Estimations of cavity-to-cavity coupling

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Cavity-to-cavity coupling

- A single cell has the usual mode spectrum
  - TE_{mnp} , TM_{mnp}
- Coupled cells (i.e. multi-cell cavity)
  - Modes split into passbands
  - Characterised by differing phase advance per cell
- Multi-cavity installations (i.e. a cryomodule)
  - Modes below cutoff, so coupling disregarded
    - But this neglects the evanescent field!
  - Investigations of cavity geometry led us to consider cavity-cavity coupling

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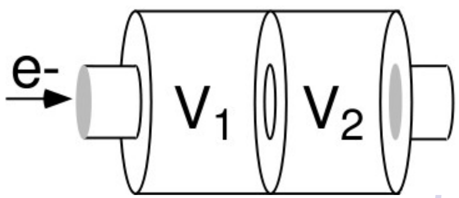
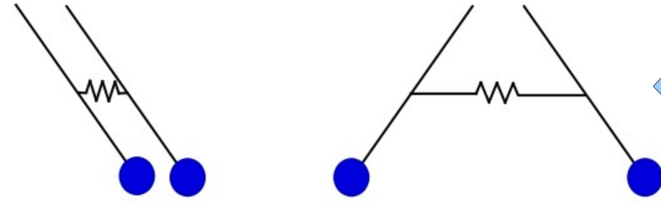
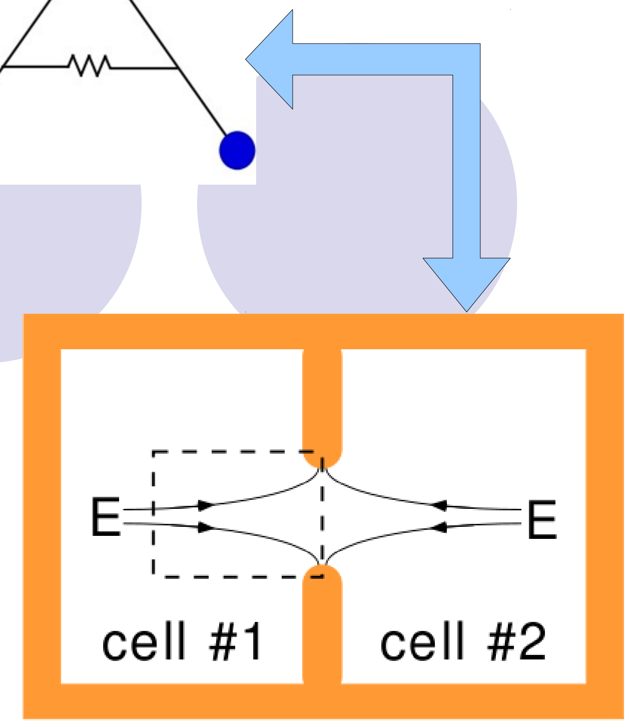
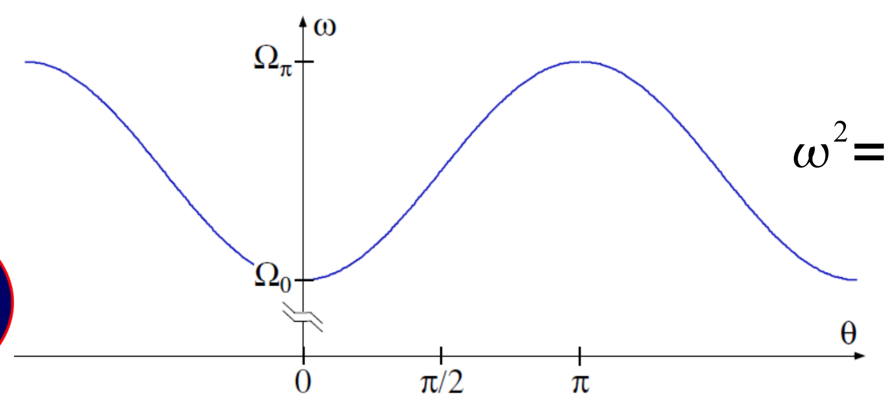
Coupled oscillators







- Eigenmodes of coupled oscillators split according to the phase difference
    - “0-mode”, “π-mode”, etc.
- For N+1 coupled oscillators
    - *iπ/N* radians phase advance (*i*=0,1,...*N*)
  - Frequency also splits
    - Dependent on the coupling strength
    - Each new mode may be plotted on a Brillouin curve
        - For N<∞ the modes are equally spaced along the curve



