



# Shape and deformation in nuclei: quantum phase transition in Zr isotopes studied via lifetime measurements

**Giorgia Pasqualato**



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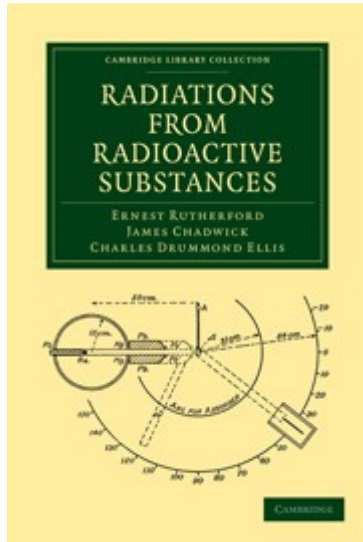


- INTRODUCTION :
  - Deformation and shapes in nuclei.
  - How can we study them from experiments?
  - Quantum phase transition in Zr isotopes.
- EXPERIMENT :
  - The experimental setup AGATA+VAMOS+plunger.
  - Lifetime measurements with the RDDS technique.
- ANALYSIS :
  - Lifetime results for 98-104Zr.
  - Comparison with theoretical predictions: MCSM, IBM-CM and SCCM-HFB.

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# Why nuclear shapes ?

- The nucleus is a complex many-body quantum system.
- Need of a **simplification** to describe its behavior.
- Where does nuclear shape come from ?

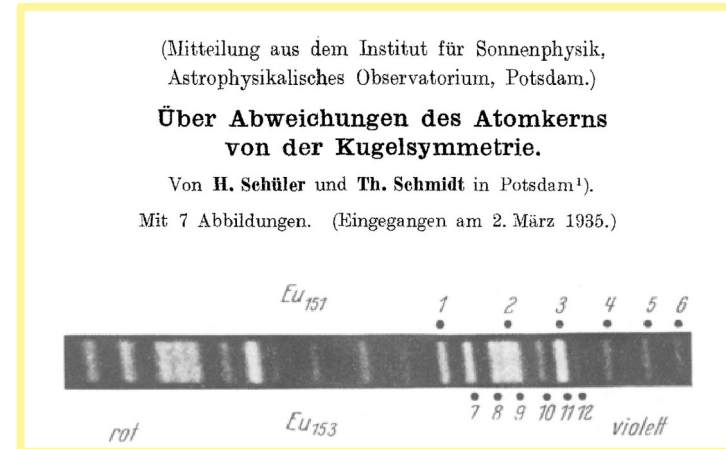


**1911**

In the famous scattering experiment Rutherford shows that the nucleus has a small but sizable spatial extension → (spherical) shape.

**1935**

Schüler and Schmidt : via atomic spectroscopy experiments, first clear indications of nuclear electric quadrupole moments (perturbation on the hyper-fine structure images in Eu reveals a deviation from the spherical symmetry.)

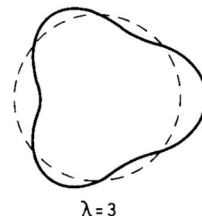
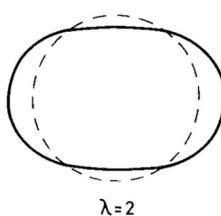




# Why nuclear shapes ?

- Parametrization of the **radius** of the nuclear surface : 
$$R(\theta, \phi) = R_0 \left\{ 1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{\lambda} a_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi) \right\}$$

- Important basic shapes :
  - $\lambda = 2 \rightarrow$  quadrupole deformation
  - $\lambda = 3 \rightarrow$  octupole deformation



- In the principal axis frame  $a_{2,1} = a_{2,-1} = 0$  and we define the Hill-Wheeler coordinates parameters  $\beta$  and  $\gamma$  for the mass surface :

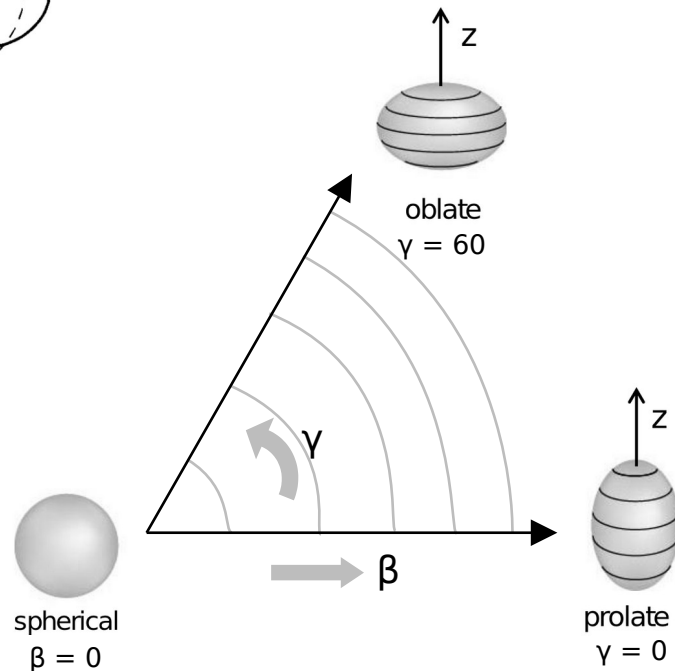
$$a_{2,0} = \beta \cos(\gamma)$$

$$a_{2,2} = \beta \sin(\gamma) / \sqrt{2}$$



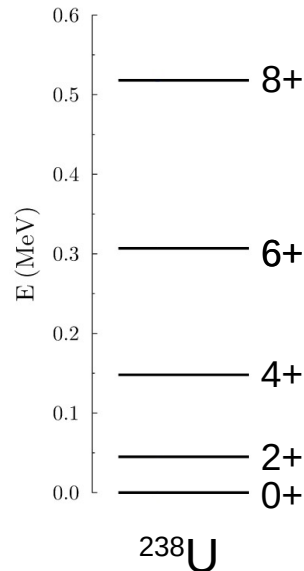
rather simple  
description !

- ... but nuclear shape is not an observable.

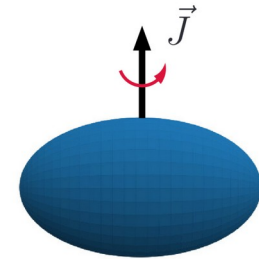


# Phenomenological manifestation of nuclear shapes

- The use of an intrinsic deformation explain many phenomenon :
- energy of excited states (e.g. energy of the first  $2^+$ ,  $3^-$  states, rotational bands)



$$E(J) = \frac{J(J+1)}{2\mathcal{I}}$$

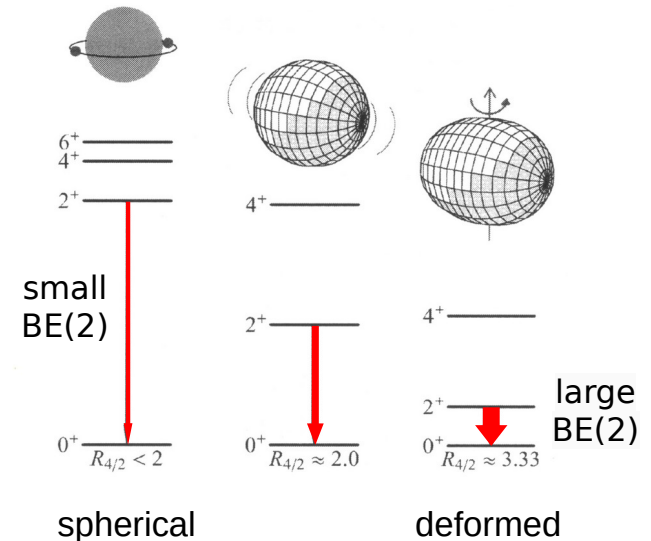


Semi-classical picture of a deformed nucleus rotating on its symmetry axis

(RIGID ROTOR = constant momentum of inertia  $\mathcal{I}$ )

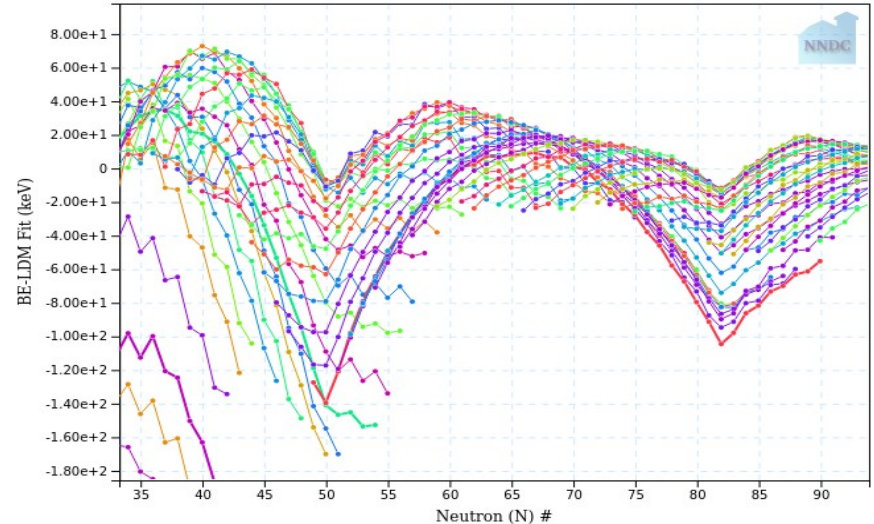
# Phenomenological manifestation of nuclear shapes

- The use of an intrinsic deformation explain many phenomenon :
  - energy of excited states
  - transition probabilities (e.g.  $B(E2; 2^+ \rightarrow 0^+)$ ,  $B(E3; 3^- \rightarrow 0^+)$  )



# Phenomenological manifestation of nuclear shapes

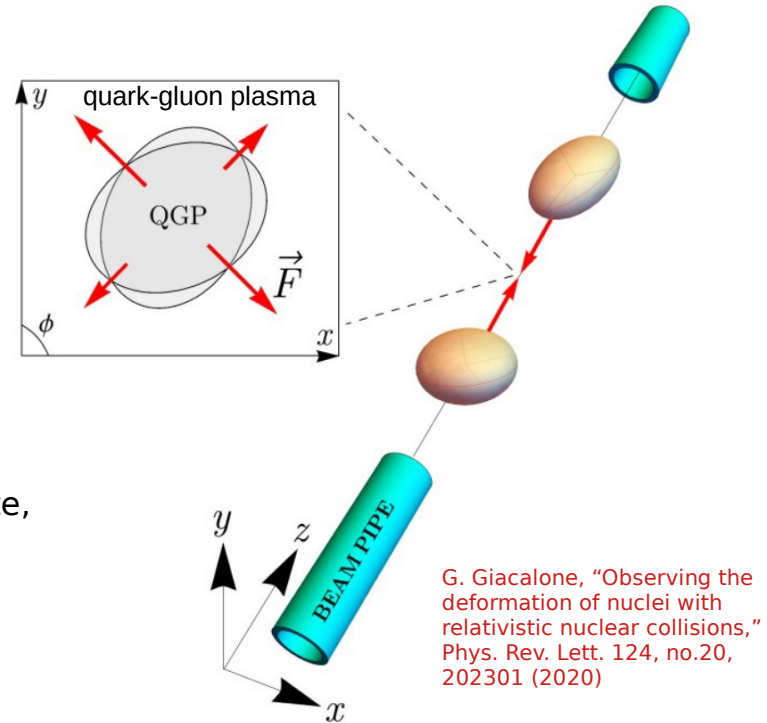
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  - energy of excited states
  - transition probabilities
  - nuclear properties varying as a function of  $N$ ,  $Z$



# Phenomenological manifestation of nuclear shapes

- The use of an intrinsic deformation explain many phenomenon :
  - energy of excited states
  - transition probabilities
  - nuclear properties varying as a function of  $N, Z$
  - high-energy heavy ion collision
  - ...

The nuclei, deformed in their ground state, are randomly oriented and influence the shape of their area of overlap. A quark-gluon plasma (QGP) is formed in the this area.

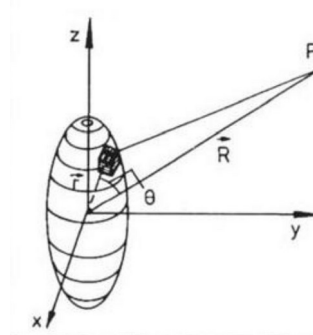


# How can we “measure” nuclear shapes ?

- We are sensitive to the **charge distribution  $\rho(\mathbf{r})$**  via the electromagnetic interaction

Intrinsic electric quadrupole moments :

$$Q_0 = \int \rho(\vec{r})(3z^2 - r^2)dV$$



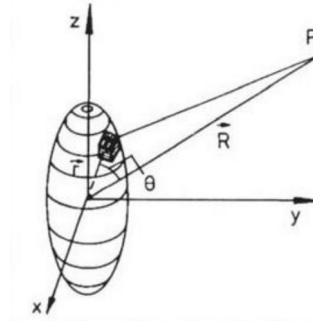
$Q_0 \neq 0 \rightarrow$  non-spherical charge distribution

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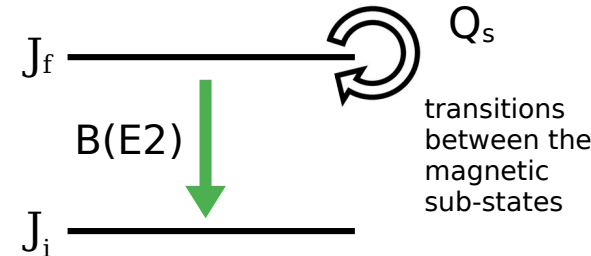


$Q_0 \neq 0 \rightarrow$  non-spherical charge distribution

And in the laboratory frame ?

- E2 transition probabilities (off-diagonal matrix elements)

$$B(E2; J_i \rightarrow J_f) = \frac{1}{2J_i + 1} |\langle J_f || M(E2) || J_i \rangle|^2$$



- Spectroscopic electric quadrupole moments (diagonal matrix elements)

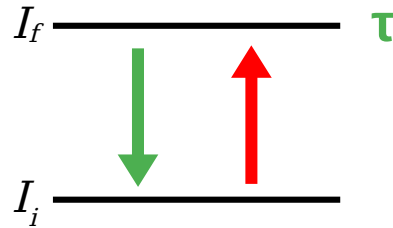
$$Q_s(J) = \sqrt{\frac{16\pi}{5}} \frac{\langle J J 20 | J J \rangle}{\sqrt{2J+1}} \langle J || E2 || J \rangle$$

**$Q_0 \neq Q_s$  !**

# How can we “measure” nuclear shapes ?



Lifetime  
measurements



$$\frac{1}{\tau} \propto B(\pi\lambda; I_i \rightarrow I_f)$$

reduced transition  
probabilities

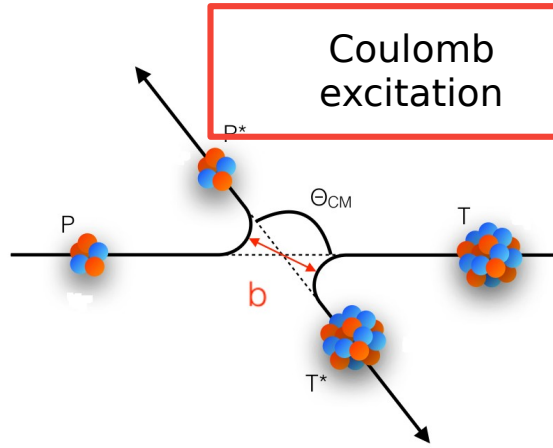
$$B(\sigma\lambda; I_i \rightarrow I_f)$$

This give us the shape?  
NO  
But ...

Coulomb  
excitation

reduced transition  
probabilities

to first order:  $\sigma_{\pi\lambda} \propto B(\pi\lambda; I_i \rightarrow I_f)$

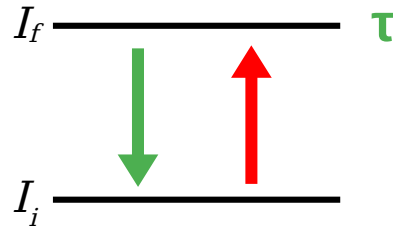




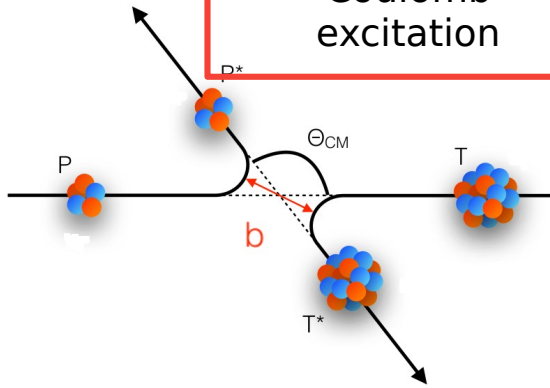
# How can we “measure” nuclear shapes ?



Lifetime measurements



Coulomb excitation



to first order:  $\sigma_{\pi\lambda} \propto B(\pi\lambda; I_i \rightarrow I_f)$

$$\frac{1}{\tau} \propto B(\pi\lambda; I_i \rightarrow I_f)$$

reduced transition probabilities

$$B(\sigma\lambda; I_i \rightarrow I_f)$$

reduced transition probabilities

$$B(\sigma\lambda; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f || \hat{O}(\sigma\lambda) || I_i \rangle|^2$$

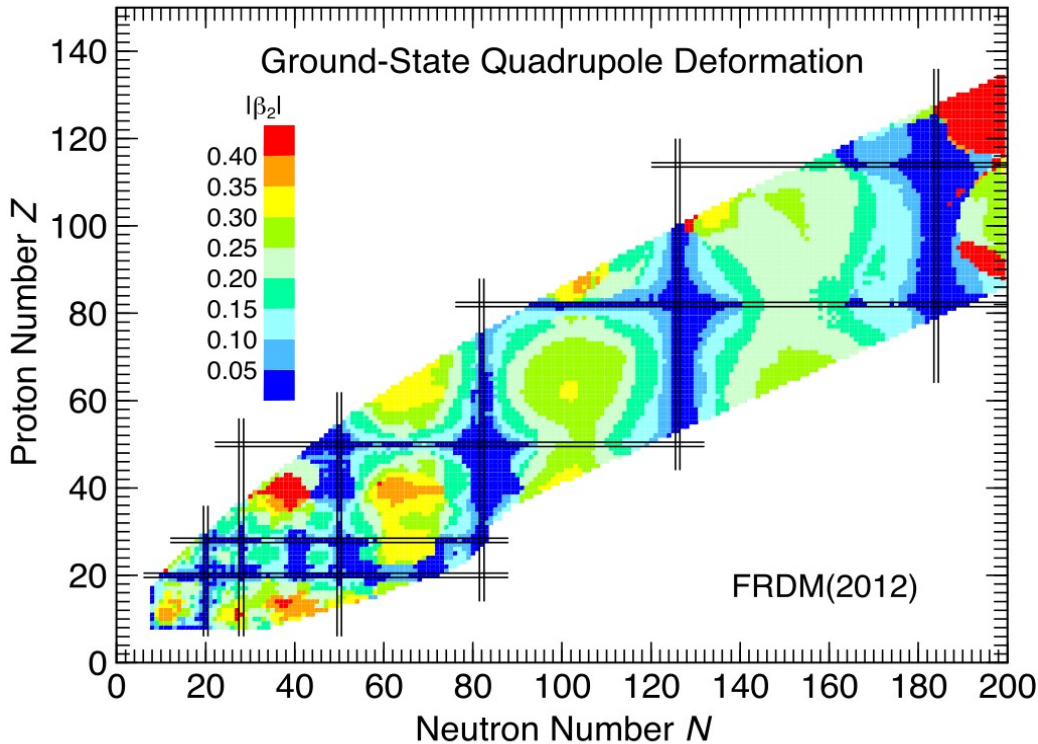
Matrix elements

Comparison with theoretical predictions

Quantitative test of nuclear models

# Where do we encounter nuclear deformation ?

- The large amount of deformation is observed **far from magic lines**

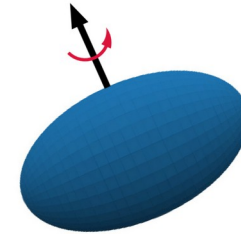


P. Möller, A.J. Sierk, T. Ichikawa, H. Sagawa, *Nuclear ground-state masses and deformations: FRDM(2012)*, Atomic Data and Nuclear Data Tables, 109-110, (2016),

