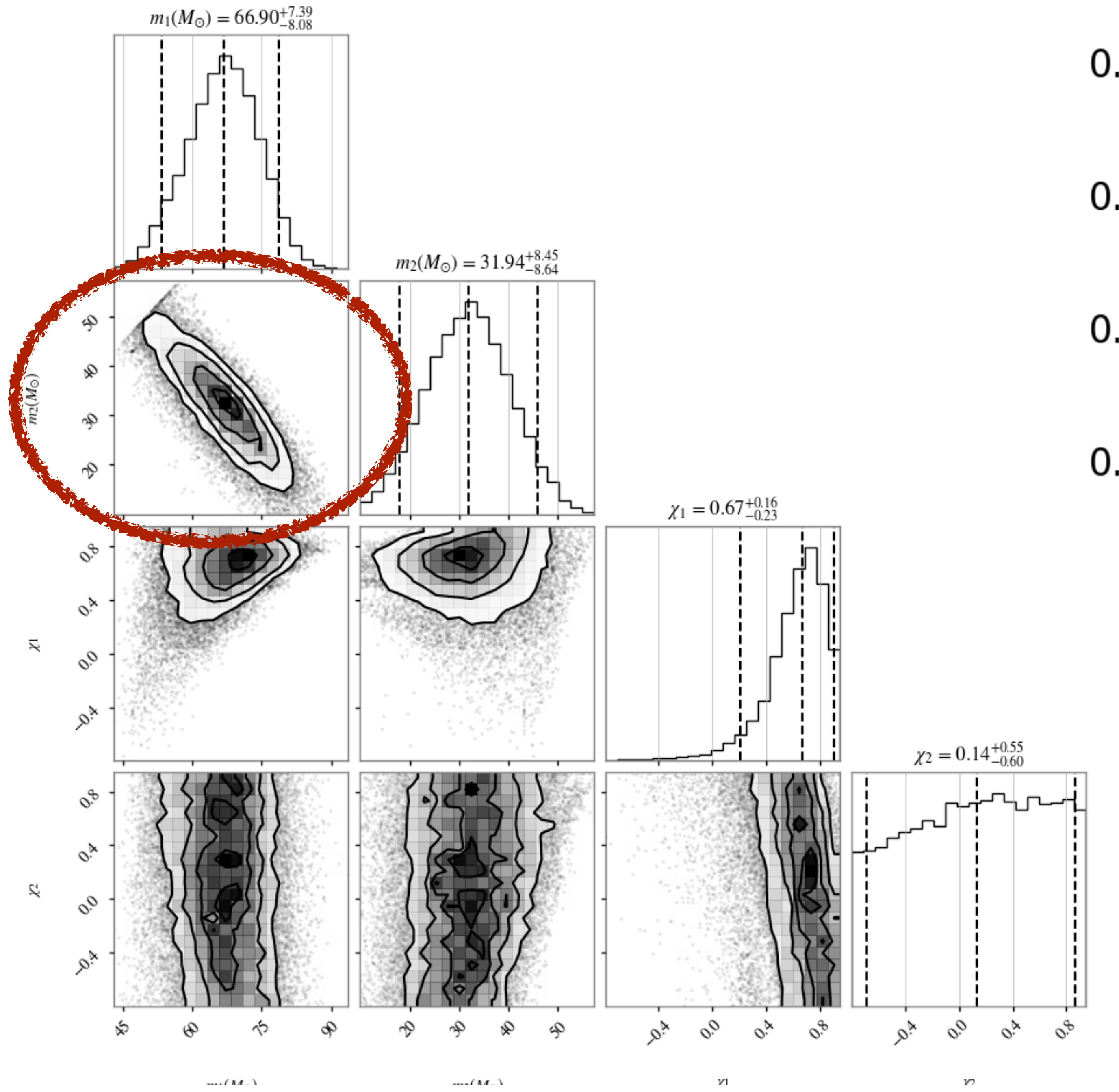
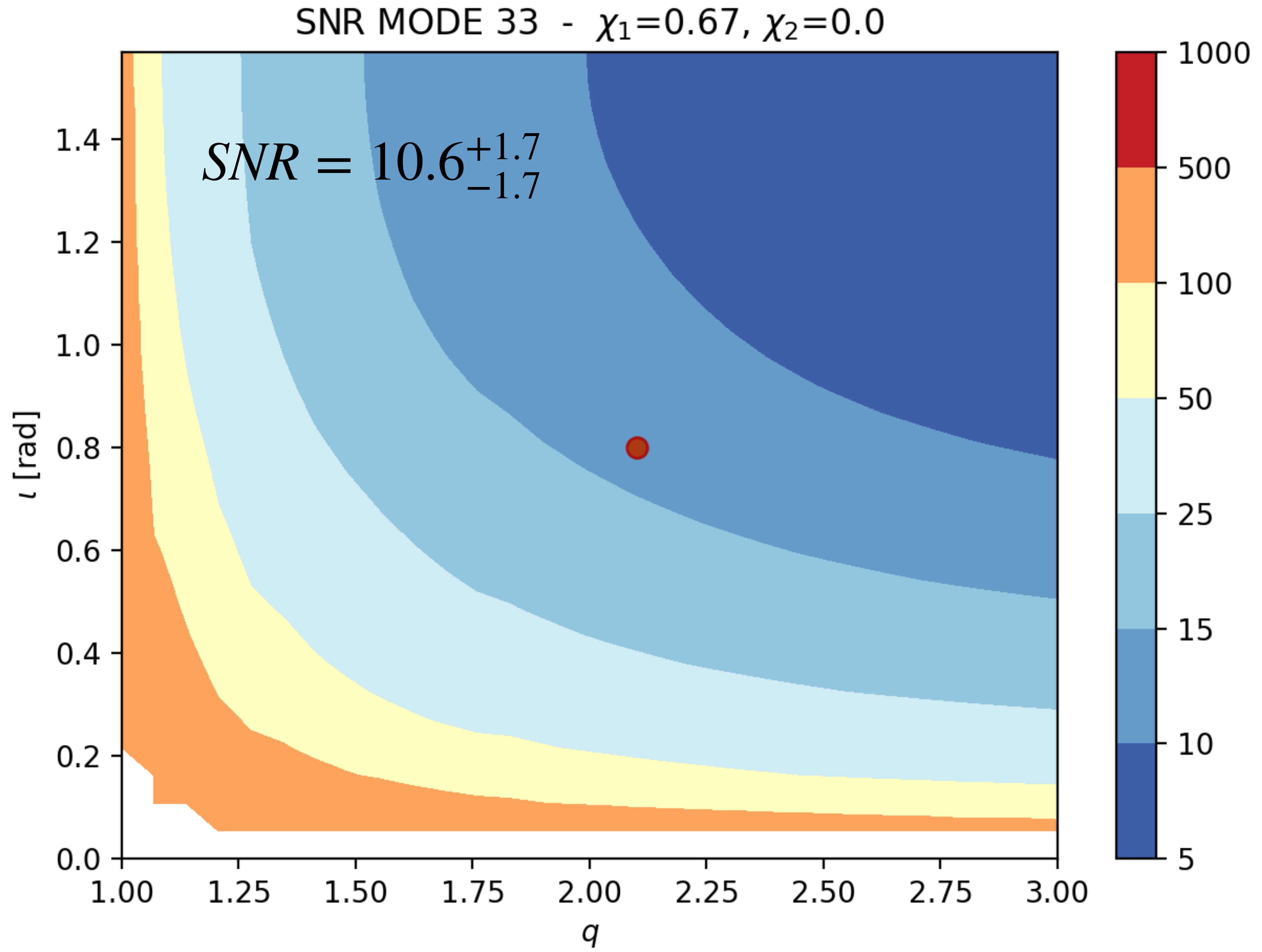




