Third Gravi-Gamma Workshop

A Bayesian approach to pulsar timing and its applications to multi-messenger astrophysics

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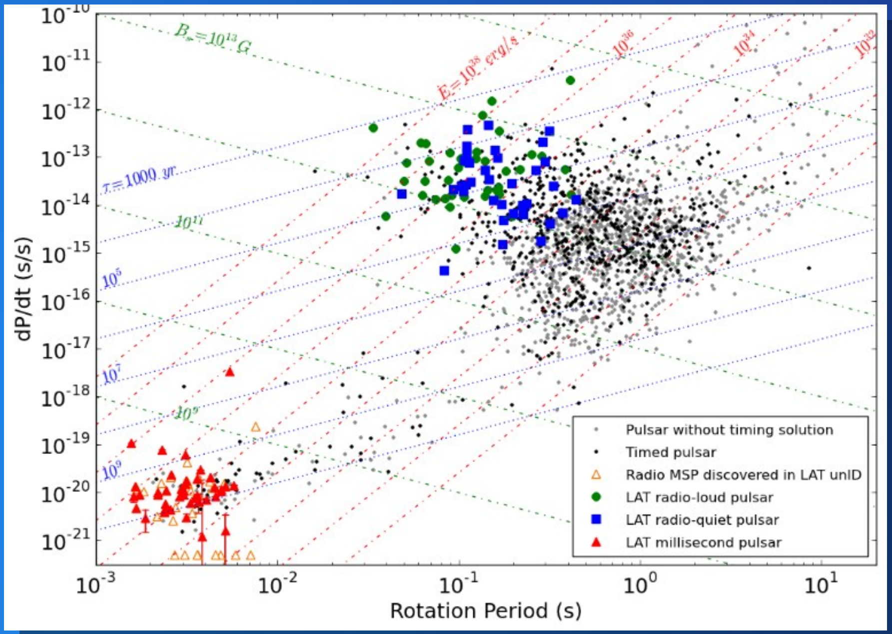
An overview on pulsars

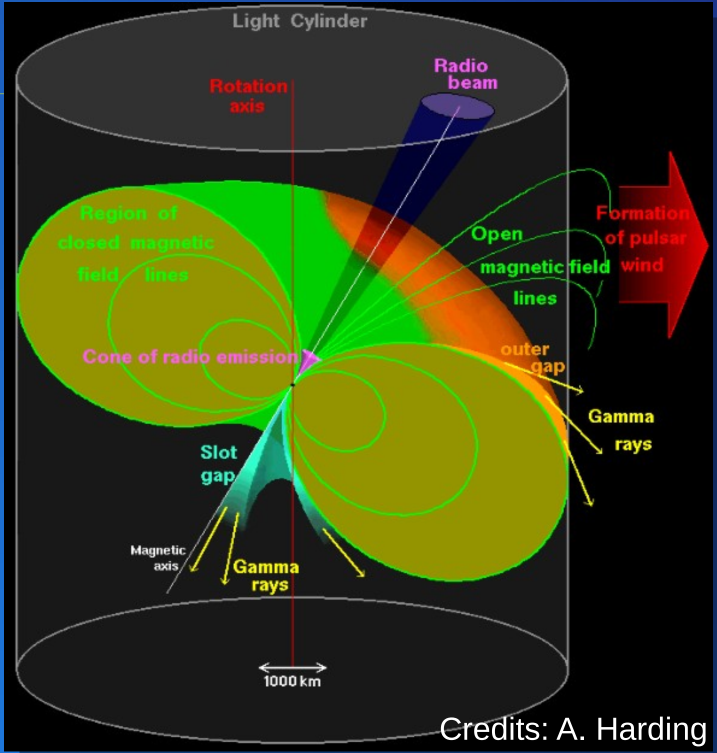
- Highly magnetized and rapidly spinning neutron stars (NSs, Pacini, 1967)
- First discovered as sources of pulsed radio signals (Hewish et al., 1968)
- **Multiwavelength emission**
  - Radio (~3000, Manchester et al., 2005)
  - Optical (~10)
  - X rays (~100 rotation-powered)
  - Gamma rays (~300 *Fermi*-LAT pulsars*)
- Potential emitters of **continuous gravitational waves** (Abbott et al., 2017, 2019)

Abdo et al., 2013 (2PC)