$$\omega^2 = \omega_{\pi/2}^2 \left(1 - \kappa \cos(\theta)\right)$$

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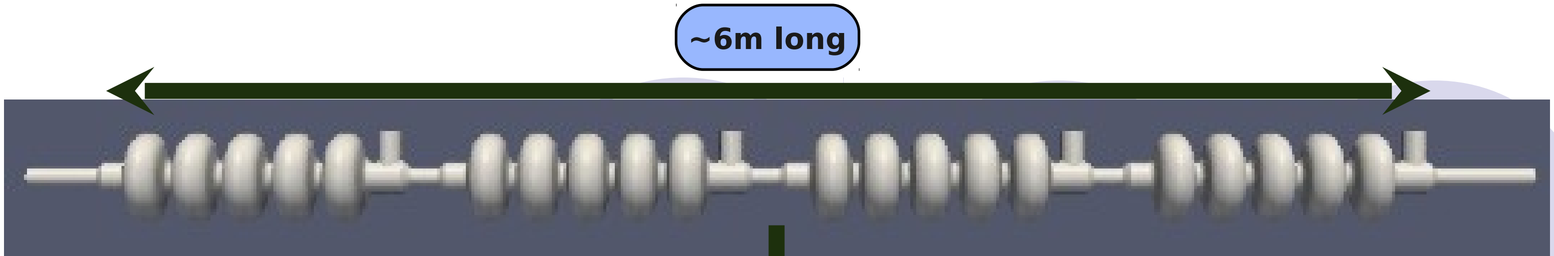
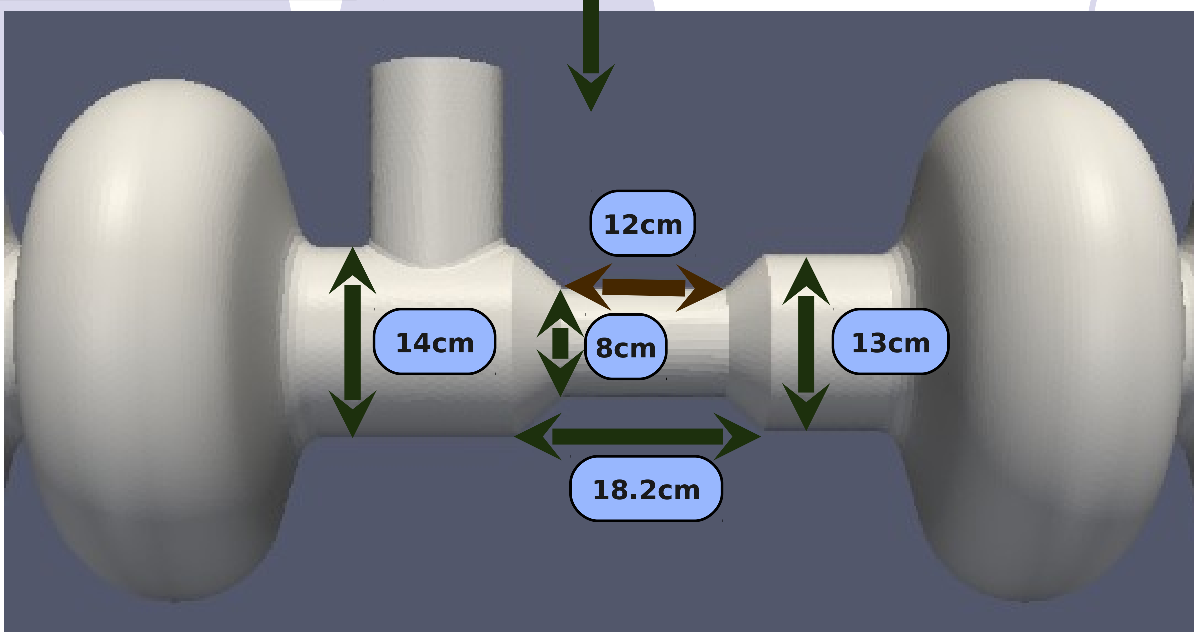
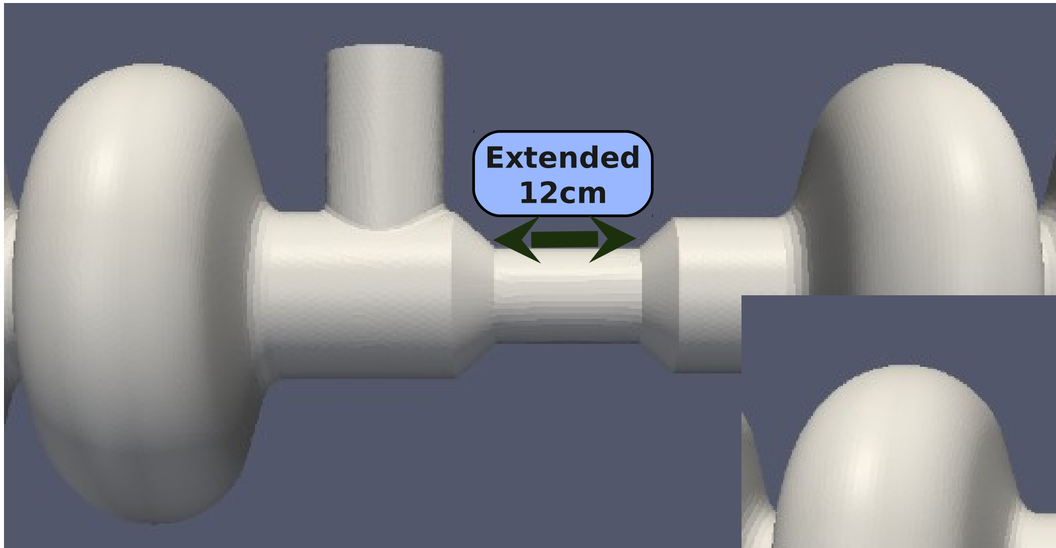
Eigensolve 4 full cavities