considering large uncertainties, agreement with predictions ✔

SUMMARY

- ringdown is the sum of different quasinormal modes of vibration, characterised by ω_{lm} and τ_{lm}
- modelling the amplitudes on NR, the TEOBPM model is particularly suited to study higher modes
- higher modes can be detected if the system has high mass-ratio or inclinations
- we have verified TEOBPM over GWTC-3 and found results consistent with previous analyses
- marginal evidence of the mode (3,3) on one event, but further studies are needed
- observing higher modes is crucial to test no-hair theorem and predictions from GR

BACKUP SLIDES

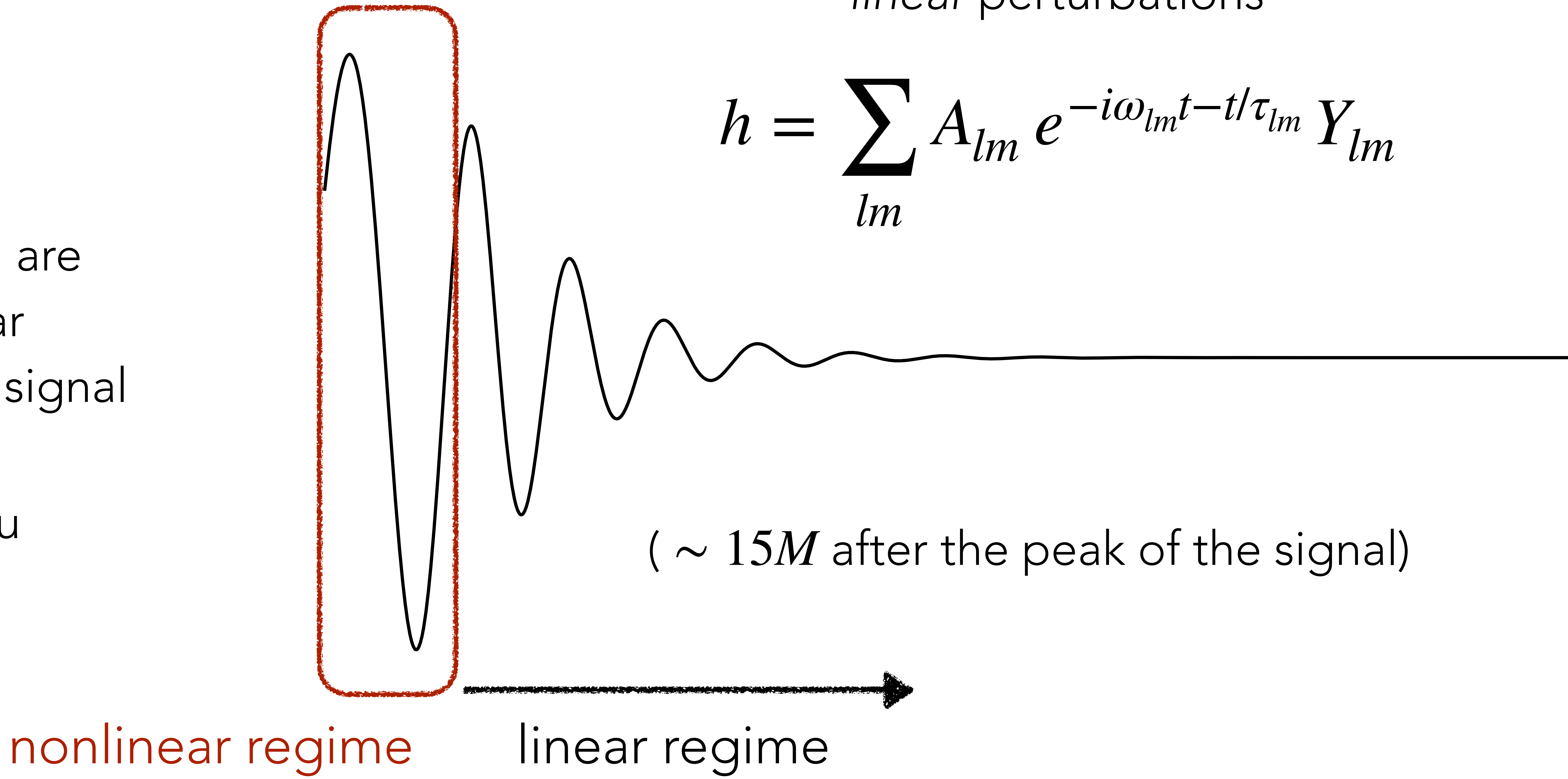
RD STARTING TIME

when the RD starts?