* https://confluence.slac.stanford.edu/display/GLAMCOG/Public+List+of+LAT-Detected+Gamma-Ray+Pulsars/pulsars

Spin-down evolution



Credits: A. Harding

- Power emission occurs at the expense of rotational energy
- The evolution is parameterized through a **braking index**

$$\dot{f} = -K f^n \qquad n = \frac{f\ddot{f}}{\dot{f}^2}$$

- Measured only for ~10 pulsars (Lyne et al., 2015)
- Carries information on emission mechanisms

Particle acceleration (n=1)

$$\dot{E}_{\mathrm{wind}} = -\frac{2\mu(2\pi f)^2\kappa\Delta\phi\cos^2\alpha}{c}$$

Dipole emission (n=3)

$$\dot{E}_{\mathrm{dipole}} = -\frac{2\mu^2(2\pi f)^4\sin^2\alpha}{3c^3}$$

Gravitational waves (n=5)

$$\dot{E}_{\mathrm{GW}} = -\frac{32G}{5c^5}\epsilon^2 I^2(2\pi f)^6$$

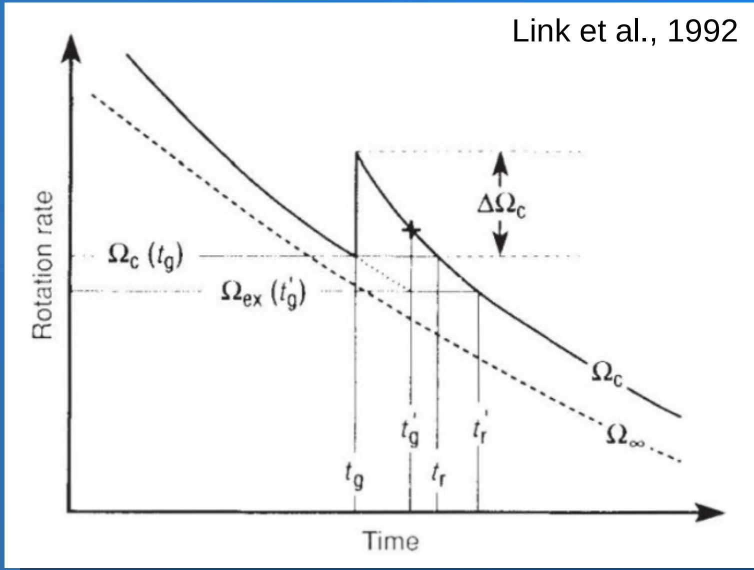
Timing irregularities

Red noise

- Dominated by **low frequencies**
- Produced by stochastic torques linked to the magnetosphere (Lyne et al., 2010) or to the NS interior (Melatos & Link, 2014)

Shannon et al., 2016

$$P_r(f) = \frac{A(f_c/f_{\rm yr})^{-\beta}}{[1+(f/f_c)^{-\beta/2}]^2}$$



Rotational glitches

- Sudden spin-ups with Δf / f between 10^{-9} and 10^{-5} (Manchester et al., 2005)
- Observed in most radio and gamma-ray pulsars
- Probes of NS interiors (Link et al., 1992)
- Possible sources of **transient GWs** (Bennett et al., 2010)