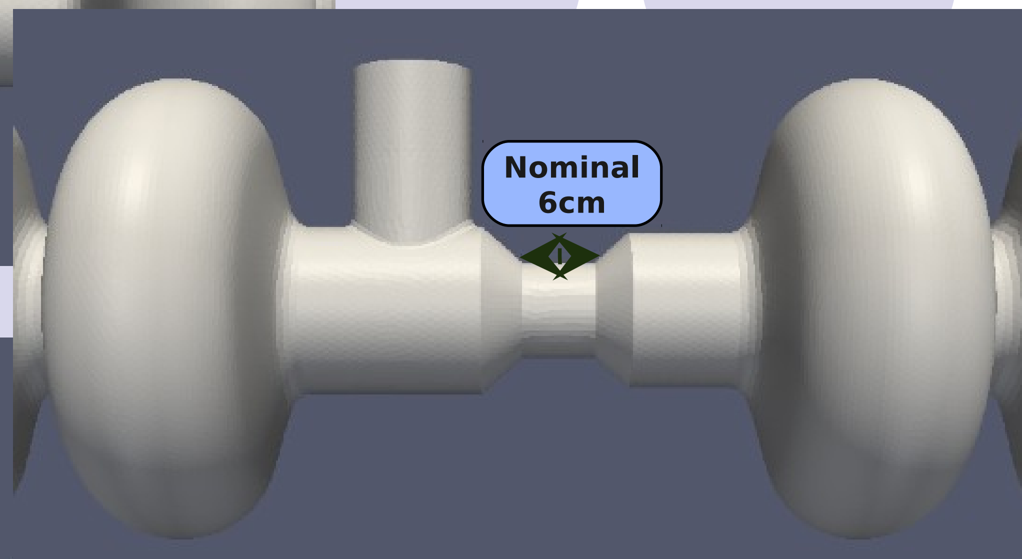


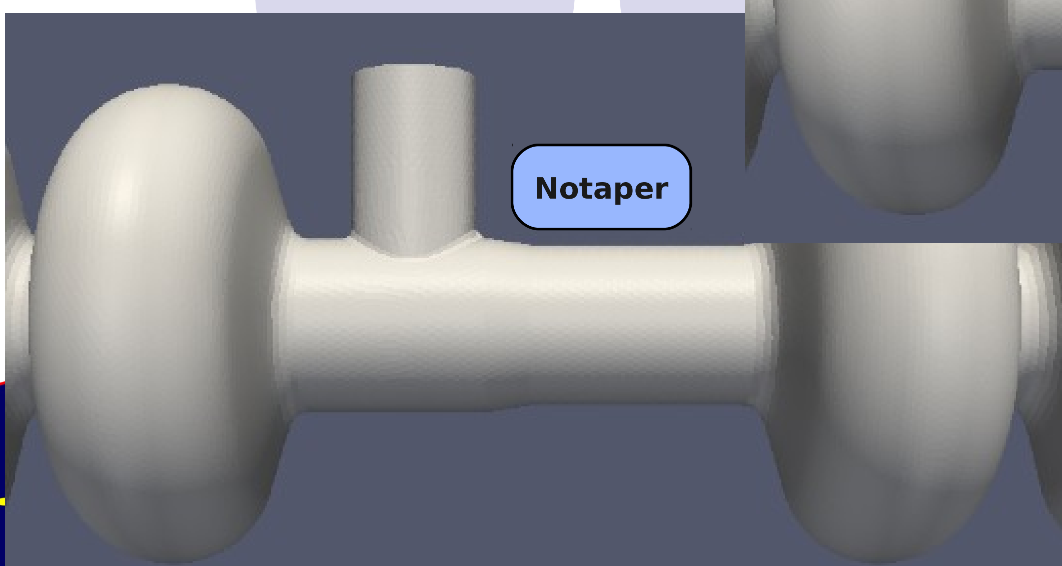
~880k elements
Average volume = 1.96 x 10^{-7} m^3
Min edge length = 2mm
Max edge length = 24mm
Magnetic symmetry plane