- Magic** numbers of Z and N over-stabilize the nucleus → spherical shape

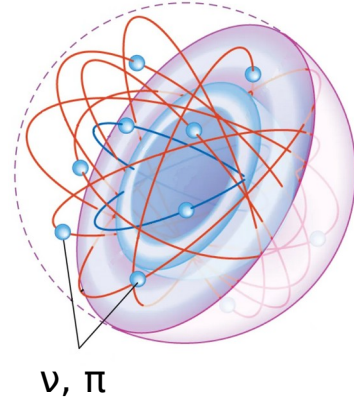


*Magic numbers are a clear evidence of the existence of an internal shell structure: a different picture with respect to a macroscopic view introduced by nuclear deformation*



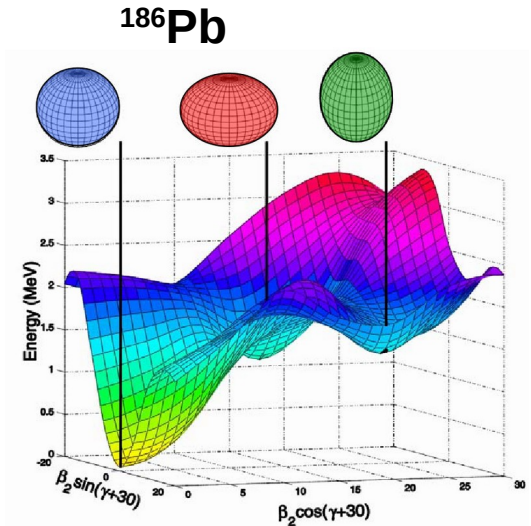
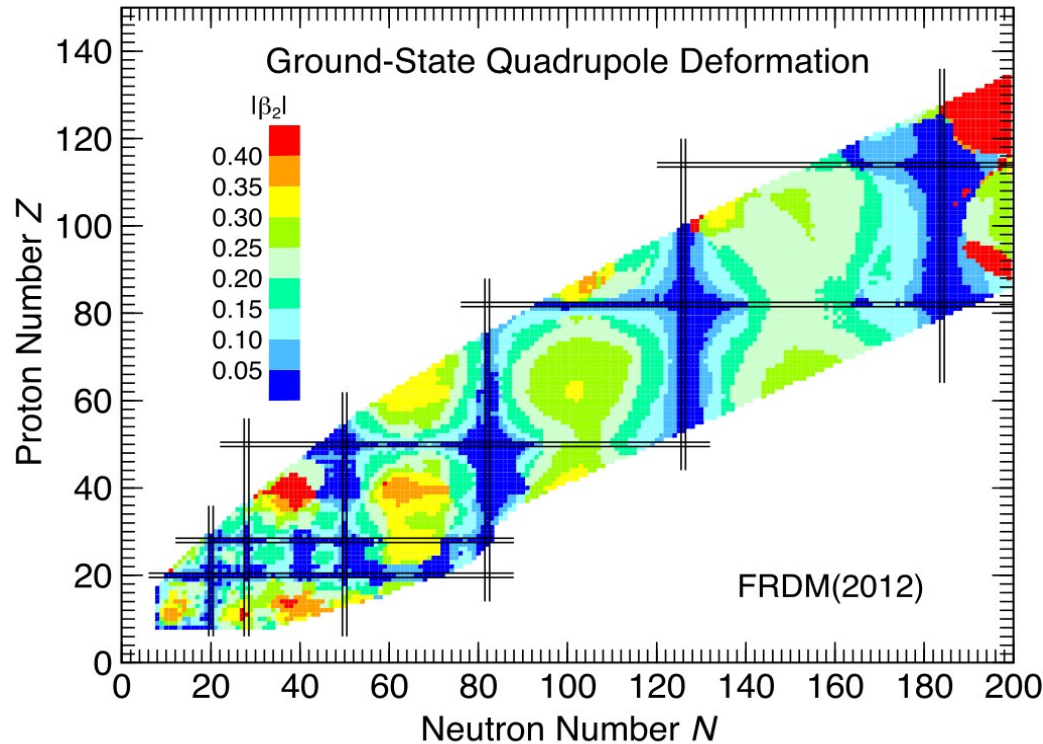
↔  
microscopic structure

many nucleons outside a closed core → large collectivity = deformation



# Shape coexistence

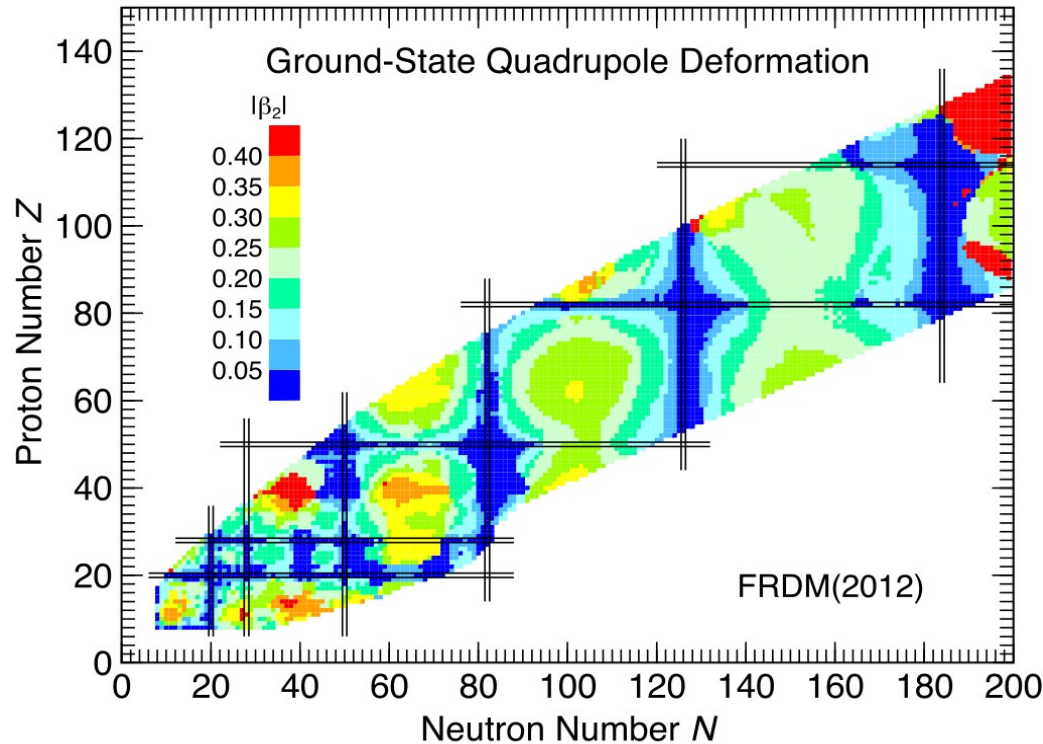
- **Shape coexistence** is a phenomenon where distinct shapes occur within the same nucleus and at a similar energy.



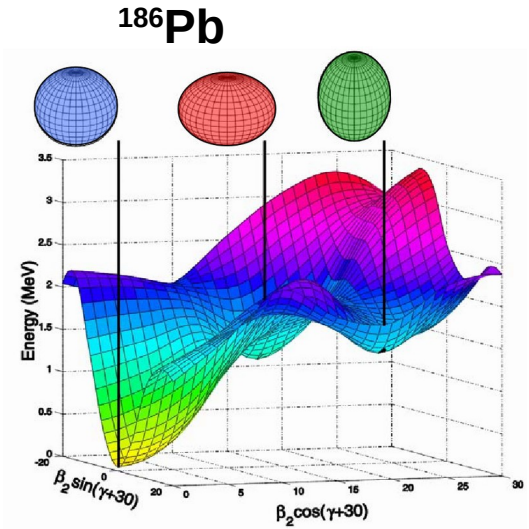
Often found in the boundaries of deformed regions in the nuclear chart as a consequence of the shape changing.

# Shape coexistence

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P. Möller, A.J. Sierk, T. Ichikawa, H. Sagawa, *Nuclear ground-state masses and deformations: FRDM(2012)*, Atomic Data and Nuclear Data Tables, 109-110, (2016),



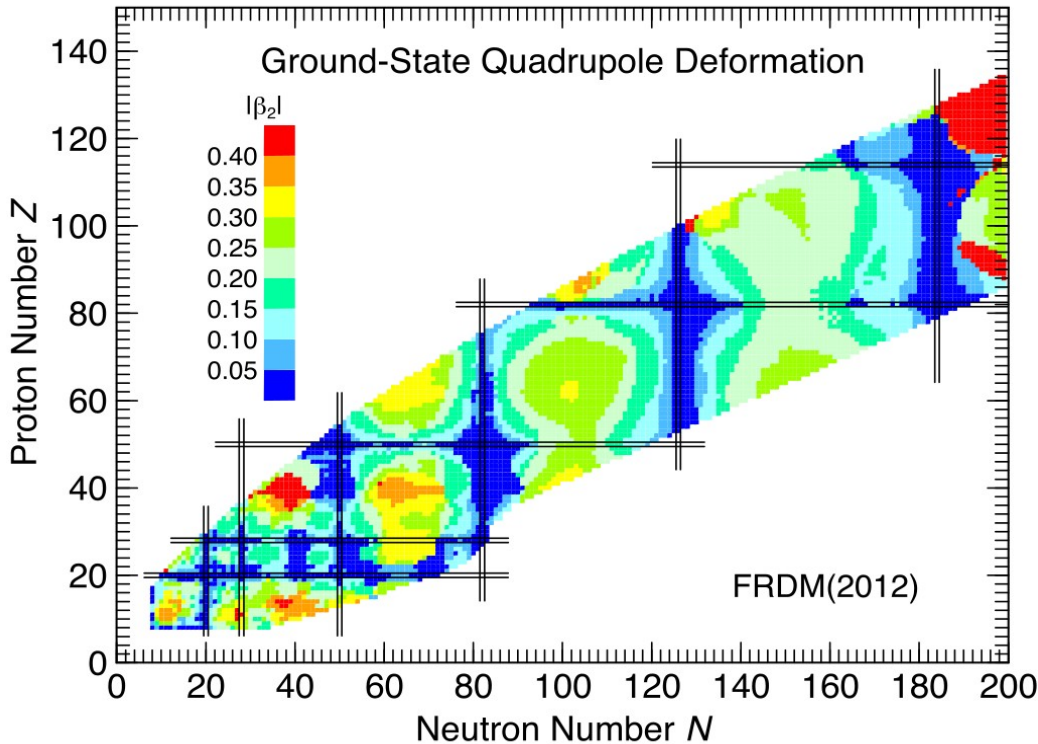
Coexistence of shapes?

not really

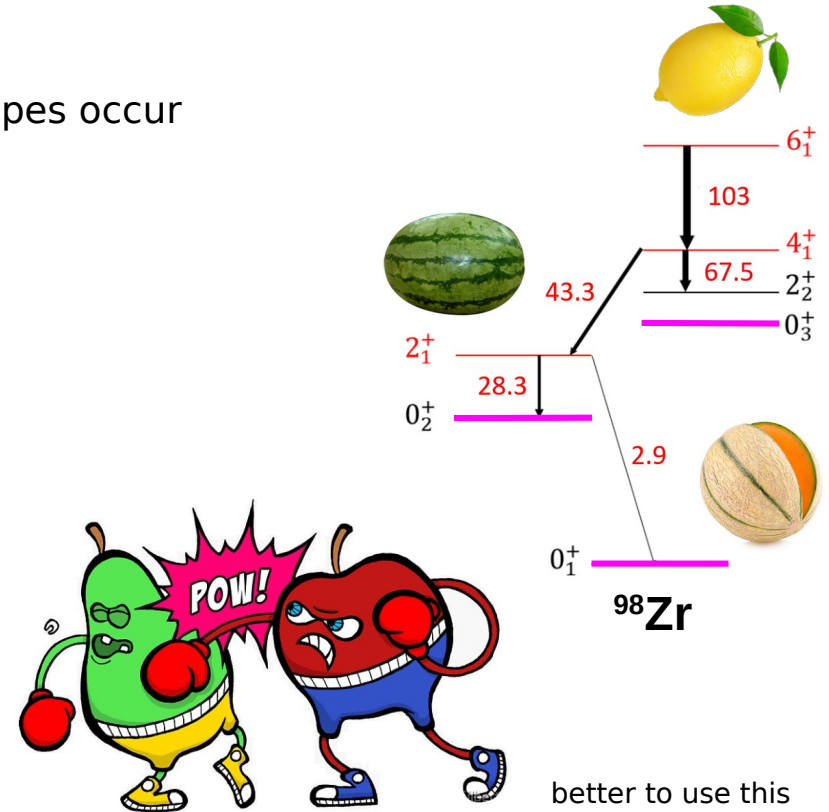


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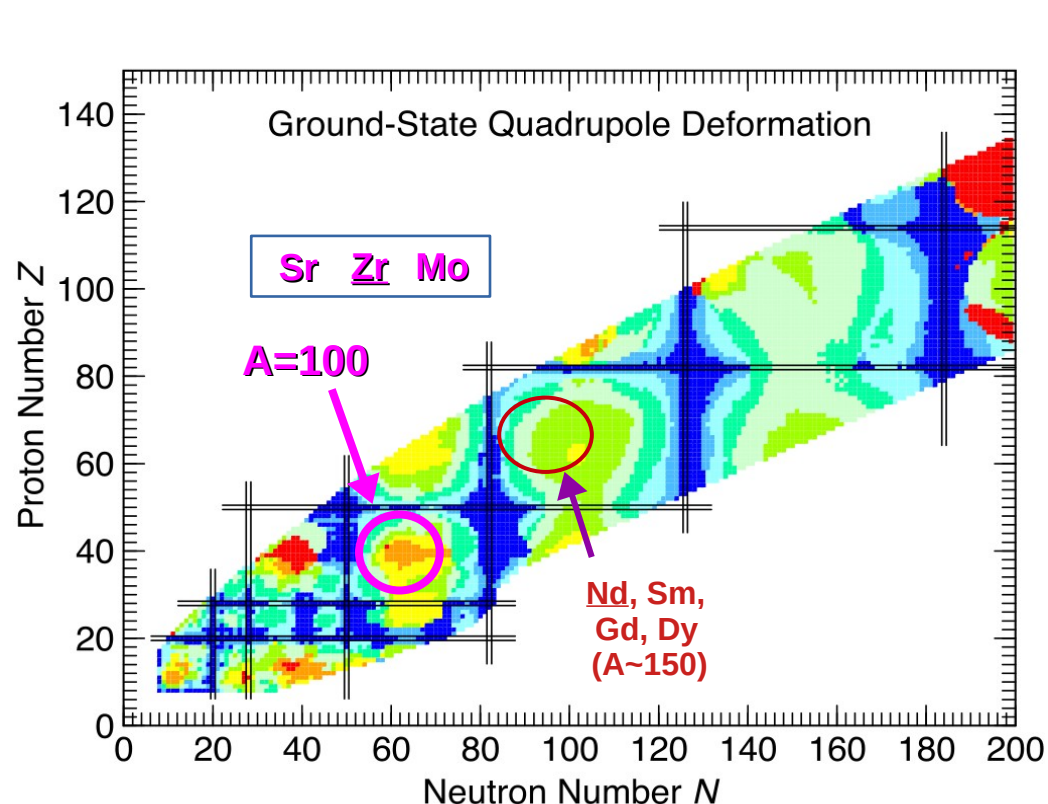
P. Möller, A.J. Sierk, T. Ichikawa, H. Sagawa, *Nuclear ground-state masses and deformations: FRDM(2012)*, Atomic Data and Nuclear Data Tables, 109-110, (2016),



Different configuration characterizing excited structure compete in energy to be the g.s. one

# Shape evolution in neutron-rich nuclei around A=100

- Drastic onset of deformation in the the rare-earth region at  $N \sim 90$  or in Zr-Sr region around  $N=60$ .



Manifested via the rapid change of several nuclear observables as a function of  $N$

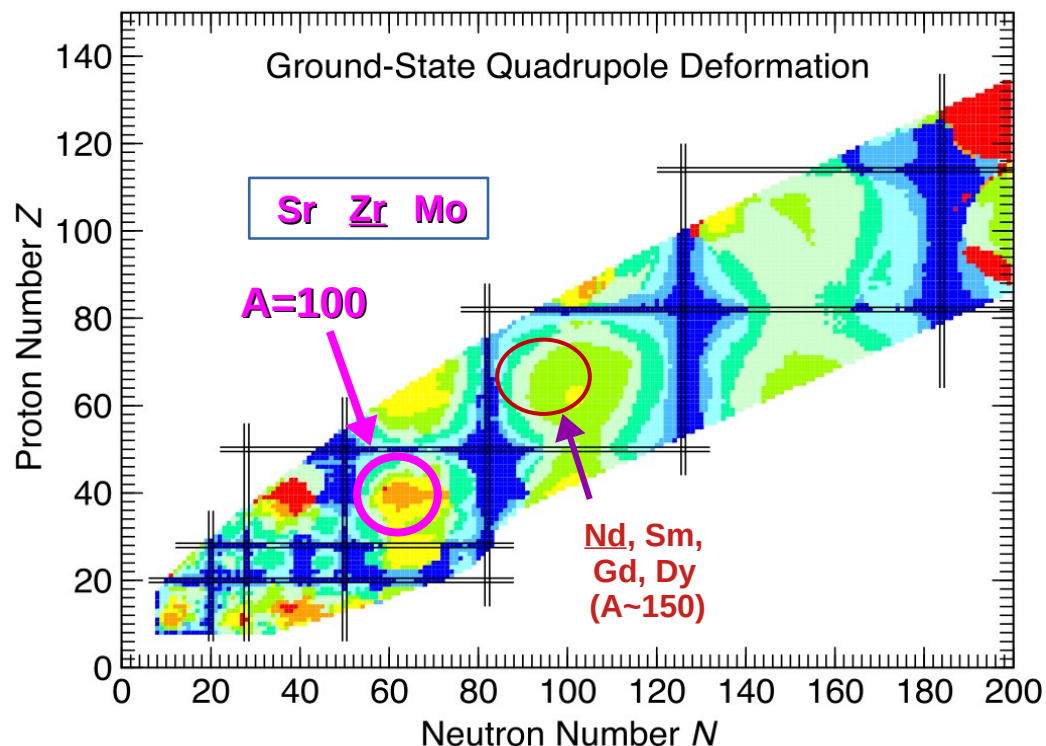
e.g:

two-neutron separation energies,  
energies  $E(J)$ ,  
energy ratios  $E(4_1^+)/E(2_1^+)$ ,  
transition probabilities  $B(E2:4_1^+ \rightarrow 2_1^+)$ ,

...

