

An hydrogen atom, an α -particle and a $T = 0$ p+n pair walk into a scattering chamber

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University of Catania (Italy) , Laboratori Nazionali del Sud – INFN (Italy)
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Selected Topics in Nuclear and Atomic Physics
Fiera di Primiero, September 2022

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Role of ${}^6\text{Li}$ ground-state structure on the ${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$ sub-barrier reaction

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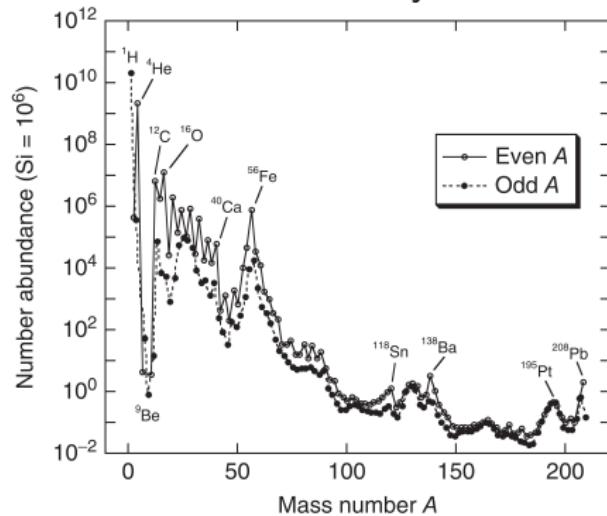
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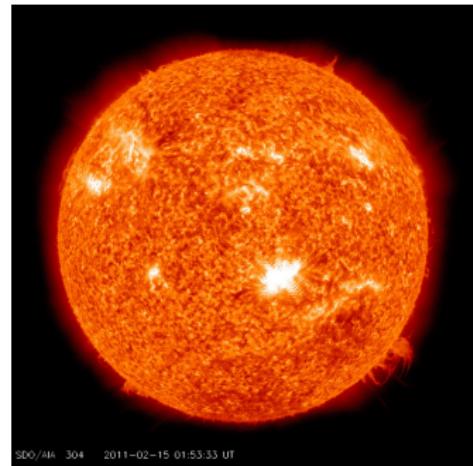
- **Introduction**
- **Barrier penetration and clustering**
- **The ${}^6\text{Li}(\text{p}, {}^3\text{He})\alpha$ transfer reaction**

Theoretical investigation on nuclear reactions between light charged particles at energies below the Coulomb barrier.

Focus on systems of astrophysical interest



C. Iliadis. *Nuclear Physics of Stars.*
2015, fig. 1.2



[sdo.gsfc.nasa.gov/
gallery](http://sdo.gsfc.nasa.gov/gallery)

Astrophysical factor $S(E)$

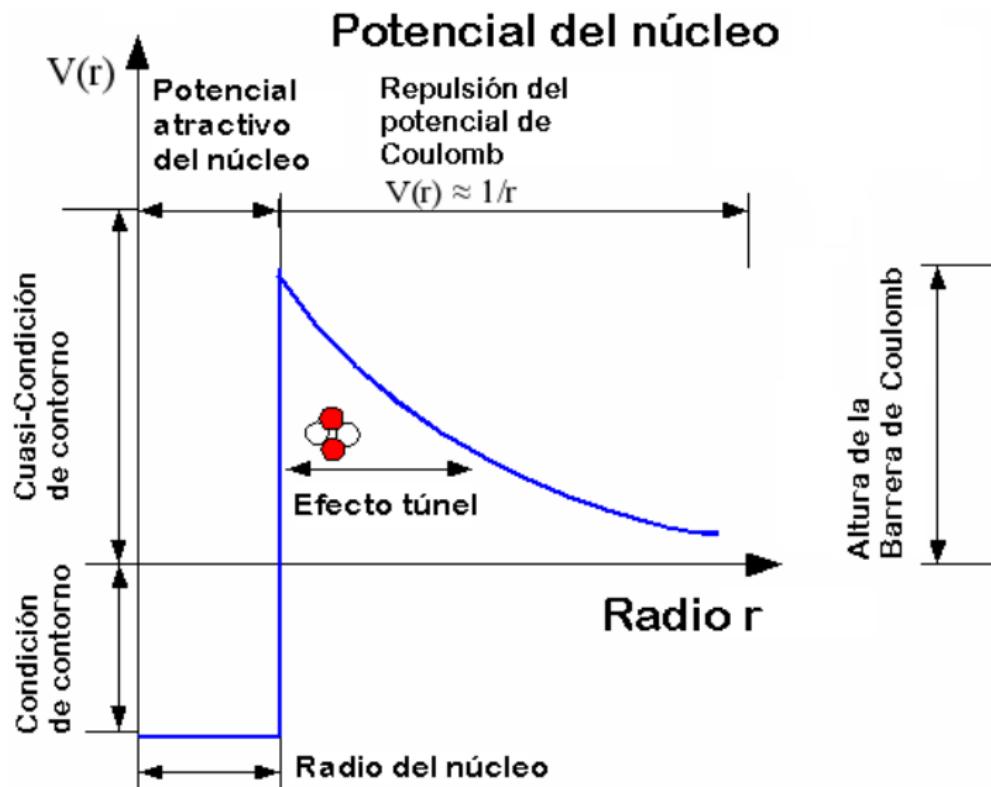
Process dominated by quantum tunnelling of the Coulomb barrier.

Astrophysical S-factor:

$$S(E) = E e^{2\pi\eta(E)} \sigma(E) \quad , \quad \eta(E) = \alpha_e Z_1 Z_2 \sqrt{\frac{\mu c^2}{2E}}$$

(σ angle-integrated cross-section, E center-of-mass collision energy,
 Z_i reactants charge number, α_e fine-structure constant,
 μ reactants reduced mass, c speed of light).

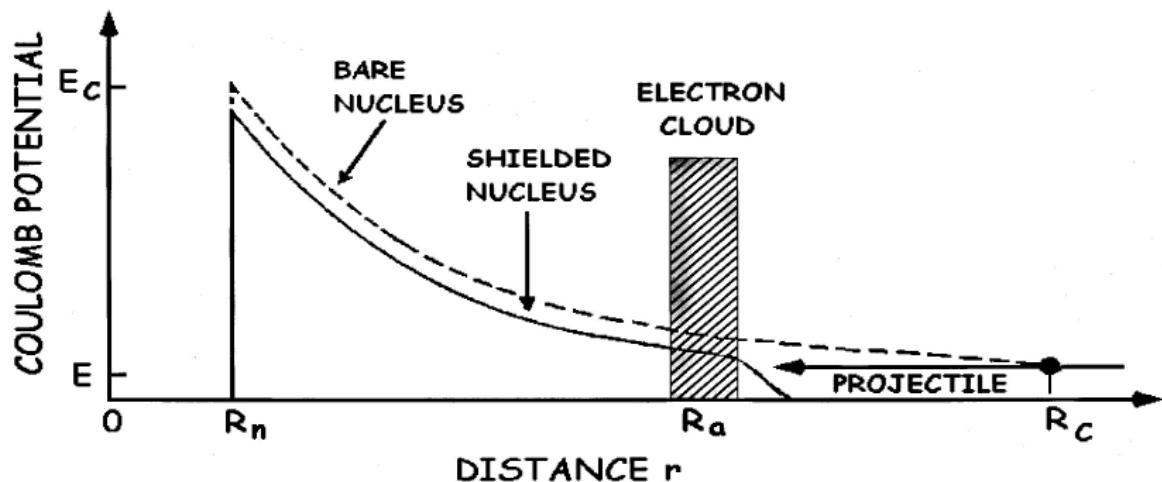
- Small variations of the effective E_{cm} are important for σ due to exp trend.



[commons.wikimedia.org/wiki/File:Coulomb-Barriere_\(es\).png](https://commons.wikimedia.org/wiki/File:Coulomb-Barriere_(es).png)

Electron screening

Atomic electrons lower the Coulomb barrier: (fig. from
H. J. Assenbaum et al. *Zeitschrift für Physik A: Atomic Nuclei* 327.4
(1987))



Cross-section enhancement for $E \rightarrow 0$.

See e.g. L. Bracci et al. *Nuclear Physics A* 513.2 (1990).

Goal

Study the influence of the reactant's **structure** on reaction dynamics in a **quantum framework**.

- Explicit evaluation of the cross-section in terms of the properties and interactions of reactants.
- No adjusting on reaction experimental data.