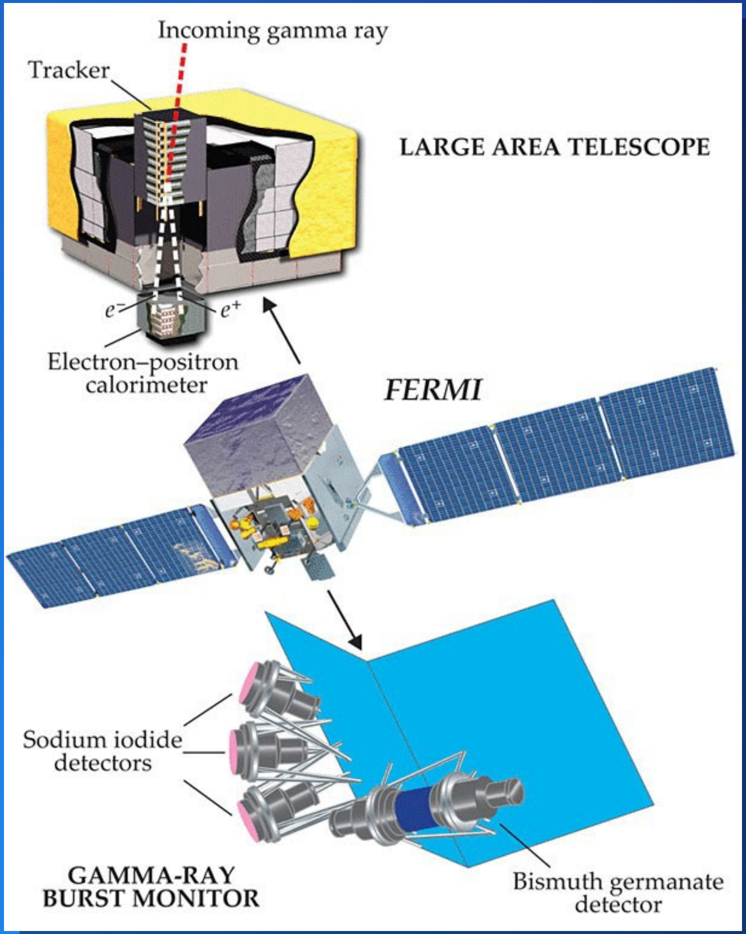
Motivation

- Most pulsar timing techniques rely on fitting a model to pulse **Times of Arrival** (ToAs)
- Unlike the radio, gamma-ray ToAs require **long integration times** (up to months) due to the **low photon flux**
- This implies a **low resolution on glitch parameters**, e.g. few days or worse for glitch epochs (Kerr et al., 2015)
- More than 70 pulsars are **radio quiet** and can only be studied in the gamma rays

- We attempted an independent approach for *Fermi*-LAT pulsars:
    - Based on **Bayesian parameter estimation**
    - **Unbinned**, i.e. uses LAT photon times rather than ToAs
- The goal is to improve our sensitivity to glitch parameters in gamma-ray pulsars and infer the amplitude of the **expected GW signals**

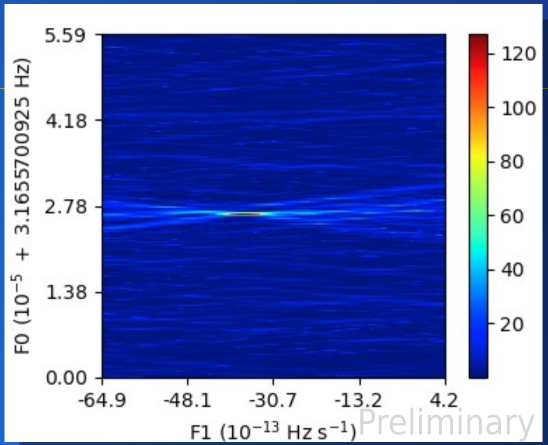
The *Fermi* Large Area Telescope

- Main instrument of the *Fermi* Gamma-ray Space Telescope
- NASA mission, operative since June 2008
- **Pair-conversion** telescope (Atwood et al., 2009)
- Sensitive to gamma rays **above 20 MeV**
- **Large field of view** (> 2 sr)
- **>6000 gamma-ray sources** detected (4FGL-DR3; Abdollahi et al., 2022)
- **~300 gamma-ray pulsars**
- Standard analysis software: *Fermitools* *