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Manifested via the rapid change of several nuclear observables as a function of N

e.g:

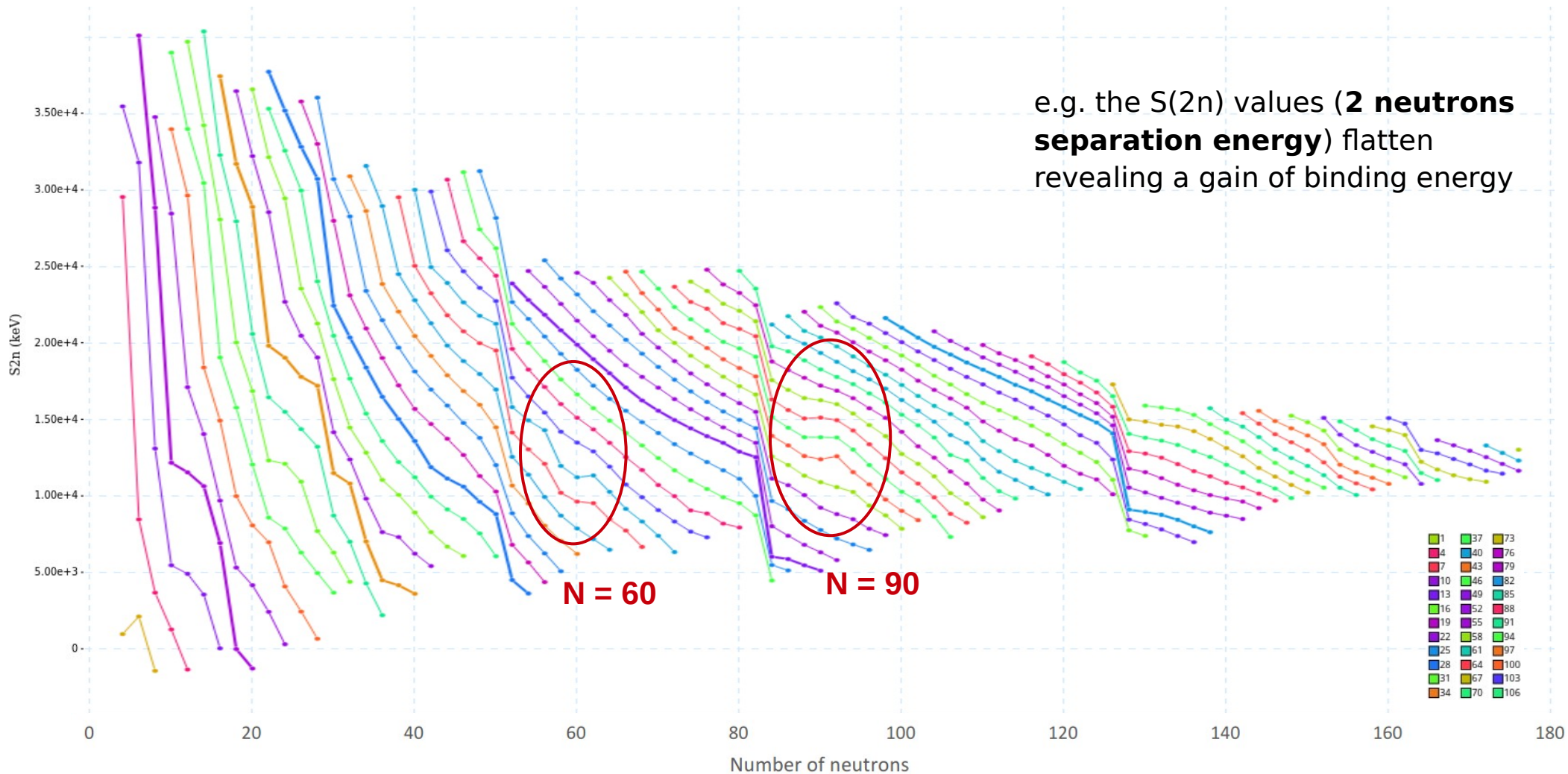
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...

The term **quantum phase transition** (QPT) is used to describe this phenomenon, due to the similarities with the thermodynamic phase transitions.

P. Cejnar, J. Jolie and R.F. Casten, *Rev. Mod. Phys.* 82, 2155 (2010).

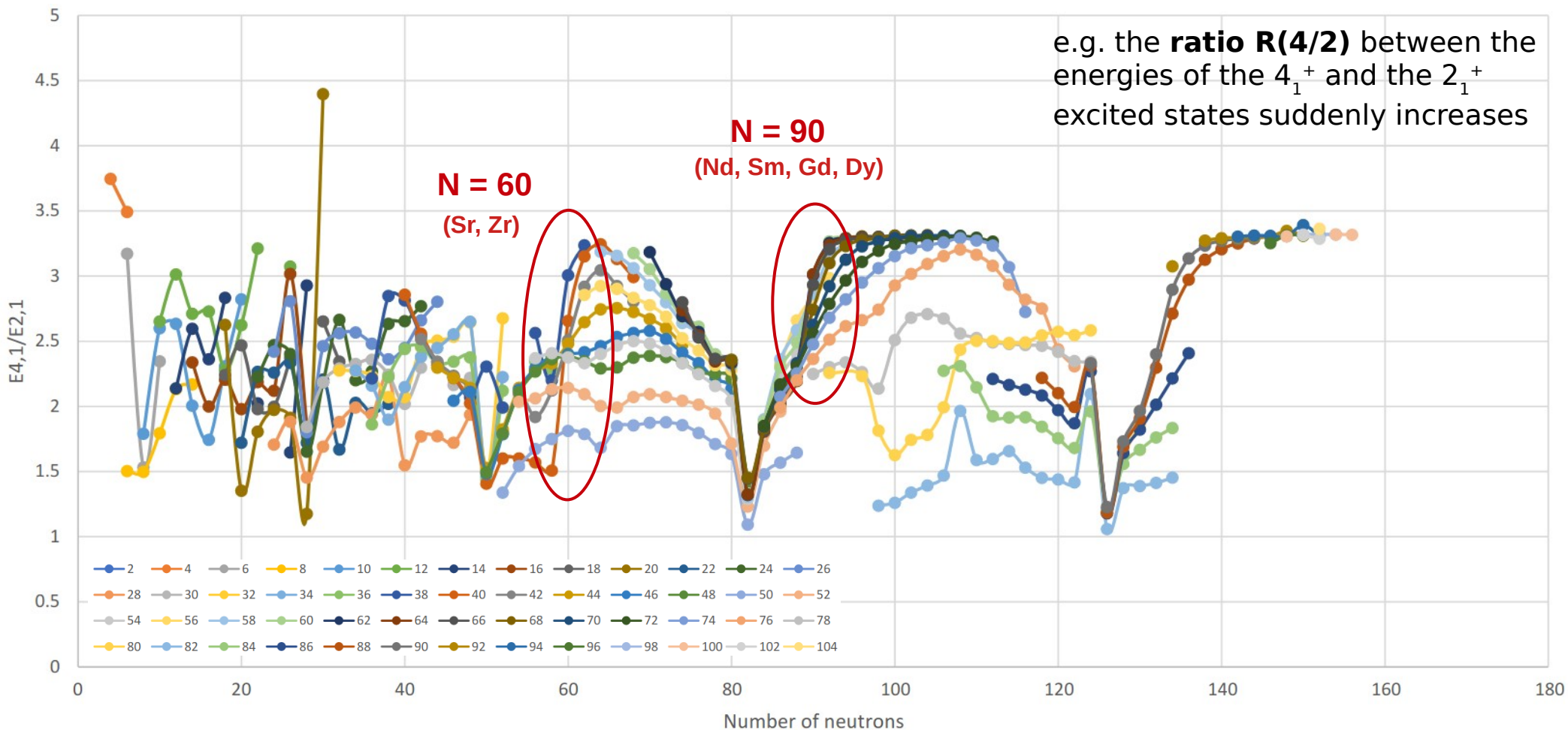
# Shape evolution in neutron-rich nuclei around A=100





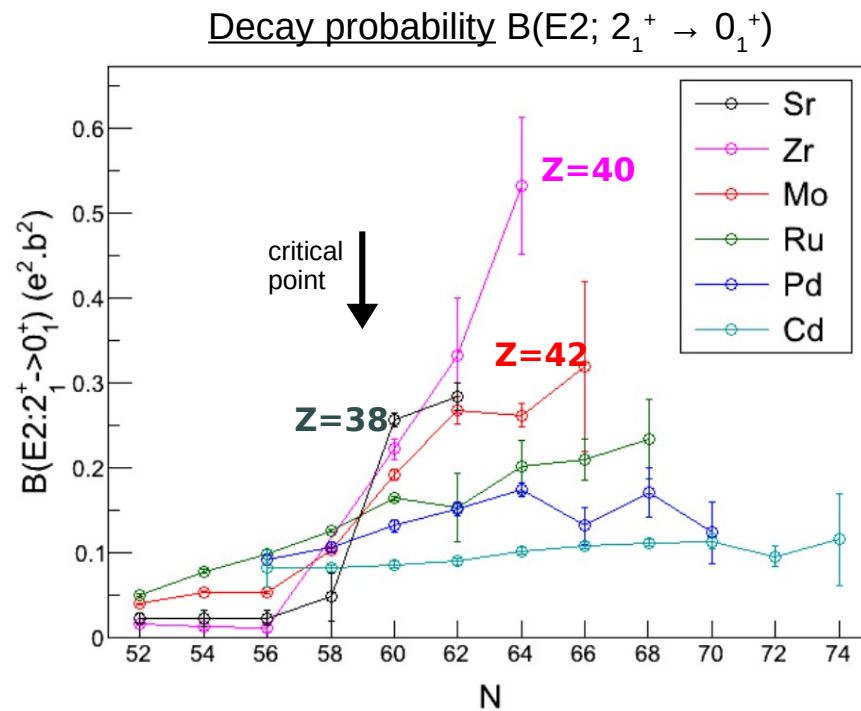
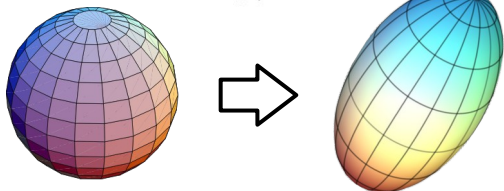
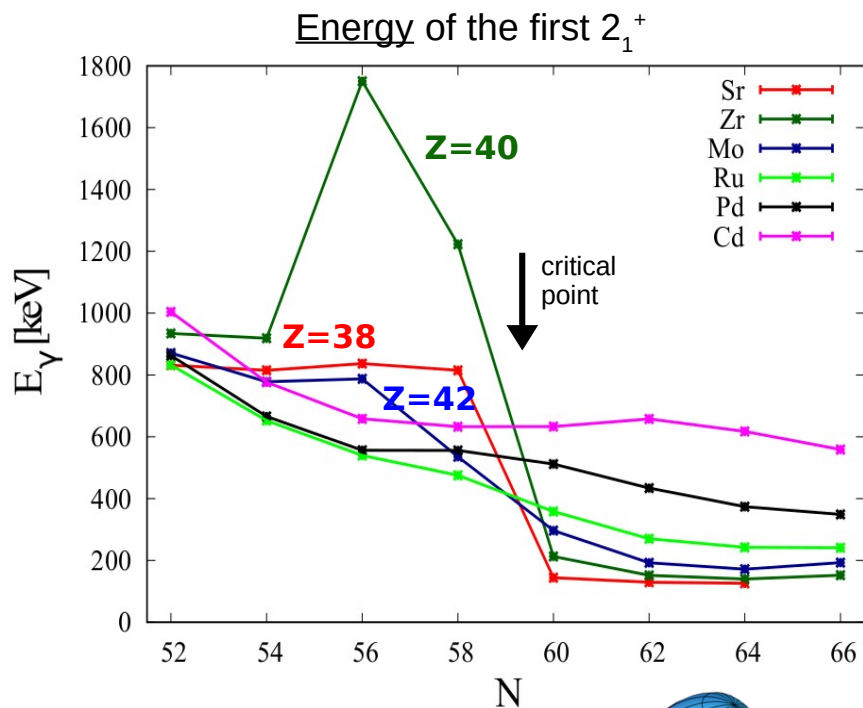
# Shape evolution in neutron-rich nuclei around A=100

e.g. the **ratio  $R(4/2)$**  between the energies of the  $4_1^+$  and the  $2_1^+$  excited states suddenly increases



# Shape evolution in neutron-rich nuclei around A=100

- **Drastic change** of  $2^+$  energies and  $B(E2)$  values as a function of N (and Z)



The strong dependence of nuclear properties on both N and Z makes the A~100 region an interesting test ground for various theoretical models.

# Microscopic interpretation

- Federman, Pittel and co-workers pointed out for the first time a microscopic approach within the framework of the shell-model

P. Federman and S. Pittel, "Towards a unified microscopic description of nuclear deformation," Phys. Lett. B 69, 385-388 (1977).

P. Federman, S. Pittel, and R. Campos, "Microscopic study of the shape transition in the zirconium isotopes," Phys. Lett. B 82, 9-12 (1979).

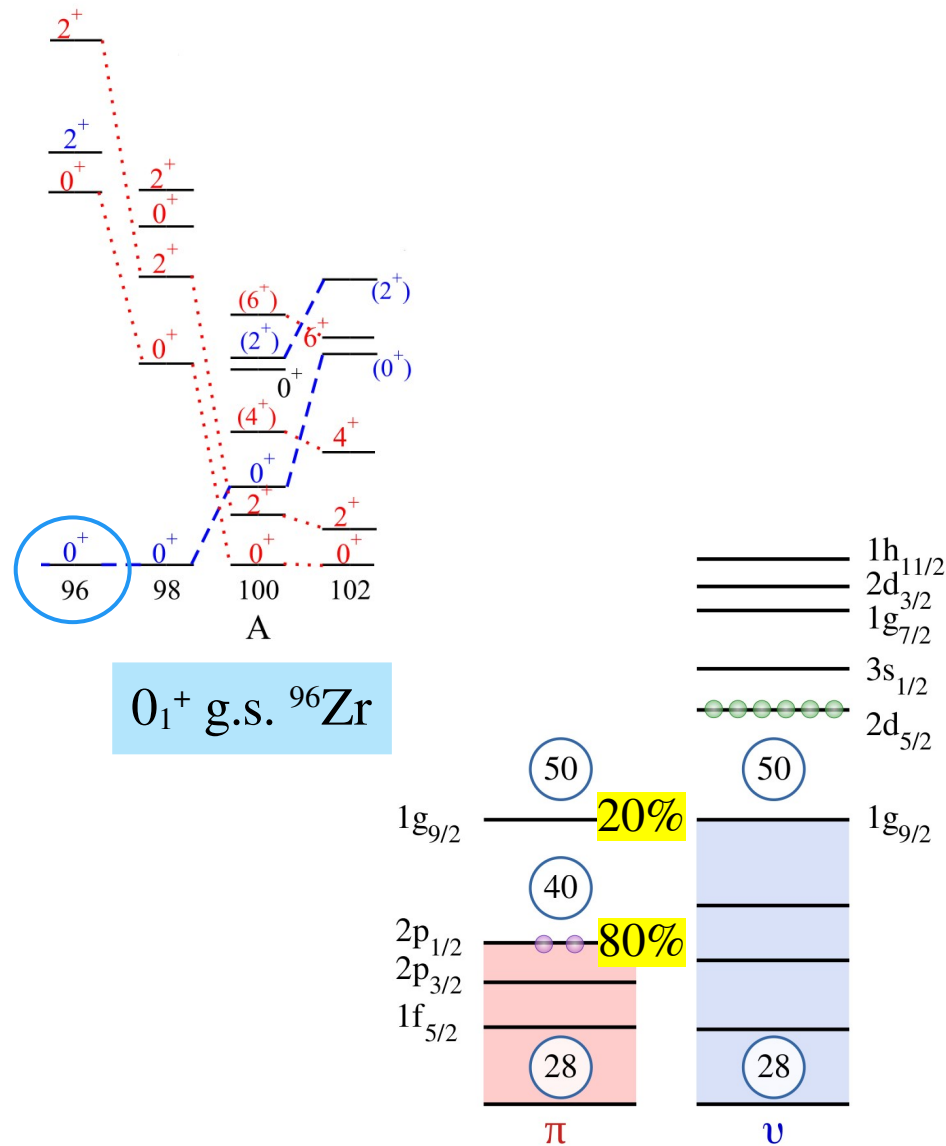
P. Federman and S. Pittel, "Unified shell-model description of nuclear deformation," Phys. Rev. C 20, 820-829 (1979).

P. Federman, S. Pittel, and A. Etchegoyen, "Quenching of the  $2p_{1/2}2p_{3/2}$  proton spin-orbit splitting in the Sr Zr region," Phys. Lett. B 140, 269-271 (1984).

K. Heyde, E. D. Kirchuk, and P. Federman, "Coexistence or strong-mixing of intruder  $0^+$  states in even-even Zr nuclei," Phys. Rev. C 38, 984-992 (1988).

A. Etchegoyen, P. Federman, and E. G. Vergini, "Importance of the neutron-proton interaction for Zr isotopes," Phys. Rev. C 39, 1130-1133 (1989).

S. Pittel, P. Federman, G. E. Arenas Peris, R. F. Casten, and W.-T. Chou, "Semiempirical determination of effective p-n monopole matrix elements." Phys. Rev. C 48, 1050 (1993).