Focus on ${}^6\text{Li}$ structure:

- Two-cluster models: $|{}^6\text{Li} \text{ } \begin{array}{c} \text{blue} \\ \text{red} \\ \text{blue} \end{array} \rangle = |\alpha d \text{ } \begin{array}{c} \text{red} \\ \text{blue} \end{array} \rangle$
- Three-cluster models: $|{}^6\text{Li} \text{ } \begin{array}{c} \text{blue} \\ \text{red} \\ \text{blue} \end{array} \rangle = |\alpha p n \text{ } \begin{array}{c} \text{red} \\ \text{blue} \\ \text{blue} \end{array} \rangle$

Static deformation, strength of clustering, dynamical reorientation...

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Coulomb barrier penetrability and clustering

- At energies well below the Coulomb barrier, the dynamics of a reaction between charged particles is dominated by the quantum tunneling process.

Qualitative description of such reaction as:

probability to overcome the barrier \times factor selecting the exit channel

- For a central potential, radial transmission coefficient easily evaluated in WKB.

Idea: improve semi-classical model in C. Spitaleri et al. *Physics Letters B* 755 (2016) and evaluate penetrability of ${}^6\text{Li}$ -projectile form factor from quantum cluster model.

Form-factor construction

Potential between structureless projectile p and cluster j : $V_{pj}(\vec{r}_{pj})$.

Two-cluster target ${}^6\text{Li} = \alpha + d$. Projectile-target potential:

$$V_{tp}(\vec{R}_{p\text{Li}}, \vec{r}_{\alpha d}) = V_{p\alpha} + V_{pd}$$

If ${}^6\text{Li}$ is confined to the ground-state

${}^6\text{Li}-p$ scattering governed by the form factor

$$V(\mu, \mu', \vec{R}_{p\text{Li}}) = \langle \text{Li}_{1\mu'} | V_{tp} | \text{Li}_{1\mu} \rangle = \langle I=1, \mu' | \tilde{V}_{tp} | I, \mu \rangle$$

($|I, \mu\rangle$ is a state in only the spin space).

If V_{pj} are central, $\left(\hat{\vec{I}} \text{ spin operator of } {}^6\text{Li} \right)$,

$$\tilde{V}_{tp} = U_C(R) + U_T(R) \frac{1}{\hbar^2} \left[\left(\hat{\vec{I}} \cdot \hat{\vec{R}} \right)^2 - \frac{1}{3} \hat{\vec{I}}^2 \right]$$

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$$\tilde{V}_{tp} = U_C(R) + \textcolor{blue}{U_T(R)} \frac{1}{\hbar^2} \left[\left(\hat{\underline{I}} \cdot \hat{\underline{R}} \right)^2 - \frac{1}{3} \hat{\underline{I}}^2 \right]$$

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^6Li ground-state structure data

Nuclide	J^π	$\sqrt{< r^2 >} \text{ [fm]}$	$Q \text{ [mb]}$	$\mu \text{ [\mu_N]}$
α	$0+$	1.676(3)	0	0
d	$1+$	2.142(9)	+2.86(2)	+0.857 438 234(2)
^6Li	$1+$	2.59(4)	-0.806(6)	+0.822 043(3)

I. Angeli et al. *Atomic Data and Nuclear Data Tables* 99.1 (2013)

www-nds.iaea.org/nuclemoments

$\langle \alpha d | {}^6\text{Li} \rangle$ overlap function

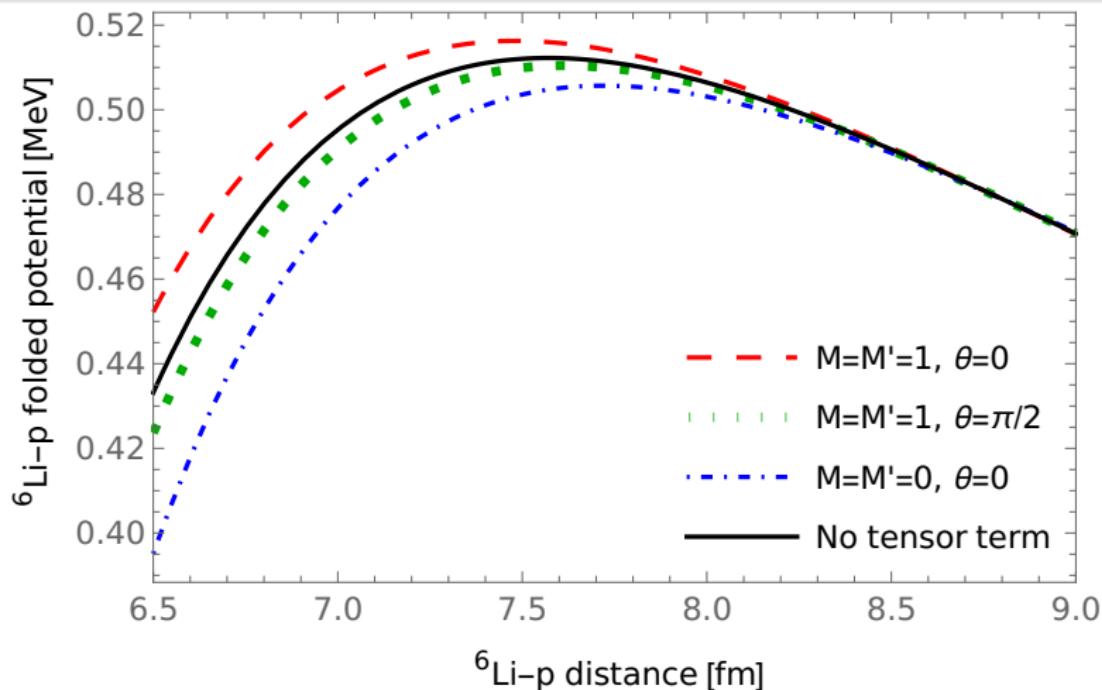
${}^6\text{Li}$ g.s. $J^\pi = 1^+$ in inert two-cluster model = $\alpha + d$.

$$\begin{aligned} |{}^6\text{Li}_{1+, \mu}\rangle = & \sum_{I=0,2} \sum_m c_I \langle (I, m), (1, \mu - m) | 1, \mu \rangle \cdot \\ & \cdot |\alpha_{0+, 0}\rangle |d_{1+, \mu-m}\rangle |Y_{Im} \chi_I\rangle \end{aligned}$$

Phenomenological $2s$ and $1d$ radial WFs
as in H. Nishioka et al. *Nuclear Physics A* 415.2 (1984).

$\alpha + d$	Experimental	$I = 0$ only	$I = 2$: 0.8 %
g.s. rms radius	2.59 fm	2.66 fm	2.66 fm
g.s. quadrupole moment	-0.806 mb	2.86 mb	-0.806 mb
g.s. dipole moment	$0.8220 \mu_N$	$0.8574 \mu_N$	$0.8530 \mu_N$

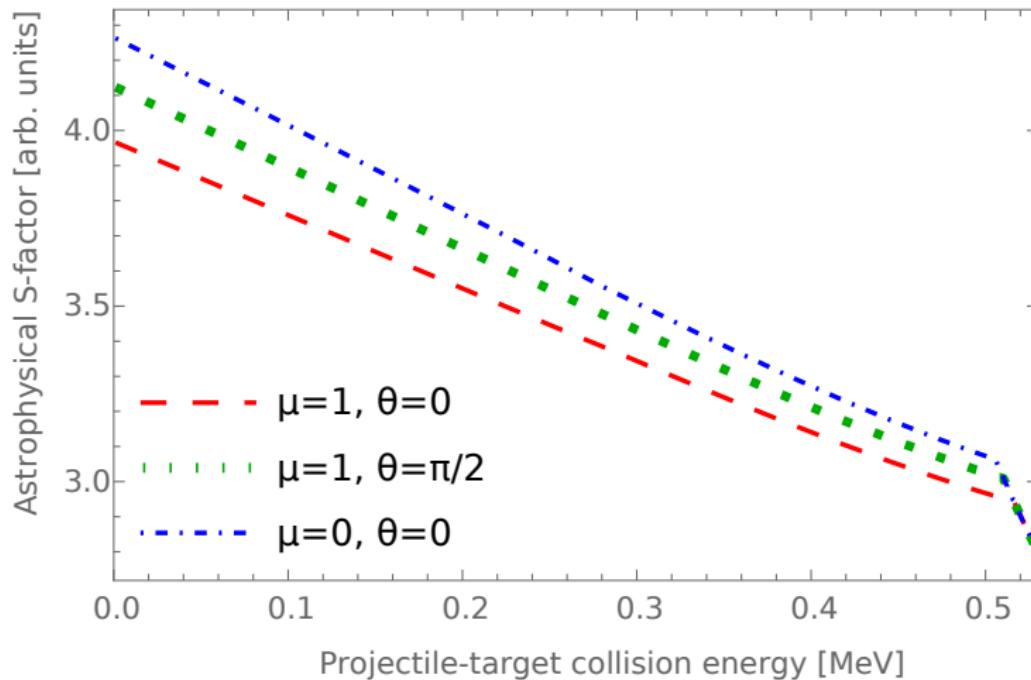
Tensorial ${}^6\text{Li}$ -p potential for deformed ${}^6\text{Li}$



From central p-cluster interactions and phenomenological ${}^6\text{Li}$.