* https://fermi.gsfc.nasa.gov/ssc/data/analysis/scitools/references.html

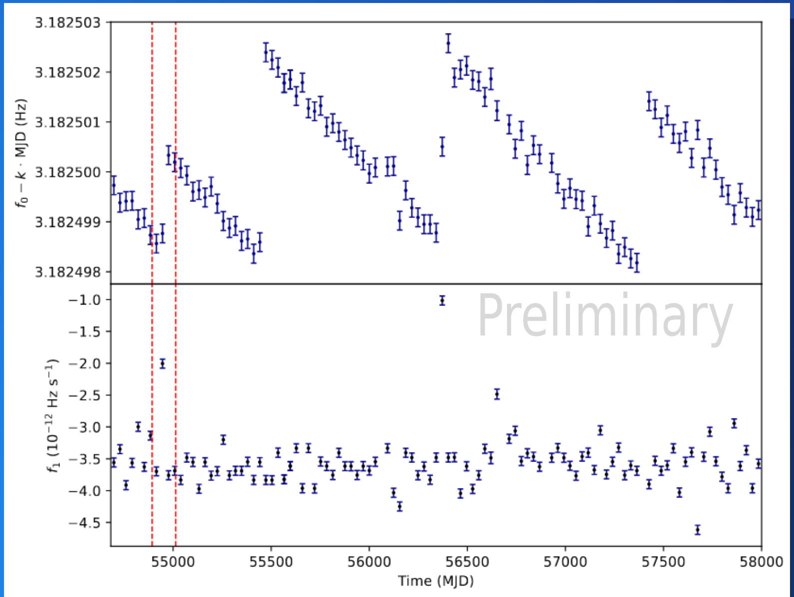
Step 0: evolution over long time scales



- We start with LAT photon times spanning the full mission
- Example analysis on PSR J0007+7303 (Li et al., 2016)
- **Search for periodicity** over short intervals assuming no glitch
- We scan the f – ḟ parameter space to find the optimal pair that maximizes the **H-statistic** (de Jager et al., 2010)

$$Z_m^2 = \frac{2}{N}\sum_{k=1}^{m}\left[\left(\sum_{i=1}^{N}\cos\, 2\pi k\varphi_i\right)^2 + \left(\sum_{i=1}^{N}\sin\, 2\pi k\varphi_i\right)^2\right]$$

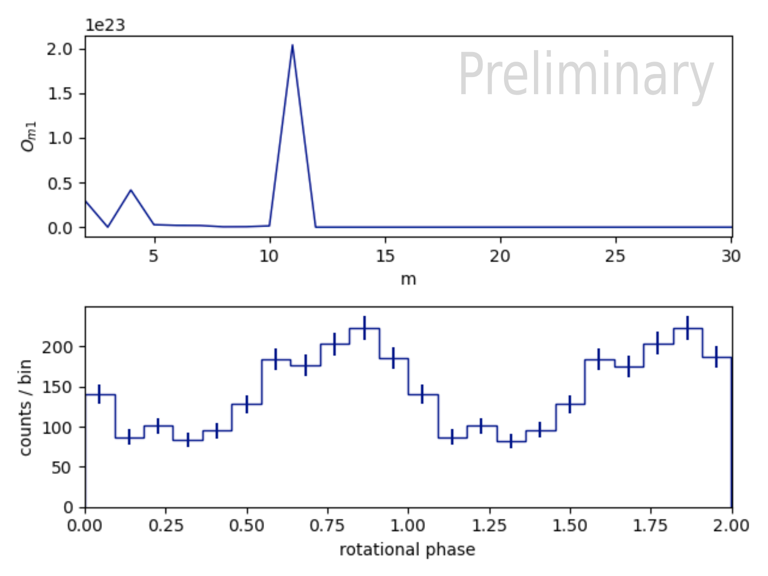
$$H = \max_{1\leq m\leq 20}\left(Z_m^2 - 4m + 4\right)$$



- We vary the selection radius and energies to determine the **optimal photon cuts**
- We produce a time series of frequency and spin-down to look at high-significance glitches

Analysis setup: pulse profile

- Bayesian parameter estimation based on the work by Gregory and Loredo (1992)
- We model the pulse profile as a **piecewise constant function** with *m* bins
- The optimal number of bins is obtained by maximizing the odds ratio (using f – ḟ from H-test)



Bayes' theorem

Posterior ∝ Likelihood × Prior

$$P(\theta \mid D, M) \propto P(D \mid \theta, M)\, P(\theta \mid M)$$

Unbinned Poisson Likelihood

$$\mathcal{L}(r) = \prod_{i=1}^{N} p_1(t_i) \prod_{k=1}^{Q} p_0(t_k)$$

$$p_n(t) = \frac{[r(t)\Delta t]^n e^{-r(t)\Delta t}}{n!}$$

Odds ratio

$$O_{m1}(\boldsymbol{\theta}) = \frac{1}{m_{\max} - 1} \binom{N + m + 1}{N}^{-1} \frac{m^N}{W_m(\boldsymbol{\theta})}$$

$$W_m(\boldsymbol{\theta}) = \frac{N!}{n_1! n_2! \dots n_m!}$$

Analysis setup: timing model

- Parameterization of the rotational phase as a function of time

$$\phi(t) = \phi_0 + \sum_{k=0}^{k_{\max}} \frac{1}{k!} f_k (t - t_0)^{k+1}$$

Spin-down evolution as a **Taylor series** around the epoch of the observation

$$\Delta\phi(t) = \Delta\Phi + \Delta f_p (t - t_g) + \frac{1}{2}\Delta \dot{f}_p (t - t_g)^2 + \Delta f_t \tau \left[1 - \exp\left(-\frac{t - t_g}{\tau}\right)\right]$$