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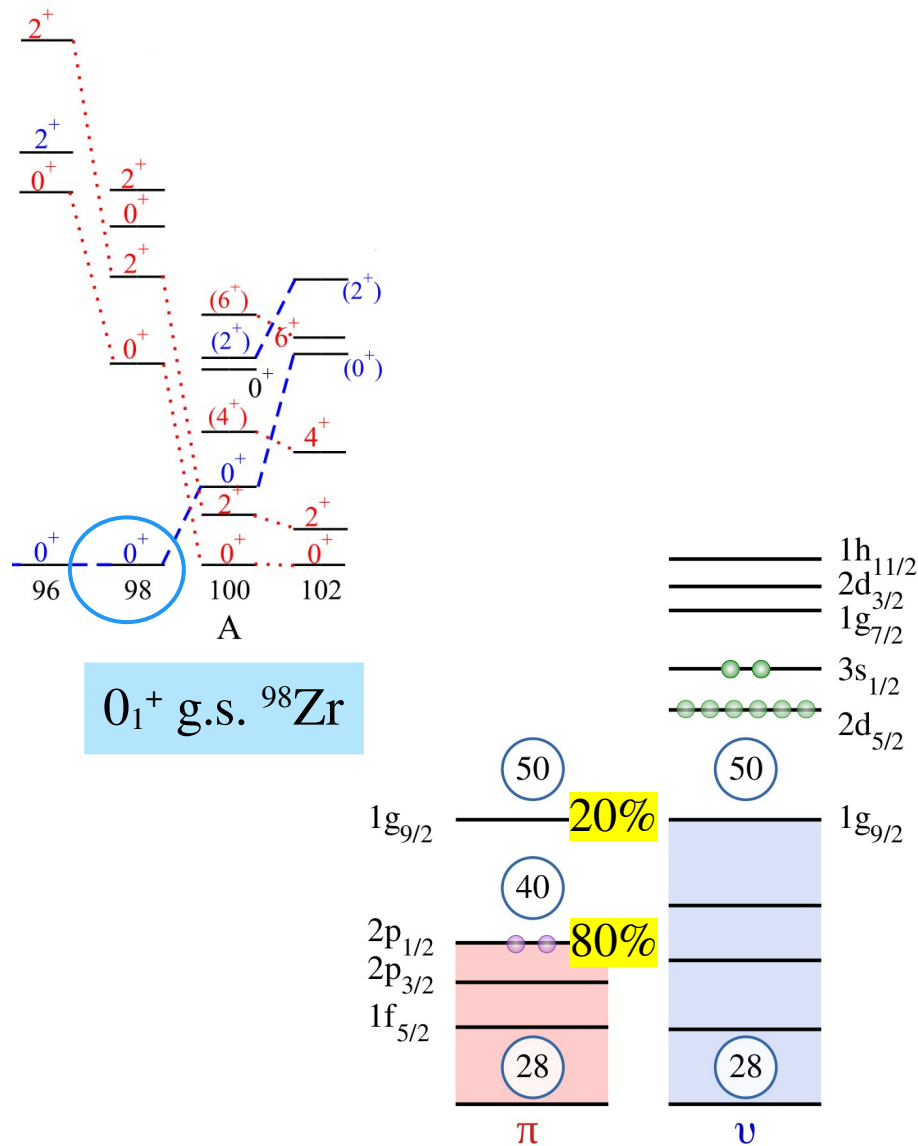
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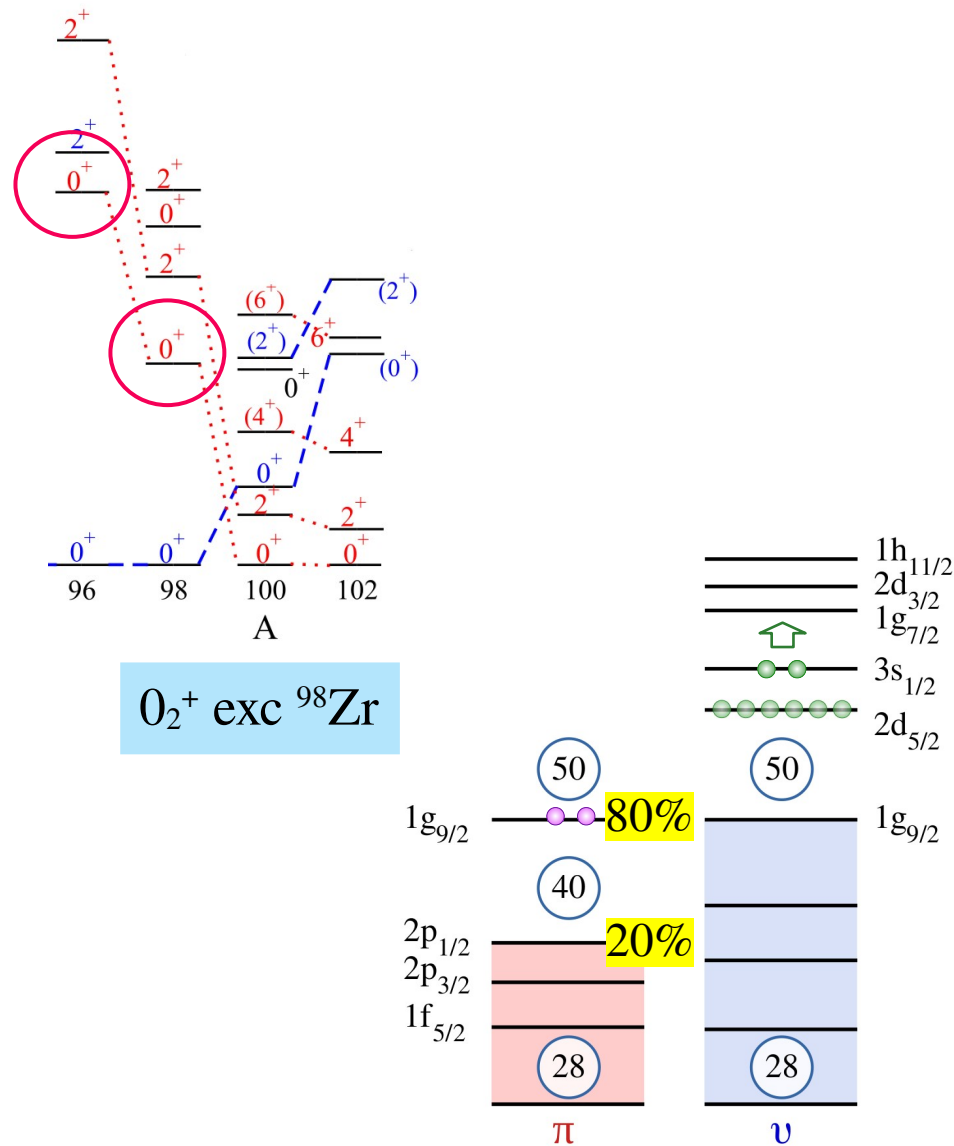
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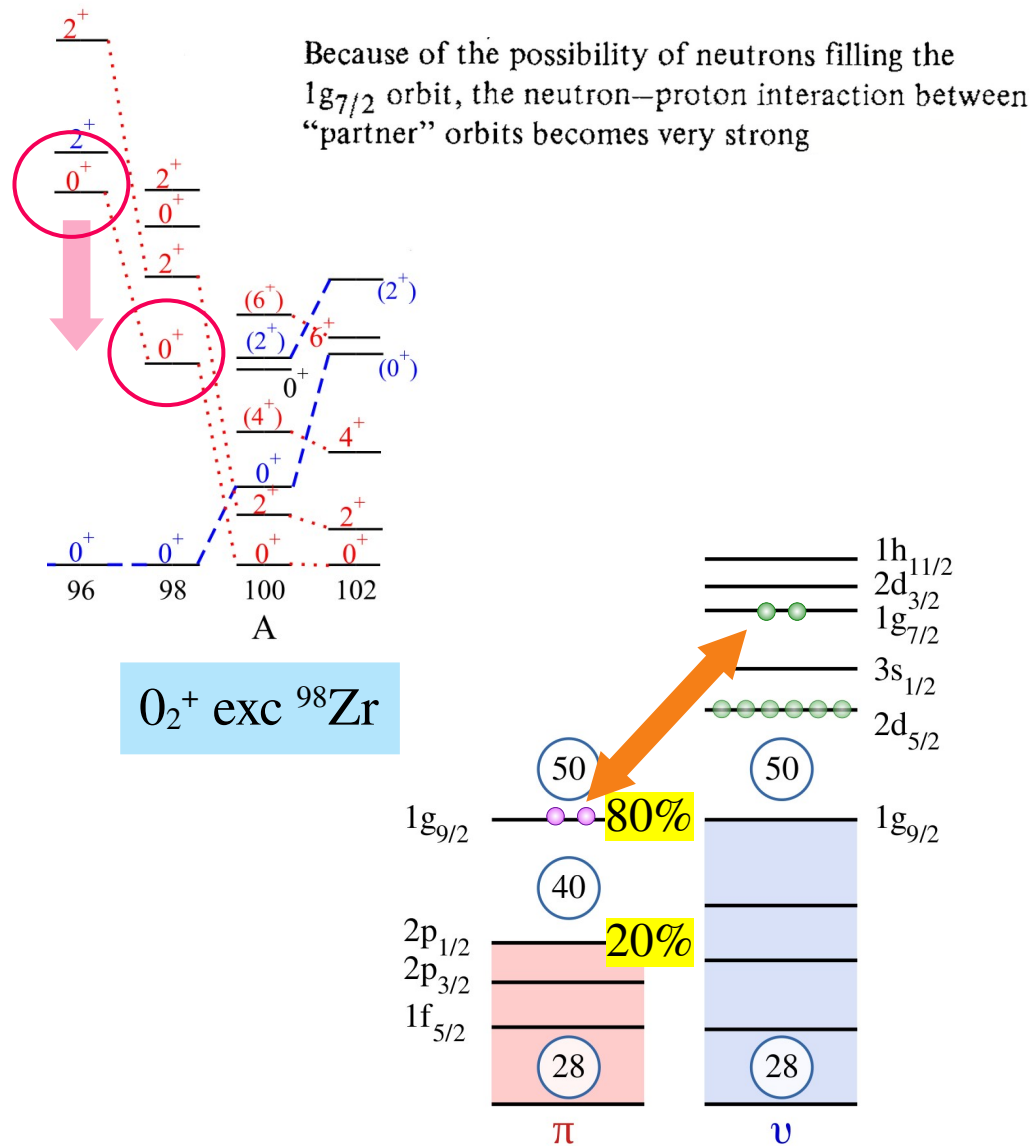
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The possibility that the lowering of the first excited  $0^+$  level might be closely related to the observed shape transition has motivated several efforts to understand its origin [5,6]. A coexistence of deformed and spherical shapes has been suggested for  $^{98}\text{Zr}$ ,

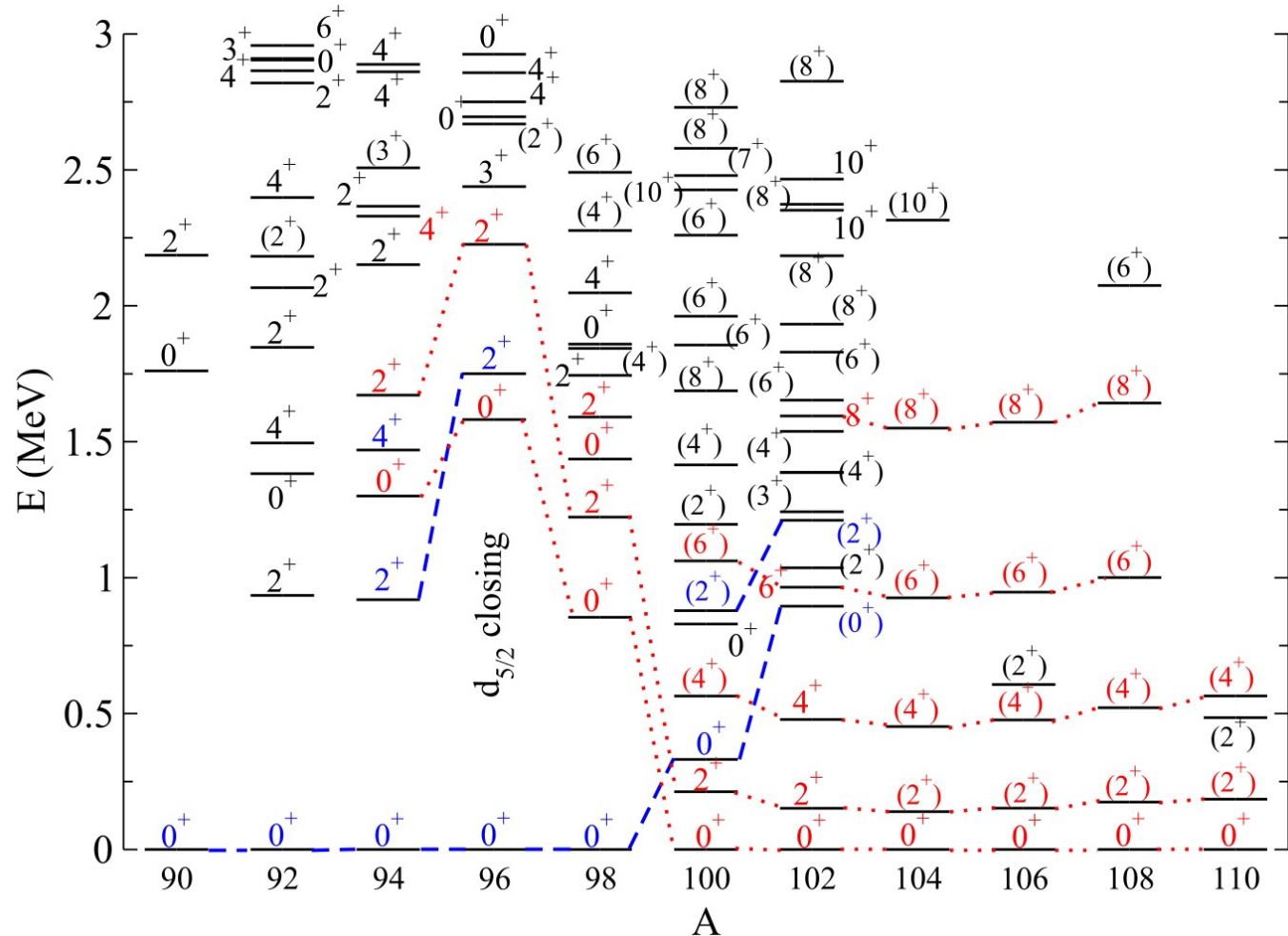
- Lowering of excited  $0^+$  states: shape transition and shape coexistence



Configuration A

Configuration B

- The drastic shape transition originates from the crossing of the **normal configuration** with an **intruder configuration** having a very different degree of deformation.
- Shape coexistence is predicted and observed in the transitional region, for  $^{96}\text{Zr}$ ,  $^{98}\text{Zr}$ ,  $^{100}\text{Zr}$ .





- Large theoretical effort have been invested to study this phenomenon

generator coordinate method (GCM):

J. Skalski, P.-H. Heenen, and P. Bonche, Nucl. Phys. A 559, 221 (1993).  
J.-P. Delaroche et al., PRC 81, 014303 (2010).

macroscopic-microscopic method:

J. Skalski, S. Mizuturo, and W. Nazarewicz, Nucl. Phys. A 617, 282 (1997).

Shell Model:

P. G. Reinhard, et al., PRC 60, 014316 (1999).  
A. Holt, T. Engeland, M. Hjorth-Jensen, and E. Osnes, PRC 61, 064318 (2000).  
K. Sieja, F. Nowacki, K. Langanke, and G. Martínez-Pinedo, PRC 79, 064310 (2009).  
Y.-X. Liu et al., Nucl. Phys. A 858, 11 (2011).

Shell Model Monte Carlo

C. Özen and D. J. Dean, PRC 73, 014302 (2006).

Monte Carlo Shell Model:

T. Togashi, Y. Tsunoda, T. Otsuka and N. Shimizu, PRL 117, 172502 (2016).

interacting boson model (IBM & IBM-CM):

J. E. García-Ramos, et al., Eur. Phys. J. A 26, 221 (2005).  
M. Büyükat, P. Van Isacker and İ. Uluer, J. Phys. G: Nucl. Part. Phys. 37, 105102 (2010).  
K. Nomura, R. Rodríguez-Guzmán, and L. M. Robledo, PRC 94, 044314 (2016).  
A. Vitturi, L. Fortunato, I. Inci, and J.A. Lay, JPS Conf. Proc. 23, 012013 (2018).  
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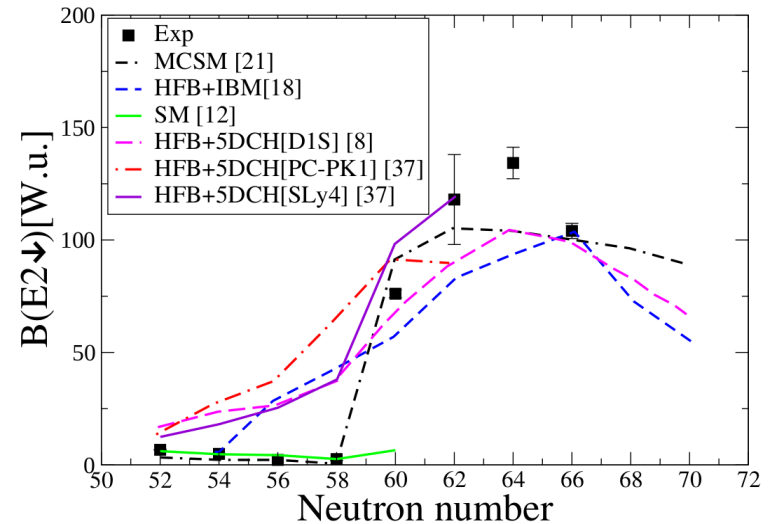
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- Large theoretical effort have been invested to study this phenomenon

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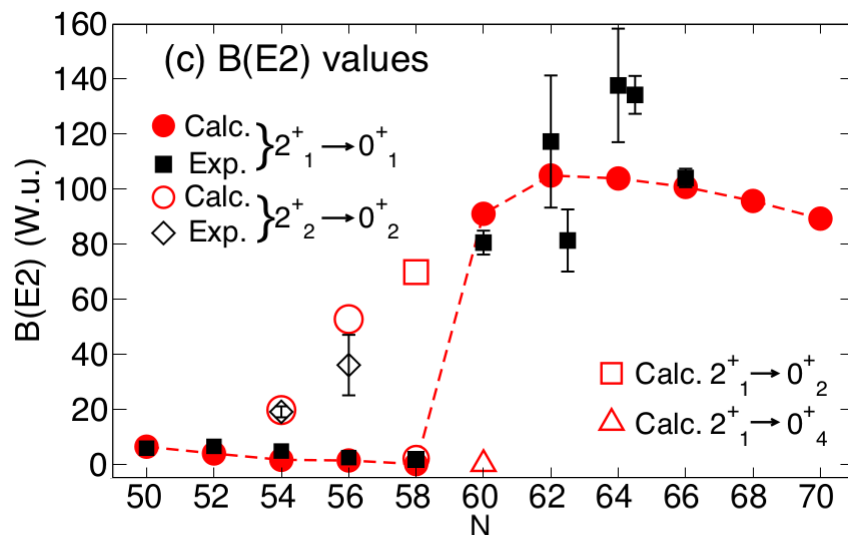
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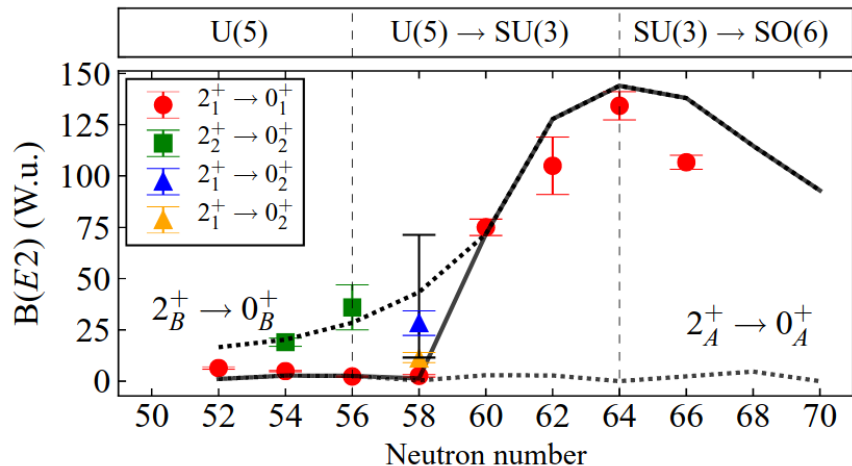
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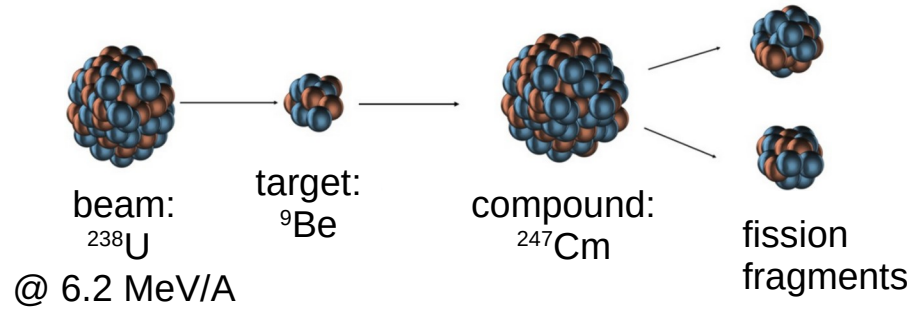
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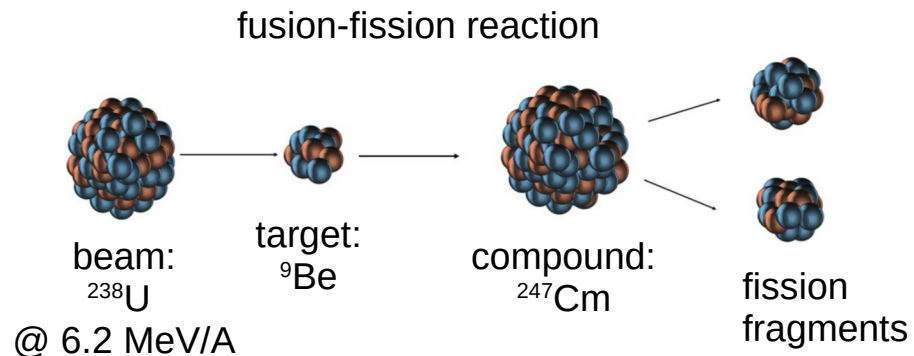
- Interacting Boson Model with Configuration Mixing (2022)