S. S. Perrotta, L. Fortunato, J. A. Lay and M. Colonna. *Il Nuovo Cimento C* 45.123 (2022)

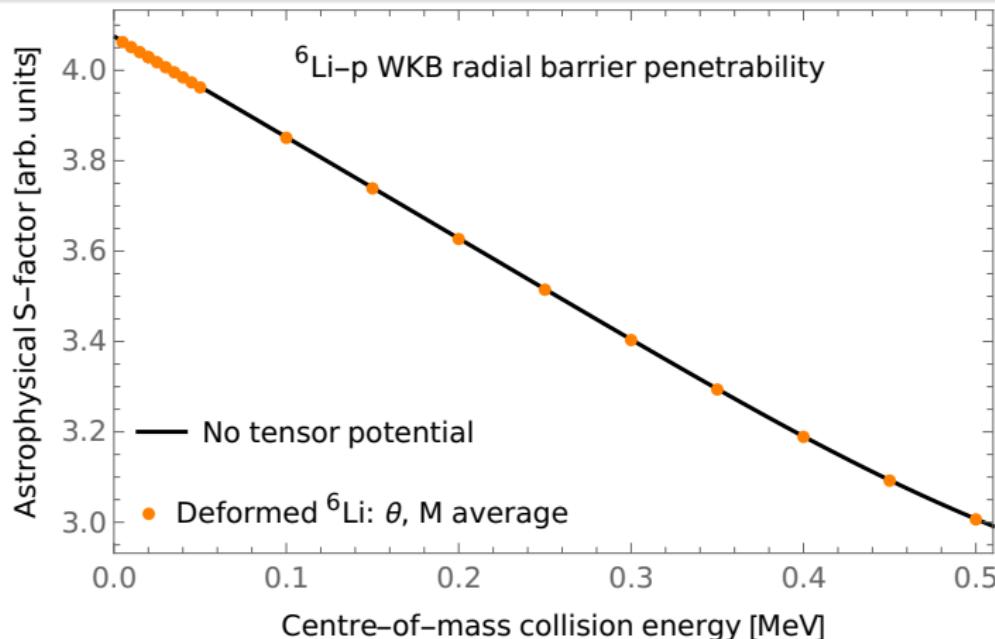
${}^6\text{Li}$ -p radial barrier penetrability for deformed ${}^6\text{Li}$



Simplified calculation: semi-classical WKB *radial* barrier penetrability for each direction using tensorial ${}^6\text{Li}$ -p potential.

S. S. Perrotta. PhD thesis. 2022

${}^6\text{Li}$ -p radial barrier penetrability for deformed ${}^6\text{Li}$

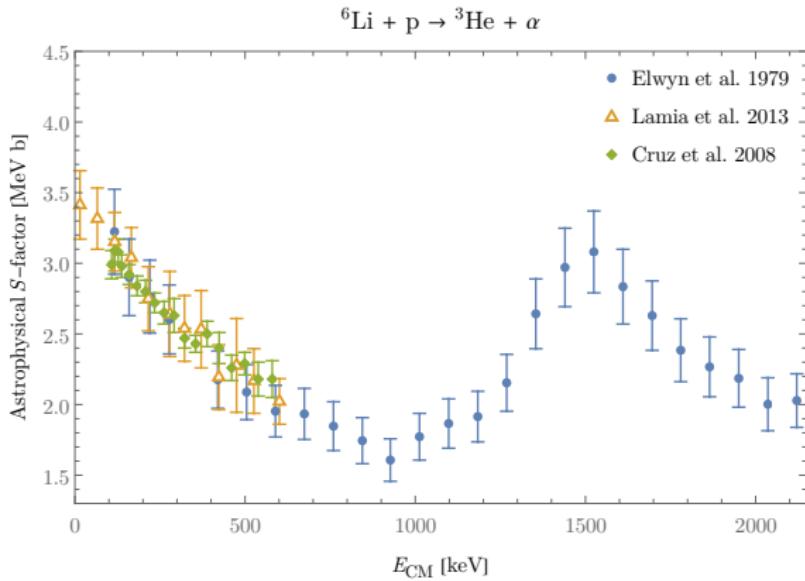


In the simplified model, role of tensor disappears after averaging over all directions. Full-quantum calculation advisable.

S. S. Perrotta, L. Fortunato, J. A. Lay and M. Colonna. *Il Nuovo Cimento C* 45.123 (2022)

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${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$ astrophysical S -factor



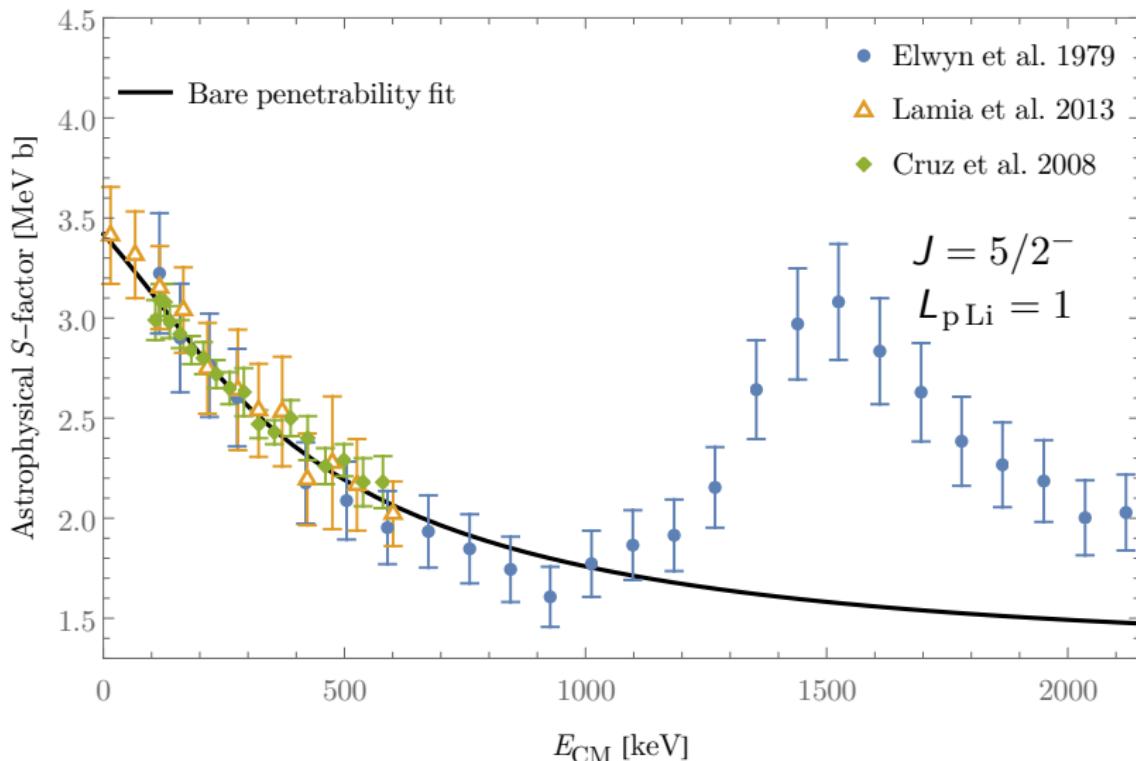
Points: bare-nucleus experimental data.

A. J. Elwyn et al. *Phys. Rev. C* 20.6 (1979)

J. Cruz et al. *Journal of Physics G* 35.1 (2007)

L. Lamia et al. *The Astrophysical Journal* 768.1 (2013)

${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$ astrophysical *S*-factor



Line: non-resonant Coulomb penetrability phenomenological fit.