Glitch **permanent changes** in absolute phase, frequency and spin-down

Glitch **transient change** in frequency, with **exponential decay**

- **Phase folding:** we take the fractional phase in the range [0, 1) and make the histogram

$$\varphi(t) = \mathrm{frac}\left[\phi(t)\right]$$

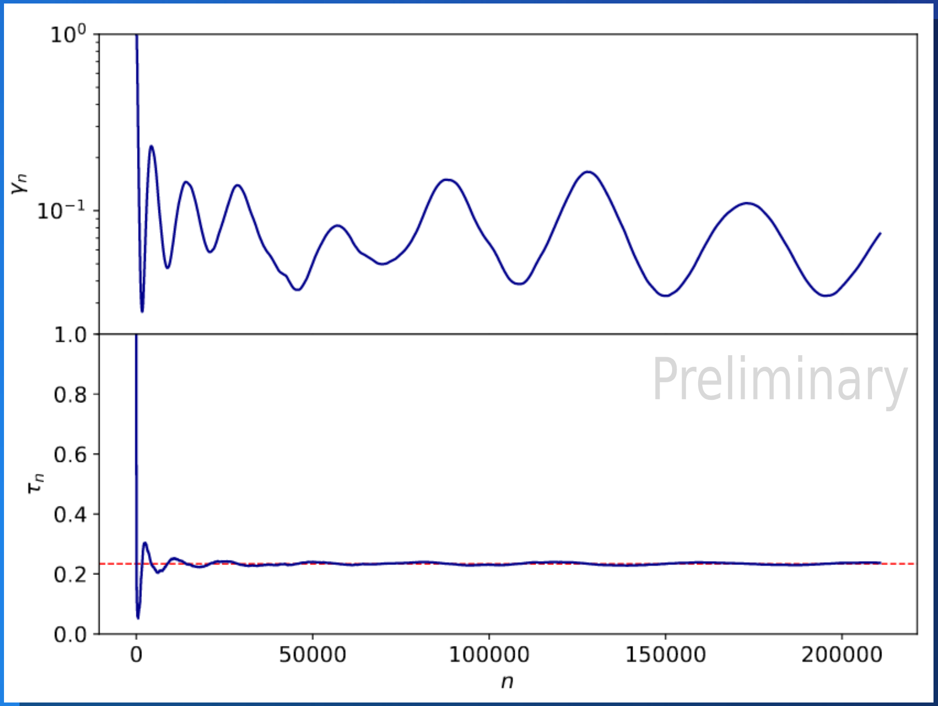
Adaptive Metropolis-Hastings sampling

- MCMC sampling usually requires a preliminary **fine tuning** of the proposal
- We use the adaptive Metropolis-Hastings proposed by Atchadé & Rosenthal, 2005
- Based on **iteratively adapting the proposal scale** to achieve the convergence of the acceptance rate

- We generate a trial point $\boldsymbol{y}_{n+1}$ using $\pi(\,\cdot\,;\Sigma_n)$ and a random number r from a uniform distribution in the range $[0, 1]$.
- We calculate the ratio

$$w = \frac{\mathcal{L}(\boldsymbol{y}_{n+1})\, p(\boldsymbol{y}_{n+1})\, \pi(\boldsymbol{y}_{n+1} - \boldsymbol{x}_n; \Sigma_n)}{\mathcal{L}(\boldsymbol{x}_n)\, p(\boldsymbol{x}_n)\, \pi(\boldsymbol{x}_n - \boldsymbol{y}_{n+1}; \Sigma_n)}\,. \tag{28}$$

- If $r \leq w$, we accept the trial point and set $\boldsymbol{x}_{n+1} = \boldsymbol{y}_{n+1}$; otherwise, we reject it and set $\boldsymbol{x}_{n+1} = \boldsymbol{x}_n$.
- We estimate τ_n and compute $\gamma_{n+1} = \gamma_n[1 - \epsilon(\tau_n - \bar{\tau})]$.
- We set $\Sigma_{n+1} = \gamma_{n+1}\Sigma_0$ if $\gamma_{n+1} \in [\gamma_{\min}, \gamma_{\max}]$; otherwise, $\Sigma_{n+1} = \Sigma_n$.