- INTRODUCTION :
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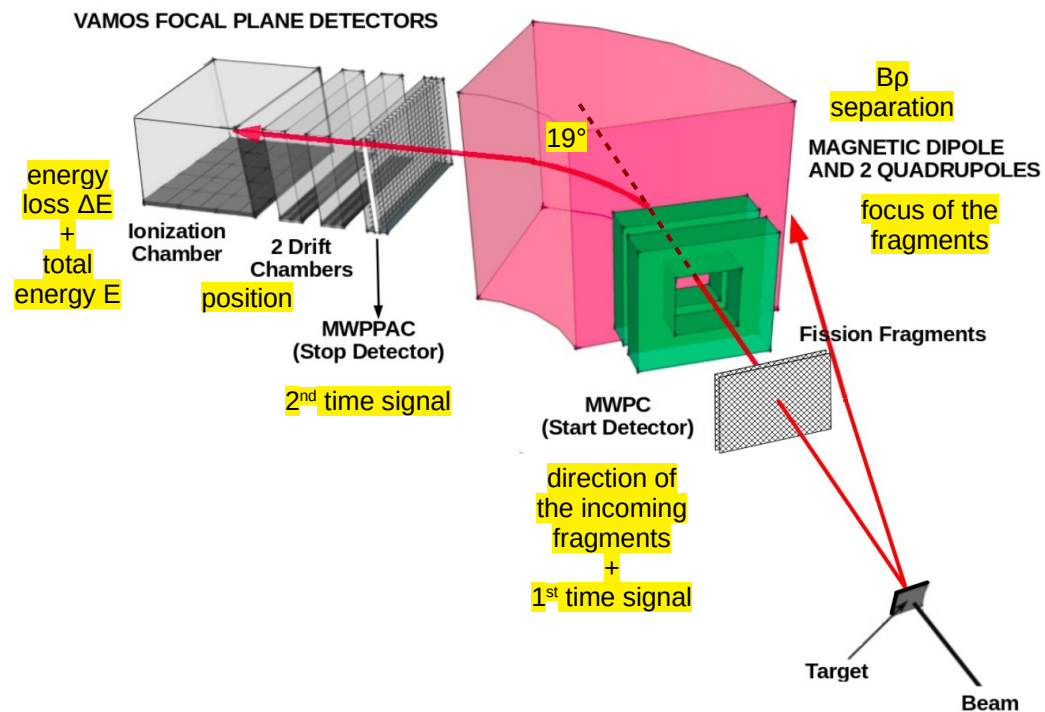
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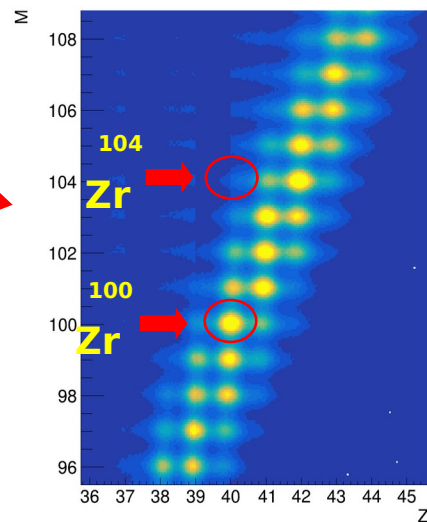
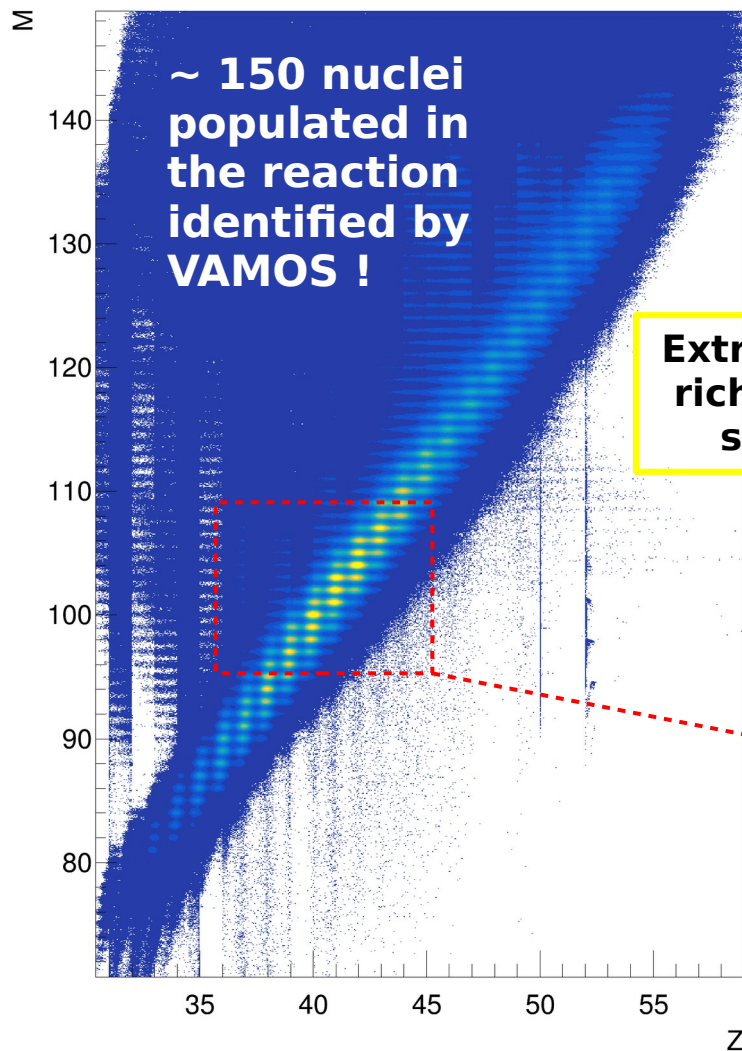




## VAMOS large acceptance magnetic spectrometer

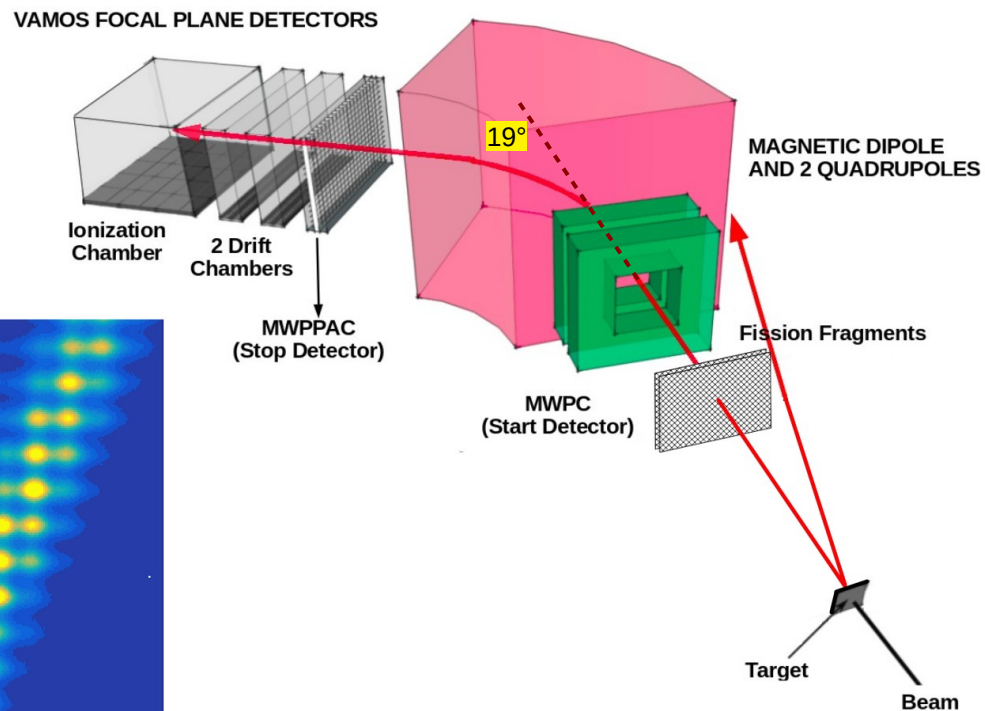
fragment identification based on atomic number  $Z$ , mass  $M$  and charge  $Q$





## VAMOS large acceptance magnetic spectrometer

fragment identification based on atomic number  $Z$ , mass  $M$  and charge  $Q$





Cable trays to preprocessing electronics

VAMOS electronics

AGATA LVPS

LN2 autofill

AGATA digitizers

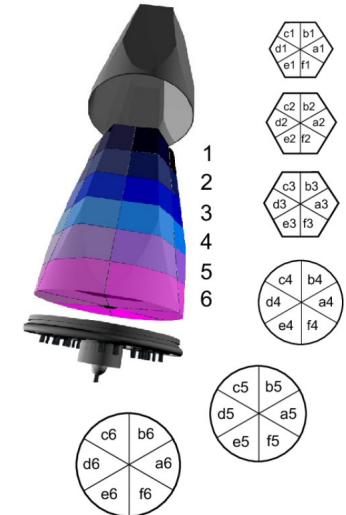
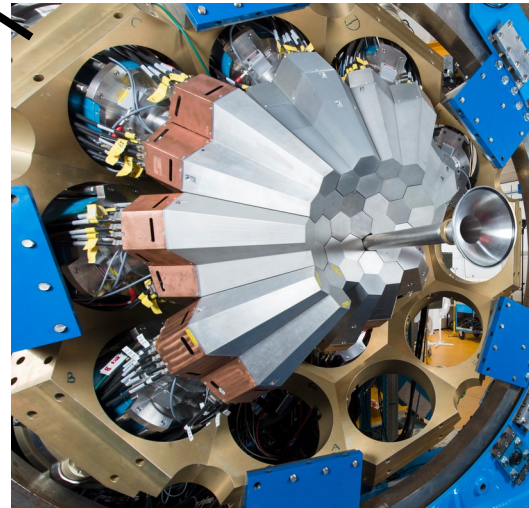
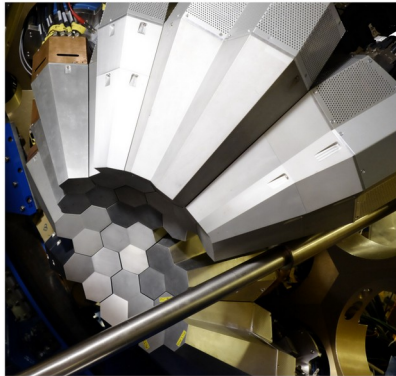
Beam line

LN2 distribution and ancillary racks

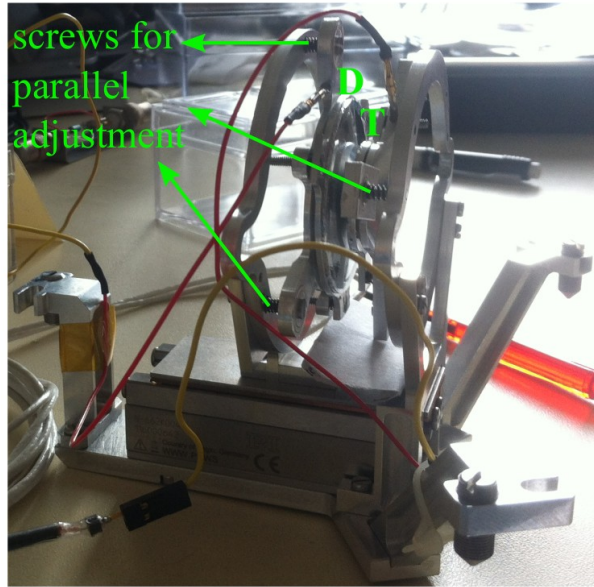


## Advanced Gamma Tracking Array

Excellent energy resolution of HPGe detectors  
+  
unprecedented photo-peak efficiency  
+  
Pulse Shape Analysis (PSA) and  $\gamma$ -ray tracking

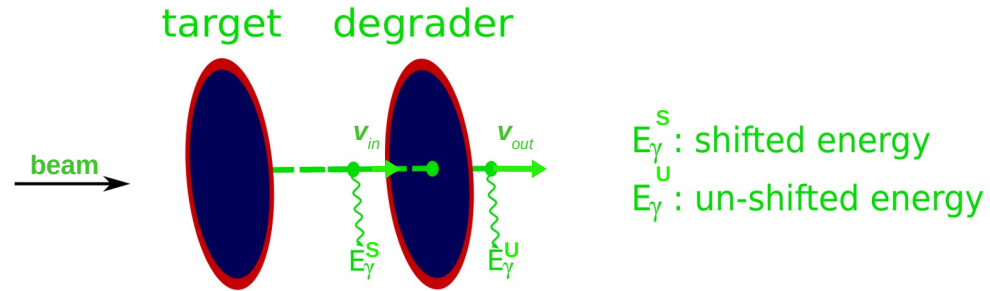


# Recoil Distance Doppler Shift technique



OUPS

Orsay Universal Plunger System





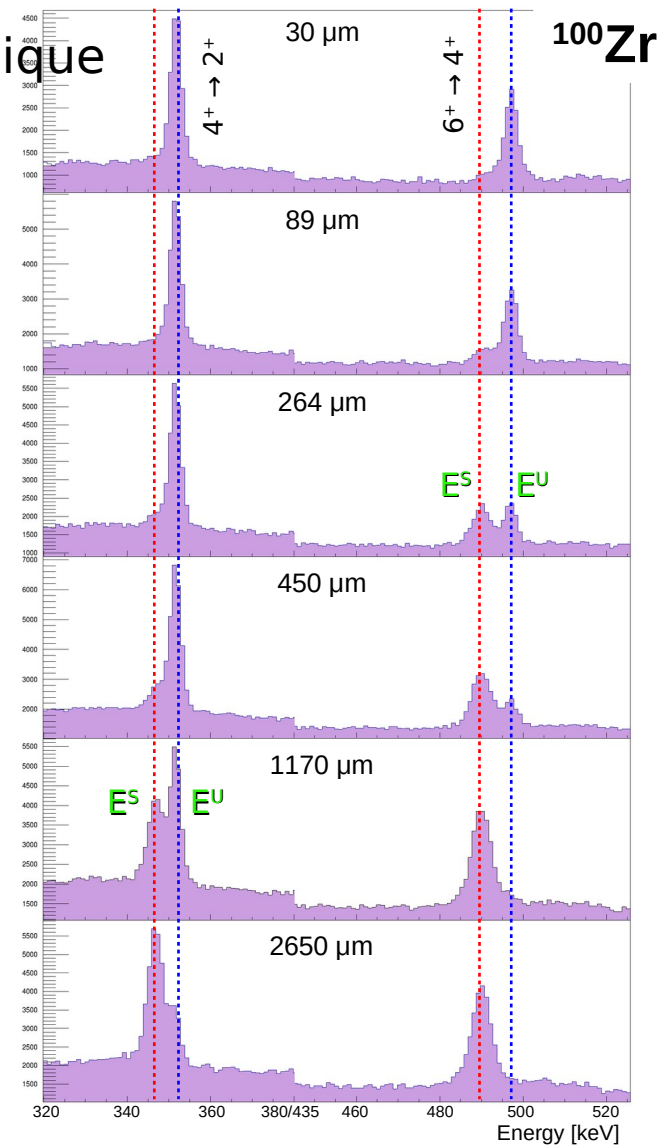
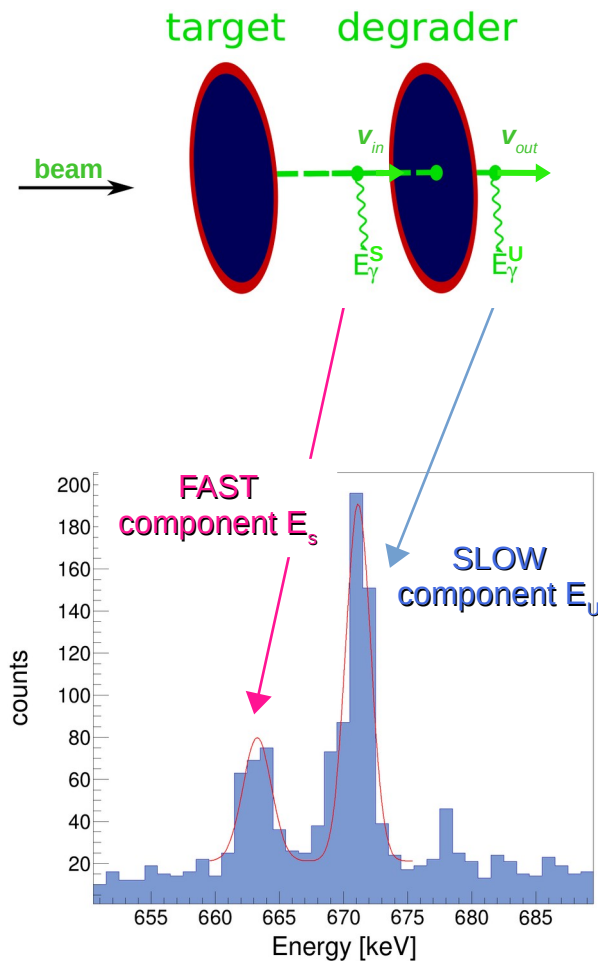
# Recoil Distance Doppler Shift technique

- the energy of the  $\gamma$  ray emitted in-flight is Doppler shifted:

$$E = E_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \Theta}$$

recoil  
velocity

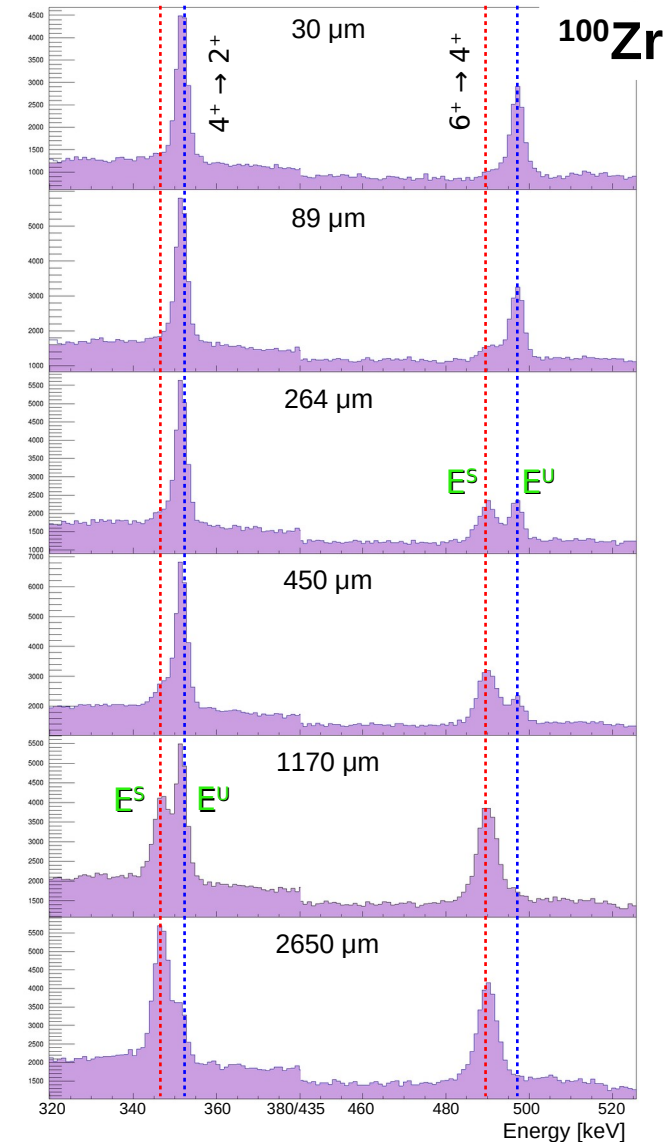
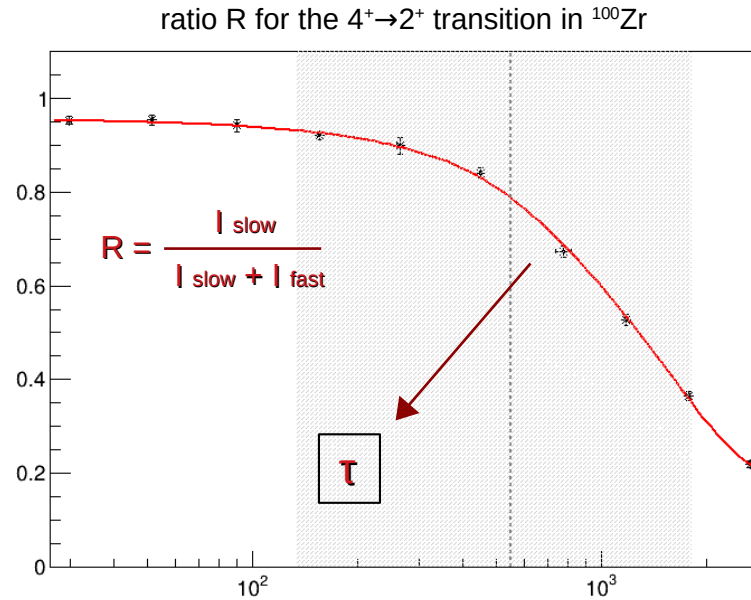
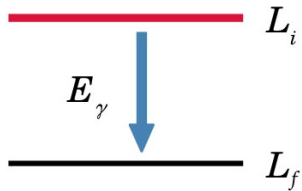
angle of  
emission