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Transition amplitude in plane-waves

Exact transition amplitude T ($|T|^2 \propto$ cross-section σ) in post-form

for ${}^6\text{Li}$  + p  \rightarrow α  + ${}^3\text{He}$  as direct transfer of a d 



$$T = \langle \underline{k}_f \phi_f | V_{\alpha d} + V_{\alpha p} | \Psi_+ \rangle$$

- $|\phi_f\rangle$: internal state wave-functions for ${}^3\text{He} = \text{p} + \text{d}$ and α .
- $|\underline{k}_f\rangle$: plane-wave for ${}^3\text{He}-\alpha$ relative motion.
- $V_{\alpha d} + V_{\alpha p}$: “microscopic” potential between α and $(\text{d} + \text{p}) = {}^3\text{He}$.
- $|\Psi_+\rangle$: exact solution of full quantum scattering problem.
 (“full” = e.g. $\alpha + \text{d} + \text{p}$) Unknown!

Transition amplitude in distorted-waves

Exact transition amplitude T ($|T|^2 \propto$ cross-section σ) in post-form

for ${}^6\text{Li}$  + p  \rightarrow α  + ${}^3\text{He}$  as transfer of a d 



$$T = \left\langle \chi_f \phi_f \mid V_{\alpha d} + V_{\alpha p} - U_{\alpha {}^3\text{He}} \mid \Psi_+ \right\rangle$$

- $U_{\alpha {}^3\text{He}}$ phenomenological potential
(usually) describing elastic scattering of structureless ${}^3\text{He}-\alpha$.

Now the operator has e.g. no long-range Coulomb.
Approximations on $|\Psi_+\rangle$ less critical.

- $|\chi_f\rangle$: solution of $U_{\alpha {}^3\text{He}}$ for ${}^3\text{He}-\alpha$ relative motion.

Simpler transition potential, more complex wave-function.

Distorted-wave Born approximation



DWBA post-form transition amplitude T ($|T|^2 \propto$ cross-section σ):



- $|\phi_i\rangle$: internal state wave-functions for p and ${}^6\text{Li} = d + \alpha$.
- $U_{{}^6\text{Li} p}$ phenomenological potential
(usually) describing elastic scattering of structureless ${}^6\text{Li}-\text{p}$.
- $|\chi_i\rangle$: solution of $U_{{}^6\text{Li} p}$ for ${}^6\text{Li}-\text{p}$ relative motion.

Practical calculation through iterative method (FRESCO code)

$\langle \alpha d | {}^6\text{Li} \rangle$ overlap function

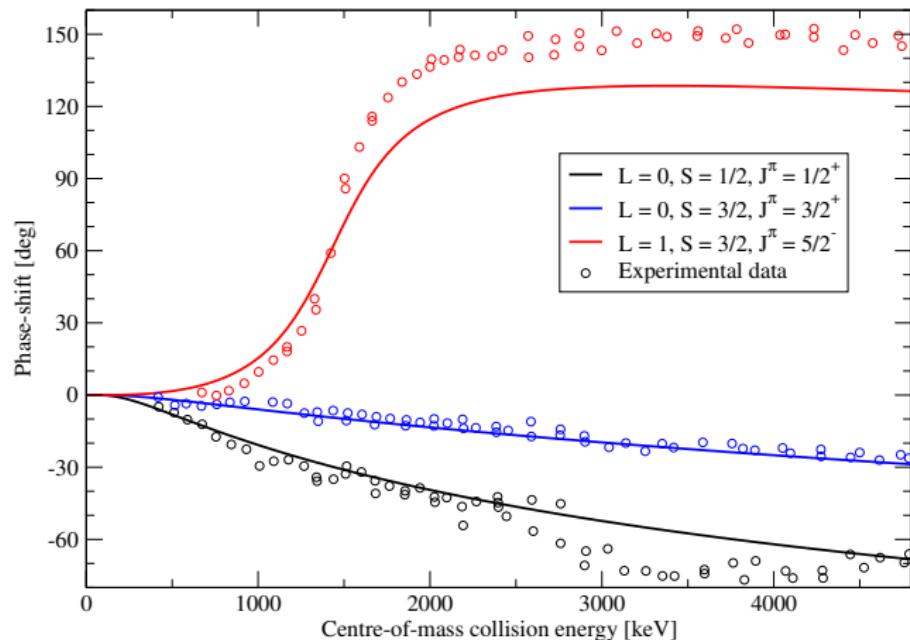
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$^6\text{Li} + \text{p}$ elastic scattering description

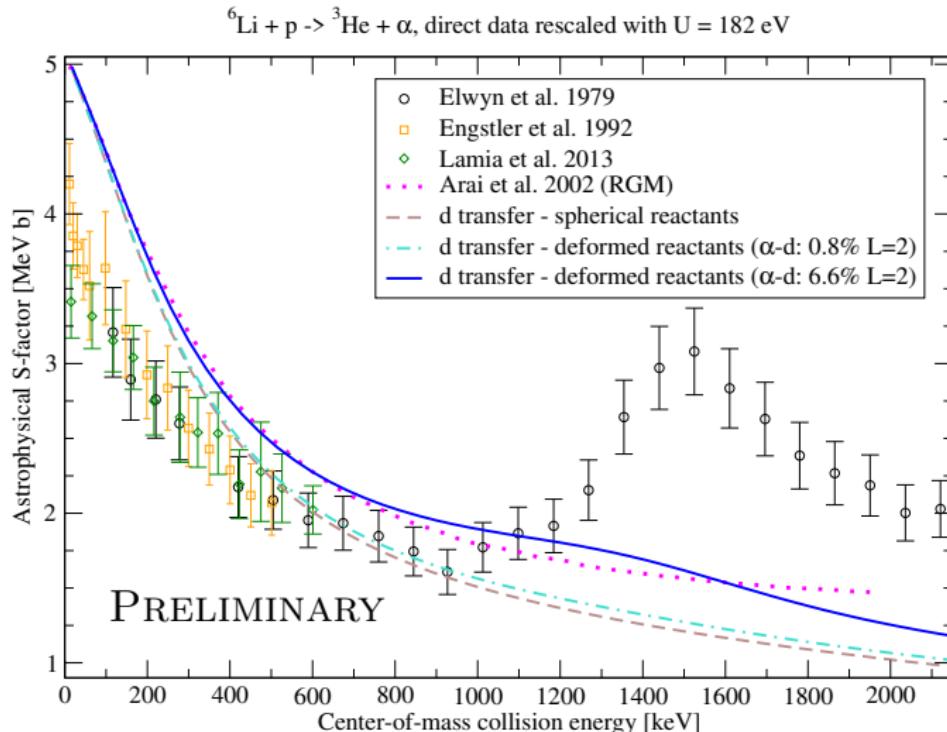


S. S. Perrotta. PhD thesis. 2022

Data collected in C. Petitjean et al. *Nuclear Physics A* 129.1 (1969).

Imaginary part added on top to improve elastic cross-section.

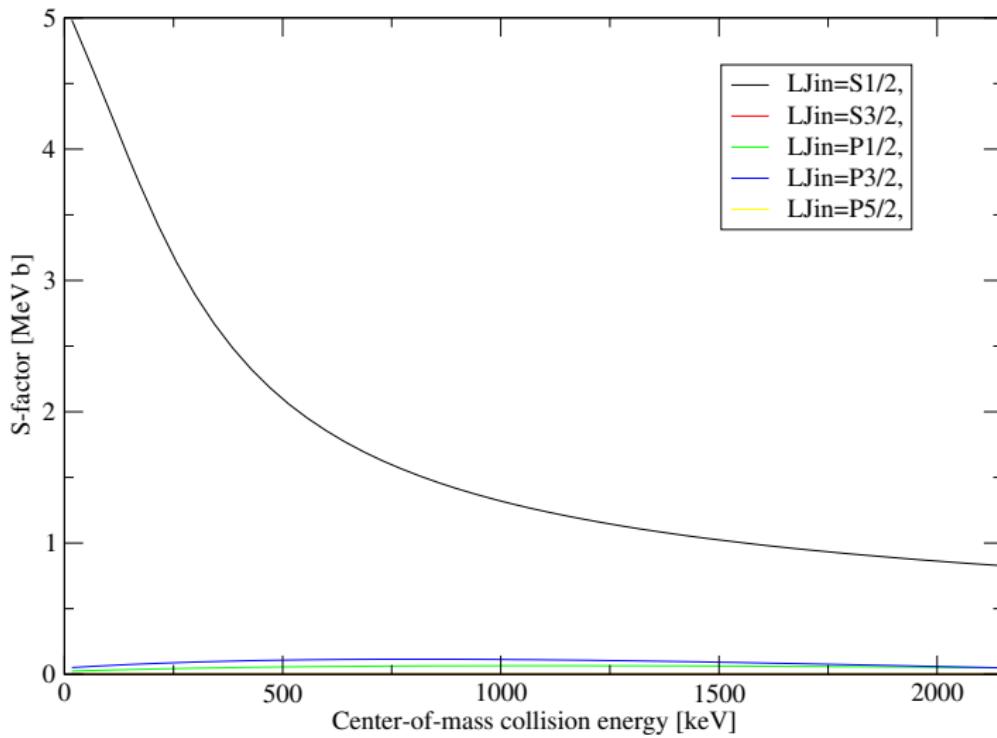
$^6\text{Li} + \text{p} \rightarrow ^3\text{He} + \alpha$: deuteron transfer