Inter-cavity geometry







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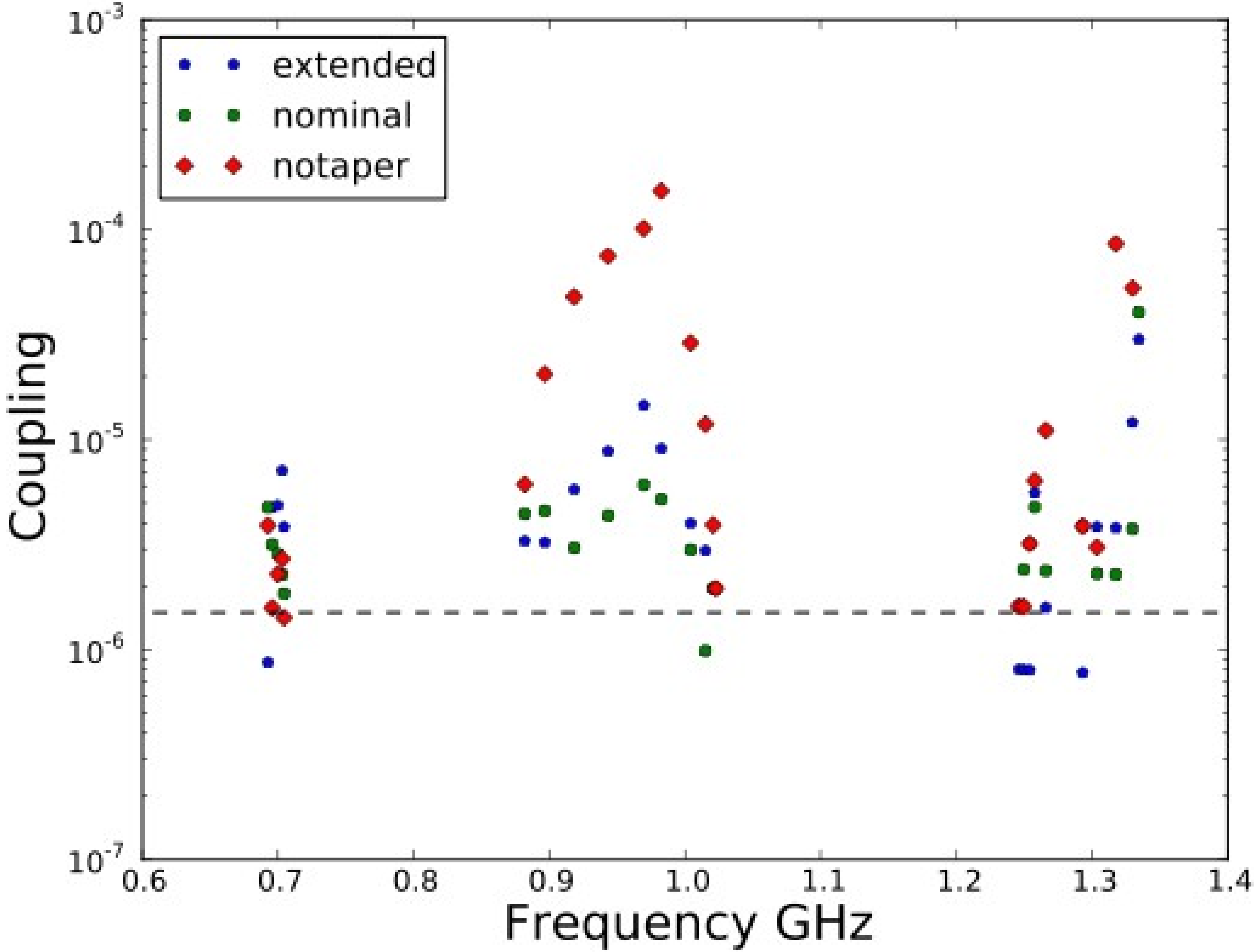