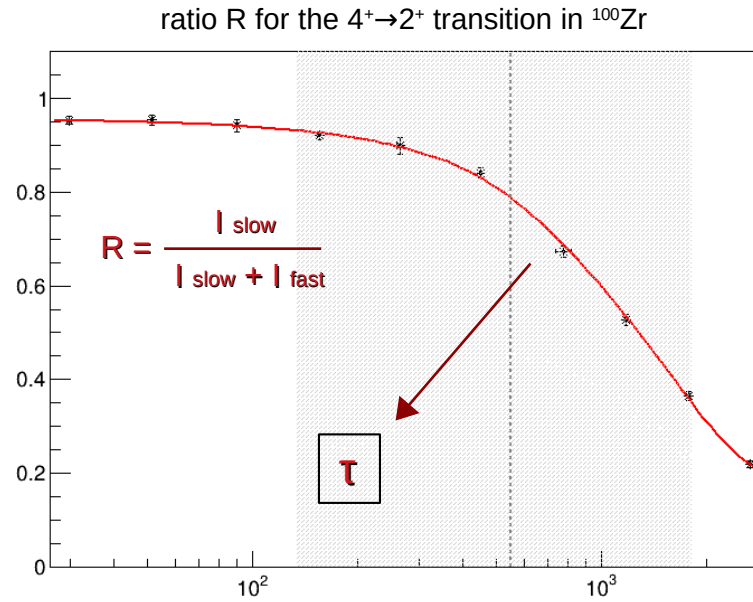
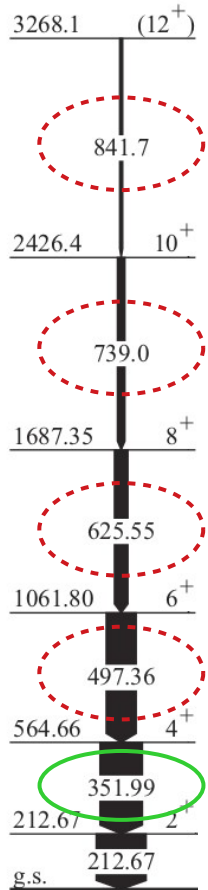
# Decay Curve Method DCM

- the evolution of fast and slow components as a function of the distance provides the lifetime of the state
- simple two-level case:

$$n(t) = N_0 e^{-\frac{t}{\tau}}$$

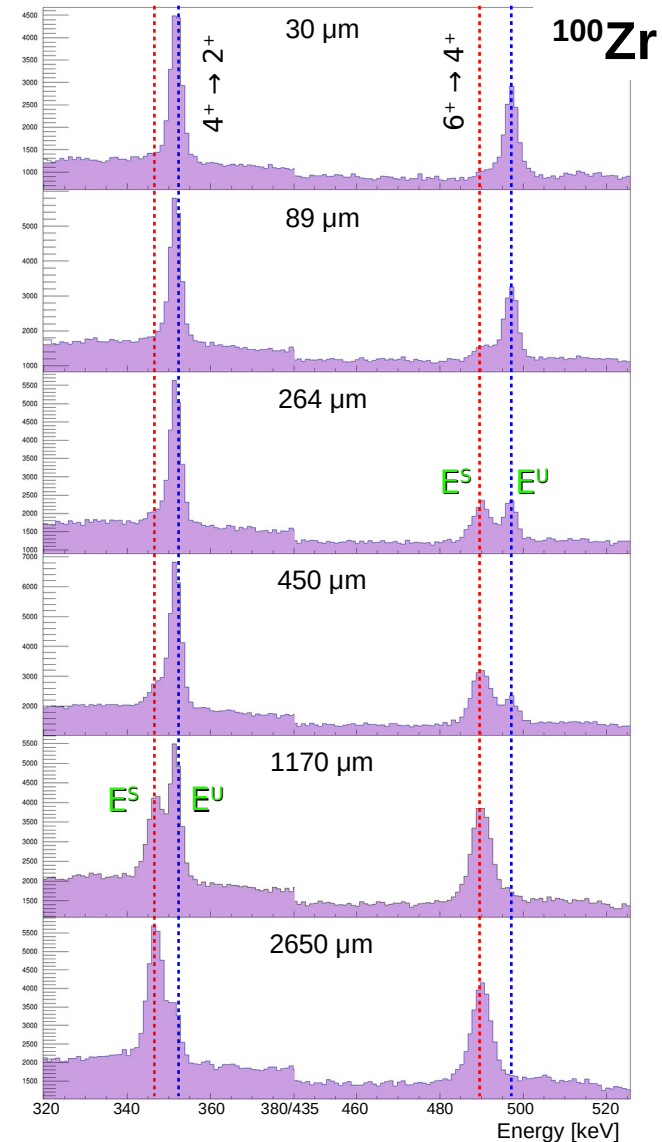


# Decay Curve Method DCM

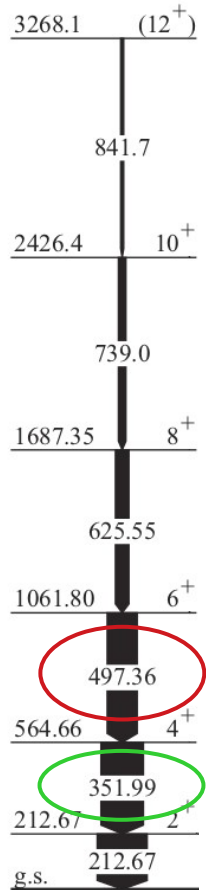


- using the Bateman equation as solution :

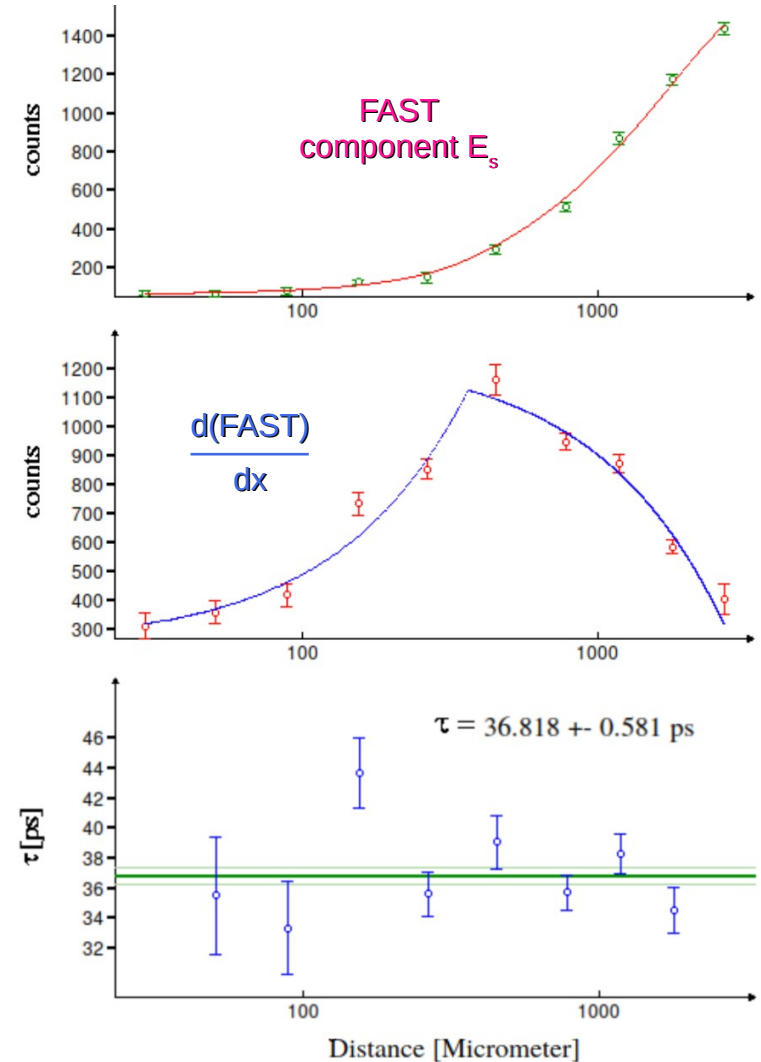
$$n_i(t) = \sum_{k=1}^i \left[ n_k(0) \times \left( \prod_{j=k}^{i-1} \lambda_j \right) \times \left( \sum_{j=k}^i \frac{e^{-\lambda_j t}}{\prod_{p=k, p \neq j}^i (\lambda_p - \lambda_j)} \right) \right]$$



# Differential Decay Curve Method DDCM



- Lifetime are analyzed with the **Differential Decay Curve Method (DDCM)** :
  - it deal with feeding problems
  - existence of a sensitive region
  - no assumption on the fitting curve shape
  - absolute distances are not needed



# Differential Decay Curve Method DDCM

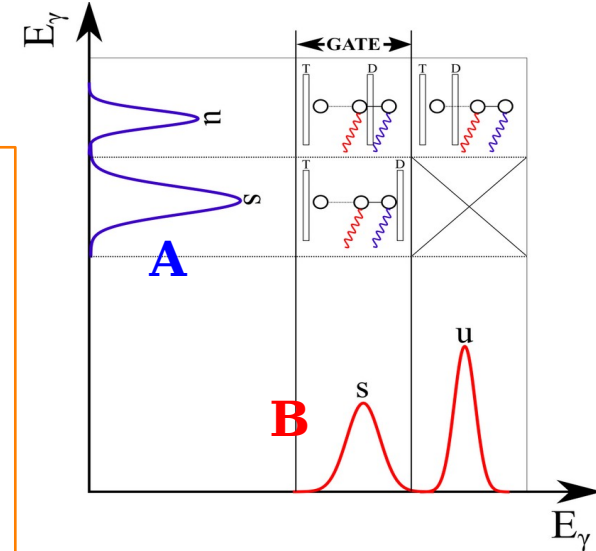
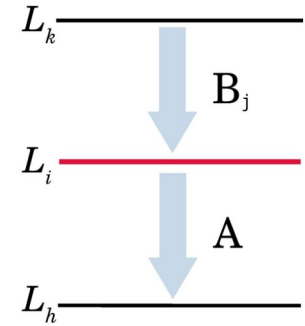
- Lifetime in single  $\gamma$  measurements are analyzed with the DDCM by using the following experimental information:

- the fast (S) and the slow (U) components of the transition A depopulating the state of interest  $L_i$
- the fast and the slow components of all observed feeding transitions  $B_j$  in the  $\gamma$ -rays spectrum (the  $\alpha$  coefficient take into account the feeding contribution form all  $B_j$  in the lifetime calculation).
- the velocity of the recoiling fragment before the target  $v_{in}$ .

$$\tau(x_p) = \frac{-A^U(x_p) + \sum_j b_j \alpha_j B_j^U(x_p)}{v_{in} \frac{dA^U(x_p)}{dx}} \quad \alpha_j(x_p) = \frac{B_j^U(x_p) + B_j^S(x_p)}{A^U(x_p) + A^S(x_p)} \cdot \frac{\epsilon_A}{\epsilon_B}$$

- Lifetime in coincidence  $\gamma\gamma$  are analyzed with the DDCM by gating in the shifted component of a direct feeding transition B of the state of interest. No other information about the feeding are needed

$$\tau(x_p) = \frac{\{A_S, B_U\}(x_p)}{\frac{d}{dx} \{A_S, B_S\}(x_p)} \cdot \frac{1}{\beta c}$$



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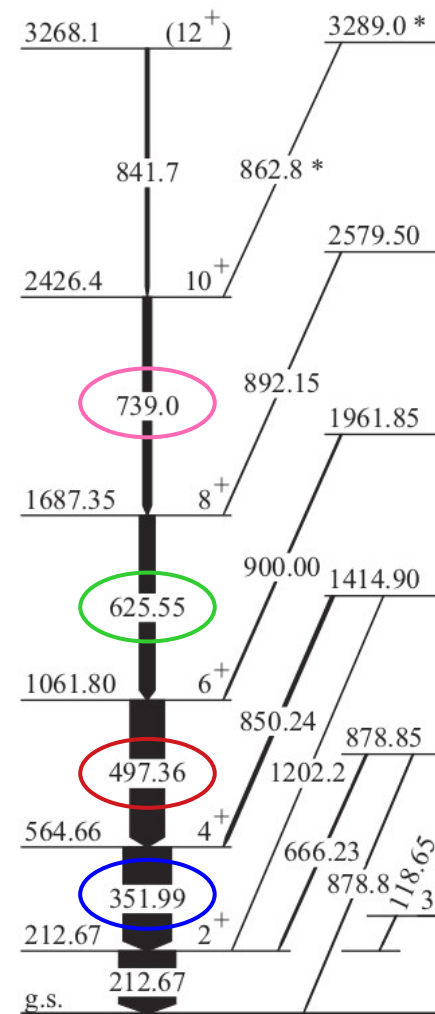
# Lifetime measurements in 100Zr

- Preliminary results: lifetime in gamma single and gamma-gamma coincidence.
- Comparison with previous results for the levels 4<sup>+</sup>, 6<sup>+</sup>, 8<sup>+</sup>, 10<sup>+</sup> of the yrast band :
  - The adopted value for the 4<sup>+</sup> may be overestimated. } unseen feeding ?
  - The 6<sup>+</sup> and 8<sup>+</sup> adopted lifetimes also result larger. }
  - Measurements in yy gives shorter lifetime for the 4<sup>+</sup>, as expected.
  - The lifetime of the 8<sup>+</sup> is accurate in single  $\gamma$  due to the short-living feeding.

J <sup><math>\pi</math></sup>	Energy [keV]	$\tau$ [ps] adopted*	$\tau$ [ps] single $\gamma$	$\tau$ [ps] coincid yy
2 <sup>+</sup>	212.7	574 (15)	/	/
4 <sup>+</sup>	352.0	53.4 (6)	36.9 (6) **	34 (3)
6 <sup>+</sup>	497.4	7.5 (1.6)	6.1 (3)	6.4 (8)
8 <sup>+</sup>	625.6	2.5 (2)	1.3 (2)	1.7 (4)
10 <sup>+</sup>	739.0	0.53 (6)	0.7 (2)	/

\* EVALUATED NNDC, <https://www.nndc.bnl.gov/nudat3/>

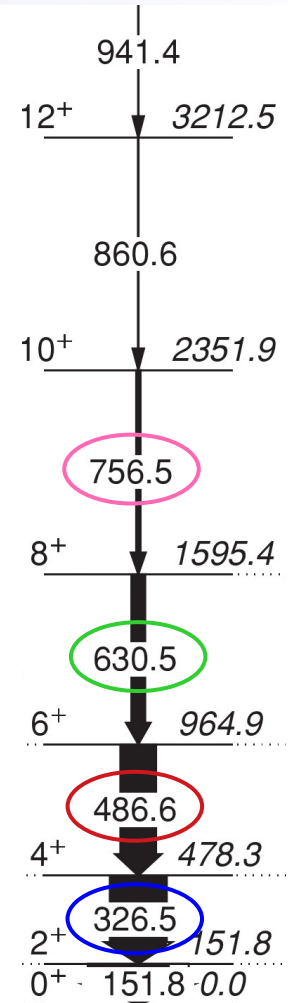
\*\* The feeding transition at 850.2 keV is not considered : difficult to resolve its shifted component from the 841.7 keV 12<sup>+</sup> → 10<sup>+</sup>.



# Lifetime measurements in $^{102}\text{Zr}$

- Preliminary results: lifetime in gamma single and gamma-gamma coincidence.
- Comparison with previous results for the levels  $4^+$ ,  $6^+$ ,  $8^+$ ,  $10^+$  of the yrast band :
  - New results for the  $4^+$  and  $6^+$  excited states.
  - Measurements in  $\gamma\gamma$  gives shorter but compatible lifetime for the  $4^+$ .
  - Lifetime of the  $8^+$  and the  $10^+$  states agree with the adopted values.

$J^\pi$	Energy [keV]	$\tau$ [ps] adopted*	$\tau$ [ps] single $\gamma$	$\tau$ [ps] coincid $\gamma\gamma$
<b>2+</b>	151.8	1800 (400)	/	/
<b>4+</b>	326.5	/	46 (1)	<b>42 (4)</b>
<b>6+</b>	486.5	/	5.5 (3)	<b>6 (1)</b>
<b>8+</b>	630.1	1.39 (21)	1.2 (2)	<b>2.5 (10)</b>
<b>10+</b>	756.6	0.53 (10)	<b>1.3 (5)</b>	/



\* EVALUATED NNDC, <https://www.nndc.bnl.gov/nudat3/>

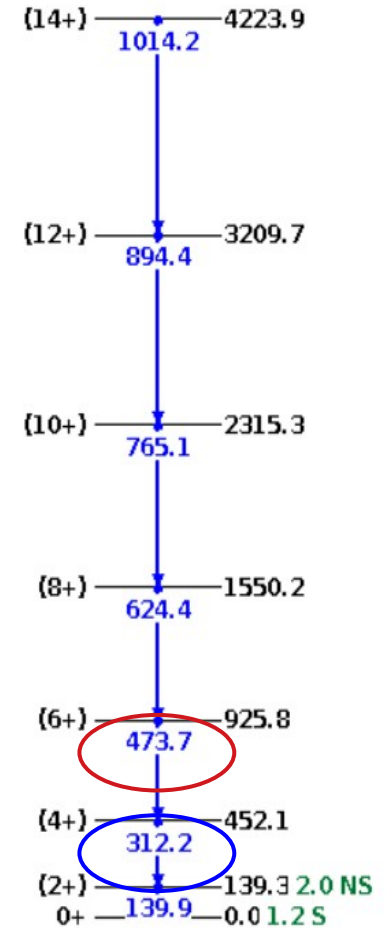


# Lifetime measurements in $^{104}\text{Zr}$

- Preliminary results: lifetime in gamma single.
- Limited amount of statistics (especially at higher energy)
- Measurements possible only in  $\gamma$  single.
- New results for the  $4+$  and the  $6+$  excited states.

$J^\pi$	Energy [keV]	$\tau$ [ps] adopted*	$\tau$ [ps] single $\gamma$	$\tau$ [ps] coincid $\gamma\gamma$
<b>2+</b>	139.9	2000 (300)	/	/
<b>4+</b>	312.3	/	<b>43 (5)</b>	/
<b>6+</b>	473.7	/	<b>4 (2)</b>	/

PRELIMINARY

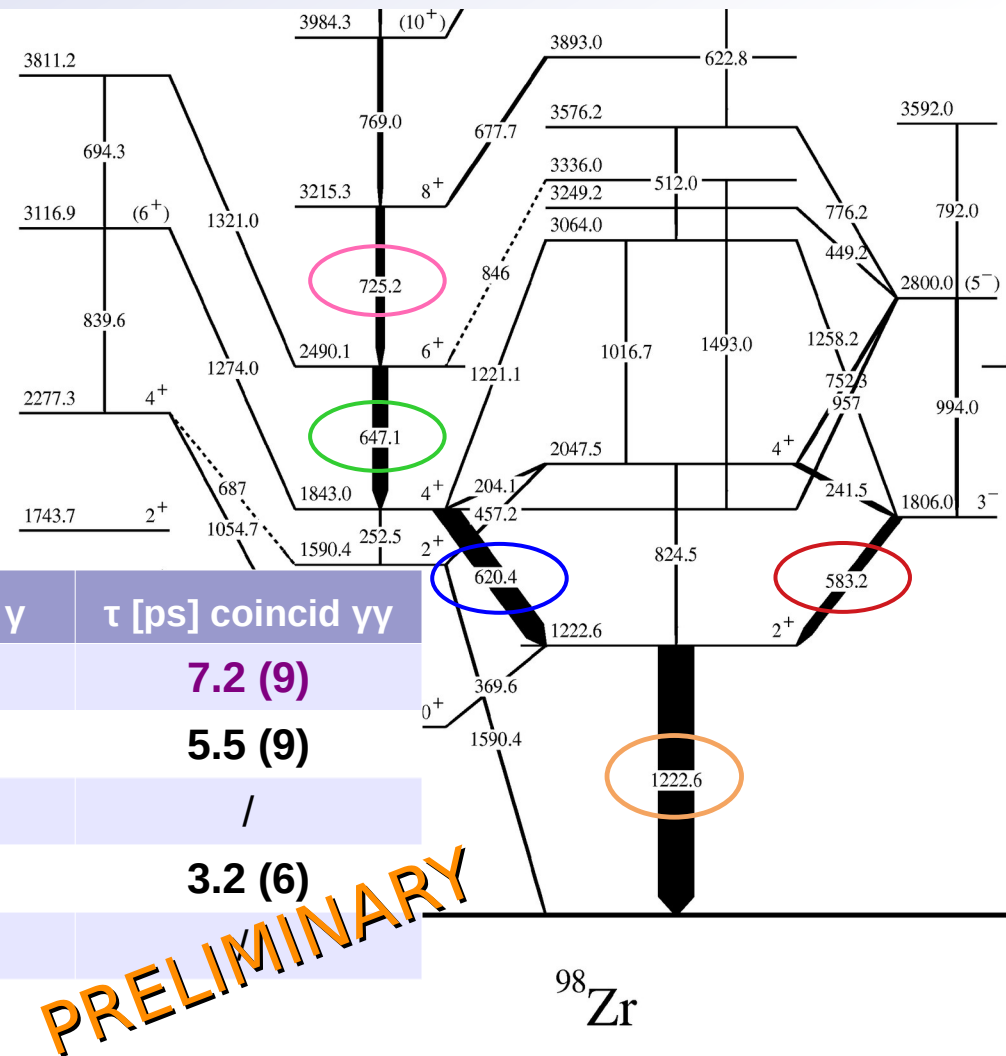


\* EVALUATED NNDC, <https://www.nndc.bnl.gov/nudat3/>

# Lifetime measurements in $^{98}\text{Zr}$

- Preliminary results: lifetime in gamma single and gamma-gamma coincidence.
- Complex level scheme.
- Comparison with previous results.
  - The lifetime of the  $2^+$  and the  $4^+$  are difficult to estimate in  $\gamma$  single because of feedings.
  - The  $6^+$  and  $8^+$  lifetimes are in agreement with the adopted values.