- if you start too late, you are sure to work in the linear regime, but lose all the signal
- if you start too early, you apply a linear model to nonlinear data

linear perturbations

$$h = \sum_{lm} A_{lm}\, e^{-i\omega_{lm}t - t/\tau_{lm}}\, Y_{lm}$$



($\sim 15M$ after the peak of the signal)

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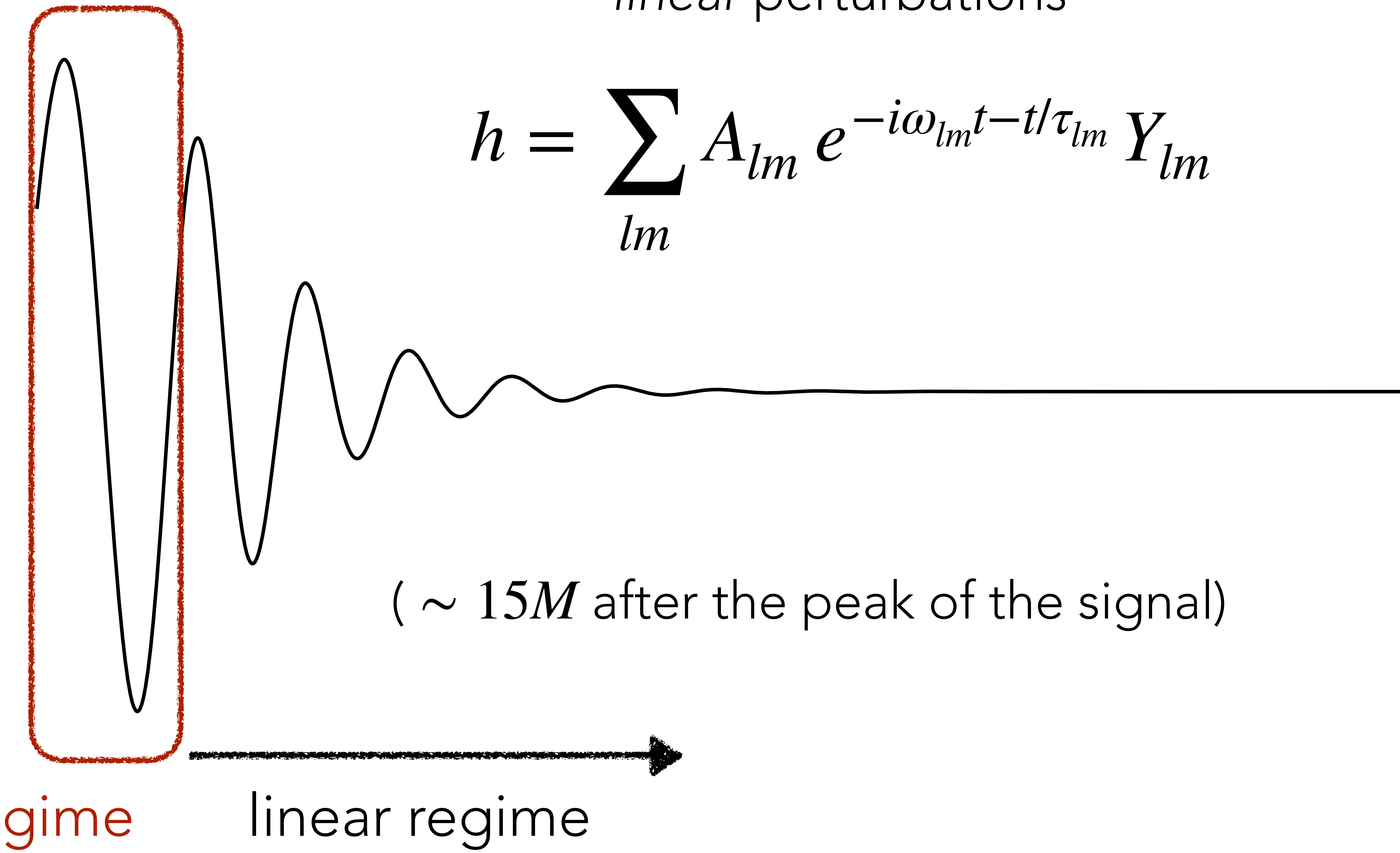


($\sim 15M$ after the peak of the signal)

difficult to choose the starting time → results depend on the starting time

TEOBPM MODEL

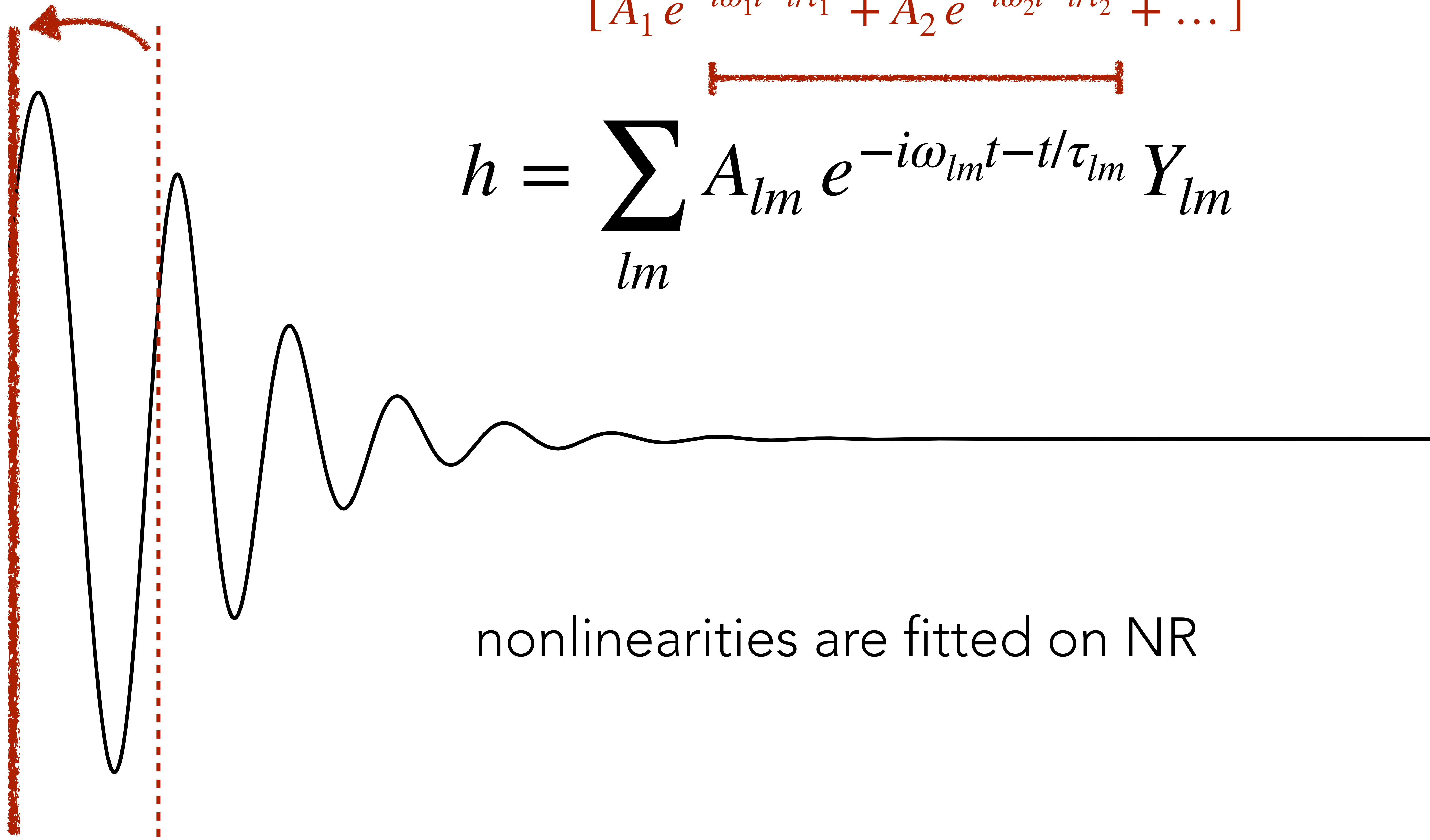
- effective one-body models can model early times nonlinearities
- the RD is expressed in terms of the progenitors m_1, m_2, χ_1, χ_2