Brown dashed line: point-like d transfer, α -d motion in $L = 0$

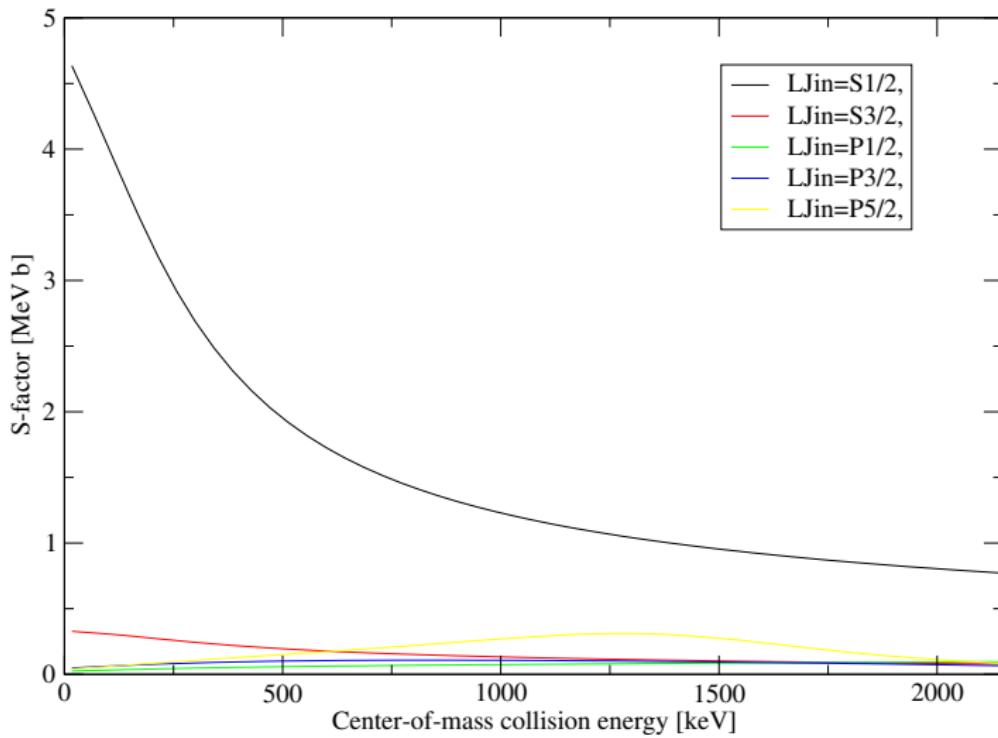
Blue solid line: point-like d transfer, 6.6 % of $L = 2$ in α -d motion

Astrophysical S-factor partial-wave decomposition



Point-like d transfer, α -d motion in $L = 0$

Astrophysical S-factor partial-wave decomposition



Point-like d transfer, 5% of $L = 2$ in α -d motion

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2nd order Distorted-wave Born approximation

Coupled reaction channels with specific choice of couplings.

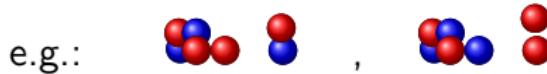
First-order DWBA term, for a system going from an initial projectile-target state i  to a final one f :

$$T = \langle \chi_f \phi_f | \hat{V} | \chi_i \phi_i \rangle$$

Second-order DWBA term:

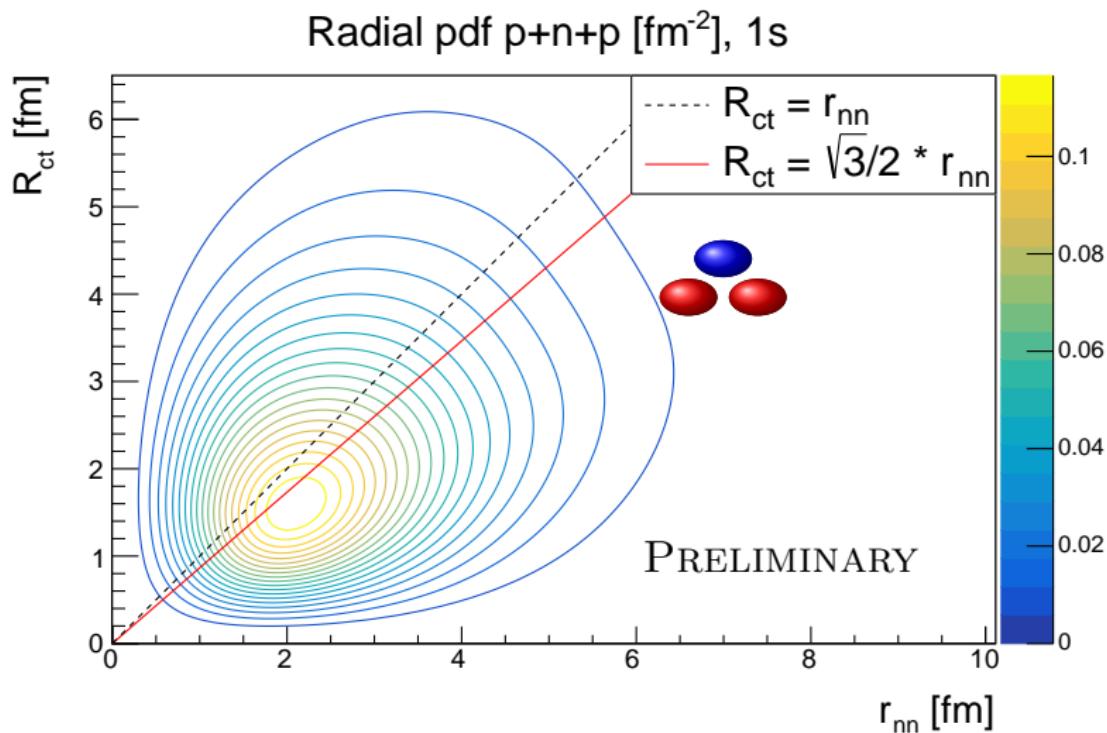
$$T = \sum_{\gamma} \langle \chi_f | \langle \phi_f | \hat{V} | \phi_{\gamma} \rangle \hat{G}_{\gamma} \langle \phi_{\gamma} | \hat{V} | \phi_i \rangle | \chi_i \rangle$$

γ : any possible intermediate state different from i and f ,



\hat{G}_{γ} : Green's function ("propagator") for the intermediate state.

p+n+p reduced probability density function



Constructing two-body bound states (e.g. $\alpha + p + n$)

Total Hamiltonian for initial state (${}^5\text{Li} = \alpha + p$, ${}^6\text{Li} = \alpha + p + n$)

$$\left[(K_{\alpha p} + V_{\alpha p}) + (K_{{}^5\text{Li} n} + V_{\alpha n} + V_{p n}) \right] + K_{{}^6\text{Li} \tilde{p}} + V_{\alpha \tilde{p}} + V_{p \tilde{p}} + V_{n \tilde{p}}$$

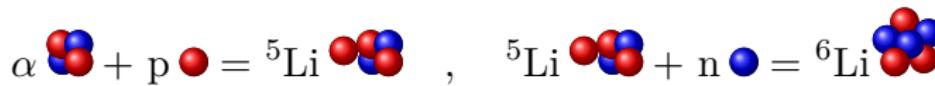
K_{ij} : kinetic energy of relative $i-j$ motion. V_{ij} : $i-j$ potential.