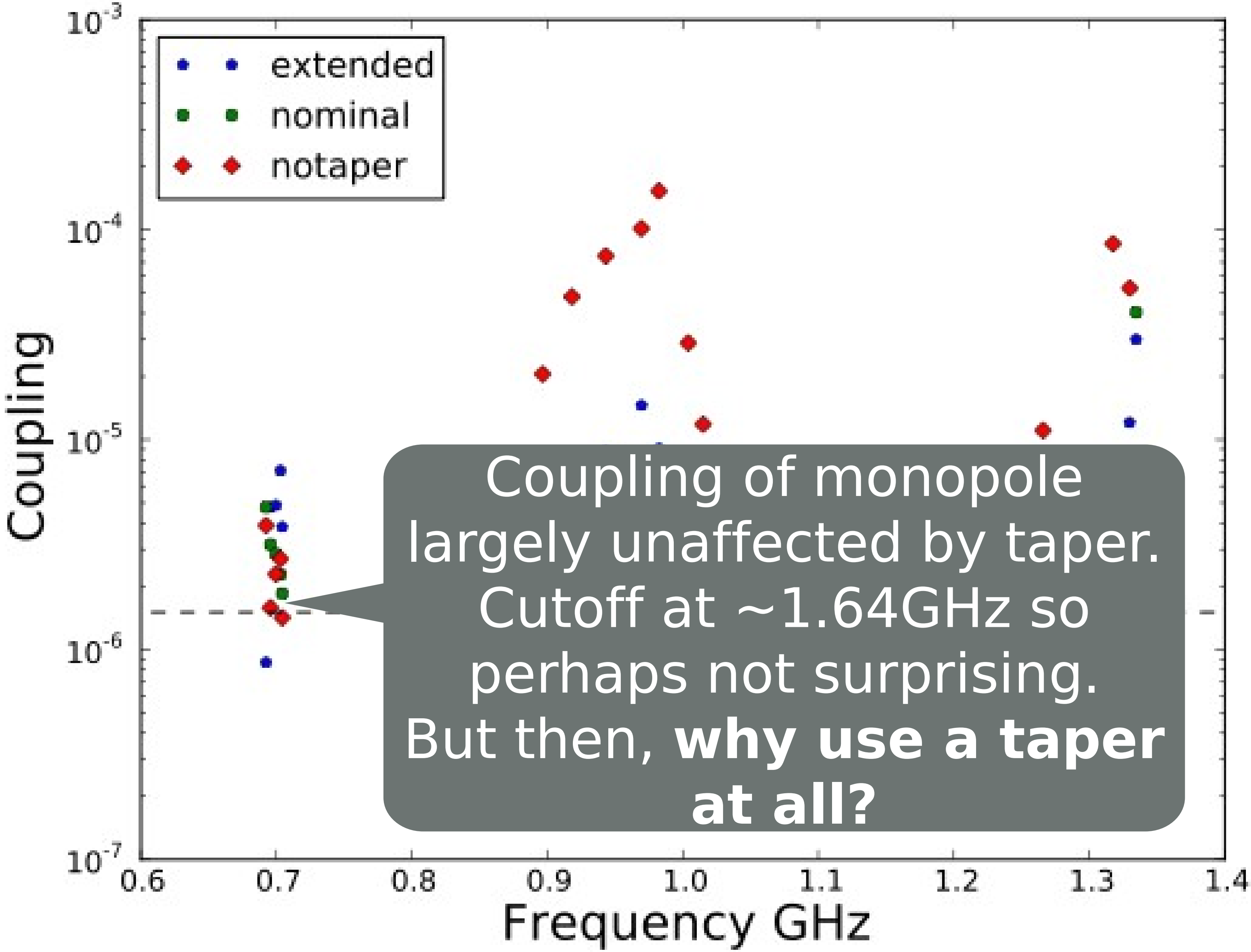
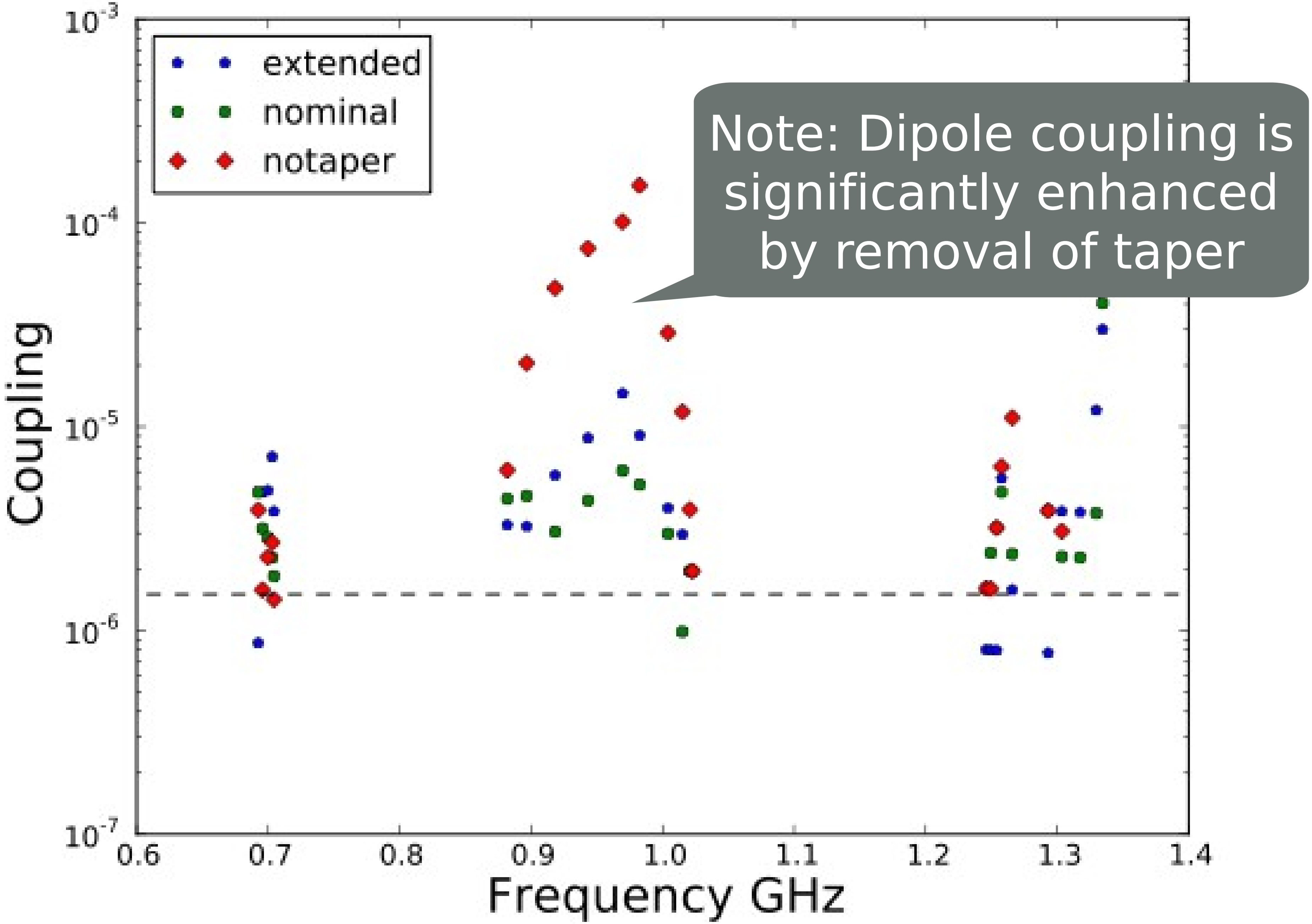