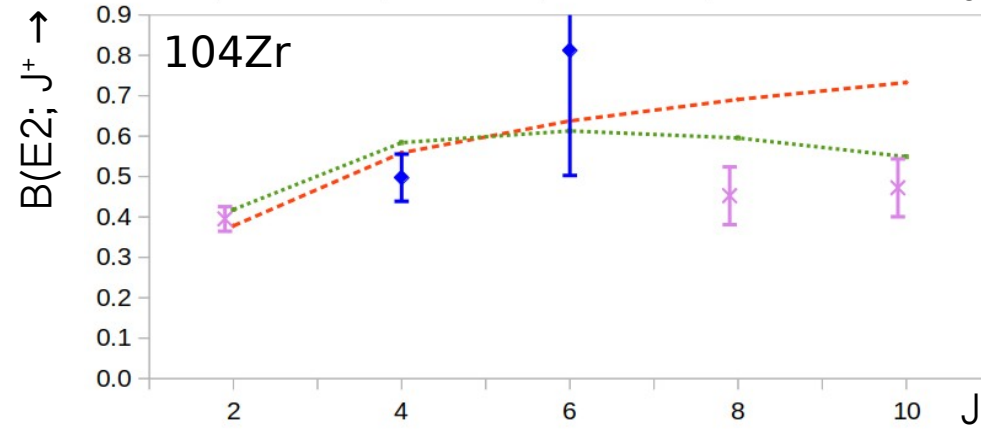
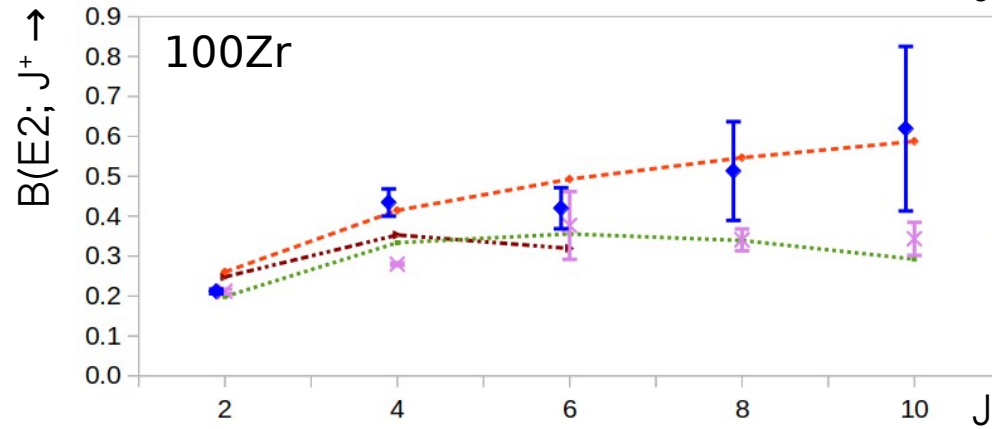
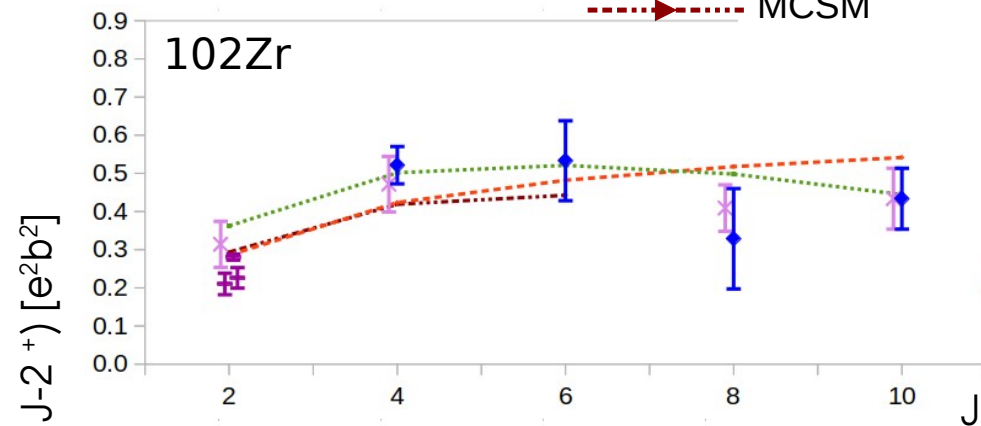
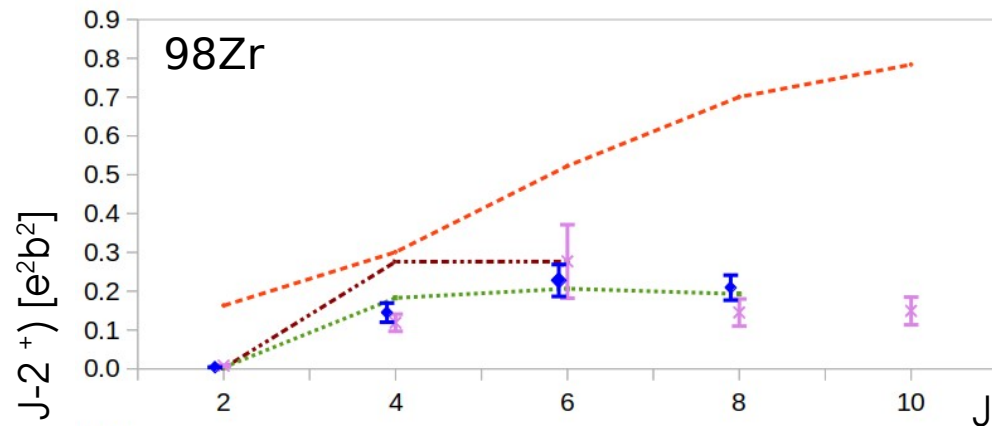
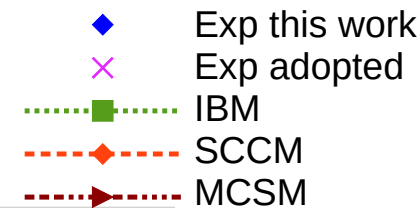
$J^\pi$	Energy [keV]	$\tau$ [ps] adopted*	$\tau$ [ps] single $\gamma$	$\tau$ [ps] coincid $\gamma\gamma$
<b>2+</b>	1222.9	3.79 (79)	...	<b>7.2 (9)</b>
<b>4+</b>	620.5	7.5 (14)	/	<b>5.5 (9)</b>
<b>3-</b>	583.3	/	<b>13 (4)</b>	/
<b>6+</b>	647.6	2.60 (89)	2.8 (3)	<b>3.2 (6)</b>
<b>8+</b>	725.4	2.81 (68)	<b>2.0 (3)</b>	



\* EVALUATED NNDC, <https://www.nndc.bnl.gov/nudat3/>

# Comparison with theoretical predictions

- Interacting Boson Model with Configuration Mixing (IBM-CM) [N. Gavrielov et al., PRC 105, 014305 \(2022\).](#)
- Monte-Carlo Shell Model (MCSM) [T. Togashi et al, PRL 117, 172502 \(2016\).](#)
- BMF within the Symmetry Conserving Configuration Mixing method (HFB-SCCM) [T. Rodriguez. \( L.M. Robledo, T.R. Rodríguez, and R.R. Rodríguez-Guzmán, Nuclear and Particle Physics 46.1, 013001 \(2018\).\)](#)





# Thanks for listening

G. Pasqualato<sup>1</sup>, A. Görgen<sup>2</sup>, J.S. Heines<sup>2</sup>, J. Ljungvall<sup>1</sup>,  
V. Modamio<sup>2</sup>, L.G.. Pedersen<sup>2</sup>, and W. Korten<sup>3</sup>

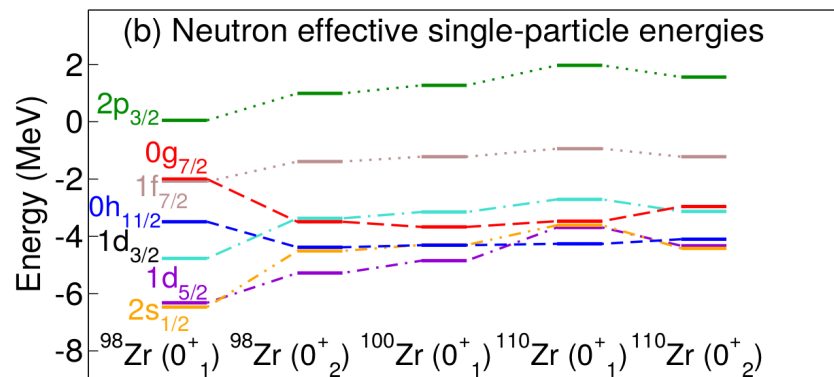
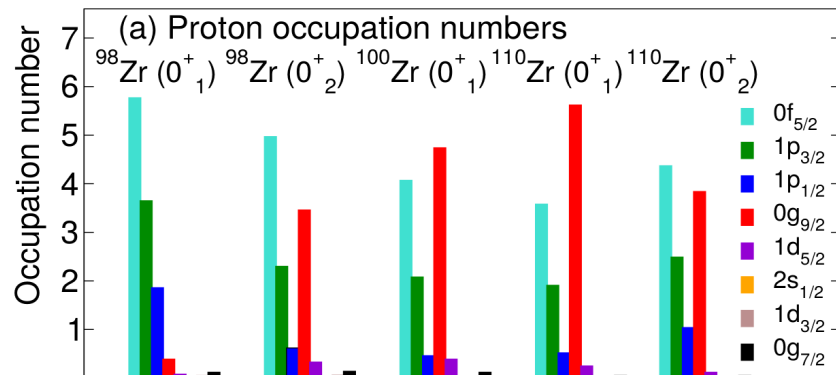
<sup>1</sup> *IJCLab, IN2P3/CNRS, Université Paris-Saclay, Orsay, France.*

<sup>2</sup> *Department of Physics, University of Oslo, Norway. and*

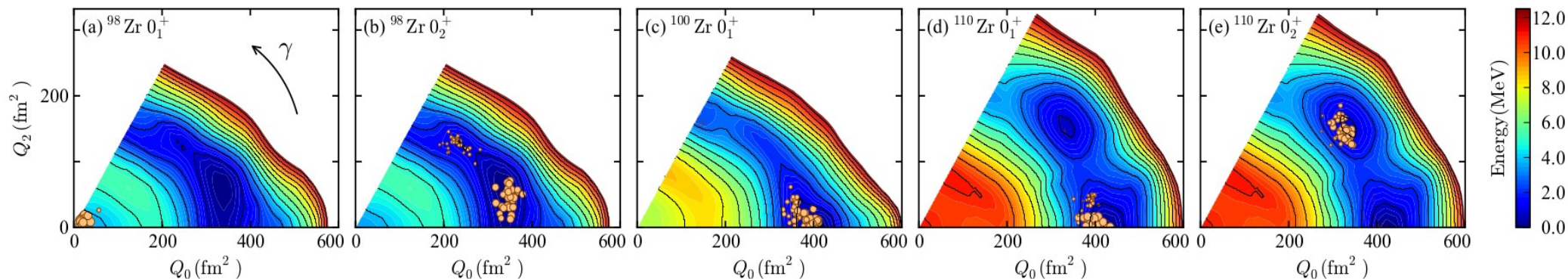
<sup>3</sup> *CEA Paris-Saclay, DRF/IRFU/DPhN, Gif-sur-Yvette, France.*

# Shape-phase transition in Zr isotopes

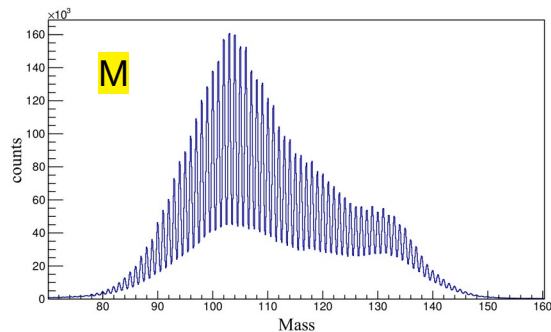
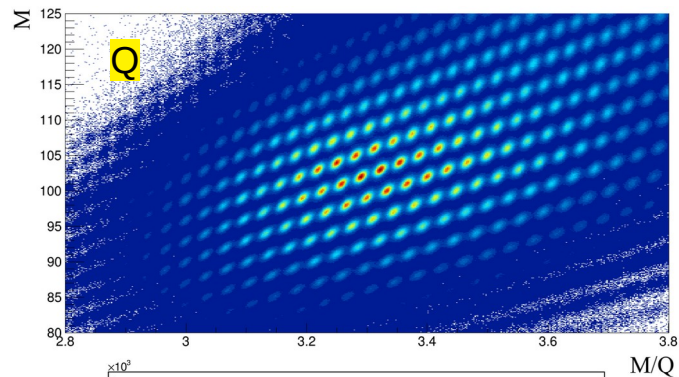
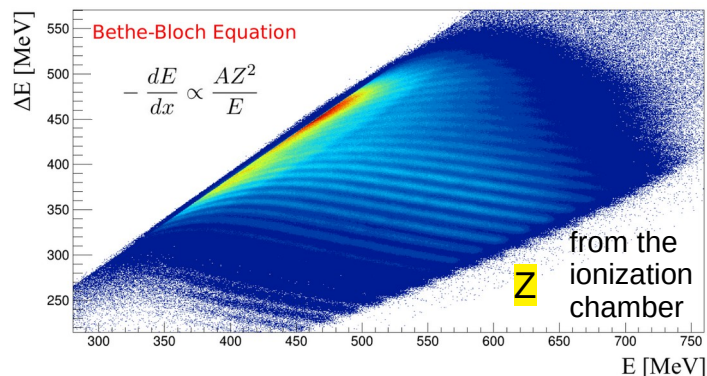
Results from recent MCSM calculations: T. Togashi, Y. Tsunoda, T. Otsuka and N. Shimizu, Phys. Rev. Lett. 117, 172502 (2016).



T-plots for  $0_{1,2}^+$  states of  $^{98,100,110}\text{Zr}$  isotopes to analyze the intrinsic shape of SM eigenstates:

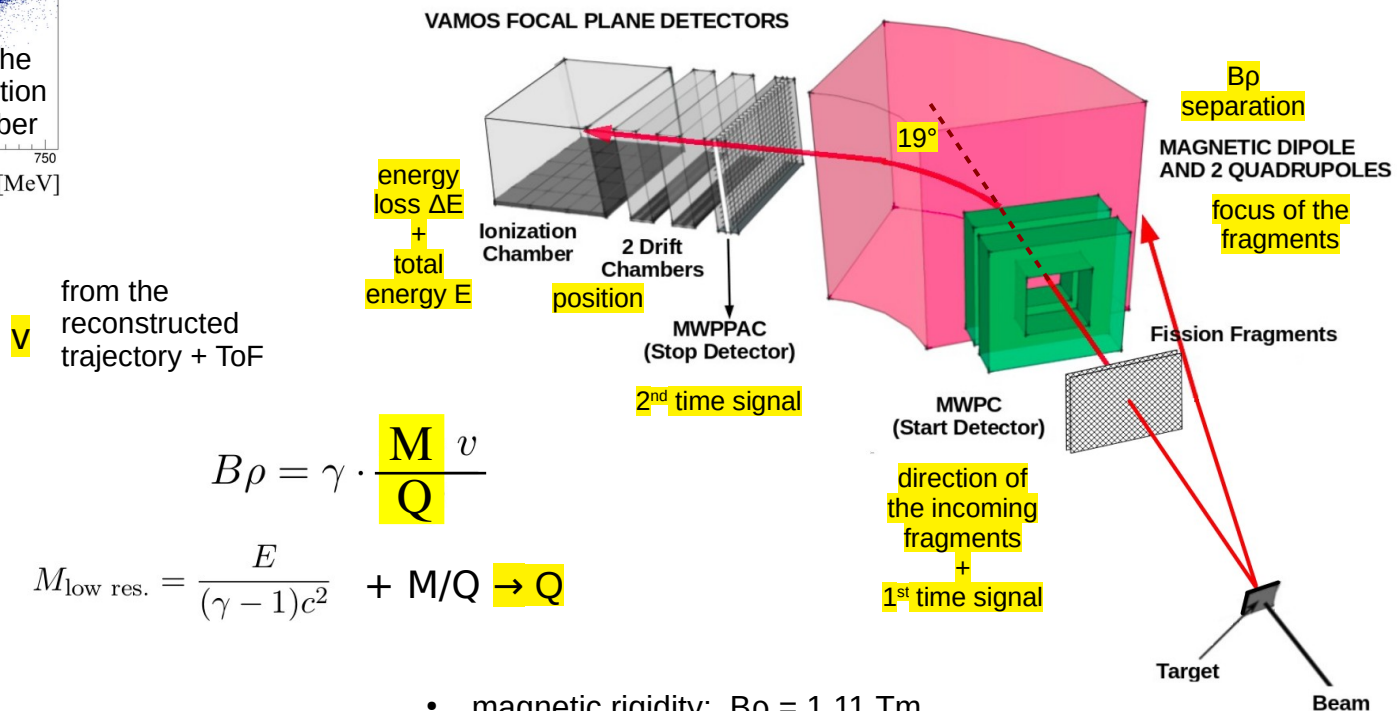


# • Production and identification of fission fragments with VAMOS



## VAMOS large acceptance magnetic spectrometer

fragment identification based on atomic number Z, mass M and charge Q



$$B\rho = \gamma \cdot \frac{M}{Q} \frac{v}{c}$$

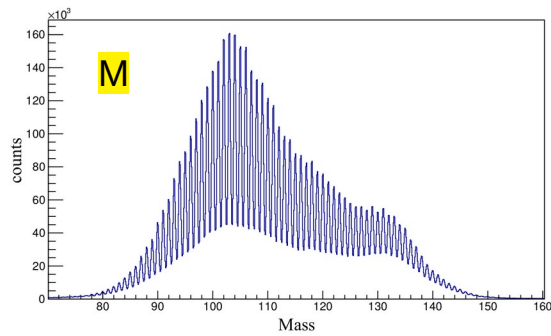
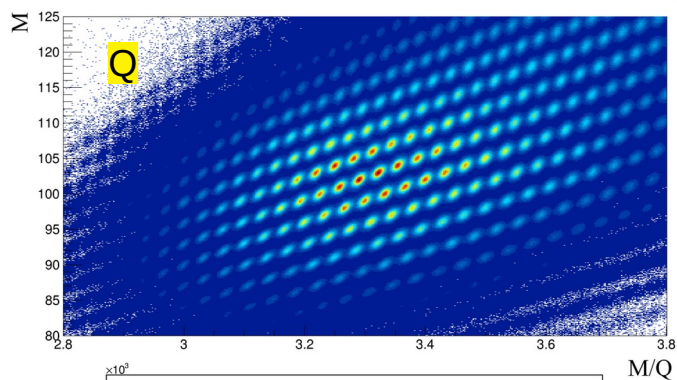
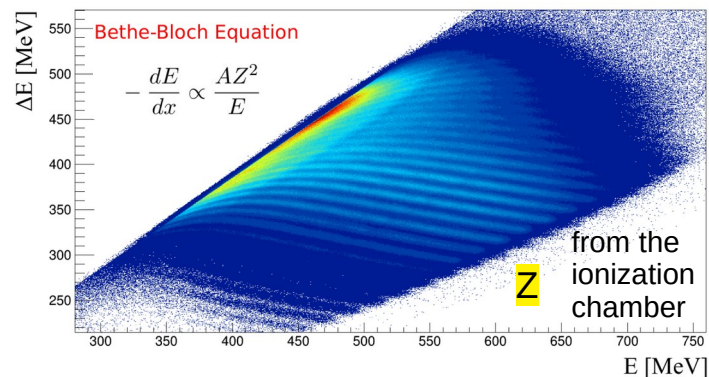
$$M_{\text{low res.}} = \frac{E}{(\gamma - 1)c^2} + M/Q \rightarrow Q$$

$$M/Q + Q \rightarrow M$$

- magnetic rigidity:  $B\rho = 1.11 \text{ Tm}$
- acceptance =  $\pm 10\%$
- VAMOS position =  $19^\circ$
- (scattered beam  $2^\circ$  on  $^9\text{Be}$ ,  $6^\circ$  on  $\text{Mg}$ )