Approximated to:

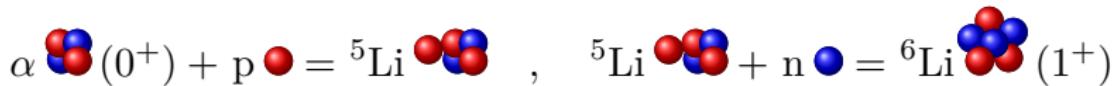
$$\left[(K_{\alpha p} + V_{\alpha p}) + (K_{{}^5\text{Li} n} + V_{{}^5\text{Li} n}) \right] + K_{{}^6\text{Li} \tilde{p}} + V_{\alpha \tilde{p}} + V_{n \tilde{p}} + V_{\tilde{d} p}$$

The bound state $\Phi_{\alpha p n}$ can be written as:

$$\Phi_{\alpha p n} = \sum \phi_{\alpha p} (\underline{r}_{\alpha p}) \phi_{{}^5\text{Li} n} (\underline{r}_{{}^5\text{Li} p})$$



Constructing $\alpha + p + n$ bound states

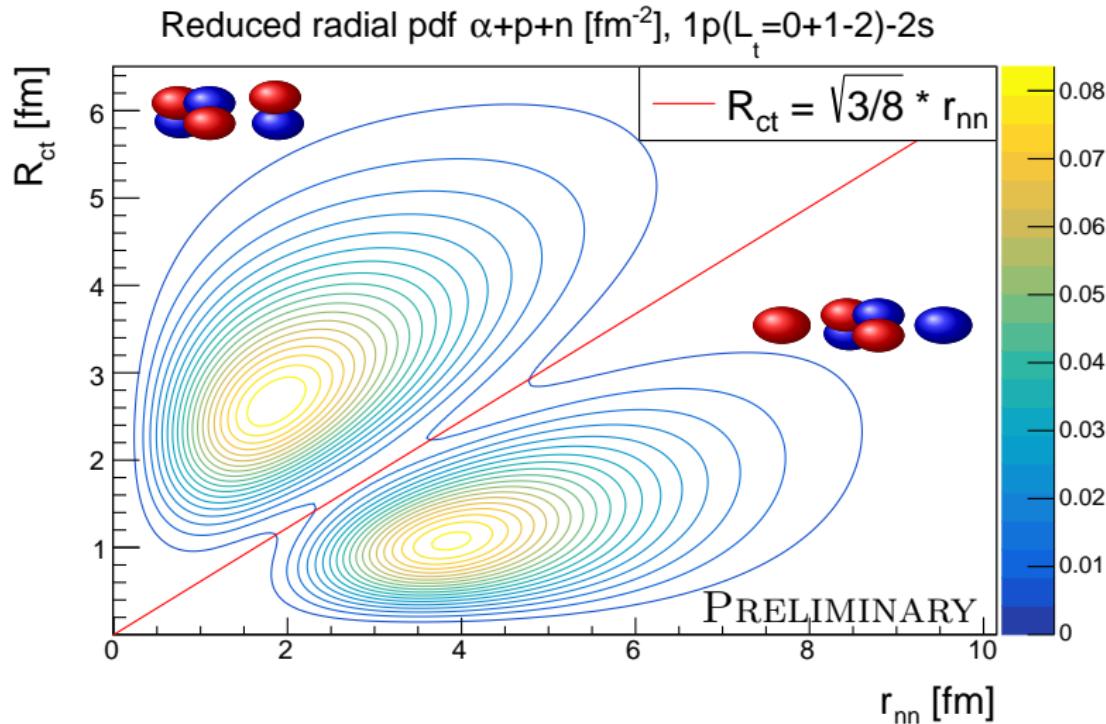


core-n sp shell	tot S	tot L	sq. norm
$1p \times 1p$	1	0	84.5 %
$1p \times 1p$	0	1	5.3 %
$1p \times 1p$	1	2	0.4 %
$2s \times 2s$	1	0	3.4 %

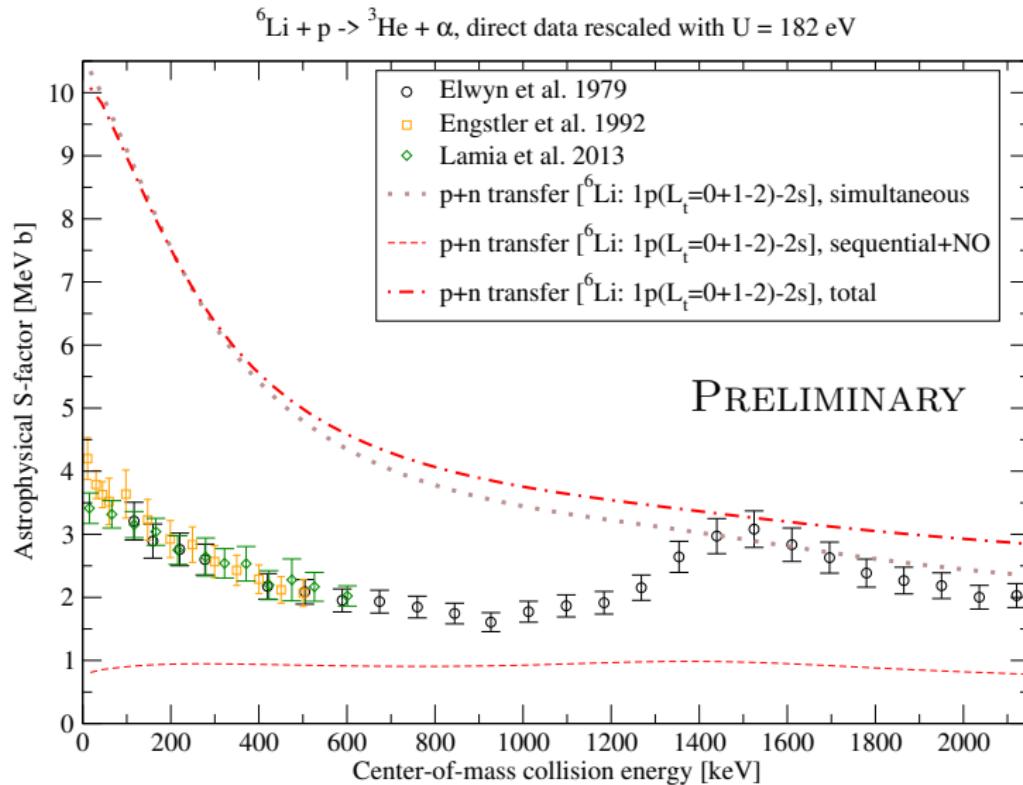
weights from J. Bang et al. *Nuclear Physics A* 313.1 (1979)

Relative phases found by comparison with 3-body WF
in Hyperspherical Harmonics formalism
from J. Casal, M. Rodríguez-Gallardo, priv. comm.