extended
nominal
notaper
Coupling
Frequency GHz
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4


extended
nominal
notaper
Coupling
Frequency GHz
10-3
10-4
10-5
10-6
10-7
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
Coupling of monopole largely unaffected by taper. Cutoff at ~1.64GHz so perhaps not surprising. But then, **why use a taper at all?**



extended
nominal
notaper
Note: Dipole coupling is significantly enhanced by removal of taper
Coupling
Frequency GHz
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0.6
0.7
0.8
0.9
1.0
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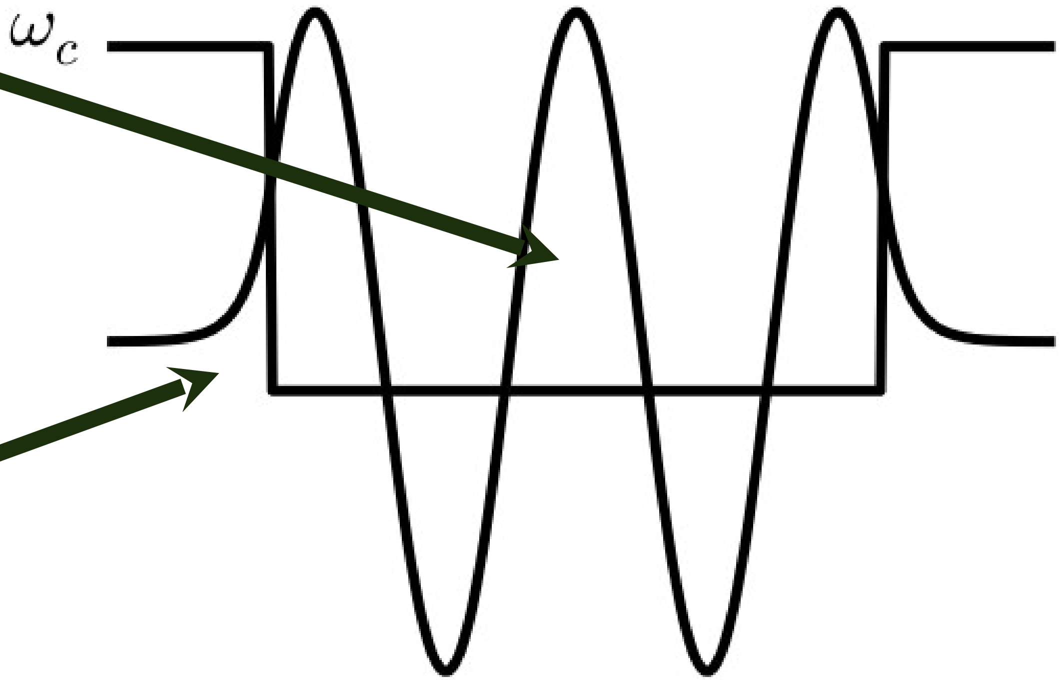
1.4

Simplified Model

Oscillation inside cavity



Decays exponentially inside beam pipe

Finite potential well

$$E > V$$

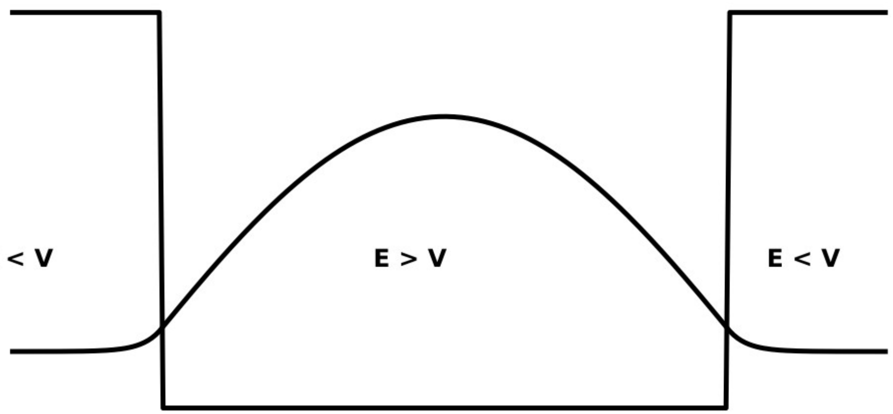
$$\psi_j = A_j \cos(k_j z) + B_j \sin(k_j z)$$

$$k = \frac{\sqrt{2m_j E}}{\hbar^2}$$

$$E < V$$

$$\psi_j = A_j e^{k_j z} + B_j e^{-k_j z}$$

$$k = \frac{\sqrt{2m_j (V - E)}}{\hbar^2}$$



Finite potential well

$\psi, \dfrac{d\psi}{dz}$ must be continuous at each boundary

Rewrite wave equations in terms of matrices

$$^{m}M_j = \begin{pmatrix} e^{k_j z_m} & e^{-k_j z_m} \\ k_j e^{k_j z_m} & -k_j e^{-k_j z_m} \end{pmatrix} \qquad E < V$$

$$^{m}M_j = \begin{pmatrix} \cos(k_j z_m) & \sin(k_j z_m) \\ -k_j \sin(k_j z_m) & k_j \cos(k_j z_m) \end{pmatrix} \qquad E > V$$

Therefore, at each boundary

$$^{j}M_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} = ^{j} M_{j+1} \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix}$$

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Finite potential well

At boundary I

$${}^{0}M_0 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = {}^{0}M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

At boundary II

$${}^{1}M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = {}^{1}M_2 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

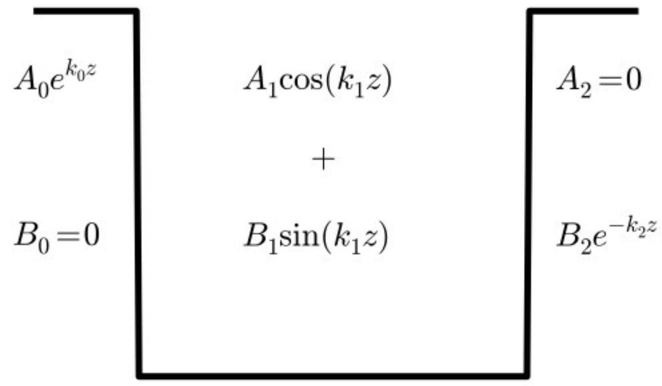
Therefore

$$[({}^{1}M_2)^{-1} * {}^{1}M_1 * ({}^{0}M_1)^{-1} * {}^{0}M_0] \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_2 \\ B_2 \end{pmatrix}$$

Looking for a bound state.

Set A_0=1 and B_0=0 (no leftward wave in first region).