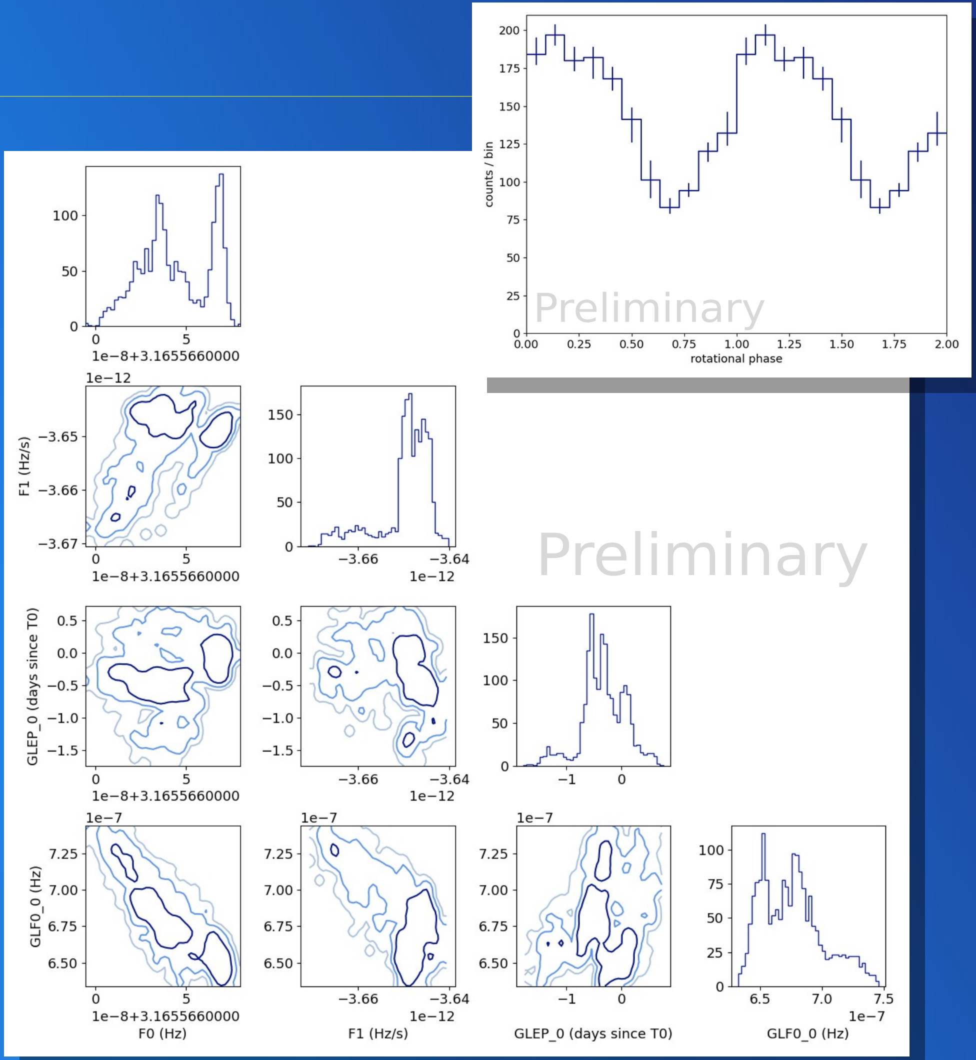
Preliminary test: PSR J0007+7303

- **Bright radio-quiet LAT pulsar**
  (flux ~ 4.3 × 10^{-10} erg cm^{-2} s^{-1}; 4FGL-DR3)
  in the CTA-1 supernova remnant

- 3 significant glitches during the first 7 years of *Fermi*-LAT mission (Li et al., 2016)

- We tested our pipeline
  on the **MJD 54953 glitch**

- **Model**
  - spin-down + permanent frequency change
  - pulse profile with 11 bins
  - 16 free parameters



Inferring GW parameters

- We expect two GW signals using the vortex unpinning model (Anderson and Itoh, 1975)