$\alpha+n+p$ reduced probability density function

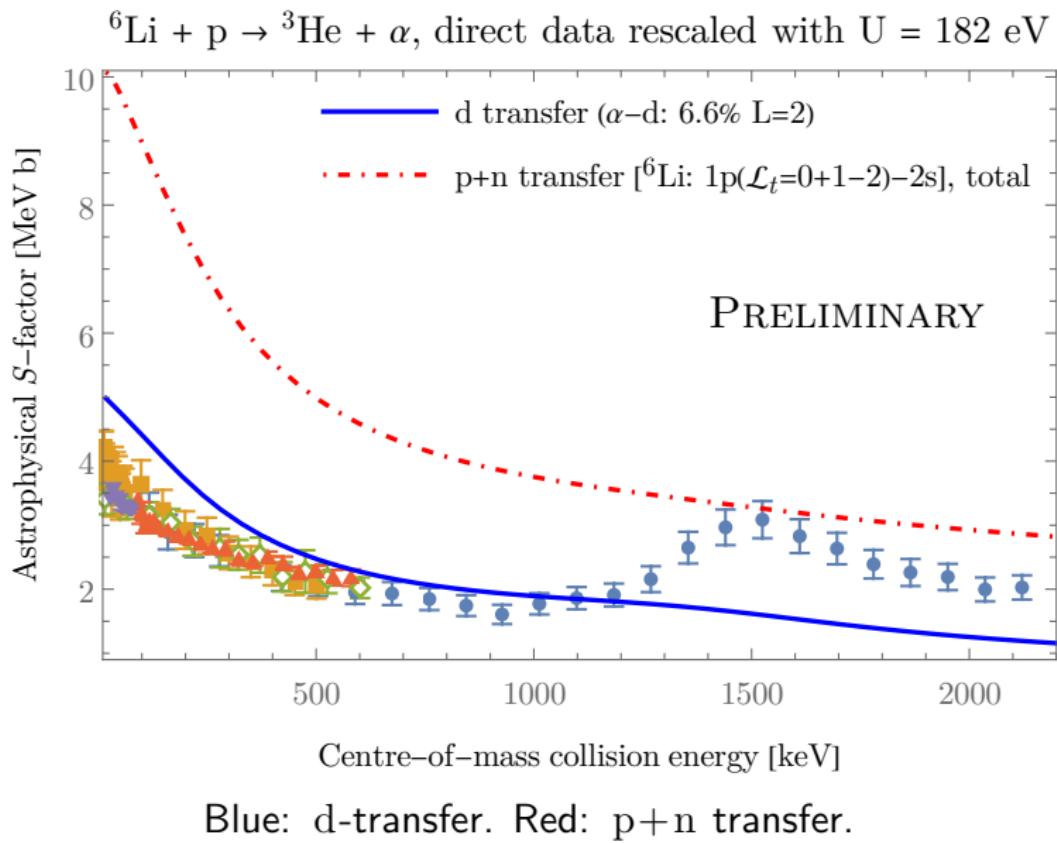


$^6\text{Li} + \text{p} \rightarrow ^3\text{He} + \alpha$: two-particle transfer

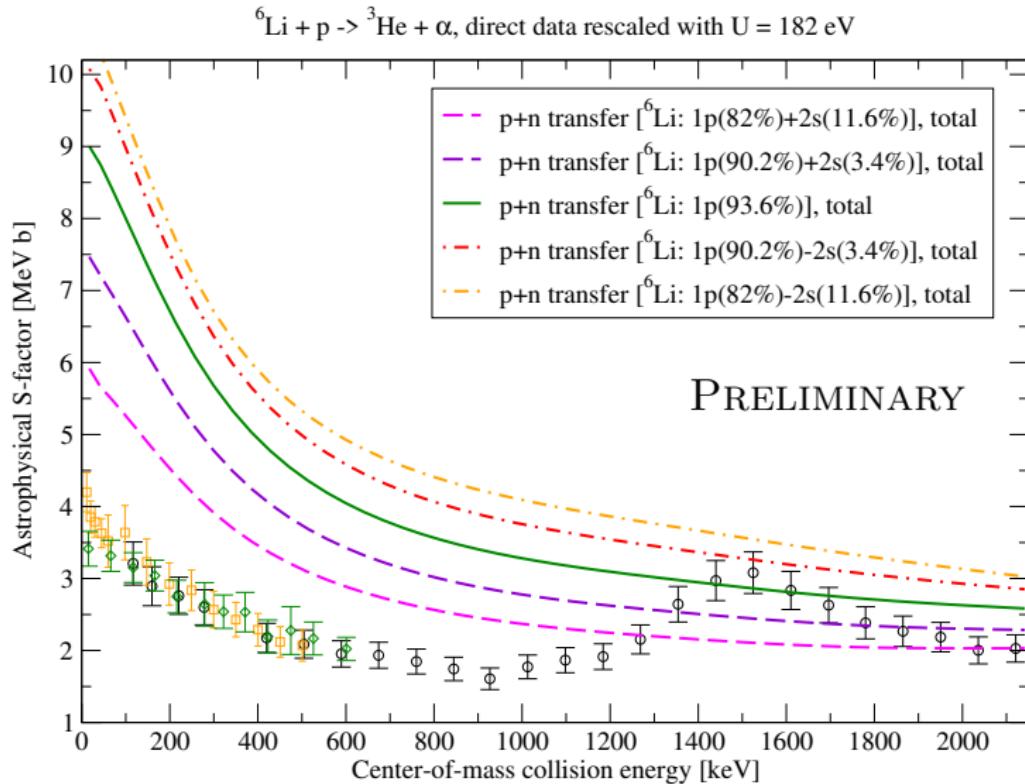


Red dot-dashed: total p+n, present-work wave-function.

$^6\text{Li} + \text{p} \rightarrow ^3\text{He} + \alpha$: one- vs. two-particle transfer

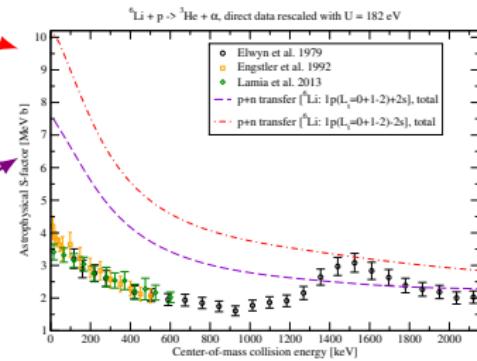
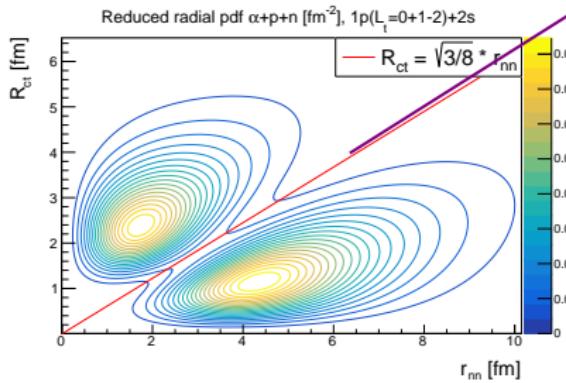
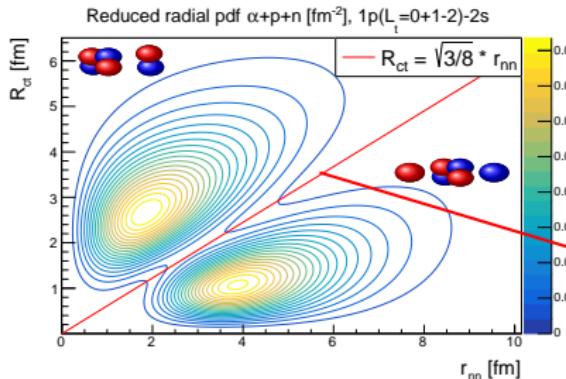


${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$: role of ${}^6\text{Li}$ ($2s$) 2 contribution

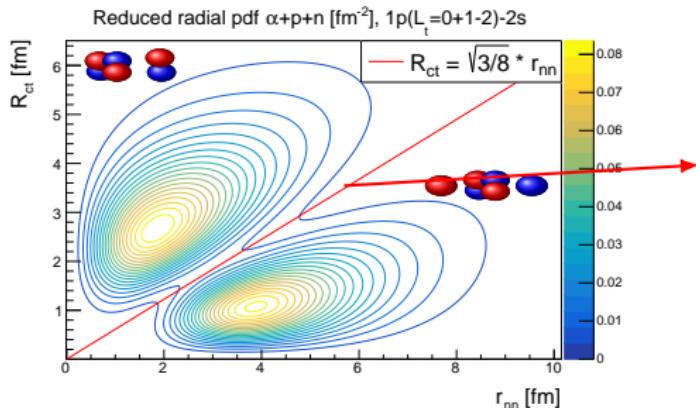


Different relative amplitude and sign of ${}^6\text{Li}$ ($2s$) 2

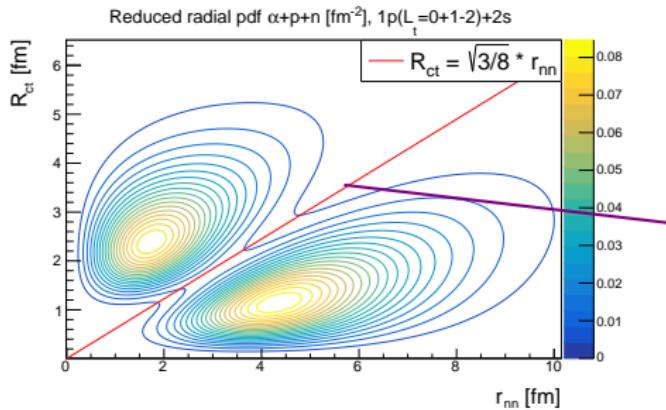
$\alpha + p + n$ reduced probability density functions