- Production and identification of fission fragments with VAMOS

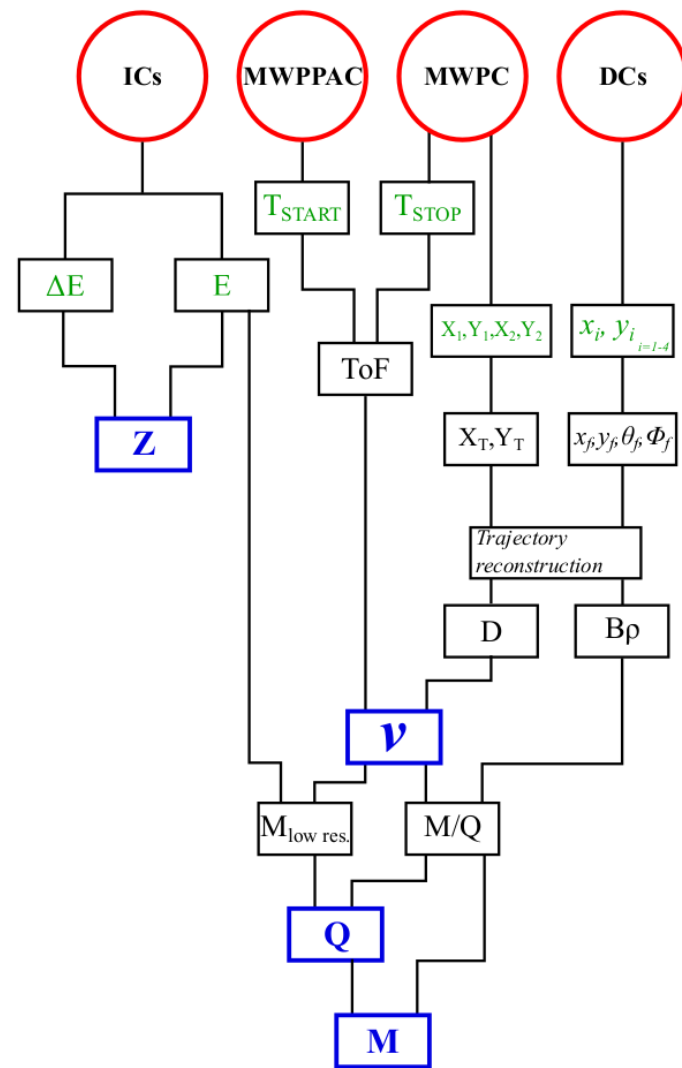


from the  
reconstructed  
trajectory + ToF

$$B\rho = \gamma \cdot \frac{M}{Q} v$$

$$M_{\text{low res.}} = \frac{E}{(\gamma - 1)c^2} + M/Q \rightarrow Q$$

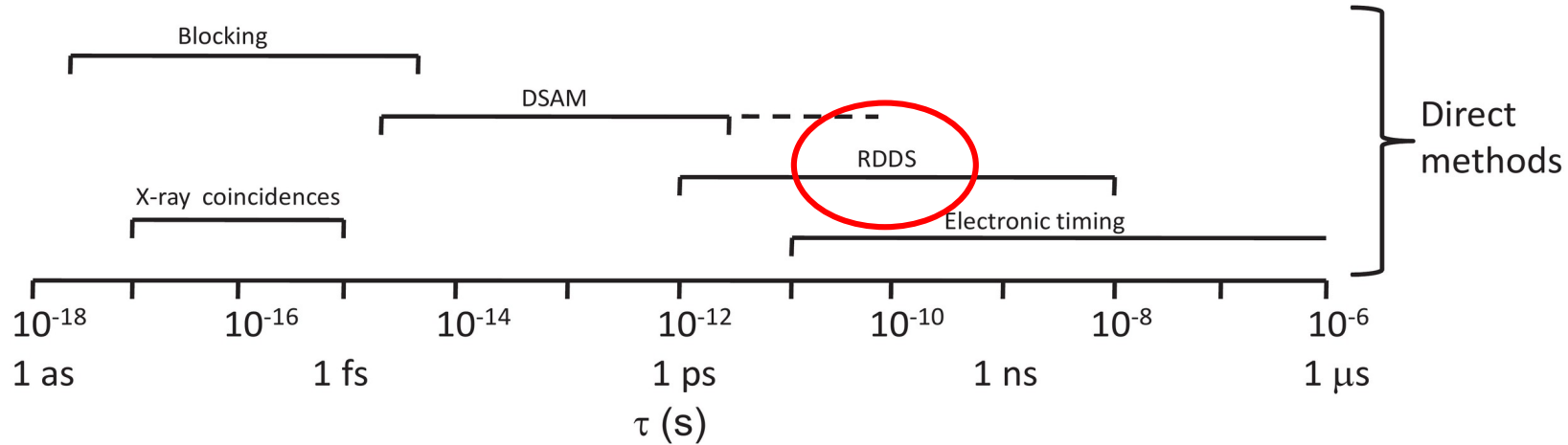
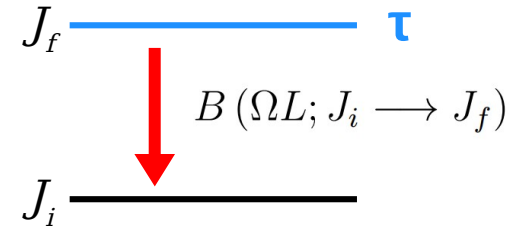
$$M/Q + Q \rightarrow M$$





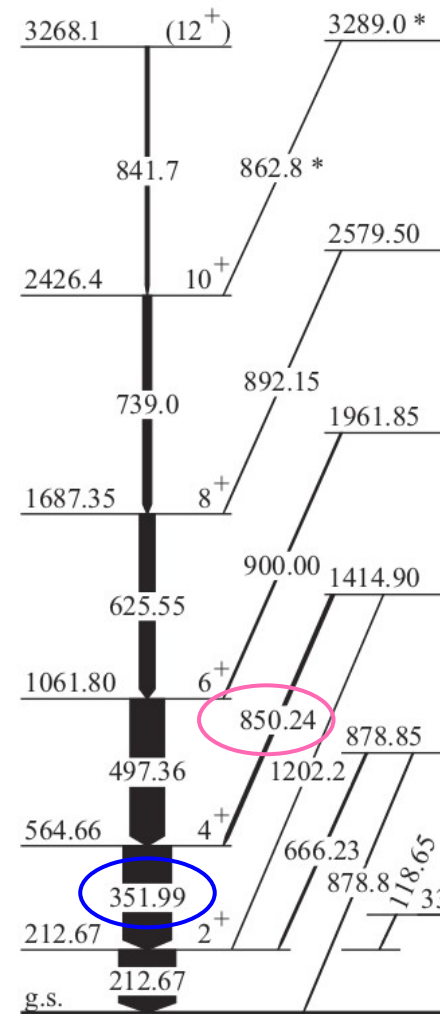
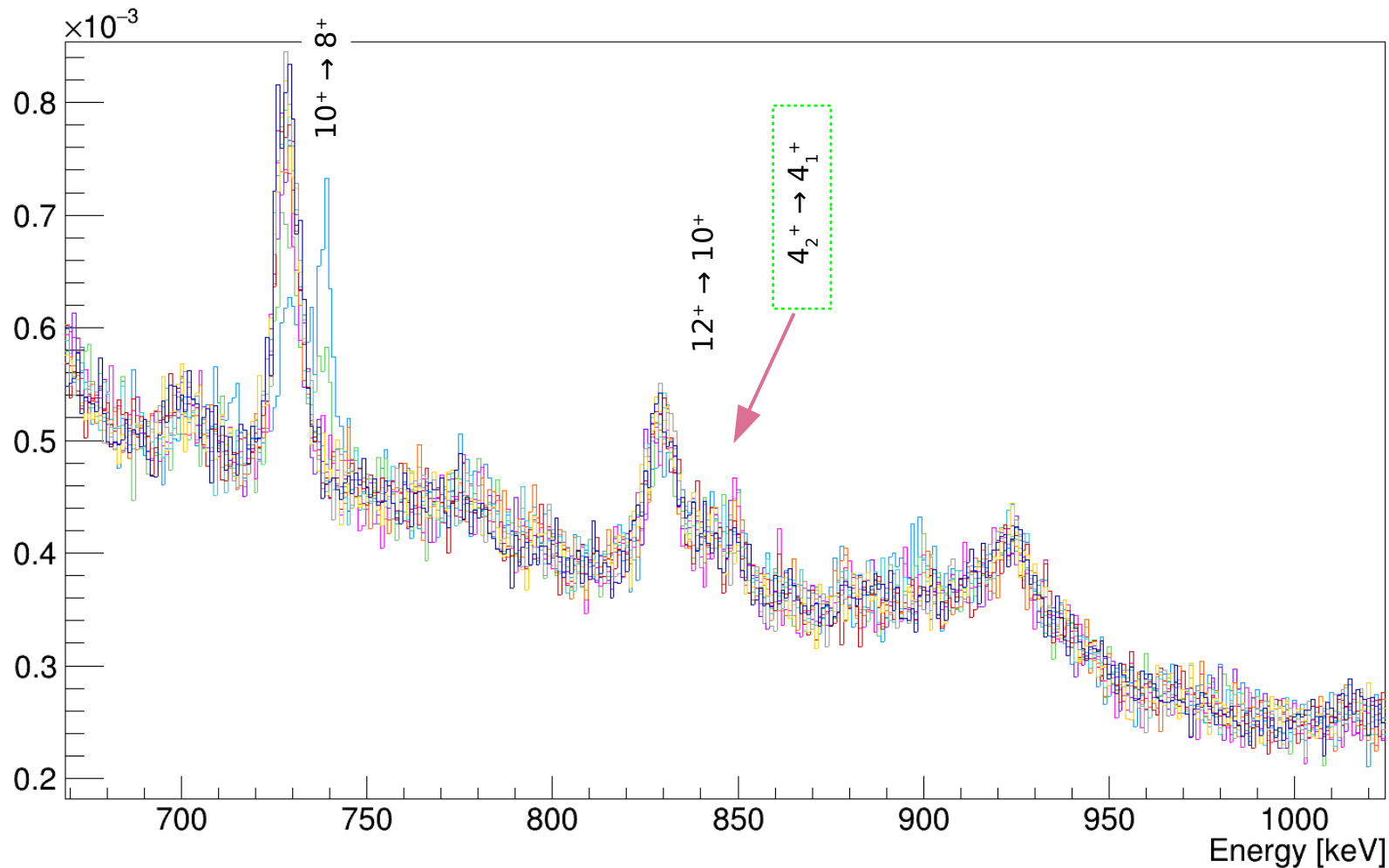
## LIFETIME MEASUREMENTS

- The lifetime of a nuclear state can range from  $10^{-20}$  seconds to many years . . .
- Different techniques have been implemented



**RDDS** : Recoil Distance Doppler Shift technique

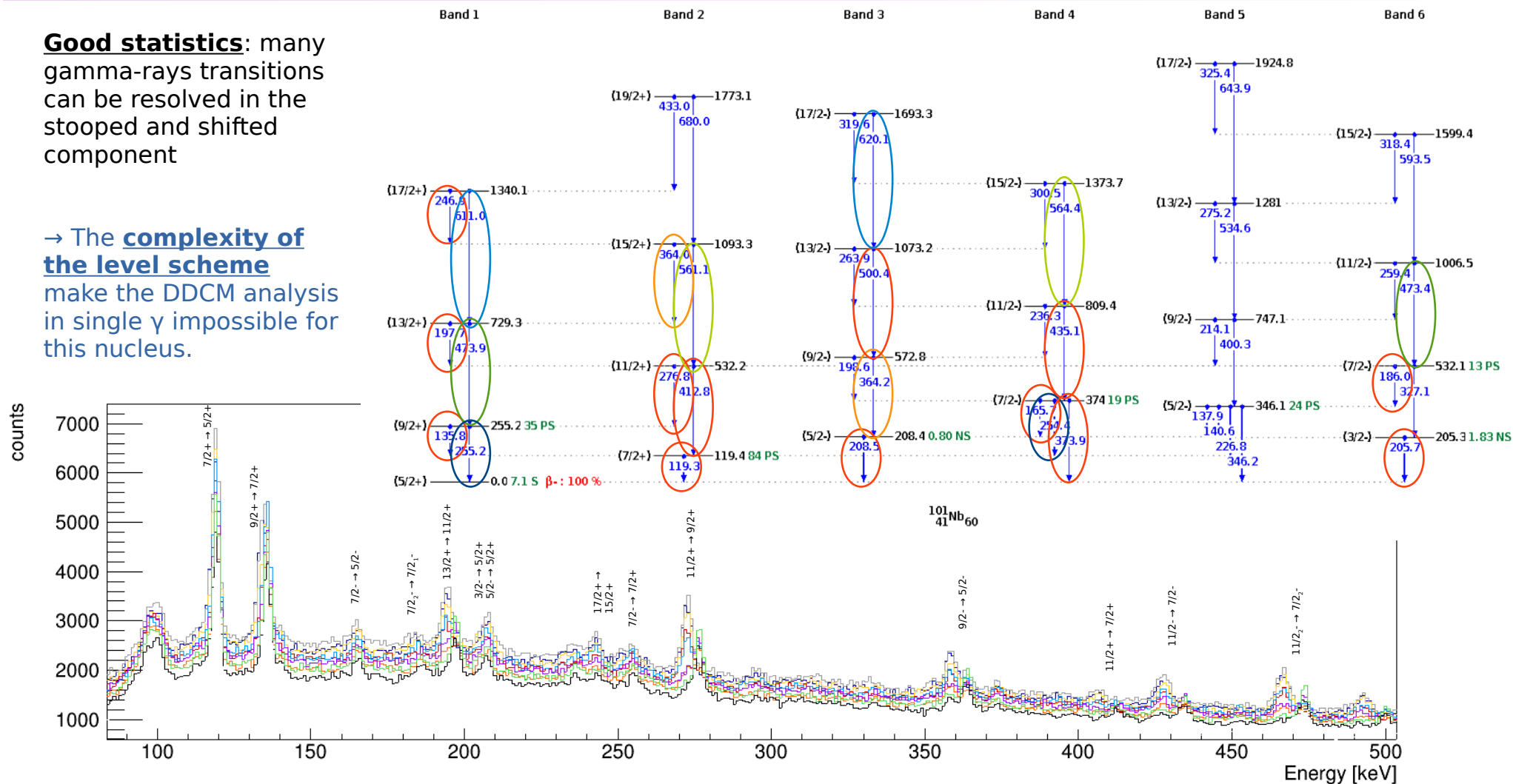
# Feeding for the $4^+$ state in $^{100}\text{Zr}$



# Example of an odd-even system: $^{101}\text{Nb}$ in single $\gamma$

**Good statistics:** many gamma-rays transitions can be resolved in the stooped and shifted component

→ The **complexity of the level scheme** make the DDCM analysis in single  $\gamma$  impossible for this nucleus.



<sup>98</sup> Zr					
J <sup>π</sup>	τ <sub>lit.</sub> [ps]	τ <sub>γγ</sub> [ps]	τ <sub>γ</sub> [ps]	E <sub>γ</sub> [keV]	B(E2↓) [e <sup>2</sup> b <sup>2</sup> ]
2 <sub>1</sub> <sup>+</sup>	3.79(79) [23], 10(2) [43], ≤6.0 [40], ≥0.68 [41]	7.2(10)		1222.9	0.0041(6)
4 <sub>1</sub> <sup>+</sup>	7.5(14) [23], 13(5) [43], ≤15.0 [40], 29(7) [42]	5.51(94)		620.5(2)	0.145(25)
6 <sub>1</sub> <sup>+</sup>	2.63(89) [23], ≤14 [42]	3.16(57)	2.82(31)	647.58(3)	0.228(41)
8 <sub>1</sub> <sup>+</sup>	2.82(68) [47]		1.95(30)	725.4(1)	0.209(32)
10 <sub>1</sub> <sup>+</sup>	2.05(48) [47]			768.4(1)	
<sup>100</sup> Zr					
J <sup>π</sup>	τ <sub>lit.</sub> [ps]	τ <sub>γγ</sub> [ps]	τ <sub>γ</sub> [ps]	E <sub>γ</sub> [keV]	B(E2↓) [e <sup>2</sup> b <sup>2</sup> ]
2 <sub>1</sub> <sup>+</sup>	1020(40) [48] 928(75) [45] 840(20) [40]			212.61(4)	
4 <sub>1</sub> <sup>+</sup>	53.4(5) [45] 37(4) [40]	34.4(27)	36.9(6)	351.97(1)	0.510(40)
6 <sub>1</sub> <sup>+</sup>	7.0(16) [45] 12(5) [40]	6.37(78)	6.11(33)	497.36(5)	0.540(65)
8 <sub>1</sub> <sup>+</sup>	2.55(30) [47, 48] 2.49(25) [45]	1.66(40)	1.32(19)	625.55(5)	0.500(120)
10 <sub>1</sub> <sup>+</sup>	1.08(12) [47, 48]		0.72(15)	739.0(1)	0.600(200)
<sup>102</sup> Zr					
J <sup>π</sup>	τ <sub>lit.</sub> [ps]	τ <sub>γγ</sub> [ps]	τ <sub>γ</sub> [ps]	E <sub>γ</sub> [keV]	B(E2↓) [e <sup>2</sup> b <sup>2</sup> ]
2 <sub>1</sub> <sup>+</sup>	2600(500) [49] 3610(430) [50] 2914(87) [40]			151.8	
4 <sub>1</sub> <sup>+</sup>	46.0(7.1) [40]	41.6(39)	45.9(13)	326.5(2)	0.510(50)
6 <sub>1</sub> <sup>+</sup>	≤12 [40]	5.6(11)	5.52(33)	486.5(2)	0.540(100)
8 <sub>1</sub> <sup>+</sup>	2.01(30) [47, 48]	2.5(10)	1.18(21)	630.1(5)	0.330(130)
10 <sub>1</sub> <sup>+</sup>	0.77(12) [47, 48]		1.27(52)	756.6(5)	0.260(100)
<sup>104</sup> Zr					
J <sup>π</sup>	τ <sub>lit.</sub> [ps]	τ <sub>γγ</sub> [ps]	τ <sub>γ</sub> [ps]	E <sub>γ</sub> [keV]	B(E2↓) [e <sup>2</sup> b <sup>2</sup> ]
2 <sub>1</sub> <sup>+</sup>	2900(250) [51]			139.3	
4 <sub>1</sub> <sup>+</sup>			43.4(51)	312.2(3)	0.450(40)
6 <sub>1</sub> <sup>+</sup>			4.2(16)	473.7(3)	0.850(400)
8 <sub>1</sub> <sup>+</sup>	1.91(29) [47, 48]			624.4(3)	
10 <sub>1</sub> <sup>+</sup>	0.67(10) [47, 48]			765.1(3)	

Lifetime results  
from this work.  
Comparison with  
all existent  
measurements.

G. Pasqualato et al., PRC to  
be submitted (2022)

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