peak of the WF

TEOBPM

$$[\, A_1\, e^{-i\omega_1 t - t/\tau_1} + A_2\, e^{-i\omega_2 t - t/\tau_2} + \dots \,]$$

$$h = \sum_{lm} A_{lm}\, e^{-i\omega_{lm} t - t/\tau_{lm}}\, Y_{lm}$$



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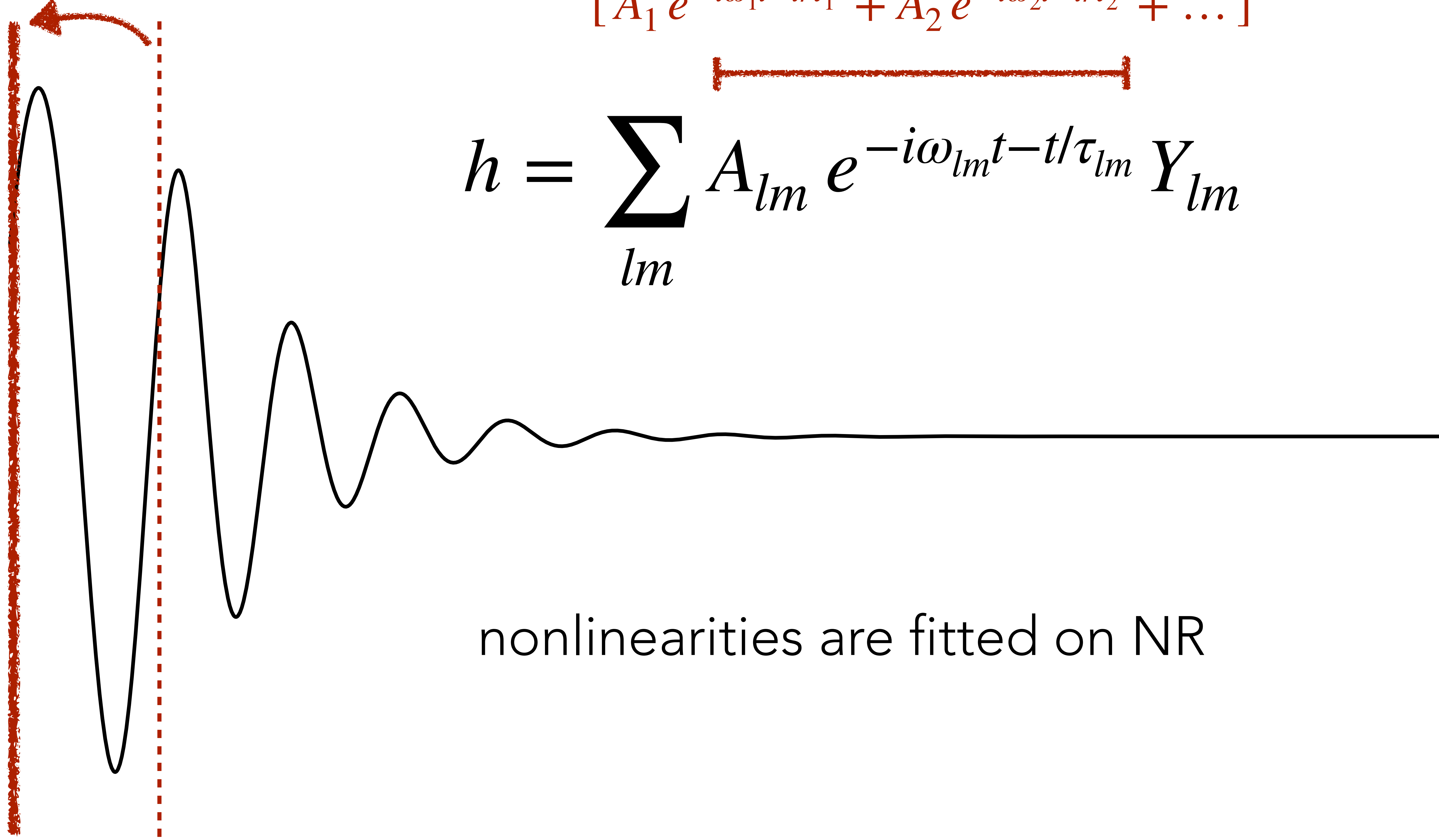


$$h = \sum_{lm} A_{lm}\, e^{-i\omega_{lm}t - t/\tau_{lm}}\, Y_{lm}$$

nonlinearities are fitted on NR

advantages:

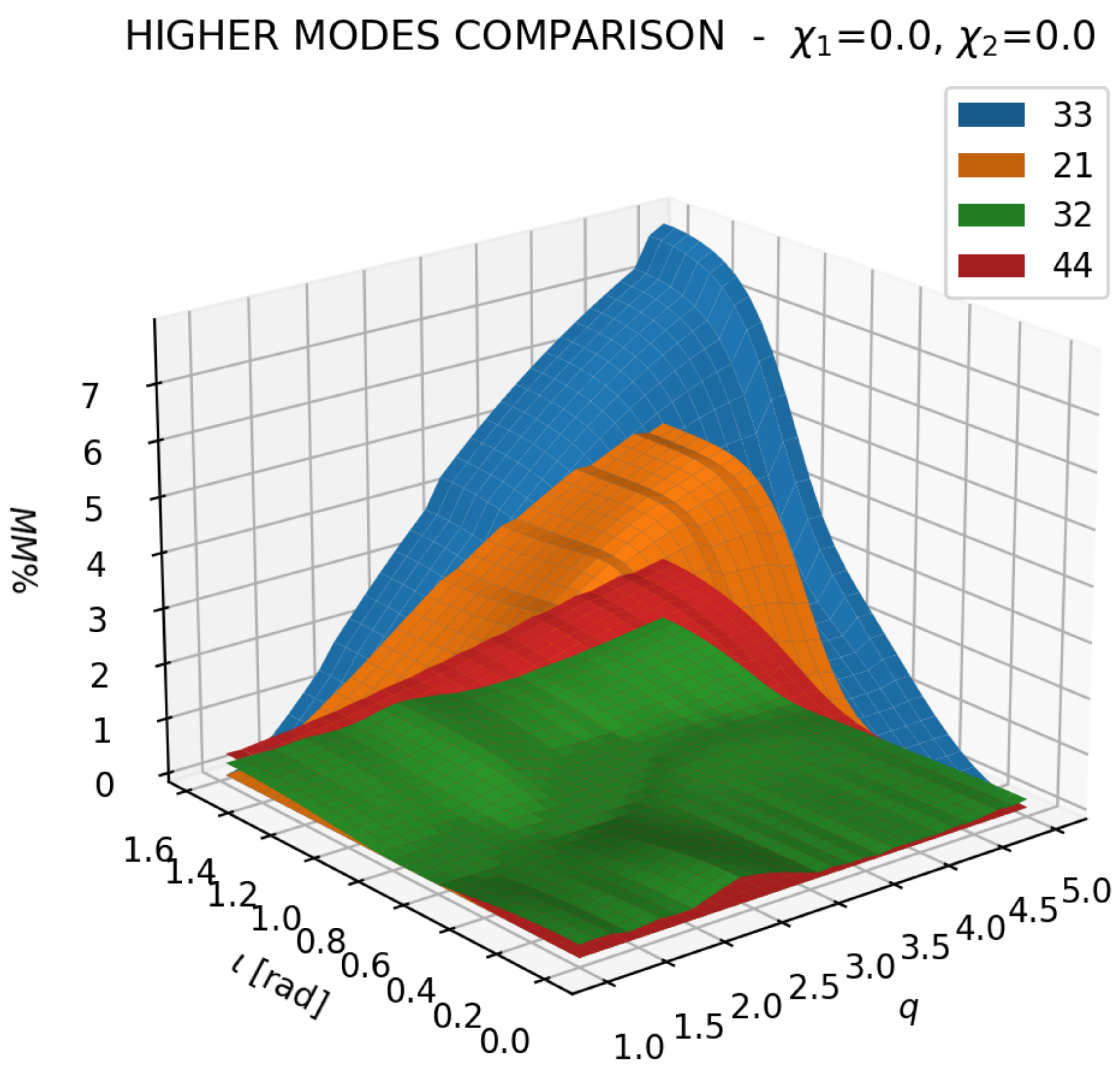
- fix the problem of the starting time
- use more data with high SNR

→

- unambiguous results
- more accurate results

($\sim 20\,\%$ more SNR)

HMS MISMATCH



we quantify the contribution of HMs (l, m) wrt the (2,2) through the mismatch

$$\mathrm{FF} \equiv \max_{\boldsymbol{\theta} \in \Theta_{lm}} \left\{ \frac{\langle \mathbf{h}_{lm} | \mathbf{h}_{22} \rangle^2}{\langle \mathbf{h}_{lm} | \mathbf{h}_{lm} \rangle \langle \mathbf{h}_{22} | \mathbf{h}_{22} \rangle} \right\}, \quad \mathrm{MM} = 1 - \mathrm{FF},$$

- (3,3) is typically dominant
- for $q \sim 2$ and $\iota \sim 1$, the (3,3) is $\sim 1\,\%$ of the (2,2) in the RD