$\alpha + p + n$ reduced probability density functions

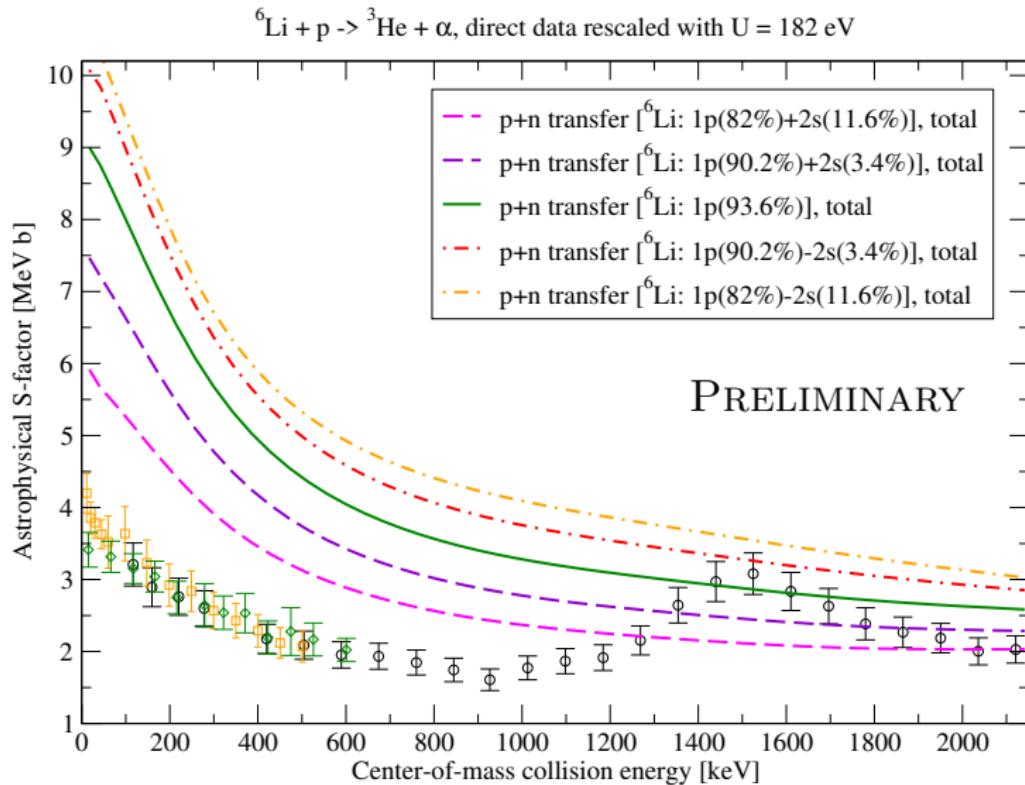


(Nominal) fraction of
“clustered” norm: 54 %



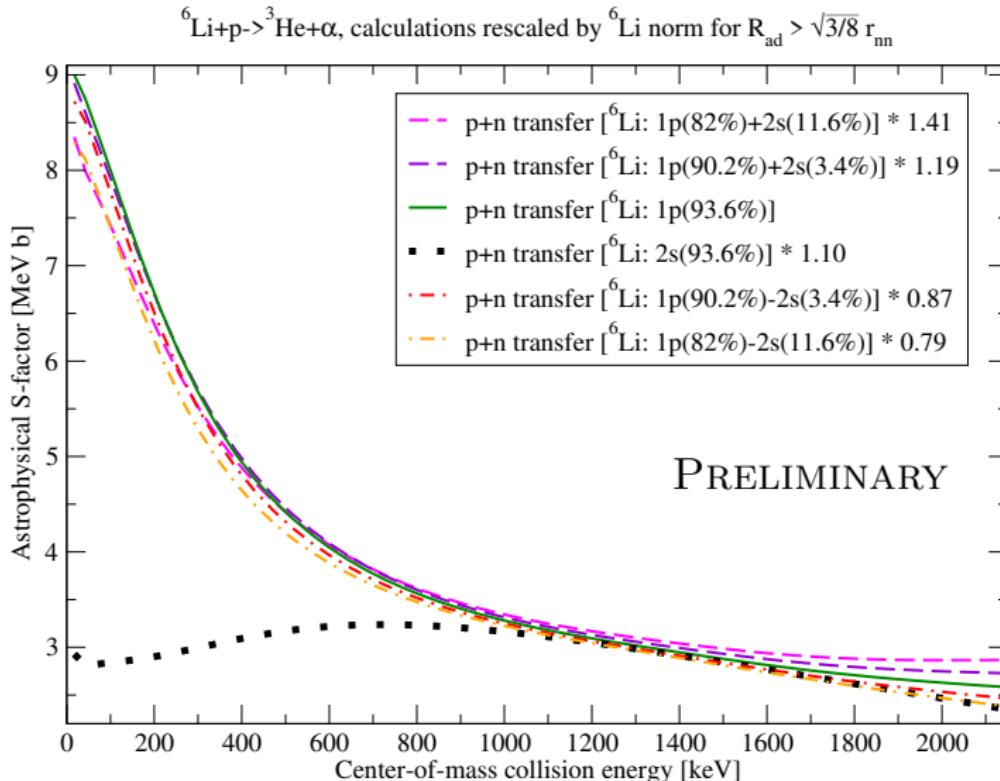
(Nominal) fraction of
“clustered” norm: 39 %

${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$: role of ${}^6\text{Li}$ ($2s$) 2 contribution



Different relative amplitude and sign of ${}^6\text{Li}$ ($2s$) 2

$^6\text{Li} + \text{p} \rightarrow ^3\text{He} + \alpha$: role of d-clustering strength

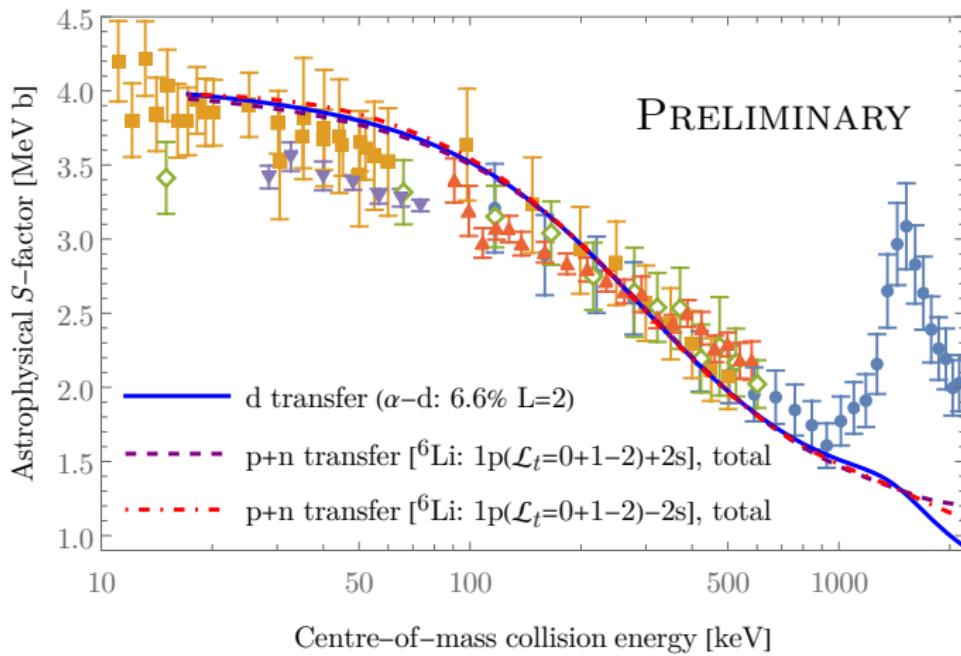


Calculations rescaled by norm of “clustered” region

${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$: role of ${}^6\text{Li}$ ($2s$) 2 contribution

${}^6\text{Li} + \text{p} \rightarrow {}^3\text{He} + \alpha$, direct data rescaled with $U = 182$ eV

Calculations rescaled on data



Calculations rescaled to fit data (0.1 MeV to 1 MeV)

What: Nuclear reactions between light charged particles at sub-Coulomb energies.

How:

- Cross-section explicit evaluation.
- Emphasis on the possible role of clustering.
- No adjusting on reaction experimental data.
- Quantum framework whenever possible

So far:

- WKB barrier penetration with quantum structure model
- Fully quantum DWBA deuteron transfer
- Fully quantum DWBA two-nucleon transfer

Within the models tested so far,

^6Li ground-state ("static") quadrupole deformation alone

- In the dynamics at astrophysical energies (non-resonant), seems to only affect details.
- Is relevant to describe resonant behaviour

"Clustering strength"

- Relevant at all energies for cross-section absolute value.
- Seems unimportant for energy trend at astrophysical energies.

Dynamical effects?

Possible improvements

- Two-nucleon transfer:
 - Microscopic construction for three-particle WFs.
 - Better treatment of unbound ${}^5\text{Li}$ in sequential transfer.
- Study dynamical excitations.
- Barrier penetrability:
 - Go beyond one-channel WKB.
 - Add spin coupling in projectile-cluster potentials
(→ use ${}^6\text{Li} + \text{p}$ potential also for transfer).