Burst produced by the spin-up event

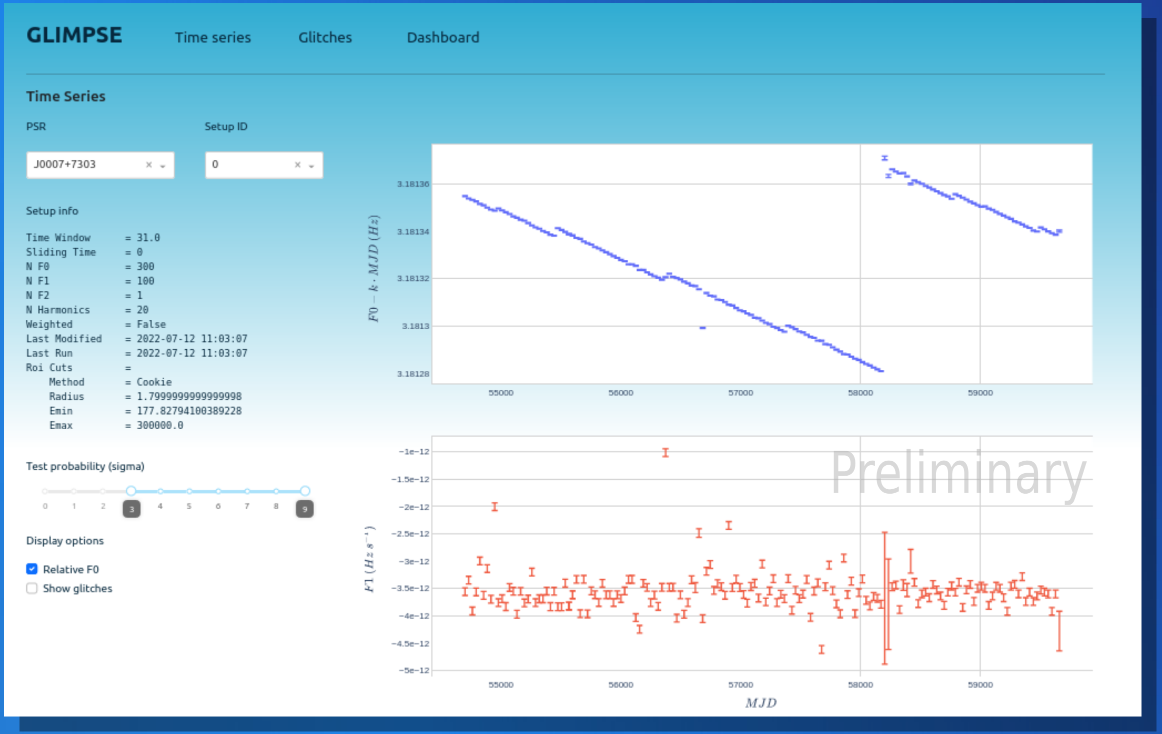
Decaying continuous wave during the follow-up relaxation

$$h_0 = 6 \times 10^{-26} \left(\frac{\delta\Omega/\Omega}{10^{-4}} \right) \left(\frac{f_*}{10^2\ \mathrm{Hz}} \right)^3 \left(\frac{D}{1\ \mathrm{kpc}} \right)^{-1}$$

- Assumptions on model parameters required
- Possible detections with the **Einstein Telescope** (Bennett et al., 2009)
- We plan to use our estimated parameters to infer the expected GW amplitude
- Results may be used to plan future **targeted searches for transient GWs** from *Fermi*-LAT pulsars

The GLIMPSE package

- **Python toolkit** for GLItch Monitoring and Periodicity SEarch
- Includes modules for all the analysis steps presented
- Use cases:
  - **Parameter estimation** for glitches in Fermi-LAT pulsars
  - **Continuous monitoring** of pulsar timing parameters
- Interactive access to results via Plotly Dash **web application** (work in progress)
- We plan to make results available to the multi-messenger community



Conclusions

- Pulsar glitches are probes of NS interiors and pulsar magnetospheres
- Radio-quiet pulsars can only be observed in the gamma-rays and allow to investigate many aspects of pulsar physics
- GW transients from pulsar glitches still remain undetected
- Multi-messenger astrophysics may benefit of:
    - A **detailed characterization** of glitches in gamma-ray pulsars
    - A **continuous monitoring** for pulsar timing
- **Stay tuned for updates...**

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Thank you for listening!

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