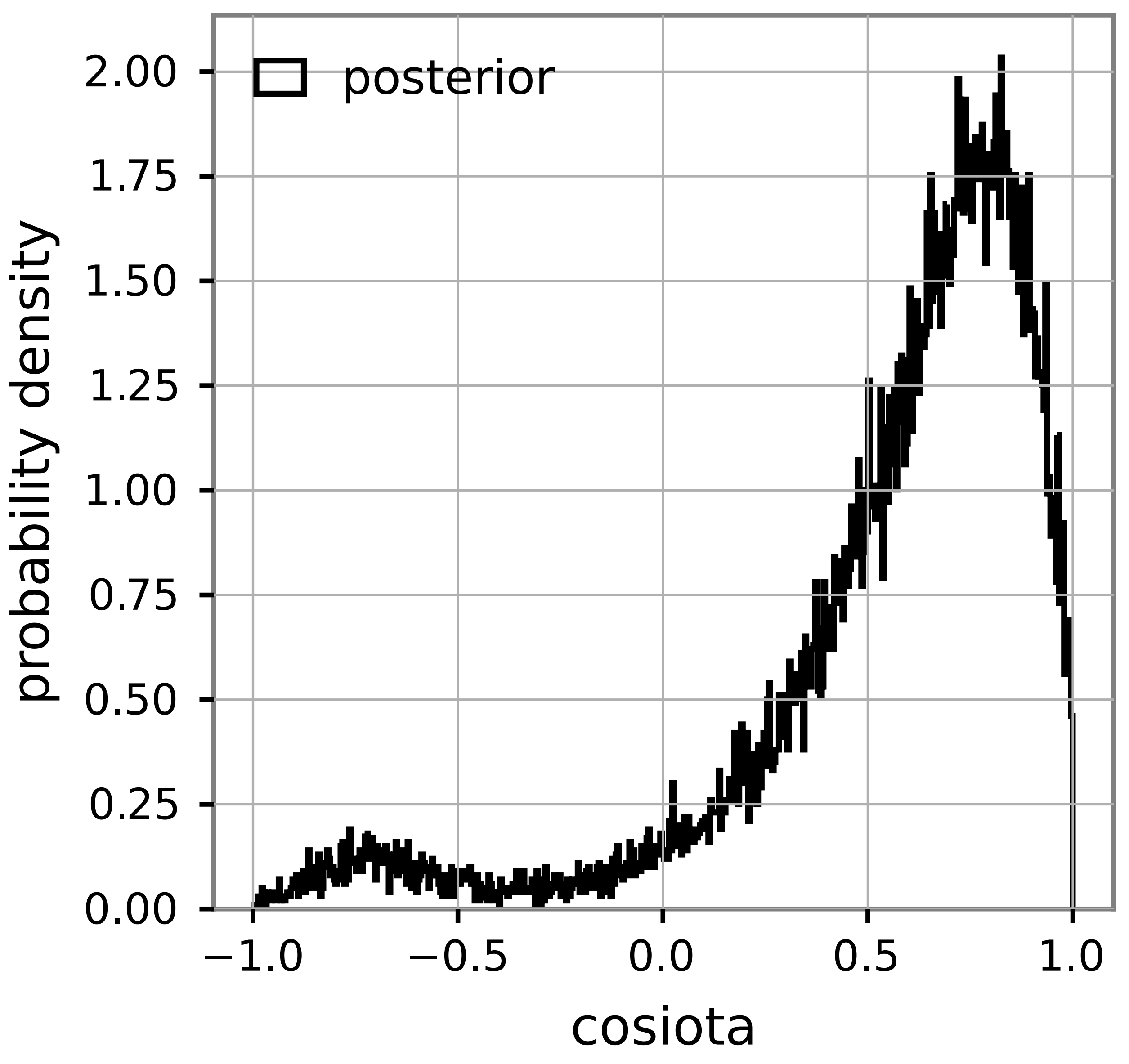
GW190521B PARAMETER ESTIMATION

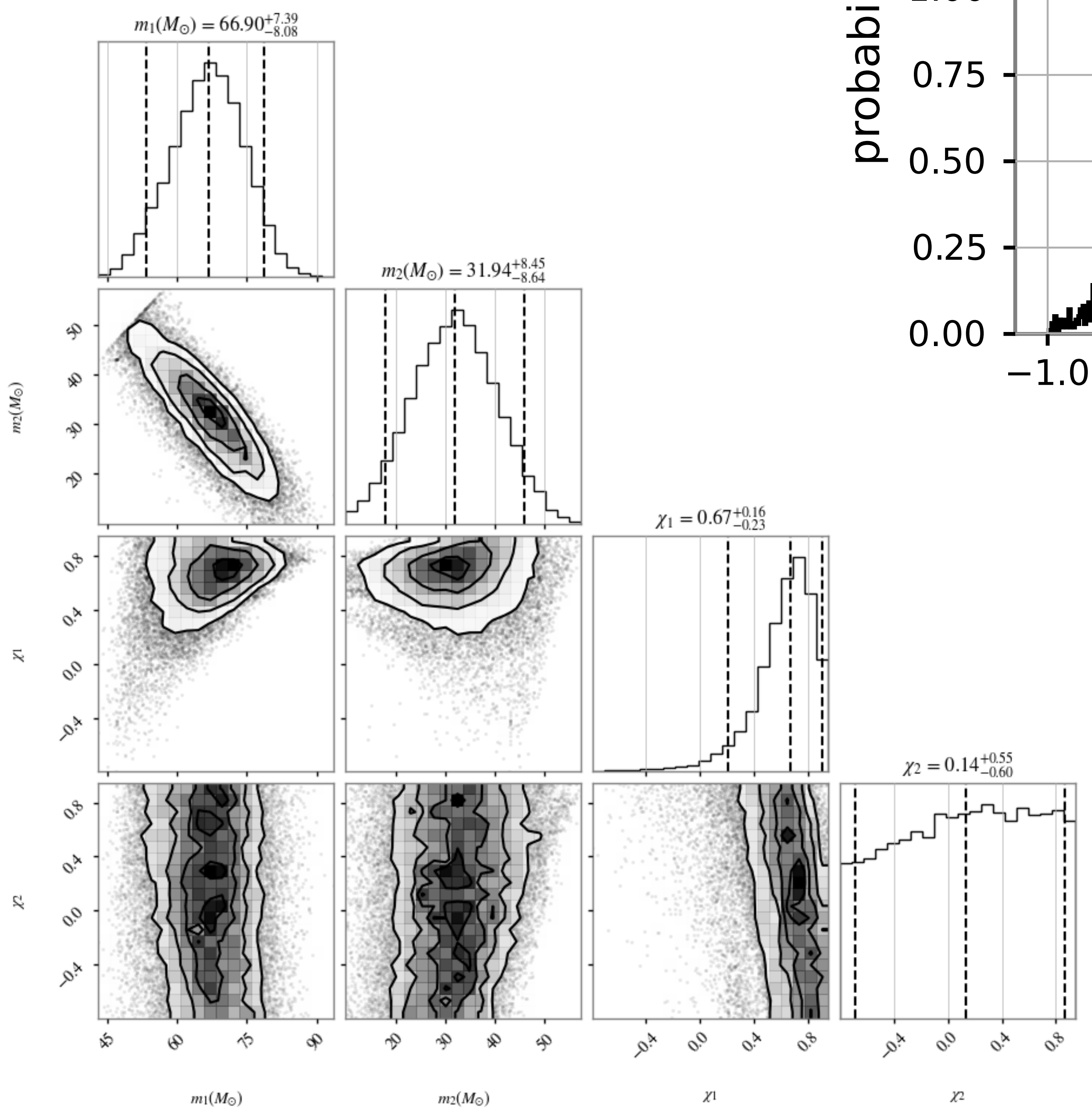
parameter estimation:

- $m_1 = 66.9^{+11.9}_{-13.4}\, M_\odot$
- $m_2 = 31.9^{+13.8}_{-14.0}\, M_\odot$

$q \sim 2$

- $\iota = 0.84^{+1,37}_{-1.19}$
- $\chi_1 = 0.67^{+0.24}_{-0.46}$
- $SNR = 10.6^{+1.7}_{-1.7}$





4. TESTS OF NO-HAIR

TEST OF NO-HAIR THEOREM I

why are **higher modes** important?

TEST OF NO-HAIR THEOREM I

BHs have no hairs

why are higher modes important?

recall that ω_{lm} and τ_{lm} are determined by only M and a of the final BH

TEST OF NO-HAIR THEOREM I

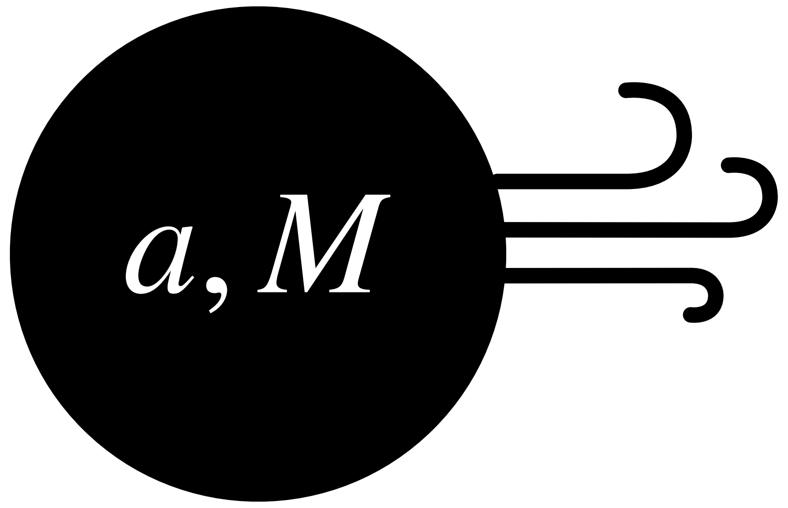
BHs have no hairs

why are higher modes important?

recall that ω_{lm} and τ_{lm} are determined by only M and a of the final BH

↓

every additional hair of the BH will change the values of ω_{lm} and τ_{lm}



TEST OF NO-HAIR THEOREM I

BHs have no hairs

why are higher modes important?