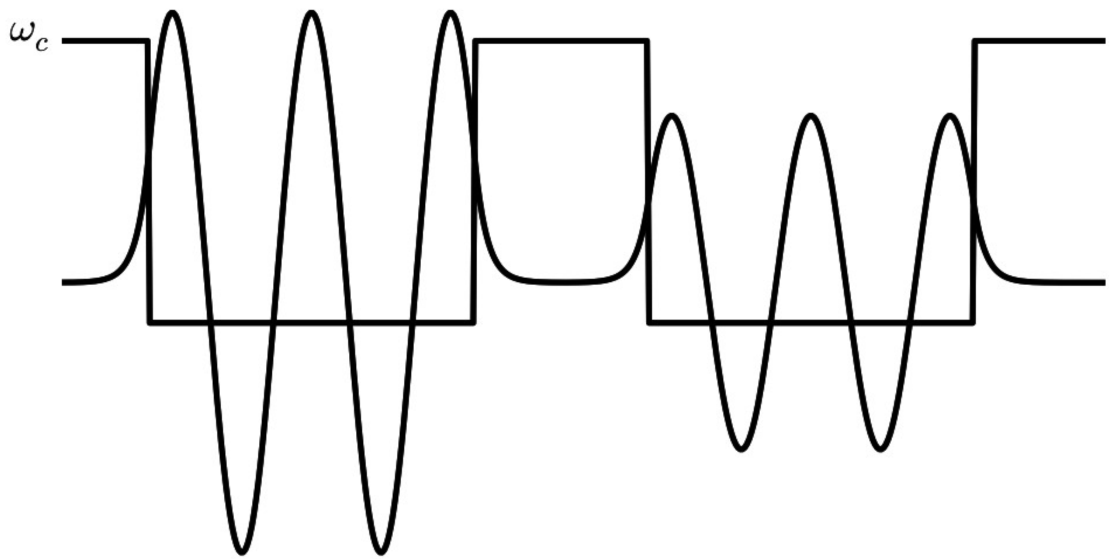
Solve for A_2=0



N>1 coupled wells

$$\left(\prod_{2N-1}^{0}\left[\left({}^{j}M_{j+1}\right)^{-1} * {}^{j}M_{j}\right]\right)\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_{2N} \\ B_{2N} \end{pmatrix}$$