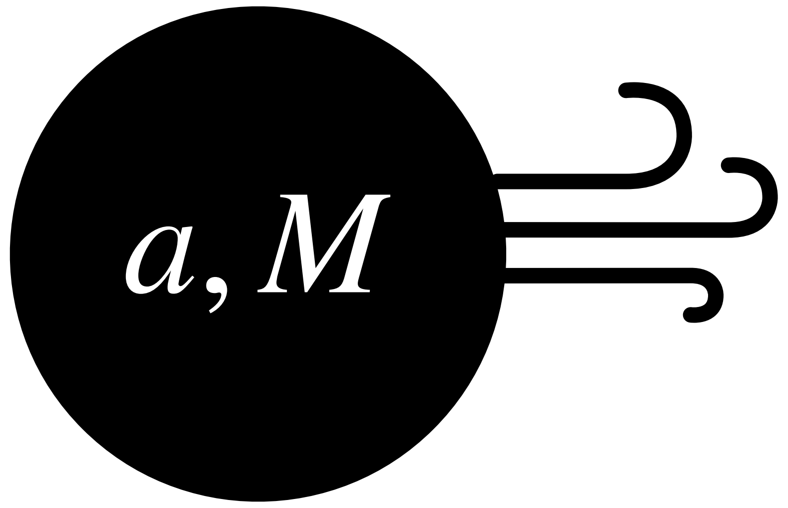
recall that ω_{lm} and τ_{lm} are determined by only M and a of the final BH