Again, solve for $A_0=1$, $B_0=0$, and $A_{2N}=0$



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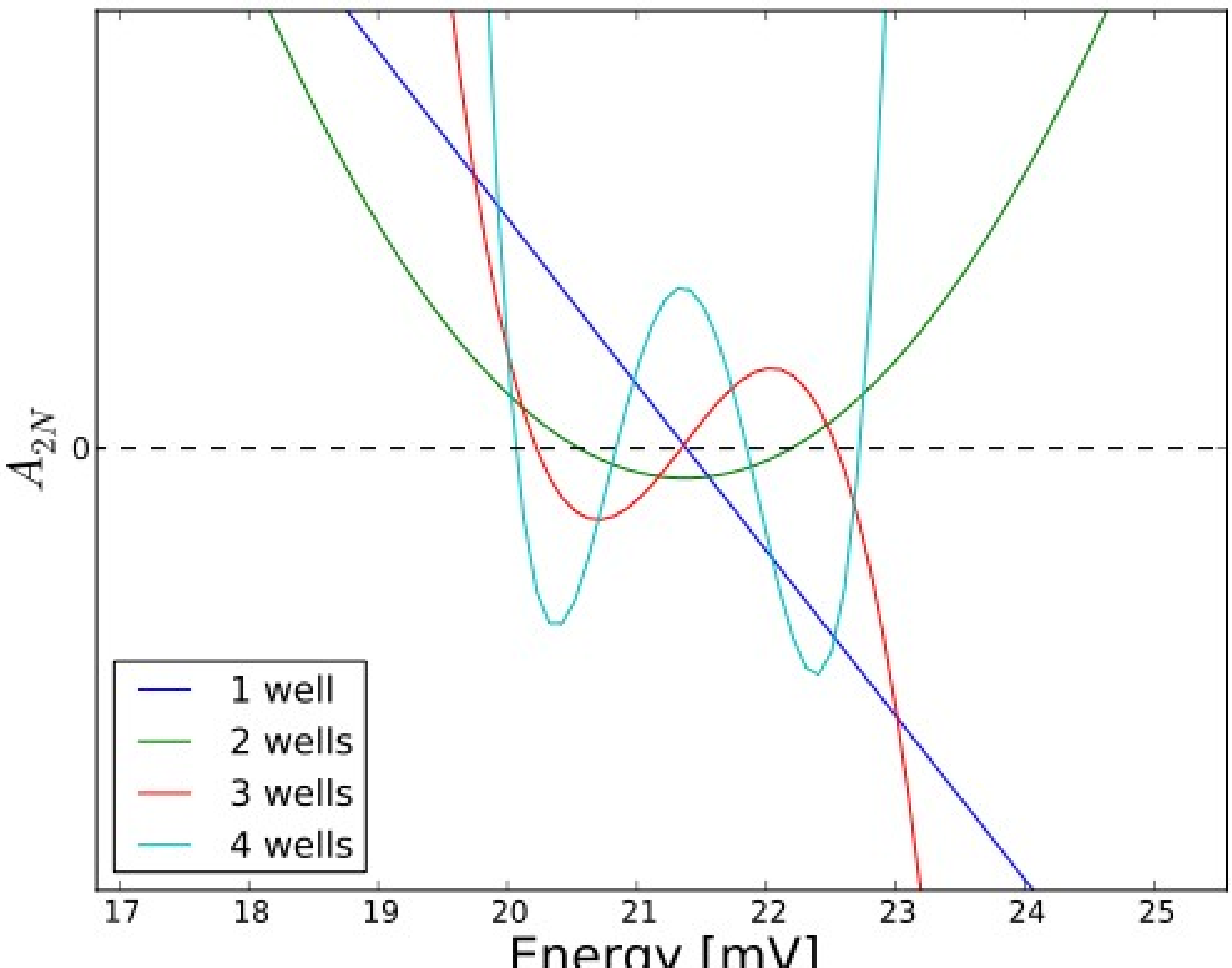
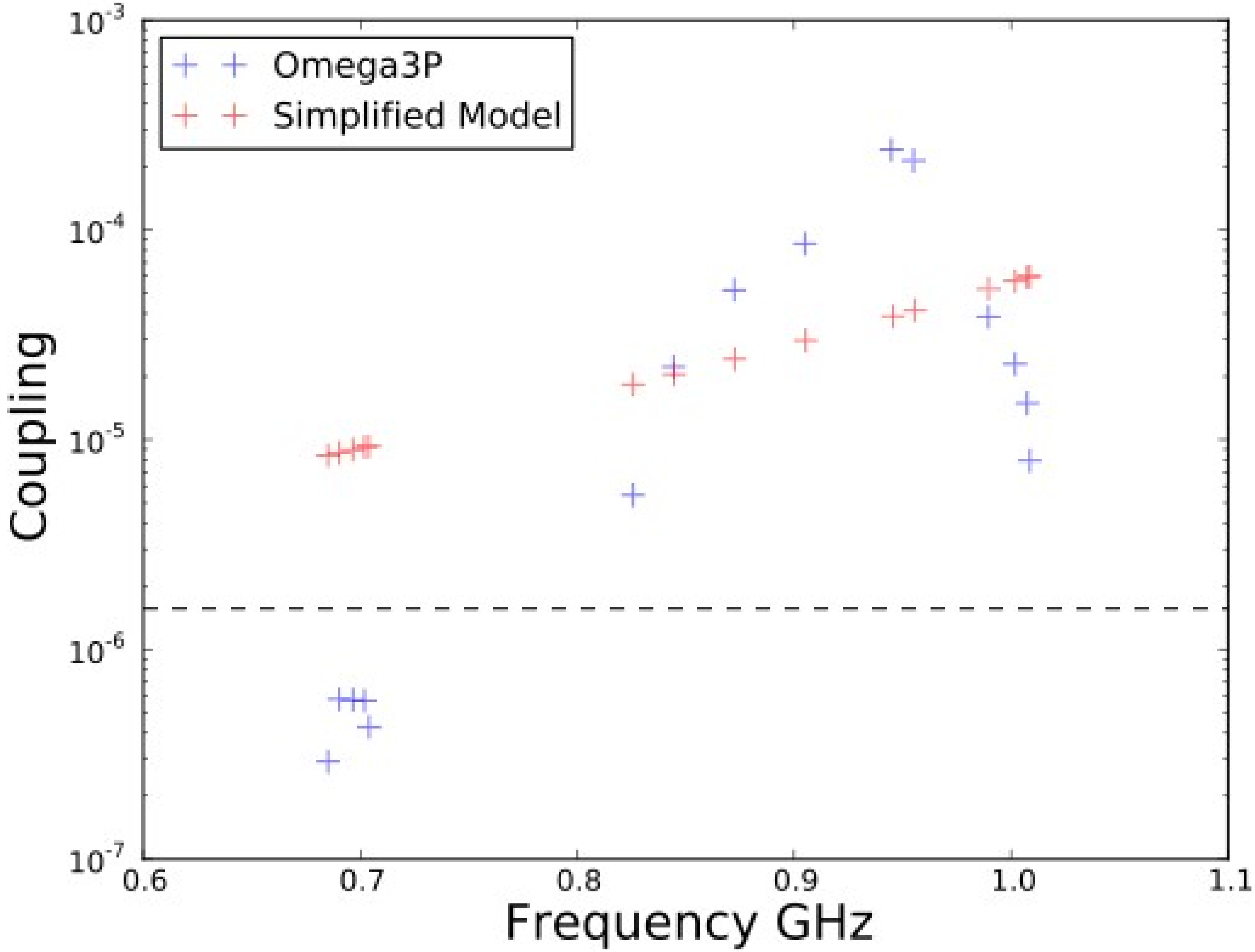
Discrete energy levels



Comparison of results



How to calculate wavenumber?

$$k = \frac{\omega}{c} \qquad \omega > \omega_c$$

$$k = \sqrt{\left(\frac{p_{nm}}{a}\right)^2 - \left(\frac{\omega}{c}\right)^2} \qquad \omega < \omega_c$$

- Each passband mode is characterised by its phase advance
  - Should *k* be redefined to encode this?
- Should model be extended to 3D?
  - But isn't this just rewriting Omega3P?
  - How to deal with couplers?

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Summary

- QM “particle in box” model developed
  - Shows some success in calculating coupling
  - Takes <1 minute on modern laptop
    - Compare with 2000CPU.hours using Omega3P!!!
  - Considering various improvements
    - But am keen not to re-write Omega3P, Ansys, etc.
- Coupling calculated
  - Is a taper necessary?
    - Increases loss factor, negligible effect on monopole coupling
    - However, it may limit field emitted particles

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