↓

every additional hair of the BH will change the values of ω_{lm} and τ_{lm}

↓

by measuring ω_{lm} and τ_{lm} we can test the predictions of general relativity



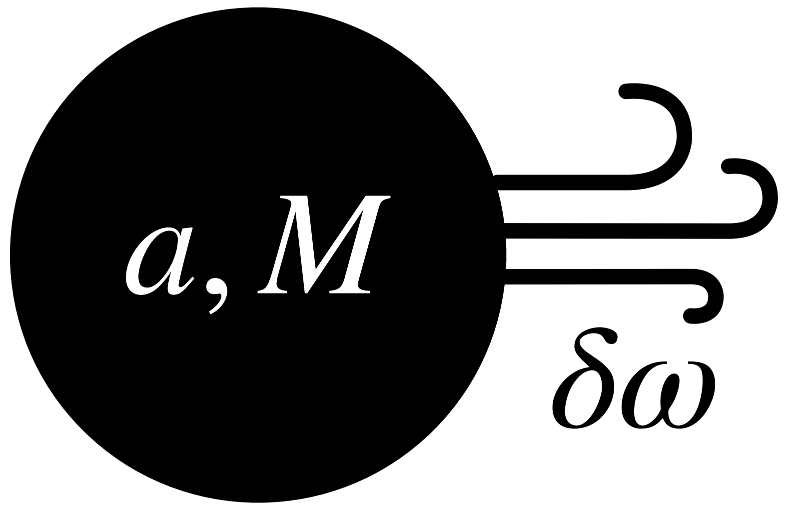
test of no-hair theorem

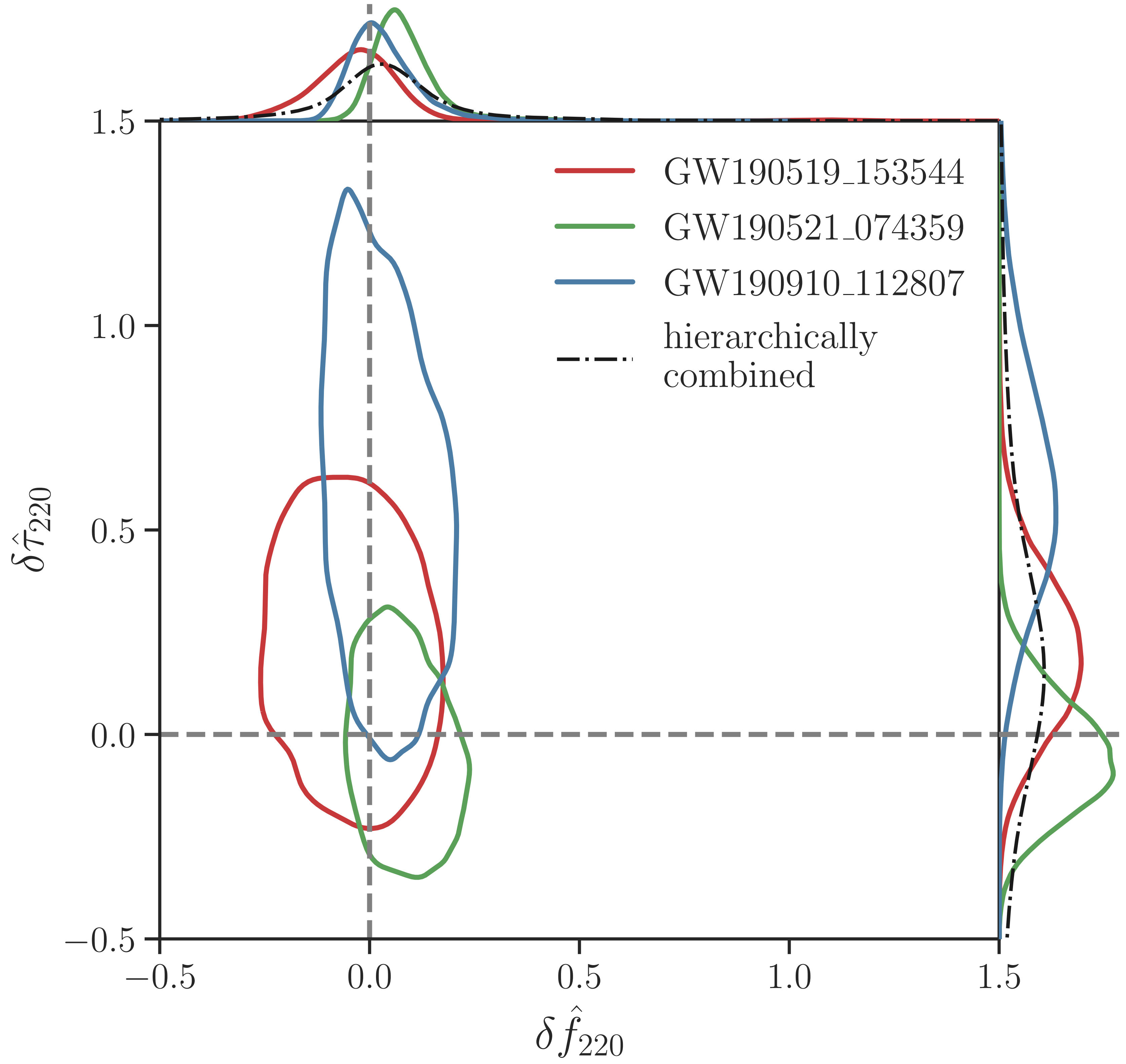
TEST OF NO-HAIR THEOREM II

consider fractional deviations from GR

$$\omega_{lm} = \omega_{lm}^{GR}(1 + \delta\omega_{lm})$$

$$\tau_{lm} = \tau_{lm}^{GR}(1 + \delta\tau_{lm})$$





TEST OF NO-HAIR THEOREM II

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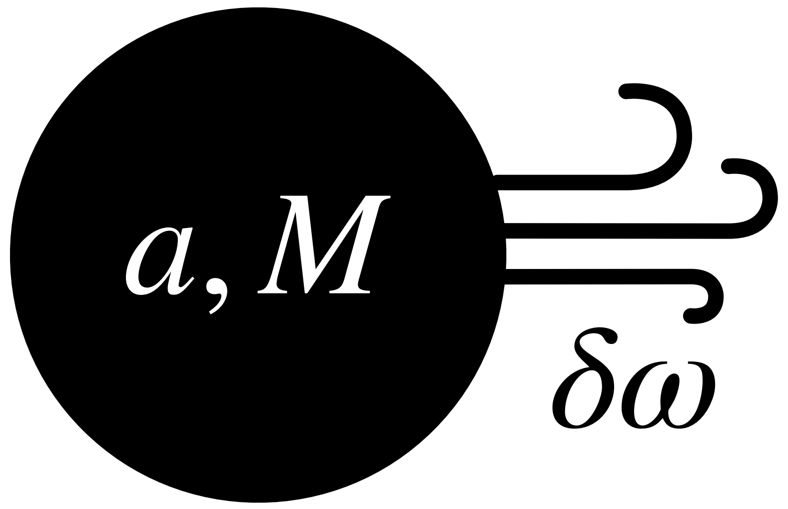

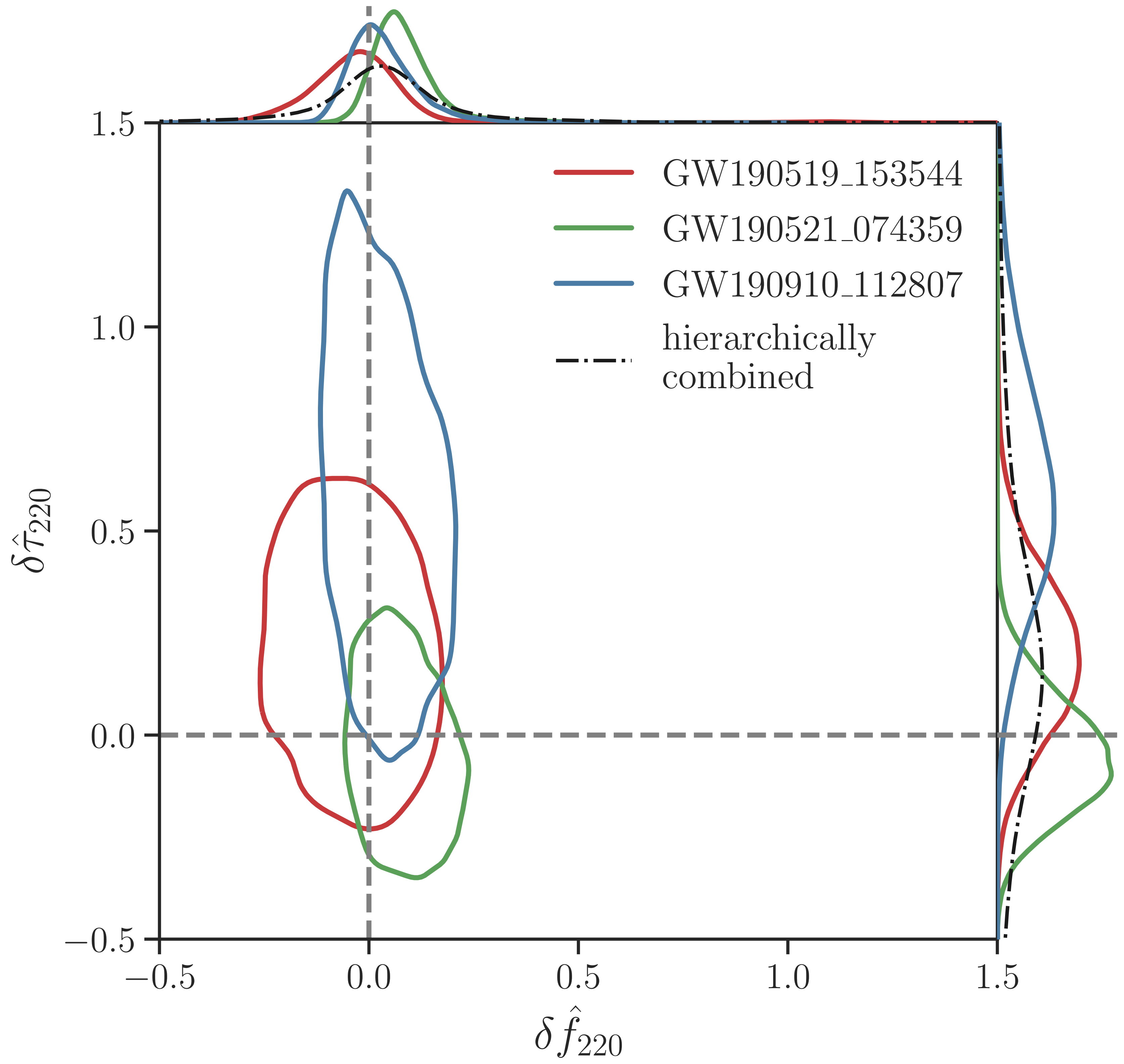

with the measurement of 2 modes we can:

- use one mode to set M and a
- use the second to compare predictions of GR → authentic test of no-hair